فصل ۶
خمش

ابتدا دیاگرام های V و M (استاتیک)،
سپس فرمول خمش برای تنش

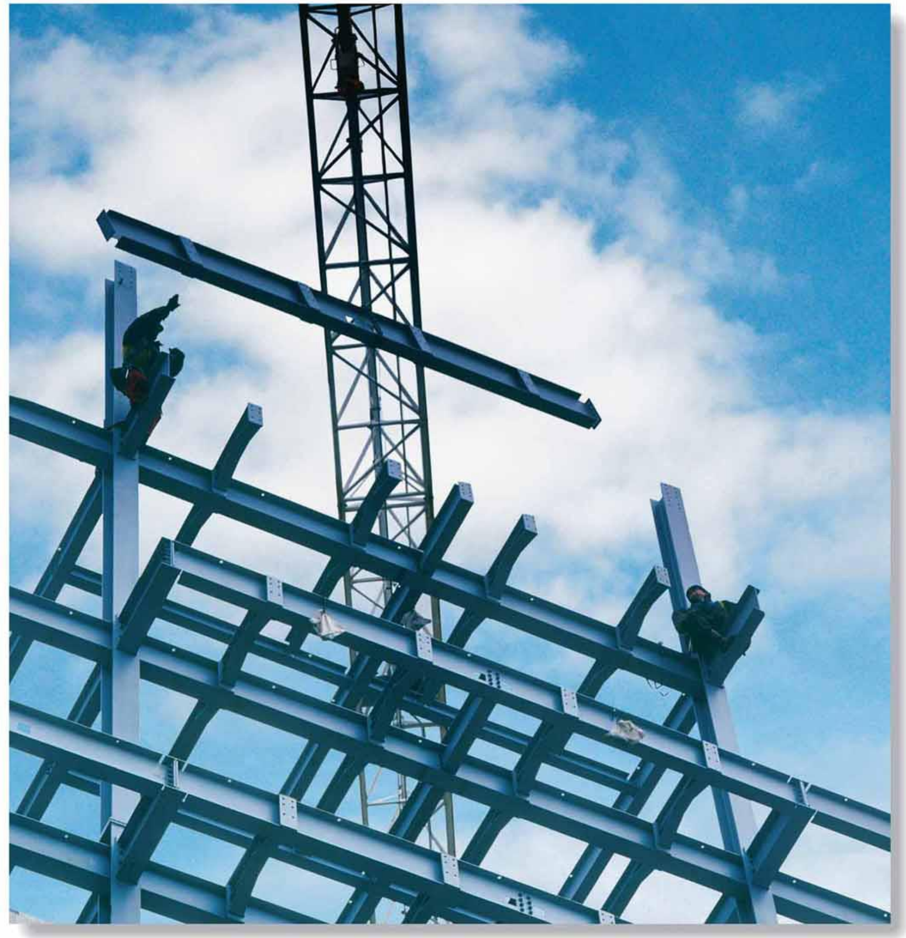


Figure: 06-01-COC
Beams are important structural members used in building construction. Their design is often based upon their ability to resist bending stress, which forms the subject matter of this chapter.

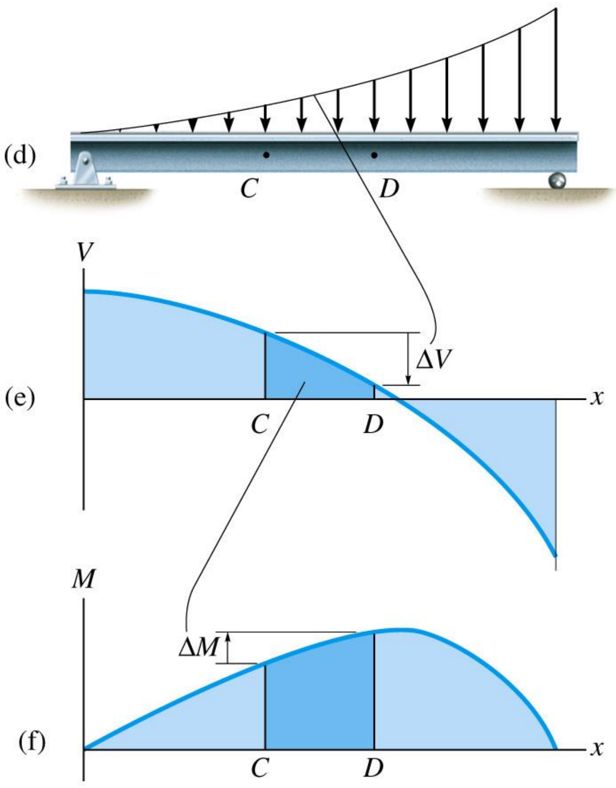
فصل ۶- بخش ۱ و ۲

- سه روش برای ترسیم دیاگرام های نیروی برشی V و ممان خمشی M وجود دارد:
  ۱. برش زدن مقاطع و برآیند گیری نیروها و ممان ها. یعنی:

$$\sum Fy = 0$$

$$\sum M_{cut} = 0$$

۲- به روش ترسیم سریع (روش بررسی / بازرسی)

