11 th IOAA Team
I.R. IRAN

Exam Set

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Physical and Astronomical Constants		
Gravitational constant	G	$6.67 \times 10^{-11} \ \frac{N\,m^2}{kg}$
Planck constant	h	$6.63 \times 10^{-34} \ J\,s$
Stefan Boltzmann constant	σ	$5.67 \times 10^{-8} \ \frac{W^2}{m^2K^4}$
Speed of light in vacuum	c	$3 \times 10^5 \ \frac{km}{s}$
Boltzmann constant	k_B	$1.38 \times 10^{-23} \ \frac{J}{K}$
Radiation constant	a	$7.56 \times 10^{-16} \ J\,m^{-3}K^{-4}$
Atom mass unit	u	$1.660539 \times 10^{-27} \ kg$
Electron volt	EV	$1.6 \times 10^{-19} \ J$
Bohr's atom radius	r_B	$5.292 \times 10^{-11} \ m$
Hydrogen atom mass	m_H	$1.674 \times 10^{-27} \ kg$
Reduced Planck constant	$\hbar$	$1.06 \times 10^{-34} \ js$
Hubble constant	H_0	$72 \ \frac{km}{s.Mpc}$
Parsec	pc	$3.09 \times 10^{16} \ m$
Astronomical unit	$A.U.$	$1.5 \times 10^{11} \ m$
Light year	ly	$9.46 \times 10^{15} \ m$
Radius of sun	$R_\odot$	$6.96 \times 10^8 \ m$
Mass of sun	$M_\odot$	$1.99 \times 10^{30} kg$
Luminosity of sun	$L_\odot$	$3.85 \times 10^{26} \ W$
Effective temperature of sun	$T_\odot$	$5700 \ K$
Apparent magnitude of sun	$m_\odot$	-26.78
Absolute magnitude of sun	$M_\odot$	4.72
Sun's particle mean mass	$\mu_\odot$	0.5
Radius of earth	$R_\oplus$	$6.378 \times 10^6 \ m$
Mass of earth	$M_\oplus$	$5.97 \times 10^{24} \ kg$
Earth's equator inclination	ϵ	$23.5\,°$
Mass of galaxy	M_{Glx}	$10^{11} \ M_\odot$

Exam1

Celestial Mechanics
Spherical Astronomy

1. The spherical tube of a telescope is filled with mercury (Hg). The car that transport this telescope is moving with the acceleration of $a_0 = 3\frac{m}{s^2}$ on the equator counter-clockwise. The telescope also rotates around its axis with angular speed of $\omega = 1.5\frac{rad}{s}$ counter-clockwise. Note that the axis of the telescope is always pointed to the zenith. The initial speed of the car is zero. At this time (The time of start) the telescope records the altitude and azimuth of the star S $h = 89.3°$ $A = 129°$. What is the distance between the lowest point of the surface of mercury and the axis of the telescope when the star S sets?

2. Suppose that the gravity force has the form of $F = -\frac{k}{r^n}\hat{r}$ instead of its Newtonian form. Prove that the angle between the equipotential line of a two-body system can be calculated from the formula:

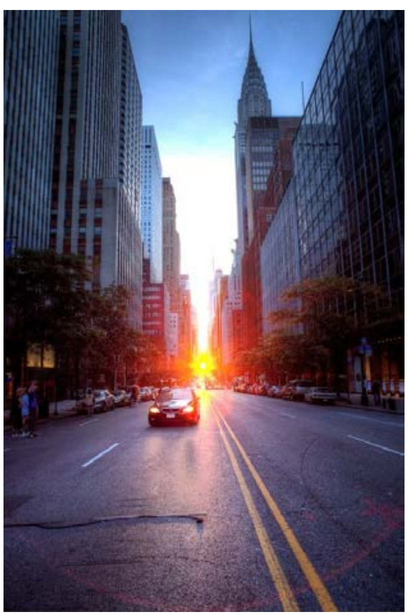
$$\tan^2\frac{\theta}{2} = n$$

3. The density of the steam is ρ. Suppose that a water drop falls from a very-high place and its shape is approximately a sphere with the radius R (R may change with time). Prove that the acceleration of this drop is constant.

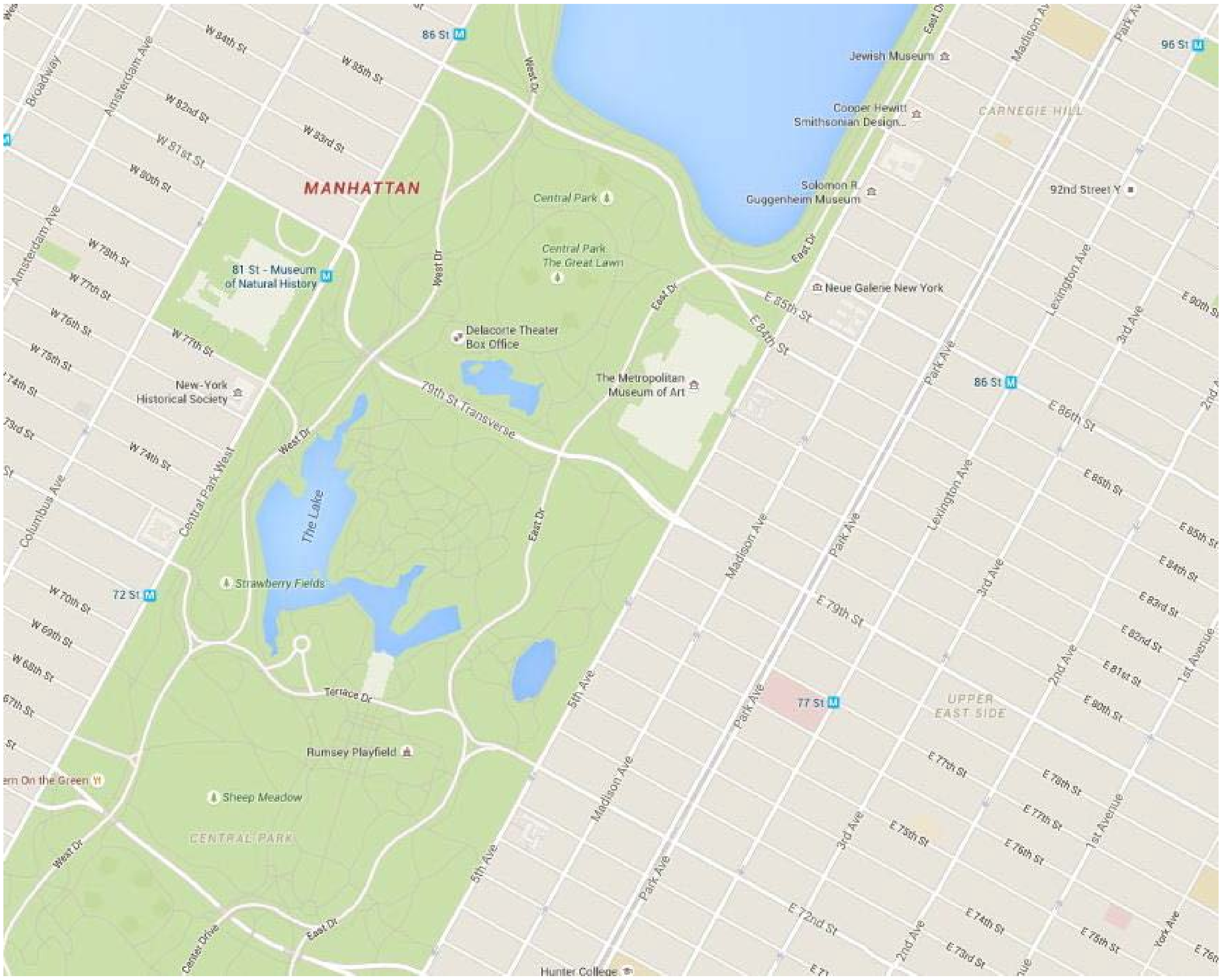
4. In a specific time , what percentage of stars have altitude higher than 70° and hour angle between zero and one hour for an observer in Ali-Abad-Katool ($latitude = 29.5°$)?

5. Manhattanhenge, also called the Manhattan Solstice, is an event during which the setting sun or the rising sun is aligned with the east–west streets of the main street grid of Manhattan, New York City. Calculate the dates and the times this event happens using the map below.

