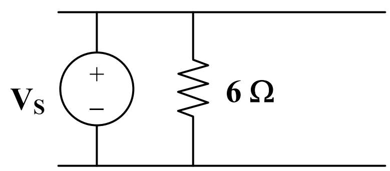
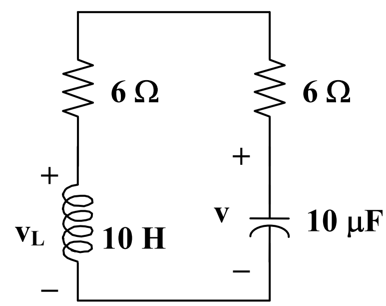
Chapter 8, Solution 1.

(a) At t = 0-, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A, \quad v(0-) = 12V$$

At t = 0+, $i(0+) = i(0-) = \underline{\mathbf{2A}}, \quad v(0+) = v(0-) = \underline{\mathbf{12V}}$

(b) For t > 0, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \text{ or } di/dt = v_L/L$$

Applying KVL at t = 0+, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 = \underline{\mathbf{-4\ A/s}}$

Similarly, $i_C = Cdv/dt, \text{ or } dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

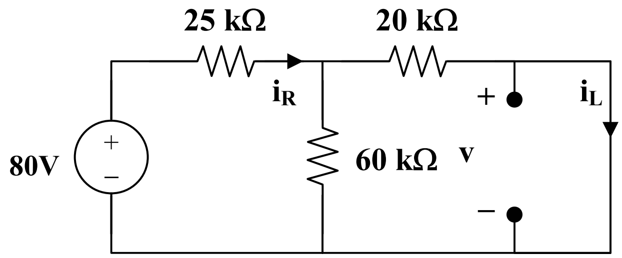
$$dv(0+)/dt = -2/0.4 = \underline{\mathbf{-5\ V/s}}$$

(c) As t approaches infinity, the circuit reaches steady state.

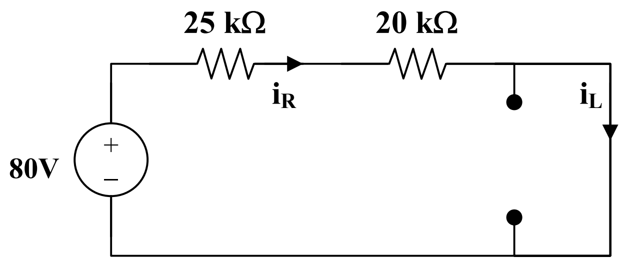
$$i(\infty) = \underline{\mathbf{0\ A}}, \quad v(\infty) = \underline{\mathbf{0\ V}}$$

Chapter 8, Solution 2.

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$60||20 = 15 \text{ kohms}, \; i_R(0\text{-}) = 80/(25 + 15) = 2\text{mA}.$$

By the current division principle,

$$i_L(0\text{-}) = 60(2\text{mA})/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0\text{-}) = 0$$

At t = 0+,

$$v_C(0+) = v_C(0\text{-}) = 0$$

$$i_L(0+) = i_L(0\text{-}) = \underline{\mathbf{1.5\ mA}}$$

$$80 = i_R(0+)(25 + 20) + v_C(0\text{-})$$

$$i_R(0+) = 80/45\text{k} = \underline{\mathbf{1.778\ mA}}$$

But, $\quad i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \text{ or } i_C(0+) = \underline{\mathbf{0.278\ mA}}$$

(b) $v_L(0+) = v_C(0+) = 0$

But, $v_L = Ldi_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$di_L(0+)/dt =$ **<u>0</u>**

Again, $80 = 45i_R + v_C$

$0 = 45di_R/dt + dv_C/dt$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278$ mohms/1 μF = 278 V/s

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$di_R(0+)/dt =$ **<u>-6.1778 A/s</u>**

Also, $i_R = i_C + i_L$

$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$

$-6.1788 = di_C(0+)/dt + 0$, or $di_C(0+)/dt =$ **<u>-6.1788 A/s</u>**

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$i_R(\infty) = i_L(\infty) = 80/45k =$ **<u>1.778 mA</u>**

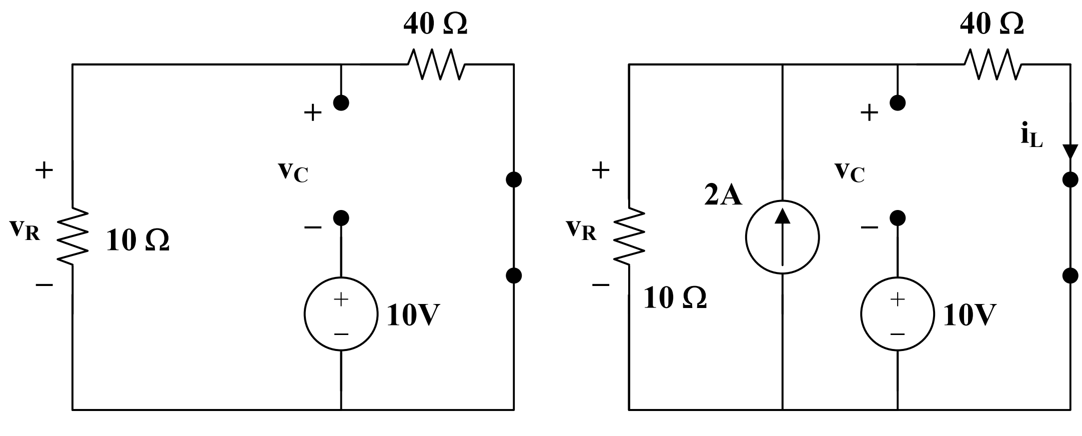
$i_C(\infty) = Cdv(\infty)/dt =$ **<u>0</u>**.

Chapter 8, Solution 3.

At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10V$.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **<u>0A</u>**, the capacitor has a voltage equal to **<u>–10V</u>**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) =$ **<u>0 V</u>**.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $Ldi_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt =$ **<u>0A/s</u>**, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C =$ **<u>8 V/s</u>**. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 =$ **<u>8 V/s</u>**.



(a)　　　　(b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \underline{\textbf{400 mA}}$$

$$v_C(\infty) = 2[10||40] - 10 = 16 - 10 = \underline{\textbf{6V}}$$

$$v_R(\infty) = 2[10||40] = \underline{\textbf{16 V}}$$

Chapter 8, Solution 4.

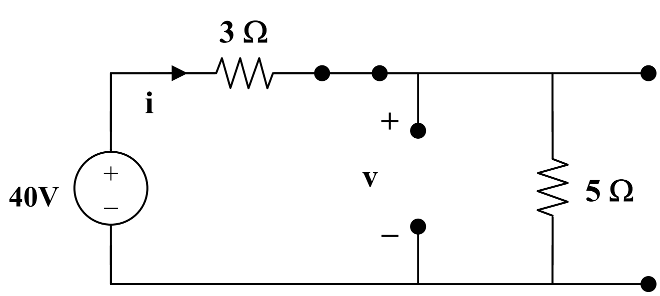
(a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

$$i(0^-) = 40/(3 + 5) = 5A, \text{ and } v(0^-) = 5i(0^-) = 25V.$$

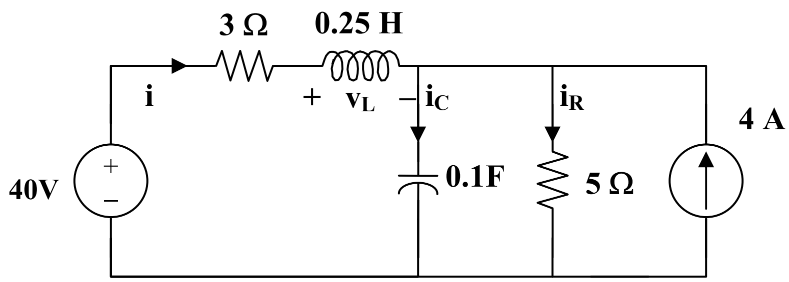
Hence,

$$i(0^+) = i(0^-) = \underline{\textbf{5A}}$$

$$v(0^+) = v(0^-) = \underline{\textbf{25V}}$$



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A, \ i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

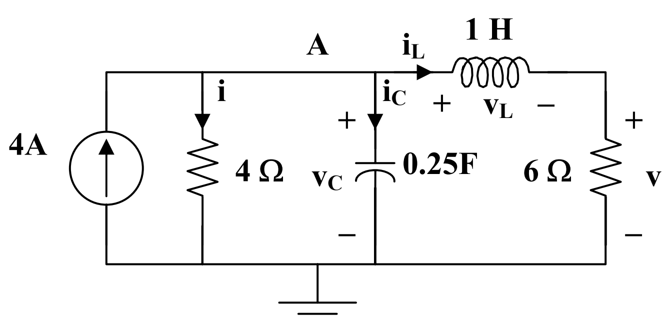
$$dv(0^+)/dt = 4/0.1 = \underline{\textbf{40 V/s}}$$

Chapter 8, Solution 5.

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \underline{\textbf{0 A}}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = \underline{\textbf{0V}}$.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \underline{\mathbf{4\ A/s}}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

Therefore $dv(0+)/dt = \underline{\mathbf{0\ V/s}}$

(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \underline{\mathbf{2.4\ A}}$$

$$v(\infty) = 6(4 - 2.4) = \underline{\mathbf{9.6\ V}}$$

Chapter 8, Solution 6.

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that
$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.
$$v_R(0+) = Ri(0+) = \underline{\mathbf{0\ V}}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \underline{\mathbf{0\ V}}$.
(1)

(b) Since $i(0+) = 0$, $i_C(0+) = V_S/R_S$

But, $i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$ (2)

From (1), $dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$ (3)
$v_R = iR$ or $dv_R/dt = Rdi/dt$ (4)

But, $v_L = Ldi/dt$, $v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $dv_R(0+)/dt = \underline{\mathbf{0\ V/s}}$

From (2) and (3), $dv_L(0+)/dt = dv(0+)/dt = \underline{\mathbf{V_s/(CR_s)}}$

(c) As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \underline{\mathbf{[R/(R + R_s)]V_s}}$$

$$v_L(\infty) = \underline{\mathbf{0\ V}}$$

Chapter 8, Solution 7.

$$s^2 + 4s + 4 = 0, \text{ thus } s_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4x4}}{2} = -2, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-2t}], \; v(0) = 1 = A$$

$$dv/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$dv(0)/dt = -1 = B - 2A = B - 2 \text{ or } B = 1.$$

Therefore, $v(t) = $ **$[(1 + t)e^{-2t}]$ V**

Chapter 8, Solution 8.

$s^2 + 6s + 9 = 0$, thus $s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 36}}{2} = -3$, repeated roots.

$$i(t) = [(A + Bt)e^{-3t}], \; i(0) = 0 = A$$

$$di/dt = [Be^{-3t}] + [-3(Bt)e^{-3t}]$$

$$di(0)/dt = 4 = B.$$

Therefore, $i(t) = $ **$[4te^{-3t}]$ A**

Chapter 8, Solution 9.

$$s^2 + 10s + 25 = 0, \text{ thus } s_{1,2} = \frac{-10 \pm \sqrt{10 - 10}}{2} = -5, \text{ repeated roots.}$$

$$i(t) = [(A + Bt)e^{-5t}], \; i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t) = $ **$[(10 + 50t)e^{-5t}]$ A**

Chapter 8, Solution 10.

$$s^2 + 5s + 4 = 0, \text{ thus } s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = -4, -1.$$

$$v(t) = (Ae^{-4t} + Be^{-t}), \; v(0) = 0 = A + B, \text{ or } B = -A$$

$$dv/dt = (-4Ae^{-4t} - Be^{-t})$$

$$dv(0)/dt = 10 = -4A - B = -3A \text{ or } A = -10/3 \text{ and } B = 10/3.$$

Therefore, $v(t) =$ **$\underline{(-(10/3)e^{-4t} + (10/3)e^{-t})\text{ V}}$**

Chapter 8, Solution 11.

$$s^2 + 2s + 1 = 0, \text{ thus } s_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = -1, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-t}], \; v(0) = 10 = A$$

$$dv/dt = [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt = 0 = B - A = B - 10 \text{ or } B = 10.$$

Therefore, $v(t) =$ **$\underline{[(10 + 10t)e^{-t}]\text{ V}}$**

Chapter 8, Solution 12.

(a) Overdamped when $C > 4L/(R^2) = 4 \text{x} 0.6/400 = 6 \text{x} 10^{-3}$, or $C >$ **<u>6 mF</u>**

(b) Critically damped when $C =$ **<u>6 mF</u>**

(c) Underdamped when $C <$ **<u>6mF</u>**

Chapter 8, Solution 13.

Let R||60 = R_o. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01x4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to R = **120 ohms**

Chapter 8, Solution 14.

This is a series, source-free circuit. 60||30 = 20 ohms

$$\alpha = R/(2L) = 20/(2x2) = 5 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.04}} = 5$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 2 = A$$

$$v = Ldi/dt = 2\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$v(0) = 6 = 2B - 10A = 2B - 20 \text{ or } B = 13.$$

Therefore, $i(t) = \mathbf{[(2 + 13t)e^{-5t}]\ A}$

Chapter 8, Solution 15.

This is a series, source-free circuit. 60||30 = 20 ohms

$$\alpha = R/(2L) = 20/(2x2) = 5 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.04}} = 5$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 2 = A$$

$$v = Ldi/dt = 2\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$v(0) = 6 = 2B - 10A = 2B - 20 \text{ or } B = 13.$$

Therefore, $i(t) = \mathbf{[(2 + 13t)e^{-5t}]\ A}$

Chapter 8, Solution 16.

At t = 0, i(0) = 0, $v_C(0)$ = 40x30/50 = 24V

For t > 0, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x2.5}} = 20$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-20t}], \ i(0) = 0 = A$$

$$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$$

but $di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$

Hence, B = -9.6 or i(t) = **$[-9.6te^{-20t}]$ A**

Chapter 8, Solution 17.

$$i(0) = I_0 = 0, \ v(0) = V_0 = 4x15 = 60$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+60) = -240$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20, \text{ which is} > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.68, -37.32$$

$$i(t) = A_1e^{-2.68t} + A_2e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2, \ \frac{di(0)}{dt} = -2.68A_1 - 37.32A_2 = -240$$

$$\text{This leads to } A_1 = -6.928 = -A_2$$

$$i(t) = 6.928\left(e^{-37.32t} - e^{-268t}\right)$$

$$\text{Since, } v(t) = \frac{1}{C}\int_0^t i(t)dt + 60, \text{ we get}$$

v(t) = **$(60 + 64.53e^{-2.68t} - 4.6412e^{-37.32t})$ V**

Chapter 8, Solution 18.

