Chapter 2 Geometrical Optics

2-1. $t = \dfrac{\sum d_{\text{op}}}{c} = \dfrac{\sum_i n_i x_i}{c}$

2-2. Referring to Figure 2-12 and with lengths in cm,

$$n_0 \left(x^2 + y^2\right)^{1/2} + n_i \left(y^2 + \left(s_o + s_i - x\right)^2\right)^{1/2} = n_o s_o + n_i s_i$$

$$(1) \left(x^2 + y^2\right)^{1/2} + 1.5 \left(y^2 + (30 - x)^2\right)^{1/2} = 20 + 1.5\,(10) = 35$$

$$2.25 \left(y^2 + (30 - x)^2\right) = \left(35 - \left(x^2 + y^2\right)^{1/2}\right)^2$$

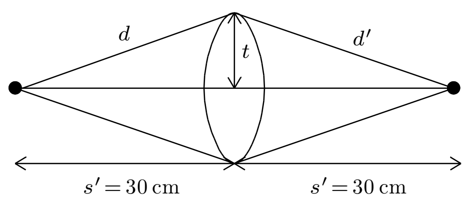
$$1.25 \left(x^2 + y^2\right) + 70 \left(x^2 + y^2\right)^{1/2} - 135\,x + 800 = 0$$

Using a calculator to guess and check or using a computer algebra system, (like the free program Maxima, for example) one can numerically solve this equation for x for given y-values. Doing so results in,

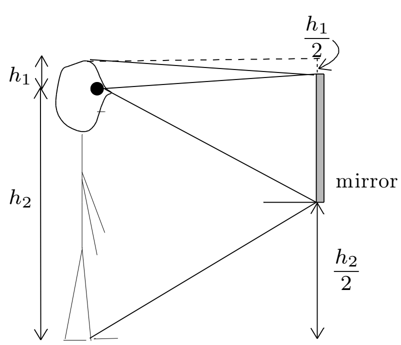
x (cm)	20	20.2	20.4	20.8	21.6	22.4	23.2	24.0	24.8	25.6	26.4	27.2
y (cm)	0	± 1.0	± 1.40	± 1.96	± 2.69	± 3.20	± 3.58	± 3.85	± 4.04	± 4.14	± 4.18	± 4.13

2-3. Refer to the figure for the relevant parameters.

$d = d' = \sqrt{30^2 + 2.5^2} = 30.104\,\text{cm}$
Fermat: $d + d' = s + s' - t + m\,t$
$d + d' = s + s' + t\,(m - 1)$
$2\,(30.10399) = 60 = t\,(1.52 - 1)$
$t = 4\,\text{mm}$
$n = 1.52$



2-4. See the figure below. Let the height of the person be $h = h_1 + h_2$.

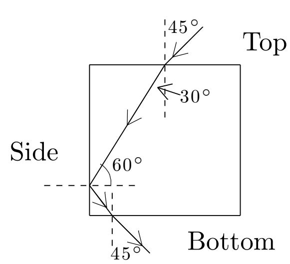


The person must be able to see the top of his head and the bottom of his feet. From the figure it is evident that the mirror height is:

$$h_{\text{mirror}} = h - h_1/2 - h_2/2 = h/2$$

The mirror must be half the height of the person. So for a person of height six ft person, the mirror must be 3 ft high.

2-5. Refer to the figure.



At Top: $(1)\sin 45 = \sqrt{2}\sin\theta' \Rightarrow \theta' = 30$
At Side: $\sqrt{2}\sin 60° = (1)\sin\theta', \sin\theta' = \sqrt{1.5} > 1$
Thus total internal reflection occurs.
At Bottom: reverse of Top: $\theta' = 45°$

2-6. The microscope first focuses on the scratch using direct rays. Then it focuses on the image I_2 formed in a two-step process: (1) reflection from the bottom to produce an intermediate image I_1 and (2) refraction through the top surface to produce an image I_2. Thus, I_1 is at $2t$ from top surface, and I_2 is at the apparent depth for I_1, serving as the object: $s' = \frac{2t}{n}$ or $n = \frac{2t}{s'} = \frac{3}{1.87} = 1.60$

