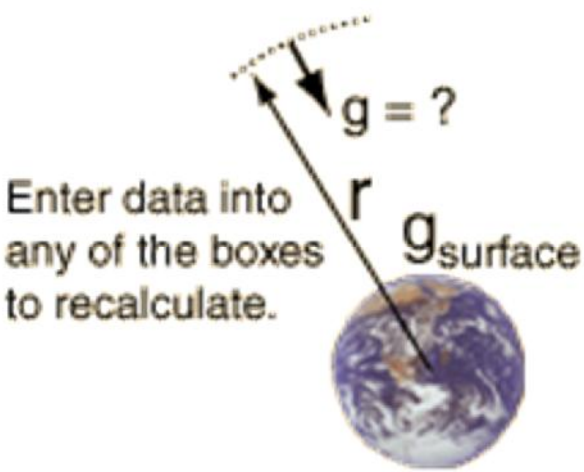
زمین و میدان ثقل ان

میدان ثقل:

$$F = G\frac{Mm}{\Delta r^2}$$



$$F_{gravity} = m\frac{GM_{Earth}}{R_{Earth}^2} = mg$$

$$M_E = 5.98x10^{24}kg$$

$$R_E = 6.38x10^6 m \quad (Average)$$

$$G = 6.67259x10^{-11} Nm^2 / kg^2$$

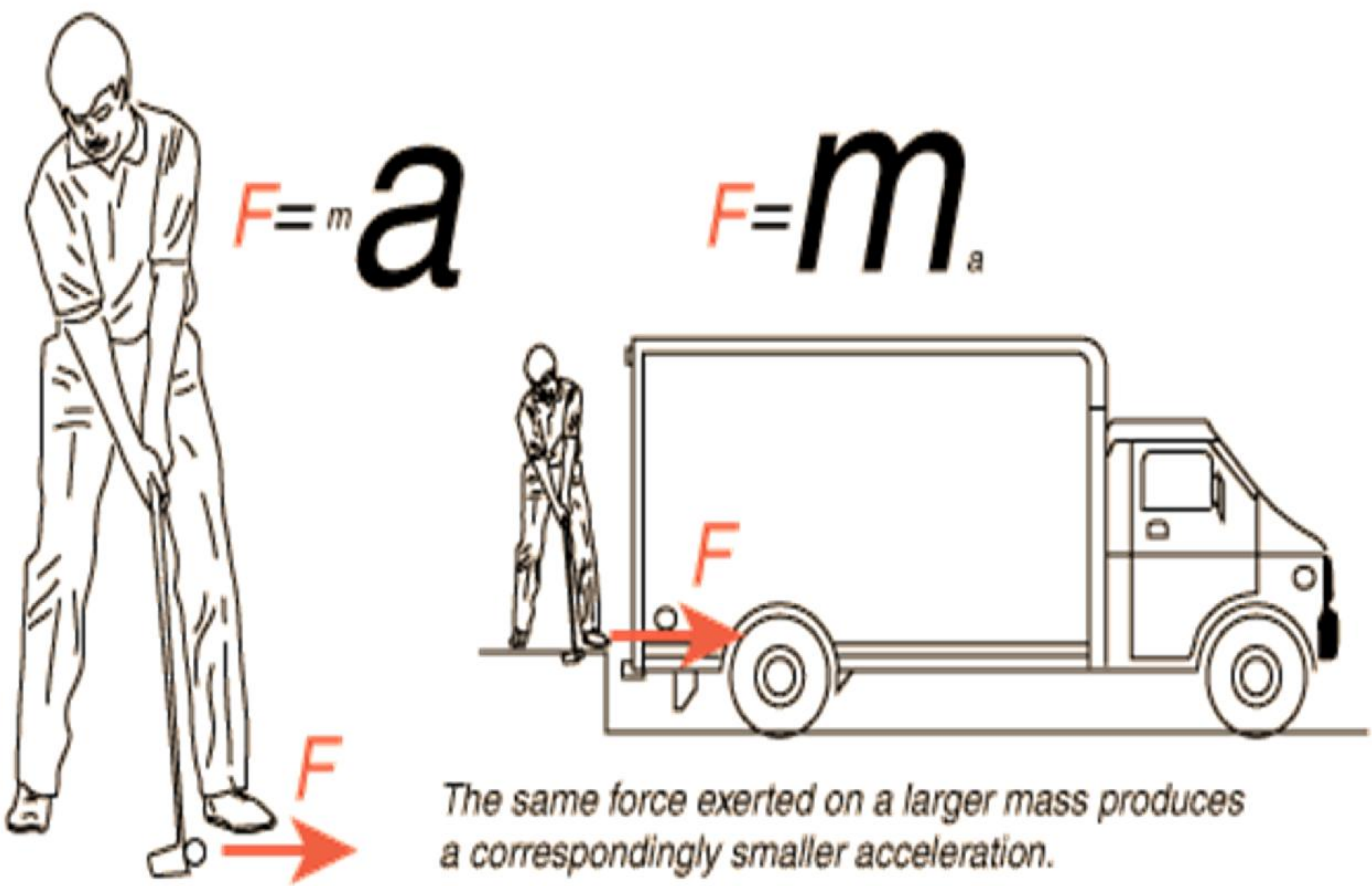
Newton's First Law

- Newton's First Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force. It may be seen as a statement about inertia, that objects will remain in their state of motion unless a force acts to change the motion. Any change in motion involves an acceleration, and then Newton's Second Law applies; in fact, the First Law is just a special case of the Second Law for which the net external force is zero.

