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Vector Calculus:

Math with Vectors

EE3321

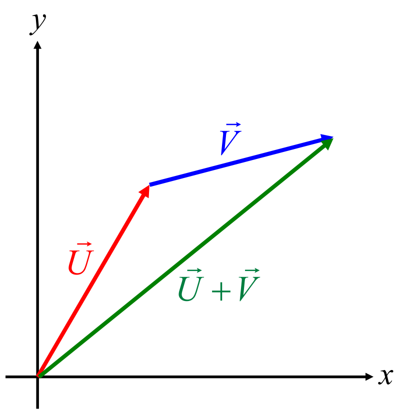
Electromagnetic Field Theory

Outline

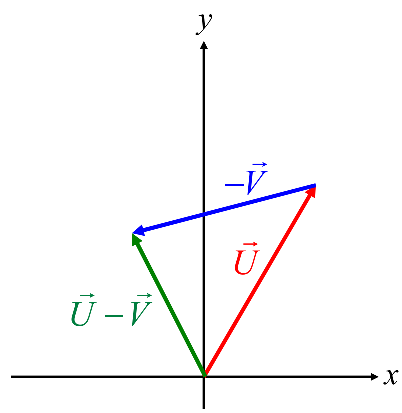
- Vector Addition and Subtraction
- Dot Product
- Projections
- Cross Product
- Area
- Vector Algebra Rules
- Triple Products

Visualization of Vector Addition & Subtraction

Vector Addition



Vector Subtraction



Vector Addition & Subtraction

Cartesian

Starting Vectors

$$\vec{U} = U_x\hat{a}_x + U_y\hat{a}_y + U_z\hat{a}_z$$
$$\vec{V} = V_x\hat{a}_x + V_y\hat{a}_y + V_z\hat{a}_z$$

Addition

$$\vec{U}+\vec{V} = (U_x+V_x)\hat{a}_x + (U_y+V_y)\hat{a}_y + (U_z+V_z)\hat{a}_z$$

Subtraction

$$\vec{U}-\vec{V} = (U_x-V_x)\hat{a}_x + (U_y-V_y)\hat{a}_y + (U_z-V_z)\hat{a}_z$$

Cylindrical

Starting Vectors

$$\vec{U} = U_\rho\hat{a}_\rho + U_\phi\hat{a}_\phi + U_z\hat{a}_z$$
$$\vec{V} = V_\rho\hat{a}_\rho + V_\phi\hat{a}_\phi + V_z\hat{a}_z$$

Addition

$$\vec{U}+\vec{V} = (U_\rho+V_\rho)\hat{a}_\rho + (U_\phi+V_\phi)\hat{a}_\phi + (U_z+V_z)\hat{a}_z$$

Subtraction

$$\vec{U}-\vec{V} = (U_\rho-V_\rho)\hat{a}_\rho + (U_\phi-V_\phi)\hat{a}_\phi + (U_z-V_z)\hat{a}_z$$

Spherical

Starting Vectors

$$\vec{U} = U_r\hat{a}_r + U_\theta\hat{a}_\theta + U_\phi\hat{a}_\phi$$
$$\vec{V} = V_r\hat{a}_r + V_\theta\hat{a}_\theta + V_\phi\hat{a}_\phi$$

Addition

$$\vec{U}+\vec{V} = (U_r+V_r)\hat{a}_r + (U_\theta+V_\theta)\hat{a}_\theta + (U_\phi+V_\phi)\hat{a}_\phi$$

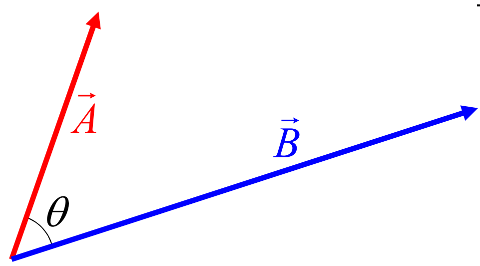
Subtraction

$$\vec{U}-\vec{V} = (U_r-V_r)\hat{a}_r + (U_\theta-V_\theta)\hat{a}_\theta + (U_\phi-V_\phi)\hat{a}_\phi$$

