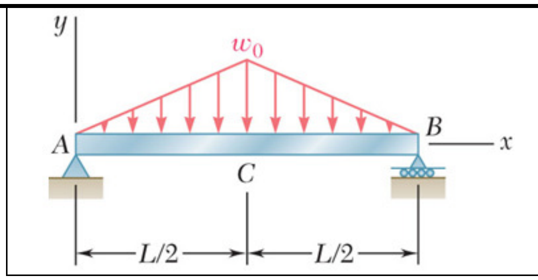
In the Name of GOD

Problem 7-1 For the prismatic beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at A, (c) the maximum deflection.



Answer:

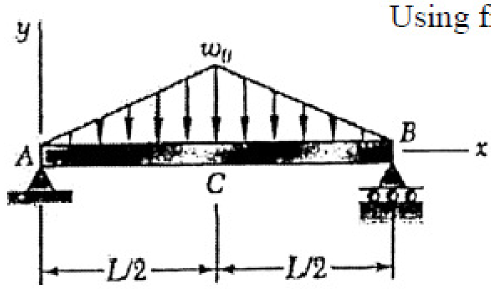
$$EIy = -\frac{\omega_0}{60L}x^5 + \frac{\omega_0}{30L}\left\langle x - \tfrac{1}{2}L\right\rangle^5 + \frac{\omega_0}{24L}x^3 - \frac{5}{192}\omega_0 L^3 x$$

$$\theta_A = -\frac{5\omega_0 L^3}{192EI} \quad , \quad y_{\max} = -\frac{\omega_0 L^4}{120EI}$$

SOLUTION

Use symmetry boundary conditions at C.

Using free body ACB and symmetry, $\quad R_A = R_B = \frac{1}{4}w_0 L$



For $0 < x < \frac{L}{2}$, $\quad w = \frac{2w_0 x}{L} \qquad \frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$

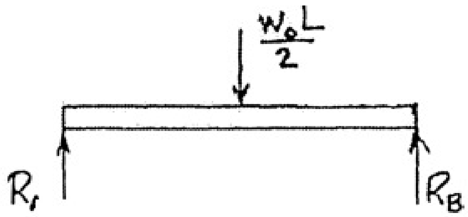
$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L}\left(\frac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right) + C_M$$

But $\quad M = 0 \quad$ at $\quad x = 0; \qquad$ hence $C_M = 0$

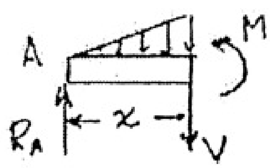
$[x = 0,\ y = 0] \quad [x = L,\ y = 0]$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$$



$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4\right) + C_1$$



$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$$

$$0 = \frac{w_0}{L}\left(\frac{1}{32}L^4 - \frac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\frac{5}{192}w_0 L^3$$

$$EIy = \frac{w_0}{L}\left(\frac{1}{24}L^2 x^3 - \frac{1}{120}x^5\right) - \frac{5}{192}w_0 L^3 x + C_2$$

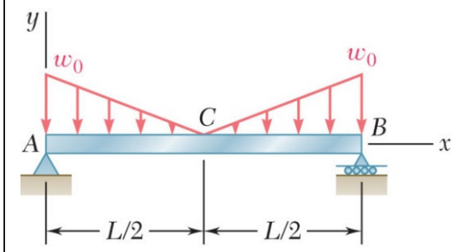
$[x = 0,\ y = 0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4 - \frac{5}{192}L^4\right) \qquad x = 0 \qquad \theta_A = -\frac{5\omega_0 L^3}{192EI} \ ◀$$

$$y = \frac{w_0}{EIL}\left(\frac{1}{24}L^2 x^3 - \frac{1}{60}x^5 - \frac{5}{192}L^4 x\right) \qquad x = \frac{L}{2} \qquad y_C = -\frac{\omega_0 L^4}{120EI} \ ◀$$