Newton's Second Law

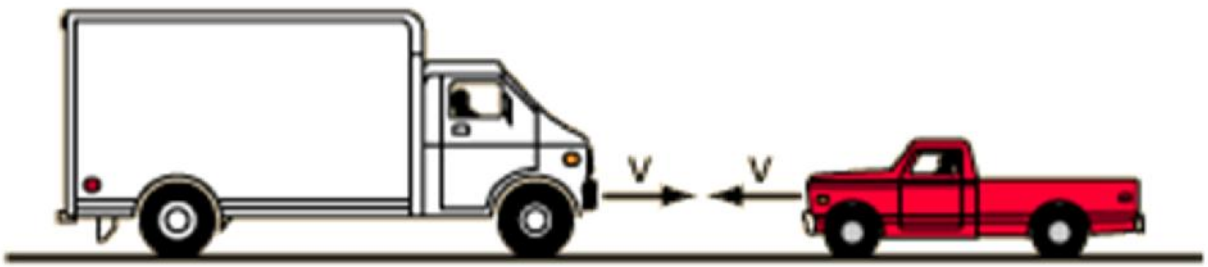
$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration



The same force exerted on a larger mass produces a correspondingly smaller acceleration.

Newton's Third Law



Force	F	$=$	F
Impulse	F_t	$=$	F_t
Change in momentum	$m_{\Delta v}$	$=$	$_m\Delta v$
Acceleration	m_a	$=$	$_m a$

وزن جسم

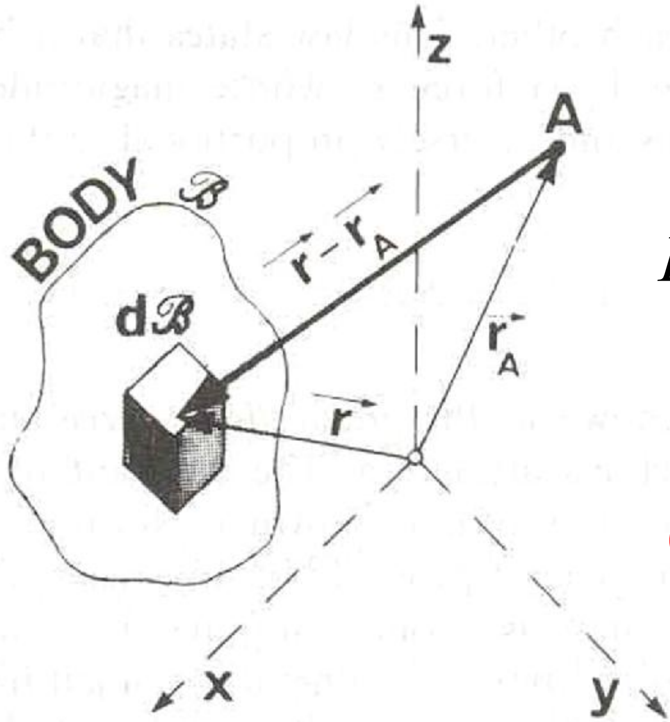
$$W = mg$$

Weight of object = mass of object x acceleration of gravity

- **gm is the weight of the mass, and g is the acceleration due to gravity, or just gravity. g = 9.80 ms-2 on average.**