2-7. Refer to Figure 2-33 in the text. By geometry, $\tan\theta_c = \frac{7.60/4}{2.25}$ so $\theta_c = 40.18°$

Snell's law: $n\sin\theta_c = (1)\sin 90° \Rightarrow n = \frac{1}{\sin 40.18°} = 1.55$

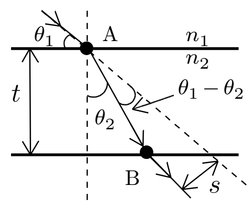
2-8. Referring to the figure one can see that,

$s = \text{AB}\sin(\theta_1 - \theta_2)$ and $\text{AB} = \frac{t}{\cos\theta_2}$. Therefore,

$s = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$. For $t = 3$ cm, $n_2 = 1.50$, $\theta_1 = 50°$,

Snell's law gives, $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{1.5}\sin 50°$.

Then, $\theta_2 = 30.71°$ and $s = \frac{3\sin(50° - 30.71°)}{\cos 30.71°} = 1.153\text{cm}$.



2-9. Image of near end: $s = 60\text{cm}$, $\frac{1}{60} = \frac{1}{s'} = \frac{1}{-40}$, $s' - 24\text{cm}$

Image of far end: $s = 60 + 100$ cm, $\frac{1}{160} + \frac{1}{s'} = \frac{1}{-40}$, $s' = -32\text{cm}$. So, $L' = \Delta s' = -24 - (-32) = 8\text{cm}$

2-10. (a) See Figure 2-34 in the text. Image due to rays directly from bubble through plane interface:
$\frac{n_1}{s} + \frac{n_2}{s'} = 0$ or $\frac{1.5}{s} + \frac{1}{s'} = 0 \Rightarrow s' = -3.33$ cm.

(b) Image due to rays first reflected in spherical mirror, then refracted through plane interface:

reflection: $\frac{1}{2} + \frac{1}{s_1'} = -\frac{2}{R}$ and $\frac{1}{2.5} + \frac{1}{s_1'} = -\frac{2}{-7.5}$ $s_1' = -7.5\text{cm}$

refraction: $\frac{n_1}{s} + \frac{n_2}{s_2'} = 0$ or $\frac{1.5}{15} + \frac{1}{s_2'} = 0$ $s_2' = -10\text{cm}$

Thus the images are at 3.33 cm and 10 cm behind the interface.

2-11. There are 5 unknowns: s_1 and s_1' in position (1), s_2 and s_2' in position (2), and the focal length f of the mirror. The five equations that, solved simultaneously, yield the results are:

(1) linear magnification: $s_1'/s_1 = 2$ (2) linear magnification: $s_2'/s_2 = 3$

(3) focal length from mirror equation: $f = \frac{s_1 s_1'}{s_1 + s_1'}$ (4) focal length from mirror equation: $f = \frac{s_2 s_2'}{s_2 + s_2'}$

(5) image distance relation: $s_2' = s_1' + 75$

One finds $s_1 = 112.5$ cm, $s_2 = 100$ cm, $s_1' = 225$ cm, $s_2' = 300$ cm, $f = 75\text{cm}$

2-12. The object distance from the front surface is the diameter of the sphere, 5 cm. Then,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.5}{5} + \frac{1}{s'} = \frac{1 - 1.5}{-2.5} \Rightarrow s' = -10\text{ cm} \text{ and, } m = -\frac{n_1 s'}{n_2 s} = -\frac{(1.5)(-10)}{(1)(5)} = +3.$$

2-13. Generally, $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

(a) $\frac{n_1}{f} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R}$ or $f = \frac{n_1 R}{n_2 - n_1}$ (b) $n_2 > n_1$: then $R > 0$ (convex), $n_2 < n_1$: then $R < 0$ (concave)

2-14. (a) In this position the object distance is $s = 15\,\text{cm}$. Then, using, $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$ and $m = -\frac{n_1 s'}{n_2 s}$,

$$\frac{4}{3\,(15)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -15\,\text{cm (center)} \quad \text{and, } m = -\frac{(4/3)\,(-15)}{(1)\,(15)} \Rightarrow m = 4/3.$$