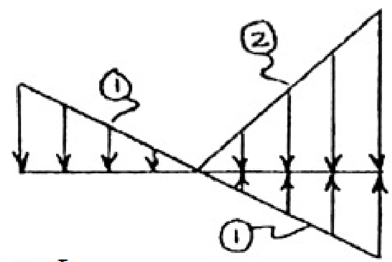
Problem 7-2 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the midpoint *C*. **Answer:** (*b*) $-\dfrac{3\omega_0 L^4}{640EI}$

(*a*) $y = \dfrac{\omega_0}{960EIL}\left\{16x^5 - 32\left\langle x - \tfrac{1}{2}L\right\rangle^5 - 40Lx^4 + 40L^2x^3 - 15L^4x\right\}$



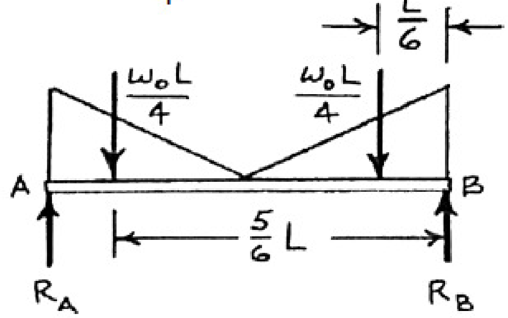
SOLUTION

Distributed loads: $k_1 = \dfrac{2w_0}{L} \qquad k_2 = \dfrac{4w_0}{L}$



(1) $w_1(x) = w_0 - k_1 x$ (2) $w_2(x) = k_2\left\langle x - \dfrac{L}{2}\right\rangle^1$

$$+\circlearrowleft \Sigma M_B = 0: \left(\frac{w_0 L}{4}\right)\left(\frac{5}{6}L\right) + \left(\frac{w_0 L}{4}\right)\left(\frac{L}{6}\right) + R_A L = 0 \qquad R_A = \frac{w_0 L}{4} \uparrow$$



$$w(x) = w_0 - k_1 x + k_2\left\langle x - \frac{L}{2}\right\rangle^1$$

$$= w_0 - \frac{2w_0}{L}x + \frac{4w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L}x - \frac{4w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{4} - w_0 x + \frac{w_0}{L}x^2 - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = \frac{1}{4}w_0 Lx - \frac{1}{2}w_0 x^2 + \frac{1}{3}\frac{w_0}{L}x^3 - \frac{2}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$EI\frac{dy}{dx} = \frac{1}{8}w_0 Lx^2 - \frac{1}{6}w_0 x^3 + \frac{1}{12}\frac{w_0}{L}x^4 - \frac{1}{6}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1$$

$$EIy = \frac{1}{24}w_0 Lx^3 - \frac{1}{24}w_0 x^4 + \frac{1}{60}\frac{w_0}{L}x^5 - \frac{1}{30}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $\quad C_2 = 0$

$[x = L,\ y = 0]$: $\quad \dfrac{1}{24}w_0 L^4 - \dfrac{1}{24}w_0 L^4 + \dfrac{1}{60}w_0 L^4 - \dfrac{1}{30}\dfrac{w_0}{L}\left(\dfrac{L}{2}\right)^5 + C_1 L = 0$

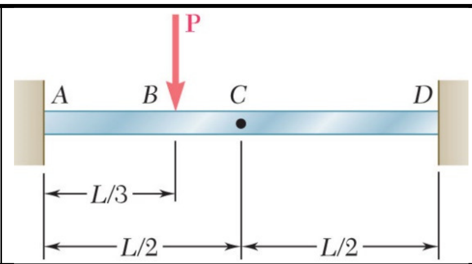
$$C_1 = -\frac{1}{64}w_0 L^3$$

$$y = w_0\left[16x^5 - 32\left\langle x - \frac{L}{2}\right\rangle^5 - 40Lx^4 + 40L^2x^3 - 15L^4x\right] \Big/ 960\,EIL \blacktriangleleft$$

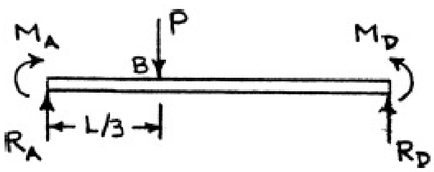
$$\left(y \text{ at } x = \frac{L}{2}\right) \quad y_C = \frac{w_0L^4}{960EI}\left(\frac{1}{2} - 0 - \frac{5}{2} + 5 - \frac{15}{2}\right) = -\frac{3w_0L^4}{640EI}$$

Problem 7-3 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at midpoint C.

