# PREFACE

This solutions manual was made available when *An Introduction to Mechanics* was published but it soon went out of print. Nevertheless, a number of copies have been circulated privately and over the years some errors have been discovered. We have corrected all the errors that have come to our attention. A word of caution: These solutions are often terse, more terse that we would provide today. In the Second Edition of our book, which is under preparation, we plan to provide solutions in a more discursive style.

We thank Gilbert Hawkins for preparing the original solutions.

Daniel Kleppner
Robert J. Kolenkow

March, 2010

# Chapter 1

**1.1**

(a) $\vec{A}+\vec{B} = (2+5)\hat{i} + (-3+1)\hat{j} + (7+2)\hat{k} = 7\hat{i} - 2\hat{j} + 9\hat{k}$

(b) $\vec{A}-\vec{B} = (2-5)\hat{i} + (-3-1)\hat{j} + (7-2)\hat{k} = -3\hat{i} - 4\hat{j} + 5\hat{k}$

(c) $\vec{A}\cdot\vec{B} = (2)(5) + (-3)(1) + (7)(2) = 21$

(d) $$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 2 & -3 & 7 \\ 5 & 1 & 2 \end{vmatrix} = -13\hat{i} + 31\hat{j} + 17\hat{k}$$

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**1.2**

$$\cos(\vec{A},\vec{B}) = \frac{AB\cos(\vec{A},\vec{B})}{AB} = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(3)(-2)+(1)(-3)+(1)(-1)}{\sqrt{(3)^2+(1)^2+(1)^2}\ \sqrt{(-2)^2+(-3)^2+(-1)^2}}$$

$$= -0.806$$

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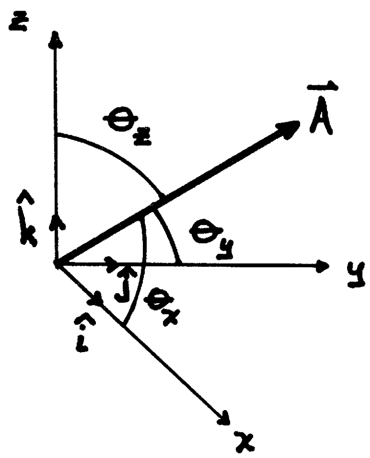
**1.3**

$A_x = \vec{A}\cdot\hat{i} = A\cos\theta_x = A\alpha$

$A_y = \vec{A}\cdot\hat{j} = A\cos\theta_y = A\beta$

$A_z = \vec{A}\cdot\hat{k} = A\cos\theta_z = A\gamma$




$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = A\sqrt{\alpha^2+\beta^2+\gamma^2}$

Hence $\alpha^2+\beta^2+\gamma^2 = 1$

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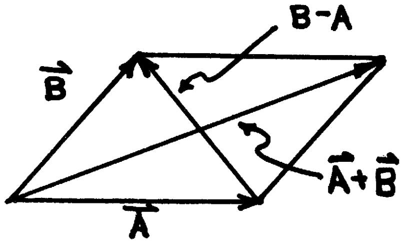
**1.4** $|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|$ hence

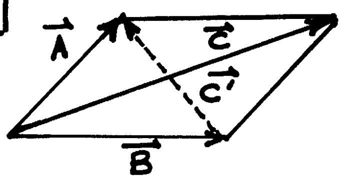
$(\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B}) = (\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})$

$A^2 - 2\vec{A}\cdot\vec{B} + B^2 = A^2 + 2\vec{A}\cdot\vec{B} + B^2$

$-2\vec{A}\cdot\vec{B} = +2\vec{A}\cdot\vec{B} \Rightarrow \vec{A}\cdot\vec{B} = 0$



$\boxed{1.5}$



$$A = B$$
$$\vec{C} = \vec{A} + \vec{B} \neq 0$$
$$\vec{C}' = \vec{A} - \vec{B} \neq 0$$
$$\vec{C}\cdot\vec{C}' = (\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = A^2 - B^2 = 0$$

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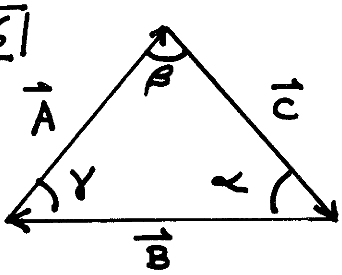
$\boxed{1.6}$



$$\vec{A}+\vec{B}+\vec{C} = 0$$
$$0 = \vec{A}\times(\vec{A}+\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$$

Hence $|\vec{A}\times\vec{B}| = |\vec{A}\times\vec{C}|$

$$AB\sin\gamma = AC\sin\beta$$
$$\frac{B}{\sin\beta} = \frac{C}{\sin\gamma} \quad \text{etc.}$$

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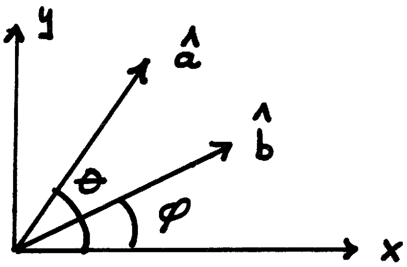
$\boxed{1.7}$



$$\hat{a} = (\hat{a}\cdot\hat{i})\,\hat{i} + (\hat{a}\cdot\hat{j})\,\hat{j}$$
$$\hat{a} = (1)(1)\cos\theta\,\hat{i} + (1)(1)\sin\theta\,\hat{j}$$

Similarly $\hat{b} = \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}$

$$\hat{a}\cdot\hat{b} = (1)(1)\cos(\theta-\varphi)$$

But $\hat{a}\cdot\hat{b} = (\cos\theta)(\cos\varphi) + (\sin\theta)(\sin\varphi)$

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$\boxed{1.8}$

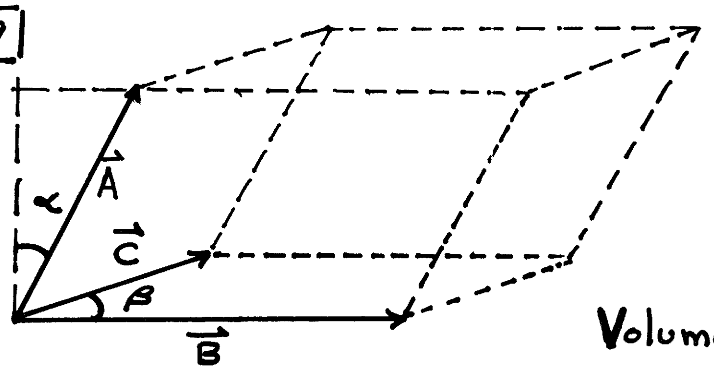
$$\vec{A}\cdot\hat{n} = 0 \qquad (1)\,n_x + (1)\,n_y - (1)\,n_z = 0$$
$$\vec{B}\cdot\hat{n} = 0 \qquad (2)\,n_x - (1)\,n_y + (3)\,n_z = 0$$

Hence $n_x = -2/3\ n_z$, and $n_y = 5/3\ n_z$

$$|\hat{n}| = 1 = \sqrt{n_x^2+n_y^2+n_z^2} = |n_z|\sqrt{(2/3)^2+(5/3)^2+(1)^2}$$

$$n_z = \pm\frac{3}{\sqrt{38}} \quad \text{and} \quad \hat{n} = \pm\frac{1}{\sqrt{38}}(-2\,\hat{i} + 5\,\hat{j} + 3\,\hat{k})$$

**1.9**




$$\text{Volume} = (A\cos\alpha)(BC\sin\beta)$$
$$= \vec{A}\cdot(\vec{B}\times\vec{C})$$

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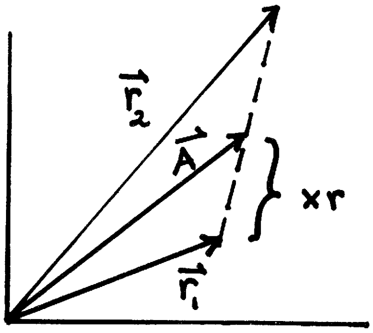
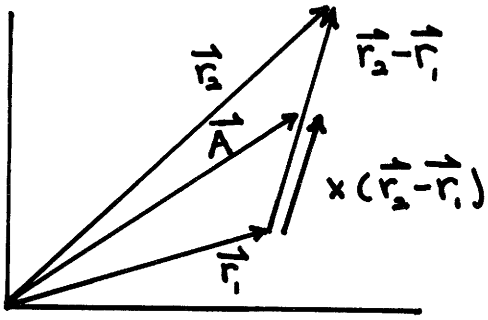
**1.10**




$$\vec{A} = \vec{r}_1 + x(\vec{r}_2-\vec{r}_1) = (1-x)\,\vec{r}_1 + x\,\vec{r}_2$$

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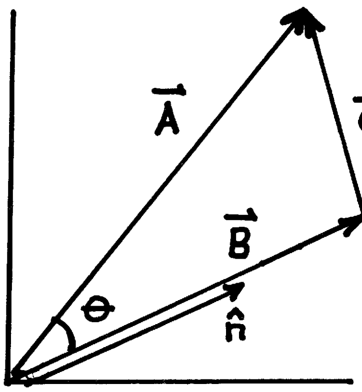
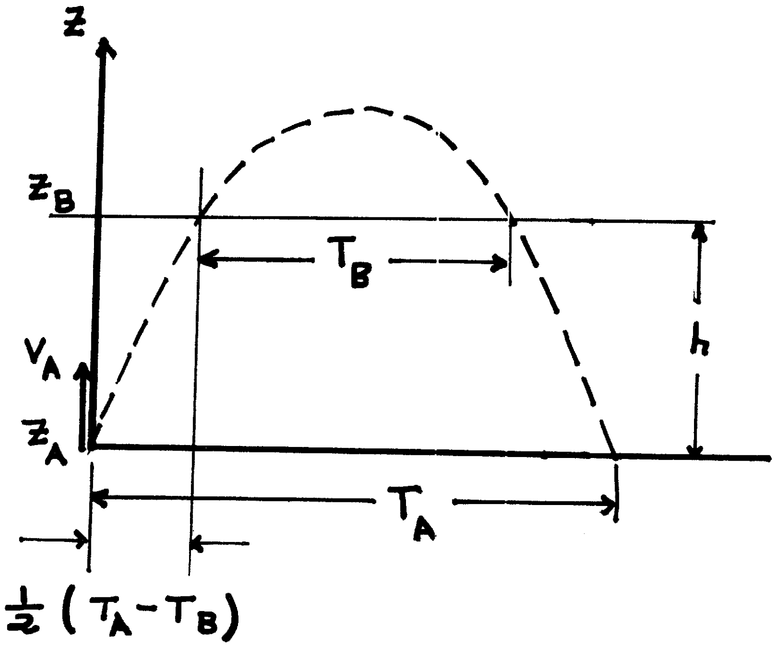
**1.11**




$$\vec{A} = \vec{B}+\vec{C}$$
$$B = A\cos\theta$$
$$\vec{B} = (\vec{A}\cdot\hat{n})\,\hat{n}$$
$$C = A\sin\theta$$
$$\vec{C} = (\hat{n}\times\vec{A})\times\hat{n}$$

**1.12**




$$z = z_A + v_A t - \tfrac{1}{2} g t^2$$

$$z_A = z_A + v_A T_A - \tfrac{1}{2} g T_A^2$$

$$\Rightarrow \quad v_A = \tfrac{1}{2} g T_A$$

$$h = v_A \frac{(T_A - T_B)}{2} - \frac{1}{2} g \left(\frac{T_A - T_B}{2}\right)^2 = \frac{1}{4} g \left[T_A^2 - T_A T_B - \tfrac{1}{2} T_A^2 + T_A T_B - \tfrac{1}{2} T_B^2\right]$$

$$\Rightarrow \quad g = \frac{8h}{T_A^2 - T_B^2}$$

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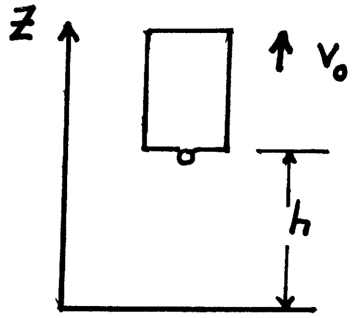
**1.13**



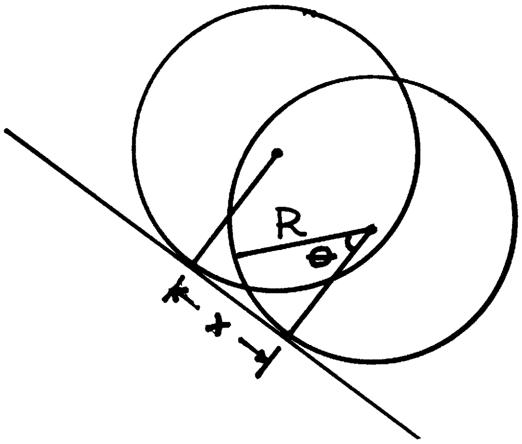

$$z = h + v_0 (t - T_1) - \tfrac{1}{2} g (t - T_1)^2$$

$$0 = h + v_0 T_2 - \tfrac{1}{2} g T_2^2$$

But $v_0 = h / T_1$

Hence $$h = \frac{\frac{1}{2} g T_2^2 T_1}{(T_2 + T_1)}$$

$\boxed{1.14}$




Distance moved by center is related to angle of rotation by $\quad x = R\theta$

Hence $\quad \ddot{x} = a = R\ddot{\theta} = R\alpha$

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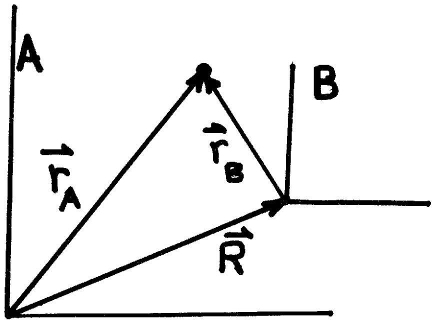
$\boxed{1.15}$

(a)

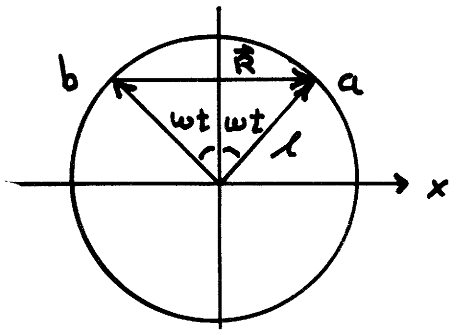



$$\vec{r}_A = \vec{r}_B + \vec{R}$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \dot{\vec{R}}$$

OR $\quad \vec{v}_B = \vec{v}_A - \dot{\vec{R}}$

(b)



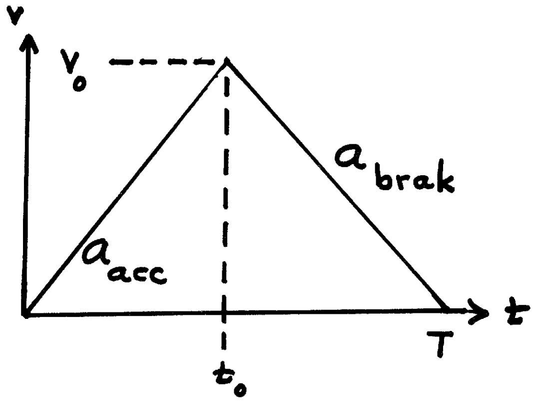

$$R = 2\ell \sin\omega t \;\hat{i}$$

$$\dot{\vec{R}} = 2\ell\omega\cos\omega t \;\hat{i}$$

Hence

$$v_A = v_B + 2\ell\omega\cos\omega t \;\hat{i}$$

$\boxed{1.16}$




$$v_0 = a_{acc} t_0 = a_{brak}(T - t_0)$$

Hence $$t_0 = \frac{a_{brak} T}{a_{acc} + a_{brak}}$$

But $$s = \frac{1}{2} v_0 T$$

Hence $$s = \frac{1}{2} \frac{a_{acc}\, a_{brak}}{a_{acc} + a_{brak}} T^2$$

$$T = \left[ \frac{2s\,(a_{acc} + a_{brak})}{a_{acc}\, a_{brak}} \right]^{\frac{1}{2}}$$

$a_{acc} = 1.11 \times 10^{-3}$ mi/s$^2$ $\qquad$ $a_{brak} \le 4.24 \times 10^{-3}$ mi/s$^2$

T is minimized by making $a_{brak}$ as large as possible. $T_{min} = 33.7$ s

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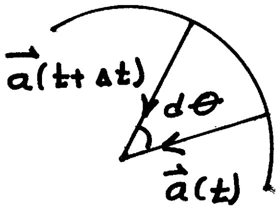
$\boxed{1.17}$

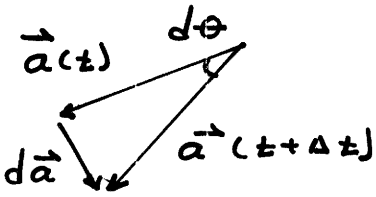
(a) $$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = 4\hat{r} + 2r\hat{\theta}$$

$$v = \sqrt{(4)^2 + (2.3)^2} = \sqrt{52} \text{ m/s}$$

(b) $$\vec{a} = (\ddot{r} - r^2\dot{\theta})\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$
$$= -2r^2\hat{r} + (2)(4)(2)\hat{\theta}$$
$$a = \sqrt{(2.9)^2 + (16)^2} = 24.1 \text{ m/s}$$

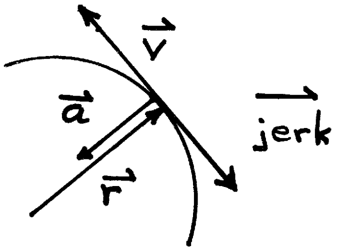
**1.18**




$$\vec a = -R\omega^2\,\hat r$$

$$d\vec a = -|\vec a|\,d\theta\,\hat\theta$$




$$\vec{jerk} = \frac{d\vec a}{dt} = -|\vec a|\,\frac{d\theta}{dt}\,\hat\theta$$

$$\vec{jerk} = -R\omega^3\,\hat\theta$$

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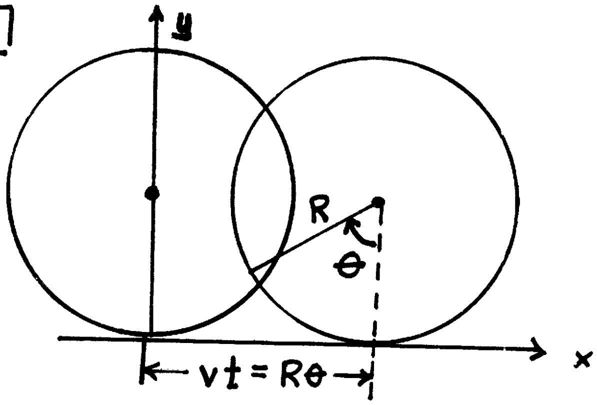
**1.19**




$$x = R\theta - R\sin\theta$$

$$y = R(1-\cos\theta)$$

$$\dot x = R\dot\theta - R\cos\theta\,\dot\theta$$

$$\dot x = V - V\cos\theta$$

$$\dot y = R\sin\theta\,\dot\theta = V\sin\theta$$

$$\Rightarrow\quad \ddot x = V\omega\sin\theta\,,\qquad \ddot y = V\omega\cos\theta$$

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**1.20**

$$r = \frac{1}{\pi}\,\theta = \frac{\alpha t^2}{2\pi}$$

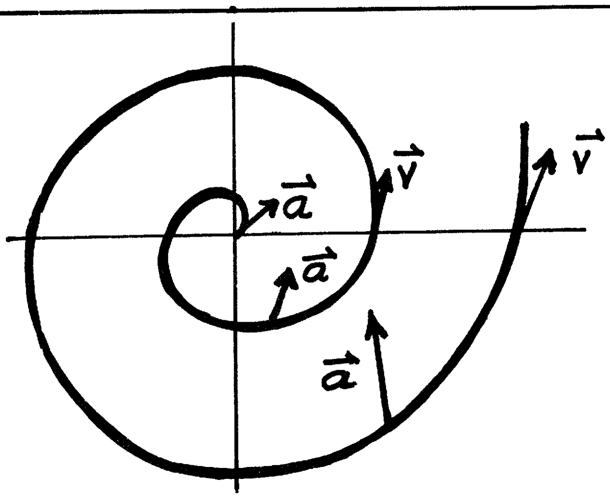
(a)

$$\dot r = \frac{\alpha t}{\pi}\qquad \dot\theta = \alpha t$$

$$\ddot r = \frac{\alpha}{\pi}\qquad \ddot\theta = \alpha$$

$$\vec a = (\ddot r - r\dot\theta^2)\,\hat r + (r\ddot\theta + 2\dot r\dot\theta)\,\hat\theta$$

$$\vec a = \left(\frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi}\right)\hat r + \frac{5}{2}\,\frac{\alpha^2 t^2}{\pi}\,\hat\theta$$



(b) $a_r = \frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi} = 0$ when $t^4 = \frac{2}{\alpha^2}$

Hence $a_r = 0$ when $\theta = \frac{\alpha t^2}{2} = \frac{\alpha\sqrt{2}}{2\alpha} = \frac{1}{\sqrt{2}}$ rad

(c) $\vec{a} = \frac{\alpha}{\pi}\left[(1-2\theta^2)\hat{r} + 5\theta\hat{\theta}\right]$

$|a_r| = |a_\theta|$ becomes $|1-2\theta^2| = |5\theta|$

For $\theta < \frac{1}{\sqrt{2}}$ $\begin{cases} 1-2\theta^2 = 5\theta \\ \theta = \frac{-5+\sqrt{33}}{4} = 0.19 \text{ rad} = 11° \end{cases}$

For $\theta > \frac{1}{\sqrt{2}}$ $\begin{cases} 2\theta^2 - 1 = 5\theta \\ \theta = \frac{5+\sqrt{33}}{4} = 2.69 \text{ rad} = 154° \end{cases}$

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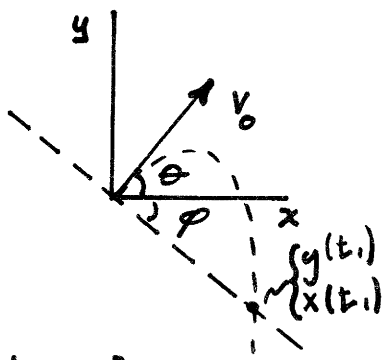
$\boxed{1.21}$ Let $v_0$ be the initial velocity




$x = (v_0 \cos\theta)t$

$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$

Let rock hit slope at $t = t_1$.

Find value of $\theta$ which maximizes $x(t_1)$.