(b) Similarly, in this position $s = 7.5\,\text{cm}$ so that,

$$\frac{4}{3\,(15/2)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -6.4\,\text{cm},\ m = -\frac{n_1 s'}{n_2 s} = -\frac{(4/3)\,(-45/7)}{(1)\,(15/2)} = 8/7.$$

2-15. See Figure 2-35 in the text. Rays from the object are (a) refracted through the spherical window, (b) then reflected from the back plane mirror, (c) then refracted out again through the spherical window. Taking these in turn:

(a) $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{30} + \frac{4}{3s'} = \frac{4/3 - 1}{5} \Rightarrow s' = 40\,\text{cm}$. Then, $m = \frac{n_1 s'}{n_2 s} = \frac{(1)\,(40)}{(4/3)\,(30)} = -1$

(b) $s = 25 - 40 = -15\,\text{cm}$ (virtual object), $s' = -s = 15\,\text{cm}$, $m = -s'/s = 1$

(c) $\frac{4/3}{10} + \frac{1}{s'} = \frac{1 - 4/3}{-5} \Rightarrow s' = -15\text{cm}$. Then, $m = \frac{-(4/3)\,(-15)}{(1)\,(10)} = +2$.

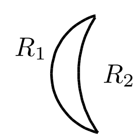
The overall magnification is $m = (-1)\,(+1)\,(+2) = -2$. Thus a virtual, inverted, double-sized image appears 15 cm behind (right) the spherical window.

2-16. The plane side of the lens has $R_1 = \infty$. The radius of curvature R_2 of the convex side is then found from the lensmaker's equation:

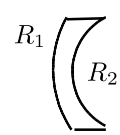
$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{25} = \frac{1.52 - 1}{1}\left(\frac{1}{\infty} - \frac{1}{R_2}\right) \Rightarrow R_2 = -13\,\text{cm}$$

2-17. In general the lensmaker's equation gives, $\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

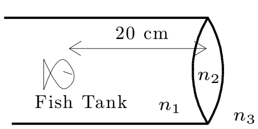
For the positive meniscus lens shown to the right, $R_1 = 5\,\text{cm}$ and $R_2 = 10\,\text{cm}$.
Then, $\frac{1}{f} = \frac{1.50 - 1}{1}\left(\frac{1}{5} - \frac{1}{10}\right) \Rightarrow f = +20\,\text{cm}$



For the negative meniscus lens shown to the right, $R_1 = 10\,\text{cm}$ and $R_2 = 5\,\text{cm}$.
For this case, $\frac{1}{f} = \frac{1.50 - 1}{1}\left(\frac{1}{10} - \frac{1}{5}\right) \Rightarrow f = -20\,\text{cm}$



2-18. The thin lens equation assumes identical, refractive indices on both sides. In this case we can modify the procedure, beginning with Eq. (2-23), to allow for three distinct media as shown.



$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R_1}\ ,\ \text{left lens surface}$$

$$\frac{n_2}{s_2} + \frac{n_3}{s_2'} = \frac{n_3 - n_2}{R_2}\ ,\ \text{right lens surface}$$

For a thin lens, $s_2 \approx -s_1'$. Adding the equations, $\frac{n_1}{s_1} + \frac{n_3}{s_2'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$. Or, simply,

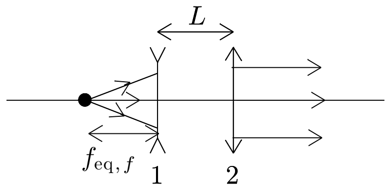
$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \Rightarrow \frac{4}{3\,(20)} + \frac{1}{s'} = \frac{3/2 - 4/3}{30} + \frac{1 - 3/2}{-30}\ ,\ \text{so that } s' = -22.5\text{cm}.$$

