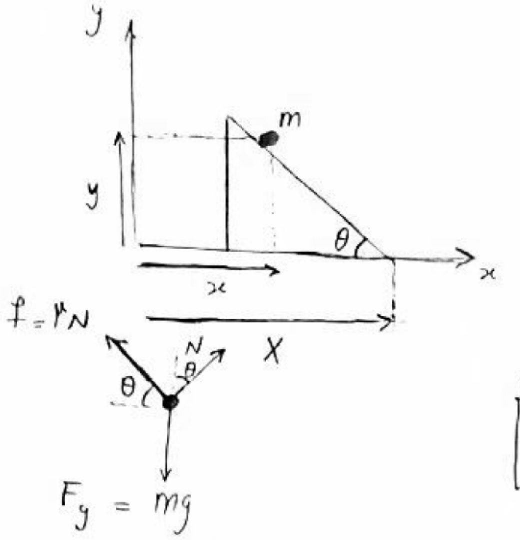
1-الف)



$$\frac{y}{X-x} = \tan\theta \Rightarrow a_y = -a_x \tan\theta \quad (1)$$

$$\begin{cases} N\cos\theta + \mu N\sin\theta - mg = ma_y \\ N\sin\theta - \mu N\cos\theta = ma_x \end{cases} \Rightarrow \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{a_x}{a_y + g} \quad (2)$$

$$(1), (2) \Rightarrow \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{a_x}{g - a_x\tan\theta} \Rightarrow a_x(\cos\theta + \mu\sin\theta) = g(\sin\theta - \mu\cos\theta) - a_x\tan\theta(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow a_x = g\cos\theta(\sin\theta - \mu\cos\theta) \overset{(1)}{\Rightarrow} a_y = -g\sin\theta(\sin\theta - \mu\cos\theta)$$

$$\Delta\vec{r} = \left(\frac{1}{2}a_x t^2 + V t\right)\hat{i} + \left(\frac{1}{2}a_y t^2\right)\hat{j}$$

$$\Rightarrow \boxed{\Delta\vec{r} = \left[\frac{gt^2}{2}\cos\theta(\sin\theta - \mu\cos\theta) + Vt\right]\hat{i} - \frac{gt^2}{2}\sin\theta(\sin\theta - \mu\cos\theta)\hat{j}}$$

ب)

$$\boxed{\vec{v} = \left[gt\cos\theta(\sin\theta - \mu\cos\theta) + V\right]\hat{i} - gt\sin\theta(\sin\theta - \mu\cos\theta)\hat{j}}$$

پ)

$$K_1 = \frac{1}{2}mV^2$$

$$K_2 = \frac{1}{2}mv^2 = \frac{1}{2}m\left[V^2 + g^2t^2(\sin\theta - \mu\cos\theta)^2 + 2Vgt\cos\theta(\sin\theta - \mu\cos\theta)\right]$$

$$\Delta K = K_2 - K_1 \Rightarrow \boxed{\Delta K = \frac{mg^2t^2}{2}(\sin\theta - \mu\cos\theta)^2 + mVgt\cos\theta(\sin\theta - \mu\cos\theta)}$$

ت)

$$\boxed{\vec{F_g} = -mg\hat{j}}$$

$$|\vec{N}| = mg\cos\theta \Rightarrow \boxed{\vec{N} = mg\cos\theta(\cos\theta\hat{j} + \sin\theta\hat{i})}$$

$$|\vec{f}| = \mu mg\cos\theta \Rightarrow \boxed{\vec{f} = \mu mg\cos\theta(\sin\theta\hat{j} - \cos\theta\hat{i})}$$

ت)

$$W_{F_g} = -mg\Delta y \Rightarrow \boxed{W_{F_g} = \frac{mg^2t^2}{2}\sin\theta(\sin\theta - \mu\cos\theta)}$$

$$W_N = mg\cos\theta\,(\cos\theta\,\Delta y + \sin\theta\,\Delta x) \Rightarrow \boxed{W_N = mgVt\sin\theta\cos\theta}$$

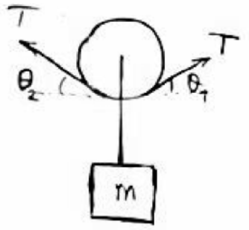
$$W_f = \mu mg\cos\theta(\sin\theta\,\Delta y - \cos\theta\,\Delta x) \Rightarrow \boxed{W_f = -\mu mg\cos\theta\left(Vt\cos\theta + \frac{gt^2}{2}(\sin\theta - \mu\cos\theta)\right)}$$

ج)

$$N = W_{F_g} + W_f + W_N = -\mu mgVt\cos^2\theta + \frac{mg^2t^2}{2}(\sin\theta - \mu\cos\theta)^2 + mgVt\sin\theta\cos\theta$$