$t_1 = \frac{x(t_1)}{v_0\cos\theta}$

$y(t_1) = (v_0 \sin\theta)t_1 - \frac{1}{2}gt_1^2 = (\tan\theta)x(t_1) - \frac{\frac{1}{2}g\,x^2(t_1)}{v_0^2\cos^2\theta}$

But $y(t_1) = -\tan\varphi\ x(t_1)$

$\Rightarrow x(t_1) = \frac{2v_0^2}{g}(\cos^2\theta\tan\varphi + \cos\theta\sin\theta)$

Maximizing $x(t_1)$ as a function of $\theta$ yields

$\tan(2\theta_{max}) = \tan(\frac{\pi}{2} - \varphi)$

$\theta_{max} = \pi/4 - \varphi/2$

# Chapter 2

$\boxed{2.1}$

$$\vec{a} = \frac{\vec{F}}{5} = \left(\frac{4t^2}{5}\hat{i} - \frac{3t}{5}\hat{j}\right) m/s^2$$

(a) $$\vec{v} = \int \vec{a}\,dt = \left(\frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}\right) m/s$$

(b) $$\vec{r} = \int \vec{v}\,dt = \left(\frac{t^4}{15}\hat{i} - \frac{t^3}{10}\hat{j}\right) m$$

(c)

$$\vec{r}\times\vec{v} = \begin{vmatrix} i & j & k \\ \frac{t^4}{15} & \frac{-t^3}{10} & 0 \\ \frac{4t^3}{15} & \frac{-3t^2}{10} & 0 \end{vmatrix} = \left(-\frac{t^6}{50} + \frac{2t^6}{75}\right)\hat{k}$$

$$\vec{r}\times\vec{v} = \frac{t^6}{150}\,\hat{k}\ m^2/s$$

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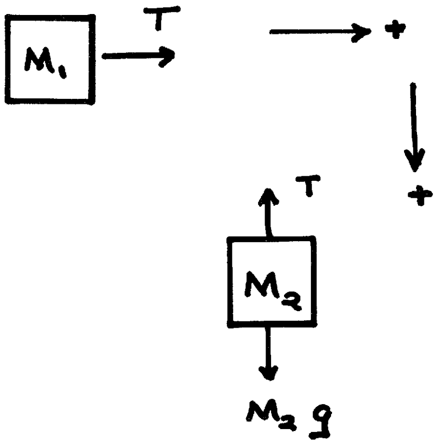
$\boxed{2.2}$




$T = M_1 \ddot{x}_1$

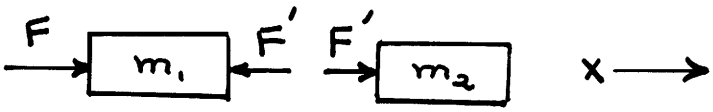
$M_2 g - T = M_2 \ddot{x}_2$

$\ddot{x}_1 = \ddot{x}_2$ since the length of the string is const.

$M_2 g = (M_1 + M_2)\ddot{x}_1$

$$\ddot{x} = \frac{M_2 g}{M_1 + M_2} \Rightarrow x = \frac{M_2 g t^2}{2(M_1 + M_2)}$$

$\boxed{2.3}$




$$F - F' = m_1 \ddot{x}$$

$$F' = m_2 \ddot{x}$$

$$\ddot{x} = \frac{F - F'}{m_1} = \frac{F'}{m_2}$$

Hence $F' = \dfrac{m_2 F}{m_1 + m_2} = \dfrac{(1)(3)}{(3)} = 1\ N$

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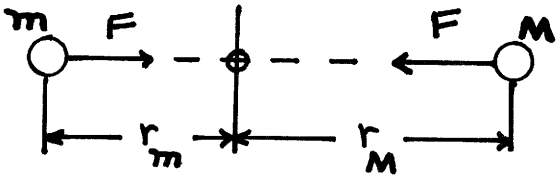
$\boxed{2.4}$




$$a_m = r_m \omega^2 = F/m$$

$$a_M = r_M \omega^2 = F/M$$

$$R = r_m + r_M = \frac{F}{\omega^2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

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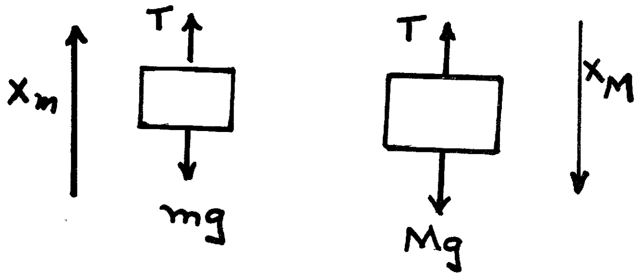
$\boxed{2.5}$




$$T - mg = m \ddot{x}_m$$

$$Mg - T = M \ddot{x}_M$$

$$\ddot{x}_m = \ddot{x}_M = a$$

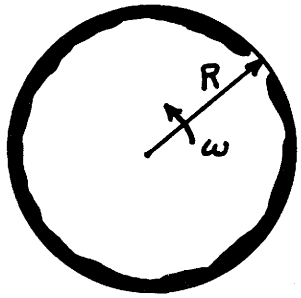
$$(M - m) g = (M + m) a$$

$$a = \left(\frac{M - m}{M + m}\right) g$$

$$T = \left(\frac{2Mm}{M + m}\right) g$$

$\boxed{2.6}$

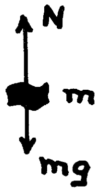
Ingredients most likely to fall from top




$mg - N = mR\omega^2$

$N = mg - mR\omega^2$

$N = 0$ at critical speed




$\Rightarrow \omega_{crit} = \sqrt{g/R}$

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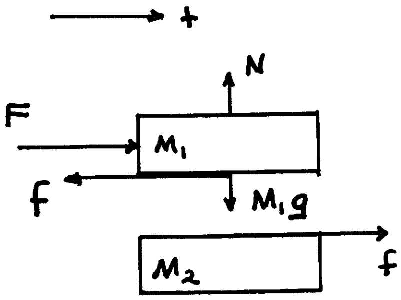
$\boxed{2.7}$

(a)



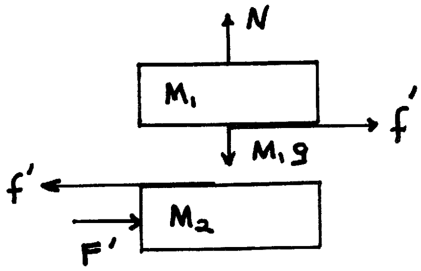

$F - f = M_1 a$

$f = M_2 a$

$F = \left(\frac{M_1 + M_2}{M_2}\right) f$

But $f \le \mu N = \mu M_1 g$

Hence $F \le \left(\frac{M_1 + M_2}{M_2}\right) \mu M_1 g$




(b)

$F' - f' = M_2 a$

$f' = M_1 a$

$F' = \left(\frac{M_1 + M_2}{M_1}\right) f'$

But $f' \le \mu N = \mu M_1 g \Rightarrow F' \le \left(\frac{M_1 + M_2}{M_1}\right) \mu M_1 g$

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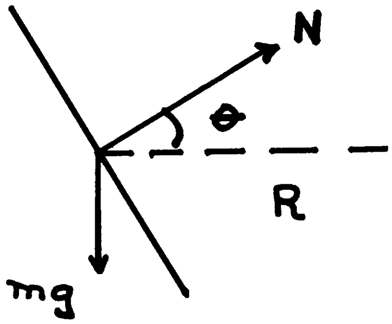
$\boxed{2.8}$ Analysis in Prob 2.7 $\frac{F}{F'} = \frac{M_1}{M_2}$ $F = \left(\frac{4}{5}\right) 27 = 21.6\,N$

$\boxed{2.9}$

$$N\sin\theta = mg$$
$$N\cos\theta = mv_0^2/R$$
$$\tan\theta = gR/v_0^2$$
$$R = \frac{v_0^2}{g}\tan\theta$$

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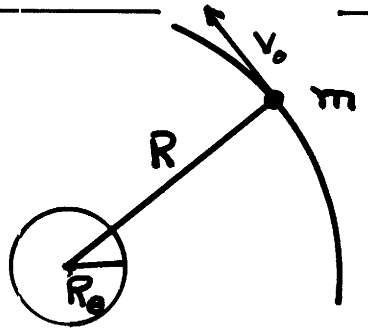
$\boxed{2.10}$




$$mR\omega^2 = \frac{GmM_e}{R^2}$$
$$R^3 = GM_e/\omega^2$$

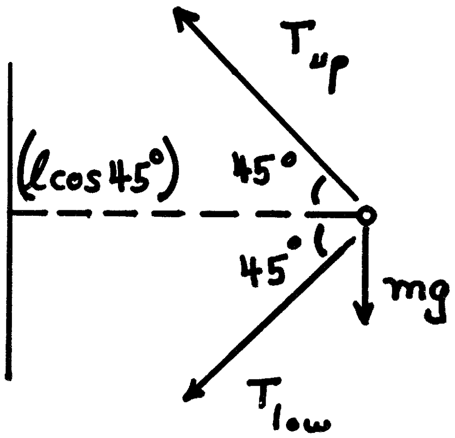
But $\dfrac{GM_e}{R_e^2} = g$ , $\omega = \dfrac{2\pi}{8.64\times10^4}\ \text{sec}^{-1}$

$$\frac{R}{R_e} = \left(\frac{g}{R_e}\frac{1}{\omega^2}\right)^{1/3} = \left[\frac{(9.8)(8.64\times10^4)^2}{(6.4\times10^6)\,4\pi^2}\right]^{\frac{1}{3}} = 6.62$$

---

$\boxed{2.11}$

$$T_{up}\frac{1}{\sqrt{2}} = mg + T_{low}\frac{1}{\sqrt{2}}$$
$$(T_{up}+T_{low})\frac{1}{\sqrt{2}} = m\ell\frac{1}{\sqrt{2}}\omega^2$$
$$T_{up} = \frac{m\ell\omega^2}{2} + \frac{mg}{\sqrt{2}}$$
$$T_{low} = \frac{m\ell\omega^2}{2} - \frac{mg}{\sqrt{2}}$$

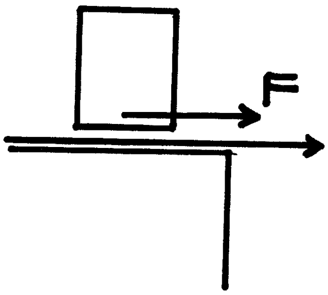


## 2.12

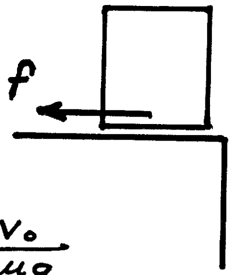
$F = \mu m g$

Hence $a = \frac{F}{m} = \mu g$.

$v_0 = \mu g T$, where $T$ is the time for cloth to be pulled out.




$f = \mu m g$ in direction shown.




$a' = -\mu g$

$v = v_0 - \mu g t$

Glass comes to rest at $t_0 = \frac{v_0}{\mu g}$

It travels a distance $s_0 = v_0 t_0 - \frac{1}{2}\mu g t_0^2$

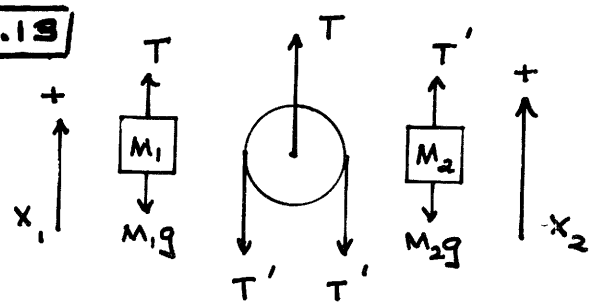
$$= \frac{1}{2}\frac{v_0^2}{\mu g} = \frac{1}{2}\mu g T^2$$

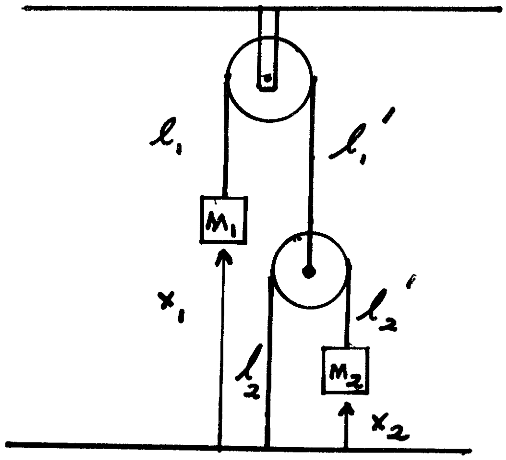
For $s_0 < \frac{1}{2}$ foot

$$T^2 < \frac{(\frac{1}{2})}{(\frac{1}{2})(0.5)(32)} \qquad T < \frac{1}{4} \text{ second}$$

---

## 2.13



$$T - M_1 g = M_1 \ddot{x}_1$$

$$T' - M_2 g = M_2 \ddot{x}_2$$

$$2T' - T = 0 \quad (\text{Massless pulley})$$

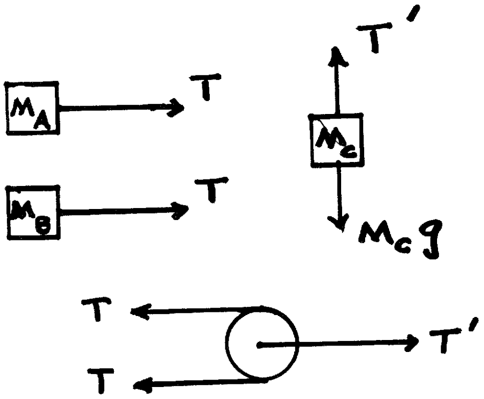
$$x_1 + l_1 + l_1' + \frac{l_2 + l_2' + x_2}{2} = \text{const.}$$

Hence $\quad \ddot{x}_1 = -\ddot{x}_2/2$

Solving, $\quad \ddot{x}_1 = \dfrac{(2M_2 - M_1)\, g}{4M_2 + M_1}$

---

$\boxed{2.14}$



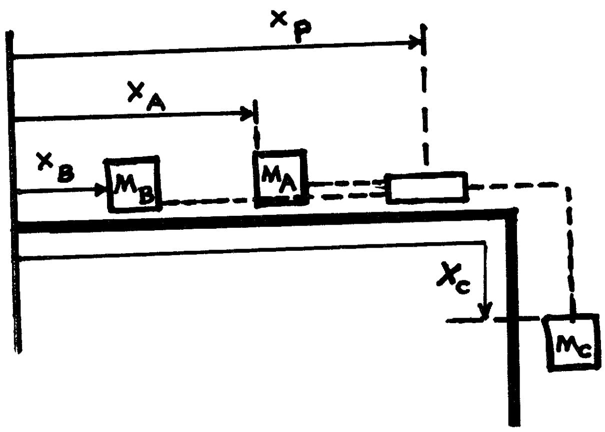

$$T = M_A \ddot{x}_A$$

$$T = M_B \ddot{x}_B$$

$$M_c g - T' = M_c \ddot{x}_c$$

$$T' - 2T = 0 \quad (\text{Massless pulley})$$

Constraint:




$$x_c - x_p = \text{const.}$$

Hence $\quad \ddot{x}_c = \ddot{x}_p$

$$(x_p - x_A) + (x_p - x_B) = \text{const.}$$

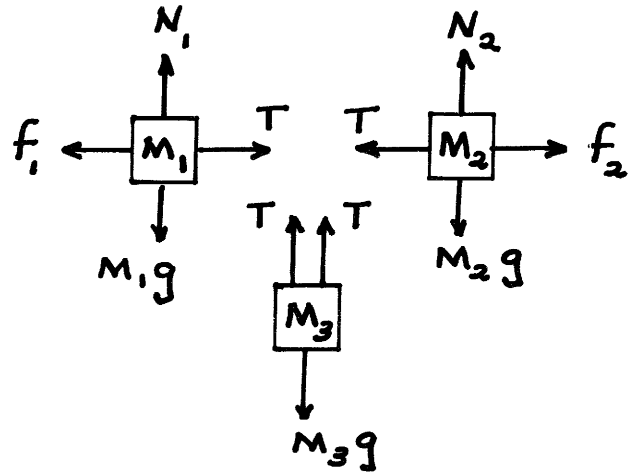
Hence $\quad \ddot{x}_A + \ddot{x}_B = 2\ddot{x}_p = 2\ddot{x}_c$

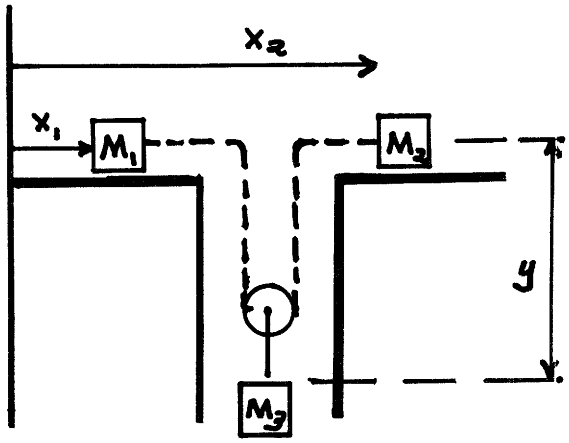
Solving,

$$\ddot{x}_A = \frac{M_B M_c g}{\left[2M_A M_B + \frac{M_c M_B}{2} + M_A M_c\right]} \equiv \frac{M_B M_c g}{D}$$

$$\ddot{x}_B = \frac{M_A M_c g}{D} \qquad \ddot{x}_c = \tfrac{1}{2}(M_A + M_B) M_c g / D$$

## $\boxed{2.15}$




$$x_2 - x_1 + 2y = \text{length of string} = \text{const.}$$

$$\ddot{x}_2 - \ddot{x}_1 + 2\ddot{y} = 0$$

$$T - f_1 = M_1 \ddot{x}_1 \qquad f_1 = \mu M_1 g$$

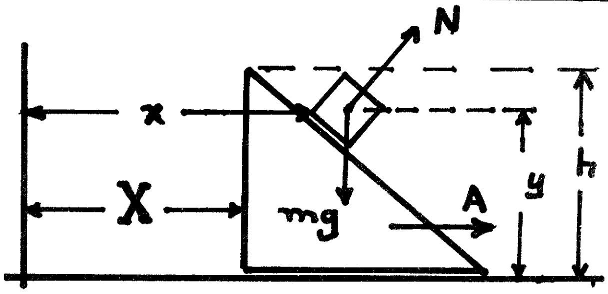
$$f_2 - T = M_2 \ddot{x}_2 \qquad f_2 = \mu M_2 g$$

$$M_3 g - 2T = M_3 \ddot{y}$$

Solving, $\quad T = \dfrac{2(\mu + 1)g}{\left(\dfrac{1}{M_1} + \dfrac{1}{M_2} + \dfrac{4}{M_3}\right)}$

---

## $\boxed{2.16}$

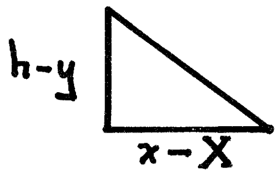



$$\frac{N}{\sqrt{2}} = m\ddot{x}$$

$$\frac{N}{\sqrt{2}} - mg = m\ddot{y}$$

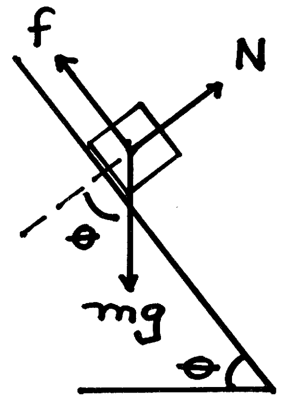
$$x - X = h - y$$

$$\ddot{x} - A = -\ddot{y}$$




Solving, $\quad \ddot{x} = \dfrac{A+g}{2} \qquad \ddot{y} = \dfrac{A-g}{2}$

$\boxed{2.17}$ (a)




$N = mg\cos\theta$

$mg\sin\theta = f \le \mu N = \mu mg\cos\theta$

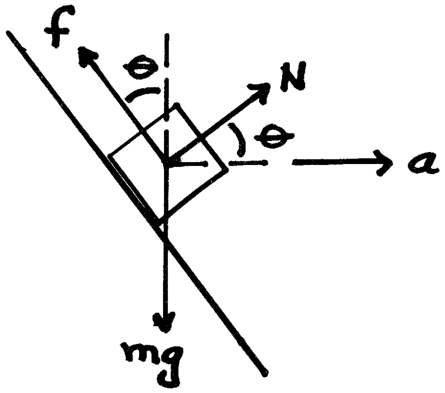
Hence $\tan\theta \le \mu$

$\tan\theta_{max} = \mu$

(b) $f\cos\theta + N\sin\theta - mg = 0$

$N\cos\theta - f\sin\theta = ma$

$f = \mu N$ at slipping limit



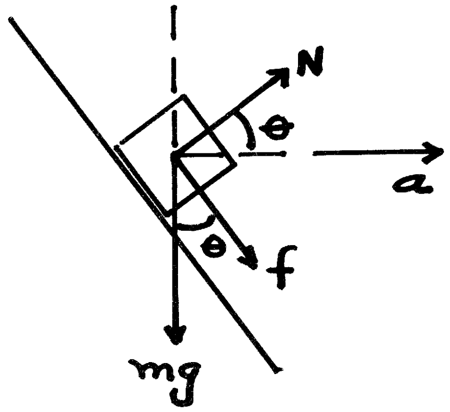

Solving,

$$a_{min} = \left(\frac{\cos\theta - \mu\sin\theta}{\mu\cos\theta + \sin\theta}\right) g$$

(c) $N\sin\theta - f\cos\theta - mg = 0$

$N\cos\theta + f\sin\theta = ma$

$f = \mu N$ at limit

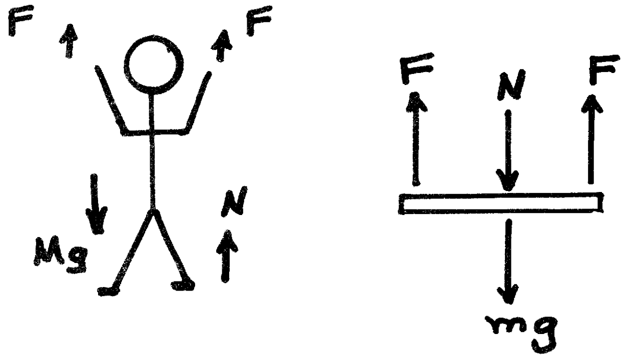



Solving,

$$a_{max} = \left(\frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}\right) g$$

---

$\boxed{2.18}$



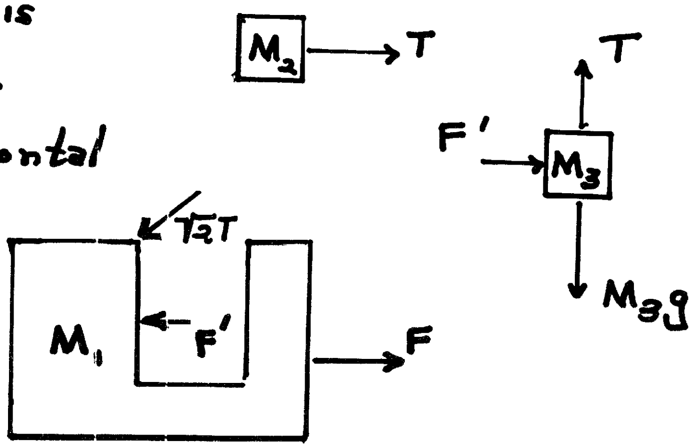

$2F + N - Mg = Ma$

$2F - N - mg = ma$

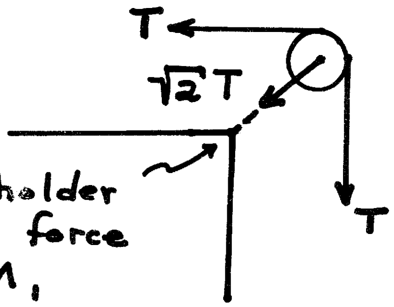
Solving,

$$a = \frac{4F}{M+m} - g$$

**2.19** Because $M_3$ is motionless, all bodies have the same horizontal acceleration $a$.




$$T = M_2 a$$
$$M_3 g - T = 0$$
$$F' = M_3 a$$
$$F - F' - T = M_1 a$$



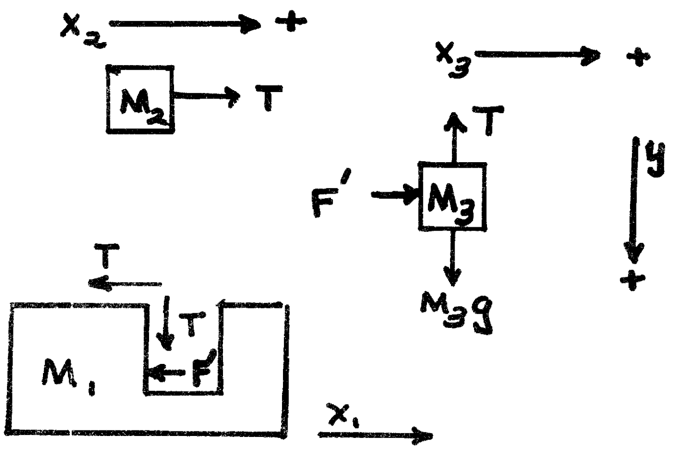

Eliminating $\{F', T, a\}$

$$F = \frac{M_3}{M_2}(M_1+M_2+M_3)g$$

---

**2.20**

$$F' + T = -M_1 \ddot{x}_1$$
$$T = M_2 \ddot{x}_2$$
$$F' = M_3 \ddot{x}_3$$
$$M_3 g - T = M_3 \ddot{y}$$