The Dot Product, $\vec{A} \bullet \vec{B}$

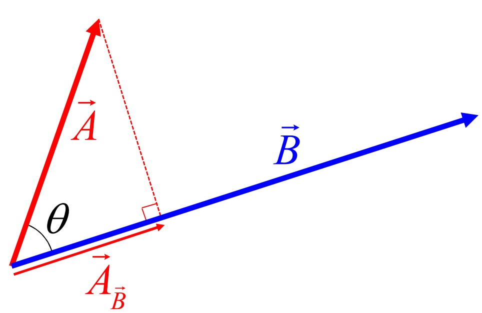
The dot product is all about projections. That is, calculating how much of one vector lies in the direction of another vector.

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$$
$$= A_x B_x + A_y B_y + A_z B_z$$



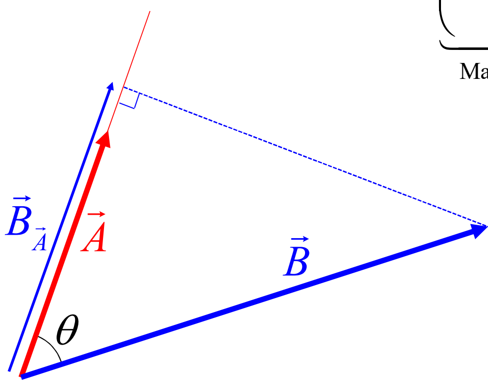
Projection of $\vec{A}$ onto $\vec{B}$

$$\vec{A}_{\vec{B}} = \underbrace{\left(\vec{A} \bullet \frac{\vec{B}}{\left|\vec{B}\right|}\right)}_{\substack{\text{Magnitude} \\ \left|\vec{A}_{\vec{B}}\right|}} \underbrace{\frac{\vec{B}}{\left|\vec{B}\right|}}_{\substack{\text{Direction} \\ \hat{B} = \frac{\vec{B}}{\left|\vec{B}\right|}}} = \frac{\vec{A} \bullet \vec{B}}{\left|\vec{B}\right|^2} \vec{B}$$



Projection of $\vec{B}$ onto $\vec{A}$

$$\vec{B}_{\vec{A}} = \underbrace{\left(\vec{B} \bullet \frac{\vec{A}}{\left|\vec{A}\right|} \right)}_{\substack{\text{Magnitude} \\ \left|\vec{B}_{\vec{A}}\right|}} \underbrace{\frac{\vec{A}}{\left|\vec{A}\right|}}_{\substack{\text{Direction} \\ \hat{A}=\frac{\vec{A}}{\left|\vec{A}\right|}}} = \frac{\vec{B} \bullet \vec{A}}{\left|\vec{A}\right|^2} \vec{A}$$



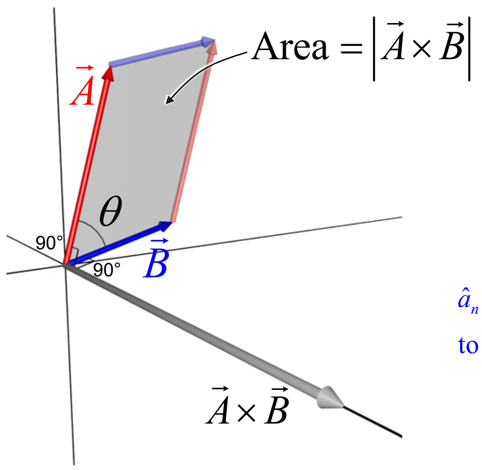
The Dot Product Test

The dot produce can be used to test of two vectors are perpendicular. If they are, the component of one along the other must be zero so the dot product must be zero.

$$\vec{A} \bullet \vec{B} = 0 \quad \text{when } \vec{A} \perp \vec{B}$$

The Cross Product, $\vec{A} \times \vec{B}$

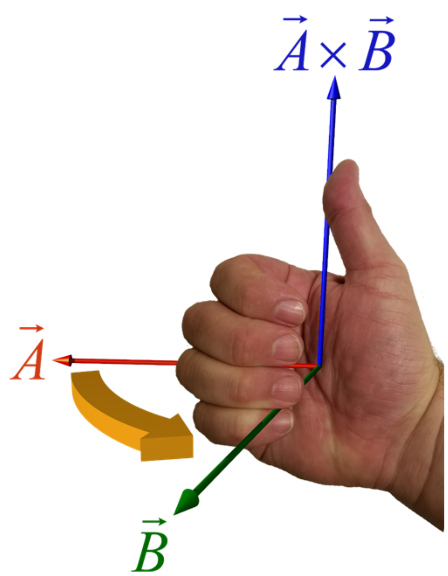
The cross product is all about area and calculating vectors that are perpendicular to $\vec{A}$ and $\vec{B}$.