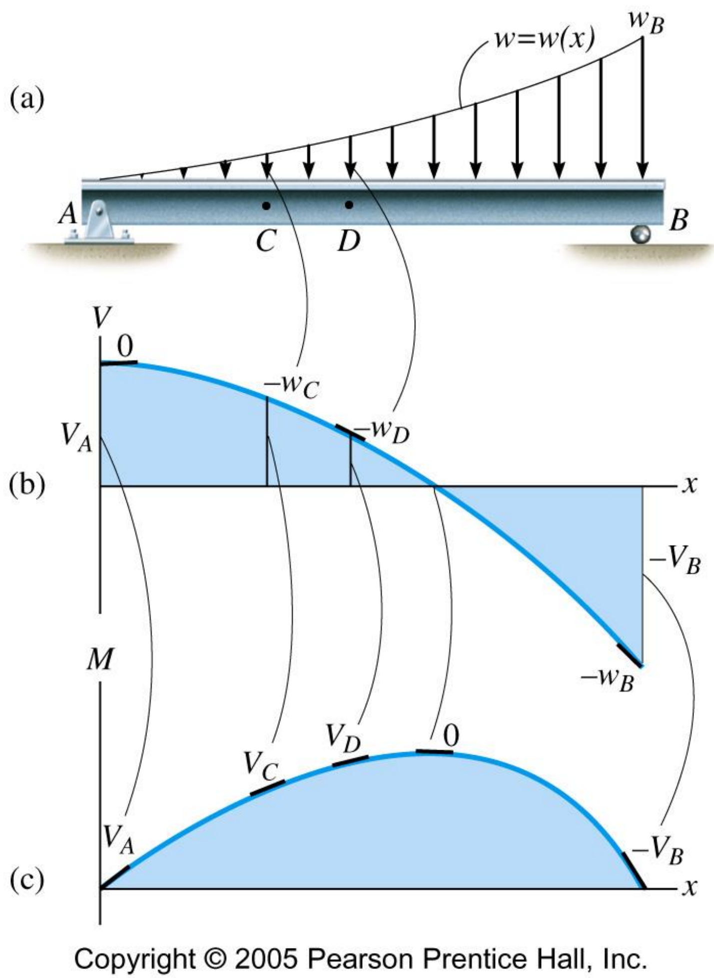


$$\Delta V = -\int w(x)dx$$

$$\Delta M = \int V(x)dx$$

در فاصله بین هر دو نقطه از تیر:
مساحت زیر نمودار بار (با یک علامت منفی) = تغییرات نیروی برشی
مساحت زیر نمودار نیروی برشی= تغییرات ممان خمشی

۳- انتگرال گیری مستقیم



$$dV = -w(x)dx$$

$$dM = V(x)dx$$

زمانی که w(x) تابع پیچیده ای باشد این روش مناسب است.

مثال:

w(x) = 4sin3x

$w(x) = 5X^3 + 3X^2 + 2X + 9$

در هر نقطه:

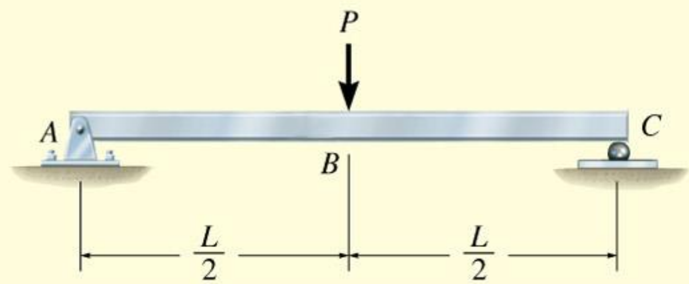
شیب خط مماس بر نمودار نیروی برشی= مقدار بار

شیب خط مماس بر نمودار ممان خمشی= مقدار نیروی برشی

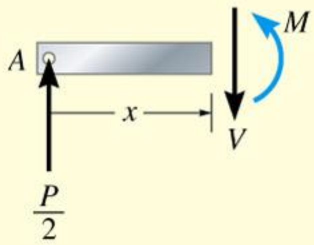
مثال ها

EXAMPLE 6.1

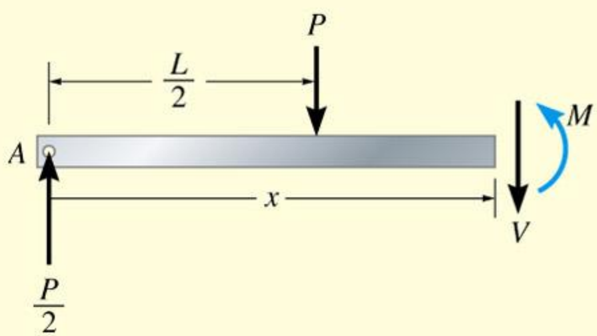
Draw the shear and moment diagrams for the beam shown in Fig. 6–4*a*.