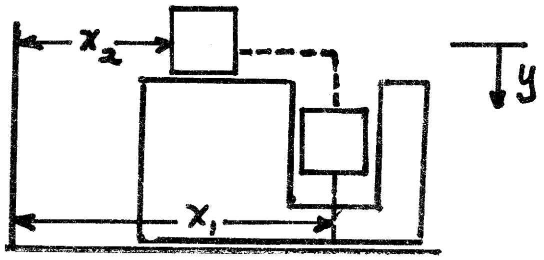



Constraints:

$$\ddot{x}_1 = \ddot{x}_3$$

$x_1 - x_2 + y =$ *length of string*

Hence $\ddot{x}_1 - \ddot{x}_2 + \ddot{y} = 0$

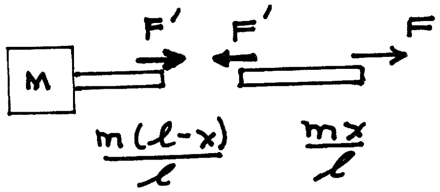



$$\ddot{x}_1 = \frac{-M_2 M_3 g}{[M_1 M_2 + M_1 M_3 + 2M_2 M_3 + M_3^2]}$$

$\boxed{2.21}$

$$F - F' = \left(\frac{mx}{\ell}\right)a$$

$$F' = \left[M + \frac{m(\ell-x)}{\ell}\right]a$$



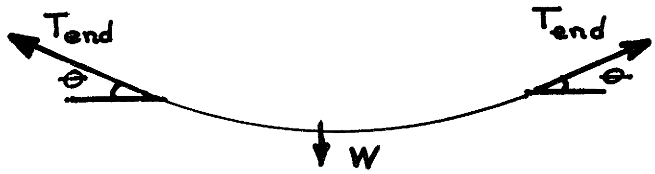

Solving, $F' = \dfrac{(M+m)\ell - mx}{(M+m)\ell}F = \left(1 - \dfrac{m}{M+m}\dfrac{x}{\ell}\right)F$

---

$\boxed{2.22}$

(a) $2T_{end}\sin\theta = W$

$$T_{end} = \frac{W}{2\sin\theta}$$



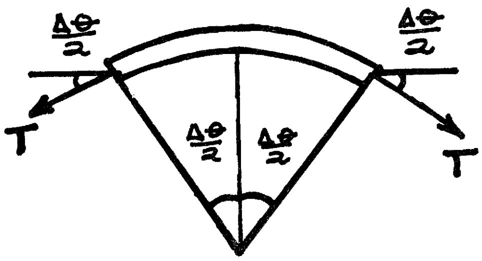

(b) $T_{middle} = T_{end}\cos\theta = \dfrac{W}{2\tan\theta}$


---

$\boxed{2.23}$

Inward force $= 2T\sin\dfrac{\Delta\theta}{2}$

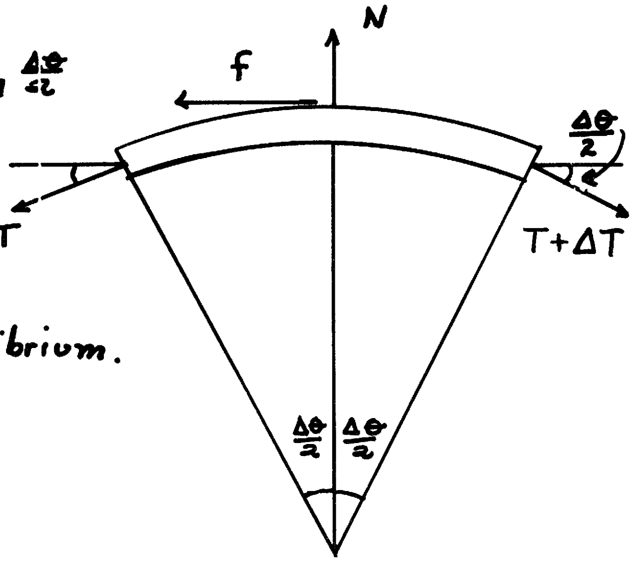
$$\cong 2T\frac{\Delta\theta}{2} = T\Delta\theta$$




$$T\Delta\theta = (\Delta M)r\omega^2 = M\frac{\Delta\theta}{2\pi}\frac{\ell}{2\pi}\omega^2$$

Hence $T = \dfrac{M\ell\omega^2}{(2\pi)^2}$

$\boxed{2.24}$




Inward force $= T\sin\frac{\Delta\theta}{2} + (T+\Delta T)\sin\frac{\Delta\theta}{2}$

$\cong 2T\frac{\Delta\theta}{2} = T\Delta\theta$

Hence $N = T\Delta\theta$

Horizontal forces are also in equilibrium.

$(T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - f = 0$

$f \cong \Delta T$

Assume rope is on verge of slipping.

Then

$f = \mu N$

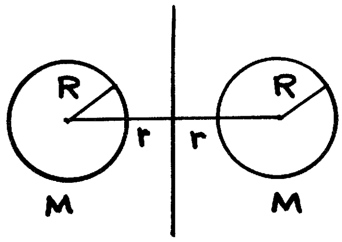
$\Delta T = \mu T\Delta\theta$

$\frac{dT}{d\theta} = \mu T$

$\int_{T_B}^{T_A}\frac{dT}{T} = \mu\int_0^{\theta} d\theta \Rightarrow \ln\frac{T_A}{T_B} = \mu\theta \qquad T_A = T_B e^{\mu\theta}$

---

$\boxed{2.25}$




$\frac{GM^2}{(2r)^2} = Mr\omega^2$

$\omega = \sqrt{\frac{GM}{4r^3}} = \sqrt{\frac{\pi G\rho}{3}\left(\frac{R}{r}\right)^3}$

Period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3}{\pi\rho G}\left(\frac{r}{R}\right)^3}$ $\qquad M = \frac{4}{3}\pi R^3\rho$

We want $r$ as small as possible; hence make $r = R$

Then $T_{min} = \sqrt{12\pi/G\rho}$

$\rho_{max} = 22 g/cm^3$ for available materials. Then $T_{min} = 5000$ seconds.

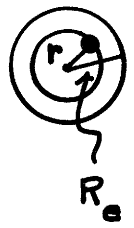
---

$\boxed{2.26}$

Gravitational force on m at radius r = $GMm/r^2$

$$M = M_e \left(\frac{r}{R_e}\right)^3$$

$$\text{Force on } m = \frac{Gm}{r^2} M_e \frac{r^3}{R_e^3} = \frac{GM_e mr}{R_e^3} = \frac{mg}{R_e} r$$




Hence $m\ddot{r} = -\dfrac{mg}{R_e} r$

SHM Period $= 2\pi \sqrt{R_e/g} = 84$ minutes

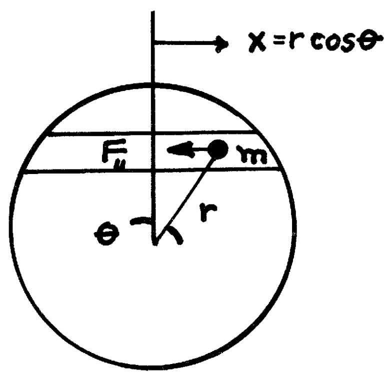
$$\frac{GM_e m}{R_e^2} = mR_e\omega^2$$

$$\omega = \frac{GM_e}{R_e^3} = \frac{g}{R_e}$$


$T = 2\pi\sqrt{R_e/g}$ as above.

---

$\boxed{2.27}$ (See problem 2.26)




$$F_{\parallel} = \frac{mg}{R_e} r\cos\theta = \frac{mg}{R_e} x$$

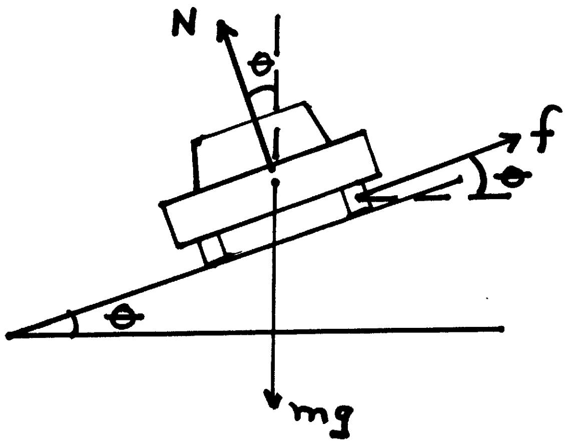
$$m\ddot{x} = -\frac{mg}{R_e} x$$

$\Rightarrow$ SHM, period $T = 2\pi\sqrt{R_e/g}$

$$F = \frac{mgr}{R_e}$$

**2.28**

Minimum speed:
(forces as shown)



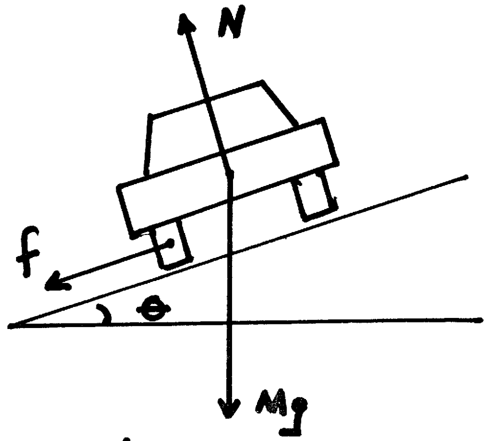

$$N\cos\theta + f\sin\theta = Mg$$
$$N\sin\theta - f\cos\theta = Mv^2/R$$

At limit, $f_{max} = \mu N$

Solving,

$$V_{min} = \left[Rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)\right]^{\frac{1}{2}}$$

Maximum speed:




$$N\cos\theta - f\sin\theta = Mg$$
$$N\sin\theta + f\cos\theta = Mv^2/R$$
$$f_{max} = \mu N$$

Solving,

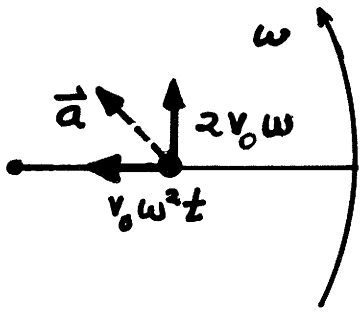
$$V_{max} = \left[Rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)\right]^{\frac{1}{2}}$$

---

**2.29**

(a) $$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

But $r = v_0 t \Rightarrow \dot{r} = v_0$
$\ddot{r} = 0$



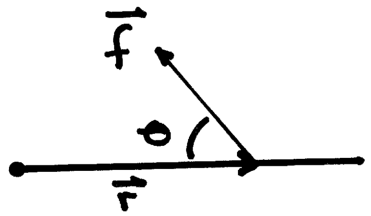

Hence $\vec{a} = -v_0\omega^2 t\,\hat{r} + 2v_0\omega\,\hat{\theta}$

(b) Starts to skid when $Ma = \mu N$ OR $a = \mu g$

$$4v_0^2\omega^2 + v_0^2\omega^4 t^2 = (\mu g)^2$$

Hence $t = \dfrac{1}{v_0\omega^2}\sqrt{(\mu g)^2 - 4v_0^2\omega^2}$

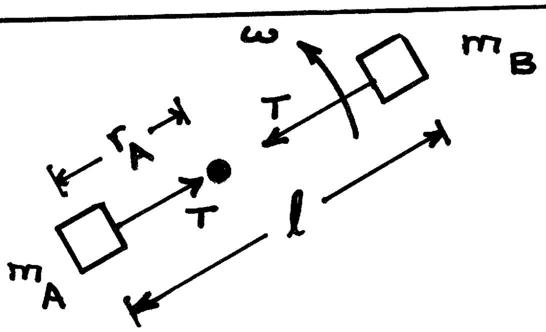
(c) $\tan\theta = \dfrac{2v_0\omega}{v_0\omega^2 t} = \dfrac{2}{\omega t}$




$$\tan\theta = \frac{2v_0\omega}{\sqrt{(\mu g)^2 - 4v_0^2\omega^2}}$$

---

$\boxed{2.30}$




$$-T = m_A(\ddot{r}_A - r_A\omega^2)$$

$$-T = m_B(\ddot{r}_B - r_B\omega^2)$$

$$r_B = l - r_A$$

$$-T = m_B\left[-\ddot{r}_A - (l - r_A)\,\omega^2\right]$$

Solving, $\ddot{r}_A = r_A\omega^2 - \dfrac{l\omega^2}{\left(1 + \dfrac{m_A}{m_B}\right)}$

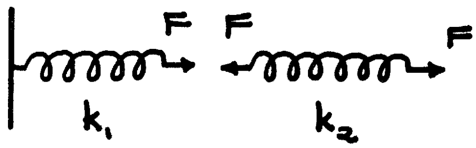
## 2.31

(a)

$\Delta x_1 = F/k_1$

$\Delta x_2 = F/k_2$

$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = F\,(1/k_1 + 1/k_2)$




Hence $k_{eff} = F/\Delta x_{tot} = \dfrac{1}{(1/k_1 + 1/k_2)}$

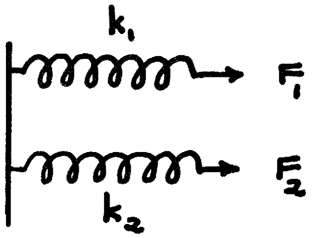
$\omega_a = \sqrt{\dfrac{k_{eff}}{m}} = \sqrt{\dfrac{1}{m}\dfrac{1}{(1/k_1 + 1/k_2)}}$

(b)

$F_1 = k_1 \Delta x$

$F_2 = k_2 \Delta x$

$F_{tot} = F_1 + F_2 = (k_1 + k_2)\,\Delta x$



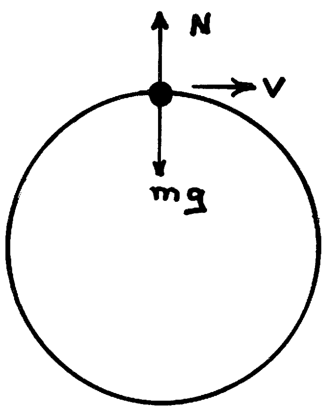

Hence $k_{eff} = k_1 + k_2$ and $\omega_b = \sqrt{\dfrac{k_1 + k_2}{m}}$

---

## 2.32

(a)

$mg - N = mv^2/R$

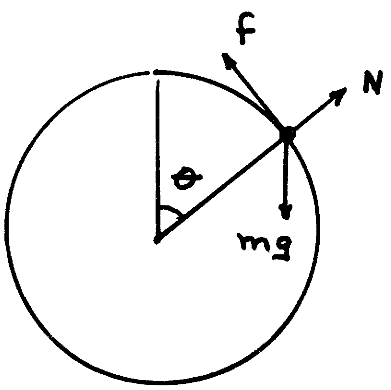



N cannot be negative

Hence pebble flies off if

$\dfrac{mv^2}{R} > mg$ or $v > \sqrt{Rg}$

(b)




While in contact

$$N - mg\cos\theta = -mv^2/R$$

With the condition $N=0$,

we must have $\cos\theta_m = v^2/Rg$

There is a more stringent condition based on $f \le \mu N$.

$$f = mg\sin\theta \le \mu m\left(g\cos\theta - v^2/R\right)$$

$$\sin\theta \le \mu\cos\theta - \mu v^2/Rg$$

Let $\mu = 1$

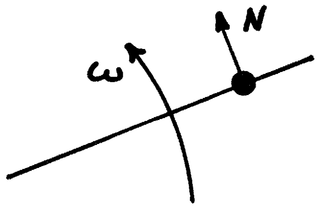
Use $\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$

Then

$$\cos\left(\theta_m + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\frac{v^2}{Rg}$$

---

$\boxed{2.33}$




$$0 = m(\ddot{r} - r\omega^2)$$

$$\frac{d^2r}{dt^2} = r\omega^2$$

This equation is satisfied by

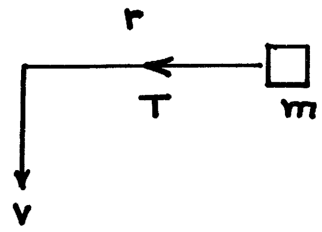
$$r = Ae^{-\omega t} + Be^{\omega t}$$

$$r(0) = A + B$$

$$v(0) = -\omega A + \omega B$$

To make $B=0$, choose $v(0) = -\omega r(0)$ Then $r = r_0 e^{-\omega t}$

$\boxed{2.34}$




(a) $0 = a_\theta = r\dot{\omega} + 2\dot{r}\omega$

Hence $\displaystyle\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -\int_{r_0}^{r} \frac{2dr}{r}$

which yields $\omega = \omega_0 \left(\frac{r_0}{r}\right)^2 = \omega_0 \left(\frac{r_0}{r_0 - vt}\right)^2$

(b) $-T = m(\ddot{r} - r\omega^2)$

$T = mr\omega^2 = m\omega_0^2 \dfrac{r_0^4}{r^3}$

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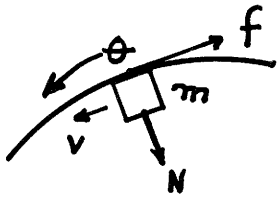
$\boxed{2.35}$




(a) $-N = -\dfrac{mv^2}{\ell}$

$-f = m\ell\ddot{\theta} = m\dfrac{dv}{dt}$ because $v = \ell\dot{\theta}$

$f = \mu N$

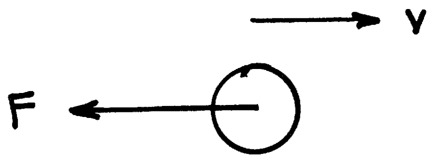
Hence $\dfrac{dv}{dt} = -\mu\dfrac{v^2}{\ell}$

$\displaystyle\int_{v_0}^{v} \frac{dv}{v^2} = -\int_0^t \frac{\mu}{\ell}\,dt \Rightarrow \frac{1}{v_0} - \frac{1}{v} = -\frac{\mu}{\ell}t$

$v = \dfrac{v_0}{\left(1 + \frac{\mu v_0 t}{\ell}\right)}$

(b) $\dfrac{d\theta}{dt} = \dfrac{v}{\ell} = \dfrac{v_0/\ell}{\left(1 + \frac{\mu v_0 t}{\ell}\right)} \Rightarrow \theta = \theta_0 + \dfrac{1}{\mu}\ln\left(1 + \dfrac{\mu v_0}{\ell}t\right)$

$\boxed{2.36}$




$$m\frac{dv}{dt} = -F = -be^{\alpha v}$$

$$\int_{v_0}^{v} e^{-\alpha v}dv = -\frac{b}{m}\int_0^t dt$$

$$\frac{1}{\alpha}\left(e^{-\alpha v} - e^{-\alpha v_0}\right) = \frac{b}{m}t$$

$$v = \frac{1}{\alpha}\ln\left(\frac{1}{\frac{\alpha b}{m}t + e^{-\alpha v_0}}\right)$$

---

$\boxed{2.37}$

$$N\cos\theta - mg = 0$$

$$N\sin\theta = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

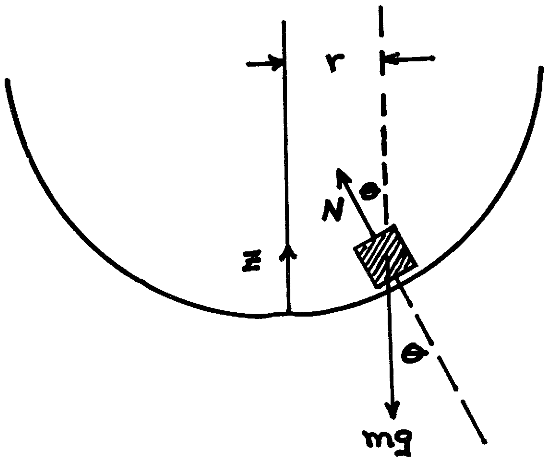
$$\tan\theta = \frac{r}{g}\left(\frac{2\pi}{T}\right)^2$$

$$\frac{dz}{dr} = \frac{r}{g}\left(\frac{2\pi}{T}\right)^2$$

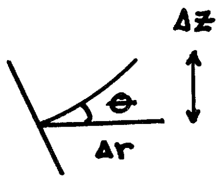
Hence

$$g\left(\frac{T}{2\pi}\right)^2 z = \frac{1}{2}(r^2 - r_0^2)$$

$$r^2 = r_0^2 + 2g\left(\frac{T}{2\pi}\right)^2 z$$



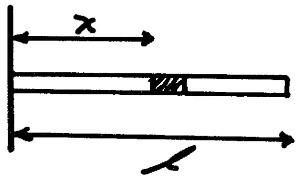

The bowl is a parabola of revolution.