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$\alpha < \omega_o \quad \longrightarrow \quad$ underdamped case $\quad \omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4 - 0.25} = 1.936$

$I_o(0) = i(0)$ = initial inductor current = 20/5 = 4A

$V_o(0) = v(0)$ = initial capacitor voltage = 0 V

$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t) = e^{-0.5\alpha t}(A_1 \cos 1.936t + A_2 \sin 1.936t)$$

$v(0) = 0 = A_1$

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1 \cos 1.936t + A_2 \sin 1.936t) + e^{-0.5\alpha t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t)$$

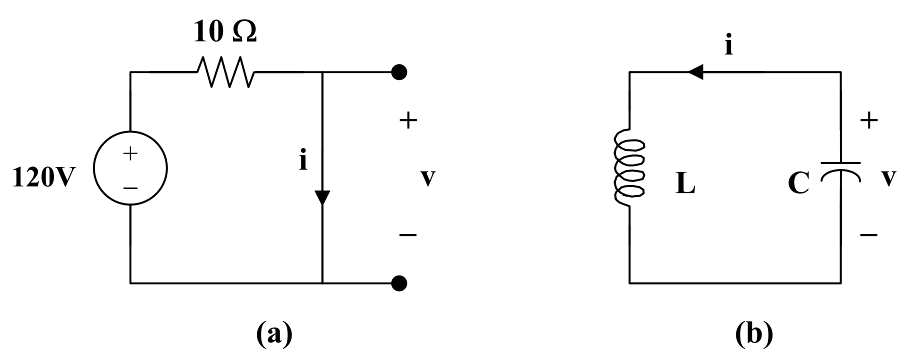
$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+4)}{1} = -4 = -0.5A_1 + 1.936A_2 \quad \longrightarrow \quad A_2 = -2.066$$

Thus,

$$\underline{v(t) = -2.066e^{-0.5t} \sin 1.936t}$$

Chapter 8, Solution 19.

For t < 0, the equivalent circuit is shown in Figure (a).



i(0) = 120/10 = 12, v(0) = 0

For $t > 0$, we have a series RLC circuit as shown in Figure (b) with $R = 0 = \alpha$.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \quad i(0) = 12 = A$$

$$v = -Ldi/dt, \text{ and } -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

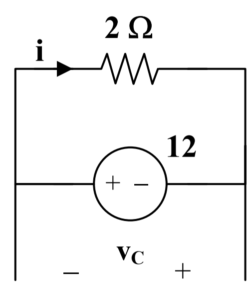
$$\text{which leads to } -v(0)/L = 0 = B$$

$$\text{Hence,} \qquad i(t) = 12\cos 0.5t \text{ A and } v = 0.5$$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] =$ **24sin0.5t V**

Chapter 8, Solution 20.

For $t < 0$, the equivalent circuit is as shown below.



$$v(0) = -12\text{V and } i(0) = 12/2 = 6\text{A}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos 2t + B\sin 2t)e^{-2t}$$

$$i(0) = 6 = A$$

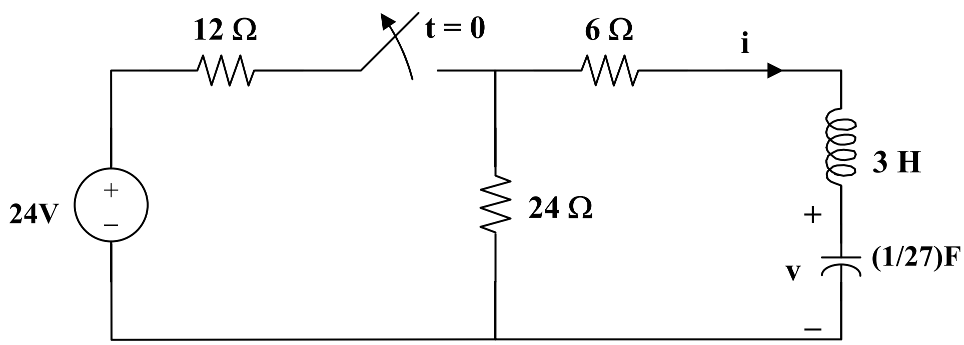
$$di/dt = -2(6\cos 2t + B\sin 2t)e^{-2t} + (-2x6\sin 2t + 2B\cos 2t)e^{-\alpha t}$$

$$di(0)/dt = -12 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[12 – 12] = 0$$

Thus, $B = 6$ and $i(t) =$ $\underline{\mathbf{(6\cos 2t + 6\sin 2t)e^{-2t}\ A}}$

Chapter 8, Solution 21.

By combining some resistors, the circuit is equivalent to that shown below.
$60||(15 + 25) = 24$ ohms.



At $t = 0-$, $i(0) = 0$, $v(0) = 24x24/36 = 16V$

For $t > 0$, we have a series RLC circuit. $R = 30$ ohms, $L = 3$ H, $C = (1/27)$ F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}],\ v(0) = 16 = A + B \qquad (1)$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A – 9B] \text{ or } A = -9B \qquad (2)$$

From (1) and (2), $B = -2$ and $A = 18$.

Hence, $v(t) =$ $\underline{\mathbf{(18e^{-t} – 2e^{-9t})\ V}}$

Chapter 8, Solution 22.

$\alpha = 20 = 1/(2RC)$ or $RC = 1/40$ (1)

$\omega_d = 50 = \sqrt{\omega_o^2 - \alpha^2}$ which leads to $2500 + 400 = \omega_o^2 = 1/(LC)$

Thus, LC 1/2900 (2)

In a parallel circuit, $v_C = v_L = v_R$

But, $i_C = Cdv_C/dt$ or $i_C/C = dv_C/dt$

$$= -80e^{-20t}\cos 50t - 200e^{-20t}\sin 50t + 200e^{-20t}\sin 50t - 500e^{-20t}\cos 50t$$
$$= -580e^{-20t}\cos 50t$$

$i_C(0)/C = -580$ which leads to $C = -6.5x10^{-3}/(-580) = \underline{\textbf{11.21 } \mu\textbf{F}}$

$R = 1/(40C) = 10^6/(2900x11.21) = \underline{\textbf{2.23 kohms}}$

$L = 1/(2900x11.21) = \underline{\textbf{30.76 H}}$

Chapter 8, Solution 23.

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$\alpha = 1/(2RC_o)$, $\omega_o = 1/\sqrt{LC_o}$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$\omega_o = 1/\sqrt{0.5x0.5} = 6.32 > \alpha$ (underdamped)

$C_o = C + 10$ mF $= 50$ mF or **40 mF**

Chapter 8, Solution 24.

For $t < 0$, u(-t) 1, namely, the switch is on.

$v(0) = 0$, $i(0) = 25/5 = 5A$

For $t > 0$, the voltage source is off and we have a source-free parallel RLC circuit.

$\alpha = 1/(2RC) = 1/(2x5x10^{-3}) = 100$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.1x10^{-3}} = 100$$

$$\omega_o = \alpha \text{ (critically damped)}$$

$$v(t) = [(A_1 + A_2t)e^{-100t}]$$

$$v(0) = 0 = A_1$$

$$dv(0)/dt = -[v(0) + Ri(0)]/(RC) = -[0 + 5x5]/(5x10^{-3}) = -5000$$

But, $dv/dt = [(A_2 + (-100)A_2t)e^{-100t}]$

Therefore, $dv(0)/dt = -5000 = A_2 - 0$

$v(t) = $ **$-5000te^{-100t}$ V**

Chapter 8, Solution 25.

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.

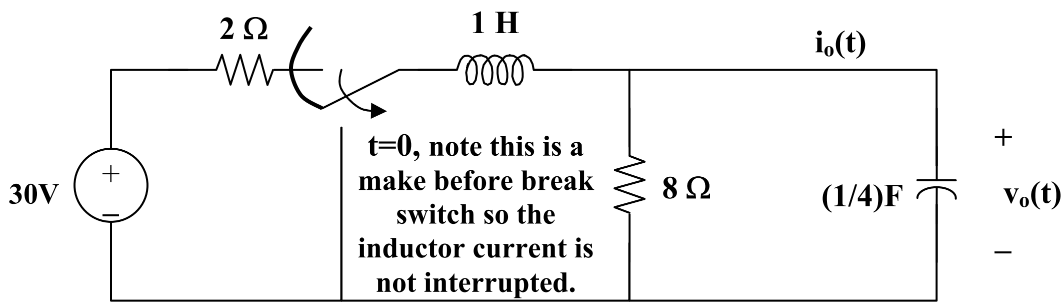


Figure 8.78 For Problem 8.25.

At $t = 0^-$, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = ¼$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$v_o(0) = 24 = A_1$ and $i_o(t) = C(dv_o/dt) = 0$ when t = 0.

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

at t = 0, we get $dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **$(24\cos\omega_d t + 3.024\sin\omega_d t)e^{-t/4}$ volts**

Chapter 8, Solution 26.

$$s^2 + 2s + 5 = 0, \text{ which leads to } s_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1\pm j4$$

$$i(t) = I_s + [(A_1\cos 4t + A_2\sin 4t)e^{-t}], \ I_s = 10/5 = 2$$

$$i(0) = 2 = = 2 + A_1, \text{ or } A_1 = 0$$

$$di/dt = [(A_2\cos 4t)e^{-t}] + [(-A_2\sin 4t)e^{-t}] = 4 = 4A_2, \text{ or } A_2 = 1$$

$i(t) =$ **$2 + \sin 4te^{-t}$ A**

Chapter 8, Solution 27.

$$s^2 + 4s + 8 = 0 \text{ leads to } s = \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$$

$$v(t) = V_s + (A_1\cos 2t + A_2\sin 2t)e^{-2t}$$

$8V_s = 24$ means that $V_s = 3$

$v(0) = 0 = 3 + A_1$ leads to $A_1 = -3$

$$dv/dt = -2(A_1\cos 2t + A_2\sin 2t)e^{-2t} + (-2A_1\sin 2t + 2A_2\cos 2t)e^{-2t}$$

$0 = dv(0)/dt = -2A_1 + 2A_2$ or $A_2 = A_1 = -3$

$v(t) =$ **$[3 - 3(\cos 2t + \sin 2t)e^{-2t}]$ volts**

Chapter 8, Solution 28.

The characteristic equation is $s^2 + 6s + 8$ with roots

$$s_{1,2} = \frac{-6 \pm \sqrt{36-32}}{2} = -4,-2$$

Hence,

$$i(t) = I_s + Ae^{-2t} + Be^{-4t}$$

$$8I_s = 12 \longrightarrow I_s = 1.5$$

$$i(0) = 0 \longrightarrow 0 = 1.5 + A + B \qquad (1)$$

$$\frac{di}{dt} = -2Ae^{-2t} - 4Be^{-4t}$$

$$\frac{di(0)}{dt} = 2 = -2A - 4B \longrightarrow 0 = 1 + A + 2B \qquad (2)$$

Solving (1) and (2) leads to A=-2 and B=0.5.

$$i(t) = \underline{1.5 - 2e^{-2t} + 0.5e^{-4t}} \text{ A}$$

Chapter 8, Solution 29.

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

$dv(0)/dt = 2 = 2B$ or $B = 1$, therefore $v(t) =$ **<u>(3 – 3cos2t + sin2t) V</u>**

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get A = -4 and B = 1

$$i(t) = \underline{\mathbf{(2 - 4e^{-t} + e^{-4t})\ A}}$$

(c) $s^2 + 2s + 1 = 0$, $s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}],\ V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \underline{\mathbf{[3 + (2 + 3t)e^{-t}]\ V}}$$

Chapter 8, Solution 30.

$$s_1 = -500 = -\alpha + \sqrt{\alpha^2 - \omega_o^{\ 2}}, \qquad s_2 = -800 = -\alpha - \sqrt{\alpha^2 - \omega_o^{\ 2}}$$

$$s_1 + s_2 = -1300 = -2\alpha \quad \longrightarrow \quad \alpha = 650 = \frac{R}{2L}$$

Hence,

$$L = \frac{R}{2\alpha} = \frac{200}{2x650} = \underline{153.8\text{ mH}}$$

$$s_1 - s_2 = 300 = 2\sqrt{\alpha^2 - \omega_o^{\ 2}} \quad \longrightarrow \quad \omega_o = 623.45 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{(632.45)^2 L} = \underline{16.25\mu\text{F}}$$

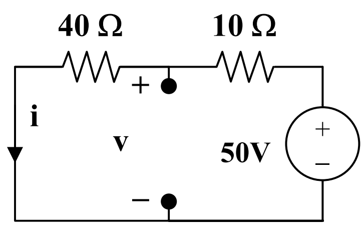
Chapter 8, Solution 31.

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

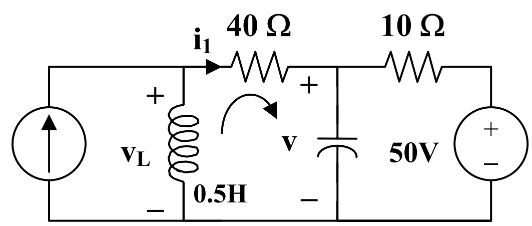
$$v(0+) = v(0-) = 40, \; i(0+) = i(0-) = 1$$

By KCL, 2 = $i(0+) + i_1 = 1 + i_1$ which leads to $i_1 = 1$. By KVL, $-v_L + 40i_1 + v(0+) = 0$ which leads to $v_L(0+) = 40x1 + 40 = 80$

$$v_L(0+) = \underline{\mathbf{80\ V}}, \quad v_C(0+) = \underline{\mathbf{40\ V}}$$



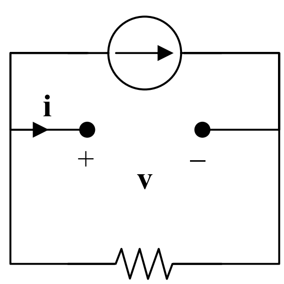
(a)



(b)

Chapter 8, Solution 32.

For t = 0-, the equivalent circuit is shown below.



$$i(0-) = 0, \; v(0-) = -2x6 = -12V$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

$$s = -3 \pm \sqrt{9 - 25} = -3 \pm j4$$

Thus, $v(t) = V_f + [(A\cos 4t + B\sin 4t)e^{-3t}]$

$$\text{where } V_f = \text{final capacitor voltage} = 50 \text{ V}$$

$$v(t) = 50 + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$$v(0) = -12 = 50 + A \text{ which gives } A = -62$$

$$i(0) = 0 = Cdv(0)/dt$$

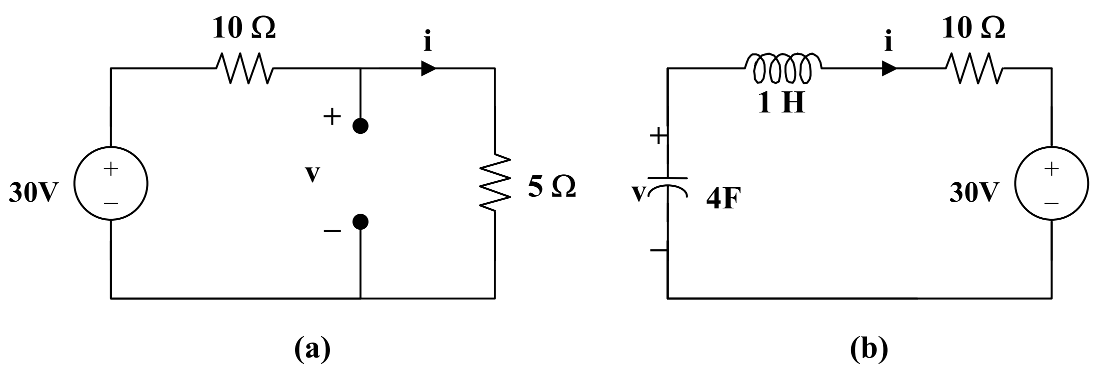
$$dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$0 = dv(0)/dt = -3A + 4B \text{ or } B = (3/4)A = -46.5$$

$$v(t) = \underline{\mathbf{\{50 + [(-62\cos 4t - 46.5\sin 4t)e^{-3t}]\}\ V}}$$

Chapter 8, Solution 33.