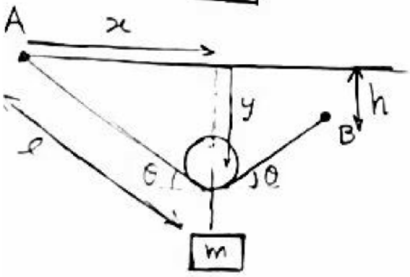
$$\Rightarrow \boxed{W = \frac{mg^2t^2}{2}(\sin\theta - \mu\cos\theta)^2 + mgVt\cos\theta(\sin\theta - \mu\cos\theta)} \Rightarrow \boxed{W = \Delta K}$$

2-آ)



$$T\cos\theta_1 = T\cos\theta_2 \Rightarrow \theta_1 = \theta_2 = \theta$$

$$\Rightarrow 2T\sin\theta = mg \Rightarrow T = \frac{mg}{2\sin\theta} \quad (1)$$



$$\cos\theta = \frac{x}{\ell} = \frac{d-x}{L-\ell} \Rightarrow xL - x\ell = d\ell - x\ell \Rightarrow \frac{x}{\ell} = \frac{d}{L}$$

$$\Rightarrow \cos\theta = \frac{d}{L} \Rightarrow \sin\theta = \frac{\sqrt{L^2-d^2}}{L} \overset{(1)}{\Rightarrow} \boxed{T = \frac{mgL}{2\sqrt{L^2-d^2}}}$$

ب)

$$\sin\theta = \frac{y}{\ell} = \frac{y-h}{L-\ell} \Rightarrow yL - \ell y = \ell y - h\ell \Rightarrow y(L-2\ell) = -h\ell \Rightarrow y = \frac{h\ell}{2\ell - L}$$

$$x^2 + y^2 = \ell^2 \Rightarrow \frac{d^2}{L^2} + \frac{h^2}{(2\ell-L)^2} = 1 \Rightarrow \frac{h}{2\ell-L} = \frac{\sqrt{L^2-d^2}}{L} \Rightarrow 2\ell - L = \frac{hL}{\sqrt{L^2-d^2}}$$

$$\Rightarrow \ell = \frac{L}{2}\left(1 + \frac{h}{\sqrt{L^2-d^2}}\right) \Rightarrow \boxed{(x,y) = \left(\frac{d}{2}\left(1 + \frac{h}{\sqrt{L^2-d^2}}\right), \frac{\sqrt{L^2-d^2}}{2}\left(1 + \frac{h}{\sqrt{L^2-d^2}}\right)\right)}$$

پ)

$$L - L_0 = \frac{T}{R}L_0 \Rightarrow R = \frac{TL_0}{L-L_0} \Rightarrow \boxed{R = \frac{mgLL_0}{2(L-L_0)\sqrt{L^2-d^2}}}$$

3 - آ)

$$\boxed{T = \frac{x}{v} + \frac{\sqrt{x^2+d^2}}{w}}$$

ب)

$$\boxed{T = t + \frac{\sqrt{d^2+v^2t^2}}{w}}$$

پ)

$$\boxed{X = vt + \frac{v}{w}\sqrt{d^2+v^2t^2}}$$

ت)

$$\left.\frac{dT}{dx}\right|_{x=x_0} = 0 \Longrightarrow \frac{1}{v} + \frac{x_0}{w\sqrt{x_0^2+d^2}} = 0 \Longrightarrow \frac{x_0^2}{x_0^2+d^2} = \frac{w^2}{v^2} \Longrightarrow \boxed{x_0 = \frac{-dw}{\sqrt{v^2-w^2}}}$$

$$T_0 = -\frac{dw}{v\sqrt{v^2-w^2}} + \frac{dv}{w\sqrt{v^2-w^2}} \Longrightarrow \boxed{T_0 = \frac{d\sqrt{v^2-w^2}}{wv}}$$

ث)

$$d = \frac{wvT_0}{\sqrt{v^2-w^2}}$$

$$T - \frac{x}{v} = \frac{\sqrt{x^2+d^2}}{w} \Longrightarrow x^2\,\frac{w^2-v^2}{w^2v^2} - \frac{2T}{v}x + T^2 + \frac{v^2T_0^2}{w^2-v^2} = 0$$

$$\Longrightarrow x = \frac{\frac{T}{v} \pm \sqrt{\frac{T^2}{v^2} - \frac{T_0^2}{w^2} - T^2\,\frac{w^2-v^2}{w^2v^2}}}{\frac{w^2-v^2}{w^2v^2}} \Longrightarrow \boxed{x = \frac{w^2v^2}{w^2-v^2}\left[\frac{T}{v} \pm \frac{\sqrt{T^2-T_0^2}}{w}\right]}$$

