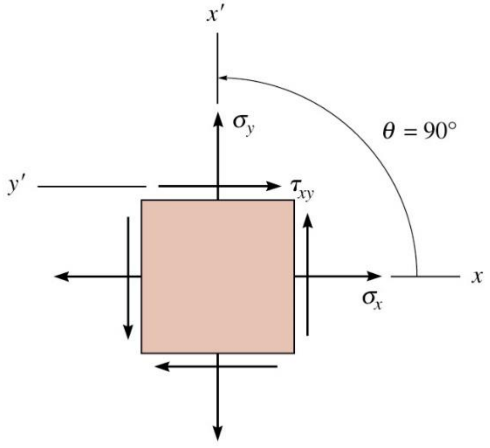
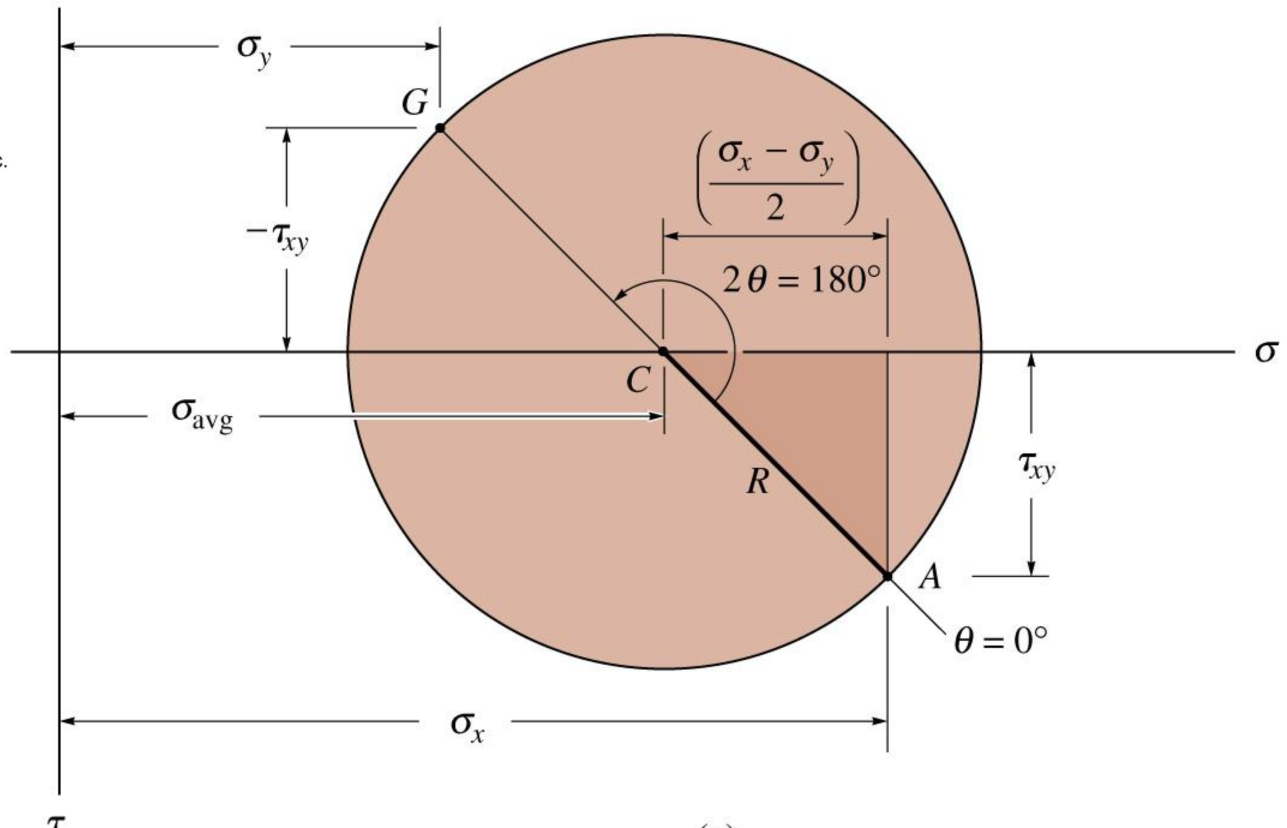
۹-۴: دایره مور (روش گرافیکی)

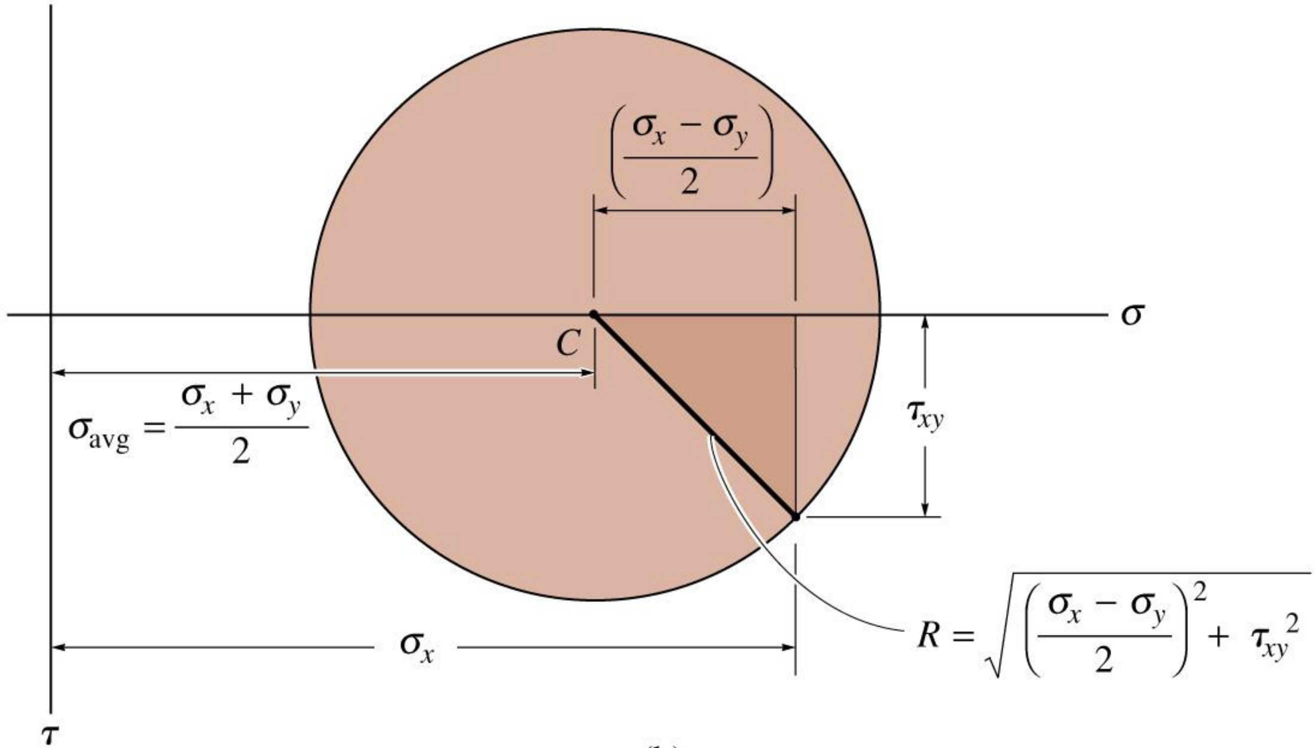


(b)