Answer: $(a)\ \frac{20P}{27}\ ;\ \frac{4PL}{27} \qquad (b)\ -\frac{5PL^3}{1296EI}$



SOLUTION



$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_Ax - P\left\langle x - \frac{L}{3}\right\rangle^1 \qquad EI\frac{dy}{dx} = M_Ax + \frac{1}{2}R_Ax^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_Ax^2 + \frac{1}{6}R_Ax^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 + C_1x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad \therefore\ C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 - 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad M_AL + \frac{1}{2}R_AL^2 - \frac{1}{2}P\left(\frac{2L}{3}\right)^2 = 0 \qquad (1)$$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_AL^2 + \frac{1}{6}R_AL^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 = 0 \qquad (2)$$

(a) Solving Eqs. (1) and (2) simultaneously,

$$R_A = \frac{20}{27}P \qquad M_A = -\frac{4}{27}PL \qquad R_A = \frac{20}{27}P\uparrow \blacktriangleleft \qquad M_A = \frac{4}{27}PL \circlearrowright \blacktriangleleft$$

Elastic curve.

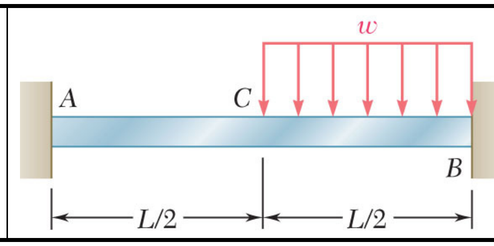
$$y = \frac{P}{EI}\left[-\frac{2}{27}Lx^2 + \frac{10}{81}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3\right]$$

(b) Deflection at midpoint C. $\left(y \text{ at } x = \frac{L}{2}\right)$

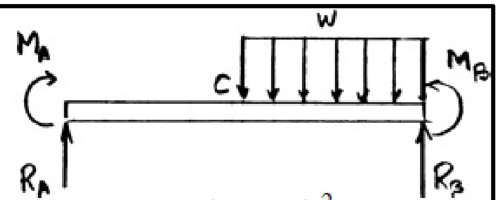
$$y_C = \frac{P}{EI}\left[-\frac{2}{27}L\left(\frac{L}{2}\right)^2 + \frac{10}{81}\left(\frac{L}{2}\right)^3 - \frac{1}{6}\left(\frac{L}{6}\right)^3\right] = -\frac{5PL^3}{1296EI} \qquad y_C = \frac{5PL^3}{1296EI}\downarrow \blacktriangleleft$$

Problem 7-4 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at midpoint C.

Answer: $(a)\ \frac{3\omega L}{32}\ ;\ \frac{5\omega L^2}{192} \qquad (b)\ -\frac{\omega L^4}{768EI}$



SOLUTION



$$w(x) = w\left\langle x - \frac{L}{2}\right\rangle^0 \qquad \frac{dV}{dx} = -w(x) = -w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\left\langle x - \frac{L}{2}\right\rangle^1 \qquad EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 \qquad \left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad C_1 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + C_1 x + C_2 \qquad [x = 0,\ y = 0] \qquad C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \qquad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 = 0 \qquad (1)$$

$$[x = L,\ y = 0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, $(a)\quad R_A = \frac{3wL}{32} \qquad M_A = -\frac{5wL^2}{192}$

$$R_A = \frac{3wL}{32}\uparrow \blacktriangleleft \qquad M_A = \frac{5wL^2}{192} \circlearrowright \blacktriangleleft$$

$$EIy = -\frac{5}{384}wL^2x^2 + \frac{3}{192}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4$$