Coordinates of Manhattan : $\begin{cases} longtitude = 73.9712\ °W \\ latitude = 40.7831\ °N \end{cases}$



The East coast Time (ET) is 5 hours behind the Coordinated Universal Time (UTC) (UTC-05:00).


MANHATTAN
86 St
96 St
Jewish Museum
Cooper Hewitt Smithsonian Design...
CARNEGIE HILL
Solomon R. Guggenheim Museum
92nd Street Y
Central Park
Central Park The Great Lawn
81 St - Museum of Natural History
Neue Galerie New York
Delacorte Theater Box Office
New-York Historical Society
The Metropolitan Museum of Art
79th St Transverse
The Lake
Strawberry Fields
72 St
77 St
UPPER EAST SIDE
Terrace Dr
Rumsey Playfield
Sheep Meadow
CENTRAL PARK
Hunter College
Broadway
Amsterdam Ave
Columbus Ave
Central Park West
West Dr
East Dr
Center Drive
5th Ave
Madison Ave
Park Ave
Lexington Ave
3rd Ave
2nd Ave
1st Avenue
York Ave
W 84th St
W 85th St
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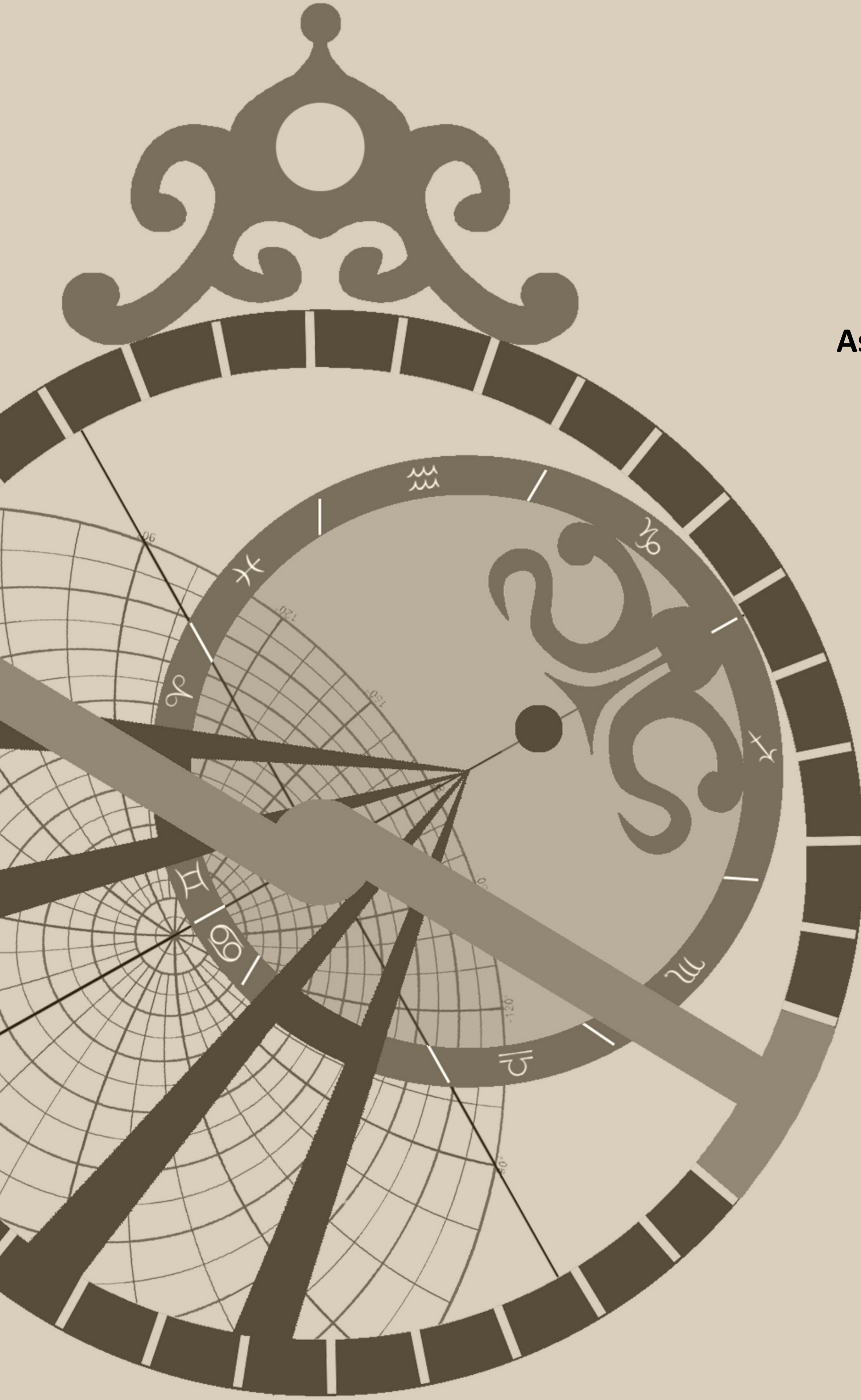


Exam2

Astrophysics

1. What would the magnitude of the sun be If all the photons made in the center of sun reached earth without any barriers (collisions and interactions with the atoms of the sun)?

2. The temperature function of a neutral hydrogen star with the radius same as the sun has the formula $T = T_0(1 - \frac{r}{R})$. Prove that this star cannot be homogenous. (suppose that the gas making the star is perfect and the star is in thermodynamic equilibrium.)

3. (equal-radius lines: the set of all points in the H-R diagram that have the same radius)

According to the formula $L = 4\pi R^2 \sigma T^4$, the main sequence stars with the same radius should make a line on the diagram. But we know that this event does not happen in reality and some of the stars are above and some of them are below this line. We want to find the reason in this problem.

a) Knowing that the stars are homogenous and are in the thermodynamic equilibrium. Prove that $L \propto M^3$ that L is the luminosity and M is the mass of the star.

b) Suppose that the stars consist of perfect gas. Find a relation between mass and radius of these stars.

c) According to the answer of the previous parts. Find a relation between the luminosity and the radius of the stars.
According to the formula found between luminosity and the radius, find that for some equal-radius main-sequence stars, which one is above the equal-radius line and which one is below it?

4. (The temperature of the core of the sun)

We discuss a simple model to find the center temperature. Suppose that this star has a low mass and is on the main sequence. If this star is made of ideal monoatomic adiabatic gas with the formula $P = K\,\rho^\gamma$.

a) Find γ for this gas.
b) In low-mass stars, the energy is transferred by convection. Therefore, the radiative temperature gradient is approximately equal to adiabatic temperature gradient. Find the slope of radiative temperature using that approximation.
c) Assuming that density is constant in the star, find $T(r)$.
d) For a ionized Hydrogen star, find the temperature of the center. (Hint: The wavelength in which the sun is most luminous is approximately $\lambda_{max} = 500\ nm$)

5. (Pulsating Stars)

In this question we discuss a model for variable stars in which the luminosity has a power relationship with its period. The star pulsates with the adiabatic speed of sound in this model. We also assume that this star is in thermodynamic equilibrium and is made of monoatomic perfect gas. The star is also isothermal.

a) Find the relationship between the period and the average density of these stars. (you can assume that the density is constant at this part.)

b) If $L \propto P^{\beta}$, find β.

$\eta - Sco$ is a $\delta - Cep$ variable star with average radius $8.13\ R_{\odot}$ and apparent magnitude 4.1 and parallax $0.00278\ arcsec$.

c) If the oscillations of this variable equal 0.3 its radius, calculate its temperature change.

d) Find the most and least apparent magnitude of this star.

The adiabatic speed of sound formula is:

$$v_S = \left(\frac{\gamma P}{\rho}\right)^{0.5}$$

γ is a constant. P is pressure and ρ is density of the gas.

6. (Discussing Quantum Mechanics in Stars)

consider a spring-box system with the box mass m and spring constant k.

a) Simply find a formula for Kinetic Energy of this system in terms of momentum of the box, the mass of the box and the spring's extension.