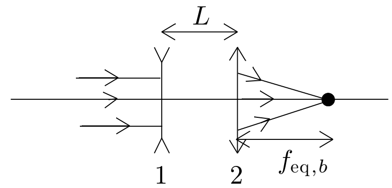
The total magnification is $m_T = m_1 m_2 = \left(-\frac{n_1 s_1'}{n_2 s}\right)\left(-\frac{n_2 s_2'}{n_3 s_2}\right)$, where $s_2 = -s_1'$. So,

$$m_T = -\frac{n_1 s'}{n_3 s} = -\frac{(4/3)\,(-22.5)}{(1)\,(20)} = 1.50.$$

2-19. (a) Using $\dfrac{1}{f_{\text{eq}}} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$ as in Eq. (2-33), $\dfrac{1}{f_{\text{eq}}} = \dfrac{1}{-5} + \dfrac{1}{20}$ or $f_{\text{eq}} = 6.67\,\text{cm}$

(b) A pair of separated lenses has a front and a back focal length. The front focal length is the object position from the first lens that leads to an image at infinity. The back focal length is the image position for an object at infinity. These cases are illustrated below. The drawings are generic and not to scale.





Working backwards, for the front focal length: Lens 2: $\dfrac{1}{s_2} + \dfrac{1}{\infty} = \dfrac{1}{f_2}$ or $s_2 = f_2$, $s_2 = L - s_1'$ or $s_1' = L - f_2$.

Lens 1: $\dfrac{1}{f_{\text{eq},f}} + \dfrac{1}{L - f_2} = \dfrac{1}{f_1}$ or $f_{\text{eq},f} = \dfrac{f_1\,(L - f_2)}{L - (f_1 + f_2)} = \dfrac{(-5)\,(10 - 20)}{10 - (-5 + 20)}\,\text{cm} = -10\,\text{cm}$

For the back focal length: for lens 1:

$\dfrac{1}{s_1} + \dfrac{1}{s_1'} = \dfrac{1}{f_1} \Rightarrow \dfrac{1}{\infty} + \dfrac{1}{s_1'} = \dfrac{1}{f_1} \Rightarrow s_1' = f_1$. Then $s_2 = L - f_1$, so that, for lens 2:

$\dfrac{1}{s_2} + \dfrac{1}{s_2'} = \dfrac{1}{f_2} \Rightarrow \dfrac{1}{L - f_1} + \dfrac{1}{f_{\text{eq},b}} = \dfrac{1}{f_2} \Rightarrow f_{\text{eq},b} = \dfrac{f_2\,(L - f_1)}{(L - f_1) - f_2} = \dfrac{(20)\,(10 - (-5))}{(10 - (-5)) - 20}\,\text{cm} = -60\,\text{cm}$

2-20. See Figure 2-36 in the text. Consider the three media as a sequence of three thin lenses. Each has a focal length given by the lensmaker's equation, and the equivalent focal length is given Eq. (2-33) as,

$\dfrac{1}{f_{\text{eq}}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$. Then, $\dfrac{1}{f_1} = (1.5 - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{-15}\right) \Rightarrow f_1 = 30\,\text{cm}$,

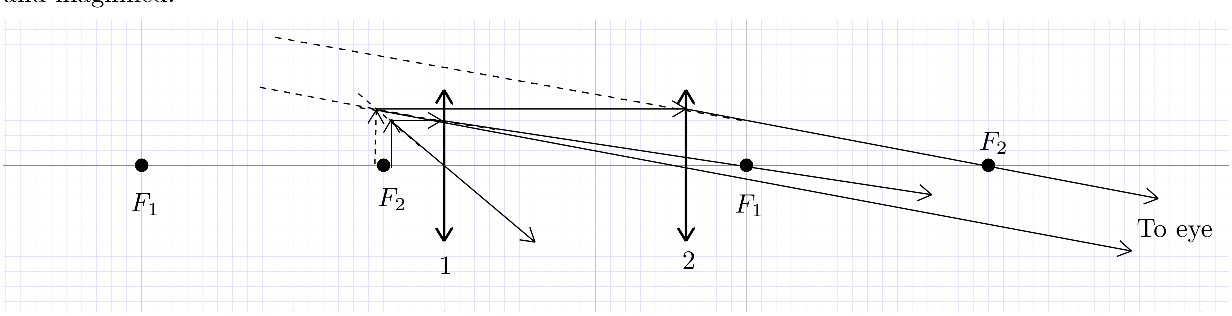
$\dfrac{1}{f_2} = (1.65 - 1)\left(\dfrac{1}{-15} - \dfrac{1}{15}\right)$ or $f_2 = -\dfrac{150}{13}$ cm, and $\dfrac{1}{f_3} = \text{same as for } f_1 : f_3 = 30\,\text{cm}$. Then,

$\dfrac{1}{f_{\text{eq}}} = \dfrac{1}{30} + \dfrac{-13}{150} + \dfrac{1}{30}$ and so $f_{\text{eq}} = -50\,\text{cm}$.