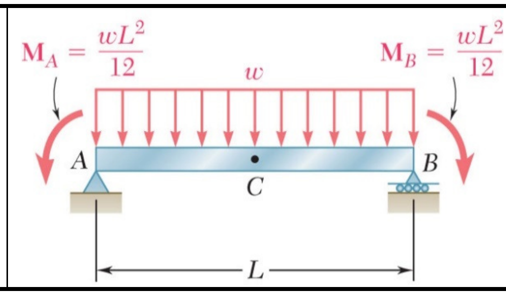
Elastic curve. $$y = \frac{w}{EI}\left\{-\frac{5}{384}L^2x^2 + \frac{3}{192}Lx^3 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4\right\}$$

(b) Deflection at midpoint C. $\left(y \text{ at } x = \frac{L}{2}\right)$

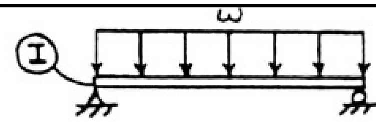
$$y_C = \frac{w}{EI}\left\{\left(-\frac{5}{384}L^2\right)\left(\frac{L}{2}\right)^2 + \left(\frac{3}{192}L\right)\left(\frac{L}{2}\right)^3 - 0\right\} = -\frac{wL^4}{768EI} \qquad y_B = \frac{wL^4}{768EI}\downarrow \blacktriangleleft$$

Problem 7-5 For the beam and loading shown, determine (a) the deflection at C, (b) the slope at end A.

Answer: $(a)\ \dfrac{\omega L^4}{384EI}\downarrow \quad (b)\ 0$.



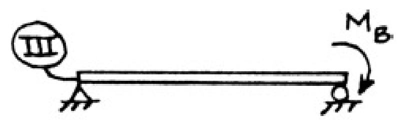
SOLUTION



Loading I: $\quad y_C = -\dfrac{5wL^4}{384EI} \qquad \theta_A = -\dfrac{wL^3}{24EI}$

Loading II: $\quad y_C = -\dfrac{M_A}{6EIL}\left[\left(\dfrac{L}{2}\right)^3 - L^2\left(\dfrac{L}{2}\right)\right] = \dfrac{1}{16}\dfrac{M_A L^2}{EI} \qquad \theta_A = \dfrac{M_A L}{3EI}$

with $\quad M_A = \dfrac{wL^2}{12} \qquad y_C = \dfrac{1}{192}\dfrac{wL^4}{EI} \qquad \theta_A = \dfrac{1}{36}\dfrac{wL^3}{EI}$



Loading III: $\quad y_C = \dfrac{1}{16}\dfrac{M_B L^3}{EI}$ (using Loading II result)

$\theta_A = \dfrac{M_B L}{6EI} \quad$ with $\quad M_B = \dfrac{wL^2}{12} \qquad y_C = \dfrac{1}{192}\dfrac{wL^4}{EI} \qquad \theta_A = \dfrac{1}{72}\dfrac{wL^3}{EI}$

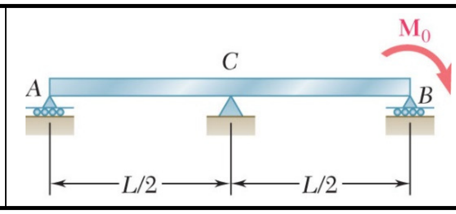
(a) Deflection at C. $\quad y_C = -\dfrac{5}{384}\dfrac{wL^4}{EI} + \dfrac{1}{192}\dfrac{wL^4}{EI} + \dfrac{1}{192}\dfrac{wL^4}{EI} = -\dfrac{1}{384}\dfrac{wL^4}{EI}$

$y_C = \dfrac{1}{384}\dfrac{wL^4}{EI}\downarrow$ ◀

(b) Slope at A. $\quad \theta_A = -\dfrac{1}{24}\dfrac{wL^3}{EI} + \dfrac{1}{36}\dfrac{wL^3}{EI} + \dfrac{1}{72}\dfrac{wL^3}{EI} = 0 \qquad \theta_A = 0$ ◀

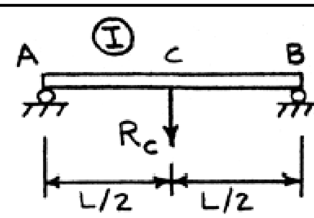
Problem 7-6 For the uniform beam shown, determine the reaction at each of the three supports.