4 - آ)

$$[a_0]^{\alpha}[\rho]^{\beta}[g]^{\gamma}[\sigma]^{\delta} = 1$$

$$[\Delta F] = [\sigma][\Delta L] \Longrightarrow [\sigma] = MT^{-2} \Longrightarrow L^{\alpha-3\beta+\gamma}\,T^{-2\gamma-2\delta}\,M^{\beta+\delta} = 1$$

$$\Longrightarrow \delta = \frac{-\alpha}{2},\ \gamma = \beta = \frac{\alpha}{2} \Longrightarrow \boxed{a_0 = \sqrt{\frac{\sigma}{\rho g}}}$$

ب)

$$(P-P_0)\pi R^2 = 2\pi R\sigma - \frac{2}{3}\pi R^3\rho g \Longrightarrow \rho g(R+h)R = 2\sigma - \frac{2\rho g R^2}{3} \Longrightarrow \frac{5}{3}R^2 + hR - 2a_0^2 = 0$$

$$\Longrightarrow \boxed{R = \frac{3}{10}\left[\sqrt{h^2+\frac{40}{3}a_0^2} - h\right]}$$

پ)

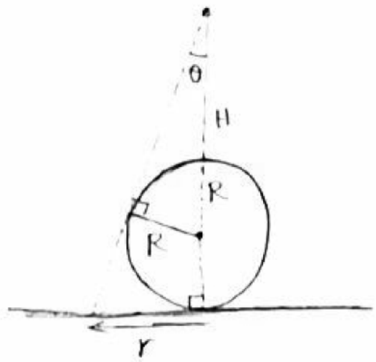
$$a_0 \ll h \Longrightarrow R = \frac{3h}{10}\left(\frac{20a_0^2}{3h^2}\right) \Longrightarrow \boxed{R = \frac{2a_0^2}{h}}$$

ت) $R = 1.2\ mm,\ h = 8\ mm$

$g = 10\ \frac{m}{s^2},\ \rho = 1\ \frac{gr}{cm^3}$

$$\sigma = \frac{\rho g R h}{2} = \frac{10^3 \times 10 \times 1.2 \times 8 \times 10^{-6}}{2} \Rightarrow \boxed{\sigma = 0.048\ \frac{N}{m}}$$

5 - آ)

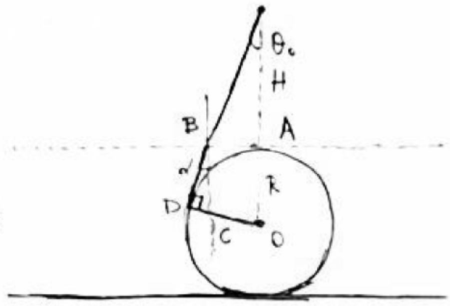


$$r = (2R+H)\tan\theta$$

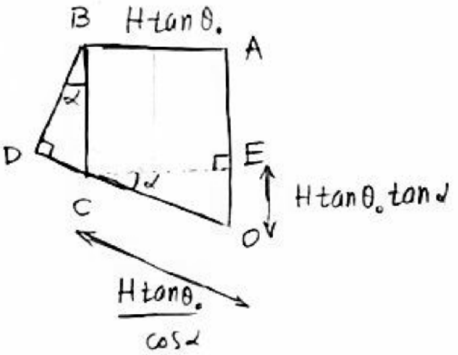
$$\sin\theta = \frac{R}{R+H} \Rightarrow \tan\theta = \frac{R}{\sqrt{H(2R+H)}} \Rightarrow r = R\sqrt{1+\frac{2R}{H}}$$

$$\Rightarrow S = \pi r^2 \Rightarrow \boxed{S = \pi R^2\left(1+\frac{2R}{H}\right)}$$

ب)



$$\sin\theta_0 = n\sin\alpha \Rightarrow \sin\alpha = \frac{\sin\theta_0}{n} \quad (1)$$



$$AE = AO - OE = R - H\tan\theta_0\tan\alpha$$

$$DC = BC\sin\alpha = R\sin\alpha - H\tan\theta_0\frac{\sin^2\alpha}{\cos\alpha}$$

$$OC + DC = OD = R \Rightarrow R(1-\sin\alpha) = H\tan\theta_0\cos\alpha \quad (2)$$

$$x := \sin\theta_0 \xRightarrow{(1),(2)} R\left(1-\frac{x}{n}\right) = H\frac{x}{\sqrt{1-x^2}}\sqrt{1-\frac{x^2}{n^2}} \Rightarrow R^2(n-x) = H^2\frac{x^2}{1-x^2}(n+x)$$

$$\Rightarrow R^2(n - nx^2 - x + x^3) = H^2(nx^2 + x^3) \Rightarrow x^3(H^2-R^2) + nx^2(H^2+R^2) + R^2x - nR^2 = 0$$

$$\Rightarrow \sin^3\theta_0 + n\frac{H^2+R^2}{H^2-R^2}\sin^2\theta_0 + \frac{R^2}{H^2-R^2}\sin\theta_0 - \frac{nR^2}{H^2-R^2} = 0 \Rightarrow \boxed{c = n\frac{H^2+R^2}{H^2-R^2}}\ ,\ \boxed{b = \frac{R^2}{H^2-R^2}}$$

$$,\ \boxed{a = -\frac{nR^2}{H^2-R^2}}$$

پ)