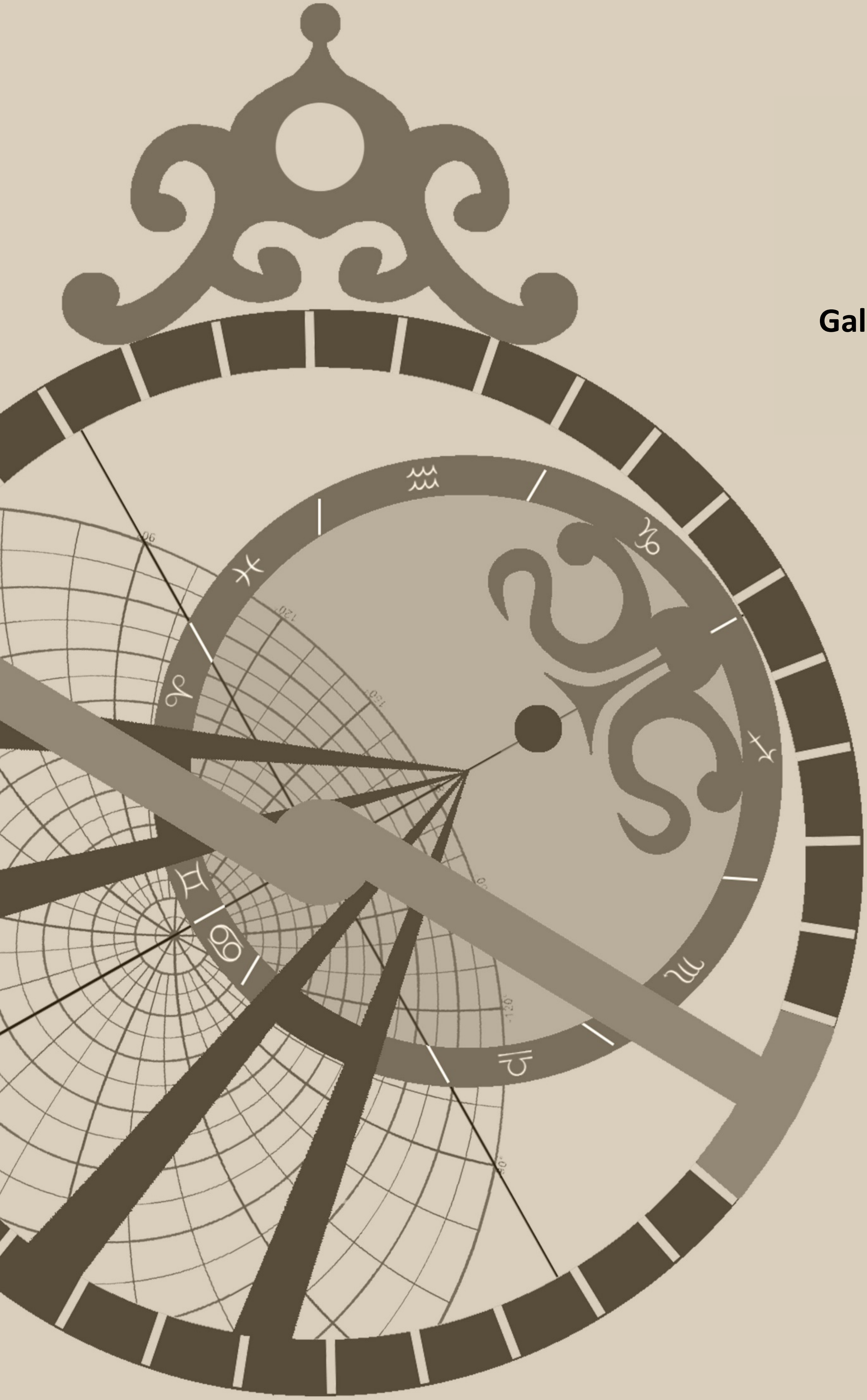
In classic Newtonian view, the kinetic energy is zero if the momentum of the box and the spring's extension is zero. But due to Heisenberg's uncertainty principle we can't determine the momentum and the position (extension here) of a mass with infinite accuracy. Therefore, if we say that the momentum is certainly zero, the spring's extension can be any number.

b) Find the minimum energy of the system (E_{base}).

c) Consider a star that has no rotation. For a particle near its surface, find its motion equation (of radial oscillation).

d) Find a formula for the ratio of E_{base} to kinetic energy of this particle. We call this ratio f.

e) Find the ratio of f of a particle on a hot neutron star (with surface temperature $100000\ K$, mass $3\ M_{\odot}$ and radius $10\ Km$) to f of a particle on sun.



Exam3

Cosmology

Galactic Dynamics

1. We put a coordination system on the center of the galaxy. The Z axis is perpendicular to the orbital plane of the sun and the X axis is towards the sun. We have the position of the sun and a globular cluster R_1 and R_2:

$$R_1 = 8.5\,\hat{\imath} \quad , \quad R_2 = 6.2\,\hat{\imath} + 1.7\hat{\jmath} - 4.2\hat{k}$$

The density function of the galaxy is $\rho = \rho_0 \frac{1}{1+\left(\frac{r}{a}\right)^2}$ where $a = 2\ kpc$ and $\rho_0 = 3 \times 10^{-20}\,\frac{kg}{m^3}$.

If we suppose that the angular momentum of this globular cluster is pointed towards the galactic latitude 20° and that the orbits are circular. Find the cluster's redshift. (Neglect earth's motion around sun.)

2. Consider having a point source of light radiating with constant intensity I_0. An observer witnesses that this source comes towards him with the velocity v. Because of special relativity, the intensity that he observes, I, is different from I_0 and is dependent to θ, the angle between the source's velocity and the direction of light.

   a) Show that $I(\theta)$ has this relationship:

   $$I(\theta) = I_0 \frac{1 + \frac{v}{c}\cos\theta}{\left(1 - \frac{v}{c}\cos\theta\right)^2}$$

   consider $\frac{v}{c} \ll 1$ for the next parts.

   b) Suppose that this light source is a black body and its effective surface temperature is T_0. What is the temperature that the observer measures from the surface of the source?

   c) If $v = 0.12\,c$, the wavelength that the source is brightest in(λ_{max}in Wien's law) is λ_0 and the wavelength the observer sees the source brightest λ, find λ and λ_0.

   d) Use Doppler's effect on λ_0 to find $\lambda_0{}'$ and compare it with λ. Explain why these two are equal or not equal.

Hint:

$$\int \frac{1+x}{(1-x)^2}\,dx = \frac{2}{1-x} + \ln|1-x| + C$$

$$\int \frac{x(1+x)}{(1-x)^2}\,dx = x + \frac{2}{1-x} + 3\ln|1-x|$$

$$\int \frac{x^2(1+x)}{(1-x)^2}\,dx = \frac{x^2}{3} + 3x + \frac{2}{1-x} + 5\ln|1-x|$$

3. The Initial Mass Function (IMF) of stars is given by Salpeter's function with the minimum mass $0.1\ M_{\odot}$ and the maximum mass $100\ M_{\odot}$. Find the constant of the IMF for the surface density of the disk. (Surface density can be calculated by integrating over a column perpendicular to the galactic plane.) Suppose that the stars are main sequence.

    a) Calculate the average mass of the stars.

    b) In what mass does the number of stars reach its maximum?

    c) How much does the star with the average luminosity weigh?

    d) Calculate the density of stars near the sun and their average distance from each other.

Hint:

The surface density of stars near the sun is $55\frac{M_{\odot}}{pc^{-2}}$. The total surface density can be calculated from Salpeter's function:

$$\xi(m)dm = \xi_0 m^{-2.35} dm$$

The width of the galaxy disk is $500\ pc$.

4. As we know well, other galaxies are moving away from us with velocity $v = Hr$. And the size of the universe is changing proportional to scale factor. The universe consists three components, matter, radiation and dark energy and the pressure from the dark energy is negative!

    We now want to discuss the universe from another view. Suppose that it's the size of us that change proportional to scale factor rather than the size of the universe. The scales of time and space also change so that we see that the distance of other galaxies change. In this view the matter and radiation are in dark energy background that is spread in the universe homogenously. The density of this dark energy is constant to all observers. If the pressure from the dark energy overcome the pressure of matter and radiation, the size of them will shrink.

    a) Find Friedmann equations for this view.

    Suppose that the wavelength of the electromagnetic waves is constant for the system $t_0$in which the scales are constant:

b) Prove that speed of light is constant for all of the observers at all times and find the relationship between redshift and scale factor.