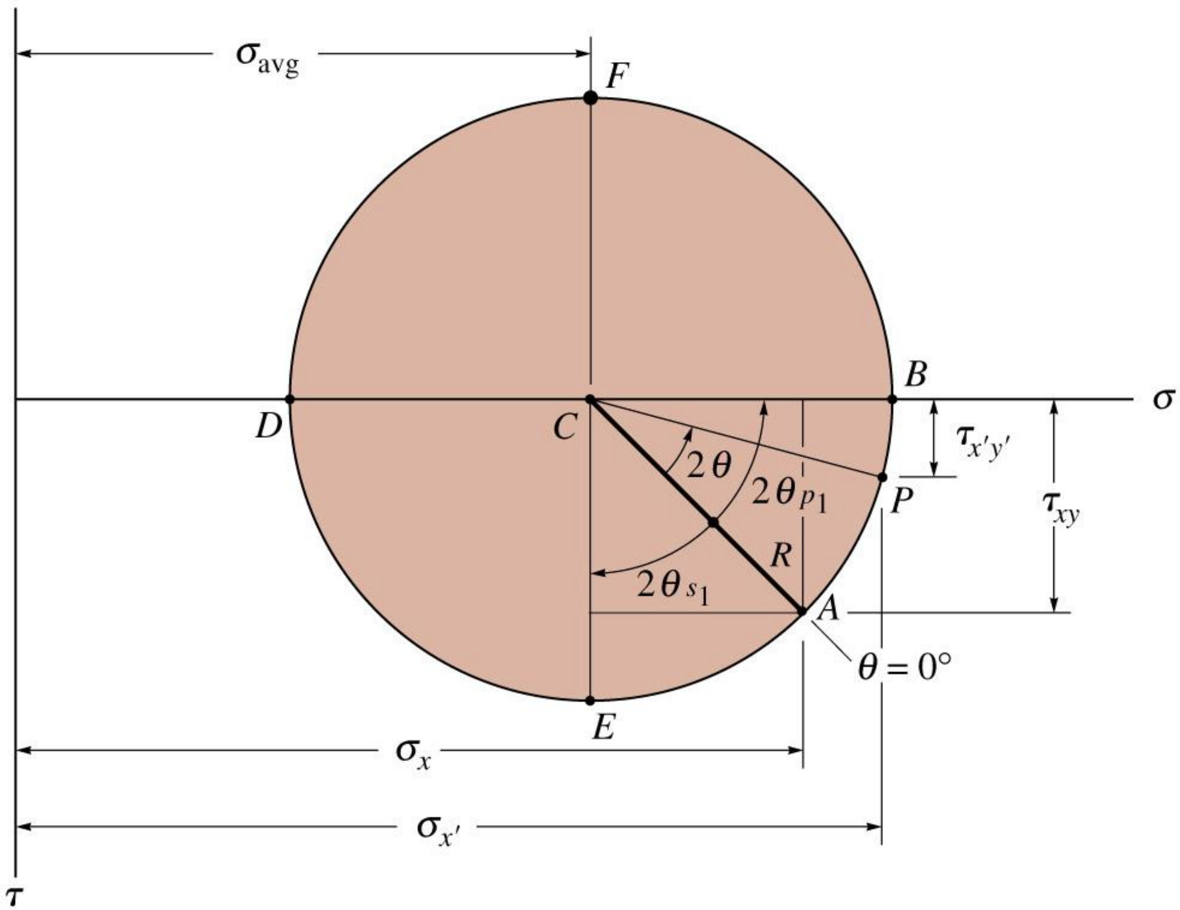
(c)

۹-۴: دایره مور (روش گرافیکی)



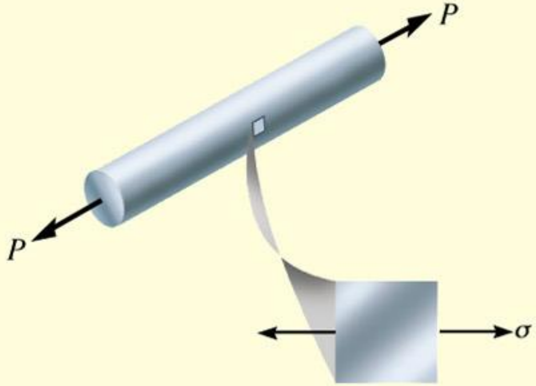
(b)

۹-۴: دایره مور (روش گرافیکی)



(a)

EXAMPLE 9.7



(a)

The axial loading P produces the state of stress in the material as shown in Fig. 9–18a. Draw Mohr's circle for this case.

Solution

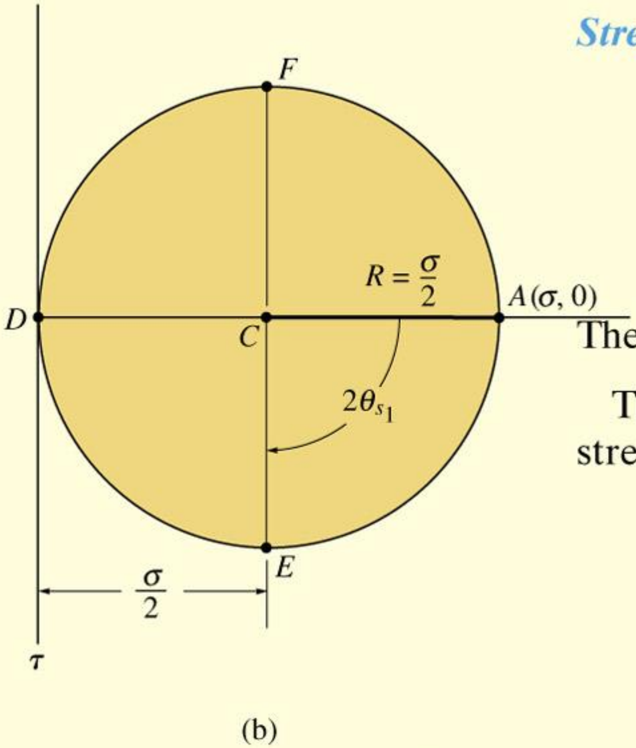
Construction of the Circle. From Fig. 9–18a,

$$\sigma_x = \sigma \qquad \sigma_y = 0 \qquad \tau_{xy} = 0$$

The σ and τ axes are established in Fig. 9–18b. The center of the circle C is on the σ axis at

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma + 0}{2} = \frac{\sigma}{2}$$

From the right-hand face of the element, Fig. 9–18a, the reference point for $\theta = 0°$ has coordinates $A(\sigma, 0)$. Hence the radius of the circle CA is $R = \sigma/2$. Fig. 9–18b.



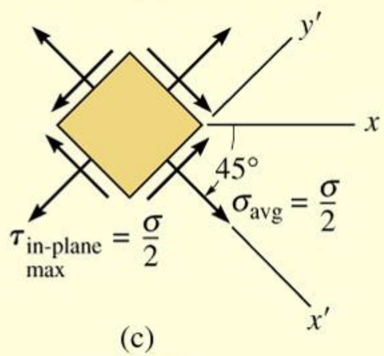


Fig. 9–18

Stresses. Note that the principal stresses are at points A and D.

$$\sigma_1 = \sigma \qquad \sigma_2 = 0$$

The element in Fig. 9–18*a* represents this principal state of stress.

The maximum in-plane shear stress and associated average normal stress is identified on the circle as point E or F, Fig. 9–18*b*. At E we have

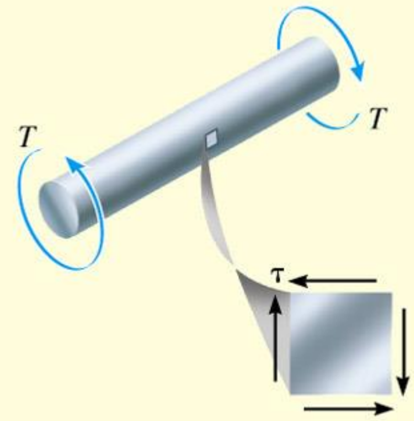
$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \frac{\sigma}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma}{2}$$

By observation, the clockwise angle $2\theta_{s_1} = 90°$. Therefore $\theta_{s_1} = 45°$, so that the x' axis is orientated 45° clockwise from the x axis; Fig. 9–18*c*. Since E has positive coordinates, then σ_{avg} and $\tau_{\substack{\text{max}\\\text{in-plane}}}$ act in the positive x' and y' directions, respectively.

EXAMPLE 9.8

The torsional loading T produces the state of stress in the shaft as shown in Fig. 9–19a. Draw Mohr's circle for this case.



(a)

Solution

Construction of the Circle. From Fig. 9–19a,

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -\tau$$

The σ and τ axes are established in Fig. 9–19b. The center of the circle C is on the σ axis at

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 0}{2} = 0$$

From the right-hand face of the element, Fig. 9–19a, the reference point for $\theta = 0°$ has coordinates $A(0, -\tau)$, Fig. 9–19b. Hence the radius CA is $R = \tau$.