$$\vec{A} \times \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta \hat{a}_n$$

$\hat{a}_n$ is a unit vector perpendicular to the plane defined by $\vec{A}$ and $\vec{B}$.

Handedness of the Cross Product



Calculating Cross Products (1 of 2)

Suppose we wish to calculate the cross product $\vec{A} \times \vec{B}$.

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \qquad \vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

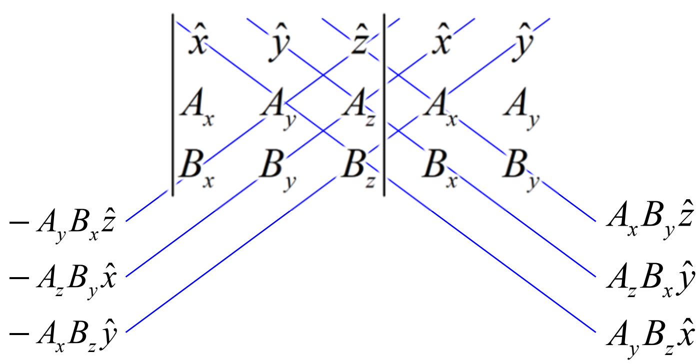
Step 1 – Construct an augmented matrix.

First two columns are repeated outside of the matrix.

$$\left|\begin{matrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{matrix}\right| \begin{matrix} \hat{x} & \hat{y} \\ A_x & A_y \\ B_x & B_y \end{matrix}$$

Calculating Cross Products (2 of 2)

Step 2 – Multiply elements along the diagonals.



Step 3 – Make left-hand side products negative.

Step 4 – Add up all of the products.

$$\vec{A} \times \vec{B} = \left(A_yB_z - A_zB_y\right)\hat{x} + \left(A_zB_x - A_xB_z\right)\hat{y} + \left(A_xB_y - A_yB_x\right)\hat{z}$$

The Cross Product Test

The cross product can be used to test if two vectors are parallel. If they are, the cross product will be zero because the angle between the vectors is zero.

$$\vec{A} \times \vec{B} = 0 \quad \text{when } \vec{A} \parallel \vec{B}$$

Vector Algebra Rules

Commutative Laws

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A} \qquad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Associative Laws

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) \neq \left(\vec{A} \times \vec{B}\right) \times \vec{C}$$

Distributive Laws

$$\vec{A} \bullet \left(\vec{B} + \vec{C}\right) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C} \qquad \vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Self-Product

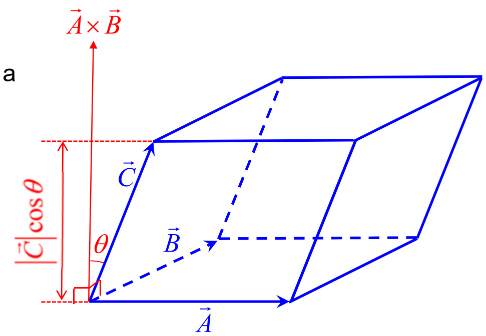
$$\vec{A} \bullet \vec{A} = \left|\vec{A}\right|^2 \qquad \vec{A} \times \vec{A} = 0$$

Vector Triple Products

Scalar Triple Product

The scalar triple product is the volume of a parallelpiped.

$$\left(\vec{B}\times\vec{C}\right)\bullet\vec{A}=\left(\vec{C}\times\vec{A}\right)\bullet\vec{B}=\left(\vec{A}\times\vec{B}\right)\bullet\vec{C}$$



Vector Triple Product

The vector triple product arises when deriving the wave equation.

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\bullet\vec{C}\right)-\vec{C}\left(\vec{A}\bullet\vec{B}\right)$$