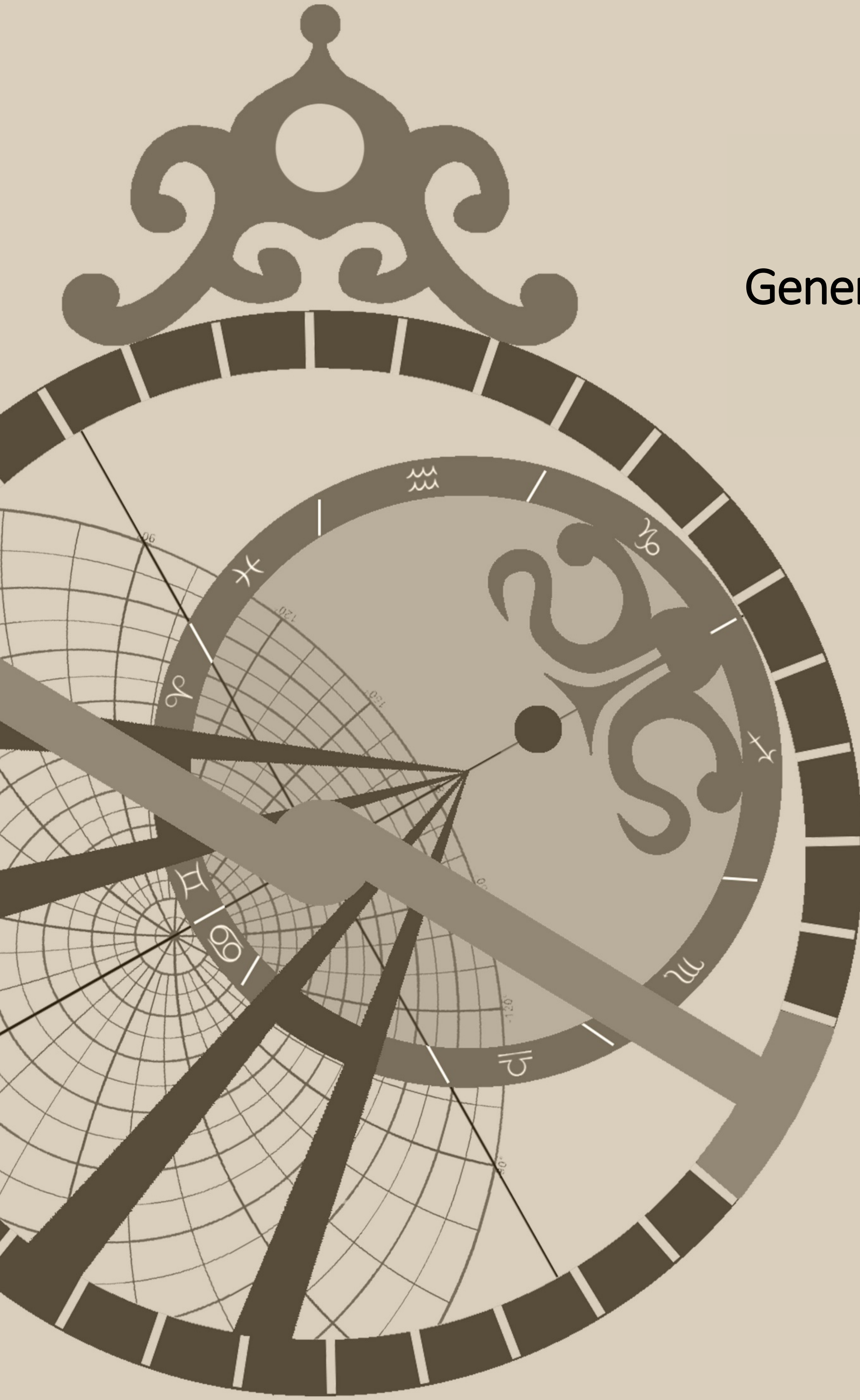
c) Prove that the density of dark energy changes if there isn't any mass production or mass consumption.

d) Is this view different than the previous view? Explain your reason.

5. A discoid galaxy has a disk with the radius R and width l $(l \ll R)$. An observer is in distance d from the center of the galaxy and the angle between the observer and the disk is θ. The observer sees the gravitational potential of the galaxy like:

$$\phi = \phi_0 \ln(r_c^2 + A_0(1 - \frac{\cos\psi}{\cos^2\eta})$$

where ψ is the angle between the center of the galaxy and a point of the disk we want to calculate the potential in and η is the azimuthal angle of the observer and the point. $A_0$is a constant that should be determined. If $d \ll R$, find the density function of the galaxy.



General Exam 1

1. Conic Sections

(Coefficients are not important in this question. You can give them any name you like!)

We can make any conic section with a cone and a plane

a) Find the equation of a cone that its main axis matches the Z axis and its vertex is on the origin.

b) Find the equation of the cone if we rotate it through the angle i around the X axis.

We know that we will have a conic section if we intersect a plane with the equation $ax + by + cz = d$ with the cone.

c) Find the equation of the tangent line in an arbitrary point on this conic section. (Hint: the tangent line is also tangent to the cone and is on the intersected plane.)

If we project the conic section on $X - Z$ plane, we will have a curve. Using previous parts:

d) Find the slope of the tangent line of the curve.

e) Substitute y in terms of x and y.

f) Using the previous part. Find the equation of the curve.

2. Stellar Perturbation

A star with constant density ρ and radius R engages some changes and its inner layers start to move. The velocity function of the layers in terms of the distance of the layer from the center of the star is $v = v_0 \sin(\frac{\pi}{2} . \frac{r}{R})$. v_0 is a constant number. After a very short time δt the initial layer with the radius between r and $r + \Delta r$ becomes the secondary layer with the radius between r' and $r' + \Delta r'$. (because $\delta t \ll 1$, $\left(\frac{r-r'}{r}\right)^2 \cong 0$)

a) Find the difference of the density of the initial and secondary layer in terms of known parameters ($r, R, v_0, \rho, \delta t$).

b) Draw the graph of the answer of the previous part in terms of r and explain the new density function.

3. Helium Main-Sequence Stars

The main opacity factor in low mass stars, in which the gas pressure is dominant and the energy transfer is done by convection, is mostly "$free - free$" interactions. The mean free path has this relationship with temperature and density

$$l \propto \frac{T^{3.5}}{\rho}$$

Luminosity can simply be calculated by dividing all the radiation energy in the star to the time it takes for energy from the center to reach the surface.

a) Find a proportional relationship for luminosity in terms of radius, temperature and density for these stars.

b) Eliminate temperature from the relationship. once again, eliminate density from the relationship. Remember that the star has thermodynamic equilibrium.

We now have mean particle weight

c) Find a formula for calculating mean particle weight in terms of X (the mass fraction of hydrogen if the star is made of hydrogen and helium.

d) For a **helium mean sequence star** with mass $0.5\ M_{\odot}$ which has lost all its hydrogen and its central temperature has reached helium burning temperature ($10^8 k$), find luminosity and radius ($X_{\odot} = 0.75$)

e) Find the effective temperature of this star.

4. The Relationship between the age and the temperature of the universe

Reason which component of the universe was dominant in the early ages of universe then prove this relationship for temperature and the age of the star for the early ages.

$$t = \left(\frac{3c^2}{32\pi G a_B T^4}\right)^{\frac{1}{2}} \cong 230s \left(\frac{10^9 K}{T}\right)^2$$

5. Pirates!

The pirates have recently found the map of the biggest treasure hidden in the deepest seas! In the found box there was a Mercator map and three pairs of coordinates of the map. One pair of these coordinates are for the treasure. One pair is the coordinate of their current position, Peaks Island with the coordinates $(43.66°N, 70.19°W)$. The other two pairs are like these:

A: $(70.88°N\ , 19.55°E)$- Norwegian Sea
B: $(34.28°S\ , 18.43°E)$ - Cape Town

The treasure is hidden in both of these coordinates. The federal officers have noticed the existence of this treasure and somehow found the coordinates of A and B. They want to destroy the pirates ship. So, they are using a fighter plane. The maps that these planes use are made using "Gnomonic Projection". In these kind of maps, the great circles on earth are projected to a straight line (Of course these planes go on great circles and the pirates go on a straight line on Mercator map)

a) To which point should the pirates go so that they don't face the federal planes.
   Hint: You can use the map of the next page. Explain your solution.

b) Suppose that the pirates chose the other way. If the speed of their ship is $20\ knots$ (each knot is equal to one nautical mile per hour and each nautical mile is equal to one arcminute on the surface of earth), what should the speed of the plane be to reach the ship and destroy it?