Stresses. Here point A represents a point of average normal stress and maximum in-plane shear stress, Fig. 9–19b. Thus,

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = -\tau$$

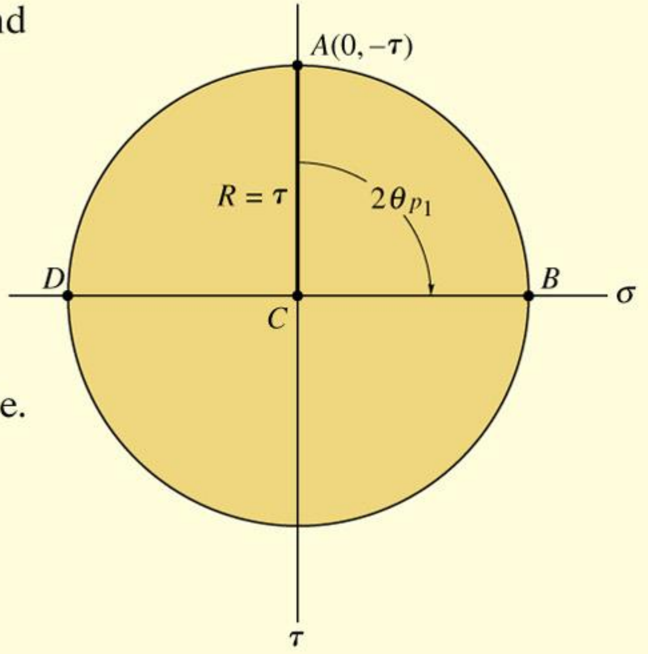
$$\sigma_{avg} = 0$$

The principal stresses are identified as points B and D on the circle. Thus,

$$\sigma_1 = \tau$$

$$\sigma_2 = -\tau$$

The clockwise angle from CA to CB is $2\theta_{p_1} = 90°$, so that $\theta_{p_1} = 45°$. This clockwise angle defines the direction of σ_1 (or the x' axis). The results are shown in Fig. 9–19c.



(b)

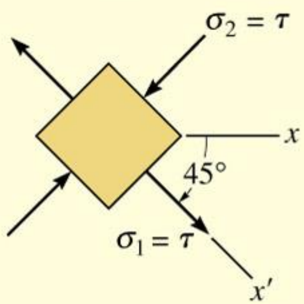
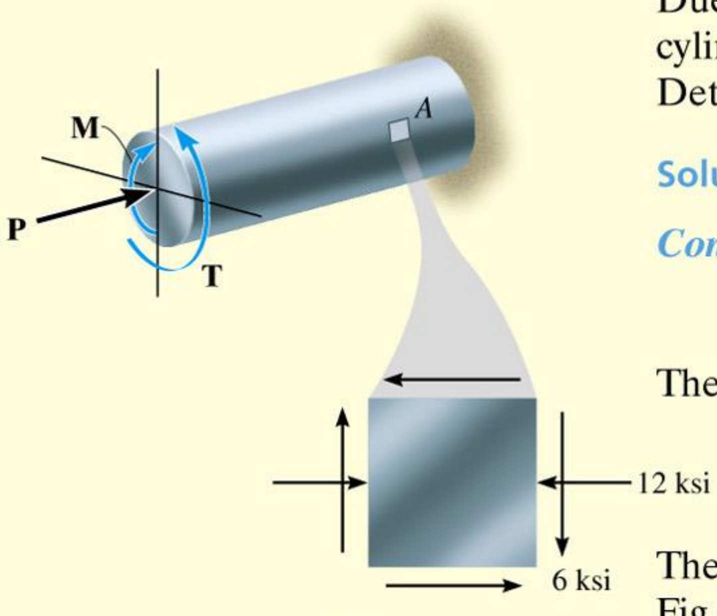


Fig. 9–19

(c)

EXAMPLE 9.9



(a)

Due to the applied loading, the element at point A on the solid cylinder in Fig. 9–20*a* is subjected to the state of stress shown. Determine the principal stresses acting at this point.

Solution

Construction of the Circle. From Fig. 9–20*a*,

$$\sigma_x = -12 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -6 \text{ ksi}$$

The center of the circle is at

$$\sigma_{avg} = \frac{-12 + 0}{2} = -6 \text{ ksi}$$

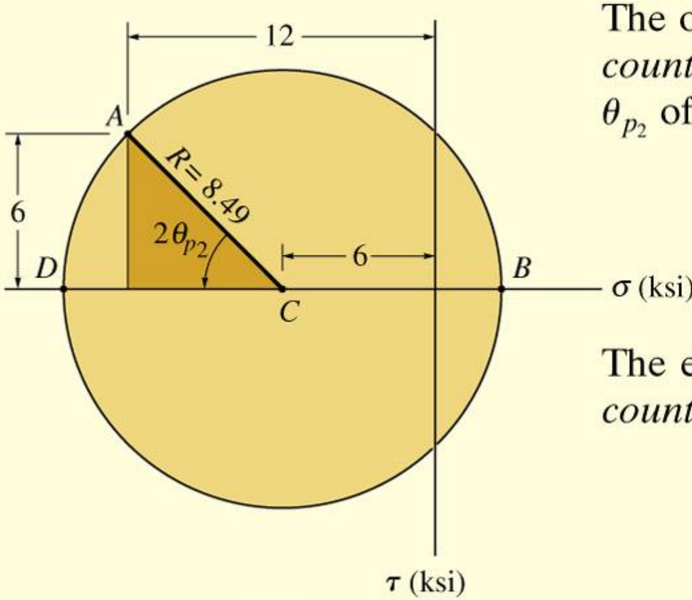
The initial point $A(-12, -6)$ and the center $C(-6, 0)$ are plotted in Fig. 9–20*b*. The circle is constructed having a radius of

$$R = \sqrt{(12 - 6)^2 + (6)^2} = 8.49 \text{ ksi}$$

Principal Stresses. The principal stresses are indicated by the coordinates of points B and D. We have, for $\sigma_1 > \sigma_2$.

$$\sigma_1 = 8.49 - 6 = 2.49 \text{ ksi} \qquad \textit{Ans.}$$

$$\sigma_2 = -6 - 8.49 = -14.5 \text{ ksi} \qquad \textit{Ans.}$$



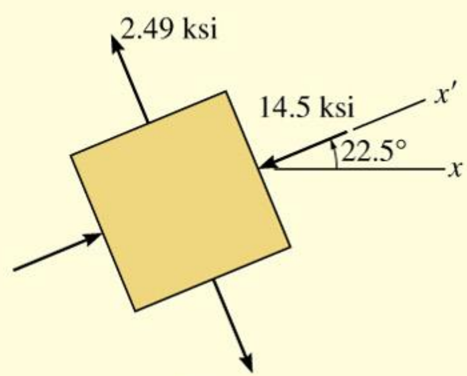
(b)

The orientation of the element can be determined by calculating the *counterclockwise* angle $2\theta_{p_2}$ in Fig. 9–20*b*, which defines the direction θ_{p_2} of σ_2 and its associated principal plane. We have

$$2\theta_{p_2} = \tan^{-1}\frac{6}{(12-6)} = 45.0^\circ$$

$$\theta_{p_2} = 22.5^\circ$$

The element is oriented such that the x' axis or σ_2 is directed 22.5° *counterclockwise* from the horizontal (x axis) as shown in Fig. 9–20*c*.

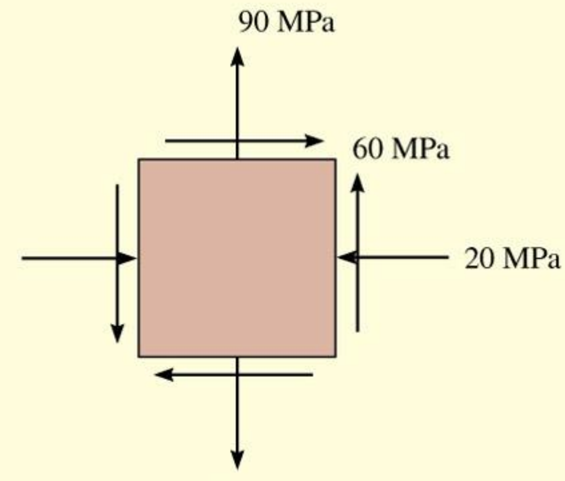


(c)

Fig. 9–20

EXAMPLE 9.10

The state of plane stress at a point is shown on the element in Fig. 9–21*a*. Determine the maximum in-plane shear stresses and the orientation of the element upon which they act.