# Chapter 3

$\boxed{3.1}$

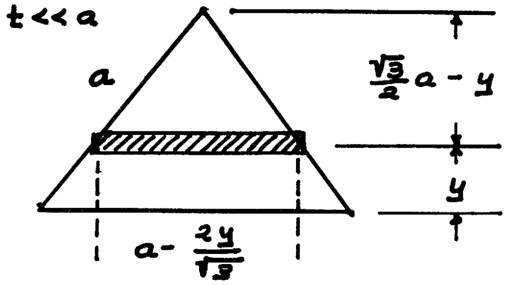
$$X = \frac{\int x\,dm}{M} = \frac{\int_0^{\ell} \frac{\rho_0}{\ell^2} x^3 dx}{\int_0^{\ell} \frac{\rho_0}{\ell^2} x^2 dx}$$




$$X = \frac{\ell^4/4}{\ell^3/3} = \frac{3}{4}\ell$$

---

$\boxed{3.2}$



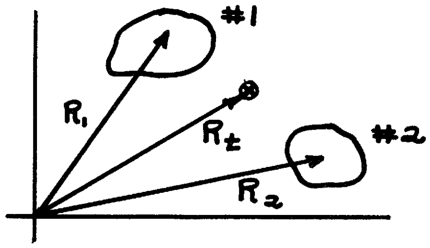

$$\text{Density} = \frac{M}{(\frac{1}{2}a)(\sqrt{3}a/2)t}$$

$$= \frac{4M}{\sqrt{3}\,a^2 t}$$

$$Y = \frac{4M}{M\sqrt{3}\,a^2 t} \int_0^{\frac{\sqrt{3}}{2}a} \left(a - \frac{2y}{\sqrt{3}}\right) y\,dy\,t = \frac{a}{2\sqrt{3}} = \frac{1}{3}\,\frac{\sqrt{3}\,a}{2}$$

---

$\boxed{3.3}$

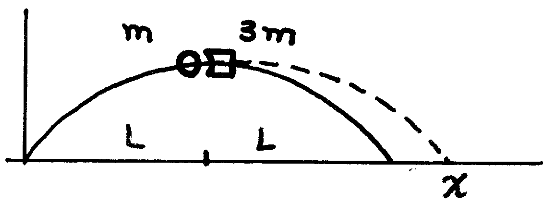



$$\vec{R}_1 = \frac{\sum \vec{r}_{1j}\, m_{1j}}{M_1}$$

$$\vec{R}_2 = \frac{\sum \vec{r}_{2j}\, m_{2j}}{M_2}$$

$$R_t = \frac{\sum \vec{r}_{1j}\, m_{1j} + \vec{r}_{2j}\, m_{2j}}{M_1 + M_2} = \frac{M_1\vec{R}_1 + M_2\vec{R}_2}{M_1 + M_2}$$

---



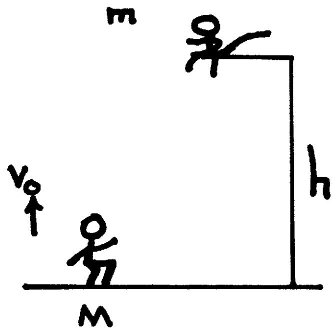

$\boxed{3.4}$ CM follows the original trajectory. CM and both masses land simultaneously because initial velocities are horizontal. When m lands, at 0, cm is at 2L. Thus $2L = \dfrac{3M(x) + M(0)}{3M + m}$

Solving, $x = 8L/3$ from the launch station

---

$\boxed{3.5}$

At height $h$ speed is $v = \sqrt{v_0^2 - 2gh}$




$Mv = (M+m)v'$

$v' = \dfrac{Mv}{M+m}$ is the new initial velocity after monkey is picked up.

Pair rise an additional height

$\dfrac{v'^2}{2g} = \dfrac{1}{2g}\left(\dfrac{M}{m+M}\right)^2 (v_0^2 - 2gh)$

---

$\boxed{3.6}$ Immediately after collision with sandbag, plane's velocity is reduced from $v_0$ to $v_0' = Mv_0/(M+m)$

$M =$ mass of plane

$m =$ mass of sandbag

Retarding force $F_r = (0.4)(250) + 300 = 400$ lbs

$v = v_0' - a_r t$ where $a_r = F_r/(M+m)$

Stops at $t' = v_0'/a_r$

Distance of travel $= s = v_0' t' - \frac{1}{2} a_r t'^2 = v_0'^2/2a_r$

$s = \left(\dfrac{M}{M+m}\right)^2 v_0^2 \,(M+m)/F_r = \dfrac{M}{M+m}\left(\dfrac{mv_0^2}{F_r}\right)$

$$s = \frac{\left(\frac{2500}{2750}\right)\left(\frac{2500}{32}\right)(120)^2}{400} = 2560 \text{ feet}$$

---

**3.7**

While block #1 is in contact, motion of block #2 is given by

$x_2 = l - \frac{l}{2}\cos\omega t$ where $\omega = \sqrt{k/m_2}$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_2}{m_1+m_2}\left(l - \frac{l}{2}\cos\omega t\right)$$

Block #1 loses contact when $x = l$. This occurs at $\omega t' = \frac{\pi}{2}$. After this time the CM moves at a constant velocity $V_{CM}$. $V_{CM}$ can be found from $\dot{x}_1$ and $\dot{x}_2$ at $t'$.

$$V_{CM} = \frac{m_1\dot{x}_1(t') + m_2\dot{x}_2(t')}{m_1+m_2} = \left(\frac{m_2}{m_1+m_2}\right)\frac{\omega l}{2}$$

---

**3.8**

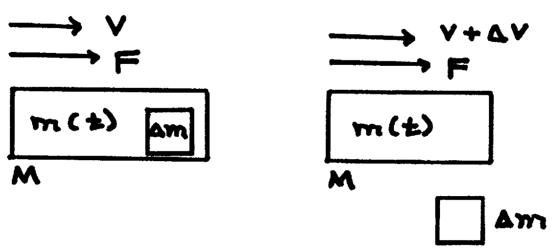
Initial velocity $v_0 = \sqrt{2gs}$

Impulse $= mv_0 = 50\sqrt{2 \times 9.8 \times 0.8} = 198$ kgm/s

---

**3.9**

Let $m(t)$ be the amount of sand in box car at time $t$. $\Delta m$ is the amount released in time $\Delta t$.



$$P_f = (M + m(t) + \Delta m)(v + \Delta v)$$
$$P_i = (M + m(t) + \Delta m)\, v$$

$$P_f - P_i = (M + m(t))\,\Delta v = F\,\Delta t \qquad \text{to first order}$$

$$F = (M + m(t))\frac{dv}{dt}$$
$$dv = \frac{F\,dt}{M + m(t)}$$

Let $\frac{dm}{dt} = -\alpha \Rightarrow m(t) = m - \alpha t$

$$\int_0^{v_f} dv = F\int_0^{m_0/\alpha} \frac{dt}{M + m - \alpha t} \Rightarrow v_f = \frac{F}{\alpha}\ln\left(1 + \frac{m}{M}\right)$$

---

$\boxed{3.10}$

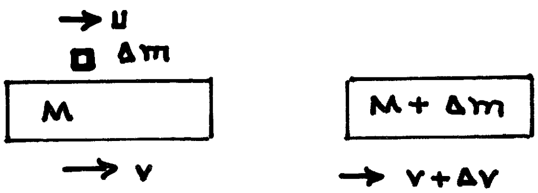
$$P_f - P_i = Ft$$
$$(M + bt)\,v - 0 = Ft$$
$$v = \frac{Ft}{M + bt}$$

(Method is valid because incoming sand contributes no horizontal momentum)

---

$\boxed{3.11}$

$$P_f = (M + \Delta m)(v + \Delta v)$$
$$P_i = Mv + (\Delta M)u$$




$$0 = P_f - P_i = M\Delta v + \Delta M(v - u)$$

Hence $M\frac{dv}{dt} + \frac{dM}{dt}(v - u) = 0$ since $\frac{dm}{dt} = \frac{dM}{dt}$

and $M\,dv = (u - v)\,dM \Rightarrow \frac{dv}{dt} = \frac{u - v}{M}\frac{dM}{dt} = \frac{u - v}{M}\,b.$

Alternatively,

$$\int_0^v \frac{dv}{u-v} = \int_{M_0}^{M_0+bt} \frac{dM}{M} \qquad \text{which yields}$$

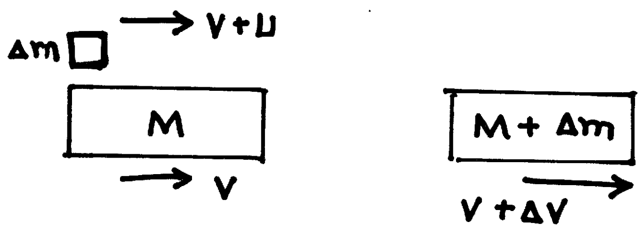
$$v = \frac{u}{1+\frac{M_0}{bt}} \qquad \frac{dv}{dt} = \frac{u\,b\,M_0}{(bt+M_0)^2}$$

---

**3.12**

$$P_f = (M+\Delta m)(v+\Delta v)$$

$$P_i = Mv + \Delta m\,(v+u)$$




$$0 = P_f - P_i = M\Delta v - \Delta m\,u$$

$$\int_0^v dv = u\int_{M_0}^{M_0+bt} \frac{dM}{M} \Rightarrow v = u \ln\left(\frac{M_0+bt}{M_0}\right)$$

$$v(100) = 5 \ln\left(\frac{2000+10(100)}{2000}\right) = 2.03 \text{ m/s}$$

---

**3.13**

Separation between skiers $= (5)(1.5) = 7.5$ m

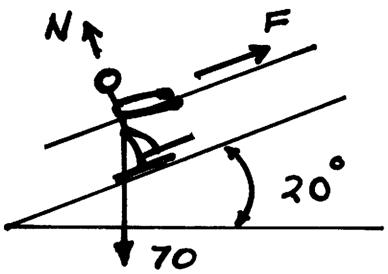
Hence there are $\frac{100}{7.5} = 13\,^1/_3$ skiers on the tow, average. Force to lift one skier is

$(70)(9.8)(\sin 20°) = 235$ N

Total force to lift skiers $= (13\tfrac{1}{3})(235) = 3130$ N

Must also add the force necessary to change momentum of skier $F = \frac{(70)(1.5)}{5} = 23$ N

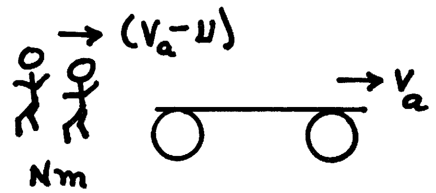
Total Force $\cong 3150$ N



$\boxed{3.14}$ In each case, the flatcar is at its final velocity as the jumper parts company ; no acceleration after jumper leaves.

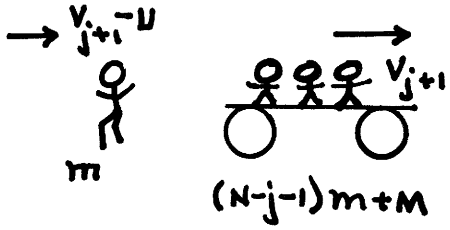
(a) $P_i = 0$

$$P_f = MV_a + Nm(V_a - u) = 0$$




Hence $V_a = \left(\frac{Nm}{Nm+M}\right)u$

(b) Assume j have already jumped so that speed of flatcar is $v_j$.




$$P_i = [(N-j)m + M]v_j$$

$$P_f = [(N-j-1)m + M]v_{j+1} + m(v_{j+1} - u)$$

$$v_{j+1} = \frac{m}{(N-j)m+M}u + v_j$$

Hence $v_b = \left[\frac{m}{Nm+M} + \frac{m}{(N-1)m+M} + \cdots \frac{m}{m+M}\right]u$

(c) But $V_a = \left[\frac{m}{Nm+M} + \frac{m}{Nm+M} + \cdots \frac{m}{Nm+M}\right]u$

Hence $V_b > V_a$

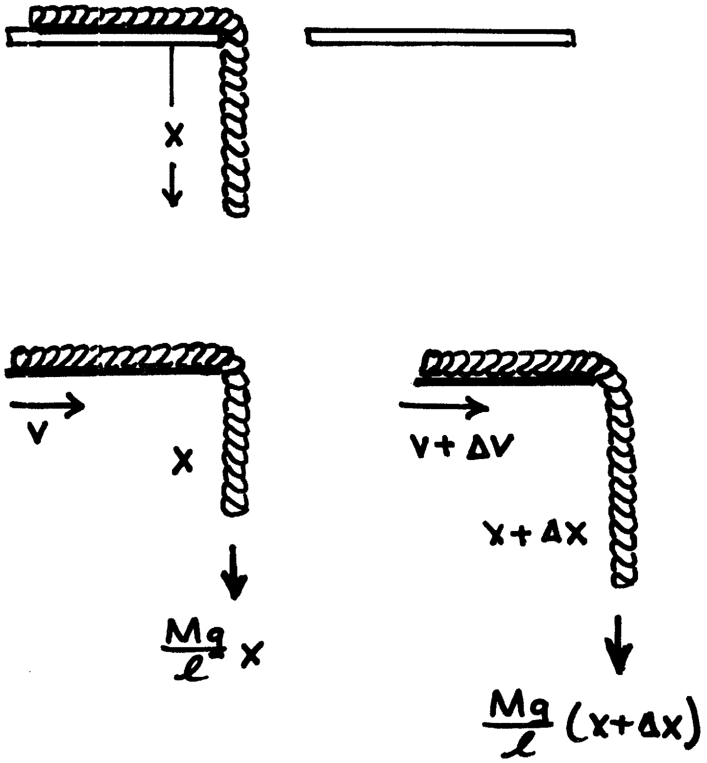
$\boxed{3.15}$

(a) Mass of hanging portion $= \dfrac{Mx}{\ell}$

$P_i = MV \qquad F_{ext}(t) = \dfrac{Mg}{\ell}x$

$P_f = M(V+\Delta V)$

$F_{ext}(t+\Delta t) = \dfrac{Mg}{\ell}(x+\Delta x)$




$$M\Delta V = \int_t^{t+\Delta t} F_{ext}\,dt = \frac{Mgx\Delta t}{\ell}$$

Hence $\quad \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = \dfrac{g\,x}{\ell}$

$$x = Ae^{g/\ell\, t} + B\,e^{-g/\ell\, t}$$

(b) $\quad x(0) = \ell_0 \qquad \Rightarrow \quad \ell_0 = A+B$

$\qquad \dot{x}(0) = 0 \qquad \rightarrow \quad \dfrac{g}{\ell}A - \dfrac{g}{\ell}B = 0$

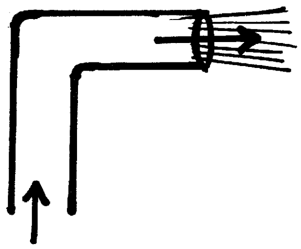
Hence $\quad A = B = \ell_0/2$

---

$\boxed{3.16}$ $\quad \lambda =$ mass/unit length $= \dfrac{\pi D^2}{4}\rho\cdot 1 = \dfrac{\pi D^2 \rho}{4}$

where $\rho$ = mass density of water

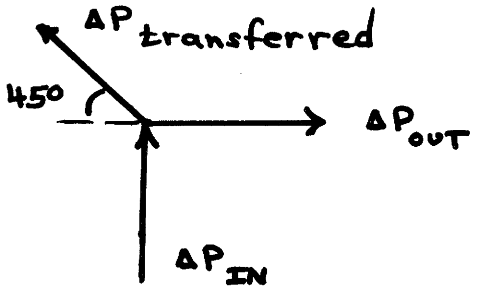
$\Delta P_{in} = \Delta P_{OUT} = \lambda v_0^2 \Delta t$

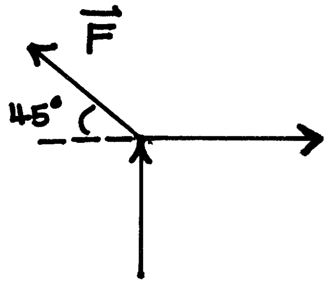


$\Delta\vec{P}$ transferred to hydrant is at $45^0$ as shown and has magnitude $\sqrt{2}\,\lambda v_0^2 \Delta t$.

Hence the force on hydrant is

$$\frac{\Delta P}{\Delta t} = \sqrt{2}\,\lambda V_0^2 = \frac{\sqrt{2}}{4}\rho D^2 V_0^2$$

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$\boxed{3.17}$ Water speed at height $h$ is

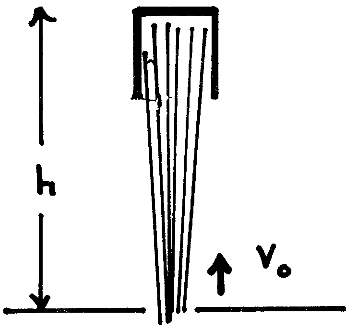
$$v = \sqrt{v_0^2 - 2gh}$$

Assume water bounces elastically from can. Then $F_{max} = 2\lambda v^2$

where $\lambda v = \lambda v_0 = (dm/dt)_0$

Hence $W = 2\left(\frac{dm}{dt}\right)_0 v$ and

$$h = \frac{v_0^2}{2g} - \frac{W^2}{8g\left(\frac{dm}{dt}\right)_0}$$

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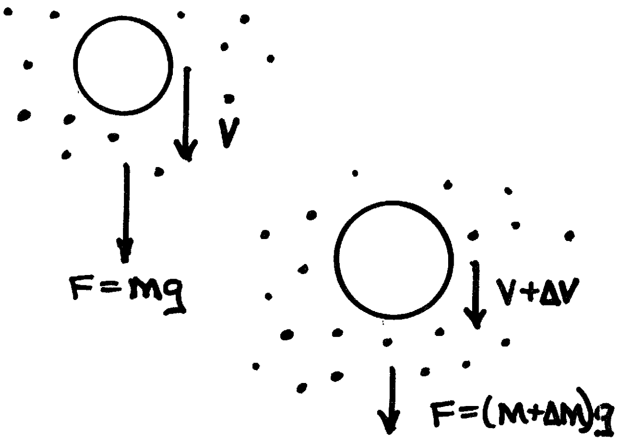
$\boxed{3.18}$ $P_i = MV$

$P_f = (M+\Delta M)(V+\Delta V)$

$$P_f - P_i = M\Delta V + (\Delta M)V = Mg\,\Delta t$$

Hence $M\frac{dV}{dt} + \frac{dM}{dt}V = Mg$

OR $M\frac{dV}{dt} + kMv^2 = Mg$



$$\frac{dv}{dt} = g - kv^2$$

As $v$ increases, $\frac{dv}{dt}$ decreases and eventually becomes zero. Hence at terminal velocity

$g = k v_{term}^2$ OR $v_{term} = \sqrt{g/k}$

---

$\boxed{3.19}$ Force on 1 square centimeter $= \lambda v^2$

$= \left(\frac{dm}{dt}\right) v = 5 \times 10^{-6} N$

$\Rightarrow$ Total force $= (500)(5\times10^{-6}) = 2.5\times10^{-3}$ N

When bowl is moving upward, $\frac{dm}{dt} = \frac{7}{5}\times10^{-6}$

and total force $= 4.9\times10^{-3} N$

---

$\boxed{3.20}$

$$m\frac{dv}{dt} + u\frac{dm}{dt} = -mg - mbv$$

$$m\frac{dv}{dt} - u\gamma m = -mg - mbv$$

$$\frac{dv}{dt} + bv = \gamma\mu - g$$

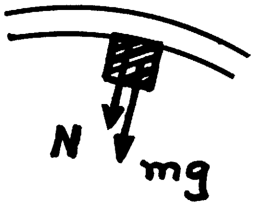
$$v = \frac{\gamma\mu - g}{b} + Ae^{-t/b}$$

Choose the constant $A$ such that $v(0) = 0$

$$v = \frac{\gamma u - g}{b}\left(1 - e^{-t/b}\right)$$

# Chapter 4

**4.1**




$E_i = mgz$

$E_f = mg(2R) + \frac{1}{2}mv^2$

$N + mg = \frac{mv^2}{R}$

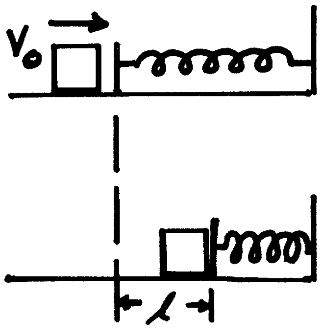
If $N = mg$, $mv^2 = 2mgR$.

Hence $E_f = 3mgR = mgz$

$z = 3R$

**4.2** $E_i = \frac{1}{2}Mv_0^2$

$E_f = \frac{1}{2}kl^2$




$E_i - E_f = W_f = \int_0^l mgbx\,dx = \frac{1}{2}Mgbl^2$

Block comes to rest when $\frac{1}{2}mv_0^2 = \frac{1}{2}(k+Mgb)l^2$

$\Rightarrow \quad l^2 = \frac{Mv_0^2}{k+Mgb}$

Loss of mechanical energy $= \frac{1}{2}Mgbl^2$

$= \frac{Mgb}{k+Mgb}\left(\frac{1}{2}Mv_0^2\right)$

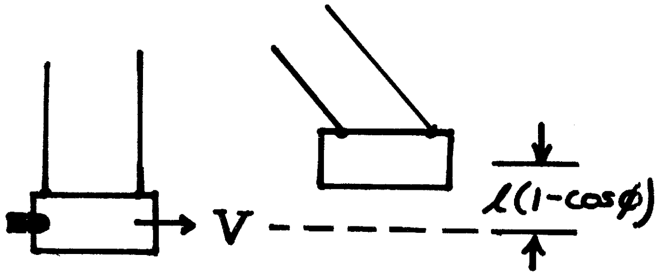
**4.3** (a) $mv = (M+m)V$

$V = \left(\frac{m}{M+m}\right)v$

(b) $E_i = \frac{1}{2}(M+m)V^2$

$E_f = (M+m)g\ell(1-\cos\phi)$

$V^2 = 2g\ell(1-\cos\phi)$



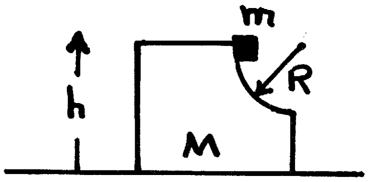

$$v = \left(\frac{M+m}{m}\right)\sqrt{2g\ell(1-\cos\phi)}$$

---

$\boxed{4.4}$

$E_i = mgh$

$E_f = mg(h-R) + \frac{1}{2}MV^2 + \frac{1}{2}mv^2$

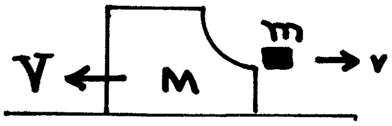



Hence

$$0 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 - mgR$$

By conservation of momentum

$MV = mv \quad \Rightarrow \quad v = \sqrt{\left(\frac{M}{M+m}\right)2gR}$

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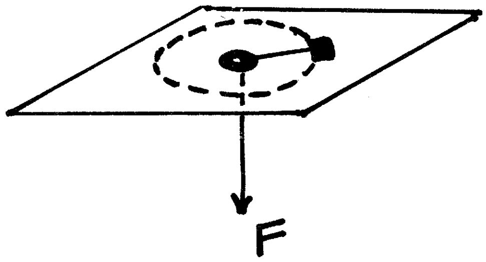
$\boxed{4.5}$

$-F = m(\ddot{r} - r\dot{\theta}^2)$

$0 = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\phantom{0} = r\dot{\omega} + 2\dot{r}\omega$




$\frac{\dot{\omega}}{\omega} = -\frac{2\dot{r}}{r}$

Hence $\frac{d\omega}{\omega} = -\frac{2dr}{r}$

$\ln\omega = -2\ln r + \ln c \quad$ OR $\quad \dot{\theta} = \frac{\text{const}}{r^2} = \frac{\ell_1^2\omega_1}{r^2}$

where $r(t_1) = \ell_1$ and $\dot{\theta}(t_1) = \omega_1$

$$W = \int \vec{F}\cdot d\vec{r} = m \int_{\ell_1}^{\ell_2} \left( \ddot{r} - r\,\frac{\ell_1^4 \omega_1^2}{r^4} \right) dr$$

$$= \tfrac{1}{2} m v_r^2(t_2) - \tfrac{1}{2} m v_r^2(t_1) + m \ell_1^4 \omega_1^2 \left( \frac{1}{2\ell_2^2} - \frac{1}{2\ell_1^2} \right)$$

$$= \tfrac{1}{2} m v_r^2(t_2) + \frac{m \ell_1^4 \omega_1^2}{2\ell_2^2} - \left( \tfrac{1}{2} m v_r^2(t_1) + \tfrac{1}{2} m \ell_1^2 \omega_1^2 \right)$$

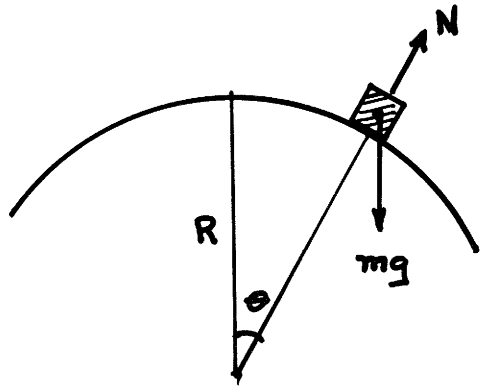
But $\dfrac{m \ell_1^4 \omega_1^2}{2\ell_2^2} = \dfrac{\ell_2^4 \omega_2^2}{\ell_2^2}\dfrac{m}{2}$

Hence $W = \left( \tfrac{1}{2} m v_r^2(t_2) + \tfrac{1}{2} m \ell_2^2 \omega_2^2 \right) - \left( \tfrac{1}{2} m v_r^2(t_1) + \tfrac{1}{2} m \ell_1^2 \omega_1^2 \right)$

$W = KE(t_2) - KE(t_1)$

---

$\boxed{4.6}$




$N - mg\cos\theta = -\dfrac{mv^2}{R}$

Contact is lost when $N = 0$

From energy considerations

$mgR = mgR\cos\theta + \tfrac{1}{2} m v^2$

$\dfrac{v^2}{R} = 2g(1-\cos\theta)$

Contact is lost when

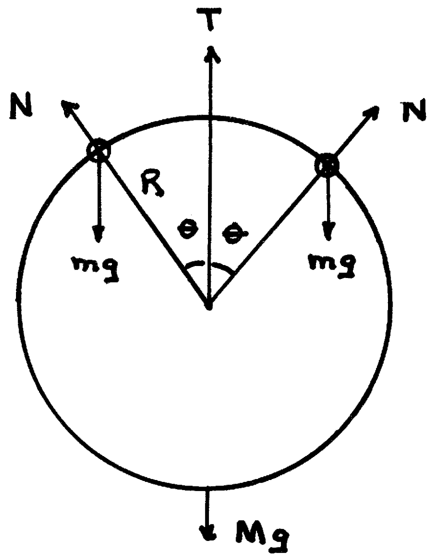
$-g\cos\theta = -2g(1-\cos\theta) \Rightarrow 3\cos\theta = 2$

Hence $x = R(1-\cos\theta) = \dfrac{R}{3}$

## 4.7

Vertical force on ring $= T - 2N\cos\theta - Mg = 0$ in equilibrium.