2-21. (a) One can use the formula derived in problem 2-19b, or do the calculation at first hand:

Second lens: $\dfrac{1}{s_2} + \dfrac{1}{\infty} = \dfrac{1}{20}$ or $s_2 = 20\,\text{cm}$, First lens: $\dfrac{1}{s_1} + \dfrac{1}{-4} = \dfrac{1}{20}$ or $s_1 = 3.33\,\text{cm}$. The object should be placed 3.33 cm before the first lens.

(b) In the figure below the dashed arrow is the intermediate image that acts as the object for the second lens. Since the image is "at infinity" it is described by an angular magnification. The image appears erect and magnified.



2-22. Refer to Figure 2-37 in the text.

(b) Lens heading towards mirror: $\frac{1}{3\,f/2}+\frac{1}{s'}=\frac{1}{-\,f}$ or $s'=-\,3\,f/5$. $m_1=-\frac{s'}{s}=-\frac{-\,3f/5}{3f/2}=2/5$

Mirror:

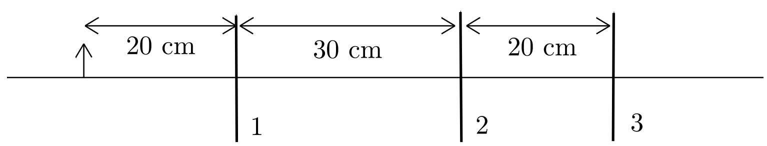
$s=3\,f+3\,f/5=18\,f/5\Rightarrow\frac{5}{18\,f}+\frac{1}{s'}=\frac{1}{f}\Rightarrow s'=18\,f/13,\ m_2=-\frac{s'}{s}=-\,(18\,f/13)/(18\,f/5)=-\,5/13$

Lens after reflection:

$s=3\,f-18\,f/13=21\,f/13\Rightarrow\frac{13}{21\,f}+\frac{1}{s'}=\frac{1}{-\,f}$ or $s'=21\,f/34, m_3=-\,s'/s=-\,(21\,f/34)/(21\,f/13)=\frac{13}{34}$

$m_T=\left(\frac{2}{5}\right)\left(-\frac{5}{13}\right)\left(\frac{13}{34}\right)=-\,17$. The image is inverted, $(21/34)\ f$ behind (right of) lens, inverted, and 1/17 original size.

2-23. The arrangement of the object and lenses is shown below.



(a) $f_1=+\,10\,\text{cm},\ f_2=+\,15\,\text{cm},\ f_3=+\,20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{10}$ $\quad s'=20 \quad m_1=-\,20/20=-\,1$

2nd lens: $\frac{1}{10}+\frac{1}{s'}=\frac{1}{15}$ $\quad s'=-\,30 \quad m_2=-\,(-\,30)/10=+\,3$

3rd lens: $\frac{1}{50}+\frac{1}{s'}=\frac{1}{20}$ $\quad s'=100/3 \quad m_3=-\,100/3\,(50)=-\,2/3$

$m_T=m_1\,m_2\,m_3=+\,2$

(b) $f_1=+\,10\,\text{cm},\ f_2=-\,15\,\text{cm},\ f_3=+\,20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{10}$ $\quad s'=20 \quad m_1=-\,20/20=-\,1$

2nd lens: $\frac{1}{10}+\frac{1}{s'}=\frac{1}{-\,15}$ $\quad s'=-\,6 \quad m_2=-\,(-\,6)/10=+\,0.6$

3rd lens: $\frac{1}{26}+\frac{1}{s'}=\frac{1}{20}$ $\quad s'=520/6 \quad m_3=-\,520/(6\times 26)=-\,\frac{10}{3}$