(a)

Solution

Construction of the Circle. From the problem data,

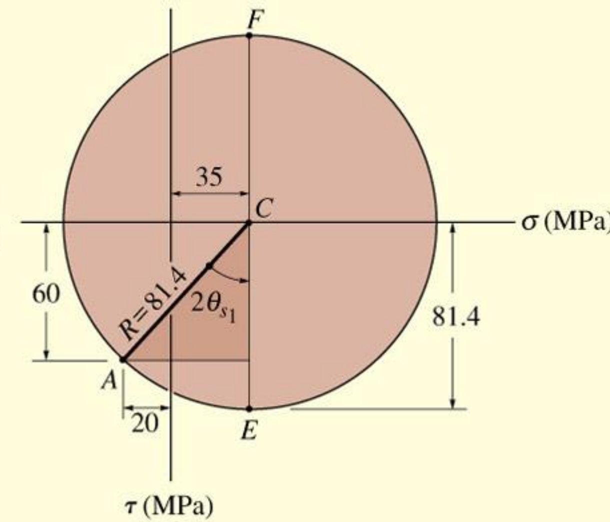
$$\sigma_x = -20 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$

The σ, τ axes are established in Fig. 9–21*b*. The center of the circle C is located on the σ axis, at the point

$$\sigma_{avg} = \frac{-20 + 90}{2} = 35 \text{ MPa}$$

Point C and the reference point $A(-20, 60)$ are plotted. Applying the Pythagorean theorem to the shaded triangle to determine the circle's radius CA, we have

$$R = \sqrt{(60)^2 + (55)^2} = 81.4 \text{ MPa}$$



(b)

Maximum In-Plane Shear Stress. The maximum in-plane shear stress and the average normal stress are identified by point E or F on the circle. In particular, the coordinates of point $E(35, 81.4)$ give

$$\tau_{\substack{\max \\ \text{in-plane}}} = 81.4 \text{ MPa} \qquad \textit{Ans.}$$

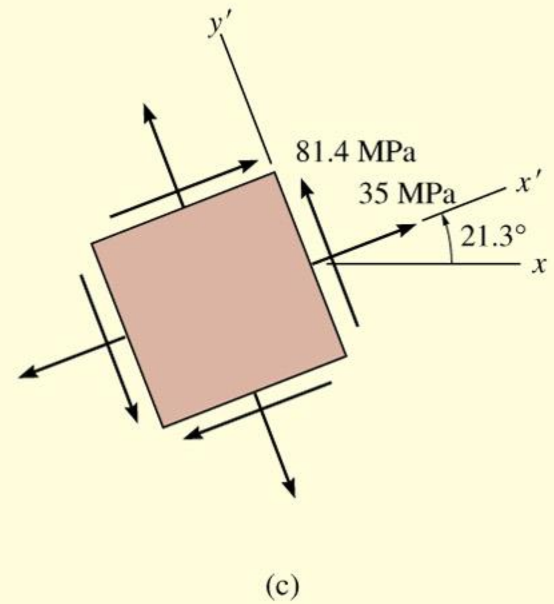
$$\sigma_{\text{avg}} = 35 \text{ MPa} \qquad \textit{Ans.}$$

The *counterclockwise* angle θ_{s_1} can be found from the circle, identified as $2\theta_{s_1}$. We have

$$2\theta_{s_1} = \tan^{-1}\left(\frac{20 + 35}{60}\right) = 42.5^\circ$$

$$\theta_{s_1} = 21.3^\circ \qquad \textit{Ans.}$$

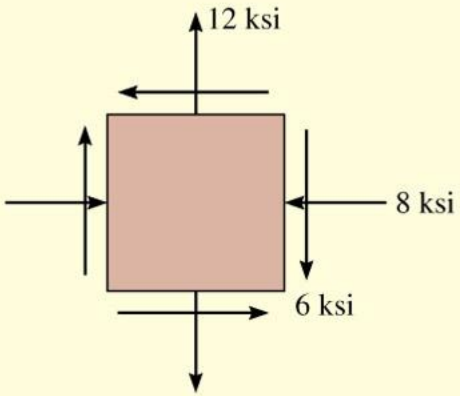
This *counterclockwise* angle defines the direction of the x' axis, Fig. 9–21c. Since point E has *positive* coordinates, then the average normal stress and the maximum in-plane shear stress both act in the *positive* x' and y' directions as shown.



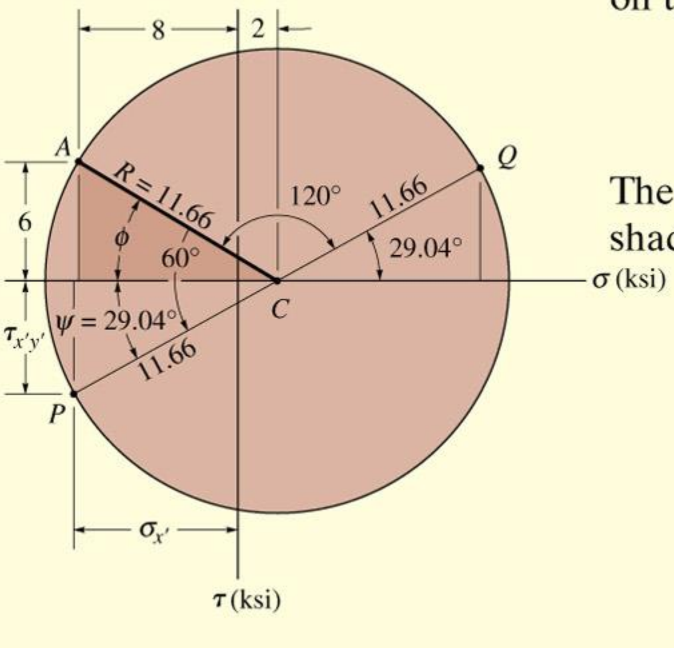
(c)

Fig. 9–21

EXAMPLE 9.11



(a)



(b)

The state of plane stress at a point is shown on the element in Fig. 9–22*a*. Represent this state of stress on an element oriented 30° counterclockwise from the position shown.

Solution

Construction of the Circle. From the problem data,

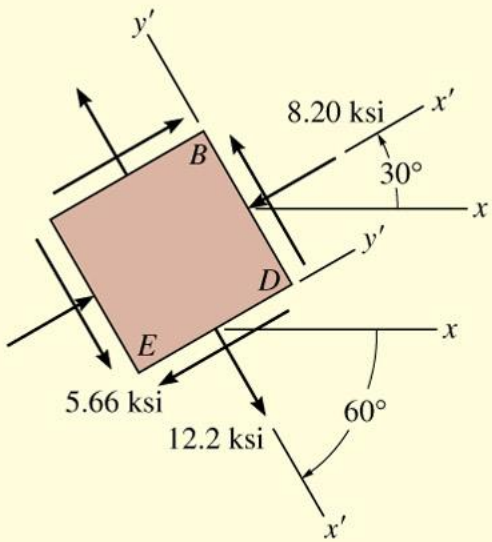
$$\sigma_x = -8 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

The σ and τ axes are established in Fig. 9–22*b*. The center of the circle C is on the σ axis at

$$\sigma_{avg} = \frac{-8 + 12}{2} = 2 \text{ ksi}$$

The initial point for $\theta = 0°$ has coordinates $A(-8, -6)$. Hence from the shaded triangle the radius CA is

$$R = \sqrt{(10)^2 + (6)^2} = 11.66$$



(c)

Fig. 9–22

Stresses on 30° Element. Since the element is to be rotated 30° *counterclockwise*, we must construct a radial line CP, $2(30°) = 60°$ *counterclockwise*, measured from $CA(\theta = 0°)$, Fig. 9–22*b*. The coordinates of point $P\ (\sigma_{x'}, \tau_{x'y'})$ must now be obtained. From the geometry of the circle,

$$\phi = \tan^{-1}\frac{6}{10} = 30.96° \qquad \psi = 60° - 30.96° = 29.04°$$

$$\sigma_{x'} = 2 - 11.66 \cos 29.04° = -8.20 \text{ ksi} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = 11.66 \sin 29.04° = 5.66 \text{ ksi} \qquad \textit{Ans.}$$

These two stress components act on face BD of the element shown in Fig. 9–22*c* since the x' axis for this face is oriented 30° counterclockwise from the x axis.