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



$$i(0) = 30/15 = 2 \text{ A}, \quad v(0) = 5\text{x}30/15 = 10 \text{ V}$$

For t > 0, we have a series RLC circuit.

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.25, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.95, -0.05$$

$$v(t) = V_s + [A_1e^{-4.95t} + A_2e^{-0.05t}], \ v = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \qquad (1)$$

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = 2/4 = 1/2$$

Hence, $\frac{1}{2} = -4.95A_1 - 0.05A_2$ (2)

From (1) and (2), $A_1 = 0$, $A_2 = -10$.

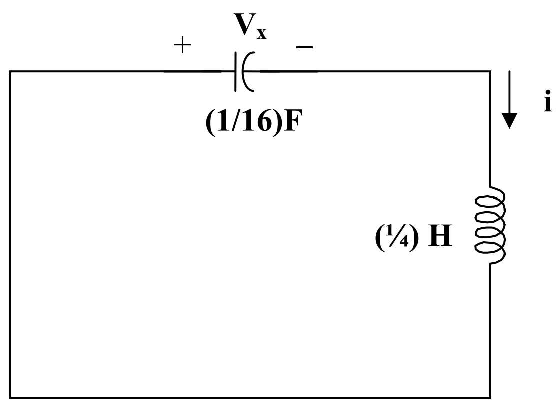
$$v(t) = \underline{\mathbf{\{20 - 10e^{-0.05t}\}\ V}}$$

Chapter 8, Solution 34.

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0, v(0) = 20 \text{ V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0, \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16} + \frac{1}{4}} = 8, \ s = \pm j8$$

Since α is less than ω_o, we have an underdamped response. Therefore,

$$i(t) = A_1\cos 8t + A_2\sin 8t \text{ where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x20 = -80$$

However, $di/dt = 8A_2\cos 8t$, thus, $di(0)/dt = -80 = 8A_2$ which leads to $A_2 = -10$

Now we have $i(t) = \underline{\mathbf{-10\sin 8t\ A}}$

Chapter 8, Solution 35.

At t = 0-, $i_L(0)$ = 0, v(0) = $v_C(0)$ = 8 V

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1,\ \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}],\ V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4,\ i(0) = Cdv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

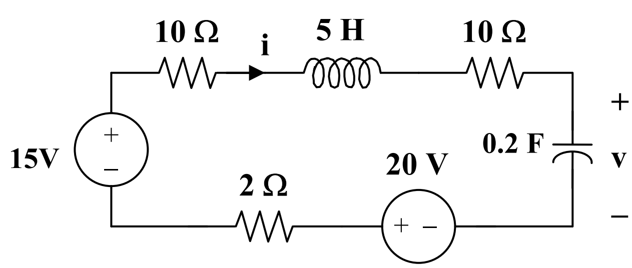
$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

v(t) = **{12 – (4cos2t + 2sin2t)e^{-t} V**.

Chapter 8, Solution 36.

For t = 0-, 3u(t) = 0. Thus, i(0) = 0, and v(0) = 20 V.

For t > 0, we have the series RLC circuit shown below.



$$\alpha = R/(2L) = (2 + 5 + 1)/(2x5) = 0.8$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos 0.6t + B\sin 0.6t)e^{-0.8t}]$$

$$V_s = 15 + 20 = 35V \text{ and } v(0) = 20 = 35 + A \text{ or } A = -15$$

$$i(0) = Cdv(0)/dt = 0$$

$$\text{But } dv/dt = [-0.8(A\cos 0.6t + B\sin 0.6t)e^{-0.8t}] + [0.6(-A\sin 0.6t + B\cos 0.6t)e^{-0.8t}]$$

$$0 = dv(0)/dt = -0.8A + 0.6B \text{ which leads to } B = 0.8x(-15)/0.6 = -20$$

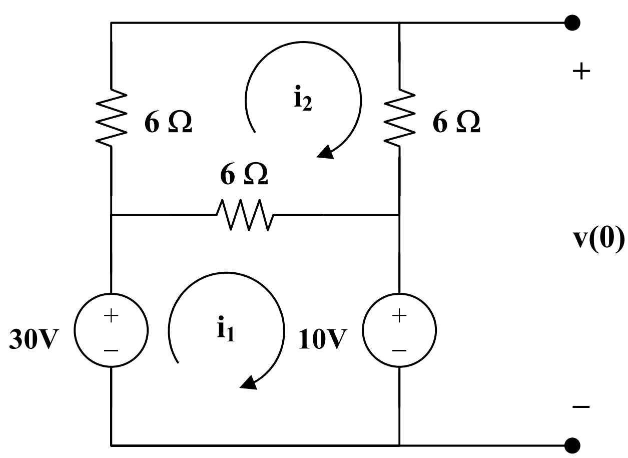
$$v(t) = \underline{\mathbf{\{35 - [(15\cos 0.6t + 20\sin 0.6t)e^{-0.8t}]\}\ V}}$$

$$i = Cdv/dt = 0.2\{[0.8(15\cos 0.6t + 20\sin 0.6t)e^{-0.8t}] + [0.6(15\sin 0.6t - 20\cos 0.6t)e^{-0.8t}]\}$$

$$i(t) = \underline{\mathbf{[(5\sin 0.6t)e^{-0.8t}]\ A}}$$

Chapter 8, Solution 37.

For t = 0-, the equivalent circuit is shown below.



$$18i_2 - 6i_1 = 0 \text{ or } i_1 = 3i_2 \qquad (1)$$

$$-30 + 6(i_1 - i_2) + 10 = 0 \text{ or } i_1 - i_2 = 10/3 \qquad (2)$$

From (1) and (2). $i_1 = 5,\ i_2 = 5/3$

$$i(0) = i_1 = 5A$$

$$-10 - 6i_2 + v(0) = 0$$

$$v(0) = 10 + 6x5/3 = 20$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2x(1/2)) = 4$$

$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = 10$$

$$v(0) = 20 = 10 + A, \text{ or } A = 10$$

$$i = Cdv/dt = -4C[(A + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

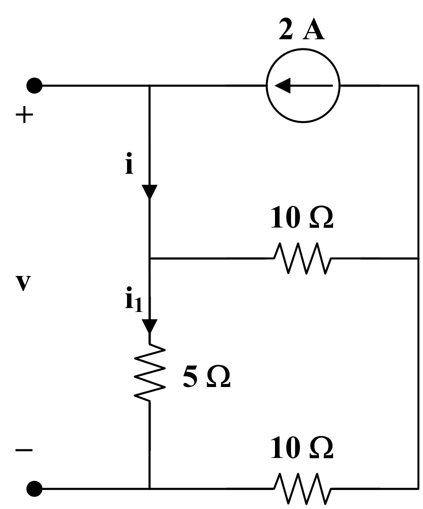
$$i(0) = 5 = C(-4A + B) \text{ which leads to } 40 = -40 + B \text{ or } B = 80$$

$$i(t) = [-(1/2)(10 + 80t)e^{-4t}] + [(10)e^{-4t}]$$

$i(t) = \underline{\mathbf{[(5 - 40t)e^{-4t}] A}}$

Chapter 8, Solution 38.

At $t = 0^-$, the equivalent circuit is as shown.



$$i(0) = 2A, \quad i_1(0) = 10(2)/(10 + 15) = 0.8 \text{ A}$$

$$v(0) = 5i_1(0) = 4V$$

For $t > 0$, we have a source-free series RLC circuit.

$$R = 5||(10 + 10) = 4 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$$

$$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -4.431, -0.903$$

$$i(t) = [Ae^{-4.431t} + Be^{-0.903t}]$$

$$i(0) = A + B = 2 \qquad (1)$$

$$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x2 + 4) = -16/3 = -5.333$$

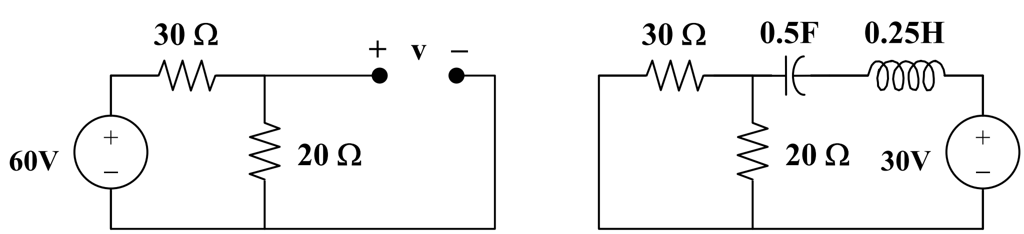
$$\text{Hence, } -5.333 = -4.431A - 0.903B \qquad (2)$$

From (1) and (2), $A = 1$ and $B = 1$.

$i(t) =$ $\underline{\mathbf{[e^{-4.431t} + e^{-0.903t}]\ A}}$

Chapter 8, Solution 39.

For $t = 0^-$, the equivalent circuit is shown in Figure (a). Where $60u(-t) = 60$ and $30u(t) = 0$.



(a) (b)

$$v(0) = (20/50)(60) = 24 \text{ and } i(0) = 0$$

For $t > 0$, the circuit is shown in Figure (b).

$$R = 20||30 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -47.833, \ -0.167$$

Thus, $\quad v(t) = V_s + [Ae^{-47.833t} + Be^{-0.167t}], \quad V_s = 30$

$$v(0) = 24 = 30 + A + B \text{ or } -6 = A + B \tag{1}$$

$$i(0) = Cdv(0)/dt = 0$$

But, $\quad dv(0)/dt = -47.833A - 0.167B = 0$

$$B = -286.43A \tag{2}$$

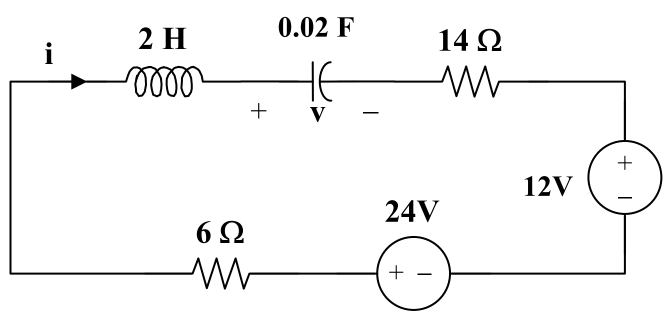
From (1) and (2), $\quad A = 0.021$ and $B = -6.021$

$$v(t) = \underline{\mathbf{30 + [0.021e^{-47.833t} - 6.021e^{-0.167t}]\ V}}$$

Chapter 8, Solution 40.

At $t = 0$-, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6 + 2))4 = 3A$

For $t > 0$, we have a series RLC circuit with a step input as shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x0.02} = 5$$

$$\alpha = R/(2L) = (6 + 14)/(2x2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = 24 - 12 = 12V$$

$$v(0) = 0 = 12 + A \text{ or } A = -12$$

$$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$i(0) = 3 = C[-5A + B] = 0.02[60 + B] \text{ or } B = 90$$

$$\text{Thus,} \quad i(t) = 0.02\{[90e^{-5t}] + [-5(-12 + 90t)e^{-5t}]\}$$

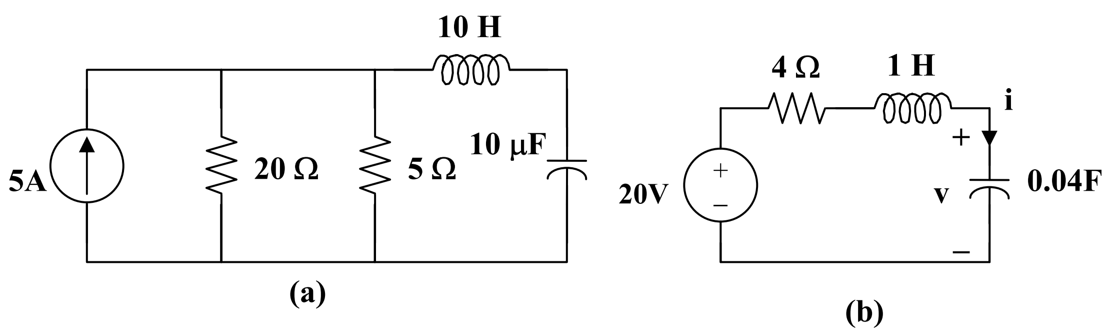
$$i(t) = \underline{\mathbf{\{(3 - 9t)e^{-5t}\}\ A}}$$

Chapter 8, Solution 41.

At t = 0-, the switch is open. i(0) = 0, and

$$v(0) = 5x100/(20 + 5 + 5) = 50/3$$

For t > 0, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (4)/(2x1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus, $v(t) = V_s + [(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}]$,

where $\omega_d = 4.583$ and $V_s = 20$

$$v(0) = 50/3 = 20 + A \text{ or } A = -10/3$$

$$i(t) = Cdv/dt = C(-2)\,[(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}] + C\omega_d[(-A\sin\omega_d t + B\cos\omega_d t)e^{-2t}]$$

$$i(0) = 0 = -2A + \omega_d B$$

$$B = 2A/\omega_d = -20/(3x4.583) = -1.455$$

$$i(t) = C\{[(0\cos\omega_d t + (-2B - \omega_d A)\sin\omega_d t)]e^{-2t}\}$$

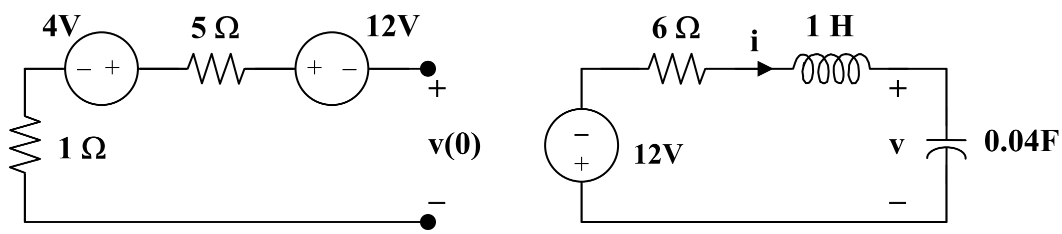
$$= (1/25)\{[(2.91 + 15.2767)\sin\omega_d t)]e^{-2t}\}$$

$$i(t) = \underline{\mathbf{\{0.7275\sin(4.583t)e^{-2t}\}\ A}}$$

Chapter 8, Solution 42.

For t = 0-, we have the equivalent circuit as shown in Figure (a).

$$i(0) = i(0) = 0, \text{ and } v(0) = 4 - 12 = -8V$$



(a) **(b)**

For t > 0, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $v(t) = V_s + [(A\cos4t + B\sin4t)e^{-3t}],\ \ V_s = -12$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos4t + B\sin4t)e^{-3t}] + [4(-A\sin4t + B\cos4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$$v(t) = \underline{\mathbf{\{-12 + [(4\cos4t + 3\sin4t)e^{-3t}]\}\ A}}$$

Chapter 8, Solution 43.