$N - mg\cos\theta = -mv^2/R$




By conservation of energy

$mgR = \frac{1}{2}mv^2 + mgR\cos\theta$

$\frac{mv^2}{R} = 2mg(1-\cos\theta)$

Hence $N = mg(3\cos\theta - 2)$

Ring starts to rise when $T=0$.

$2mg(2-3\cos\theta)\cos\theta = Mg$

The maximum value of $2\cos\theta - 3\cos^2\theta$ occurs at $\cos\theta = 1/3$. Hence the ring will not rise unless

$(2m)1/3 \geq M$

$m \geq \frac{3}{2}M$

Angle is given by $3\cos^2\theta - 2\cos\theta + \frac{M}{2m} = 0$

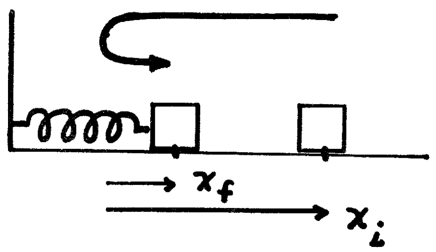
$\Rightarrow \cos\theta = \frac{1}{3} + \sqrt{\frac{1}{9} - \frac{M}{6m}}$

---

## 4.8

$\frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \cong 2f(x_i + x_f)$

$(x_i + x_f)(x_i - x_f) = \frac{4f}{k}(x_i + x_f)$




$\Delta$ amplitude $= x_i - x_f = \frac{4f}{k}$

(b) Block comes to rest when $kx_f = f$ or $x_f = f/k$

Hence $4fn/k = x_0 - f/k$

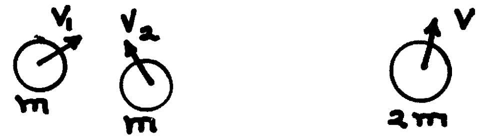
$$n = \frac{1}{4}\left(\frac{kx_0}{f} - 1\right) \cong \frac{kx_0}{4f}$$

---

$\boxed{4.9}$

$m\vec{v}_1 + m\vec{v}_2 = 2m\vec{V}$

Total energy available $= Q + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}(2m)V^2$




$V^2 = \frac{1}{2}(v_1^2 + v_2^2) + \frac{Q}{m}$, $Q = 5\text{eV}$

But $\vec{V} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)$

$V^2 = \vec{V}\cdot\vec{V} = \frac{1}{4}(v_1^2 + v_2^2 + 2\vec{v}_1\cdot\vec{v}_2)$

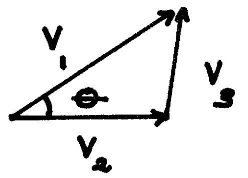
Hence we require $\frac{1}{2}(v_1^2 + v_2^2) + \frac{Q}{m} = \frac{1}{4}(v_1^2 + v_2^2 + 2\vec{v}_1\cdot\vec{v}_2)$

or $Q/m = -\frac{1}{4}(v_1^2 + v_2^2 - 2\vec{v}_1\cdot\vec{v}_2)$

This condition cannot be satisfied because $Q > 0$ and the right hand side is $\leq 0$:

Let $\vec{v}_3 = \vec{v}_1 - \vec{v}_2$

$v_3^2 = v_1^2 + v_2^2 - 2v_1v_2\cos\theta \geq 0$




$\Rightarrow \quad v_1^2 + v_2^2 \geq 2\vec{v}_1\cdot\vec{v}_2$

$\boxed{4.10}$

(a) (1) $T_f = 2\pi\sqrt{\dfrac{M+m}{k}} = \sqrt{\dfrac{M+m}{M}}\,T_0$

(2) At $v=0$, amplitude is $A_0$. Hence new amplitude $= A_0$.

(3) $E = \frac{1}{2}k\,(\text{amplitude})^2 = \frac{1}{2}kA_0^2$

Hence $E_{initial} = E_{final}$

(b) (1) $T_f = 2\pi\sqrt{\dfrac{M+m}{k}} = \sqrt{\dfrac{M+m}{M}}\,T_0$

(2) $mv = (M+m)v'$


$v' = \left(\dfrac{M}{M+m}\right)v$

$\frac{1}{2}kA_f^2 = \frac{1}{2}(M+m)v'^2 = \left(\dfrac{M}{M+m}\right)\frac{1}{2}Mv^2$

$= \left(\dfrac{M}{M+m}\right)\frac{1}{2}kA_0^2$

$\Rightarrow A_f = \sqrt{\dfrac{M}{M+m}}\,A_0$

(3) $\Delta E = \frac{1}{2}kA_0^2 - \frac{1}{2}kA_f^2 = \frac{1}{2}kA_0^2\left[1 - \dfrac{M}{M+m}\right]$

$= \left(\dfrac{m}{m+M}\right)\frac{1}{2}kA_0^2$

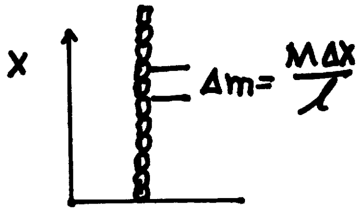
---

$\boxed{4.11}$

$\Delta m$ hits the pan with speed

$\frac{1}{2}(\Delta m)v^2 = (\Delta m)g\,x$

$v = \sqrt{2gx}$



It delivers momentum $(\Delta m) v = \frac{M}{\ell}\sqrt{2gx}\;\Delta x$

Average force due to collisions

$$= \bar{F} = \frac{\Delta P}{\Delta t} = \frac{M}{\ell}\sqrt{2gx}\,\frac{\Delta x}{\Delta t} = \frac{M}{\ell}\sqrt{2gx}\;v$$

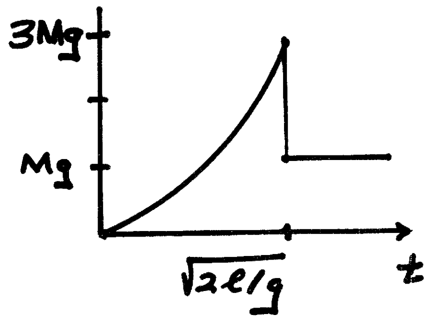
$$= 2g\frac{M}{\ell}\,x$$

Weight of chain on pan $= \frac{Mg}{\ell}x$

Reading of scale $= \frac{3Mg}{\ell}x$

$= \frac{3Mg}{\ell}\left(\frac{1}{2}gt^2\right)$ for $0 < t < \sqrt{2\ell/g}$

$= Mg$ for $t > \sqrt{2\ell/g}$

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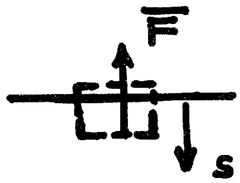
$\boxed{4.12}$ Speed at impact $= v_0 = \sqrt{2gh}$

If $\Delta t$ = duration of impact,

$\bar{F}\,\Delta t = mv_0 - 0$

$s = v_0\Delta t - \frac{1}{2}\frac{\bar{F}}{m}(\Delta t)^2$

$s = v_0\Delta t - \frac{1}{2}\frac{v_0}{\Delta t}(\Delta t)^2 = \frac{1}{2}v_0\Delta t$




Hence $\Delta t = \frac{2s}{v_0}$

Alternate Method: Use $\frac{1}{2}mv_0^2 = \bar{F}s$

$\bar{F} = \frac{\frac{1}{2}mv_0^2}{s}$

$\bar{P} = \frac{\bar{F}}{A} = \frac{mgh}{As} = \frac{(144)(150)}{(5)(2)} = (144)(15)\ lb/ft^2$

OR $\bar{P} = 15\ lb/in^2 \Rightarrow$ safe to drop

$\boxed{4.13}$

(a) $$\frac{dU}{dr} = 0 = \epsilon\left[-\frac{12 r_0^{12}}{r^{13}} + \frac{12 r_0^6}{r^7}\right]$$

$$\text{OR} \quad \frac{r_0^6}{r^6} = 1 \quad \Longrightarrow \quad r = r_0$$

$$U(r_0) = \epsilon\left[\left(\frac{r_0}{r_0}\right)^{12} - 2\left(\frac{r_0}{r_0}\right)^6\right] = -\epsilon$$

(b) $$\omega = \sqrt{k/\mu} \qquad \mu = \frac{m}{2}$$
$$= \sqrt{2k/m}$$

$$k = \left.\frac{d^2U}{dr^2}\right|_{r_0} = \epsilon\left[\frac{(12)(13)}{r_0^{14}}r_0^{12} - \frac{(12)(7)r_0^6}{r_0^8}\right] = \frac{72\epsilon}{r_0^2}$$

$$\omega = 12\sqrt{\epsilon/m r_0^2}$$

---

$\boxed{4.14}$

(a) $$U = -\frac{GMm}{r} - \frac{GMm}{r} = \frac{-2GMm}{\sqrt{a^2+x^2}}$$

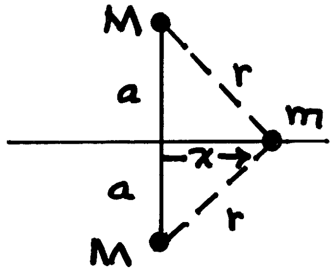
(b) $$\frac{1}{2}mv_0^2 - \frac{2GMm}{\sqrt{a^2+9a^2}} = \frac{1}{2}mv^2 - \frac{2GMm}{\sqrt{a^2}}$$

$$v^2 = v_0^2 + \frac{4GM}{a}\left(1 - \frac{1}{\sqrt{10}}\right) = v_0^2 + \frac{2.74\,GM}{a}$$

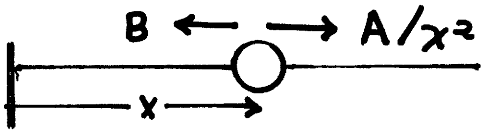
(c) $$\omega = \sqrt{k/m}$$

$$k = \left.\frac{d^2U}{dx^2}\right|_0 = \frac{2GMm}{a^3}$$

$$\omega = \sqrt{2GM/a^3}$$



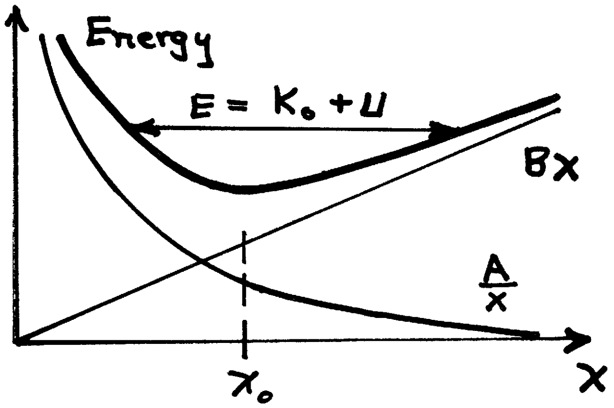
**4.15**




(a) $U = -\int \vec{F} \cdot \vec{d}x$

$= Bx + A/x + \text{const}$

(b) Let const = 0




(c) $0 = \left.\frac{dU}{dx}\right|_{x_0} = B - \frac{A}{x_0^2}$

$\Rightarrow \quad x_0 = \sqrt{A/B}$

(d) $\omega = \sqrt{k/m}$ where $k = \frac{2A}{x_0^3} = 2\sqrt{\frac{B^3}{A}}$

$\omega = \sqrt{\frac{2}{m}}\left(\frac{B^3}{A}\right)^{\frac{1}{4}}$

---

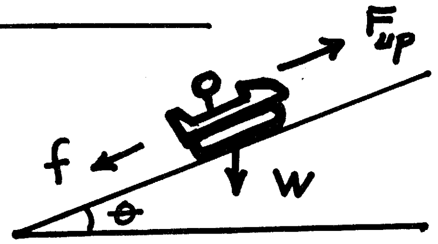
**4.16** $\bar{P} = \frac{\Delta E}{T} = \frac{\frac{1}{2}mv^2}{T} = \frac{(\frac{1}{2})\left(\frac{1800}{32}\right)(88\,ft/s)^2}{8}$

$= 27{,}225$ ft-lb/s

$\bar{P} = \frac{27{,}225}{550} = 49.5$ horsepower

---

**4.17**

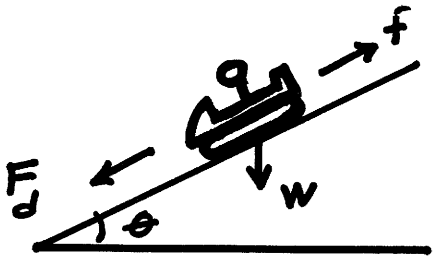



$F_{up} = f + W\sin\theta$

$= W(0.05 + \sin\theta)$

$$F_d = f - W\sin\theta$$

$$= W(0.05 - \sin\theta)$$




$$P = F_{up} V_{up} = F_d V_d$$

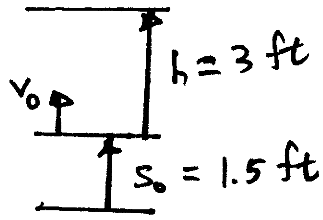
$$V_d = \left(\frac{0.05 + \sin\theta}{0.05 - \sin\theta}\right) V_{up} \cong \left(\frac{0.05 + 0.025}{0.05 - 0.025}\right) 15$$

$$= 45 \text{ mph}$$

---

$\boxed{4.18}$ He leaves the ground at time $T$.

$$V_0 = \left[\frac{F}{m} - g\right]T, \quad S_0 = \frac{1}{2}\left[\frac{F}{m} - g\right]T^2 = \frac{1}{2}V_0 T$$




METHOD 1: $P = F V_{ave} = F\left(\frac{V_0}{2}\right)$

$$\frac{1}{2}mV_0^2 = mgh \rightarrow V_0 = \sqrt{2gh} = \sqrt{(2)(32)(3)}$$

$$= 13.9 \text{ ft/s}$$

$$F = \frac{mV_0}{T} + mg = \frac{mgh}{S_0} + mg = mg\left(\frac{h}{S_0} + 1\right) = 480 \text{ lb}$$

$$P_{hp} = \frac{(480)(13.9)}{(550)(2)} = 6.1 \text{ hp}$$

METHOD 2:

$$P = \frac{\Delta E}{\Delta t} = \frac{\left(\frac{1}{2}mV_0^2 + mgS_0\right)}{T} = \frac{mg(h + S_0)}{T}$$

$$T = \frac{2S_0}{V_0} = \frac{(2)(1.5)}{13.9} = 0.216 \text{ s}$$

$$P_{hp} = \frac{(160)(3 + 1.5)}{(550)(0.216)} = 6.1 \text{ hp}$$

$\boxed{4.19}$

$$v = \int_0^t \left(\frac{F_0}{m}\cos\omega t - g\right)dt = \frac{1}{\omega}\frac{F_0}{m}\sin\omega t - gt$$

$$v_0 = \frac{1}{\omega}\left(\frac{F_0}{m} - g\frac{\pi}{2}\right)$$

$$s_0 = \int_0^{\frac{1}{\omega}\frac{\pi}{2}} \left(\frac{1}{\omega}\frac{F_0}{m}\sin\omega t - gt\right)dt = \left(-\frac{1}{\omega^2}\frac{F_0}{m}\cos\omega t - \frac{1}{2}gt^2\right)\Bigg|_0^{\frac{\pi}{2\omega}}$$

$$s_0 = \frac{1}{\omega^2}\left(\frac{F_0}{m} - \frac{g\pi^2}{8}\right)$$

Using $v_0 = \sqrt{2gh} = \sqrt{(2)(32)(3)} = 13.86$ ft/s and

$s_0 = 1.5$ ft, solve for $\omega$ and $F_0$.

$\omega = 9.96\ s^{-1}$

$F_0 = 942$ lb

Power $= P = Fv = (F_0\cos\omega t)\left(\frac{1}{\omega}\frac{F_0}{m}\sin\omega t - gt\right)$

$$= F_0\left(\frac{1}{\omega}\frac{F_0}{m}\sin\omega t\cos\omega t - gt\cos\omega t\right)$$

$$= \frac{F_0}{\omega m}\left(F_0\sin\omega t\cos\omega t - mg\,\omega t\cos\omega t\right)$$

$$= (18.92)(942\sin\omega t\cos\omega t - 160\,\omega t\cos\omega t)$$

Power is a maximum at $\omega t \cong 0.77$

$P_{max} = 7230$ ft lb/s $= 13$ horsepower

---

$\boxed{4.20}$ Consider a mass of sand $\Delta m$.

The momentum of the belt and the transported sand does not change in $\Delta t$.

$$\Delta P_x = (\Delta m) V = F \Delta t$$

$$F = \frac{dm}{dt} V$$

$$P = FV = \frac{dm}{dt} V^2$$

(b) $\Delta KE = \frac{1}{2} (\Delta m) V^2$

$$\frac{d(KE)}{dt} = \frac{1}{2} \frac{dm}{dt} V^2$$

The remaining power is needed to make up for frictional losses as the sand initially slides on the belt.

---

$\boxed{4.21}$

(a) $\Delta P_y = (\Delta m) v_0 = F \Delta t$

$F = \frac{dm}{dt} v_0$ But $\frac{dm}{dt} = \frac{dm}{dy} \frac{dy}{dt} = \lambda v_0$

$F = \lambda v_0^2$

$$F_{total} = \lambda v_0^2 + \lambda g y$$

(b) Power delivered $= F_{tot} v_0 = \lambda v_0^3 + \lambda g y v_0$

$E = \frac{1}{2} (\lambda y) v_0^2 +$ potential energy

Potential energy $= \int_0^y g y \, dm = \int_0^y g \lambda y \, dy = \frac{1}{2} g \lambda y^2$

$$E = \frac{1}{2} \lambda y v_0^2 + \frac{1}{2} g \lambda y^2$$

$$\frac{dE}{dt} = \frac{1}{2} \lambda v_0^2 \frac{dy}{dt} + g \lambda y \frac{dy}{dt} = \frac{1}{2} \lambda v_0^3 + \lambda g y v_0$$

There is no discrepancy with the gravitational part of $\frac{dE}{dt}$ (conservative force, no dissipation of mechanical energy.) Mechanical energy <u>is</u> lost in the "collision" of one part of the rope with the next; think of the rope as a chain of links.

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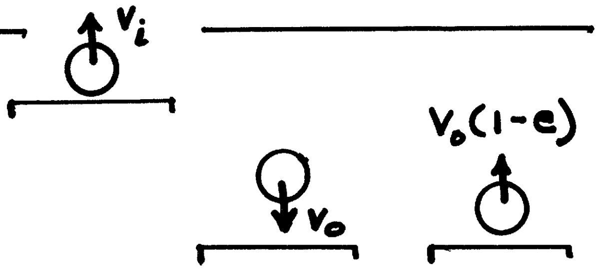
$\boxed{4.22}$ $v = v_i - gt$




$$t_{1\ cycle} = \frac{2v_i}{g}$$

$$T = \frac{2v_0(1-e)}{g} + \frac{2v_0(1-e)^2}{g} + \dots$$

$$= \frac{2v_0}{g}(1-e)\left[1 + (1-e) + (1-e)^2 + \dots\right]$$

$$T = \frac{2v_0}{g}(1-e)\left[\frac{1}{1-(1-e)}\right] = \frac{2v_0}{g}\left(\frac{1-e}{e}\right)$$

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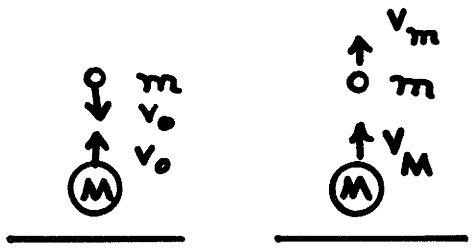
$\boxed{4.23}$ $v_0 = \sqrt{2gh}$




$$(M-m)v_0 = Mv_M + mv_m$$

$$\tfrac{1}{2}(M+m)v_0^2 = \tfrac{1}{2}Mv_M^2 + \tfrac{1}{2}mv_m^2$$

Solving $v_M = \left(1 - \frac{m}{M}\right)v_0 - \frac{m}{M}v_m$

$$(M+m)v_0^2 = M\left[\left(1-\frac{m}{M}\right)v_0 - \frac{m}{M}v_m\right]^2 + mv_m^2$$

$$= Mv_0^2 - 2mv_0^2 + \frac{m^2}{M}v_0^2 + \frac{m^2}{M}v_m^2$$

$$-2mv_0v_m + 2\frac{m^2}{M}v_0v_m + mv_m^2$$

OR $v_m^2 - 2v_0v_m - 3v_0^2 = 0$

$$v_m = \frac{2v_0 + \sqrt{4v_0^2 + 12v_0^2}}{2} = 3v_0 = 3\sqrt{2gh}$$

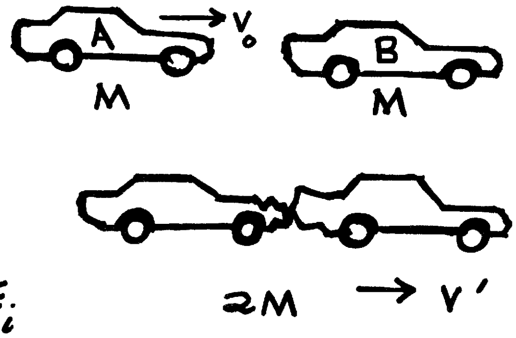
Hence marble rises to a height $H$ given by

$$mgH = \frac{1}{2}mv_m^2$$

$$H = \frac{\frac{1}{2}v_m^2}{g} = 9h$$

---

$\boxed{4.24}$ A–B collision:




$$Mv_0 = 2Mv'$$

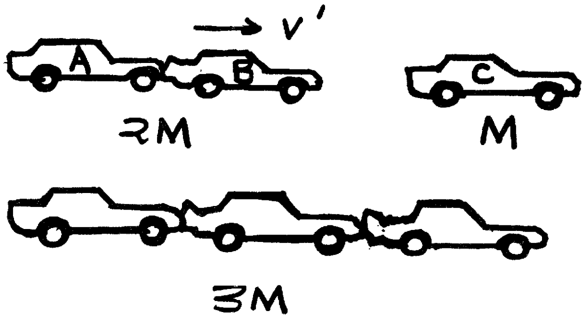
$$v' = \frac{1}{2}v_0$$

$$E_i = \frac{1}{2}Mv_0^2$$

$$E' = \frac{1}{2}(2M)v'^2 = \frac{1}{2}\left(\frac{1}{2}Mv_0^2\right) = \frac{1}{2}E_i$$

$\frac{1}{2}$ the original energy is dissipated in AB collision.