The stress components acting on the adjacent face DE of the element, which is 60° *clockwise* from the positive x axis, Fig. 9–22*c*, are represented by the coordinates of point Q on the circle. This point lies on the radial line CQ, which is 180° from CP. The coordinates of point Q are

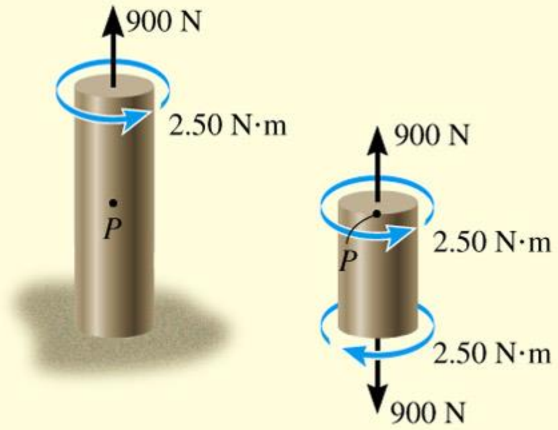
$$\sigma_{x'} = 2 + 11.66 \cos 29.04° = 12.2 \text{ ksi} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -(11.66 \sin 29.04) = -5.66 \text{ ksi} \quad \text{(check)}$$

Note that here $\tau_{x'y'}$ acts in the $-y'$ direction.

EXAMPLE 9.12

An axial force of 900 N and a torque of 2.50 N · m are applied to the shaft as shown in Fig. 9–23*a*. If the shaft has a diameter of 40 mm, determine the principal stresses at a point *P* on its surface.



(a) (b)

Solution

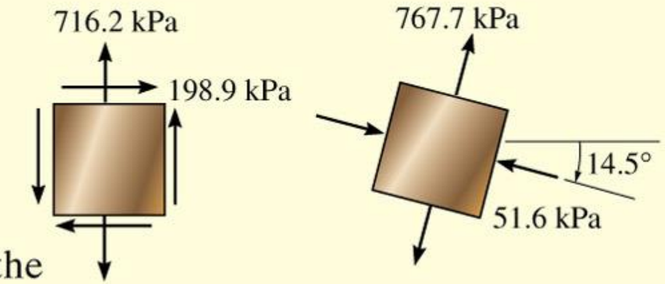
Internal Loadings. The internal loadings consist of the torque of 2.50 N · m and the axial load of 900 N, Fig. 9–23*b*.

Stress Components. The stresses produced at point *P* are therefore

$$\tau = \frac{Tc}{J} = \frac{2.50\ \text{N}\cdot\text{m}\ (0.02\ \text{m})}{\frac{\pi}{2}(0.02\ \text{m})^4} = 198.9\ \text{kPa}$$

$$\sigma = \frac{P}{A} = \frac{900\ \text{N}}{\pi(0.02\ \text{m})^2} = 716.2\ \text{kPa}$$

The state of stress defined by these two components is shown on the element at *P* in Fig. 9–23*c*.



(c) (e)

Principal Stresses. The principal stresses can be determined using Mohr's circle, Fig. 9–23*d*. Here the center of the circle C is at the point

$$\sigma_{avg} = \frac{0 + 716.2}{2} = 358.1 \text{ kPa}$$

Plotting C (358.1, 0) and the reference point A (0, 198.9), show that the radius of the circle is $R = 409.7$. The principal stresses are represented by points B and D. Therefore,

$$\sigma_1 = 358.1 + 409.7 = 767.8 \text{ kPa} \qquad \textit{Ans.}$$

$$\sigma_2 = 358.1 - 409.7 = -51.6 \text{ kPa} \qquad \textit{Ans.}$$

The clockwise angle $2\theta_{p2}$ can be determined from the circle. It is $2\theta_{p2} = 29.1°$. The element is orientated such that the x' axis or σ_2 is directed clockwise $\theta_{p1} = 14.5°$ with the x axis as shown in Fig. 9–23*e*.

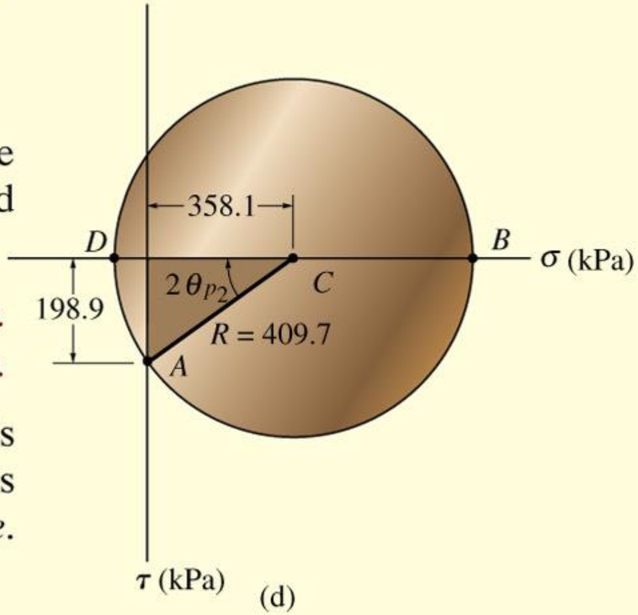
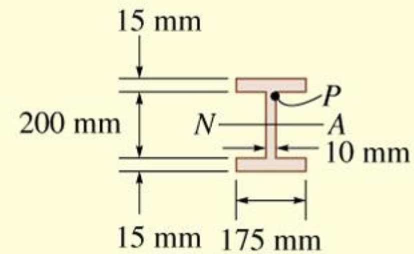
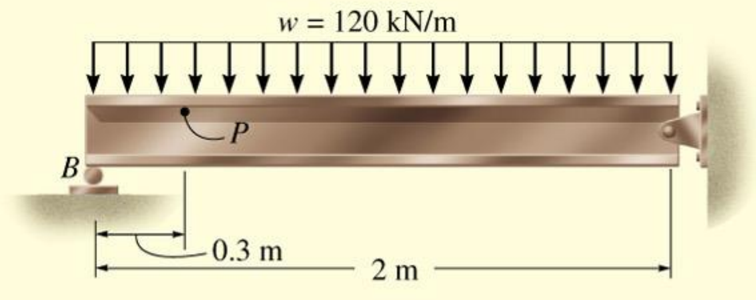


Fig. 9–23

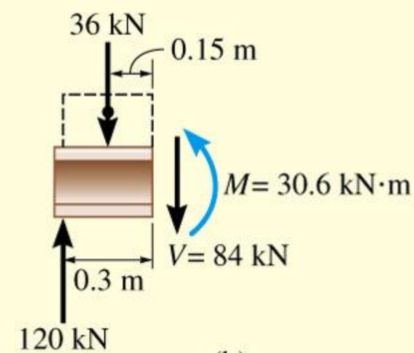
EXAMPLE 9.13

The beam shown in Fig. 9–26*a* is subjected to the distributed loading of $w = 120$ kN/m. Determine the principal stresses in the beam at point *P*, which lies at the top of the web. Neglect the size of the fillets and stress concentrations at this point. $I = 67.4(10^{-6})\ \text{m}^4$.