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \quad \longrightarrow \quad R = 2\alpha L = 2x8x0.5 = \underline{8\Omega}$$

$$\omega_d = \sqrt{\omega_o^{\,2} - \alpha^2} = 30 \quad \longrightarrow \quad \omega_o = \sqrt{900 \;\; - 64} = \sqrt{836}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad C = \frac{1}{\omega^2{}_o L} = \frac{1}{836x0.5} = \underline{2.392 \text{ mF}}$$

Chapter 8, Solution 44.

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \quad \longrightarrow$ **underdamped.**

Chapter 8, Solution 45.

$$\omega_o = 1/\sqrt{LC} \; = \; 1/\sqrt{1x0.5} \; = \; \sqrt{2}$$

$$\alpha \;\; = \; R/(2L) \; = \; (1)/(2x2x0.5) \; = \; 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} \; = \; -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \; = \; -0.5 \pm j1.323$$

Thus, $\quad i(t) \; = \; I_s + [(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}], \quad I_s \; = \; 4$

$$i(0) \; = \; 1 \; = \; 4 + A \;\; \text{or} \;\; A \; = \; -3$$

$$v \; = \; v_C \; = \; v_L \; = \; Ldi(0)/dt \; = \; 0$$

$$di/dt \; = \; [1.323(-A\sin 1.323t + B\cos 1.323t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

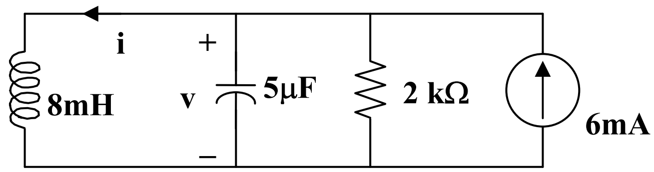
$$di(0)/dt \; = \; 0 \; = \; 1.323B - 0.5A \;\; \text{or} \;\; B \; = \; 0.5(-3)/1.323 \; = \; -1.134$$

Thus, $\quad i(t) \; = \; \underline{\mathbf{\{4 - [(3\cos 1.323t + 1.134\sin 1.323t)e^{-0.5t}]\}\ A}}$

Chapter 8, Solution 46.

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input, as shown below.



$$\alpha = 1/(2RC) = (1)/(2x2x10^3 x5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^3 x5x10^{-6}} = 5,000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5,000$$

Thus, $$i(t) = I_s + [(A\cos 5,000t + B\sin 5,000t)e^{-50t}], \quad I_s = 6mA$$

$$i(0) = 0 = 6 + A \text{ or } A = -6mA$$

$$v(0) = 0 = Ldi(0)/dt$$

$$di/dt = [5,000(-A\sin 5,000t + B\cos 5,000t)e^{-50t}] + [-50(A\cos 5,000t + B\sin 5,000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5,000B - 50A \text{ or } B = 0.01(-6) = -0.06mA$$

Thus, $$i(t) = \underline{\mathbf{\{6 - [(6\cos 5,000t + 0.06\sin 5,000t)e^{-50t}]\}\ mA}}$$

Chapter 8, Solution 47.

At $t = 0-$, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$

and $v_o(0) = 0$.

For $t > 0$, the 20-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$$

$$i(0) = 1 = 3 + A \text{ or } A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

Thus, $v_o(t) =$ **$\underline{(200te^{-10t})\ V}$**

Chapter 8, Solution 48.

For $t = 0-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2x1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = [(A + Bt)e^{-2t}], \quad i(0) = -2 = A$$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus, $i(t) =$ **$\underline{[(-2 - 2t)e^{-2t}]\ A}$**

and $v(t) =$ **$\underline{[(2 + 4t)e^{-2t}]\ V}$**

Chapter 8, Solution 49.

For $t = 0^-$, $i(0) = 3 + 12/4 = 6$ and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-2t}], \quad I_s = 3$$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

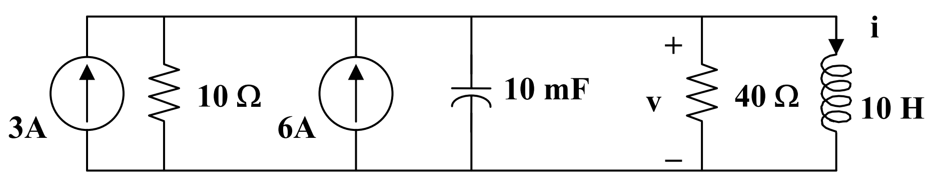
$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t) =$ **$\{3 + [(3 + 6t)e^{-2t}]\}$ A**

Chapter 8, Solution 50.

For $t = 0-$, $4u(t) = 0$, $v(0) = 0$, and $i(0) = 30/10 = 3A$.

For $t > 0$, we have a parallel RLC circuit.



$$I_s = 3 + 6 = 9A \text{ and } R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2x8x0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4x0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $i(t) = I_s + [Ae^{-10t}] + [Be^{-2.5t}]$, $I_s = 9$

$$i(0) = 3 = 9 + A + B \text{ or } A + B = -6$$

$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \text{ or } di(0)/dt = 0 = -10A - 2.5B \text{ or } B = -4A$$

Thus, $A = 2$ and $B = -8$

Clearly, $i(t) =$ $\underline{\mathbf{\{9 + [2e^{-10t}] + [-8e^{-2.5t}]\}\ A}}$

Chapter 8, Solution 51.

Let i = inductor current and v = capacitor voltage.
At t = 0, v(0) = 0 and $i(0) = i_o$.

For t > 0, we have a parallel, source-free LC circuit ($R = \infty$).

$$\alpha = 1/(2RC) = 0 \text{ and } \omega_o = 1/\sqrt{LC} \text{ which leads to } s_{1,2} = \pm j\omega_o$$

$$v = A\cos\omega_o t + B\sin\omega_o t,\ v(0) = 0\ A$$

$$i_C = Cdv/dt = -i$$

$$dv/dt = \omega_o B\sin\omega_o t = -i/C$$

$$dv(0)/dt = \omega_o B = -i_o/C \text{ therefore } B = i_o/(\omega_o C)$$

$v(t) =$ $\underline{\mathbf{-(i_o/(\omega_o C))\sin\omega_o t\ V\ \text{where}\ \omega_o = \sqrt{LC}}}$

Chapter 8, Solution 52.

$$\alpha = 300 = \frac{1}{2RC} \qquad (1)$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 400 \longrightarrow \omega_o = \sqrt{400^2 - 300^2} = 264.575 = \frac{1}{\sqrt{LC}} \qquad (2)$$

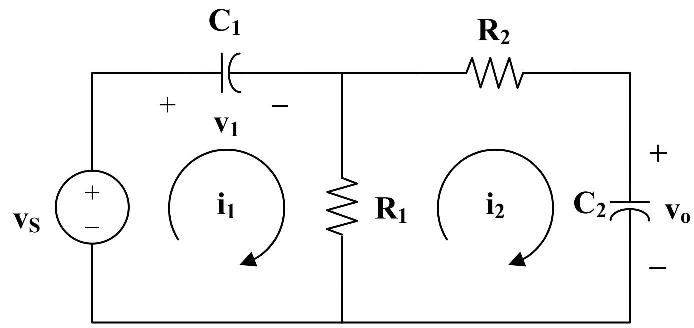
From (2),

$$C = \frac{1}{(264.575)^2 x 50x10^{-3}} = \underline{285.71\mu\text{F}}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2x300}(3500) = \underline{5.833\Omega}$$

Chapter 8, Solution 53.



$$i_2 = C_2 dv_o/dt \quad (1)$$

$$i_1 = C_1 dv_1/dt \quad (2)$$

$$0 = R_2 i_2 + R_1(i_2 - i_1) + v_o \quad (3)$$

Substituting (1) and (2) into (3) we get,

$$0 = R_2C_2dv_o/dt + R_1(C_2dv_o/dt - C_1dv_1/dt) \quad (4)$$

Applying KVL to the outer loop produces,

$$v_s = v_1 + i_2R_2 + v_o = v_1 + R_2C_2dv_o/dt + v_o, \text{ which leads to}$$

$$v_1 = v_s - v_o - R_2C_2dv_o/dt \quad (5)$$

Substituting (5) into (4) leads to,

$$0 = R_1C_2dv_o/dt + R_1C_2dv_o/dt - R_1C_1(dv_s/dt - dv_o/dt - R_2C_2d^2v_o/dt^2)$$

Hence, $\underline{\mathbf{(R_1C_1R_2C_2)(d^2v_o/dt^2) + (R_1C_1 + R_2C_2 + R_1C_2)(dv_o/dt) = R_1C_1(dv_s/dt)}}$

Chapter 8, Solution 54.

Let i be the inductor current.

$$-i = \frac{v}{4} + 0.5\frac{dv}{dt} \quad (1)$$

$$v = 2i + \frac{di}{dt} \quad (2)$$

Substituting (1) into (2) gives

$$-v = \frac{v}{2} + \frac{dv}{dt} + \frac{1}{4}\frac{dv}{dt} + \frac{1}{2}\frac{d^2v}{dt^2} \longrightarrow \frac{d^2v}{dt^2} + 2.5\frac{dv}{dt} + 3v = 0$$

$$s^2 + 2.5s + 3 = 0 \longrightarrow s = -1.25 \pm j1.199$$

$$v = Ae^{-1.25t}\cos 1.199t + Be^{-1.25t}\sin 1.199t$$

v(0) = 2=A. Let w=1.199

$$\frac{dv}{dt} = -1.25(Ae^{-1.25t}\cos wt + Be^{-1.25t}\sin wt) + w(-Ae^{-1.25t}\sin wt + Be^{-1.25t}\cos wt)$$

$$\frac{dv(0)}{dt} = 0 = -1.25A + Bw \longrightarrow B = \frac{1.25X2}{1.199} = 2.085$$

$$v = \underline{2e^{-1.25t}\cos 1.199t + 2.085e^{-1.25t}\sin 1.199t} \text{ V}$$

Chapter 8, Solution 55.

At the top node, writing a KCL equation produces,

$$i/4 + i = C_1 dv/dt,\ C_1 = 0.1$$

$$5i/4 = C_1 dv/dt = 0.1dv/dt$$

$$i = 0.08dv/dt \qquad (1)$$

But, $v = -(2i + (1/C_2)\int i dt)$, $C_2 = 0.5$

or $-dv/dt = 2di/dt + 2i \qquad (2)$

Substituting (1) into (2) gives,

$$-dv/dt = 0.16d^2v/dt^2 + 0.16dv/dt$$

$$0.16d^2v/dt^2 + 0.16dv/dt + dv/dt = 0, \text{ or } d^2v/dt^2 + 7.25dv/dt = 0$$

Which leads to $s^2 + 7.25s = 0 = s(s + 7.25)$ or $s_{1,2} = 0, -7.25$

$$v(t) = A + Be^{-7.25t} \qquad (3)$$

$$v(0) = 4 = A + B \qquad (4)$$

From (1), $i(0) = 2 = 0.08dv(0+)/dt$ or $dv(0+)/dt = 25$

But, $dv/dt = -7.25Be^{-7.25t}$, which leads to,

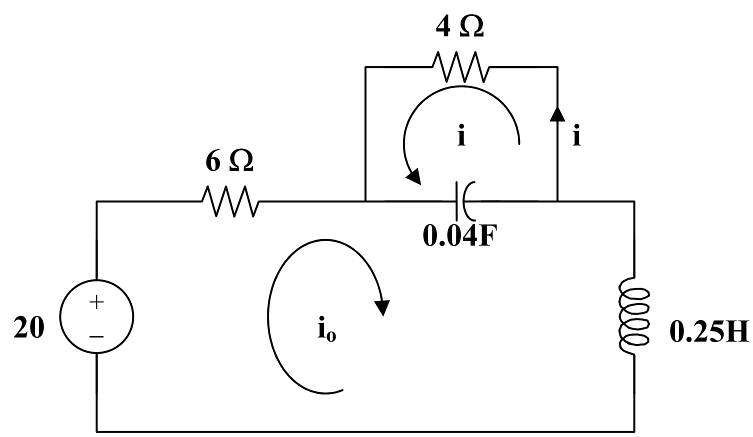
$dv(0)/dt = -7.25B = 25$ or $B = -3.448$ and $A = 4 - B = 4 + 3.448 = 7.448$

Thus, $v(t) =$ **$\{7.45 - 3.45e^{-7.25t}\}$ V**

Chapter 8, Solution 56.

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0$$

Taking the derivative,

$$6di_o/dt + 0.25d^2i_o/dt^2 + 25(i_o + i) = 0 \qquad (1)$$

For the smaller loop, $4 + 25\int(i + i_o)dt = 0$

Taking the derivative, $25(i + i_o) = 0$ or $i = -i_o$ (2)

From (1) and (2) $6di_o/dt + 0.25d^2i_o/dt^2 = 0$

This leads to, $0.25s^2 + 6s = 0$ or $s_{1,2} = 0, -24$

$$i_o(t) = (A + Be^{-24t}) \text{ and } i_o(0) = 0 = A + B \text{ or } B = -A$$

As t approaches infinity, $i_o(\infty) = 20/10 = 2 = A$, therefore $B = -2$

$$\text{Thus, } i_o(t) = (2 - 2e^{-24t}) = -i(t) \text{ or } i(t) = \underline{\mathbf{(-2 + 2e^{-24t})\ A}}$$

Chapter 8, Solution 57.

(a) Let v = capacitor voltage and i = inductor current. At t = 0-, the switch is closed and the circuit has reached steady-state.

$$v(0-) = 16V \text{ and } i(0-) = 16/8 = 2A$$

At t = 0+, the switch is open but, v(0+) = 16 and i(0+) = 2.

We now have a source-free RLC circuit.

$$R\ 8 + 12 = 20 \text{ ohms, } L = 1H, C = 4mF.$$

$$\alpha = R/(2L) = (20)/(2x1) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$$

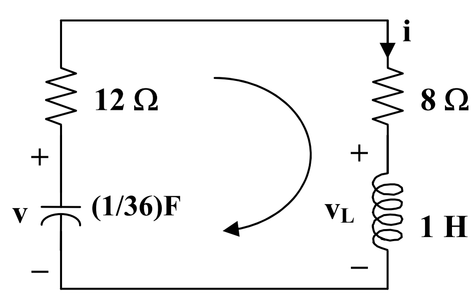
Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is $(s + 2)(s + 18) = 0$ or $\underline{\mathbf{s^2 + 20s + 36 = 0}}$.

(b) $$i(t) = [Ae^{-2t} + Be^{-18t}] \text{ and } i(0) = 2 = A + B \qquad (1)$$

To get di(0)/dt, consider the circuit below at t = 0+.



$$-v(0) + 20i(0) + v_L(0) = 0, \text{ which leads to,}$$

$-16 + 20x2 + v_L(0) = 0$ or $v_L(0) = -24$

$Ldi(0)/dt = v_L(0)$ which gives $di(0)/dt = v_L(0)/L = -24/1 = -24$ A/s

Hence $-24 = -2A - 18B$ or $12 = A + 9B$ (2)

From (1) and (2), $B = 1.25$ and $A = 0.75$

$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t)$ or $i_x(t) = \underline{\mathbf{[-0.75e^{-2t} - 1.25e^{-18t}]\ A}}$

$v(t) = 8i(t) = \underline{\mathbf{[6e^{-2t} + 10e^{-18t}]\ A}}$

Chapter 8, Solution 58.