AB – C collision:




$$2Mv' = 3Mv''$$

$$v'' = \frac{2}{3}v' = \frac{1}{3}v_0$$

$$E'' = \frac{1}{2}(3M)v''^2 = \frac{1}{3}\left(\frac{1}{2}Mv_0^2\right)$$

$$= \frac{1}{3}E_i$$

Hence $\frac{1}{2}E_i - \frac{1}{3}E_i = \frac{1}{6}E_i$ is dissipated in the second collision, divided between C and the other cars.

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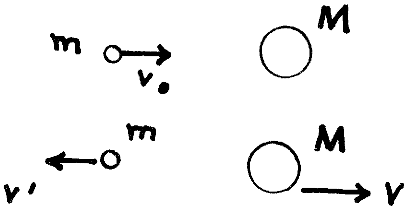
$\boxed{4.25}$ $mv_0 = mv'' - mv'$




$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv''^2 + \frac{1}{2}mv'^2$$

$$\frac{1}{2}mv'^2 = \frac{4}{9}\left(\frac{1}{2}mv_0^2\right)$$

$$v' = \frac{2}{3} v_0$$

Hence $\quad v_0 = \frac{M}{m} v'' - \frac{2}{3} v_0$

$$v_0^2 = \frac{M}{m} v''^2 + \frac{4}{9} v_0^2$$

Solving, $\frac{M}{m} = 5$

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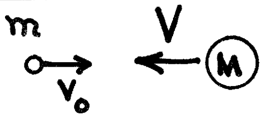
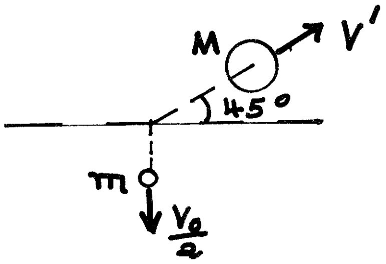
$\boxed{4.26}$ $\quad m v_0 - M V = \frac{M V'}{\sqrt{2}}$




$$\frac{M V'}{\sqrt{2}} = \frac{m v_0}{2}$$




$$\frac{1}{2} m v_0^2 + \frac{1}{2} M V^2 = \frac{1}{2} m \frac{v_0^2}{4} + \frac{1}{2} M V'^2$$

Solving, $\frac{m v_0}{2} = MV$, $\quad V = \frac{1}{2}\frac{m}{M} v_0$

$$V' = \frac{1}{\sqrt{2}} \frac{m}{M} v_0$$

Hence $\quad v_0^2 + \left(\frac{M}{m}\right)\frac{1}{4}\left(\frac{m}{M}\right)^2 v_0^2 = \frac{1}{4} v_0^2 + \left(\frac{M}{m}\right)\frac{1}{2}\left(\frac{m}{M}\right)^2 v_0^2$

OR $\quad \frac{m}{M} = 3$, $\quad \frac{M}{m} = \frac{1}{3}$

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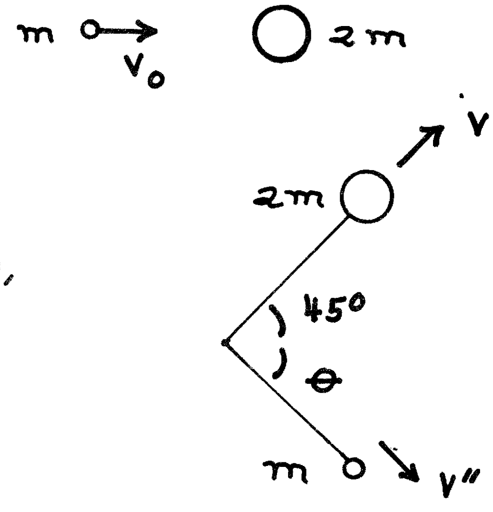
$\boxed{4.27}$




$$m v_0 = \frac{2 m v'}{\sqrt{2}} + m v'' \cos\theta$$

$$\frac{2 m v'}{\sqrt{2}} = m v'' \sin\theta$$

Assuming the collision to be elastic,

$$\frac{1}{2} m v_0^2 = \frac{1}{2} (2m) v'^2 + \frac{1}{2} m v''^2$$

Solving, $\quad v_0 = v''(\sin\theta + \cos\theta)$

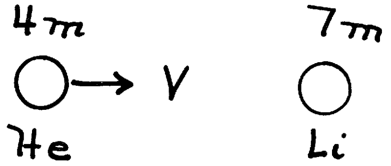
$$v_0^2 = v''^2 (\sin^2\theta + 1)$$

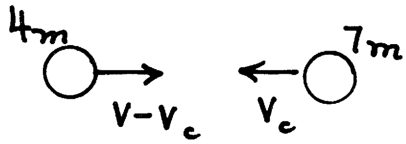
Hence $\quad \sin^2\theta + 1 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + 2\sin\theta\cos\theta$

OR $\quad \sin\theta = 2\cos\theta$, $\quad \tan\theta = 2 \quad \Longrightarrow \quad \theta \cong 63°$

$\boxed{4.28}$

(a) Transfer to the C of M system:




$$4(V-V_c) = 7V_c$$

$$V_c = \frac{4}{11}V\ , \quad V - V_c = \frac{7}{11}V$$

Energy in the C system is $E_c = \frac{1}{2}(4m)(V-V_c)^2 + \frac{1}{2}(7m)V_c^2$

$$E_c = \left[\frac{(49)(4)}{121} + \frac{(16)(7)}{121}\right]\frac{1}{2}mV^2 = \left(\frac{77}{121}\right)\frac{1}{2}(4m)V^2 = \frac{77}{121}E_0$$

All the energy in the C system can be used for the reaction. At threshhold,

$$\frac{77}{121}E_{0_t} = 2.8 \text{ MeV}$$

$$E_{0_t} = 4.4 \text{ MeV}$$

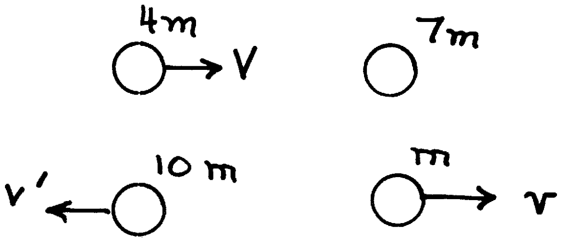
At threshhold, neutrons are produced at rest in the C system so that they have speed $V_c$ in the L system. Their energy in L is

$$E_n = \frac{1}{2}mV_c^2 = \left(\frac{16}{121}\right)\left(\frac{1}{2}mV^2\right) = \left(\frac{4}{121}\right)\frac{1}{2}(4m)V^2$$

$$= \frac{4}{121}E_{0_t}$$

$$E_n = 0.145 \text{ MeV}$$

(b) $4V = v - 100v'$




$$\frac{1}{2}(4m)V^2 = \frac{1}{2}mv^2 + \frac{1}{2}(10m)v'^2 + 2.8\ (\text{MeV})$$

Solving, we find

$$11v^2 - 8vV - 24V^2 + \frac{56}{m}(\text{MeV}) = 0$$

or $$v = \frac{8V \pm \sqrt{1120V^2 - \frac{2464}{m}(\text{MeV})}}{22}$$

There are no real roots until $1120V^2 \geq \frac{2464}{m}(\text{MeV})$

or $(560)\frac{1}{2}(4m)V^2 \geq 2464\,\text{MeV}$

$\frac{1}{2}(4m)V^2 \geq 4.4\,\text{MeV}$

(threshold)

There are two forward velocities

if $(8V)^2 \geq 1120V^2 - \frac{2464}{m}(\text{MeV})$

or $(528)\frac{1}{2}(4m)V^2 \leq 2464\,\text{MeV}$

$$\frac{1}{2}(4m)V^2 \leq 4.67\,\text{MeV} = E_{0_t} + 0.27\,\text{MeV}$$

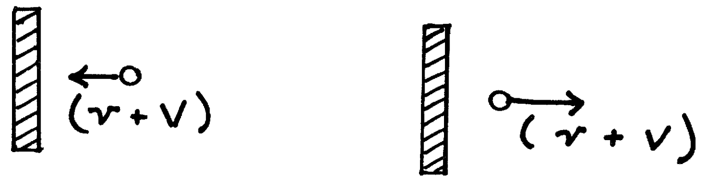
The two groups of forward projected neutrons represent neutrons projected at $0°$ and $180°$ in the C System.

$\boxed{4.29}$

(a) $\quad \bar{F} = \dfrac{\Delta P}{\Delta \tau} = \dfrac{m v_0}{\left(\frac{2\ell}{v_0}\right)} = \dfrac{m v_0^2}{\ell}$

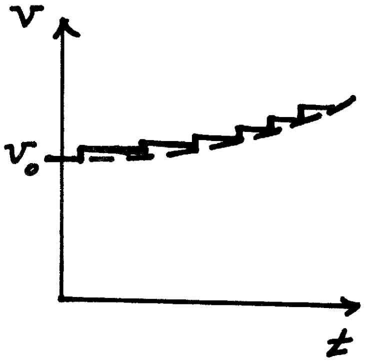
(b)


In frame in which wall is stationary, we have




Hence in L frame $\quad v' = (v + V) + V = v + 2V$

$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{2V}{2x/v} = \frac{vV}{x}$$




$$\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx} = -\frac{1}{V}\frac{dv}{dt} = -\frac{v}{x}$$

Hence $\quad \dfrac{dv}{v} = -\dfrac{dx}{x}$

$$\ln v = -\ln x + \ln c$$

$$v = \frac{v_0 \ell}{x}$$

$$\bar{F} = \frac{\Delta P}{\Delta t} = \frac{2mv}{2x/v} = \frac{mv^2}{x} = \frac{m v_0^2 \ell^2}{x^3}$$

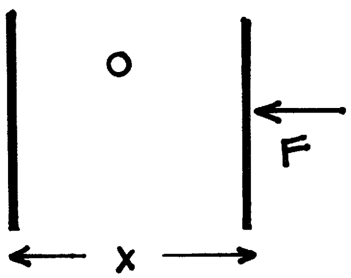
(c) $\quad W = -\displaystyle\int_\ell^x \bar{F}\,dx = -m v_0^2 \ell^2 \int_\ell^x \frac{dx}{x^3} = m v_0^2 \ell^2 \left(\frac{1}{2x^2}\right)\Bigg|_\ell^x$

$$= \frac{m v_0^2 \ell^2}{2}\left(\frac{1}{x^2} - \frac{1}{\ell^2}\right)$$

$$W = \frac{mv_0^2}{2}\left(\frac{\ell}{x}\right)^2 - \frac{mv_0^2}{2}$$

$$\Delta KE = E_f - E_i = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

$$\Delta KE = \frac{1}{2}\frac{mv_0^2\ell^2}{x^2} - \frac{1}{2}mv_0^2$$

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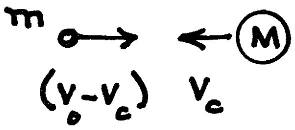
$\boxed{4.30}$


Lab frame

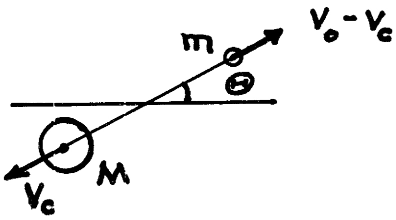



CM frame

(a) $m(v_0 - v_c) = Mv_c$

$$v_c = \frac{mv_0}{m+M}$$

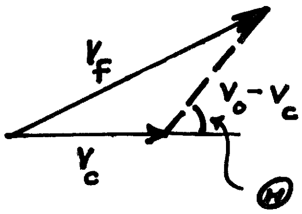
$$v_0 - v_c = \frac{Mv_0}{m+M}$$




$$v_f^2 = v_c^2 + (v_0 - v_c)^2 - 2v_c(v_0 - v_c)\cos(\pi - \Theta)$$

$$v_f^2 = \left(\frac{v_0}{m+M}\right)^2 (m^2 + M^2 + 2mM\cos\Theta)$$

$$v_f = \left(\frac{v_0}{m+M}\right)(m^2 + M^2 + 2mM\cos\Theta)^{1/2}$$




(b) $K_0 = \frac{1}{2}mv_0^2$

$$K_f = \tfrac{1}{2}mv_f^2 = \frac{\frac{1}{2}mv_0^2}{(m+M)^2}(m^2 + M^2 + 2mM\cos\Theta)$$

$$\frac{K_0 - K_f}{K_0} = 1 - \frac{m^2 + M^2 + 2mM\cos\Theta}{(M+m)^2} = \frac{2mM(1-\cos\Theta)}{(m+M)^2}$$

# Chapter 5

$\boxed{5.1}$

(a) $$\vec{F} = -\vec{\nabla}U = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$
$$= -2Ax\,\hat{i} - 2By\,\hat{j} - 2Cz\,\hat{k}$$

(b) $$\vec{F} = -\vec{\nabla}U = \frac{-2Ax}{x^2+y^2+z^2}\hat{i} - \frac{2Ay}{x^2+y^2+z^2}\hat{j} - \frac{2Az}{x^2+y^2+z^2}\hat{k}$$

(c) $$\vec{F} = -\vec{\nabla}U = -\frac{\partial U}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial U}{\partial \theta}\hat{\theta}$$
$$\vec{F} = \frac{2A\cos\theta}{r^3}\hat{r} + \frac{A\sin\theta}{r^3}\hat{\theta}$$

---

$\boxed{5.2}$

(a) $$\vec{F}_a = -Ar^3\,\hat{r} = -Ar^3\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$$
$$= Ar^2\,(x\hat{i} + y\hat{j} + z\hat{k})$$

Curl $\vec{F}_a = 0$ by direct calculation or using the result that all central forces are conservative.

$$\text{curl}\,\vec{F}_b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ By^2 & -Bx^2 & 0 \end{vmatrix} = (-2Bx - 2By)\,\hat{k} \neq 0$$

Hence $\vec{F}_b$ is not conservative.

(b) $$U_a = \frac{Ar^4}{4}$$

$$\tfrac{1}{2}mv_f^2 + U_a(0,0) - \tfrac{1}{2}mv_0^2 - U_a(1,1) = \int_{1,1}^{0,0} \vec{F}_b\cdot d\vec{r}$$

$$= B\int_{1,1}^{0,0}(y^2dx - x^2dy)$$

But $U_a(0,0) = 0$ $\qquad U_a(1,1) = \frac{A}{4}(\sqrt{2})^4 = A$

$y = x^2$ so that

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_0^2 + A + B\int_1^0 x^4dx - B\int_1^0 y\,dy$$

$$= \tfrac{1}{2}mv_0^2 + A - B/5 + B/2$$

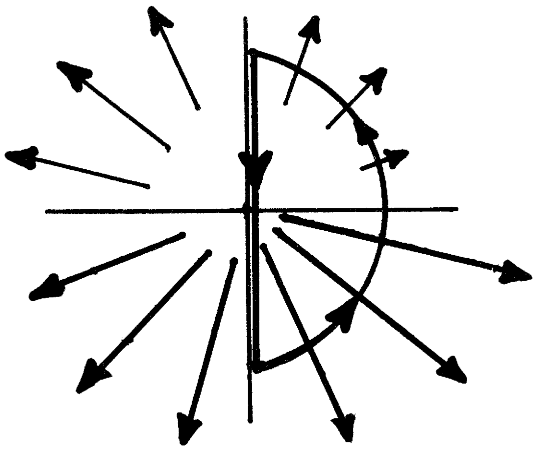
Hence $v_f = \left(v_0^2 + \frac{2A}{m} + \frac{3B}{5m}\right)^{1/2}$

---

$\boxed{5.3}$

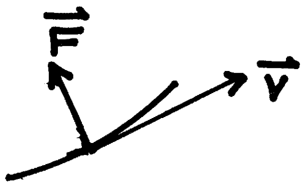
(a) Curl $\vec{F} = (\sin\omega t)$ curl $\vec{F}_0 = 0$ because the curl operator acts only on spatial coordinates.

(b)



The work around the indicated closed path is zero over the curved part but not zero over the straight part, because $\vec{F}$ has a different magnitude at 90° and 180°.

(c)




$\vec{F}$ does no work because $\vec{F}\cdot d\vec{r} = 0$. Hence $\int \vec{F}\cdot d\vec{r}$ around a closed path vanishes and the force $\vec{F}$ is conservative.

$\boxed{5.4}$

(a) $\text{curl}\,\vec{F} = A\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & z & y \end{vmatrix} = A\left[(1-1)\,\hat{i}\right] = 0$

$$U = -A\,(3x + f(y,z))$$

$$\frac{\partial U}{\partial y} = -A\frac{\partial f}{\partial y} = Az$$

$$f = zy + g(z)$$

$$\frac{\partial U}{\partial z} = -Ay - A\frac{\partial g}{\partial z} = -Ay$$

Hence $g = 0$ and $U = -A\,(3x + yz)$

(b) $\text{curl}\,\vec{F} = A\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & xyz & xyz \end{vmatrix} = A\left[(xz - xy)\,\hat{i} + \ldots\right]$

$$\neq 0$$

(c) $$\begin{aligned} \text{curl}\,\vec{F} = {} & (5\alpha A x^3 y^4 e^{\alpha z} - 5\alpha A x^3 y^4 e^{\alpha z})\,\hat{i} \\ & + (3\alpha A x^2 y^5 e^{\alpha z} - 3\alpha A x^2 y^5 e^{\alpha z})\,\hat{i} \\ & + (15 A x^2 y^4 e^{\alpha z} - 15 A x^2 y^4 e^{\alpha z})\,\hat{j} \end{aligned}$$

$$= 0$$

$$\frac{\partial U}{\partial x} = -3Ax^2y^5e^{\alpha z}, \quad U = -Ax^3y^5\alpha^{\alpha z} + f(y,z)$$

$$U = -Ax^3y^5e^{\alpha z}$$

(d) $\text{curl}\,\vec{F} = (\alpha A x \cos(\alpha y)\sin(\beta z)$

$-\beta A x \alpha \cos(\alpha y)\sin(\beta z))\,\hat{i} + \ldots$

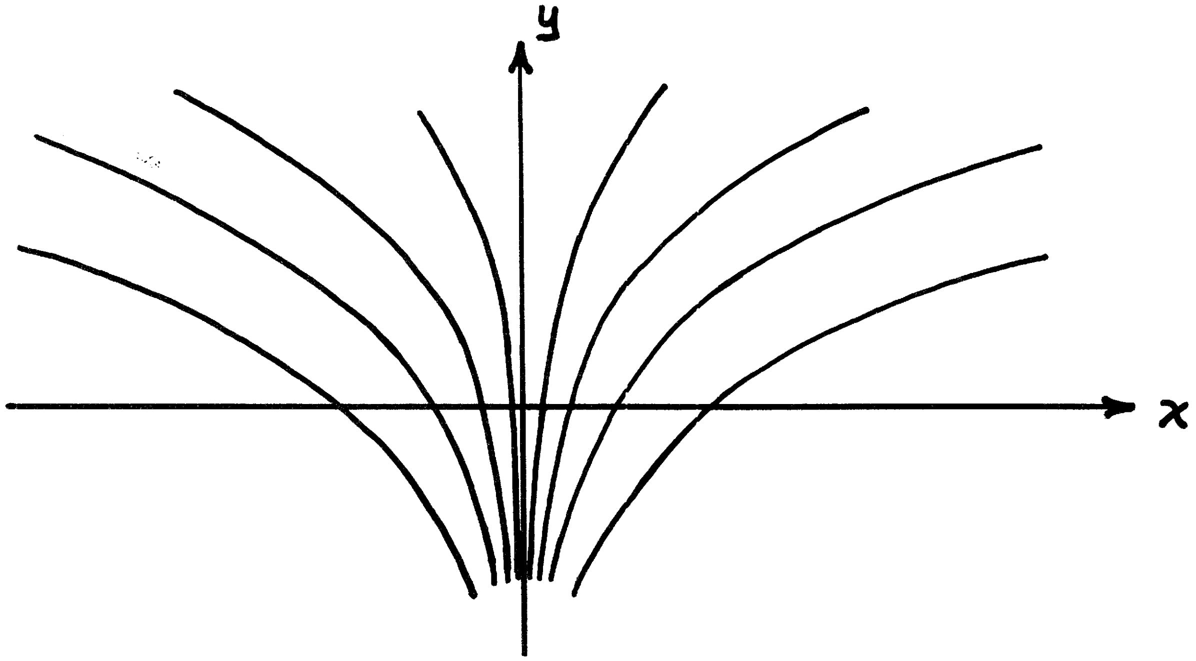
$\neq 0$

---

$\boxed{5.5}$

(a) $U = C x e^{-y} = \text{const.}$

The lines of constant potential are given by $x = K e^{y}$ where $K = \text{const.}$




(b) $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

Along a line $U = \text{const.}$, $dU = 0 = C e^{-y} dx - C x e^{-y} dy$

Hence $dy = \frac{dx}{x}$ and

$d\vec{r} = dx\,(\hat{i} + \hat{j}/x)$ along a line $U = \text{const.}$

(c) $\vec{\nabla} U = C\left[e^{-y}\hat{i} - xe^{-y}\hat{j}\right]$

$\vec{\nabla} U \cdot d\vec{r} = cdx\left[e^{-y} - e^{-y}\right] = 0$

---

**5.6**

By Stokes theorem, $\oint \vec{A}\cdot d\vec{r} = \int (\vec{\nabla}\times\vec{A})\cdot d\vec{s}$, where $d\vec{s}$ is an element of the surface bounded by the closed contour.