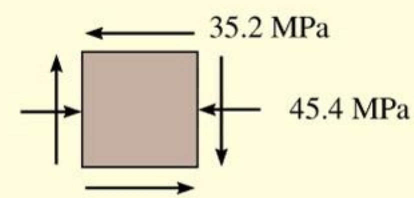




(a)



(b)



(c)

Solution

Internal Loadings. The support reaction on the beam at *B* is determined, and equilibrium of the sectioned beam shown in Fig. 9–26*b* yields

$$V = 84\ \text{kN} \qquad M = 30.6\ \text{kN} \cdot \text{m}$$

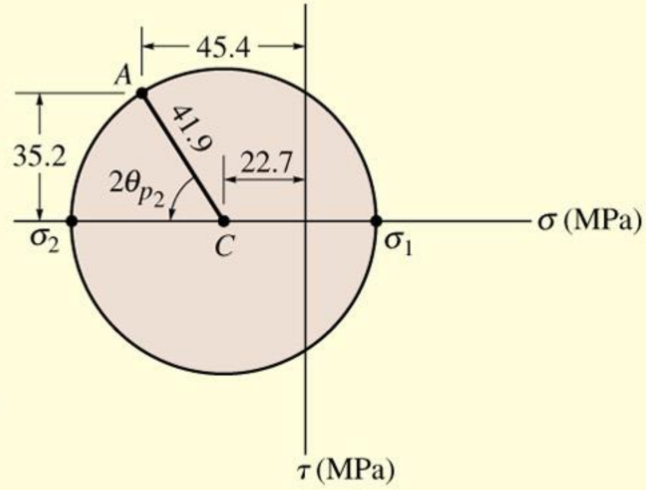
Stress Components. At point P,

$$\sigma = \frac{-My}{I} = \frac{30.6(10^3)\ \text{N}\cdot\text{m}(0.100\ \text{m})}{67.4(10^{-6})\ \text{m}^4} = -45.4\ \text{MPa} \qquad \textit{Ans.}$$

$$\tau = \frac{VQ}{It} = \frac{84(10^3)\ \text{N}[(0.1075\ \text{m})(0.175\ \text{m})(0.015\ \text{m})]}{67.4(10^{-6})\ \text{m}^4(0.010\ \text{m})}$$

$$= 35.2\ \text{MPa} \qquad \textit{Ans.}$$

These results are shown in Fig. 9–26*c*.



(d)

Principal Stresses. Using Mohr's circle the principal stresses at P can be determined. As shown in Fig. 9–26*d*, the center of the circle is at $(-45.4 + 0)/2 = -22.7$, and point A has coordinates of $A(-45.4, -35.2)$. Show that the radius is $R = 41.9$, and therefore

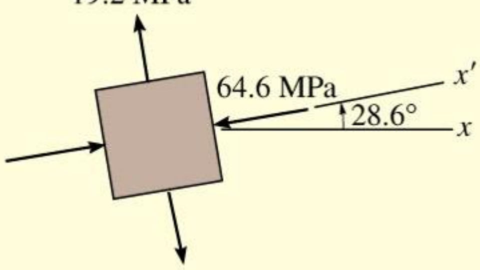
$$\sigma_1 = (41.9 - 22.7) = 19.2\ \text{MPa}$$

$$\sigma_2 = -(22.7 + 41.9) = -64.6\ \text{MPa}$$

The counterclockwise angle $2\theta_{p_2} = 57.2°$, so that

$$\theta_{p_2} = 28.6°$$

These results are shown in Fig. 9–26*e*.



(e)

Fig. 9–26

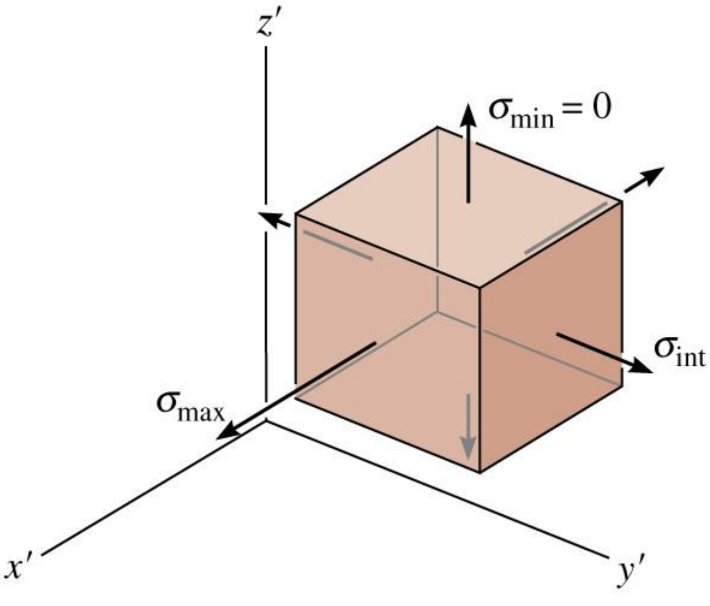
How does this compare to max normal stress at point P? How does this compare to absolute max normal stress in the beam??

۹-۶: تنش برشی ماکزیمم مطلق

- نکته: در مورد تنش های صفحه ای، اگر هر دو تنش مثبت باشند، تنش برشی ماکزیمم خارج از صفحه بوده و با رابطه مقابل بدست می آید:

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

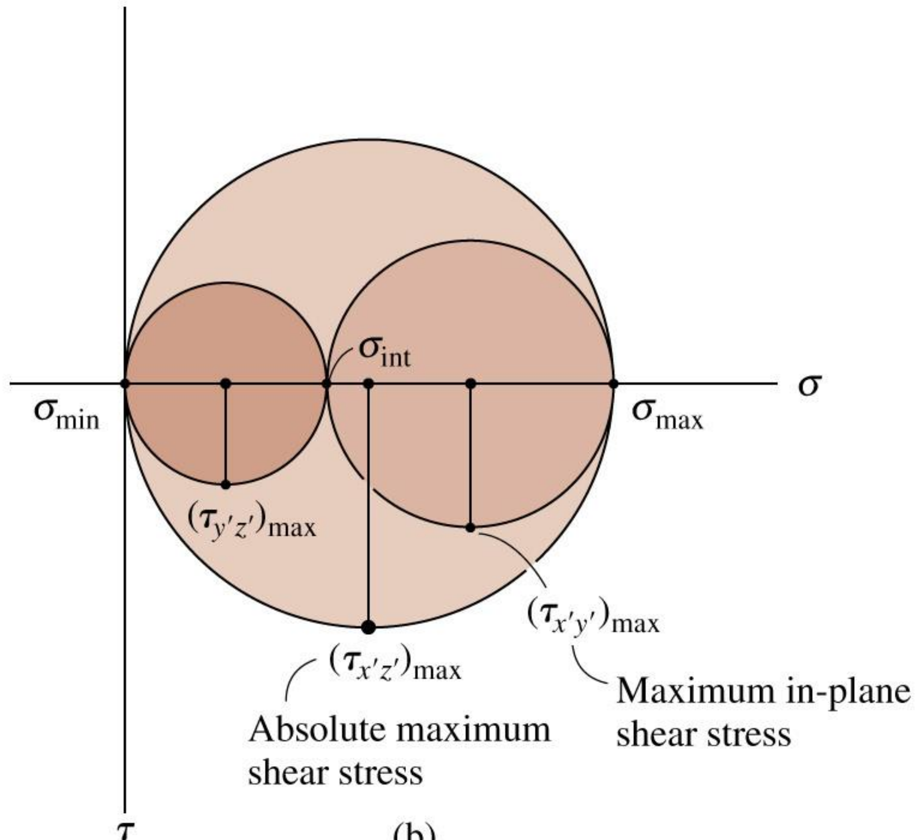
حالت اول – تنش صفحه ای



x'–y' plane stress

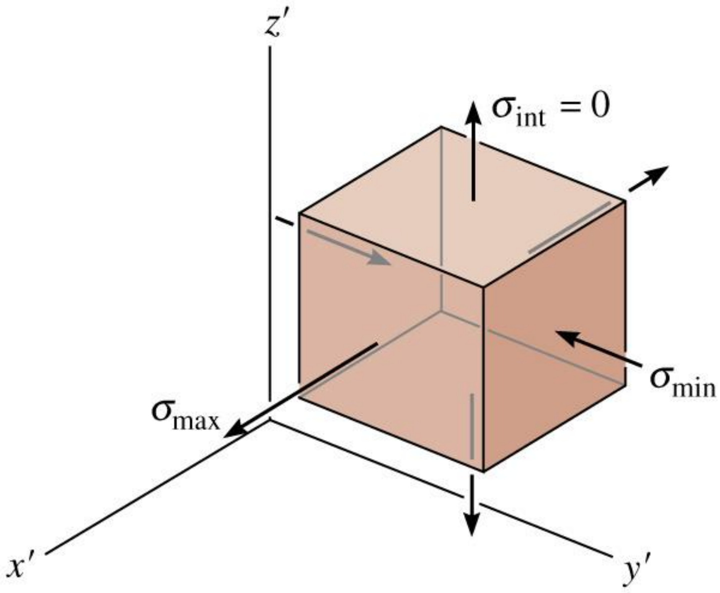
(a)

تنش برشی ماکزیمم خارج از صفحه است!