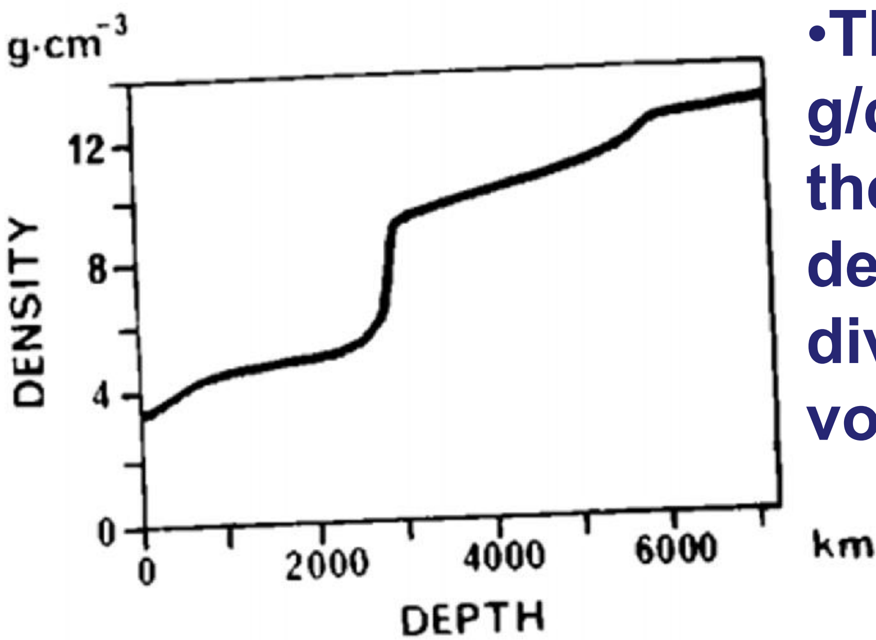
- Units of Gravity:

$1\ \text{Gal} = 1\ \text{cm s}^{-2}$



$$F_{B\to A} = F(r_A) = Gm\iiint \frac{\partial(r)}{\left|r - r_A\right|^3}(r - r_A)dB$$

•To study the gravitation, the density distribution σ(r) with in the earth must be known.

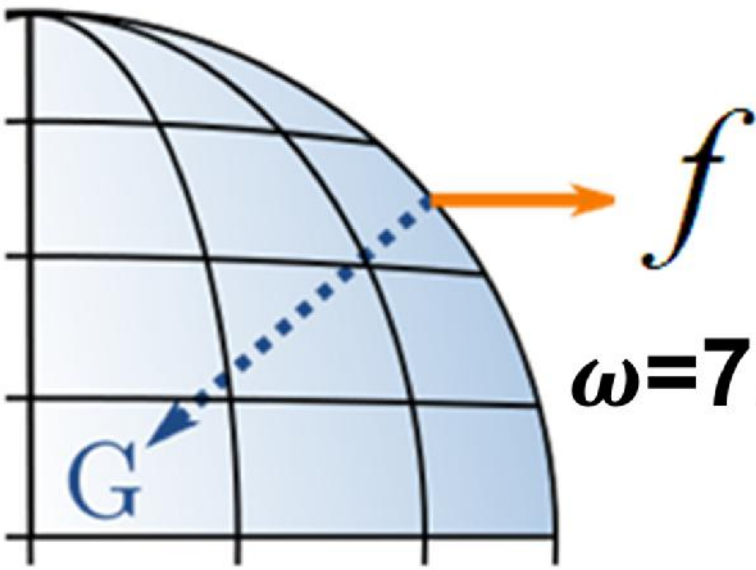


•The density of the Earth is 5.513 g/cm3. This is an average of all of the material on the planet. The density of Earth is calculated by dividing the planet's mass by its volume,

