باسمه تعالی

تکلیف سری چهارم درس سیگنال ها و سیستم ها نام استاد: خانی

1- If $c(t) = x(t) * g(t)$, then show that $Ac = AxAg$, where Ax, Ag, and Ac are the areas under $x(t)$, $g(t)$, and $c(t)$, respectively.
2- If $x(t) * g(t) = c(t)$, then show that $x(at) * g(at) = |1/a|c(at)$. This *time-scaling property* of convolution states that if both $x(t)$ and $g(t)$ are time-scaled by a, their convolution is also time-scaled by a (and multiplied by $|1/a|$).
3- Show that the convolution of an odd and an even function is an odd function and the convolution of two odd or two even functions is an even function. [Hint: Use time-scaling property of convolution in Problem 2.]
4- Using direct integration, find $e^{-at}u(t) * e^{-bt}u(t)$.
5- Using direct integration, find $u(t) * u(t)$, $e^{-at}u(t) * e^{-at}u(t)$, and $tu(t) * u(t)$.
6- Using direct integration, find $sin(t)u(t) * u(t)$ and $cos(t)u(t) * u(t)$.
7- The unit impulse response of a continuous-time LTI system is $h(t)=e^{-t}u(t)$. Find this system's response $y(t)$ if the input $x(t)$ is:
   a. $u(t)$
   b. $e^{-t}u(t)$
   c. $e^{-2t}u(t)$
   d. $sin(3t)u(t)$
8- Repeat Problem 7 for $h(t)=[(1-2t)e^{-2t}+3e^{-3t}]u(t)$ and if the input $x(t)$ is:
   a. $u(t)$
   b. $e^{-t}u(t)$
   c. $e^{-2t}u(t)$
9- An analog LTIC system with impulse response function $h(t) = u(t + 2) − u(t − 2)$ is presented with an input $x(t) = t(u(t) − u(t − 2))$.
a. Determine and plot the system output $y(t) = x(t) * h(t)$.
b. Is this system stable? Is this system causal? Justify your answers.
10- A system has an impulse response function shaped like a rectangular pulse, $h(t) = u(t) − u(t − 1)$. Is the system stable? Is the system causal?
11- Find the unit step response of the system of Problem 10.
12- The autocorrelation of a function $x(t)$ is given by $r_{xx}(t)=\int_{-\infty}^{+\infty} x(\tau)x(\tau - t)d\tau$. This equation is computed in a manner nearly identical to convolution.
a. Show $r_{xx}(t) = x(t) * x(−t)$
b. Determine and plot $r_{xx}(t)$ for the signal $x(t)$ depicted in Fig. 1. [Hint: $r_{xx}(t) = r_{xx}(−t)$.]

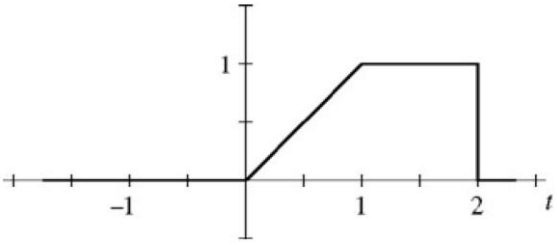

Figure 1: Analog signal $x(t)$.

13- Consider the electric circuit shown in Fig. 2. A) Determine the differential equation relating input $x(t)$ to output $y(t)$. B) Determine the output $y(t)$ in response to the input $x(t) = 4te^{-3t/2}u(t)$. Assume component values of $R = 1\ \Omega$, $C_1 = 1$ F, and $C_2 = 2$ F, and initial capacitor voltages of $V_{C1} = 2$ V and $V_{C2} = 1$ V.

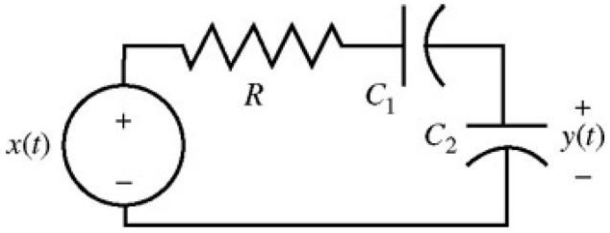

Figure 2: *RCC* circuit.