(b)

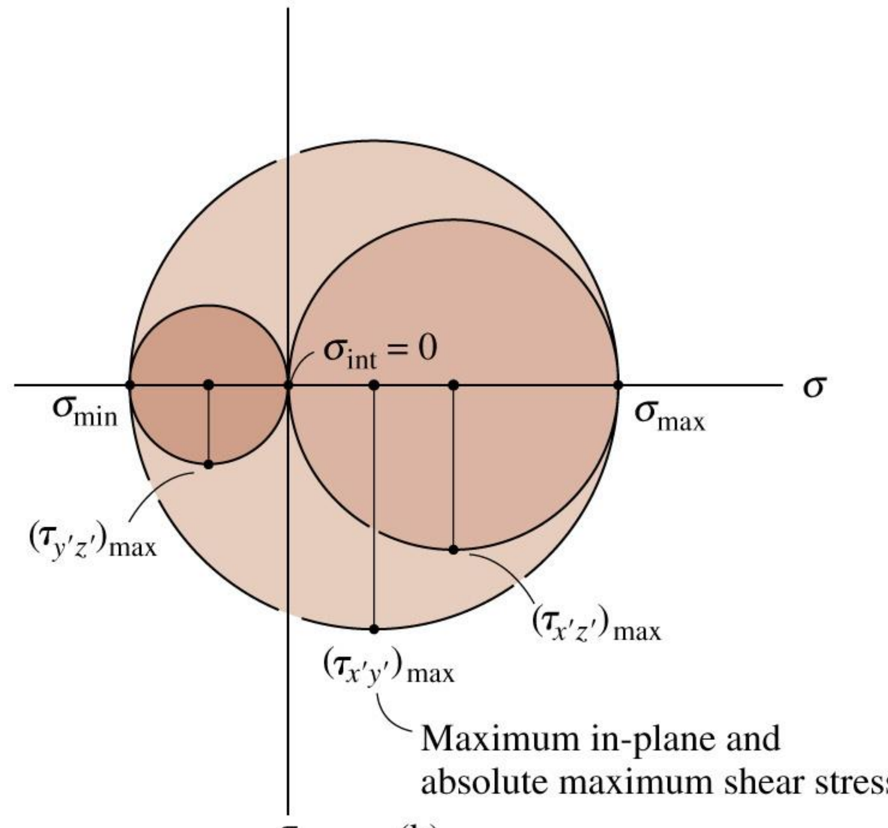
حالت دوم تنش صفحه ای



x′–y′ plane stress

(a)

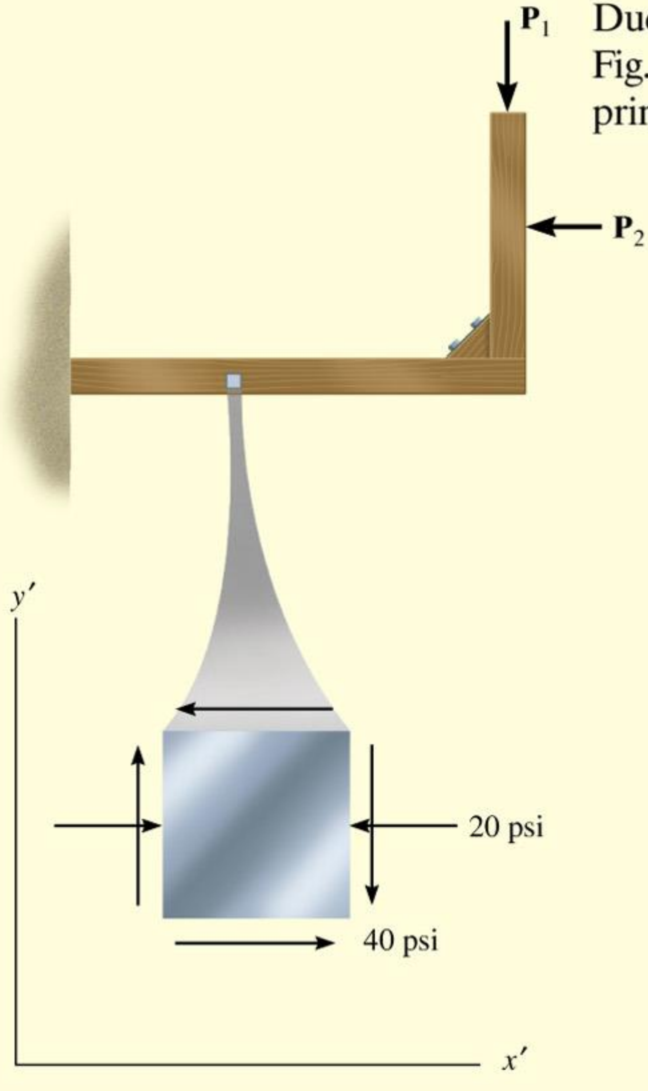
تنش برشی ماکزیمم داخل صفحه است!



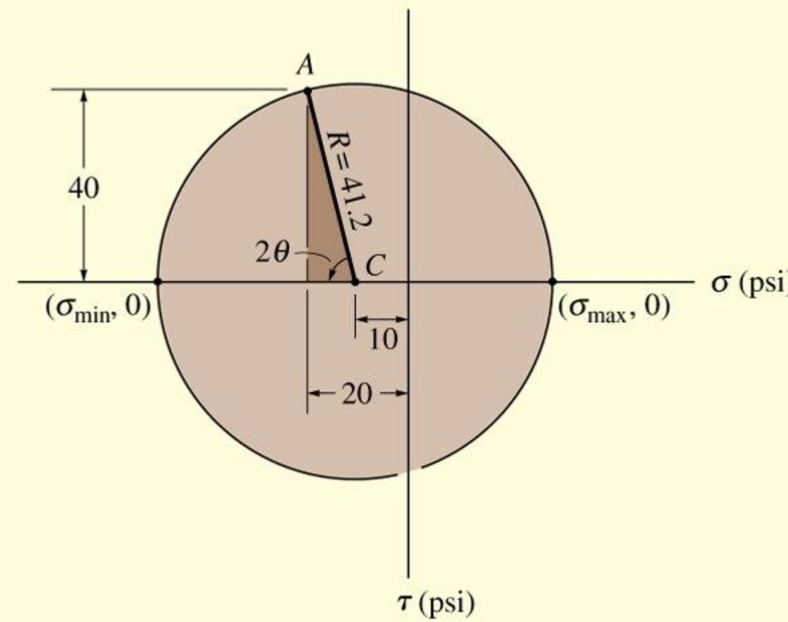
(b)

EXAMPLE 9.14

Due to the applied loading, the element at the point on the frame in Fig. 9–31*a* is subjected to the state of plane stress shown. Determine the principal stresses and the absolute maximum shear stress at the point.