Centrifugal force

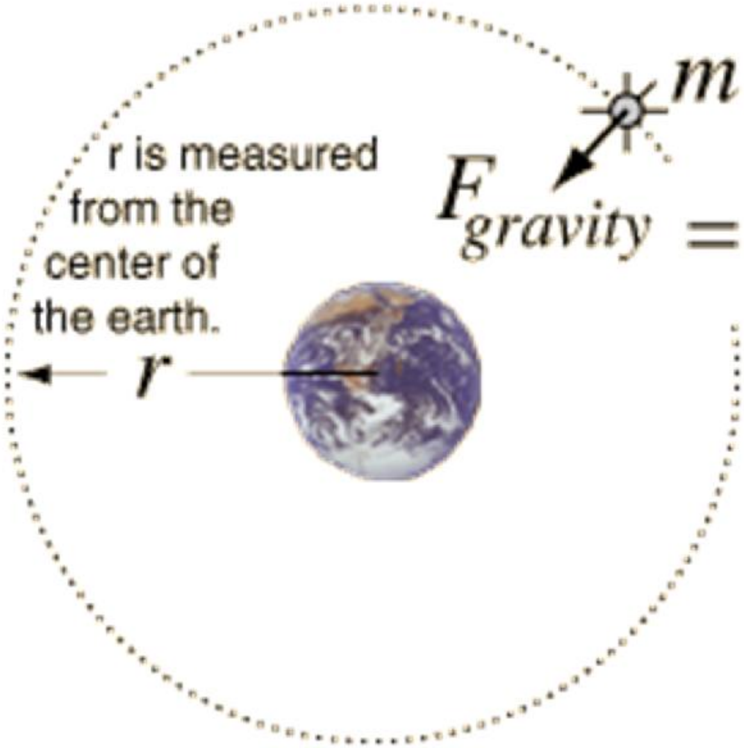
$$f = p\omega^2 m$$

نیروی فوق در حدود 0.35 درصد نیروی جاذبه



ω=72.92115*10^-6 rad/s

$$a_c = \left(\frac{2\pi}{86164\sec}\right)^2 \times 6378000\ m = 0.0339\ m/s^2 = 3.39\ cm/s^2$$



$$F_{gravity} = mg_{orbit} = m\frac{v^2}{r} = F_{centripetal}$$

$$v_{orbit} = \sqrt{rg_{orbit}}$$

$$g_{orbit} = g_{surface}\left[\frac{R_{Earth}}{r}\right]^2$$

$$m\frac{v^2}{r} = \frac{GmM_{Sun}}{r^2}$$

Applying Newton' s 2nd Law for the case of circular motion, the required centripetal force is supplied by gravity.

$$v^2 = \frac{GM_{Sun}}{r}$$

$$v = \sqrt{\frac{GM_{Sun}}{r}} = \frac{2\pi r}{T}$$

The application of Newton's 2nd law gives you the velocity. For a circular orbit, the period T can be found from the orbit velocity.

$$\frac{T}{2\pi r} = \sqrt{\frac{r}{GM_{Sun}}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{Sun}}$$

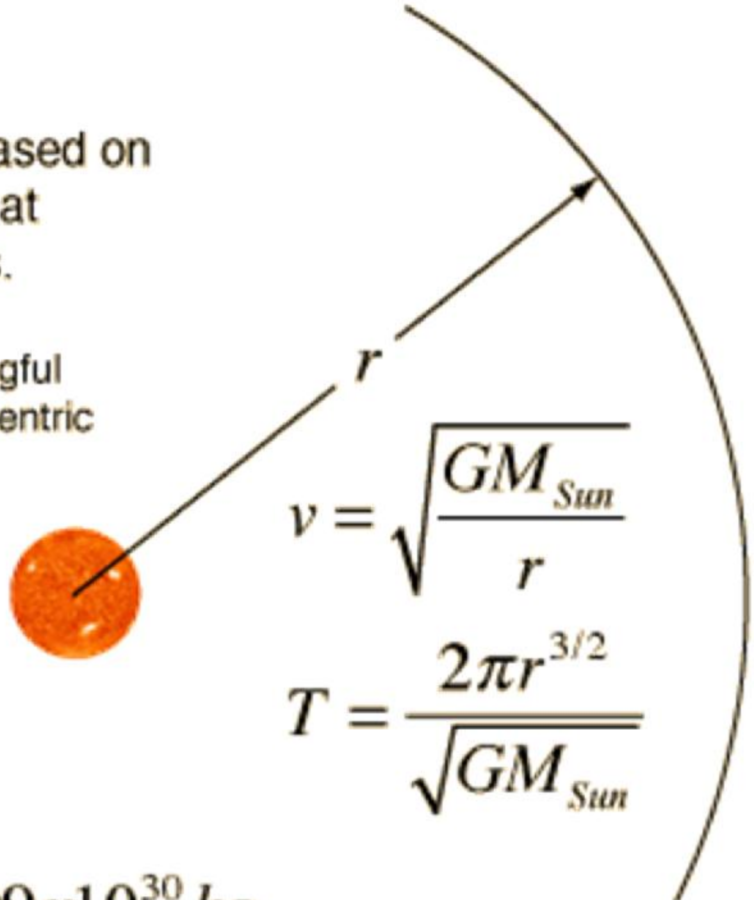
Solving for the period gives you Kepler's Law of Periods for the special case of a circular orbit.