$m_T=m_1\,m_2\,m_3=+\,2$

(c) $f_1=-\,10\,\text{cm},\ f_2=+\,15\,\text{cm},\ f_3=-\,20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{-\,10}$ $\quad s'=20/3 \quad m_1=-\,(-\,20)/3\,(20)=\frac{1}{3}$

2nd lens: $\frac{3}{110}+\frac{1}{s'}=\frac{1}{15}$ $\quad s'=330/13 \quad m_2=-\,\frac{(330)\,(3)}{(13)\,(110)}=-\,\frac{9}{13}$

3rd lens: $\frac{-\,13}{70}+\frac{1}{s'}=\frac{1}{-\,20}$ $\quad s'=140/19 \quad m_3=-\,\frac{(140)\,(13)}{(19)\,(-\,70)}=\frac{26}{19}$

$m_T=m_1\,m_2\,m_3=-\,6/19$

2-24. Using the lensmaker's formula, $\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ gives

in air: $\frac{1}{30}=\frac{1.50-1}{1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

and in the liquid: $-\,\frac{1}{188}=\frac{1.50-n_L}{n_L}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

Dividing the two equations gives, $\frac{-\,188}{30}=\frac{0.5n_L}{1.5-n_L}$ or $n_L=1.63$.

2-25. Use the lensmaker's formula to find the focal length of the lens,

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1.5-1}{1}\left(0+\frac{1}{60}\right)\Rightarrow f=120\,\text{cm}$$

The Newtonian equations are, $m=-\frac{f}{x}=-\frac{x'}{f}$. For $m=-4$, .

$$-4=-\frac{f}{x}=-\frac{120}{x} \text{ or } x=30\text{cm}$$
$$-4=-\frac{x'}{f}=-\frac{x'}{120} \text{ or } x'=480\text{cm}$$

Thus, $s=x+f=30+120=150\text{cm}$ and $s'=x'+f=480+120=600\text{cm}$.

Check: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{150}+\frac{1}{600}=\frac{1}{120}=\frac{1}{f}$

2-26. (a) $f_1=10\,\text{cm}\Rightarrow P_1=\frac{1}{0.01}=+10\,D$, $f_2=20\,\text{cm}\Rightarrow P_2=\frac{1}{0.2}=+5\,D$, $f_3=-40\,\text{cm}\Rightarrow P_3=\frac{1}{-0.4}=-2.5\,D$

Then, $P=P_1+P_2+P_3=10+5-2.5=+12.5\,D$

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, $V+V'=P$, where $V=\frac{1}{s}=\frac{1}{0.12}=+8.33\,D$,

$V'=4.167\,D$ or $s'=\frac{1}{V'}=\frac{1}{4.167}=0.24\,\text{m}=24\text{cm}$

2-27. See Figure 2-38 in the text. The applicable relations are:

Lens equations: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$,

Geometrical: $L=s_1+s_1'=s_2+s_2'$, $D=s_2-s_1=s_1'-s_2'$
Thus,

$$f=\frac{s_1\,s_1'}{s_1+s_1'}=\frac{s_1\,s_1'}{L}=\frac{s_2\,s_2'}{s_2+s_2'}=\frac{s_2\,s_2'}{L} \tag{1}$$

Because the lens equation can be satisfied the second time by simply interchanging object and image distances,

$$s_2=s_1' \text{ and } s_2'=s_1 \tag{2}$$

Adding and subtracting the equations $L=s_2+s_2$ and $D=-s_1+s_2$, we get,
$L-D=2s$ and $L+D=2s_2$. Their product is by Eq. (1), $L^2-D^2=4\,s_1\,s_2$, or by Eq. (2), $L^2-D^2=4\,fL$.
Thus, $f=\frac{L^2-D^2}{4\,L}$.