   Note: the map covers from latitude $82°N$ to $82°S$. You can use any spherical trigonometry or projection formula you want!

General Exam
2

1. Stellar Cluster

We have observed a stellar cluster with the absolute magnitude -6.6. The stars in this cluster are born 5 million years ago with the initial mass function below and their minimum and maximum mass was 0.1 and 100 respectively.

$$\xi(m) = \xi_0 m^{-\alpha} dm$$

and $\alpha < 2$
The initial mean mass of the stars was$0.4\ M_{\odot}$. In this question $A^{-B} \cong 0$ if A is large and B is positive.

a) How many stars does this cluster have now? Call the number of the stars of cluster N

b) We define *luminosity mean mass* the mass of a star that the total luminosity of N of these stars equals the luminosity of the cluster. Find the *luminosity mean free mass* for present time.

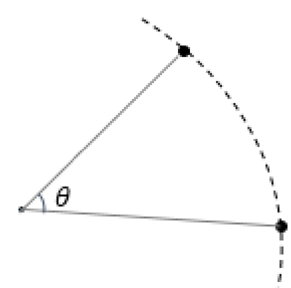
c) Calculate the mean mass of the cluster at its birth and compare it with the mean mass of present time and *luminosity mean mass.*

2. Celestial Mechanics

a) Consider two objects that move around a central object under Newtonian gravity in one orbit and the difference between their true anomaly is θ. Prove that the size of their relative speed equals:

$$\left|\vec{V}\right| = 2\frac{\mu}{h}\sin\frac{\theta}{2}$$

h is the angular momentum of the orbit.



b) A probe is orbiting around the sun in a parabola with perigee $p = 3\ AU$. In time $t = 0$ it is in its perigee. In $t = 1\ yr$ we put another satellite in this orbit. The second satellite starts its move from perigee. These two satellite will have electromagnetic communication with each other. In $t = 2\ yr$ the first satellite sends some waves to the second satellite. With what redshift does the second satellite receive the wave?
Hint: The time equation for parabola is:

$$t = \sqrt{\frac{2p^3}{\mu}}\left(\tan\frac{\theta}{2} + \frac{1}{3}\tan^2\frac{\theta}{2}\right)$$

3. Stellar Electron

Part One: Electron Energy

Suppose that due to the interactions in the stars we have electron radiation, other than electron radiation, which comes from the center of the stars. We divide the movement of electron to the surface into two parts. From 0 to $0.3\ R_{\odot}$ because of the collisions happening the electron will not go in a straight way and turn $6.33\ \times\ 10^{-20} rad$ in each collision. From $0.3\ R_{\odot}$ to the surface, the electron will go straight. Because of the electron movement and the effects of the absorption of the gas, the energy of electron reduces with the equation:

$$\frac{dE}{d\chi} = -\alpha - \frac{E}{\xi}$$

where $d\chi$ is a length element towards the electron's movement and α and ξ are two constants. If the initial energy of the photon is $2.29E_0$ (where $E_0$is the rest energy of electron) calculate:

a) The energy of the electron when exiting the star and

b) The time it takes for the electron to travel from center to the surface of the star. (remember that it does not move straight!) Suppose that the sun is completely made of ionized hydrogen
Effective collision radius :$2.81\ \times 10^{-15} m$
$\alpha = 1.78 \times 10^{-20} E_0$
$\xi = 5.62\ \times 10^{18} m$

Part Two: Van Allen Radiation Belt

Van Allen Belt is a layer of charged particles that is made because of the magnetic field of earth. We discuss a simple model to see why this belt is made.

We know that the presence of magnetic field near a charged particle applies force to it with the formula $\vec{F} = q\vec{v}\ \times \vec{B}$. We have a Cartesian coordination system that the Z axis is toward north celestial pole and the $X - Y$ plane matches the equator. We know that the magnetic field vector of earth is towards the point with latitude $79°S$ and longitude $108°E$. Suppose that all the particles in Van Allen's belt come from the sun. solve this problem for winter solstice.

a) Find a good coordination system that eases the procedure of solving the problem. (Is it possible to solve the problem in this system?)

b) Solve the system of equation to find the curve equation of the movement of the particles.

(curve equation : $\overrightarrow{R(t)} = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$)

c) Find the curve equation in the initial Cartesian system.

d) What does the curve look like? Find its parameters.

4. Protostar

   Consider a gas cloud with average temperature $20\ K$ that in some parts of it the process of collapsing and increase of density is happening. These parts, that are called *protostars,* radiate because of their gravitational free fall. Suppose that this protostar will be a star like sun. If the final radius of the system is ignorable, find the luminosity due to free fall. (The luminosity is constant during the free fall.)