The expressions for velocity and period are seen to follow from Newton's 2nd law and the law of gravity.

$$v = \sqrt{\frac{GM_{Sun}}{r}} \qquad T = \frac{2\pi r^{3/2}}{\sqrt{GM_{Sun}}}$$

Calculations based on a circular orbit at average radius.

(Not very meaningful for the highly eccentric orbit of Pluto.)



$$v = \sqrt{\frac{GM_{Sun}}{r}}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_{Sun}}}$$

Approximate expressions for velocity and period, assuming a circular orbit.

Average radii of planetary orbits in au:

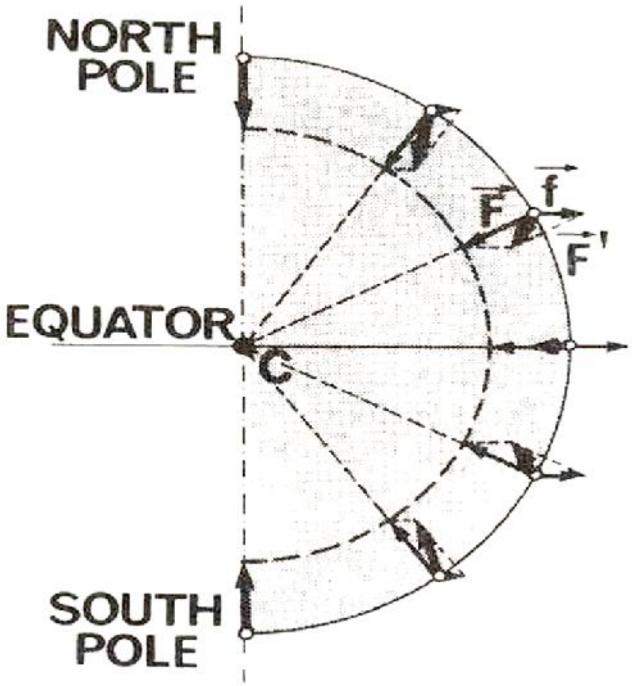
Mercury	0.389 $A_{\odot}$
Venus	0.725
Earth	1.000
Mars	1.53
Jupiter	5.22
Saturn	9.57
Uranus	19.255
Neptune	30.168
Pluto	39.597