(a) Let i =inductor current, v = capacitor voltage i(0) =0, $\underline{v(0) = 4}$

$$\frac{dv(0)}{dt} = -\frac{[v(0) + Ri(0)]}{RC} = -\frac{(4+0)}{0.5} = \underline{-8\text{ V/s}}$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_d = \sqrt{\omega^2{}_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$

v(0) = 4 = A_1

$$\frac{dv}{dt} = -e^{-t}A_1 \cos 1.732t - 1.732e^{-t}A_1 \sin 1.732t - e^{-t}A_2 \sin 1.732t + 1.732e^{-t}A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -8 = -A_1 + 1.732A_2 \quad \longrightarrow \quad A_2 = -2.309$$

$$v(t) = \underline{e^{-t}(4\cos 1.732t - 2.309\sin 1.732t)}\text{ V}$$

Chapter 8, Solution 59.

Let i = inductor current and v = capacitor voltage
v(0) = 0, i(0) = 40/(4+16) = 2A
For t>0, the circuit becomes a source-free series RLC with

$$\alpha = \frac{R}{2L} = \frac{16}{2x4} = 2, \quad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4x1/16}} = 2, \quad \longrightarrow \quad \alpha = \omega_o = 2$$

$$i(t) = Ae^{-2t} + Bte^{-2t}$$

i(0) = 2 = A

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$$

$$\frac{di(0)}{dt} = -2A + B = -\frac{1}{L}[Ri(0) + v(0)] \quad \longrightarrow \quad -2A + B = -\frac{1}{4}(32+0), \quad B = -4$$

$$i(t) = 2e^{-2t} - 4te^{-2t}$$

$$v = \frac{1}{C}\int_0^t idt + v(0) = 32\int_0^t e^{-2t}dt - 64\int_0^t te^{-2t}dt = -16e^{-2t}\Big|_0^t - \frac{64}{4}e^{-2t}(-2t-1)\Big|_0^t$$

$$v = \underline{32te^{-2t}} \text{ V}$$

Chapter 8, Solution 60.

At t = 0-, 4u(t) = 0 so that $i_1(0) = 0 = i_2(0)$ (1)

Applying nodal analysis,

$4 = 0.5di_1/dt + i_1 + i_2$ (2)

Also, $i_2 = [1di_1/dt - 1di_2/dt]/3$ or $3i_2 = di_1/dt - di_2/dt$ (3)

Taking the derivative of (2), $0 = d^2i_1/dt^2 + 2di_1/dt + 2di_2/dt$ (4)

From (2) and (3), $di_2/dt = di_1/dt - 3i_2 = di_1/dt - 3(4 - i_1 - 0.5di_1/dt)$

$= di_1/dt - 12 + 3i_1 + 1.5di_1/dt$

Substituting this into (4),

$d^2i_1/dt^2 + 7di_1/dt + 6i_1 = 24$ which gives $s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$

$$\text{Thus,}\quad i_1(t) = I_s + [Ae^{-t} + Be^{-6t}],\ 6I_s = 24 \text{ or } I_s = 4$$

$$i_1(t) = 4 + [Ae^{-t} + Be^{-6t}] \text{ and } i_1(0) = 4 + [A + B] \qquad (5)$$

$$i_2 = 4 - i_1 - 0.5di_1/dt = i_1(t) = 4 + -4 - [Ae^{-t} + Be^{-6t}] - [-Ae^{-t} - 6Be^{-6t}]$$

$$= [-0.5Ae^{-t} + 2Be^{-6t}] \text{ and } i_2(0) = 0 = -0.5A + 2B \qquad (6)$$

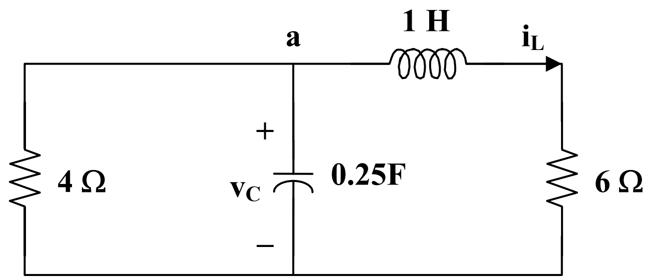
From (5) and (6), $A = -3.2$ and $B = -0.8$

$$i_1(t) = \underline{\mathbf{\{4 + [-3.2e^{-t} - 0.8e^{-6t}]\}\ A}}$$

$$i_2(t) = \underline{\mathbf{[1.6e^{-t} - 1.6e^{-6t}]\ A}}$$

Chapter 8, Solution 61.

For $t > 0$, we obtain the natural response by considering the circuit below.



At node a, $v_C/4 + 0.25dv_C/dt + i_L = 0$ (1)

But, $v_C = 1di_L/dt + 6i_L$ (2)

Combining (1) and (2),

$$(1/4)di_L/dt + (6/4)i_L + 0.25d^2i_L/dt^2 + (6/4)di_L/dt + i_L = 0$$

$$d^2i_L/dt^2 + 7di_L/dt + 10i_L = 0$$

$$s^2 + 7s + 10 = 0 = (s + 2)(s + 5) \text{ or } s_{1,2} = -2, -5$$

$$\text{Thus,}\quad i_L(t) = i_L(\infty) + [Ae^{-2t} + Be^{-5t}],$$

where $i_L(\infty)$ represents the final inductor current $= 4(4)/(4 + 6) = 1.6$

$$i_L(t) = 1.6 + [Ae^{-2t} + Be^{-5t}] \text{ and } i_L(0) = 1.6 + [A+B] \text{ or } -1.6 = A+B \qquad (3)$$

$$di_L/dt = [-2Ae^{-2t} - 5Be^{-5t}]$$

$$\text{and } di_L(0)/dt = 0 = -2A - 5B \text{ or } A = -2.5B \qquad (4)$$

From (3) and (4), $A = -8/3$ and $B = 16/15$

$$i_L(t) = 1.6 + [-(8/3)e^{-2t} + (16/15)e^{-5t}]$$

$$v(t) = 6i_L(t) = \underline{\mathbf{\{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}\ V}}$$

$$v_C = 1di_L/dt + 6i_L = [(16/3)e^{-2t} - (16/3)e^{-5t}] + \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}$$

$$v_C = \{9.6 + [-(32/3)e^{-2t} + 1.0667e^{-5t}]\}$$

$$i(t) = v_C/4 = \underline{\mathbf{\{2.4 + [-2.667e^{-2t} + 0.2667e^{-5t}]\}\ A}}$$

Chapter 8, Solution 62.

This is a parallel RLC circuit as evident when the voltage source is turned off.

$$\alpha = 1/(2RC) = (1)/(2x3x(1/18)) = 3$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x1/18} = 3$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -3$$

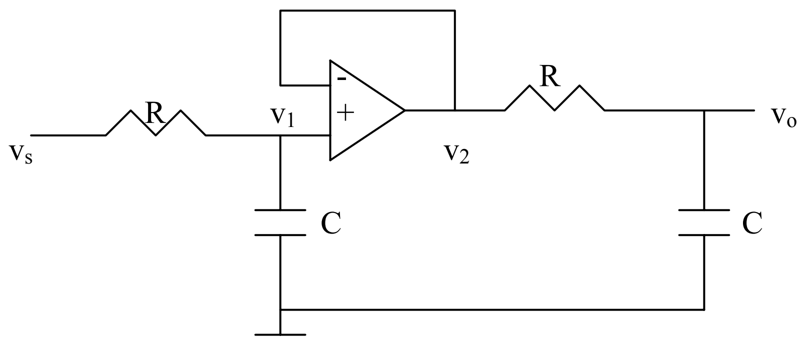
Let $v(t)$ = capacitor voltage

Thus, $v(t) = V_s + [(A + Bt)e^{-3t}]$ where $V_s = 0$

But $-10 + v_R + v = 0$ or $v_R = 10 - v$

Therefore $v_R = \underline{\mathbf{10 - [(A + Bt)e^{-3t}]}}$ where A and B are determined from initial conditions.

Chapter 8, Solution 63.



At node 1,

$$\frac{v_s - v_1}{R} = C\frac{dv_1}{dt} \qquad (1)$$

At node 2,

$$\frac{v_2 - v_o}{R} = C\frac{dv_o}{dt} \qquad (2)$$

As a voltage follower, $v_1 = v_2 = v$. Hence (2) becomes

$$v = v_o + RC\frac{dv_o}{dt} \qquad (3)$$

and (1) becomes

$$v_s = v + RC\frac{dv}{dt} \qquad (4)$$

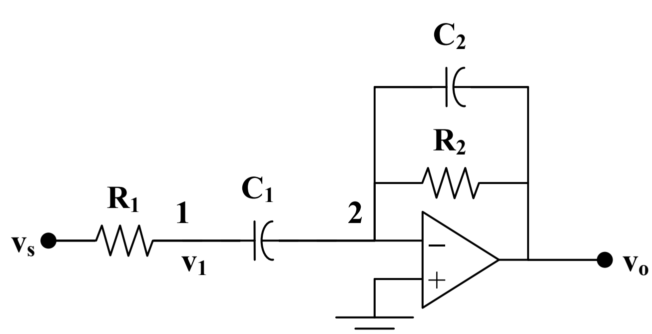
Substituting (3) into (4) gives

$$v_s = v_o + RC\frac{dv_o}{dt} + RC\frac{dv_o}{dt} + R^2C^2\frac{d^2v_o}{dt^2}$$

or

$$\underline{R^2C^2\frac{d^2v_o}{dt^2} + 2RC\frac{dv_o}{dt} + v_o = v_s}$$

Chapter 8, Solution 64.



At node 1, $(v_s - v_1)/R_1 = C_1\, d(v_1 - 0)/dt$ or $v_s = v_1 + R_1C_1dv_1/dt$ (1)

At node 2, $C_1dv_1/dt = (0 - v_o)/R_2 + C_2d(0 - v_o)/dt$

or $-R_2C_1dv_1/dt = v_o + C_2dv_o/dt$ (2)

From (1) and (2), $(v_s - v_1)/R_1 = C_1\, dv_1/dt = -(1/R_2)(v_o + C_2dv_o/dt)$

or $v_1 = v_s + (R_1/R_2)(v_o + C_2dv_o/dt)$ (3)

Substituting (3) into (1) produces,

$$v_s = v_s + (R_1/R_2)(v_o + C_2dv_o/dt) + R_1C_1d\{v_s + (R_1/R_2)(v_o + C_2dv_o/dt)\}/dt$$

$$= v_s + (R_1/R_2)(v_o) + (R_1C_2/R_2)\, dv_o/dt) + R_1C_1dv_s/dt + (R_1R_1C_1/R_2)dv_o/dt + (R_1^2\, C_1C_2/R_2)[d2vo/dt^2]$$

Simplifying we get,

$$\underline{\mathbf{d^2v_o/dt^2 + [(1/\, R_1C_1) + (1/\, C_2)]dv_o/dt + [1/(R_1C_1C_2)](v_o) = -\, [R_2/(R_1C_2)]dv_s/dt}}$$

Chapter 8, Solution 65.

At the input of the first op amp,

$$(v_o - 0)/R = Cd(v_1 - 0) \qquad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R = Cdv_2/dt \qquad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o = -v_2 \text{ or } v_2 = -v_o \qquad (3)$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2v_o}{dt^2} - \left(\frac{1}{R^2C^2}\right)v_o = \frac{d^2v_o}{dt^2} - 100v_o = 0$$

$$\text{Which leads to } s^2 - 100 = 0$$

Clearly this produces roots of –10 and +10.

And, we obtain,

$$v_o(t) = (Ae^{+10t} + Be^{-10t})V$$

$$\text{At } t = 0,\ v_o(0+) = -v_2(0+) = 0 = A + B,\ \text{thus } B = -A$$

This leads to $v_o(t) = (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) = 2V$.

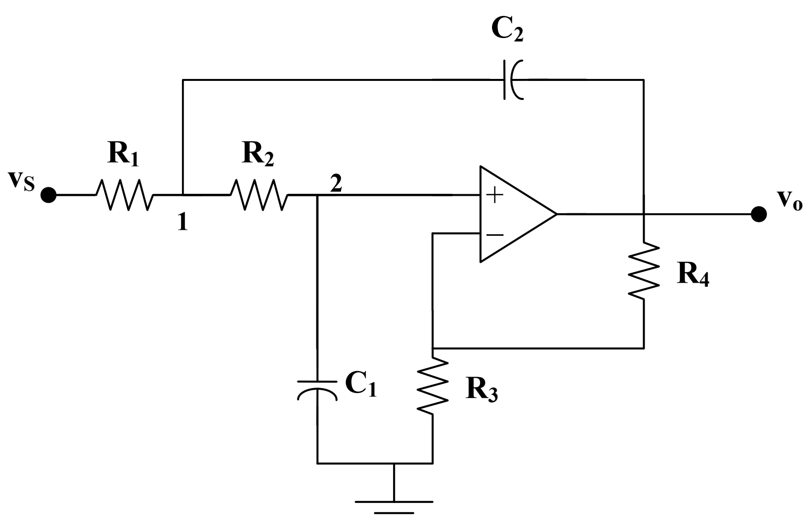
$$\text{From (2)},\ v_1 = -RCdv_2/dt = 0.1dv_o/dt = 0.1(10Ae^{+10t} + 10Ae^{-10t})$$

$$v_1(0+) = 2 = 0.1(20A) = 2A \text{ or } A = 1$$

$$\text{Thus, } v_o(t) = \underline{\mathbf{(e^{+10t} - e^{-10t})V}}$$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

Chapter 8, Solution 66.



Note that the voltage across C_1 is $v_2 = [R_3/(R3 + R_4)]v_o$

This is the only difference between this problem and Example 8.11, i.e. $v = kv$, where $k = [R_3/(R3 + R_4)]$.