(a)



(b)



(c)

Solution

Support Reactions. The support reactions have been determined, Fig. 6–4*d*.

Shear and Moment Functions. The beam is sectioned at an arbitrary distance *x* from the support *A*, extending within region *AB*, and the free-body diagram of the left segment is shown in Fig. 6–4*b*. The unknowns **V** and **M** are indicated acting in the *positive sense* on the right-hand face of the segment according to the established sign convention. Applying the equilibrium equations yields

$$+\uparrow \Sigma F_y = 0; \qquad V = \frac{P}{2} \qquad (1)$$

$$\circlearrowright + \Sigma M = 0; \qquad M = \frac{P}{2}x \qquad (2)$$

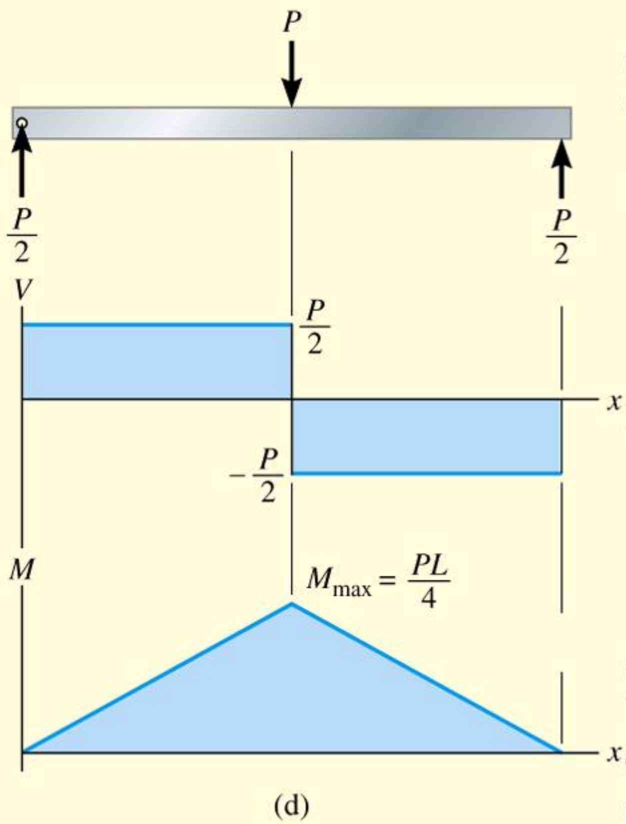


Fig. 6–4

A free-body diagram for a left segment of the beam extending a distance x within region BC is shown in Fig. 6–4c. As always, **V** and **M** are shown acting in the positive sense. Hence,

$$+\uparrow \Sigma F_y = 0; \qquad \frac{P}{2} - P - V = 0$$

$$V = -\frac{P}{2} \tag{3}$$

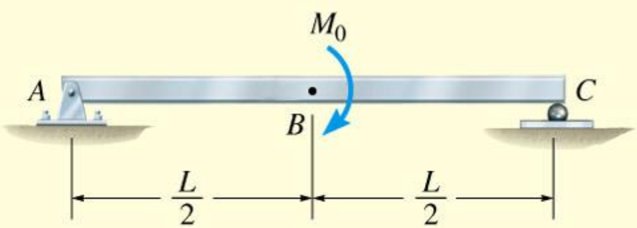
$$\circlearrowleft + \Sigma M = 0; \qquad M + P\left(x - \frac{L}{2}\right) - \frac{P}{2}x = 0$$

$$M = \frac{P}{2}(L - x) \tag{4}$$

The shear diagram represents a plot of Eqs. 1 and 3, and the moment diagram represents a plot of Eqs. 2 and 4, Fig. 6–4d. These equations can be checked in part by noting that $dV/dx = -w$ and $dM/dx = V$ in each case. (These relationships are developed in the next section as Eqs. 6–1 and 6–2.)

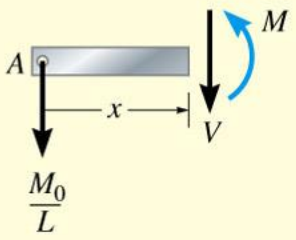
EXAMPLE 6.2

Draw the shear and moment diagrams for the beam shown in Fig. 6–5*a*.