$$M_{Sun} = 1.99x10^{30}\,kg$$

$$r_{Sun} = 6.96x10^{8}\,m$$

$A_{\odot}$ = 1 astronomical unit $= 1.49x10^{11}\,m$

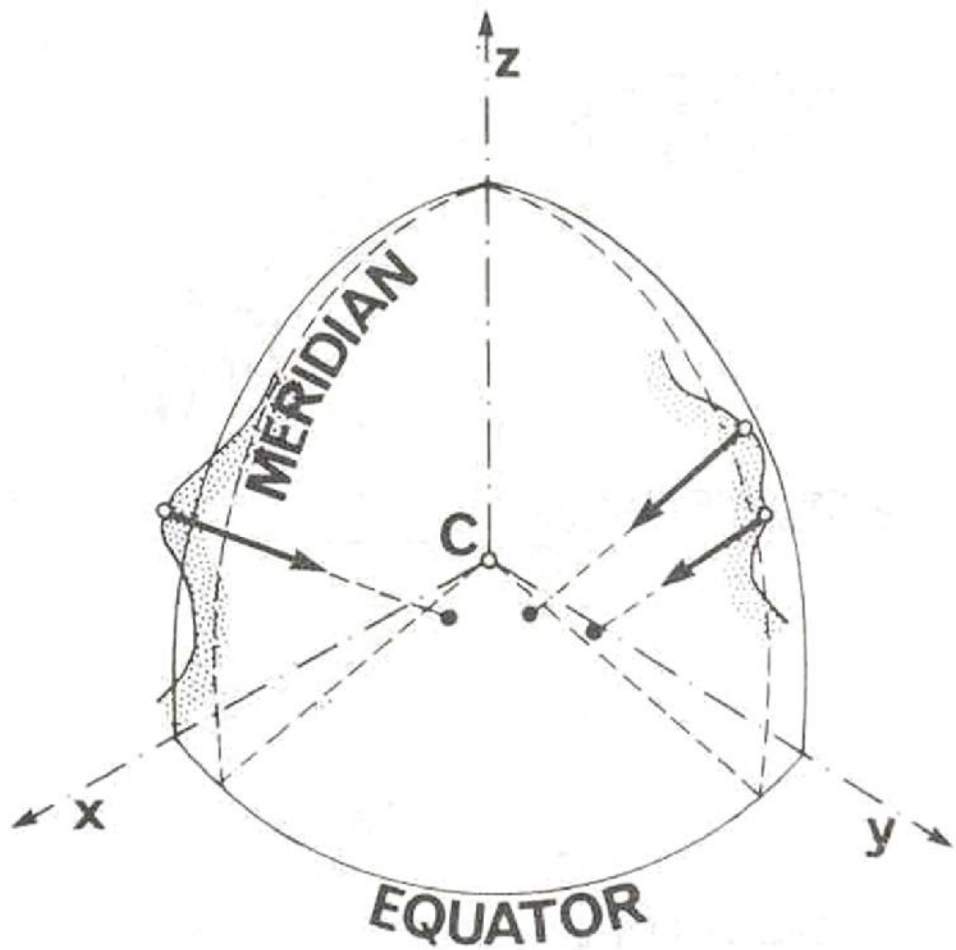
Gravity force



The sum of the gravitational and centrifugal forces is called the *gravity force.*

$$\vec{F}'(\vec{r}_A) = \vec{F}_{\mathfrak{B}\to A} + \vec{f}_A$$
$$= \left\{ G \iiint_{\mathfrak{B}} \frac{\sigma(\vec{r})}{|\vec{r}-\vec{r}_A|^3}(\vec{r}-\vec{r}_A)\,\mathrm{d}\mathfrak{B} + \vec{p}_A\omega^2 \right\} m\,.$$

Gravity force direction



Variation in gravity on the earth's surface

- These variations is more than 5 Gal
- These variations have several sources
  - Different heights of observation points
  - The oblateness of the earth
  - uneven lateral distribution of masses with in the earth
  - Other factors

Gravity anomaly

$$\Delta g = g - \gamma$$

- International Union of Geodesy and Geophysics (IUGG) is a non-governmental, scientific organization, established in 1919.

•The global range of the variations on the surface of the earth is more than 5 Gal, i.e., more than 0.5% of the avarage g.

• Modern instruments measure accurately to with in fraction of a µGal (10^-6) Gal, i.e ., to about 10^-10 “g”.

Gravity Variation with height (free-air correction)