(a)



(b)

Fig. 9–31

Solution

Principal Stresses. The in-plane principal stresses can be determined from Mohr's circle. The center of the circle is on the σ axis at $\sigma_{avg} = (-20 + 0)/2 = -10$ psi. Plotting the controlling point $A(-20, -40)$, the circle can be drawn as shown in Fig. 9–31*b*. The radius is

$$R = \sqrt{(20 - 10)^2 + (40)^2} = 41.2 \text{ psi}$$

The principal stresses are at the points where the circle intersects the σ axis; i.e.,

$$\sigma_{max} = -10 + 41.2 = 31.2 \text{ psi}$$
$$\sigma_{min} = -10 - 41.2 = -51.2 \text{ psi}$$

From the circle, the *counterclockwise* angle 2θ, measured from CA to the $-\sigma$ axis, is

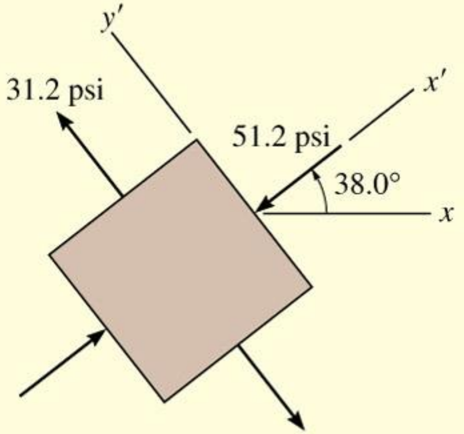
$$2\theta = \tan^{-1}\left(\frac{40}{20 - 10}\right) = 76.0^\circ$$

Thus,

$$\theta = 38.0^\circ$$

This *counterclockwise* rotation defines the direction of the x' axis or $\sigma_{\min}$ and its associated principal plane, Fig. 9–31c. Since there is no principal stress on the element in the z direction, we have

$$\sigma_{\max} = 31.2 \text{ psi} \qquad \sigma_{\text{int}} = 0 \qquad \sigma_{\min} = -51.2 \text{ psi} \qquad \textit{Ans.}$$



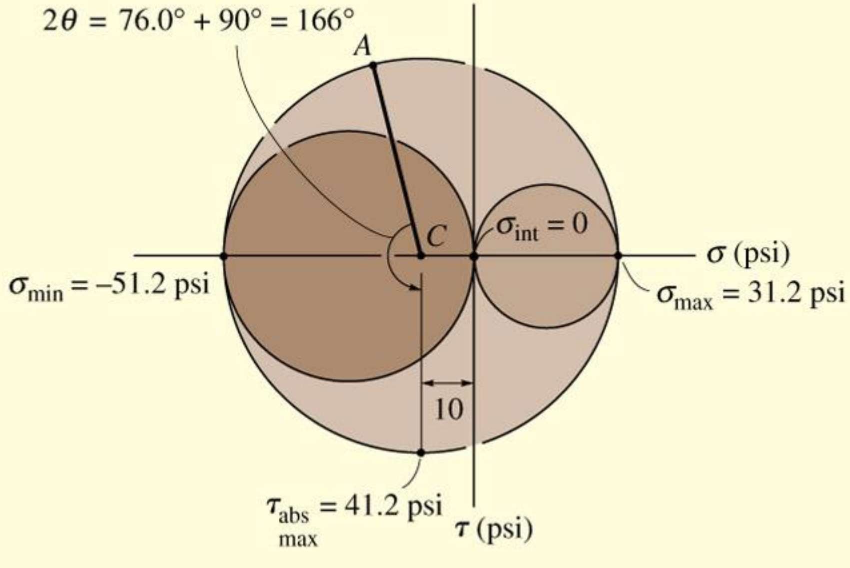
(c)

Absolute Maximum Shear Stress. Applying Eqs. 9–13 and 9–14, we have

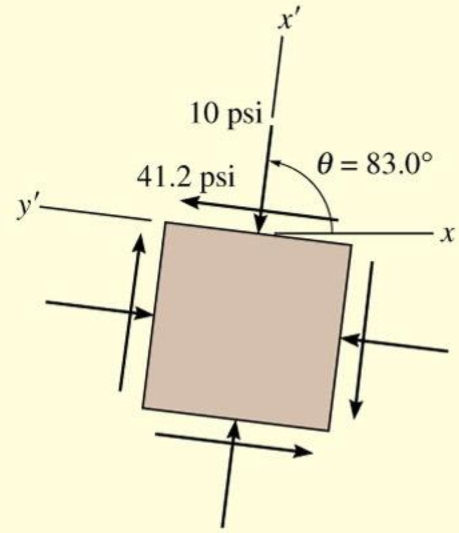
$$\tau_{\max}^{\text{abs}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{31.2 - (-51.2)}{2} = 41.2 \text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{31.2 - 51.2}{2} = -10 \text{ psi}$$

These same results can also be obtained by drawing Mohr's circle for each orientation of an element about the x', y', and z' axes, Fig. 9–31*d*. Since σ_{max} and σ_{min} are of *opposite signs*, then the absolute maximum shear stress equals the maximum in-plane shear stress. This results from a 45° rotation of the element in Fig. 9–31*c* about the z' axis, so that the properly oriented element is shown in Fig. 9–31*e*.



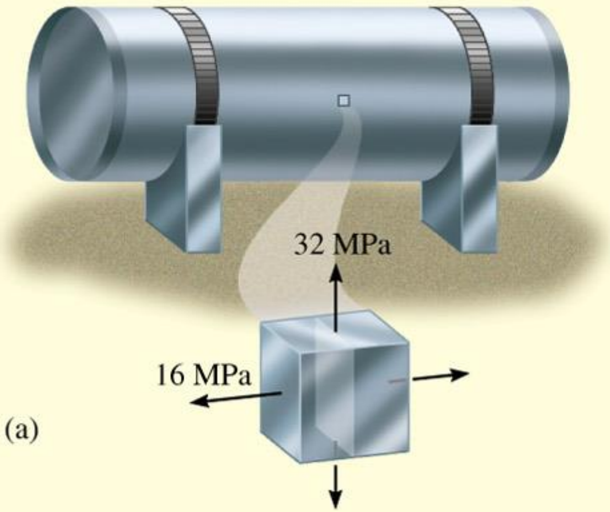
(d)



(e)

EXAMPLE 9.15

The point on the surface of the cylindrical pressure vessel in Fig. 9–32*a* is subjected to the state of plane stress. Determine the absolute maximum shear stress at this point.



(a)

Solution

The principal stresses are $\sigma_{max} = 32$ MPa, $\sigma_{int} = 16$ MPa, and $\sigma_{min} = 0$. If these stresses are plotted along the σ axis, the three Mohr's circles can be constructed that describe the stress state viewed in each of the three perpendicular planes, Fig. 9–32*b*. The largest circle has a radius of 16 MPa and describes the state of stress in the plane containing $\sigma_{max} = 32$ MPa and $\sigma_{min} = 0$, shown shaded in Fig. 9–32*a*. An orientation of an element 45° within this plane yields the state of absolute maximum shear stress and the associated average normal stress, namely,

$$\tau_{\substack{abs\\max}} = 16 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{avg} = 16 \text{ MPa}$$

These same results can be obtained from direct application of Eqs. 9–13 and 9–14; that is,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{32 - 0}{2} = 16 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{32 + 0}{2} = 16 \text{ MPa}$$

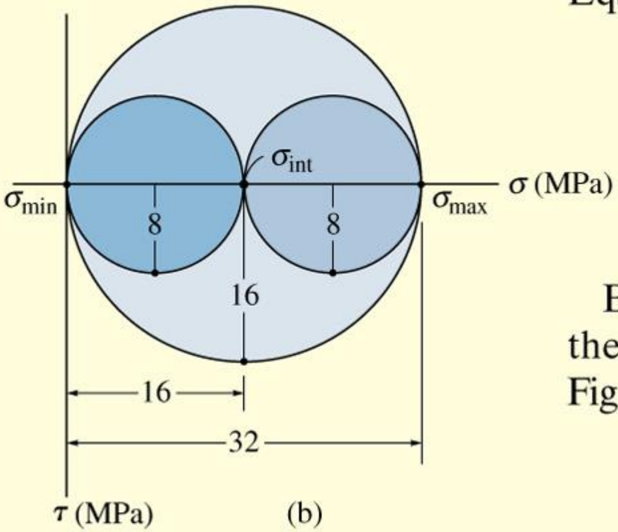


Fig. 9–32

By comparison, the maximum in-plane shear stress can be determined from the Mohr's circle drawn between $\sigma_{max} = 32$ MPa and $\sigma_{int} = 16$ MPa, Fig. 9–32*b*. This gives a value of

$$\tau_{\substack{max\\in\text{-}plane}} = \frac{32 - 16}{2} = 8 \text{ MPa}$$

$$\sigma_{avg} = 16 + \frac{32 - 16}{2} = 24 \text{ MPa}$$