If $\vec{\nabla}\times\vec{A} = 0$

$\oint \vec{A}\cdot d\vec{r} = 0$

Hence A is mathematically analogous to a conservative force. Since any conservative force can be written as the gradient of some potential,

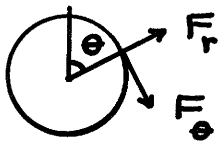
$\vec{A} = \vec{\nabla}\phi$

---

**5.7**

$$F_\theta = -\frac{1}{r}\frac{dU}{d\theta} = \frac{GM_e m}{r^2}\left[5.4\times10^{-4}\left(\frac{R_e}{r}\right)^2(6\cos\theta\sin\theta)\right]$$

The force disappears at $\theta = 0°$ and $\theta = 90°$.




$$F_\theta \Big/ \left.\frac{GM_e m}{r^2}\right|_{45°} = (5.4\times10^{-4})(6)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = 1.62\times10^{-3}$$

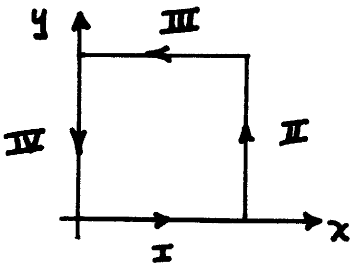
$\boxed{5.8}$

$$W_{I} = \int_0^d F_x\,dx = 0 \qquad (y=0)$$

$$W_{II} = \int_0^d F_y\,dy = A(2d^2)\int_0^d dy = 2Ad^3 \qquad (x=d)$$

$$W_{III} = \int_d^0 F_x\,dx = -Ad^3 \qquad (y=d)$$

$$W_{IV} = -\int_d^0 F_y\,dy = 0 \qquad (x=0)$$




Hence $W_{tot} = Ad^3$

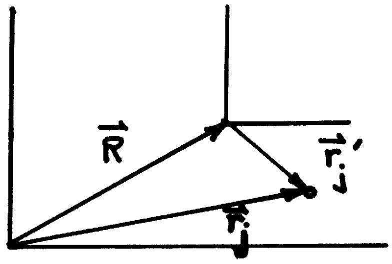
$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay^2 & 2Ax^2 & 0 \end{vmatrix} = (4Ax - 2Ay)\,\hat{k}$$

$$W = \oint \vec{F}\cdot d\vec{r} = \int \vec{\nabla}\times\vec{F}\cdot d\vec{s} = A\int(4x-2y)\,dy\,dx$$

$$= Ad\int_0^d 4x\,dx - 2Ad\int_0^d y\,dy = 2Ad^3 - Ad^3 = Ad^3 \checkmark$$

# Chapter 6

$\boxed{6.1}$ (a)




$$\vec L = \Sigma\, \vec r_j \times \vec p_j$$
$$\vec r_j = \vec r + \vec r_j'$$
$$\vec p_j = m\dot{\vec r}_j = m\dot{\vec r}_j' = \vec p_j'$$

$$\vec L' = \Sigma\, \vec r_j' \times \vec p_j' = \Sigma\, (\vec r_j - \vec R) \times \vec p_j$$
$$= \Sigma\, \vec r_j \times \vec p_j - \vec R \times \Sigma\, \vec p_j = \vec L \qquad \text{since } \Sigma\, \vec p_j = 0$$

(b)

$$\vec \tau = \Sigma\, \vec r_j \times \vec F_j$$
$$\vec F_j = \vec F_j'$$
$$\vec \tau' = \Sigma\, \vec r_j' \times \vec F_j' = \Sigma\, (\vec r_j - \vec R) \times \vec F_j$$
$$\tau' = \Sigma\, \vec r_j \times \vec F_j - \vec R \times \Sigma\, \vec F_j = \vec \tau \qquad \text{since } \Sigma\, F_j = 0$$

---

$\boxed{6.2}$

The total angular momentum remains constant.

Initial angular momentum $= (M_A + M_S)\, a^2 \omega_A(0)$

Angular momentum at time $t$ is given by

$$(M_A + M_S - \lambda t)\, a^2 \omega_A + (M_B + \lambda t)\, b^2 \omega_B$$

Note that $\omega_A = \omega_A(0)$ because the sand exerts no torque on drum A as it leaves.

Hence

$$(M_A+M_S)\,a^2\omega_A(0) = (M_A+M_S-\lambda t)\,a^2\omega_A(0) + (M_B+\lambda t)\,b^2\omega_B$$

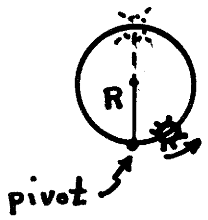
$$\Rightarrow \quad \omega_B = \frac{\lambda t a^2 \omega_A(0)}{(M_B+\lambda t)\,b^2}$$

---

$\boxed{6.3}$

The verticle component of angular momentum $\vec{L}$ is constant and equal to its initial value 0.

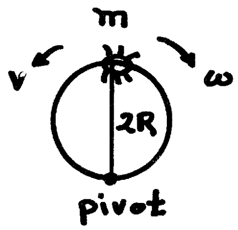
(a) Angular momentum of bug about pivot $= m(v-2R\omega)2R$




Angular momentum of ring

$$= -I_{pivot}\,\omega = -(I_0+MR^2)\,\omega = -2MR^2\omega$$

Hence




$$m(v-2R\omega)2R = 2MR^2\omega$$

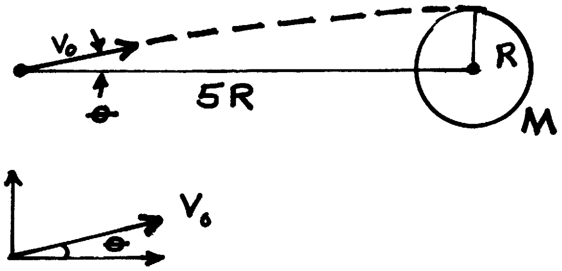
$$\omega = \frac{mv}{(M+2m)R}$$

(b) $L_{Bug} = 0$

Hence

$L_{Ring} = L - L_{Bug} = 0$ and $\omega = 0$.

$\boxed{6.4}$ The central gravitational force exerts no torque; hence $L =$ const.




$$L = (m v_0 \sin\theta)\, 5R = m v R$$

By conservation of energy
$$E = \tfrac{1}{2} m v_0^2 - \frac{GMm}{5R} = \tfrac{1}{2} m v^2 - \frac{GMm}{R}$$

Hence $\quad v = 5 v_0 \sin\theta$

$$v_0^2 - \frac{2GM}{5R} = v^2 - \frac{2GM}{R}$$

or $\quad \sin\theta = \frac{1}{5}\sqrt{1 + \frac{8}{5}\frac{GM}{R v_0^2}}$

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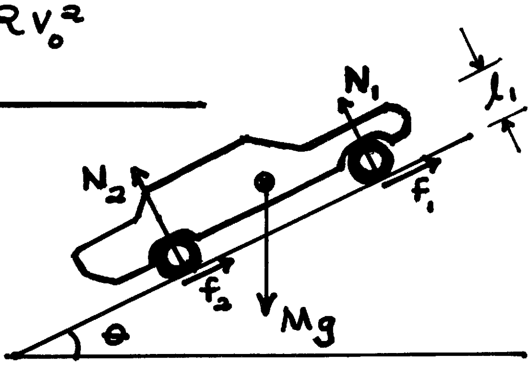
$\boxed{6.5}$ $N_1 + N_2 = Mg\cos\theta$
$f_1 + f_2 = Mg\sin\theta$




Torque about CM:

$$0 = N_1 l_2 - N_2 l_2 + f_1 l_1 + f_2 l_1$$
$$= (N_1 - N_2)\, l_2 + (Mg\sin\theta)\, l_1$$

Solving, $\quad N_1 = \tfrac{1}{2} Mg\left(\cos\theta - \frac{l_1}{l_2}\sin\theta\right)$

$$N_2 = \tfrac{1}{2} Mg\left(\cos\theta + \frac{l_1}{l_2}\sin\theta\right)$$

For the given numerical values,

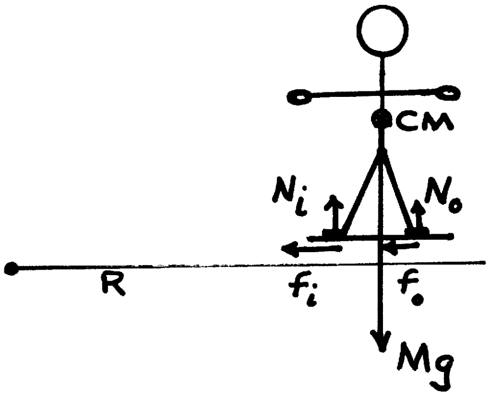
$N_1 = 924$ lbs $\qquad N_2 = 1674$ lbs

$\boxed{6.6}$

$N_i + N_o = Mg$

$f_i + f_o = Mv^2/R$

Take torques about CM:




$$0 = N_o \frac{d}{2} - N_i \frac{d}{2} - (f_i + f_o) L$$

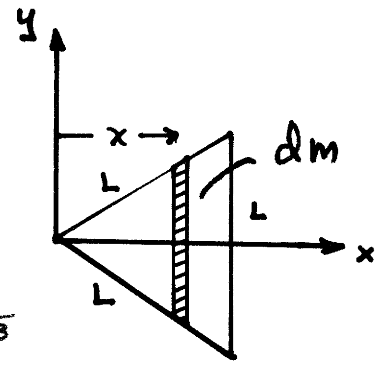
$$= (N_o - N_i)\frac{d}{2} - \frac{Mv^2 L}{R}$$

Solving, $N_{inside} = \frac{1}{2}\left[ Mg - \frac{Mv^2L}{R(d/2)} \right]$

$$N_{outside} = \frac{1}{2}\left[ Mg + \frac{Mv^2L}{R(d/2)} \right]$$

---

$\boxed{6.7}$ $dm = \left(\frac{M}{\text{Area}}\right)\frac{2x}{\sqrt{3}}\,dx = \left(\frac{8M}{3L^2}\right) x\,dx$

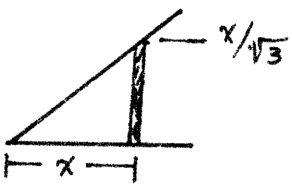



By the parallel axis theorem,

$$dI = \frac{(4x^2/3)}{12}\,dm + x^2 dm = \left(\frac{10}{9}\right) x^2\, dm$$

$$I = \left(\frac{10}{9}\right)\left(\frac{8M}{3L^2}\right)\int_0^{\sqrt{3}L/2} x^3\, dx$$

$$= \frac{5}{12} ML^2$$

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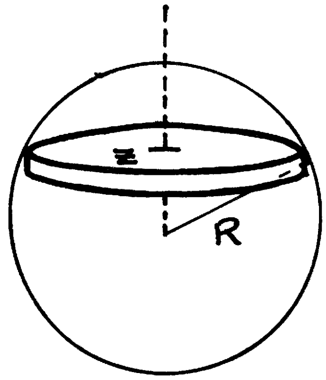
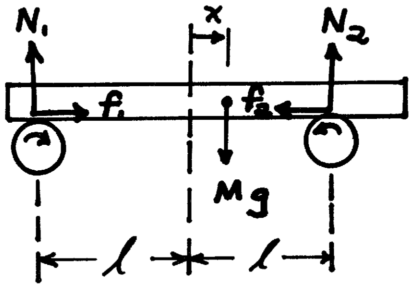
$\boxed{6.8}$ Consider the sphere to be a stack of disks.




$$dI = \frac{1}{2} dM\,(R^2 - z^2)$$

$$dM = \frac{M}{V}\pi\,(R^2 - z^2)dz = \left(\frac{M}{\frac{4}{3}R^3}\right)(R^2 - z^2)\,dz$$

$\boxed{6.9}$




$$N_1 + N_2 = Mg$$

$$N_1(l + x) = N_2(l - x)$$

Hence $\quad N_1 = \frac{Mg}{2}\left(1 - \frac{x}{l}\right)$

$$N_2 = \frac{Mg}{2}\left(1 + \frac{x}{l}\right)$$

Horizontal Force $= f_2 - f_1 = \mu(N_1 - N_2) = -\frac{\mu Mg}{l}\, x$

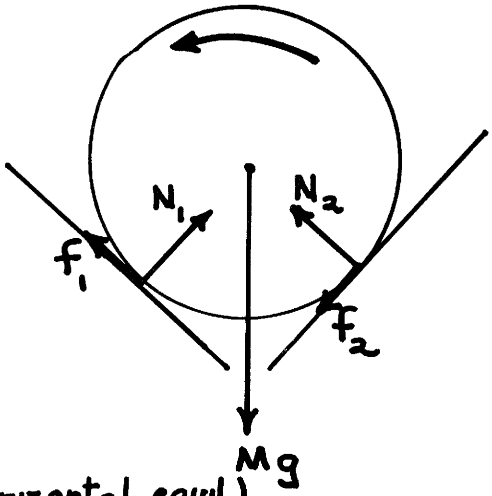
$$M\ddot{x} = -\frac{\mu Mg}{l}\, x$$

SHM $\quad x = x_0 \cos\omega t \quad$ where $\omega = \sqrt{\frac{\mu g}{l}}$

---

$\boxed{6.10}$ $\quad \tau = (f_1 + f_2)R$

$$= \mu(N_1 + N_2)R$$




$$(N_1 + N_2)\frac{1}{\sqrt{2}} = f_1\frac{1}{\sqrt{2}} - f_2\frac{1}{\sqrt{2}} = Mg$$

$$N_1(1 + \mu) + N_2(1 - \mu) = \sqrt{2}\, Mg$$

$$\frac{N_1}{\sqrt{2}} = \frac{N_2 + f_1 + f_2}{\sqrt{2}}$$

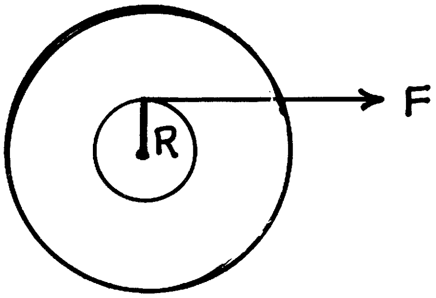
$N_1(1 - \mu) = N_2(1 + \mu) \Leftarrow$ (for horizontal equil.)

Solving, $\quad N_1 = \sqrt{2}\,\frac{Mg}{2}\left(\frac{1+\mu}{1+\mu^2}\right)$

$$N_2 = \sqrt{2}\,\frac{Mg}{2}\left(\frac{1-\mu}{1+\mu^2}\right)$$

Hence $\quad \tau = \sqrt{2}\, Mg\left(\frac{\mu}{1+\mu^2}\right) R$

$\boxed{6.11}$




$$\tau = I_0 \frac{d\omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{FR}{I_0}$$

$$\omega = \left(\frac{FR}{I_0}\right) t$$

$$\theta = \int \omega dt = \frac{1}{2}\left(\frac{FR}{I_0}\right) t^2$$

At $t = t_0$, a length $L$ has been unwound.

$$L = R\theta = \frac{1}{2}\left(\frac{FR^2}{I_0}\right) t_0^2$$

$$t_0 = \sqrt{\frac{2LI_0}{FR^2}}$$

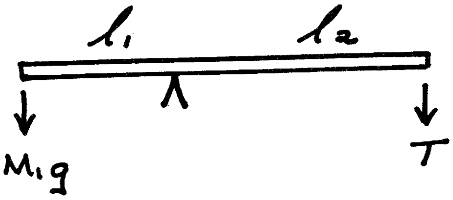
$\omega = \omega_0$ at $t = t_0$

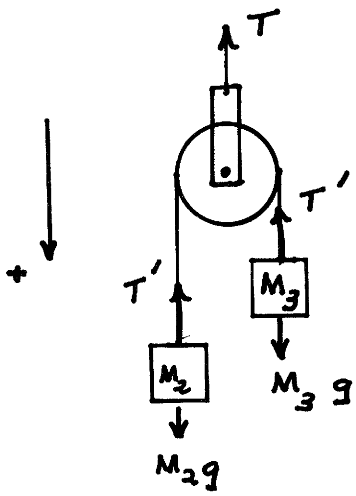
$$\omega_0 = \frac{FR}{I_0} t_0 = \sqrt{\frac{2LF}{I_0}}$$

Hence $I_0 = \frac{2LF}{\omega_0^2} = \frac{2 \times 5 \times 10}{(0.5)^2} = 400 \text{ kg m}^2$

---

$\boxed{6.12}$ If $\Sigma \tau = 0$, beam will not rotate:




Require $M_1 g l_1 = T l_2$

$$M_2 g - T' = M_2 a$$

$$T' - M_3 g = M_3 a$$

$$T = 2T' = \frac{4 g M_2 M_3}{M_2 + M_3}$$

Hence, the condition becomes

$$M_1 \ell_1 = \frac{4 M_2 M_3 \ell_2}{M_2 + M_3}$$

---

**6.13**

(a) Force is central, hence angular momentum is conserved. Momentum and energy of $m$ change because of applied force : $\int \vec{T} dt \neq 0$

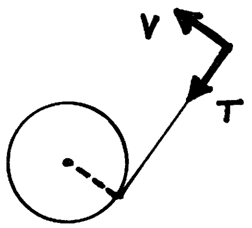
$\int \vec{T} \cdot d\vec{r} \neq 0$

$$L = m v_0 r = m v_f R$$

$$v_f = \frac{v_0 r}{R}$$

(b) Angular momentum is not conserved because force is not central. Momentum also changes since $\int \vec{T} \, dt \neq 0$ but energy is conserved because

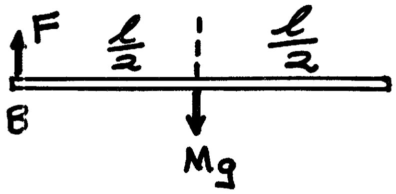
$$\int \vec{T} \cdot d\vec{r} = \int \vec{T} \cdot \vec{v} \, dt = 0$$




Hence $\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2$

$$v_f = v_0$$

---




**6.14**

(a) $\tau = Mg \frac{\ell}{2}$

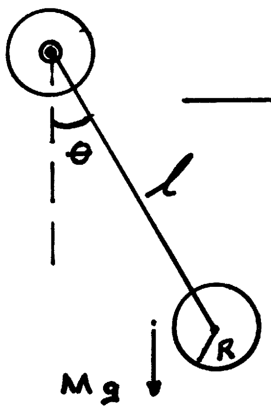
(b) $\tau = I \alpha \qquad \alpha = \frac{\tau}{I} = \frac{Mg \frac{\ell}{2}}{\frac{1}{3} M \ell^2} = \frac{3}{2} \frac{g}{\ell}$

(c) $a = \frac{\ell}{2}\alpha = \frac{3}{4} g$

(d) $Ma = Mg - F_v$

$F_v = Mg - Ma = \frac{Mg}{4}$

---




$\boxed{6.15}$ $\tau = I \frac{d\omega}{dt}$

Take torques about pivot

$-Mg\ell \sin\theta = I\ddot{\theta}$

$\ddot{\theta} + \left(\frac{Mg\ell}{I}\right)\theta \cong 0$

$\omega = \sqrt{\frac{Mg\ell}{I}}$ $\quad I = \frac{1}{2}MR^2 + \left(\frac{1}{2}MR^2 + M\ell^2\right)$

$= M(R^2 + \ell^2)$

$\omega = \sqrt{\frac{g\ell}{R^2+\ell^2}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^2+\ell^2}{g\ell}}$

---

$\boxed{6.16}$ $\omega = \sqrt{\frac{Mg\ell}{I}}$ (See problem 6.15)

Using the parallel axis theorem, $I = \frac{1}{2}MR^2 + M\ell^2$

$T = \frac{2\pi}{\omega} = \sqrt{\frac{R^2/2 + \ell^2}{g\ell}}$

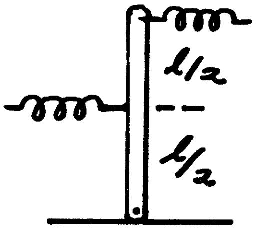
$0 = \frac{dT}{d\ell} = \frac{1}{2}\frac{1}{\sqrt{\frac{R^2/2+\ell^2}{g\ell}}}\left[\frac{2\ell}{g\ell} - \frac{\left(\frac{R^2}{2}+\ell^2\right)}{g\ell^2}\right]$

Hence $2\ell^2 = \frac{R^2}{2} + \ell^2$

$\ell = \frac{R}{\sqrt{2}}$ The pivot point is in the body of disk.

**6.17**

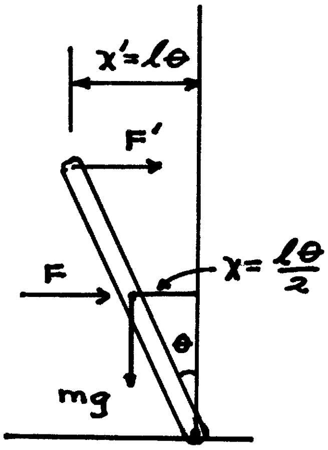
Take torques about pivot.




$\tau = I\ddot{\theta}$

$\tau = -\frac{Fl}{2} - F'l + mg\frac{l}{2}\sin\theta$

$\tau \cong -k\left(\frac{l}{2}\right)^2\theta - kl^2\theta + mg\frac{l}{2}\theta$




$$\ddot{\theta} = \left(k\frac{l^2}{4} + kl^2 - mg\frac{l}{2}\right)\frac{\theta}{I} = 0$$

$$\omega = \sqrt{\frac{\frac{kl^2}{4} + kl^2 - mg\frac{l}{2}}{\frac{ml^2}{3}}}$$

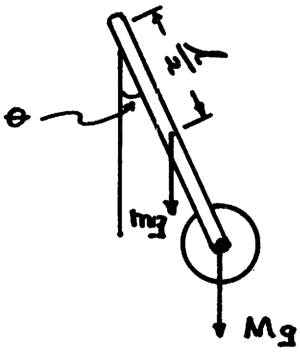
$F = \frac{kl\theta}{2}$

$F' = kl\theta$

$$\omega = \sqrt{\frac{15k}{4m} - \frac{3}{2}\frac{g}{l}} \cong \sqrt{\frac{15k}{4m}}$$ if springs are strong

---

**6.18**




$\tau = I\ddot{\theta}$

$-mg\frac{l}{2}\theta - Mgl\theta = I\ddot{\theta}$

$$\ddot{\theta} + \frac{mg\frac{l}{2} + Mgl}{I}\theta = 0$$

$I = \frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2$ $\qquad T = 2\pi\sqrt{\frac{\left(\frac{m}{3}+M\right)l^2 + \frac{M}{2}R^2}{\left(\frac{m}{2}+M\right)gl}}$

If disk is on a free bearing it does not contribute to the rotational motion. Hence

$I' = \frac{1}{3}ml^2 + Ml^2$ $\qquad T' = 2\pi\sqrt{\frac{\left(\frac{m}{3}+M\right)l^2}{\left(\frac{m}{2}+M\right)gl}}$

Energy Method: $E = \left(mg\frac{l}{2} + Mgl\right)(1-\cos\theta)$
$$+ \frac{1}{2}\left[\frac{1}{3}ml^2 + \left(\frac{1}{2}MR^2 + Ml^2\right)\right]\dot{\theta}^2$$

Since $E = \text{const.}$, $\frac{dE}{dt} = 0$

$$\Rightarrow 0 = \left(mg\frac{l}{2} + Mgl\right)\sin\theta + \left(\frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2\right)\ddot{\theta}$$

as above. If disk is free, the term $\frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2$ should not be included in $E$.