$$g = \frac{GM}{R^2}$$

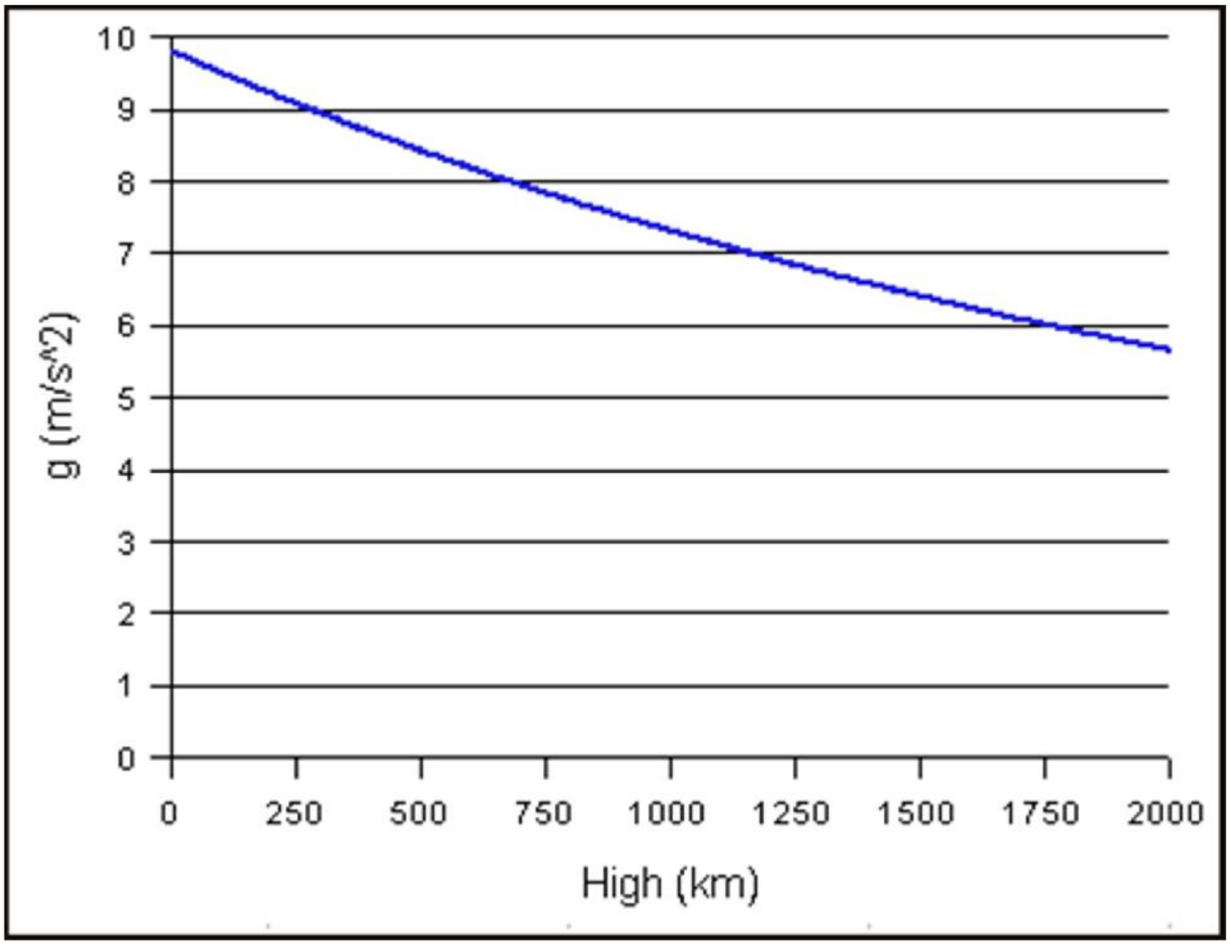
$$\frac{dg}{dR} = -\frac{2GM}{R^3} = -\frac{2g}{R}$$

- At the equator $2g/R = 0.3086$ mGal/m

Bouguer anomaly

$$dg \doteq -0.308\ [\mathrm{mGal\,m^{-1}}]\,dH.$$

- Note that dg is negetive for a positive H
- g decreases 1% with increases H about 32 KM
- The correction to gravity for the hieght effect is called " *free air correction*"



Normal gravity

- Analytically defined
- Normal gravity vector denoted by ɣ
- Function of R and φ
- Independed from λ

1914:

$$\gamma = 980.624(1 - 0.002644\cos 2\phi + 0.000007\cos^2 2\phi) - 0.3086\,h - 0.0002\,h\cos 2\phi + 7.1\times 10^{-8}h^2 \text{ Gal},$$