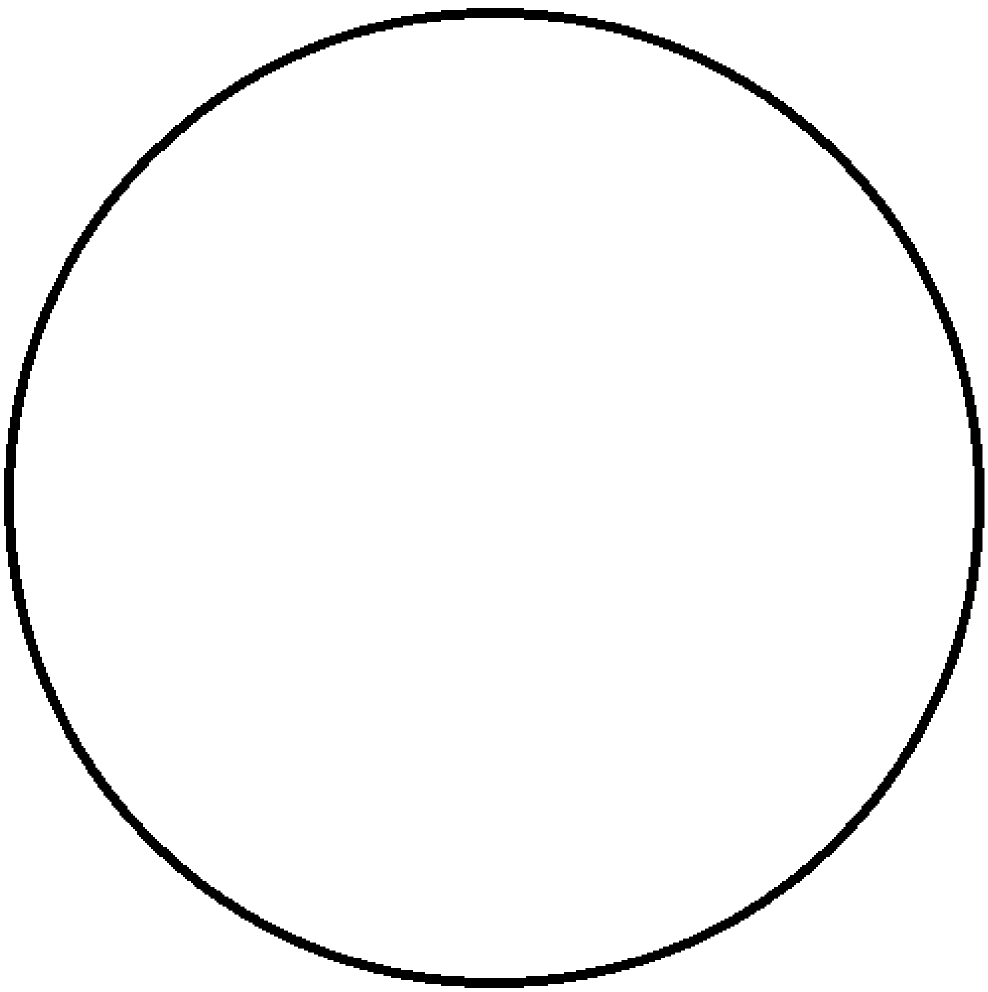
5. Birds Migration

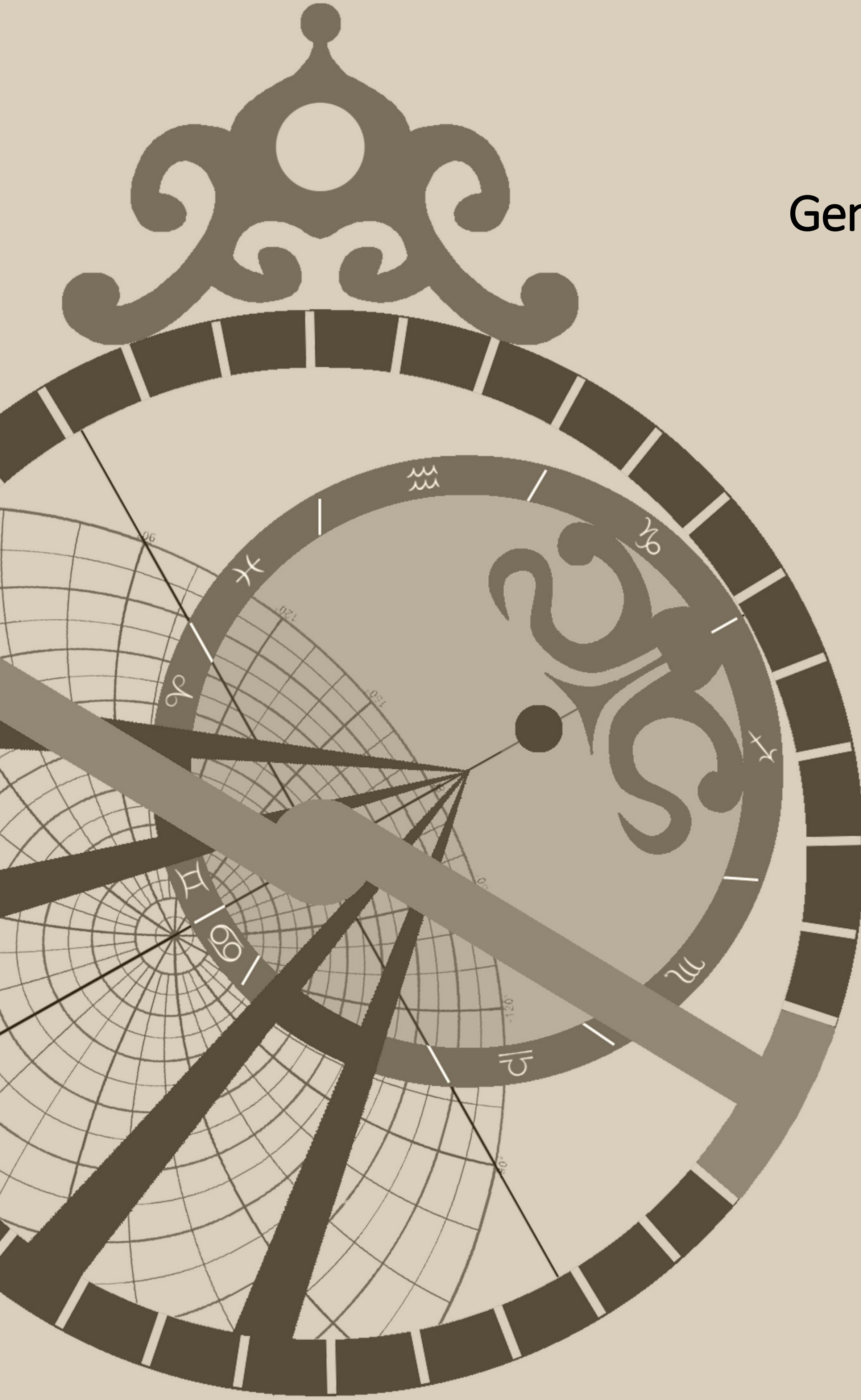
   A group of researchers want to study a flock of birds during their migration. These birds start from city A with coordinates $(16°N\ ,77.52°E)$ to city B with coordinates $(41.59°N, 133.15\ °W)$ and always move on a great circle. They can't fly in latitudes above $60°$ due to cold air. For their study, the researchers put a chip on one of the birds to follow their path. When these birds reach $60°N$ they change their direction of flying so that the angle between East and their final direction is equal to the angle between West and their initial direction. (They somehow are reflected from the $60°N$ circle!). The radar these researchers use has a limited range. So, they want to put somewhere to cover all of the path of the birds using the shortest range possible.

a) Find the position of the radar.

b) How long is the range of this radar?

   You CAN NOT use spherical trigonometry formulas. You can use the stereographic projections formulas.

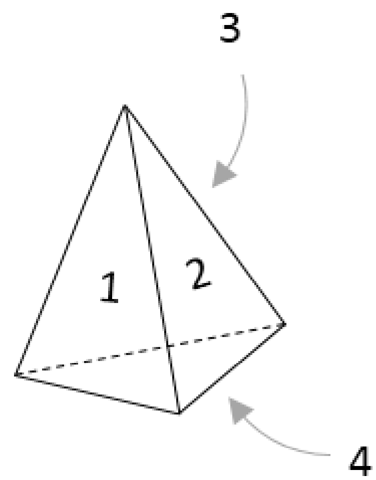




General Exam
3

1. Dice!

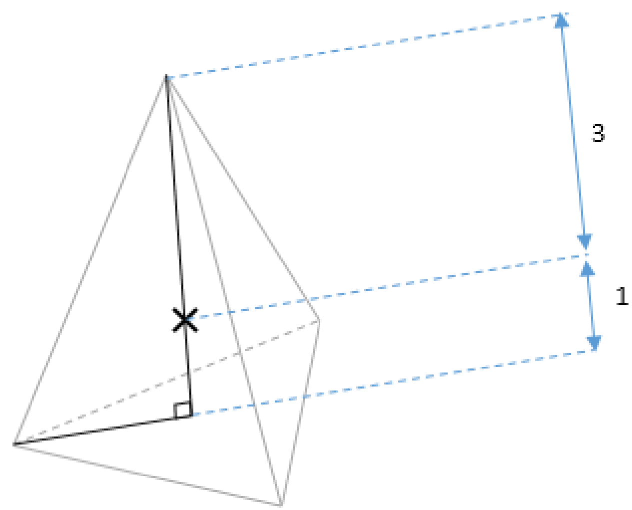
   We use a tetrahedron dice to play a game. Each face of this tetrahedron is an equilateral triangle with side length $a = 2\ cm$. One of the players cheats, so he places a small object right in the middle of the face that has number 4 on it. The mass of the object is $m = 20\ g$ and the mass of the dice without the object is $M = 40\ g$.



   a) Find the distance between dice's center of mass and the center of the tetrahedron.

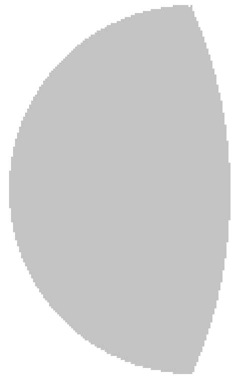
   b) Find the probability for each face of dice to be under in a toss.

      Hint 1: The gravity does NOT make torque on the dice.
      Hint 2: The distance between the center of the tetrahedron and each face is $\frac{1}{3}$ of the distance between the center and each vertex



2. Projection of Moon

   In 19th century, a man in Rafasnjan ($\phi = 30°N$, $l = 56°E$) wanted to make a map from the sky. His map is made with stereographic projection with north celestial pole in the center. The image of moon in his map is something like the picture below. Did he do his work right?



Each formula you use should be proved!

3. Compressible Fluid

   For an incompressible fluid, like water, the Archimedes force comes from the formula $F = \rho g V$. But for the stars that the density is related with radius, the fluid is compressible. Assuming that the density function is $\rho = \frac{\rho_0 R}{r}$, find the Archimedes force on a layer that is between the radius r_0 and $r_0 + \Delta r$.
   (Hint 1: divergence theorem : $\oint \vec{F} \cdot \overrightarrow{dA} = \int \vec{\nabla} \cdot \vec{F} \, dV$)
   (Hint 2: If the force (except pressure)acting on a volume unit is $\vec{f}$, for pressure, we have $\vec{\nabla} p = \vec{f}$

4. The Reason Planets Are Solid

   In hot temperatures, almost all the matter in a star is ionized. Therefore, they interact each other with Coulomb's force. We want to prove that this interaction is dominant in planets too. So, the particles are trapped in each other's potential wells and their oscillation and vibration is minimum. In this state the particles are nearly fixed. Therefore, the matter is seen as solid.