Answer: $R_A = \dfrac{M_0}{2L}\uparrow\ ;\ R_B = \dfrac{5M_0}{2L}\uparrow\ ;\ R_C = \dfrac{3M_0}{L}\downarrow$.



SOLUTION

Consider R_C as redundant and replace loading system by I and II.

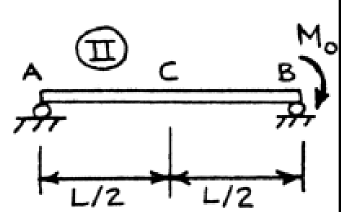


Loading I:

$y'_C = -\dfrac{R_C L^3}{48EI}$

Loading II:

$y''_C = -\dfrac{M_0}{6EIL}\left[\left(\dfrac{L}{2}\right)^3 - L^2\left(\dfrac{L}{2}\right)\right] = \dfrac{M_0 L^2}{16EI}$

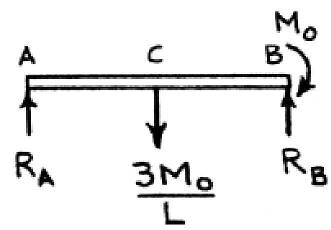


Superposition and constraint: $\quad y_C = y'_C + y''_C = 0$

$$-\frac{R_C L^3}{48EI}+\frac{M_0 L^2}{16EI}=0 \qquad R_C=\frac{3M_0}{L}\downarrow \blacktriangleleft$$

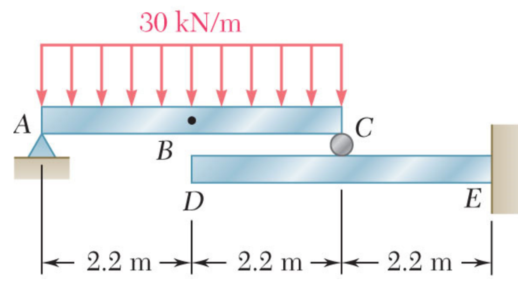
$$+\circlearrowleft \Sigma M_B=0:\quad -R_A L+\left(\frac{3M_0}{L}\right)\left(\frac{L}{2}\right)-M_0=0 \qquad R_A=\frac{M_0}{2L}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_Y=0:\quad \frac{M_0}{2L}-\frac{3M_0}{L}+R_B=0 \qquad R_B=\frac{5M_0}{2L}\uparrow \blacktriangleleft$$

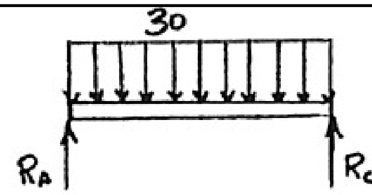


Problem 7-7 Beam *AC* rests on the cantilever beam *DE* as shown. Determine for the loading shown (a) the deflection at point *B*, (b) the deflection at point *D*. Use $E = 200$ GPa and $I = 125\times10^{-6}\ \text{m}^4$.

Answer: (*a*) 10.54 mm↓ . (*b*) 23.4 mm↓ .



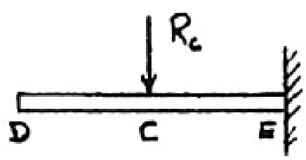
SOLUTION



Units: Forces in kN; lengths in m.

Using free body *ABC*,

$$+\circlearrowleft \Sigma M_A = 0:\ 4.4R_C-(4.4)(30)(2.2)=0 \qquad R_C = 66.0\ \text{kN}$$



$$E = 200\times10^9\ \text{Pa} \qquad I = 125\times10^6\ \text{mm}^4 = 125\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^9)(125\times10^{-6}) = 25.0\times10^{-6}\ \text{N}\cdot\text{m}^2 = 25{,}000\ \text{kN}\cdot\text{m}^2$$

For slope and defection at *C*, portion *CE* of beam *DCE*.

$$\theta_C = \frac{R_C L^2}{2EI} = \frac{(66.0)(2.2)^2}{(2)(25{,}000)} = 6.3888\times10^{-3}\ \text{rad}$$

$$y_C = -\frac{R_C L^3}{3EI} = \frac{(66.0)(2.2)^3}{(3)(25{,}000)} = -9.3702\times10^{-3}\ \text{m}$$