At node 1,

$$(v_s - v_1)/R_1 = C_2[d(v_1 - v_o)/dt] + (v_1 - v_2)/R_2$$

$$v_s/R_1 = (v_1/R_1) + C_2[d(v_1)/dt] - C_2[d(v_o)/dt] + (v_1 - kv_o)/R_2 \qquad (1)$$

At node 2,

$$(v_1 - kv_o)/R_2 = C_1[d(kv_o)/dt]$$

or

$$v_1 = kv_o + kR_2C_1[d(v_o)/dt] \qquad (2)$$

Substituting (2) into (1),

$$v_s/R_1 = (kv_o/R_1) + (kR_2C_1/R_1)[d(v_o)/dt] + kC_2[d(v_o)/dt] + kR_2C_1C_2[d^2(v_o)/dt^2] - (kv_o/R_2) + kC_1[d(v_o)/dt] - (kv_o/R_2) + C_2[d(v_o)/dt]$$

We now rearrange the terms.

$$[d^2(v_o)/dt^2] + [(1/C_2R_1) + (1/ R_2C_2) + (1/R_2C_1) - (1/ kR_2C_1)][d(v_o)/dt] + [v_o/(R_1R_2C_1C_2)] = v_s/(kR_1R_2C_1C_2)$$

If $R_1 = R_2$ 10 kohms, $C_1 = C_2 = 100\ \mu F$, $R_3 = 20$ kohms, and $R_4 = 60$ kohms,

$$k = [R_3/(R3 + R_4)] = 1/3$$

$$R_1R_2C_1C_2 = 10^4 \text{x} 10^4 \text{x} 10^{-4} \text{x} 10^{-4} = 1$$

$$(1/C_2R_1) + (1/ R_2C_2) + (1/R_2C_1) - (1/ kR_2C_1) = 1 + 1 + 1 - 3 = 3 - 3 = 0$$

Hence, $[d^2(v_o)/dt^2] + v_o = 3v_s = 6,\ t > 0,$ and $s^2 + 1 = 0$, or $s_{1,2} = \pm j$

$$v_o(t) = V_s + [A\cos t + B \sin t],\ V_s = 6$$

$$v_o(0) = 0 = 6 + A \text{ or } A = -6$$

$$dv_o/dt = -A\sin t + B\cos t, \text{ but } dv_o(0)/dt = 0 = B$$

Hence, $v_o(t) =$ **6(1 – cost)u(t) volts**.

Chapter 8, Solution 67.

At node 1,

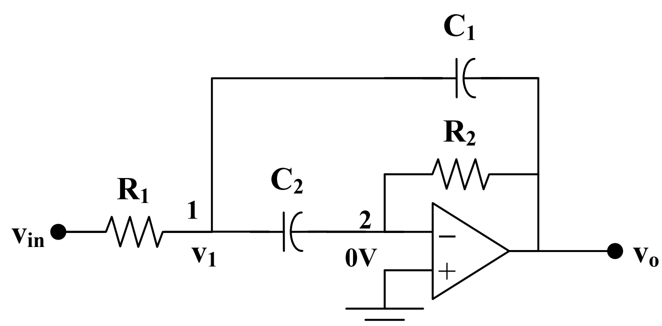
$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

At node 2, $$C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \text{ or } \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$



From (2) and (3),

$$-\frac{v_o}{C_2 R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{d^2 v_o}{dt^2} + \frac{R_1}{R_2}\frac{dv_o}{dt}$$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{C_1 C_2 R_2 R_1} = -\frac{1}{R_1 C_1}\frac{dv_{in}}{dt}$$

But $C_1C_2R_1R_2 = 10^{-4} \text{ x} 10^{-4} \text{ x} 10^{4} \text{ x} 10^{4} = 1$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2}{R_2 C_1} = \frac{2}{10^4 \text{x} 10^{-4}} = 2$$

$$\frac{d^2 v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s + 1)^2 = 0$ and $s = -1, -1$

Therefore, $v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B - Bt)e^{-t}] \qquad (4)$$

From (2), $$\frac{dv_o(0+)}{dt} = -\frac{v_o(0+)}{C_2R_2} = 0$$

From (1) at $t = 0+$,

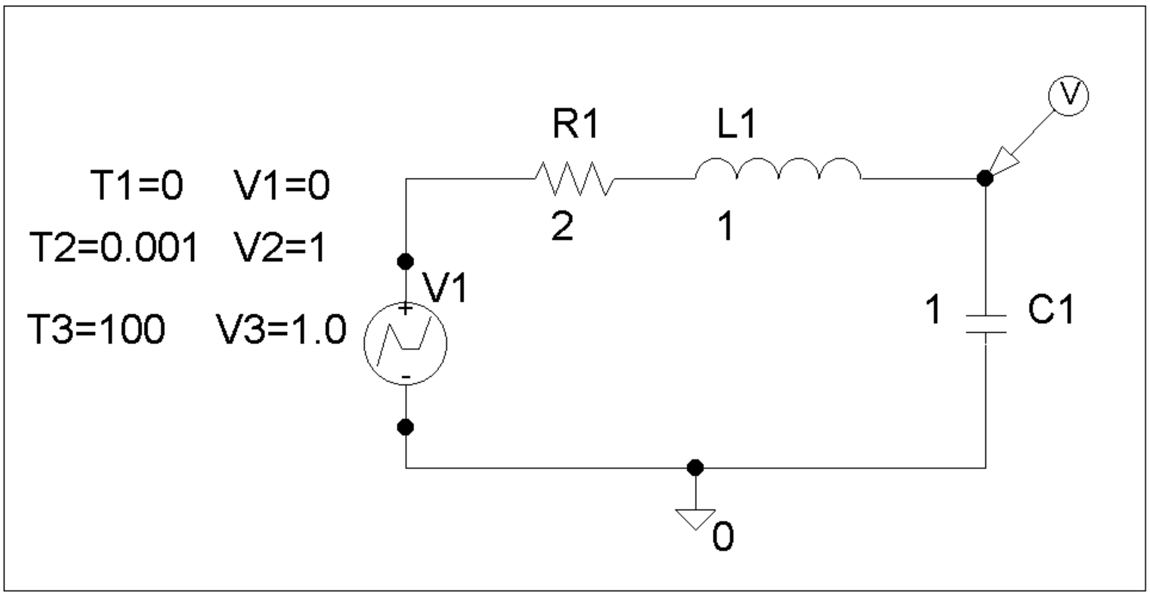
$$\frac{1-0}{R_1} = -C_1\frac{dv_o(0+)}{dt} \text{ which leads to } \frac{dv_o(0+)}{dt} = -\frac{1}{C_1R_1} = -1$$

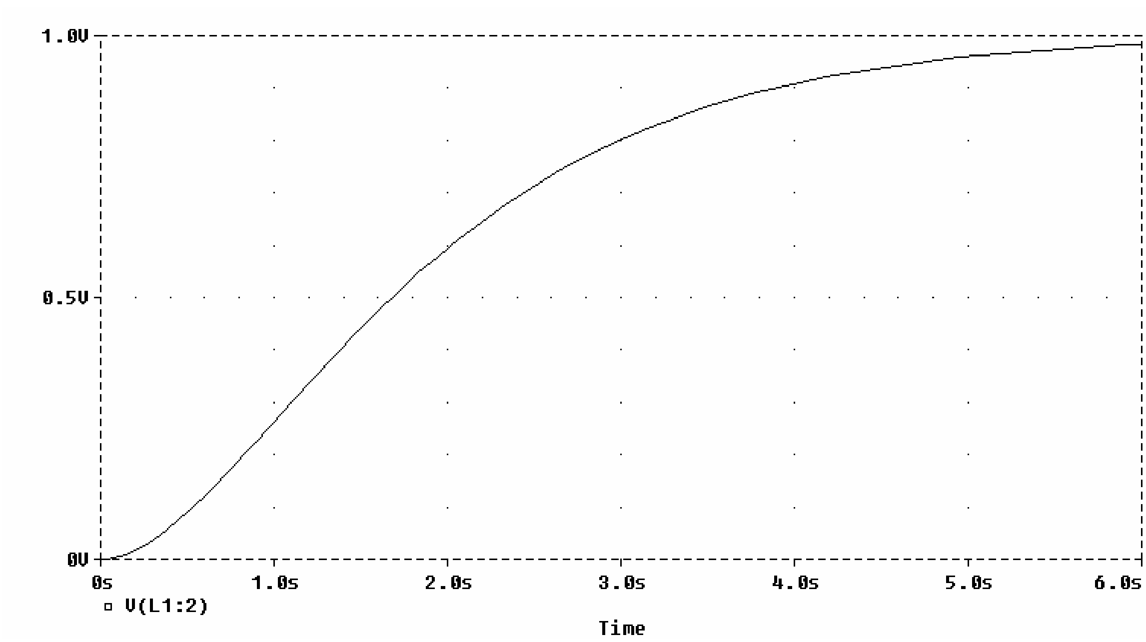
Substituting this into (4) gives $B = -1$

Thus, $v(t) = \underline{\mathbf{-te^{-t}u(t)\ V}}$

Chapter 8, Solution 68.

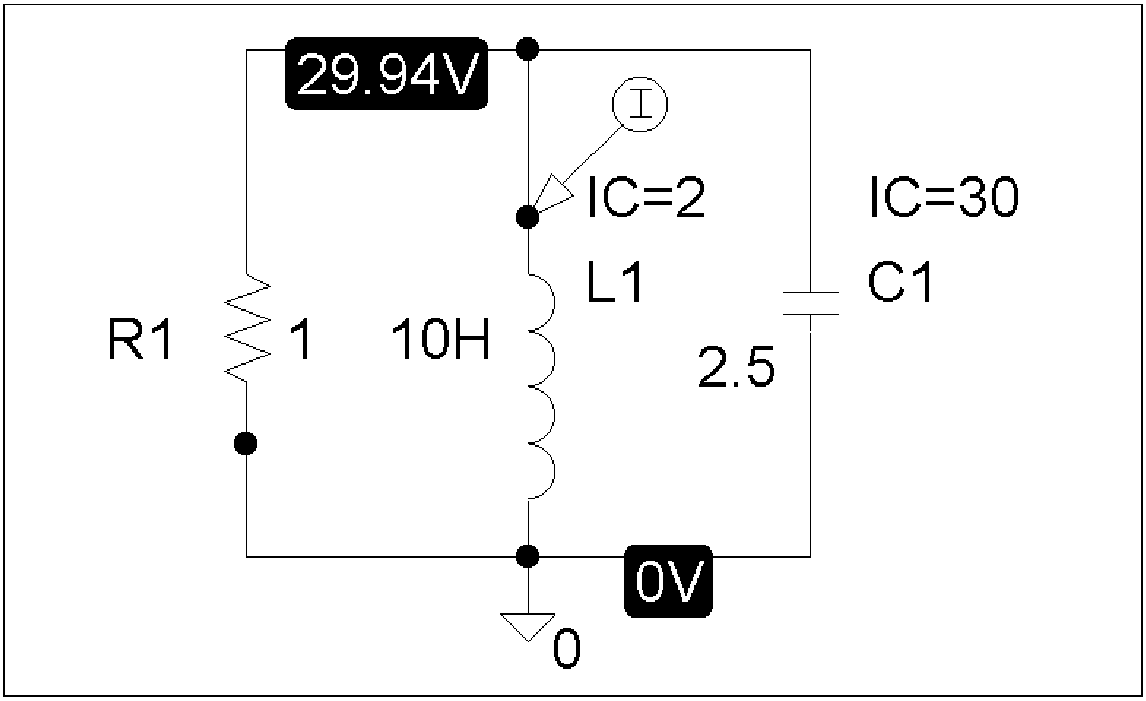
The schematic is as shown below. The unit step is modeled by VPWL as shown. We insert a voltage marker to display V after simulation. We set Print Step = 25 ms and final step = 6s in the transient box. The output plot is shown below.

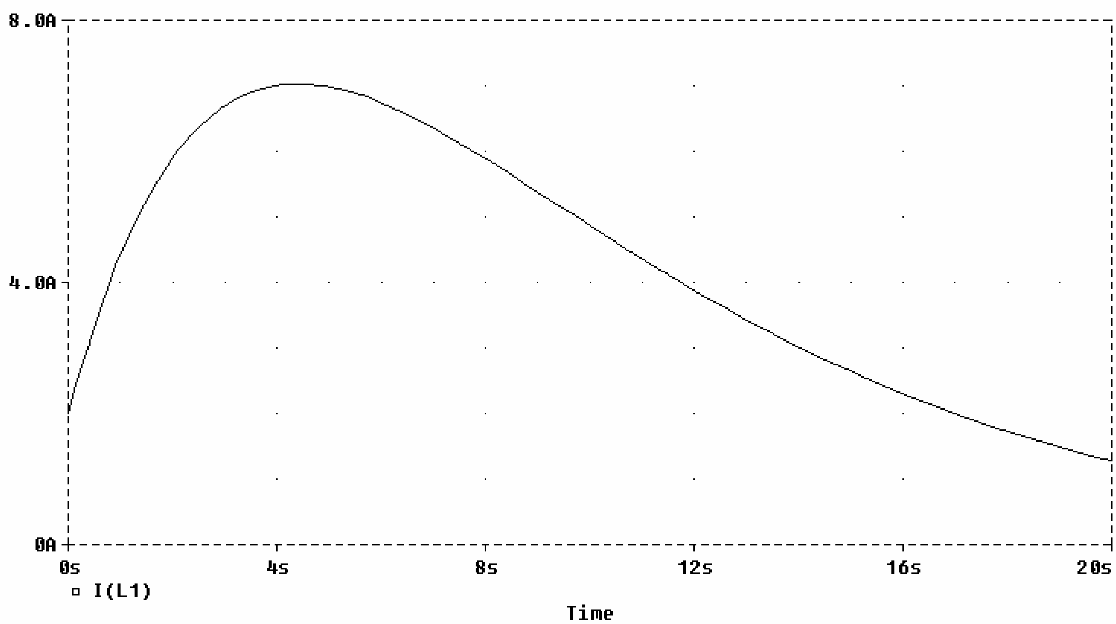




Chapter 8, Solution 69.

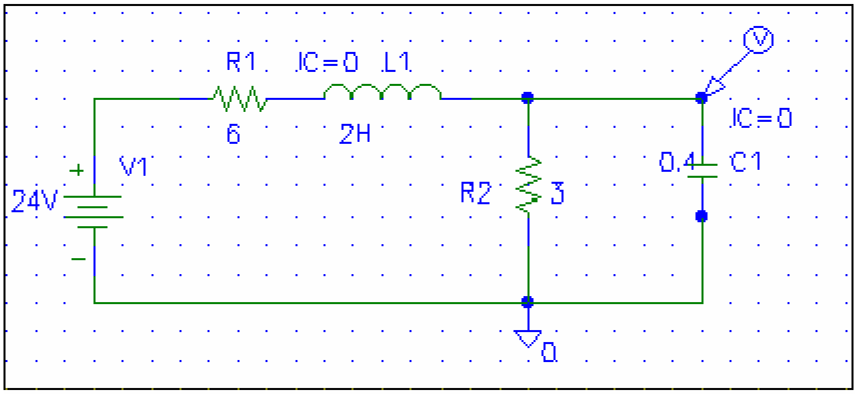
The schematic is shown below. The initial values are set as attributes of L1 and C1. We set Print Step to 25 ms and the Final Time to 20s in the transient box. A current marker is inserted at the terminal of L1 to automatically display i(t) after simulation. The result is shown below.