(a)

دیاگرام آزاد در ناحیه AB:



(b)

Solution

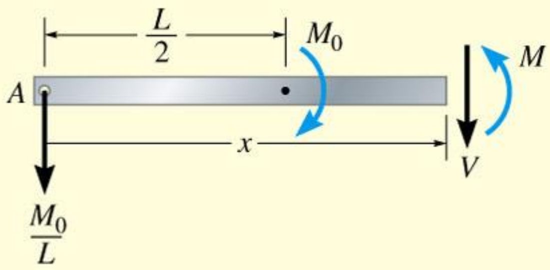
Support Reactions. The support reactions have been determined in Fig. 6–5*d*.

Shear and Moment Functions. This problem is similar to the previous example, where two *x* coordinates must be used to express the shear and moment in the beam throughout its length. For the segment within region *AB*, Fig. 6–5*b*, we have

$$+\uparrow \Sigma F_y = 0; \qquad V = -\frac{M_0}{L}$$

$$\circlearrowright + \Sigma M = 0; \qquad M = -\frac{M_0}{L}x$$

دیاگرام آزاد در ناحیه AC:



(c)

And for the segment within region BC, Fig. 6–5c,

$$+\uparrow \Sigma F y = 0; \qquad V = -\frac{M_0}{L}$$

$$\curvearrowright + \Sigma M = 0; \qquad M = M_0 - \frac{M_0}{L}x$$

$$M = M_0\left(1 - \frac{x}{L}\right)$$

Shear and Moment Diagrams. When the above functions are plotted, the shear and moment diagrams shown in Fig. 6–5d are obtained. In this case, notice that the shear is constant over the entire length of the beam; i.e., it is not affected by the couple moment $\mathbf{M}_0$ acting at the center of the beam. Just as a force creates a jump in the shear diagram, Example 6–1, a couple moment creates a jump in the moment diagram.

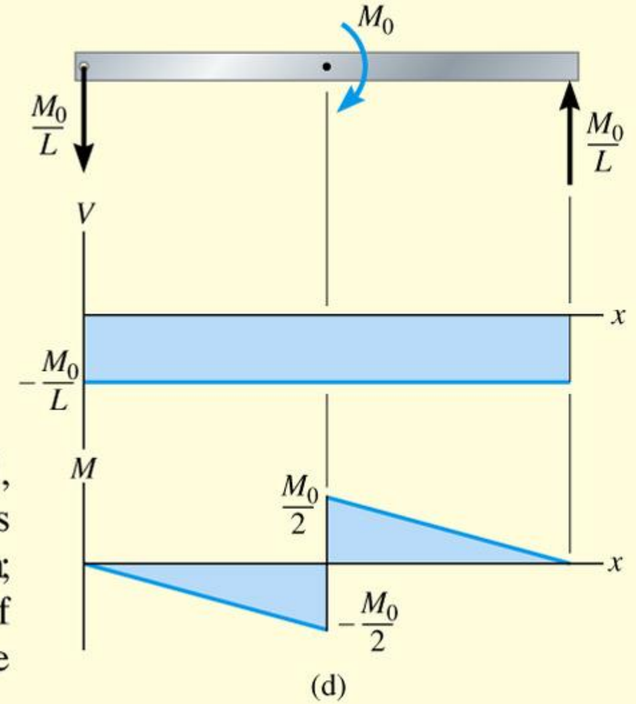
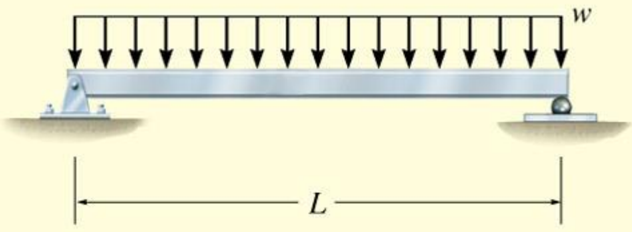
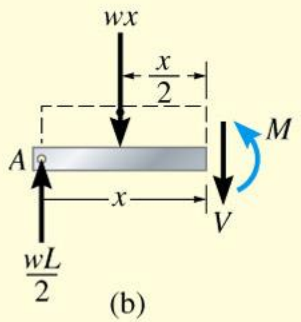


Fig. 6–5

EXAMPLE 6.3



(a)



(b)

<u>**دیاگرام آزاد در فاصله بین دو تیکه گاه:**</u>

Draw the shear and moment diagrams for the beam shown in Fig. 6–6*a*.

Solution

Support Reactions. The support reactions have been computed in Fig. 6–6*c*.