Defection at *B*, assuming that point *C* does not move.

$$(y_B)_1 = -\frac{5WL^4}{384EI} = -\frac{(5)(30)(4.4)^4}{(384)(25{,}000)} = -5.8564\times10^{-3}$$

Additional defection at *B* due to movement of point *C*: $(y_B)_2 = \frac{1}{2}y_C = -4.6851\times10^{-3}\ \text{m}$

(a) Total deflection at B. $y_B = (y_B)_1 + (y_B)_2 = -10.54\times10^{-3}\text{ m}$ $\quad y_B = 10.54\text{ mm}\downarrow$ ◀

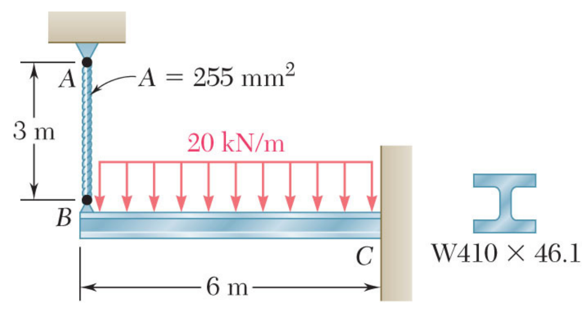
Portion DC of beam DCB remains straight.

(b) Deflection at D. $y_D = y_C - a\theta_C = -9.3702\times10^{-3} - (2.2)(6.3888\times10^{-3})$

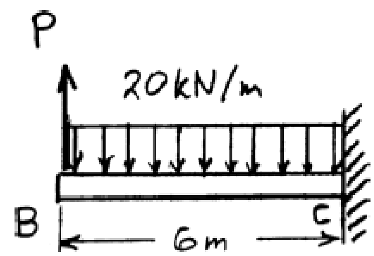
$= -23.4\times10^{-3}\text{ m}$ $\quad y_D = 23.4\text{ mm}\downarrow$ ◀

Problem 7-8 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa. (Help: $I = 156\times10^{-6}\text{ m}^4$).

Answer: 43.9 kN.



SOLUTION



Let P be the tension developed in member AB and δ_B be the elongation of that member.

Cable AB: $\quad A = 255\text{ mm}^2 = 255\times10^{-6}\text{ m}^2$

$$\delta_B = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^9)(255\times10^{-6})} = 58.82\times10^{-9}\text{P}$$

Beam BC: $\quad I = 156\times10^6\text{ mm}^4 = 156\times10^{-6}\text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\text{N}\cdot\text{m}^2$$

Loading I: 20 kN/m downward. $\quad (y_B)_1 = -\dfrac{wL^4}{8EI} = -\dfrac{(20\times10^3)(6)^4}{(8)(31.2\times10^6)} = -103.846\times10^{-3}\text{ m}$

Loading II: Upward force P at Point B. $\quad (y_B)_2 = \dfrac{PL^3}{3EI} = \dfrac{P(6)^3}{(3)(31.2\times10^6)} = 2.3077\times10^{-6}P$

By superposition, $\quad y_B = (y_B)_1 + (y_B)_2$

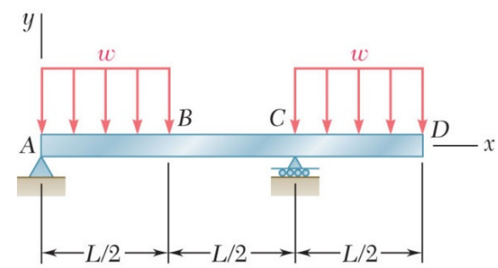
Also, matching the deflection at B, $\quad y_B = -\delta_B$

$-103.846\times10^{-3} + 2.3077\times10^{-6}P = -58.82\times10^{-9}P \qquad 2.3666\times10^{-6}P = 103.846\times10^{-3}$

$P = 43.9\times10^3\text{ N} \qquad P = 43.9\text{ kN}$ ◀

Problem 7-9 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point D.