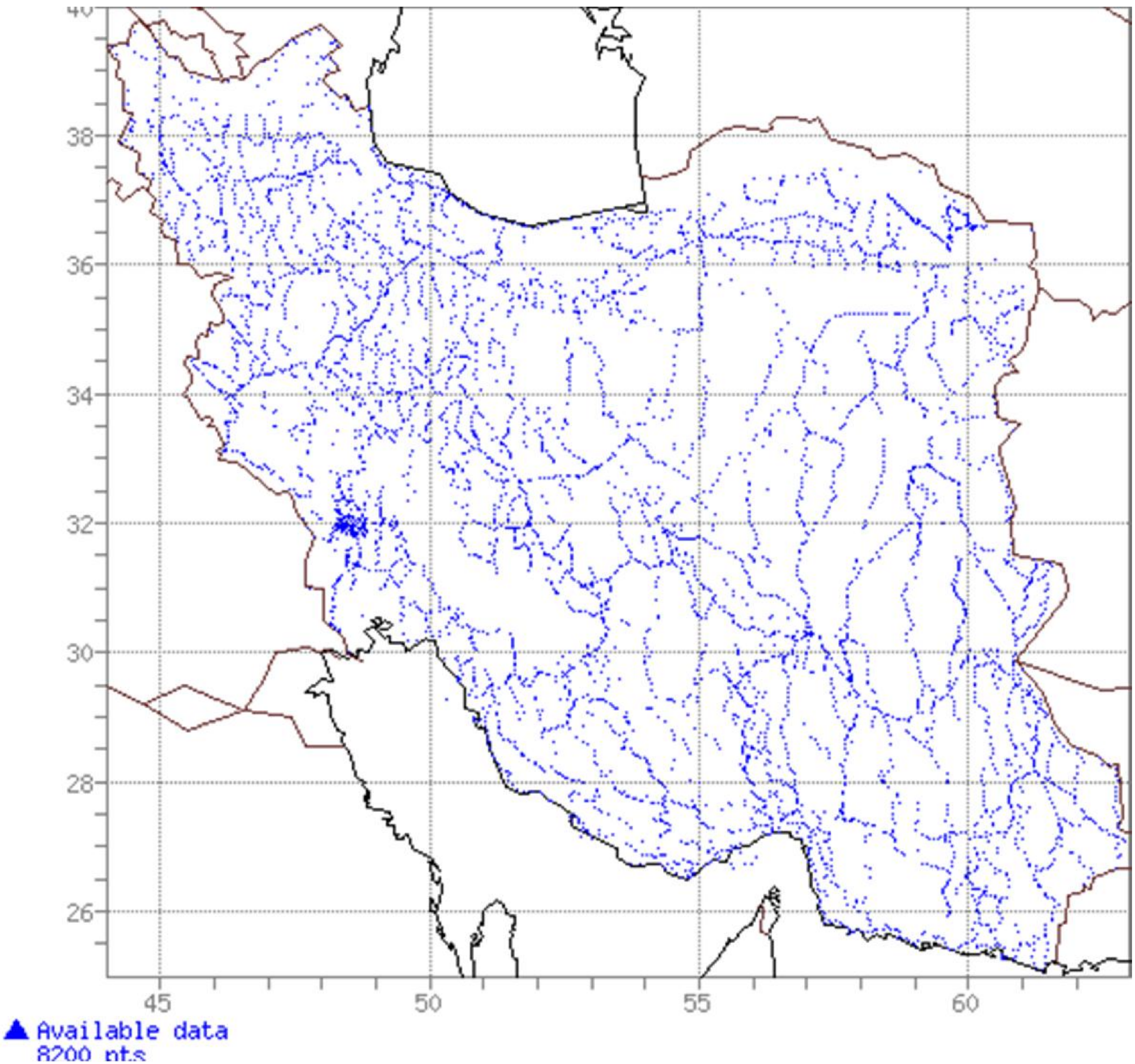
$$\gamma_0 = 980.624(1 - 0.002644\cos 2\phi + 0.000007\cos^2 2\phi)\ \text{Gal}.$$

The newest International Gravity Formula 1980 by International Association of Geodesy(IAG)

Accuracy 0.7 μGal

$$\gamma_0 \doteq 978.0327(1 + 0.0052790414\sin^2\phi + 0.0000232718\sin^4\phi + 0.0000001262\sin^6\phi)\ \text{Gal}.$$

Effect of oblateness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38
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32
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Variatons of gravity due to irregular distribution of masses

- $g > \gamma$ positive gravity anomaly shows that there are relatively denser masses
- g< γ negative gravity anomaly

$$g(r) = -\frac{GM(r)}{r^2}. \qquad g(r) = \frac{4\pi}{3}G\rho r.$$

•If the density decreased linearly with increasing radius from a density ρ_0 at the centre to ρ_1 at the surface, then $\rho(r) = \rho_0 - (\rho_0 - \rho_1)\, r / r_e$, and the dependence would be

$$g(r) = \frac{4\pi}{3}G\rho_0 r - \pi G\left(\rho_0 - \rho_1\right)\frac{r^2}{r_e}.$$