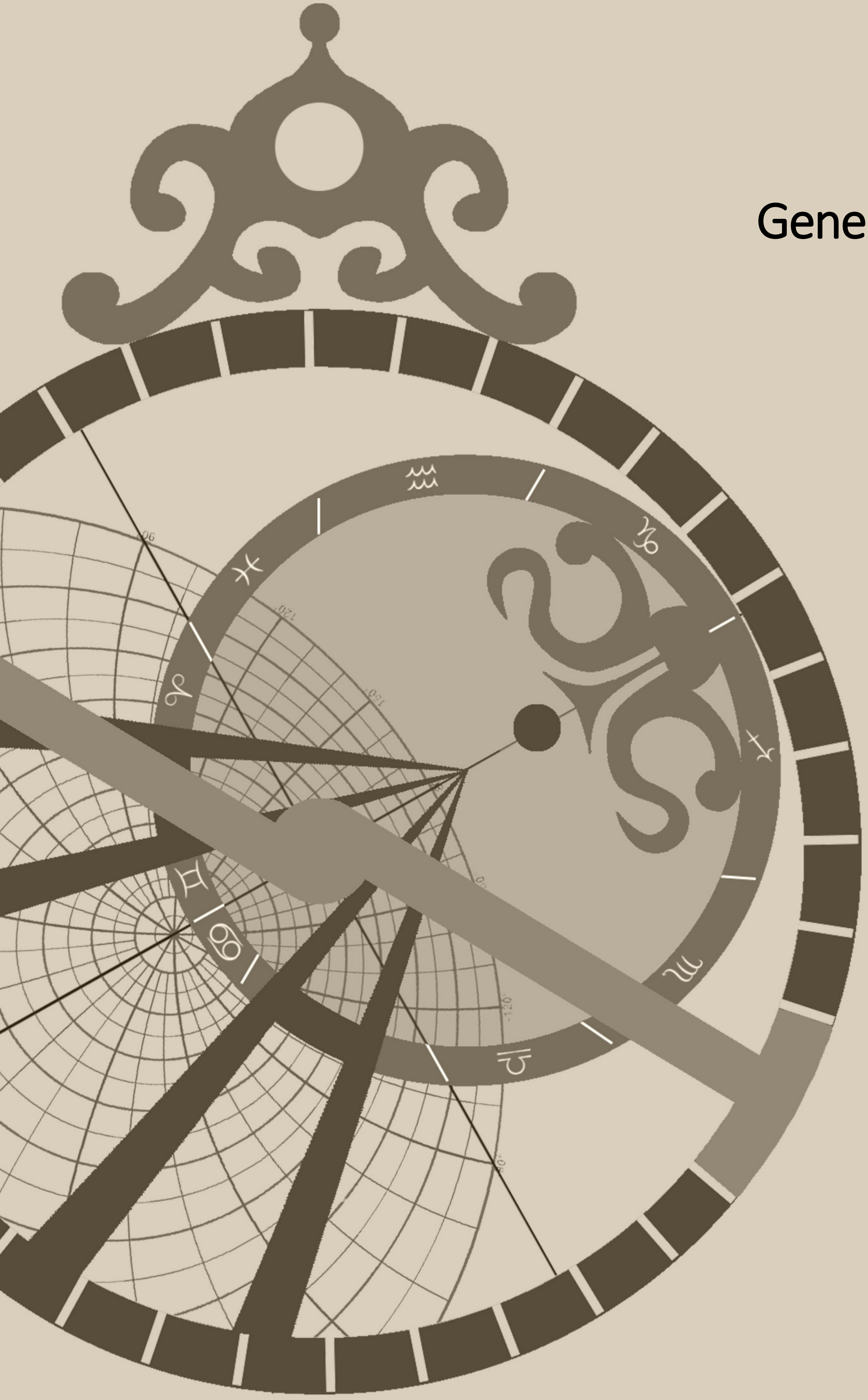
   First, find a parametric formula for $\frac{E_{Interaction}}{E_{Kinetic}}$ in terms of physical constants and the mass of the star. Then Substitute the mass with mass of a star and calculate $\frac{E_{Interaction}}{E_{Kinetic}}$. After that, substitute the mass with mass of a planet and calculate $\frac{E_{Interaction}}{E_{Kinetic}}$. Then argue why the planets are seen solid. Use logical and simplifier assumptions.

5. The Fall of Earth!

   In a horrifying event, in 21st March, earth stops in its orbit around the sun and falls towards sun. in the moment that the thermometers on earth show the temperature $100\ K$, a comet have a complete inelastic collision with earth (two objects stick together). The angle between the gas tail and the dust tail is $60°$ right before the collision. If the orbit of the comet was a parabola:

a) Find the celestial coordinates of the comet and the directions of gas tail and dust tail when earth stopped moving in its orbit.

b) Find the new orbit properties.



General Exam
4

1. Some extraterrestrial creatures have found out that there is a civilization on earth. To know more about our cultural system, they want to kidnap one of us!

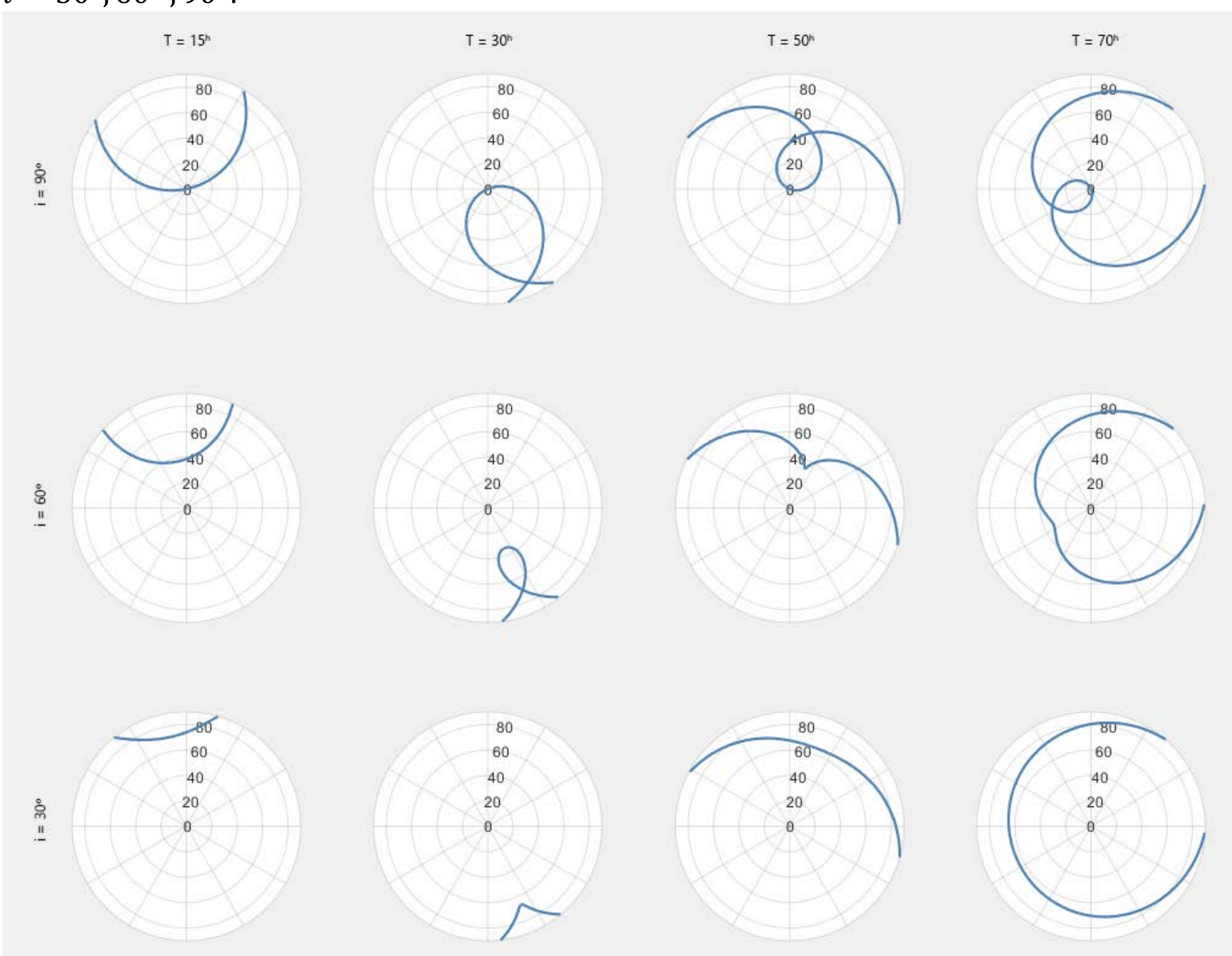
   So they should find a person that knows his planets cultural and social situations. (One of the traits a person should have if he/she wants to candidate for presidency in Iran)

   a) What is the probability for these creatures to find the appropriate person?

   b) From the time they lay a trap with radius $0.5\ m$, how long does it take to catch a person (appropriate or inappropriate)? Suppose that people move randomly on earth and their average speed is $10\,\frac{m}{s}$.