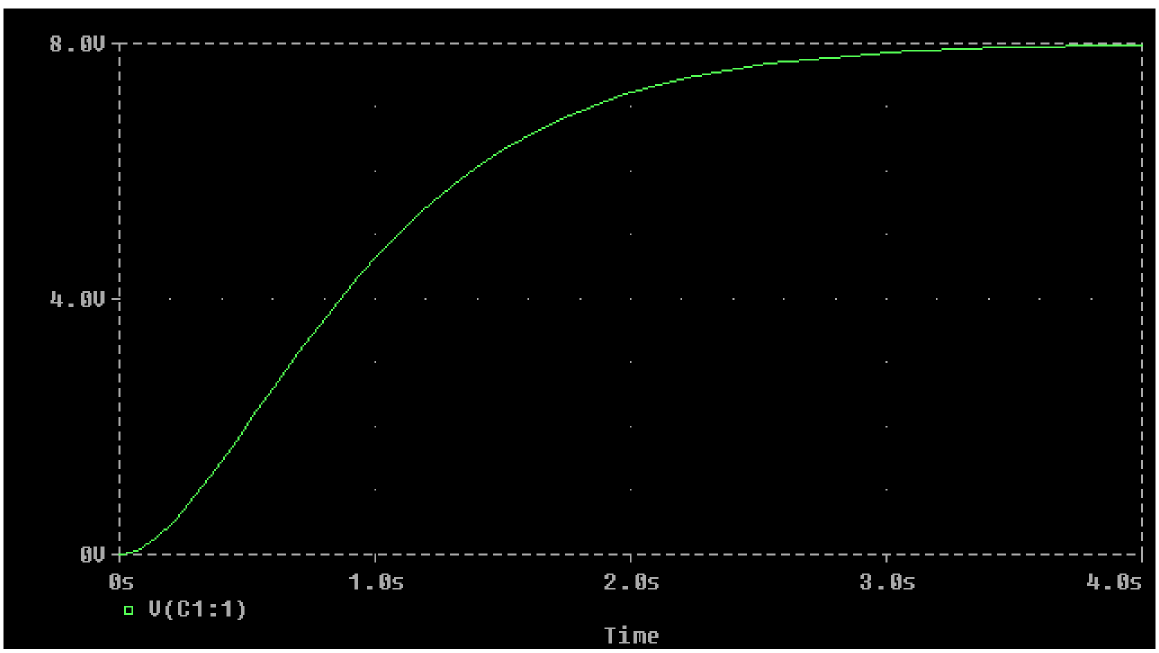


Chapter 8, Solution 70.

The schematic is shown below.

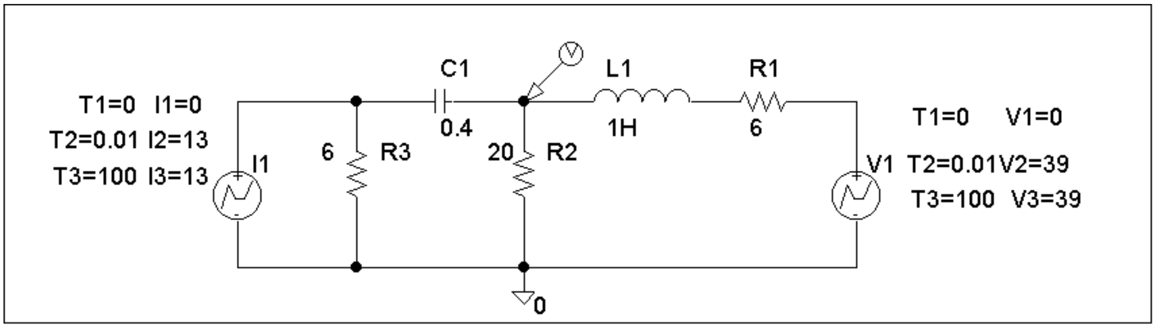


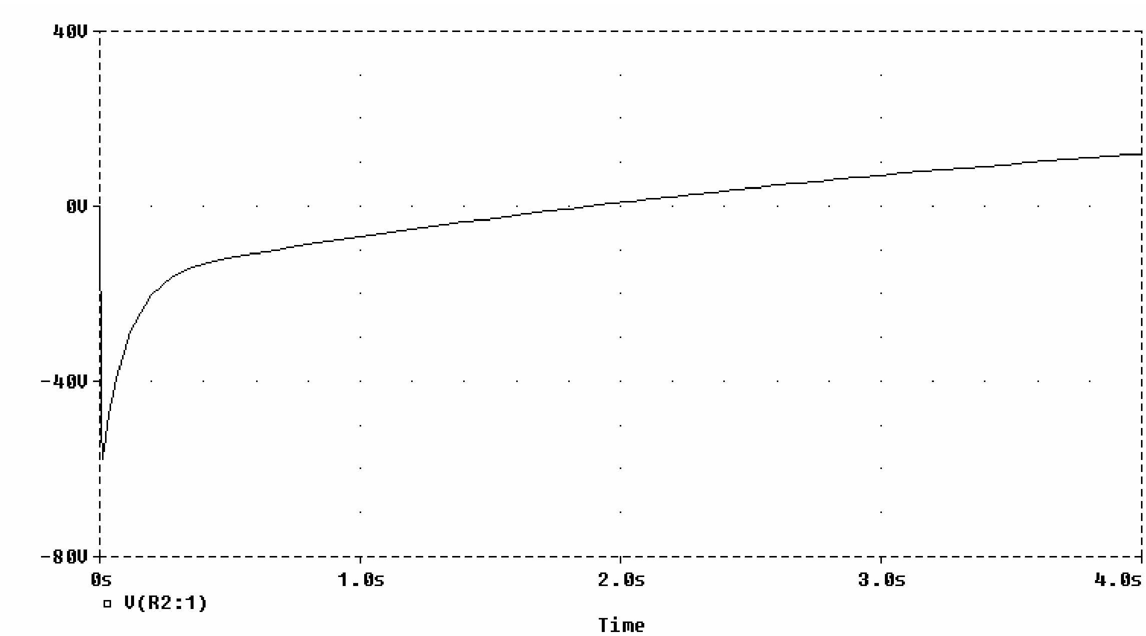
After the circuit is saved and simulated, we obtain the capacitor voltage v(t) as shown below.



Chapter 8, Solution 71.

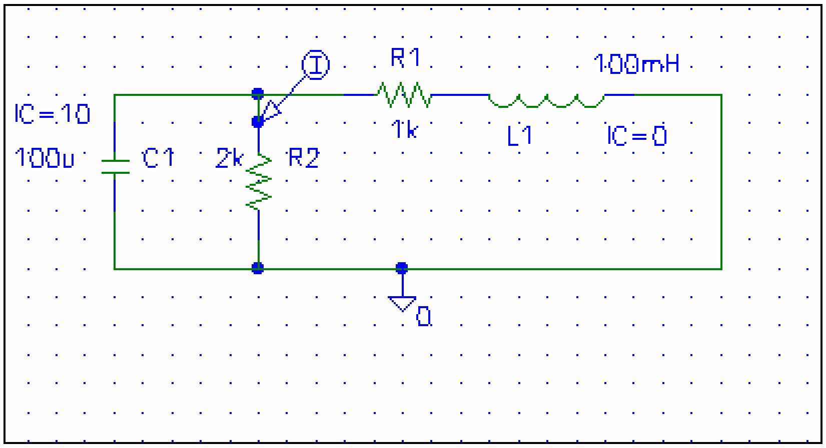
The schematic is shown below. We use VPWL and IPWL to model the 39 u(t) V and 13 u(t) A respectively. We set Print Step to 25 ms and Final Step to 4s in the Transient box. A voltage marker is inserted at the terminal of R2 to automatically produce the plot of v(t) after simulation. The result is shown below.



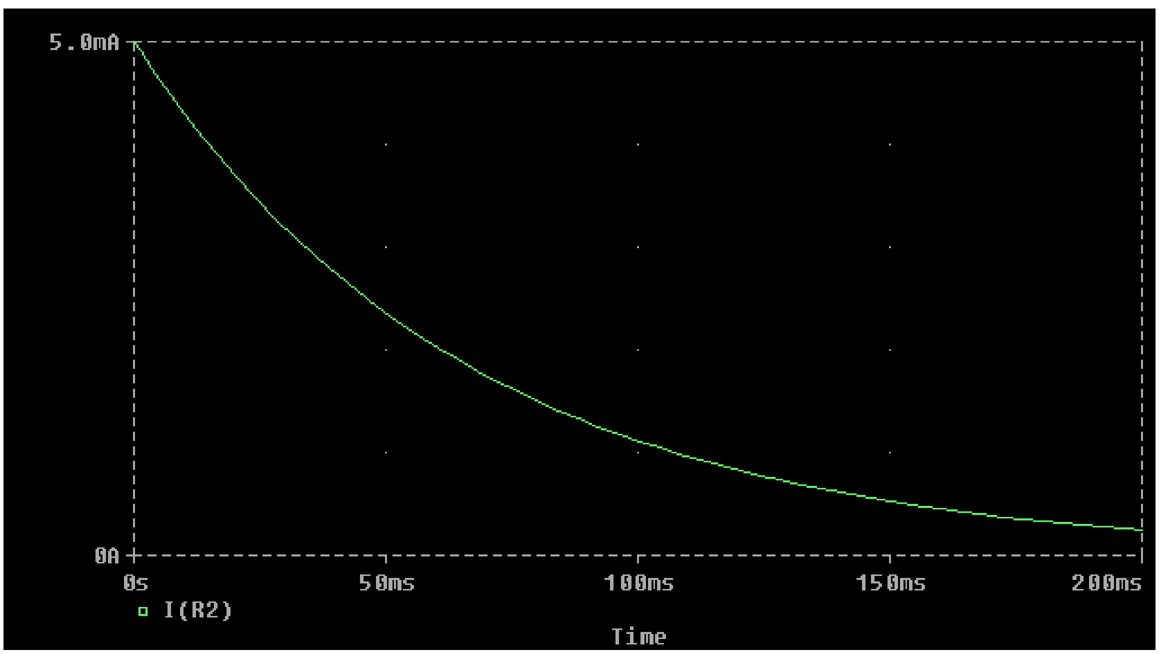


Chapter 8, Solution 72.

When the switch is in position 1, we obtain IC=10 for the capacitor and IC=0 for the inductor. When the switch is in position 2, the schematic of the circuit is shown below.



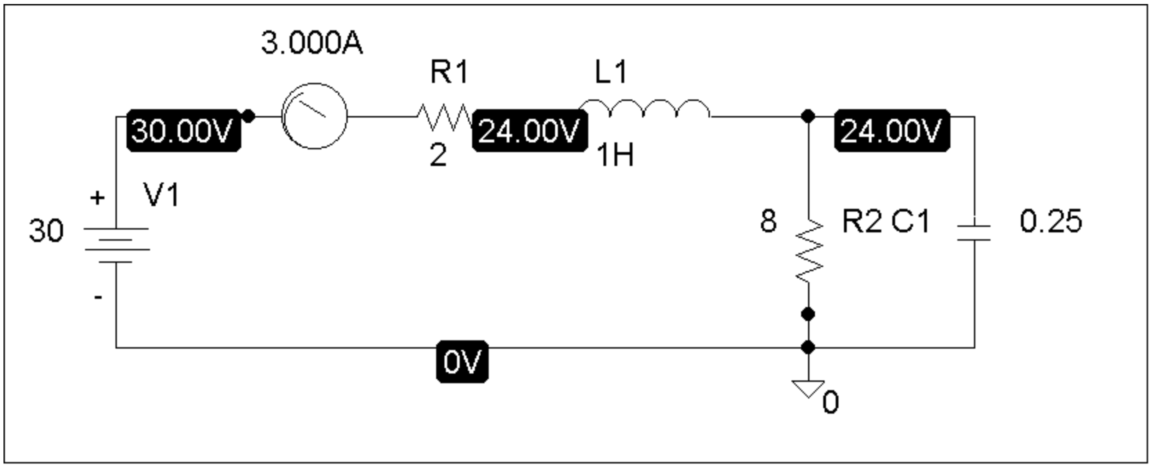
When the circuit is simulated, we obtain i(t) as shown below.



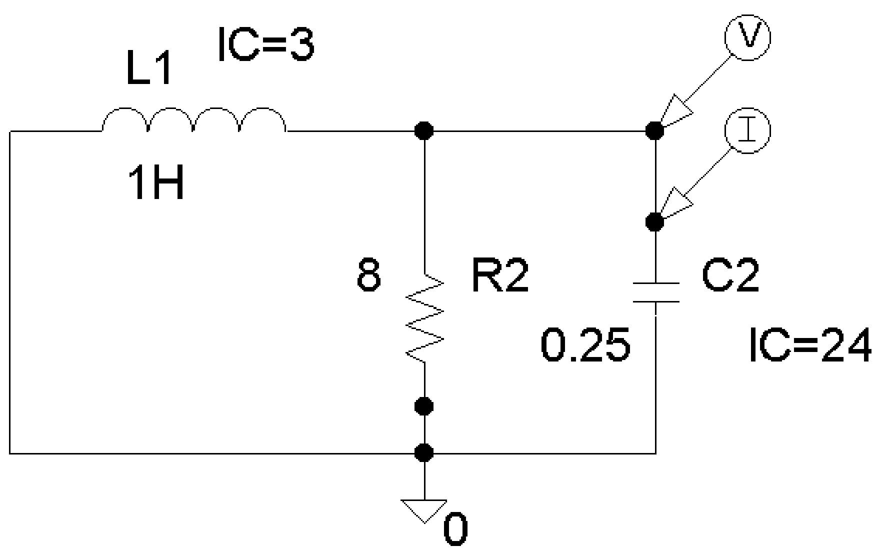
Chapter 8, Solution 73.

(a) For $t < 0$, we have the schematic below. When this is saved and simulated, we obtain the initial inductor current and capacitor voltage as

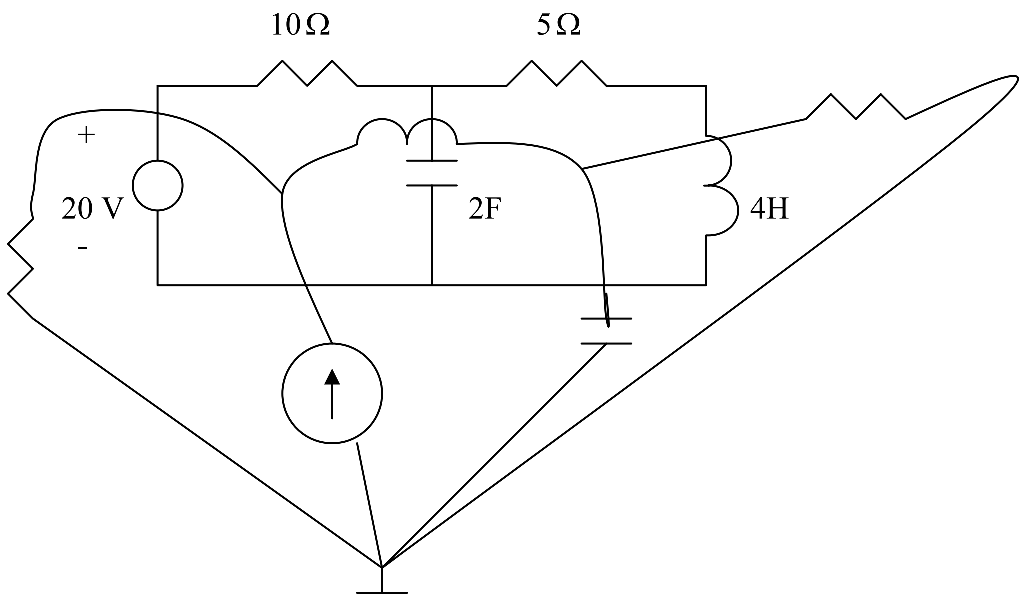
$$i_L(0) = 3 \text{ A} \quad \text{and } v_c(0) = 24 \text{ V}.$$



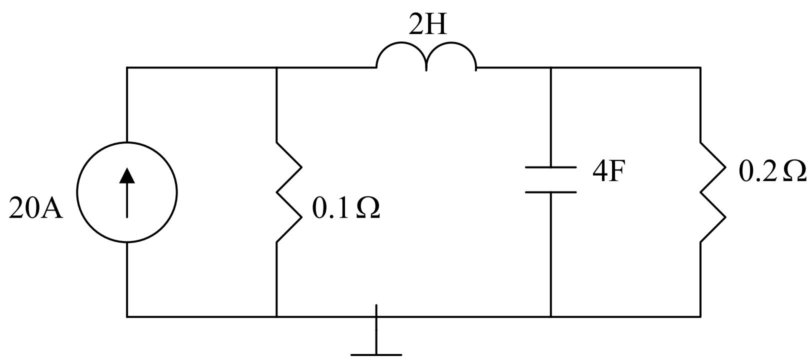
(b) For $t > 0$, we have the schematic shown below. To display i(t) and v(t), we insert current and voltage markers as shown. The initial inductor current and capacitor voltage are also incorporated. In the Transient box, we set Print Step = 25 ms and the Final Time to 4s. After simulation, we automatically have $i_o(t)$ and $v_o(t)$ displayed as shown below.