Shear and Moment Functions. A free-body diagram of the left segment of the beam is shown in Fig. 6–6*b*. The distributed loading on this segment is represented by its resultant force only *after* the segment is isolated as a free-body diagram. Since the segment has a length x, the *magnitude* of the *resultant force* is wx. This force acts through the centroid of the area comprising the distributed loading, a distance of $x/2$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \tag{2}$$

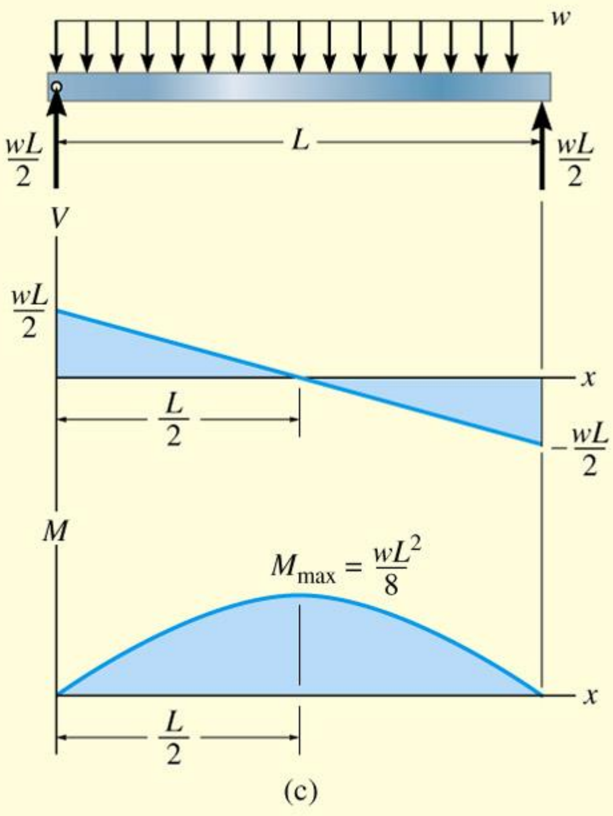


Fig. 6–6

These results for V and M can be checked by noting that $dV/dx = -w$. This is indeed correct, since positive w acts downward. Also, notice that $dM/dx = V$.

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–6*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$

جایی که نیروی برشی صفر است

From the moment diagram, this value of x happens to represent the point on the beam where the *maximum moment* occurs, since by Eq. 6–2, the slope $V = 0 = dM/dx$. From Eq. 2, we have

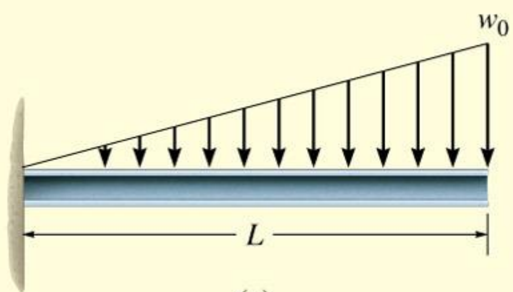
$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

جایی که ممان خمشی ماکزیمم است

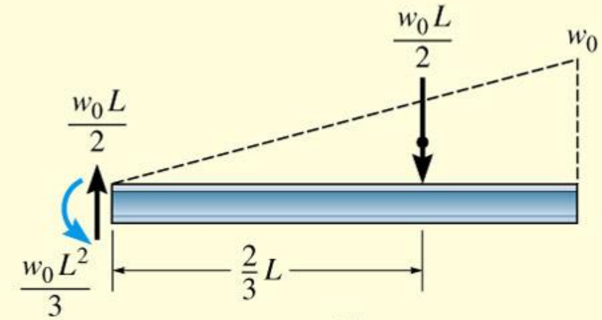
EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



(a)

دیاگرام آزاد کل تیر:



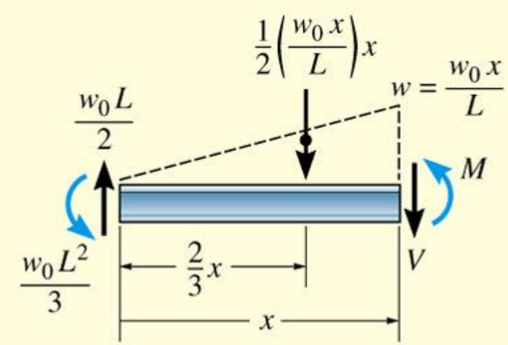
(b)

دیاگرام آزاد در بخشی از تیر:

Solution

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–7*b*.