2. A satellite moves around the earth in an orbit with inclination i and period T. An observer on north pole is watching this satellite. The path of this satellite on sky is a little complicated.

   For example we drew the path of the satellite for $T = 15^h, 30^h, 50^h, 70\text{^}h$ and $i = 30°, 60°\,, 90°$.



In the graphs, the center is zenith and the origin of Azimuth is an arbitrary point on the horizon. As you can see, the path of the satellite is complicated and is based on orbit parameters. In some situations, the path crosses itself. The other important property is the azimuth of set and rise of the satellite.

a) For a satellite with $i = 90°$ and $T = 24^h$ find the difference between set and rise azimuths.

Note that in north pole there is no special point of origin for measuring azimuth. You can choose any point you want.

b) For a satellite with $i = 90°$, find T so that the position of set and rise overlap each other.

3. The atmospheres of stars are some perfect gas that are in equilibrium because of the gravity and are mostly isothermal. The mean mass of the particles is $\overline{m}$.

a) Prove that we cannot show a border for the atmosphere and that the density falls exponentially with the height from the surface

$$\rho = \rho_{ph} e^{-\frac{h}{H}}$$

ph is the index of surface and note that H is not a parameter of problem! The final answer should not have H.
The atmosphere begins from the last layer we can see ($\tau = 1$)

b) Find a formula for calculating H in terms of effective temperature using the definition of effective temperature.

Opacity can be calculated using the formula below:

$$\kappa = \kappa_0^{ph} \rho^a T^b$$

c) Using the given formula and the perfect gas relationships, find a relationship for pressure on the surface of the star.

d) Find a relationship for P_{ph} in terms of gravity on the surface $g, a, \kappa_0^{ph}, T_{eff}, b$.

e) Calculate P_{ph} for sun.

$$\kappa_0^{ph} = 1.6 \times 10^{-23}, \quad a = 0.4, \quad b = 9.3$$

4. Astrophysicists separate stars into two generations. The first-generation stars are stars that are younger and are born from the matter second-generation stars make while they burn. Scientists believe that a little after that second-generations stars leave the main-sequence

they explode in a supernova that spreads the matter in the universe. We think that in each explosion about 60% of star's mass is lost. Suppose that the mass function for first-generation stars is:

$$\frac{dN}{dM} = \alpha M^{-3}$$

and the mass function for second-generation stars is:

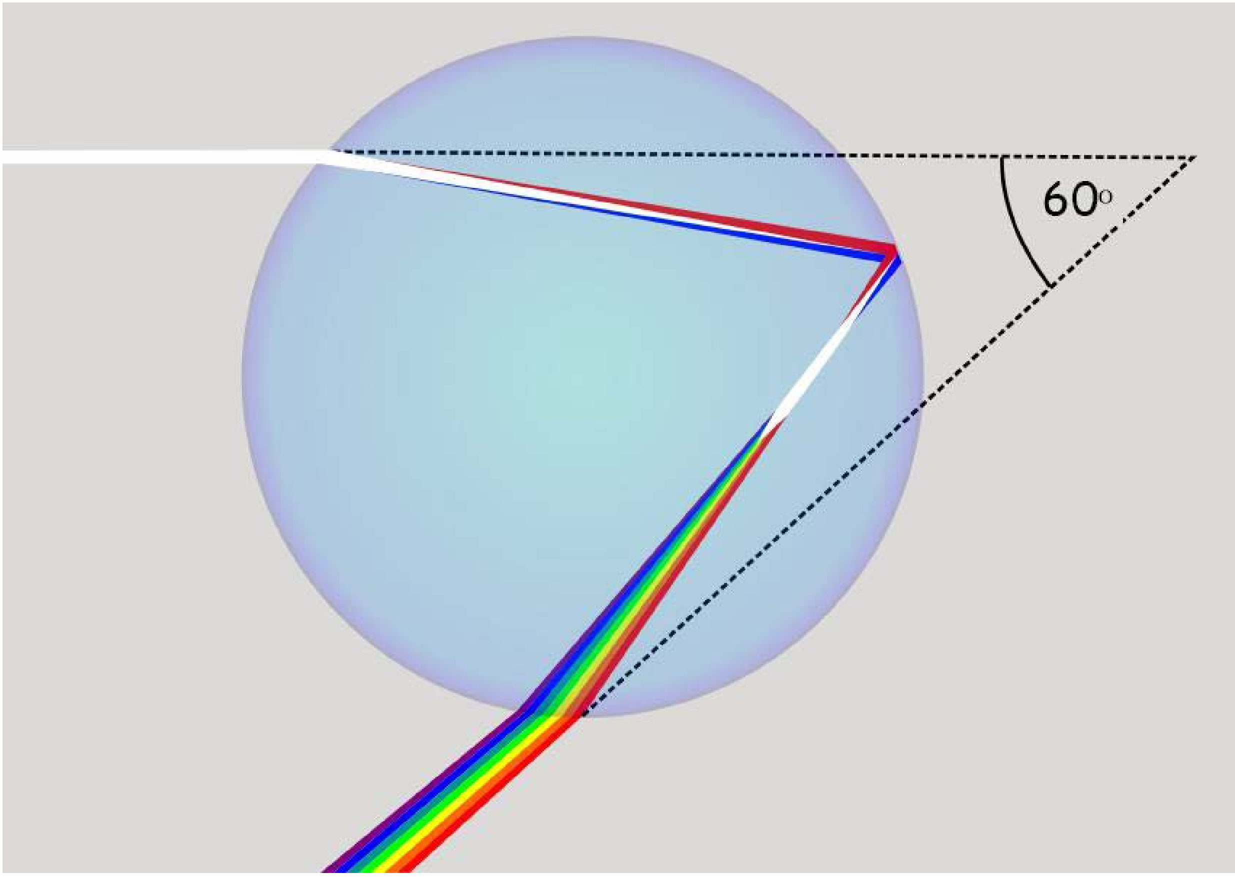
$$\frac{dN}{dM} = \beta M^{-2.35}$$

lightest main sequence stars weigh $0.1\ M_{\odot}$. Find the ratio of number of second-generation stars to first-generation stars.

5. At April 3rd, when the sun reached its highest position in sky, a terrible blizzard started in Semnan-Damghan road ($\phi = 36.2°$) that reduced the seeing of drivers and caused many car accidents.

   a) If the speed of the cars was $100\frac{km}{h}$ and their direction was to west and the speed of wind was $35\frac{km}{h}$ to south and the speed of the snowflakes speed was $4\frac{km}{h}$. In what altitude and azimuth do the drivers see the snows come from? (where is the focus point of the snowflakes?)

   A rainbow is a meteorological phenomenon that is caused by reflection, refraction and dispersion of light in water droplets resulting in a spectrum of light appearing in the sky.



The picture shows how sunlight is reflected in raindrop. If all the sun beams entering the raindrop are parallel, the exiting beams move in opposite direction of the sun beams and make an angle $60°$ with the sun beams. That's why rainbows appear in the section of sky directly opposite the sun.

b) If we see a rainbow after this blizzard(!) what is the angle between the rainbow and the horizon?