Chapter 8, Solution 74.

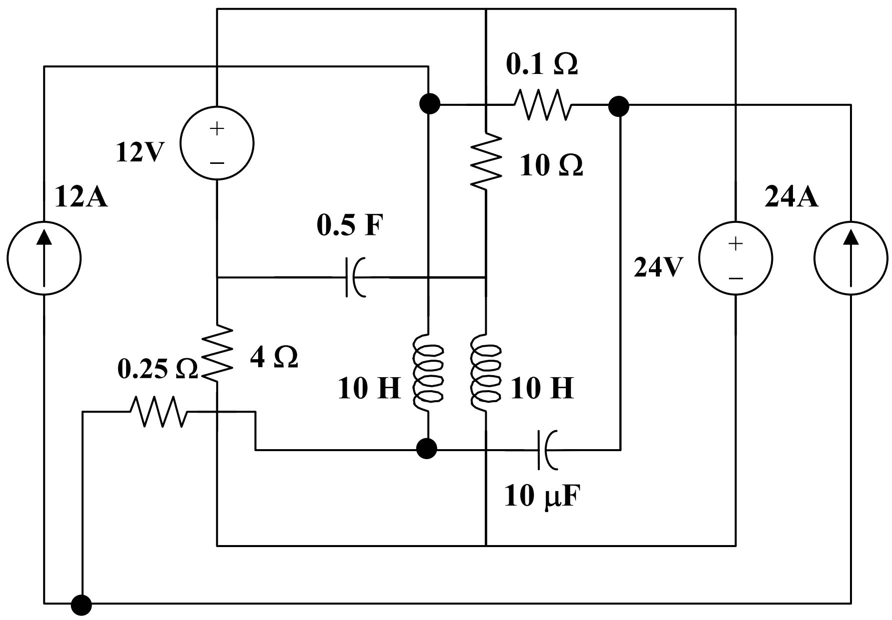


Hence the dual circuit is shown below.

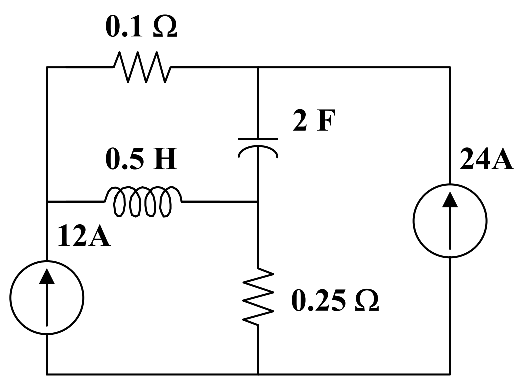


Chapter 8, Solution 75.

The dual circuit is connected as shown in Figure (a). It is redrawn in Figure (b).



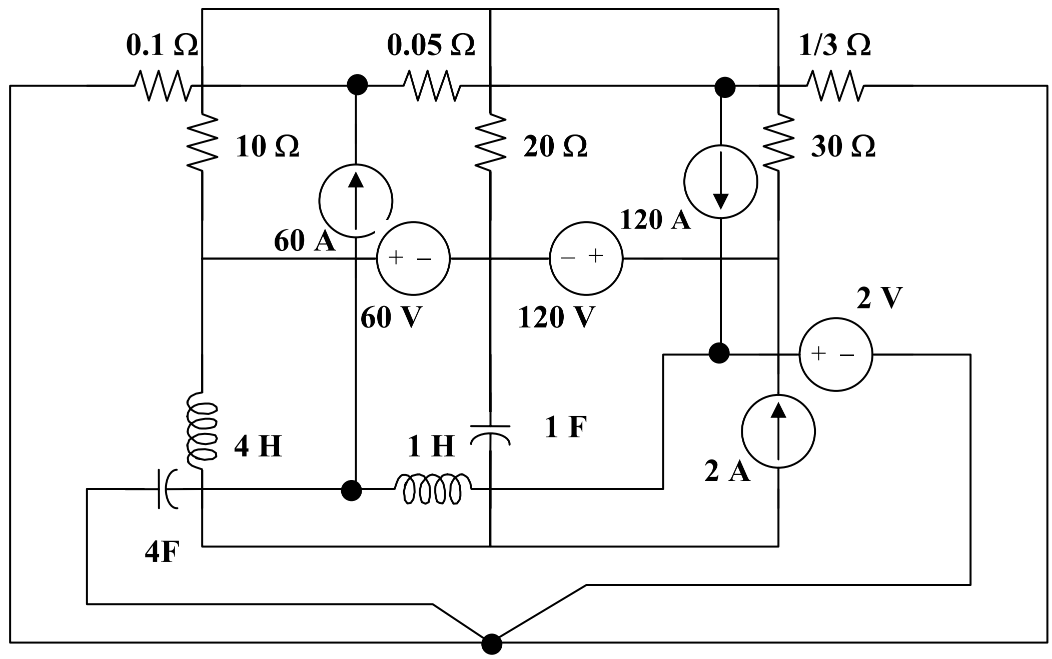
(a)



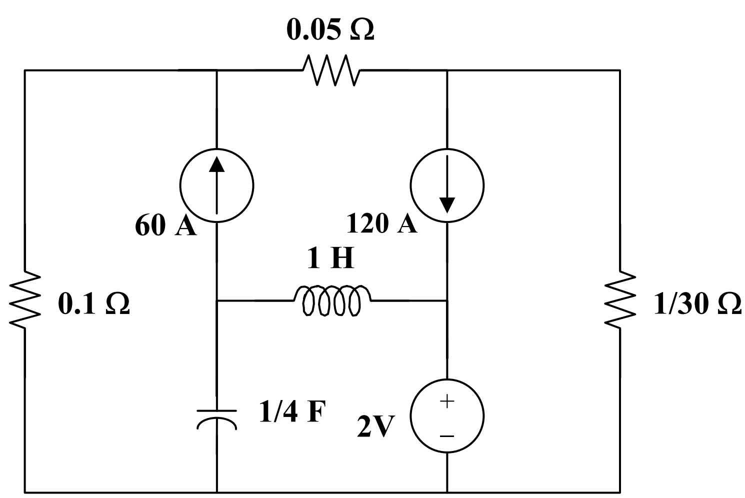
(b)

Chapter 8, Solution 76.

The dual is obtained from the original circuit as shown in Figure (a). It is redrawn in Figure (b).



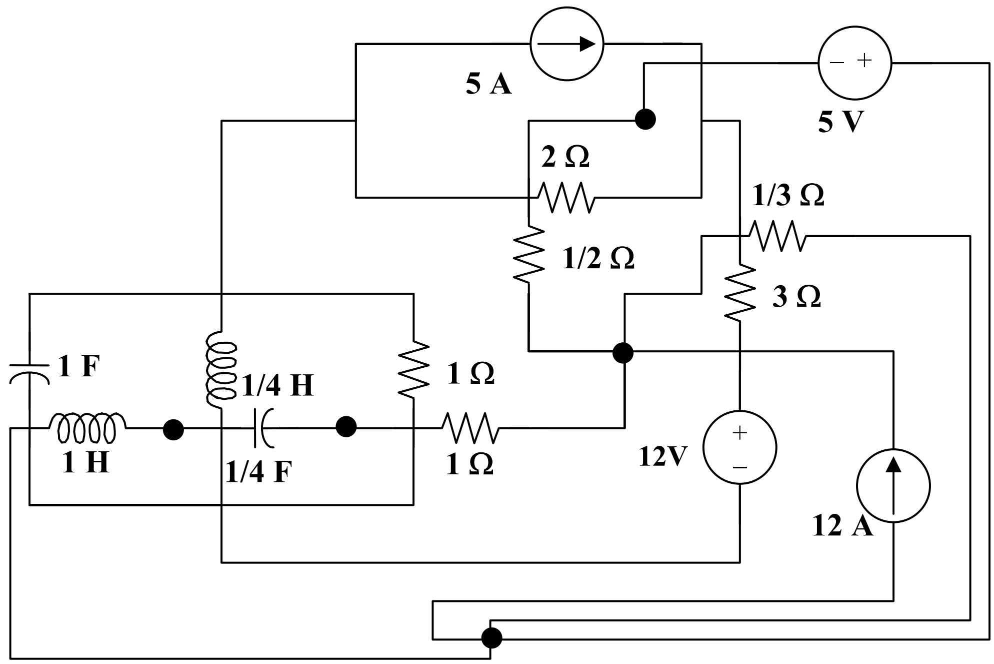
(a)



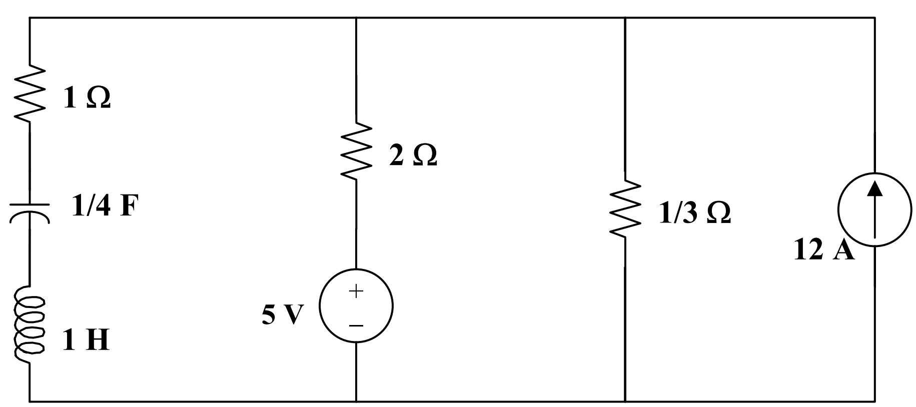
(b)

Chapter 8, Solution 77.

The dual is constructed in Figure (a) and redrawn in Figure (b).



(a)



(b)

Chapter 8, Solution 78.

The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \qquad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $\quad dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $\quad -120 = -5A + 21.794B$, leads to $B\ (5x12 - 120)/21.794 = -2.753$

At the peak value, $\quad dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $\quad 21.7945t_o = |-0.451|$

$t_o = |-0.451|/21.794 =$ **<u>20.68 ms</u>**

Chapter 8, Solution 79.

For critical damping of a parallel RLC circuit,

$$\alpha = \omega_o \quad \longrightarrow \quad \frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

Hence,

$$C = \frac{L}{4R^2} = \frac{0.25}{4x144} = \underline{434\mu\text{F}}$$

Chapter 8, Solution 80.

$$t_1 = 1/|s_1| = 0.1x10^{-3} \text{ leads to } s_1 = -1000/0.1 = -10{,}000$$

$$t_2 = 1/|s_2| = 0.5x10^{-3} \text{ leads to } s_1 = -2{,}000$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -2\alpha = -12{,}000, \text{ therefore } \alpha = 6{,}000 = R/(2L)$$

$$L = R/12{,}000 = 60{,}000/12{,}000 = \underline{\mathbf{5H}}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2{,}000$$

$$\alpha - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$6{,}000 - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$\sqrt{\alpha^2 - \omega_o^2} = 4{,}000$$

$$\alpha^2 - \omega_o^2 = 16x10^6$$

$$\omega_o^2 = \alpha^2 - 16x10^6 = 36x10^6 - 16x10^6$$

$$\omega_o = 10^3\sqrt{20} = 1/\sqrt{LC}$$

$$C = 1/(20x10^6x5) = \underline{\mathbf{10\ nF}}$$

Chapter 8, Solution 81.

$$t = 1/\alpha = 0.25 \text{ leads to } \alpha = 4$$

But, $\alpha \; 1/(2RC)$ or, $C = 1/(2\alpha R) = 1/(2x4x200) =$ **<u>625 μF</u>**

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

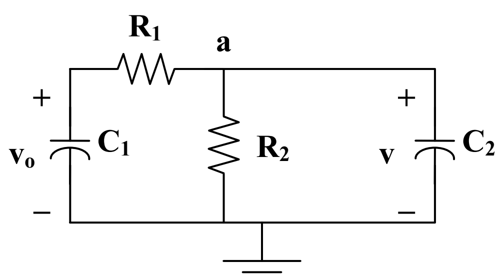
$$\omega_o^2 = \omega_d^2 + \alpha^2 = (2\pi 4x10^3)^2 + 16 \cong (2\pi 4x10^3 0^2 = 1/(LC)$$

This results in $L = 1/(64\pi^2 x10^6 x625x10^{-6}) =$ **<u>2.533 μH</u>**

Chapter 8, Solution 82.

For t = 0-, v(0) = 0.

For t > 0, the circuit is as shown below.



At node a,

$$(v_o - v/R_1 = (v/R_2) + C_2 dv/dt$$

$$v_o = v(1 + R_1/R_2) + R_1C_2 \, dv/dt$$

$$60 = (1 + 5/2.5) + (5x10^6 \, x5x10^{-6})dv/dt$$

$$60 = 3v + 25dv/dt$$

$$v(t) = V_s + [Ae^{-3t/25}]$$

where $3V_s = 60$ yields $V_s = 20$

$$v(0) = 0 = 20 + A \text{ or } A = -20$$

v(t) = **<u>$20(1 - e^{-3t/25})$V</u>**

Chapter 8, Solution 83.

$$i = i_D + C dv/dt \qquad (1)$$

$$-v_s + iR + L di/dt + v = 0 \qquad (2)$$

Substituting (1) into (2),

$$v_s = Ri_D + RC dv/dt + L di/dt + LC d^2v/dt^2 + v = 0$$

$$LC d^2v/dt^2 + RC dv/dt + Ri_D + L di/dt = v_s$$

$$\underline{\mathbf{d^2v/dt^2 + (R/L)dv/dt + (R/LC)i_D + (1/C)di/dt = v_s/LC}}$$