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–7*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0 x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram, Fig. 6–7*c*. Thus,



(c)

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

$$\circlearrowright + \Sigma M = 0; \qquad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

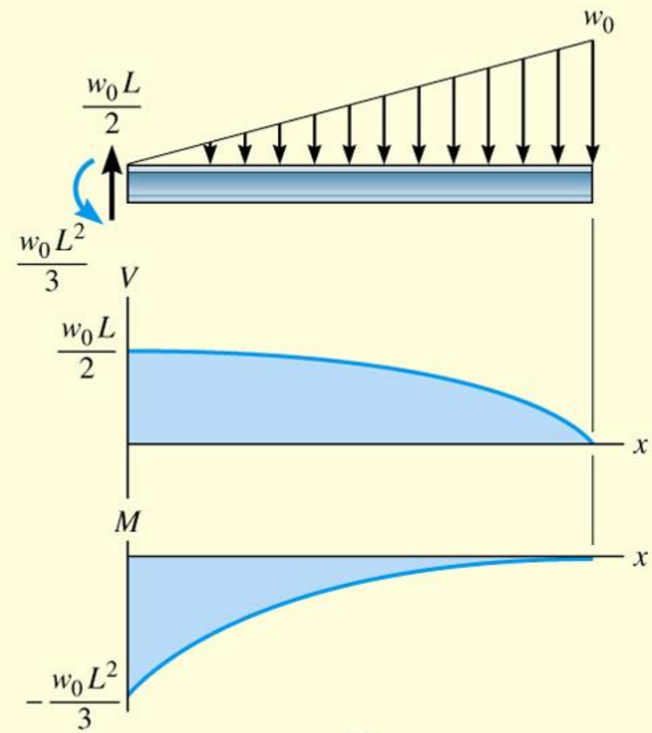
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2 x - x^3) \qquad (2)$$

These results can be checked by applying Eqs. 6–1 and 6–2, that is,

$$w = -\frac{dV}{dx} = -\frac{w_0}{2L}(0 - 2x) = \frac{w_0 x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(-0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

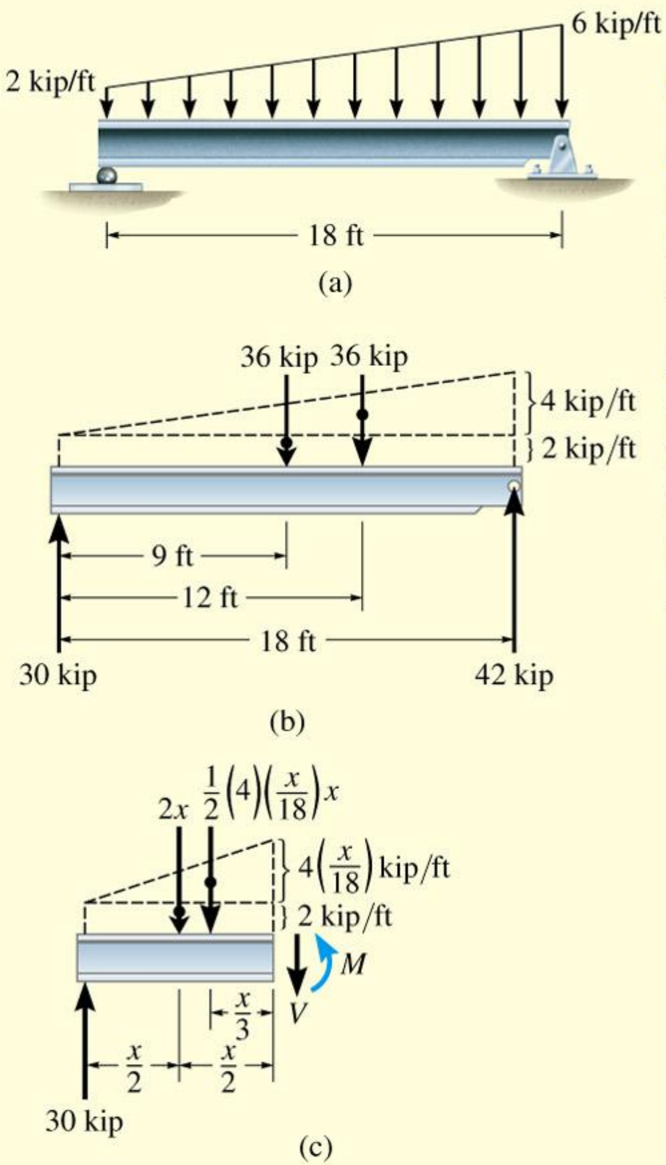
Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–7*d*.



(d)

Fig. 6–7

EXAMPLE 6.5



Draw the shear and moment diagrams for the beam shown in Fig. 6–8*a*.

Solution

Support Reactions. The distributed load is divided into triangular and rectangular component loadings and these loadings are then replaced by their resultant forces. The reactions have been determined as shown on the beam's free-body diagram, Fig. 6–8*b*.