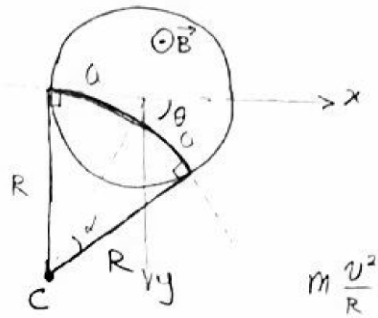
$$H = R \Rightarrow 2n\sin^2\theta_0 + \sin\theta_0 - n = 0 \Rightarrow \boxed{\sin\theta_0 = \frac{\sqrt{1+8n^2}-1}{4n}}$$

ت)

$$n=\sqrt{3} \Rightarrow \sin\theta_0=\frac{1}{\sqrt{3}} \Rightarrow \sin\alpha=\frac{1}{3} \Rightarrow \tan\theta_0=\frac{1}{\sqrt{2}}\ ,\ \tan\alpha=\frac{1}{2\sqrt{2}}$$

$$r=H\tan\theta_0+2R\tan\alpha \Rightarrow r=\frac{R}{\sqrt{2}}+2R\frac{1}{2\sqrt{2}}=R\sqrt{2} \Rightarrow \boxed{S=2\pi R^2}$$

6-آ)



$$\alpha=\frac{\pi}{2}-\left(\frac{\pi-\theta}{2}\right)=\frac{\theta}{2}$$

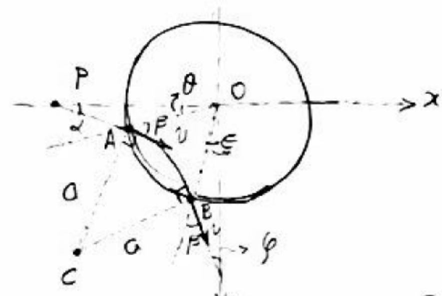
$$2R\sin\left(\frac{\theta}{2}\right)=2a\sin\left(\frac{\pi-\theta}{2}\right) \Rightarrow R=a\cot\left(\frac{\theta}{2}\right)$$

$$m\frac{v^2}{R}=qvB \Rightarrow R=\frac{mv}{qB} \Rightarrow \boxed{m=\frac{qBa}{v}\cot\left(\frac{\theta}{2}\right)}$$

ب)

$$\boxed{x_C=-a}\ ,\ \boxed{y_C=a\cot\left(\frac{\theta}{2}\right)}$$

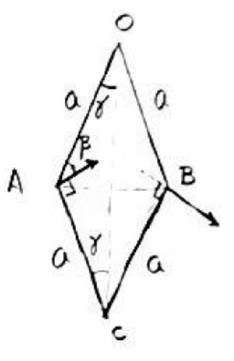
پ) چون در این بخش $\theta=\frac{\pi}{2}$ است پس $R=a$ می باشد



$$a\theta=P\alpha \Rightarrow \theta=\frac{P}{a}\alpha$$

به دلیل وجود تقارن در نقطه ی ورود و خروج زاویه با راستای شعاعی برابر و مقدار β است.

$$\beta=\alpha+\theta \Rightarrow \beta=\alpha\left(1+\frac{P}{a}\right)$$



$$2\gamma+\beta=\frac{\pi}{2} \Rightarrow \gamma=\frac{\pi}{4}-\frac{\alpha}{2}\left(1+\frac{P}{a}\right)$$

$$\epsilon+\theta+2\gamma=\frac{\pi}{2} \Rightarrow \epsilon+\theta=\beta=\alpha+\theta \Rightarrow \epsilon=\alpha$$

$$\beta=\varphi+\epsilon \Rightarrow \varphi=\frac{P\alpha}{a}$$

$$q\varphi=a\epsilon \Rightarrow \frac{qP\alpha}{a}=a\alpha \Rightarrow \boxed{q=\frac{a^2}{P}}$$

ت)

$$q'=q+a\ ,\ P'=P+a \Rightarrow (q'-a)(P'-a)=a^2 \Rightarrow q'P'=a(q'+P') \Rightarrow \boxed{\frac{1}{P'}+\frac{1}{q'}=\frac{1}{a}}$$