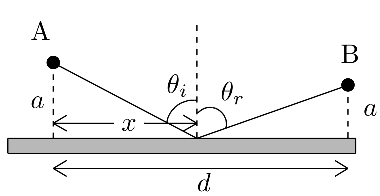
2-28. Lens equations: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$. Then calculate,

$$\frac{1}{M_1}-\frac{1}{M_2}=-\frac{s_1}{s_1'}+\frac{s_2}{s_2'}=-\frac{s_1}{s_1\,f/(s_1-f)}+\frac{s_2}{s_2\,f/(s_2-f)}=\frac{s_2-f}{f}-\frac{s_1-f}{f}=\frac{s_2-s_1}{f}$$

Thus,

$$f=\frac{s_2-s_1}{1/M_1-1/M_2}$$

2-29. Consider an arbitrary path from point A to point B by reflection from a mirror surface,

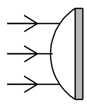


The path distance D from A to B is $D=\sqrt{a^2+x^2}+\sqrt{b^2+(d-x)^2}$

$$\frac{dD}{dx}=\frac{x}{\sqrt{a^2+x2}}+\frac{-(d-x)}{\sqrt{b^2+(d-x)^2}}=0$$

$\sin\theta_i-\sin\theta_r=0\Rightarrow\theta_i=\theta_r$

2-30. The two set-ups are illustrated below,

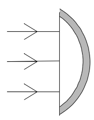


Refraction at curved side: $\frac{1}{\infty}+\frac{n}{s'}=\frac{n-1}{R}$; $s' - \frac{n\,R}{n-1}$

Reflection at plane side: $s' = -\,s = -\,\frac{n\,R}{n-1}$

Refraction at curved side: $\frac{n}{-\,n\,R/(n-1)}+\frac{1}{s'}=\frac{1-n}{-\,R}$ or $s' = R/2\,(n-1)$

Thus, $f_1 = \frac{R}{2\,(n-1)}$



Reflection at curved face: $\frac{1}{\infty}+\frac{1}{s'}=-\,\frac{2}{R}$; $s' = -\,\frac{R}{2}$

Refraction at plane face: $\frac{n}{-\,R/2}+\frac{1}{s'}=0$; $s' = \frac{R}{s\,n}$

Refraction at plane face: $\frac{n}{-\,R/2}+\frac{1}{s'}=0$; $s' = \frac{R}{s\,n}$

Thus, $f_2 = R/2\,n$

Therefore the ratio of the focal lengths is $\frac{f_1}{f_2}=\frac{R/2\,(n-1)}{R/2\,n}=\frac{n}{n-1}$.

2-31. The distance between the object and the image is $D = s + s' = s + \frac{f\,s}{s-f}$. This is minimized when,

$\frac{d\,D}{d\,s} = 1 + \frac{(s-f)\,f - f\,s}{(s-f)^2} = 0 \Rightarrow s\,(s-2\,f) = 0 \Rightarrow s = \emptyset,\ 2\,f$. The minimum distance D occurs when $s = 2f$ and has the value $D = 2f + \frac{f\,(2f)}{2f-f} = 4f$. That is, in this configuration $s = s' = 2f$.

2-32. Refer to Figure 2-39 in the text.

(a) Let the angle with the normal to the interface in each region of index of refraction n_i be θ_i. Then applying Snell's law sequentially at each interface leads to,

$$n_0\sin\theta_0 = n_1\sin\theta_1 = n_2\sin\theta_2 \ldots = n_i\sin\theta_i \ldots = n_f\sin\theta_f$$

That is,

$$n_0\sin\theta_0 = n_f\sin\theta_f$$

(b) In each medium the lateral displacement is $t_i\tan\theta_i$. The total lateral displacement y due to N media can be written as,

$$y = \sum_{i=1}^{n} t_i\tan\theta_i$$

where $\sin\theta_i = (n_0/n_i)\sin\theta_0$.

2-33. At each surface use the relation,

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

For light incident first on the plane side:

1st surface (plane): no change, 2nd surface (curved): $\frac{1.5}{\infty}+\frac{1}{s'}=\frac{1-1.5}{-\,4}$ or $s' = 8\,\text{cm}$.

For light incident first on the curved side:

1st surface (curved): $\frac{1}{\infty}+\frac{1.5}{s'}=\frac{1.5-1}{4}$ or $s' = 12\text{cm}$.