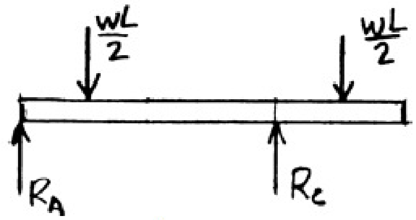
Answer: $(b)\ \dfrac{\omega L^4}{768EI}$ $(c)\ -\dfrac{5\omega L^4}{256EI}$



$$(a)\ y=\frac{\omega}{24EI}\left\{-x^4+\left\langle x-\tfrac{1}{2}L\right\rangle^4-\left\langle x-L\right\rangle^4+Lx^3+3L\left\langle x-L\right\rangle^3-\tfrac{1}{16}L^3x\right\}$$

SOLUTION

Use free body $ABCD$ with the distributed loads replaced by equivalent concentrated loads.



$$+\circlearrowleft \Sigma M_C = 0:\ -R_AL+\left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right)-\left(\frac{wL}{2}\right)\left(\frac{L}{4}\right)=0 \qquad R_A=\frac{1}{4}wL$$

$$+\circlearrowleft \Sigma M_A = 0:\ R_CL-\left(\frac{wL}{2}\right)\left(\frac{L}{4}\right)-\left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right)=0 \qquad R_C=\frac{3}{4}wL$$

$$\frac{dV}{dx}=-w=-w+w\left\langle x-\frac{L}{2}\right\rangle^0-w\langle x-L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx}=V=-wx+w\left\langle x-\frac{L}{2}\right\rangle^1-w\langle x-L\rangle^1+R_A+R_C\langle x-L\rangle^0$$

$$EI\frac{d^2y}{dx^2}=M=-\frac{1}{2}wx^2+\frac{1}{2}w\left\langle x-\frac{L}{2}\right\rangle^2-\frac{1}{2}w\langle x-L\rangle^2+R_Ax+R_C\langle x-L\rangle^1$$

$$EI\frac{dy}{dx}=-\frac{1}{6}wx^3+\frac{1}{6}w\left\langle x-\frac{L}{2}\right\rangle^3-\frac{1}{6}w\langle x-L\rangle^3+\frac{1}{2}R_Ax^2+\frac{1}{2}R_C\langle x-L\rangle^2+C_1$$

$$EIy=-\frac{1}{24}wx^4+\frac{1}{24}w\left\langle x-\frac{L}{2}\right\rangle^4-\frac{1}{24}w\langle x-L\rangle^4+\frac{1}{6}R_Ax^3+\frac{1}{6}R_C\langle x-L\rangle^3+C_1x+C_2$$

$$[x=0,\ y=0]\quad -0+0-0+0+0+0+C_2=0 \qquad C_2=0$$

$$[x=L,\ y=0]\quad -\frac{1}{24}wL^4+\frac{1}{24}w\left(\frac{L}{2}\right)^4-0+\frac{1}{6}\left(\frac{wL}{4}\right)L^3+0+C_1L+0=0$$

$$C_1=-\frac{1}{384}wL^3 \qquad EIy=-\frac{1}{24}wx^4+\frac{1}{24}w\left\langle x-\frac{L}{2}\right\rangle^4+\frac{1}{24}w\langle x-L\rangle^4$$

$$+\frac{1}{6}\left(\frac{wL}{4}\right)x^3+\frac{1}{6}\left(\frac{3wL}{4}\right)\langle x-L\rangle^3-\frac{1}{384}wL^3x$$

(a) $$y = \frac{w}{24EI}\left\{-x^4 + \left\langle x - \frac{L}{2}\right\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \frac{1}{16}L^3 x\right\} \blacktriangleleft$$

(b) $\left(y \text{ at } x = \frac{L}{2}\right)$ $$y_B = \frac{w}{24EI}\left\{-\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right)\right\}$$

$$y_B = \frac{wL^4}{768EI} \uparrow \blacktriangleleft$$

(c) $\left(y \text{ at } x = \frac{3L}{2}\right)$

$$y_D = \frac{w}{24EI}\left\{-\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right)\right\} = -\frac{5wL^4}{256EI}$$

$$y_D = \frac{5wL^4}{256EI} \downarrow \blacktriangleleft$$