Shear and Moment Functions. A free-body diagram of the left segment is shown in Fig. 6–8*c*. As above, the trapezoidal loading is replaced by rectangular and triangular distributions. Note that the intensity of the triangular load at the section is found by proportion. The resultant force and the location of each distributed loading are also shown. Applying the equilibrium equations, we have

$$+\uparrow\Sigma F_y = 0;\quad 30\text{ kip} - (2\text{ kip/ft})x - \frac{1}{2}(4\text{ kip/ft})\left(\frac{x}{18\text{ ft}}\right)x - V = 0$$

$$V = \left(30 - 2x - \frac{x^2}{9}\right)\text{kip} \qquad (1)$$

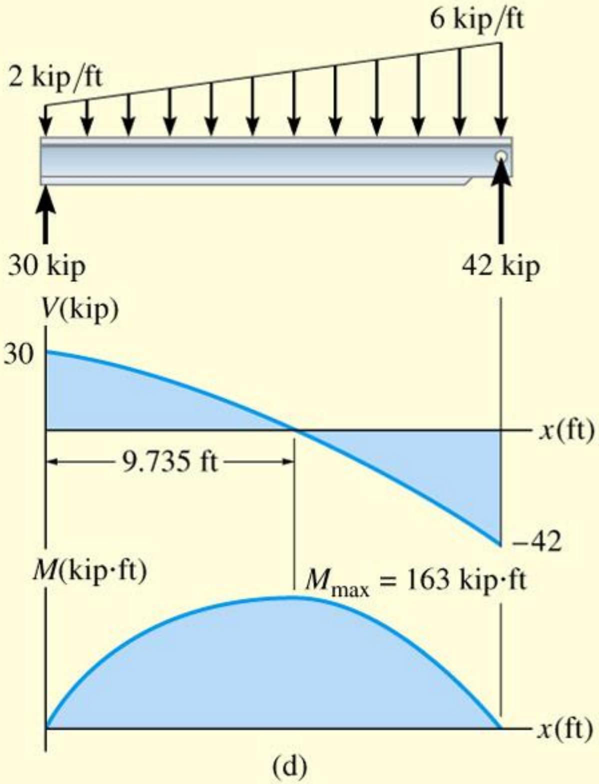


Fig. 6–8

$\circlearrowleft + \Sigma M = 0;$

$$-30\text{ kip}(x) + (2\text{ kip/ft})x\left(\frac{x}{2}\right) + \frac{1}{2}(4\text{ kip/ft})\left(\frac{x}{18\text{ ft}}\right)x\left(\frac{x}{3}\right) + M = 0$$

$$M = \left(30x - x^2 - \frac{x^3}{27}\right)\text{kip}\cdot\text{ft} \qquad (2)$$

Equation 2 may be checked by noting that $dM/dx = V$, that is, Eq. 1. Also, $w = -dV/dx = 2 + \frac{2}{9}x$. This equation checks, since when $x = 0$, $w = 2$ kip/ft, and when $x = 18$ ft, $w = 6$ kip/ft, Fig. 6–8*a*.

Shear and Moment Diagrams. Equations 1 and 2 are plotted in Fig. 6–8*d*. Since the point of maximum moment occurs when $dM/dx = V = 0$, then, from Eq. 1,