2nd surface (plane): object distance $= 4 - 12 = -\,8\,\text{cm}$ (virtual), $\frac{1.5}{-\,8}+\frac{1}{s'}=\frac{1-1.5}{\infty}$ or $s' = 5.33\,\text{cm}$

2-34. The focal length is the image position for incident parallel light rays (object at ∞). In all cases the following relation is to be used

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}\Rightarrow\frac{n_1}{\infty}+\frac{n_2}{f}=\frac{n_2-n_1}{R}\Rightarrow\frac{n_2}{f}=\frac{n_2-n_1}{R}$$

For the situation in which the center of curvature in medium with $n=4/3$:

For light incident from the medium of index 1: $\frac{4/3}{f}=\frac{4/3-1}{10}$ or $f=+40\,\text{cm}$

For light incident from the medium of index 4/3: $\frac{1}{f}=\frac{1-4/3}{-10}$ or $f=+30\,\text{cm}$

For the situation in which the center of curvature is in the medium with $n=1$,

For light incident from the medium of index 4/3: $\frac{4/3}{\infty}+\frac{1}{f}=\frac{1-4/3}{10}$ or $f=-30\,\text{cm}$

For light incident from the medium of index 1: $\frac{1}{\infty}+\frac{4/3}{f}=\frac{4/3-1}{-10}$ or $f=-40\,\text{cm}$

2-35. $|m|=\frac{s}{s'}=\frac{1}{50,000}=\frac{f}{s}=\frac{6\,\text{in}}{s}$, since $s'=f$. So $s=50,000\times 6\,\text{in}=25,000\,\text{ft}$

2-36. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.60-1.0}{5\,\text{cm}}\Rightarrow f=8.33\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{8.33\,\text{cm}}-\frac{1}{15\,\text{cm}}\Rightarrow s'=18.73\,\text{cm}$$

Then, Eq. (2-37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{15+18.73}{15}7\,\text{cm}=15.75\,\text{cm}$$

The line image is real, 18.75 cm past the lens and 15.75 cm long.

2-37. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.52-1.0}{15\,\text{cm}}\Rightarrow f=28.85\,\text{cm}$$

The image distance is then found as, $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{28.85\,\text{cm}}-\frac{1}{20\,\text{cm}}\Rightarrow s'=-65.2\,\text{cm}$

Then, Eq. (2-37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{20-65.2}{20}2.5\,\text{cm}=-5.65\,\text{cm}$$

The line image is virtual, 65.2 cm from the lens on the object side of the lens and 5.65 cm long.

2-38. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f} = \frac{n_2 - n_1}{R} = \frac{1.5 - 1.0}{-10\,\text{cm}} \Rightarrow f = -20\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-20\,\text{cm}} - \frac{1}{25\text{cm}} \Rightarrow s' = -11.11\,\text{cm}$$

Then, Eq. (2-37) gives,

$$AB = \frac{s + s'}{s} CL = \frac{25 - 11.11}{25} 5\,\text{cm} = 2.78\,\text{cm}$$

The line image is virtual, 11.11 cm from the lens on the object side of the lens and 2.78 cm long.

2-39. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f} = \frac{n_2 - n_1}{R} = \frac{1.5 - 1.0}{-20\,\text{cm}} \Rightarrow f = -40\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-40\,\text{cm}} - \frac{1}{20\text{cm}} \Rightarrow s' = -13.33\,\text{cm}$$

Then, Eq. (2-37) gives,

$$AB = \frac{s + s'}{s} CL = \frac{20 - 13.33}{20} 2\,\text{cm} = 0.667\,\text{cm}$$

The line image is virtual, 13.33 cm from the lens on the object side of the lens and 0.67 cm long.

2-40. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f} = \frac{n_2 - n_1}{R} = \frac{1.60 - 1.0}{5\,\text{cm}} \Rightarrow f = 8.33\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{8.33\,\text{cm}} - \frac{1}{6\,\text{cm}} \Rightarrow s' = -21.45\,\text{cm}$$

Then, Eq. (2-37) gives,

$$AB = \frac{s + s'}{s} CL = \frac{6 - 21.45}{6} 7\,\text{cm} = -18.0\,\text{cm}$$

The line image is virtual, 21.45 cm from the lens on the object side of the lens and 18.0 cm long.