$$V = 0 = 30 - 2x - \frac{x^2}{9}$$

Choosing the positive root,

$$x = 9.735\text{ ft}$$

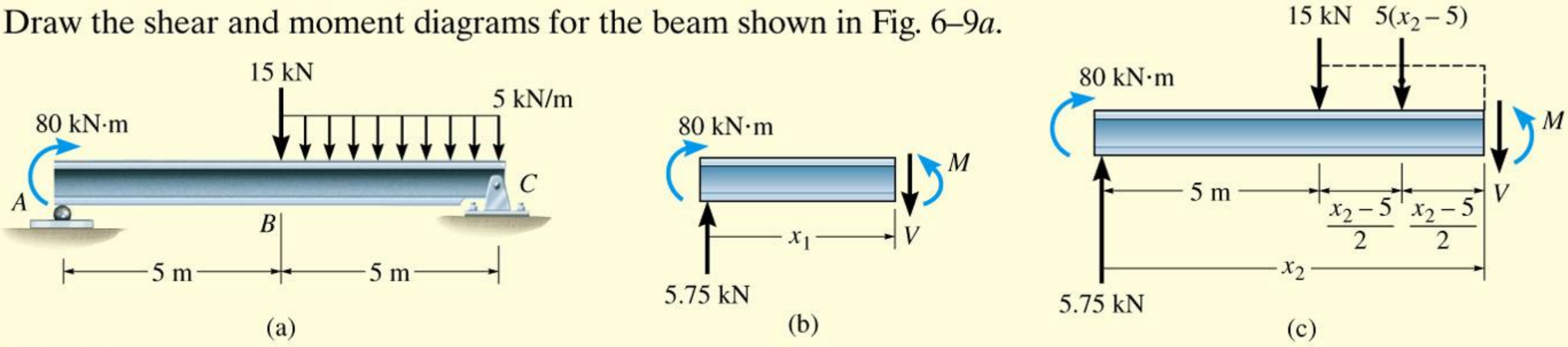
Thus, from Eq. 2,

$$M_{max} = 30(9.735) - (9.735)^2 - \frac{(9.735)^3}{27}$$

$$= 163\text{ kip}\cdot\text{ft}$$

EXAMPLE 6.6

Draw the shear and moment diagrams for the beam shown in Fig. 6–9*a*.



Solution

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–9*d*.

Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of *x* must be considered in order to describe the shear and moment functions for the entire beam.

$0 \leq x_1 < 5$ m, Fig. 6–9*b*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN} \qquad (1)$$

$\circlearrowleft + \Sigma M = 0;\qquad -80\ \text{kN}\cdot\text{m} - 5.75\ \text{kN}\ x_1 + M = 0$

$$M = (5.75x_1 + 80)\ \text{kN}\cdot\text{m} \qquad (2)$$

$5\ \text{m} < x_2 \leq 10\ \text{m}$, Fig. 6–9*c*:

$+\uparrow \Sigma F_y = 0;\quad 5.75\ \text{kN} - 15\ \text{kN} - 5\ \text{kN/m}(x_2 - 5\ \text{m}) - V = 0$

$$V = (15.75 - 5x_2)\ \text{kN} \qquad (3)$$

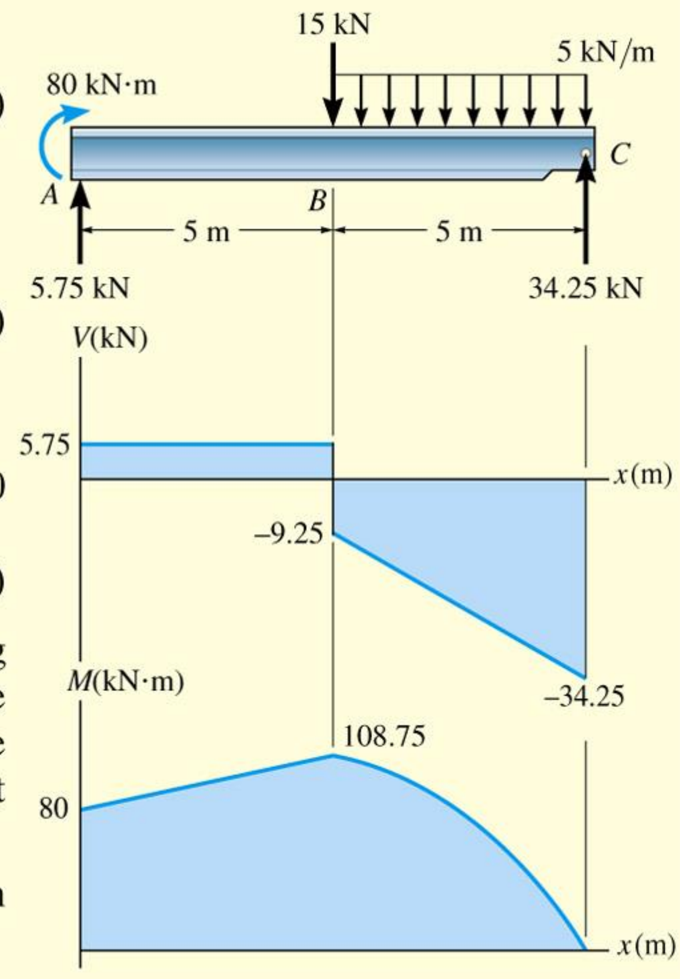
$\circlearrowleft + \Sigma M = 0;\quad -80\ \text{kN}\cdot\text{m} - 5.75\ \text{kN}\ x_2 + 15\ \text{kN}(x_2 - 5\ \text{m})$

$$+ 5\ \text{kN/m}(x_2 - 5\ \text{m})\left(\frac{x_2 - 5\ \text{m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\ \text{kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that by applying $w = -dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75\ \text{kN}$ and $M = 80\ \text{kN}\cdot\text{m}$; when $x_2 = 10\ \text{m}$, Eqs. 3 and 4 give $V = -34.25\ \text{kN}$ and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–9*d*.

Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–9*d*.



(d)

Fig. 6–9