---

$\boxed{6.19}$

(a) $\tau = I\ddot{\theta}$

$$-c\theta = \left(\frac{1}{2}MR^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{c}{\frac{1}{2}MR^2}\theta = 0$$

$$\omega = \sqrt{\frac{c}{\frac{1}{2}MR^2}}$$

(b) New moment of inertia is $\frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Hence $\omega' = \sqrt{\frac{c}{\frac{3}{2}MR^2}} = \frac{1}{\sqrt{3}}\omega$

At $t_1 = \pi/\omega$, $\theta = 0$, $\dot{\theta} = \omega\theta_0$

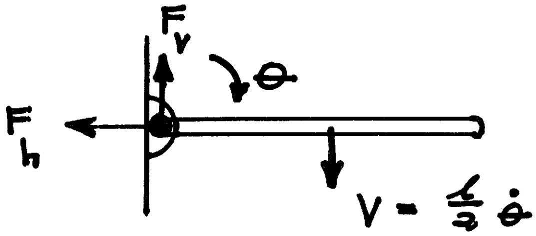
Immediately before collision, $L = I\dot{\theta} = \frac{1}{2}MR^2\omega\theta_0$

Afterward $L = I'\dot{\theta}' = \frac{3}{2}MR^2\omega'\theta_0'$ where $\theta_0'$ is the new amplitude. Because $L$ is conserved,

$$\frac{1}{2}MR^2\omega\theta_0 = \frac{3}{2}MR^2\frac{1}{\sqrt{3}}\omega\theta_0' \Rightarrow \theta_0' = \frac{1}{\sqrt{3}}\theta_0$$

$\boxed{6.20}$

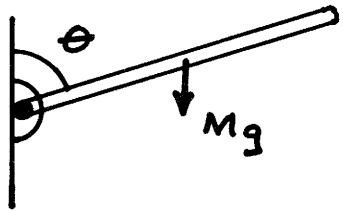
$$F_h = \frac{Mv^2}{\left(\frac{\ell}{2}\right)} = M\frac{\ell}{2}\dot{\theta}^2$$




$$Mg - F_v = Ma = M\frac{\ell}{2}\ddot{\theta}$$

Hence we need $\dot{\theta}$ and $\ddot{\theta}$ at $\theta = 90°$

$$I\ddot{\theta} = \tau = Mg\frac{\ell}{2}\sin\theta$$




$$\ddot{\theta}\Big|_{90°} = \frac{Mg\frac{\ell}{2}}{\frac{1}{3}M\ell^2} = \frac{3}{2}\frac{g}{\ell}$$

$$E(60°) = Mg\frac{\ell}{2}\cos(60°) = \frac{Mg\ell}{4}$$

$$E(90°) = \frac{1}{2}I(\dot{\theta})^2 = \frac{1}{6}M\ell^2\dot{\theta}^2$$

Hence $\frac{1}{6}M\ell^2\dot{\theta}^2 = \frac{Mg\ell}{4}$

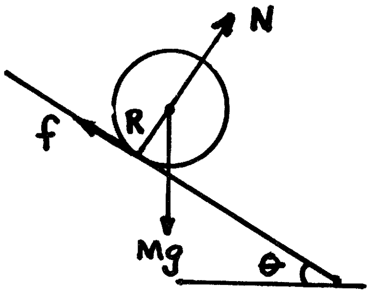
OR $\dot{\theta}\Big|_{90°} = \sqrt{\frac{3}{2}\frac{g}{\ell}} \Rightarrow F_h = \frac{3}{4}Mg$ , $F_v = \frac{1}{4}Mg$

---

$\boxed{6.21}$ $fR = (\frac{1}{2}MR^2)\alpha$

$f = \frac{1}{2}MR\alpha = \frac{1}{2}Ma$

for rolling without slipping.




$N = Mg\cos\theta$

$Mg\sin\theta - f = Ma$ , $Mg\sin\theta = \frac{3}{2}Ma$

But $f \le \mu N = \mu Mg\cos\theta$

$\frac{1}{2}Ma \le \mu gM\cos\theta$

$\frac{1}{3}Mg\sin\theta \le \mu Mg\cos\theta \Rightarrow \tan\theta \le 3\mu$

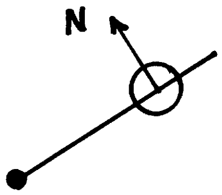
$\boxed{6.22}$

(a) $$0 = m(\ddot{r} - r\omega^2)$$

$$\ddot{r} - r\omega^2 = 0$$

$$r = A e^{\omega t} + B e^{-\omega t}$$

Hence If $B = 0$, $r = r_0 e^{\omega t}$ where $r_0 = r(0)$




(b) $$N = m a_\theta = m(2\dot{r}\omega)$$

$$= 2m\omega\,(r_0 \omega e^{\omega t})$$

$$= 2m\omega^2 r_0 e^{\omega t}$$

(c) $$P = Fv = \tau\omega = (Nr)\omega$$

$$= 2m\omega^3 r_0^2 e^{2\omega t}$$

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\dot{r}^2 + (r\omega)^2)$$

$$= \tfrac{1}{2}m(r_0^2\omega^2 e^{2\omega t} + r_0^2\omega^2 e^{2\omega t})$$

$$= m r_0^2 \omega^2 e^{2\omega t}$$

$$\frac{dKE}{dt} = 2m r_0^2 \omega^3 e^{2\omega t} = P$$

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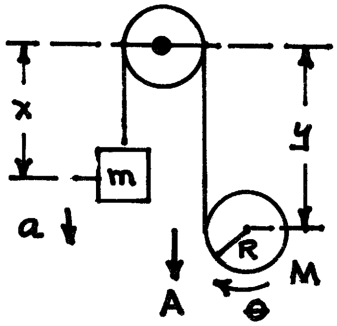
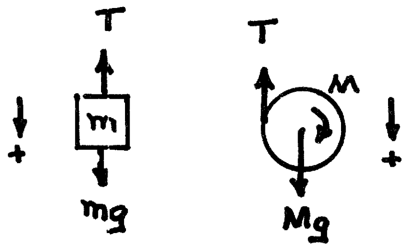
$\boxed{6.23}$




(a) $$x + y = \ell_0 + R\theta$$

$$\ddot{x} + \ddot{y} = R\ddot{\theta}$$

OR $$a + A = R\alpha$$



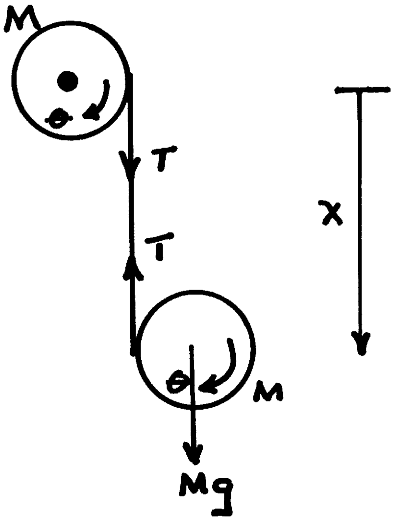

(b) $$mg - T = ma$$

$$Mg - T = MA$$

$$TR = \tfrac{1}{2}MR^2\alpha$$

$$\Rightarrow \quad a = \left(\frac{3m - M}{3m + M}\right)g \qquad A = \left(\frac{m + M}{3m + M}\right)g \qquad \alpha = \left(\frac{4m}{3m + M}\right)\frac{g}{R}$$

$\boxed{6.24}$ Both drums turn through the same angle and therefore have the same angular acceleration. (The torques about the centers are identical)




$$x = l_0 + 2R\theta$$

$$\ddot{x} = A = 2R\alpha$$

Drum A: $\quad Mg - T = MA$

$\qquad\qquad TR = \frac{1}{2}MR^2\alpha$

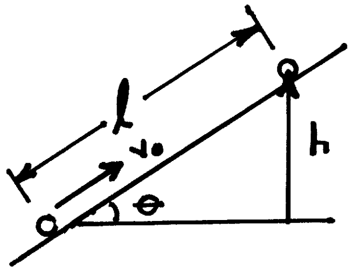
Drum B: $\quad TR = \frac{1}{2}MR^2\alpha$

Solving, $\quad Mg = 5/4\ MA$

$$A = \frac{4}{5}g$$

---

$\boxed{6.25}$




$$E_i = \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\dot{\theta}^2$$

$$= \frac{1}{2}Mv_0^2 + \frac{1}{5}Mv_0^2$$

$$= \frac{7}{10}Mv_0^2$$

$$E_f = Mgh = Mgl\sin\theta$$

Hence $\quad l = \dfrac{\frac{7}{10}Mv_0^2}{Mg\sin\theta} = \dfrac{\frac{7}{10}v_0^2}{g\sin\theta}$

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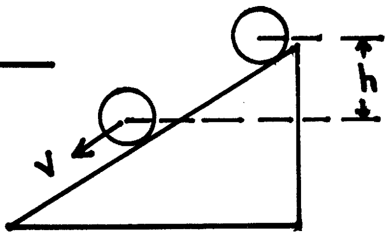



$\boxed{6.26}$ $E_i = Mgh$

After falling a height $h$, speed is $v$

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\dot{\theta}^2$$

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{I}{R^2}v^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2$$

Hence $v = \left[ \dfrac{2Mgh}{M + \frac{I}{R^2}} \right]^{\frac{1}{2}}$

For a uniform sphere, $\frac{I}{R^2} = \frac{2}{5} M$

For a uniform cylinder, $\frac{I}{R^2} = \frac{1}{2} M > \frac{2}{5} M$

Hence the speed of the sphere is always greater than the speed of the cylinder at a given location and the sphere reaches the bottom first.

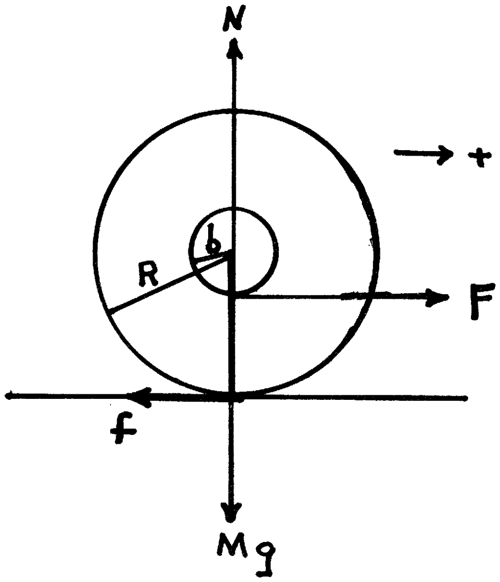
---

$\boxed{6.27}$ $F - f = MA$

$fR - Fb = (\frac{1}{2} MR^2)\alpha$

$= \frac{1}{2} MRA$

( because $A = R\alpha$ for rolling without slipping.)




Hence $f = \frac{1}{3} F \left(1 + \frac{2b}{R}\right)$

But $f \le \mu N = \mu Mg$

Thus $\frac{F}{3}\left(1 + \frac{2b}{R}\right) \le \mu Mg$

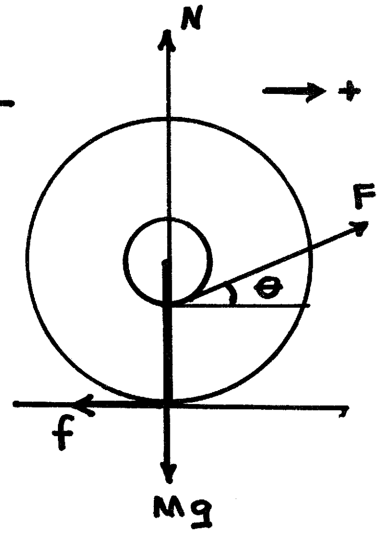
$F_{max} = \dfrac{3\mu Mg}{\left(1 + \frac{2b}{R}\right)}$

---

$\boxed{6.28}$ ( F is considered to be given)

$F \cos\theta - f = MA$ (not needed)

$F \sin\theta + N = Mg$




There is no tendency to rotate when torque = 0

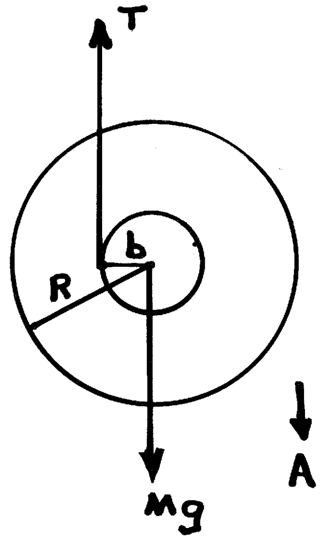
Hence $Fb = fR$

Also, $f = \mu N = \mu\,(Mg - F\sin\theta)$ because YoYo is not rotating.

Hence $Fb = \mu\,(Mg - F\sin\theta)\,R$

OR $\sin\theta = \dfrac{\mu MgR - Fb}{\mu FR}$

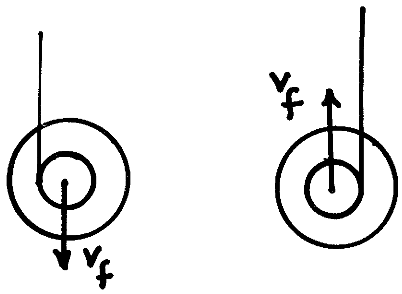
---




$\boxed{6.29}$ $Mg - T = MA$

(a) $Tb = \left(\frac{1}{2}MR^2\right)\alpha = \frac{1}{2}MR^2\,\frac{A}{b}$

$$\Rightarrow\ T = \frac{Mg}{1 + \frac{2b^2}{R^2}}$$




(b) $F = \dfrac{\Delta p}{\Delta t} = \dfrac{2MV_f}{\frac{\pi b}{V_f}}$

$$= \frac{2MV_f^2}{\pi b}$$

By conservation of energy,

$E_i = Mgh$

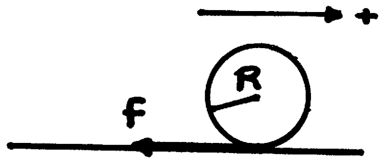
$E_f = \frac{1}{2}MV_f^2 + \frac{1}{2}I\dot{\theta}_f^2 = \frac{1}{2}MV_f^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V_f^2}{b^2}$

$= \frac{1}{2}MV_f^2\left[1 + \frac{1}{2}\left(\frac{R}{b}\right)^2\right]$

Hence $MV_f^2 = \dfrac{2Mgh}{\left[1 + \frac{1}{2}\left(\frac{R}{b}\right)^2\right]}$

$$F = \left(\frac{4Mgh}{\pi b}\right)\frac{1}{\left[1 + \frac{1}{2}\left(\frac{R}{b}\right)^2\right]}$$

$\boxed{6.30}$




$f = -MA$

$fR = I_0 \alpha$

Ball rolls without slipping when $v = R\dot{\theta}$

$$v(t) = v_0 - \int_0^t \frac{f}{M}\,dt \qquad \dot{\theta} = \int_0^t \frac{fR}{I_0}\,dt$$

v decreases and $\dot{\theta}$ increases until $v = R\dot{\theta}$

$$R\dot{\theta} = \int_0^t \frac{fR^2}{I_0}\,dt = \frac{R^2}{\frac{2}{5}MR^2}\int_0^t f\,dt = v = v_0 - \frac{1}{M}\int_0^t f\,dt$$

Hence ball rolls without slipping when

$$\frac{5}{2}\frac{1}{M}\int_0^t f\,dt = v_0 - \frac{1}{M}\int_0^t f\,dt$$

$$\frac{1}{M}\int_0^t f\,dt = \frac{2}{7}v_0$$

When rolling begins $v = v_0 - \frac{2}{7}v_0 = \frac{5}{7}v_0$. After this time, $f = 0$ and rolling without slipping continues.

<u>Simpler Solution:</u> use conservation of angular momentum about contact point.

$L_i = Mv_0R$, $\qquad L_f = MvR + I_0\omega$

$\omega = \frac{v}{R}$

$I_0 = \frac{2}{5}MR^2$

$$L_i = L_f \Rightarrow Mv_0 R = MvR + \frac{2}{5}MvR$$

$$v = \frac{5}{7}v_0$$

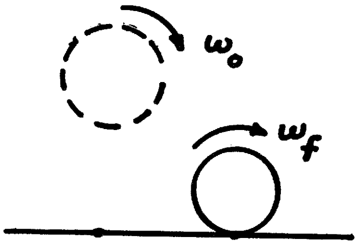
---

$\boxed{6.31}$

Angular momentum is conserved about point of impact.

$$I_0\omega_0 = I_0\omega_f + MvR$$

For pure rolling, $\omega_f = \frac{v}{R}$




$$I_0\omega_0 = I_0\omega_f + MR^2\omega_f$$

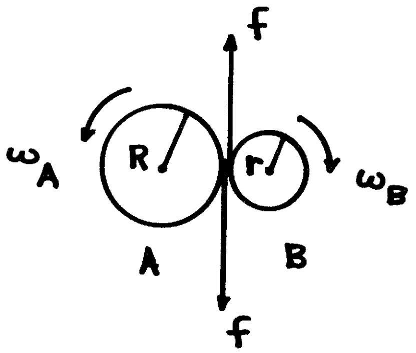
$$I_0 = \frac{1}{2}MR^2$$

$$\frac{1}{2}MR^2\omega_0 = \frac{1}{2}MR^2\omega_f + MR^2\omega_f$$

$$\omega_f = \frac{\omega_0}{3}$$

---

$\boxed{6.32}$




$$fR = -I_A\alpha_A$$

$$fr = I_B\alpha_B$$

$$\omega_A = \omega_0 - \frac{R}{I_A}\int_0^t f\,dt$$

$$\omega_B = \frac{r}{I_B}\int_0^t f\,dt$$

Hence $\omega_A = \omega_0 - \left(\frac{R}{I_A}\right)\left(\frac{I_B}{r}\right)\omega_B$

f continues to act until the surfaces of contact attain the same linear velocity: $R\omega_{A,fin} = r\omega_{B,fin}$.

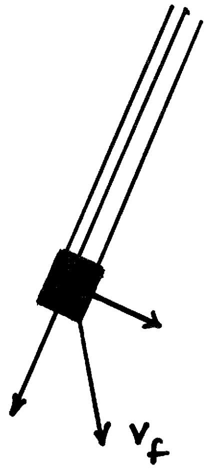
$$\omega_{A,fin} = \omega_0 - \left(\frac{R^2}{I_A}\right)\left(\frac{I_B}{r^2}\right)\omega_{A,fin}$$

$$\omega_{A,fin} = \omega_0 \Big/ \left(1 + \frac{R^2}{r^2}\frac{I_B}{I_A}\right) = \frac{\omega_0}{1+\frac{m}{M}} = \frac{M}{M+m}\,\omega_0$$

$\boxed{6.33}$

(a) Angular momentum along the vertical direction is conserved.

$$L_i = I_0 \omega_0$$
$$L_f = I_0 \omega_f + mR^2 \omega_f$$
$$\omega_f = \left(\frac{I_0}{I_0 + mR^2}\right)\omega_0$$




(b) $E_i = \frac{1}{2} I_0 \omega_0^2 + mgh$

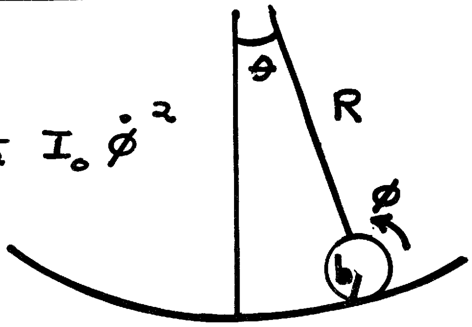
$$E_f = \frac{1}{2} I_0 \omega_f^2 + \frac{1}{2} m v_f^2$$

Hence $v_f = \sqrt{\frac{I_0}{m}\left(\omega_0^2 - \omega_f^2\right) + 2gh}$

$$v_f = \sqrt{\frac{I_0}{m}\omega_0^2\left[1 - \left(\frac{I_0}{I_0 + mR^2}\right)^2\right] + 2gh}$$

---

$\boxed{6.34}$




$$E = mgR(1 - \cos\theta) + \frac{1}{2} m v^2 + \frac{1}{2} I_0 \dot{\phi}^2$$

But $v = R\dot{\theta}$

$$b\dot{\phi} = R\dot{\theta}$$
$$I_0 = \frac{2}{5} m b^2$$

Hence $E = mgR(1-\cos\theta) + \frac{1}{2} m R^2 \dot{\theta}^2 + \frac{1}{5} m b^2 \frac{R^2}{b^2}\dot{\theta}^2$

$$= mgR(1-\cos\theta) + \frac{7}{10} m R^2 \dot{\theta}^2$$

$$0 = \frac{dE}{dt} = (mgR\sin\theta)\dot{\theta} + \frac{7}{5} m R^2 \dot{\theta}\ddot{\theta}$$

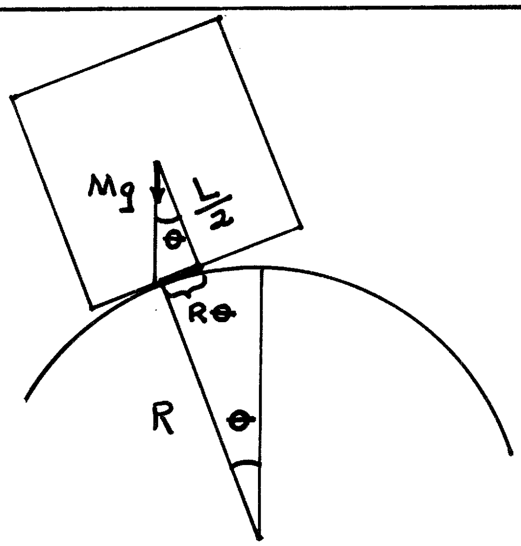
Hence $\ddot{\theta} + \frac{mgR}{\frac{7}{5} m R^2}\theta = 0$

$$\ddot{\theta} + \frac{5}{7}\frac{g}{R}\theta = 0$$

Hence $\omega = \sqrt{\frac{5}{7}\frac{g}{R}}$

---

**6.35**




As block rocks without slipping, it measures off a distance $R\theta$.

Block is stable if line of $Mg$ fall within $R\theta$ of center, so as to produce a restoring torque.

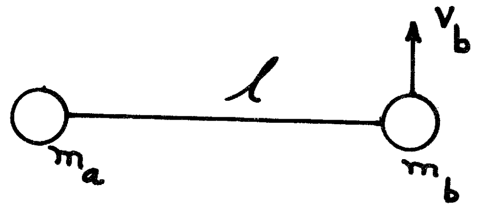
Unstable if $\frac{L}{2}\tan\theta > R\theta$

For small $\theta$, $\tan\theta \cong \theta$

Hence block is unstable if $L > 2R$

---

**6.36**

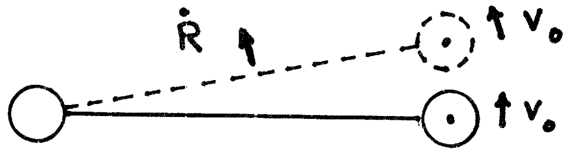



After release, the CM moves at a steady velocity

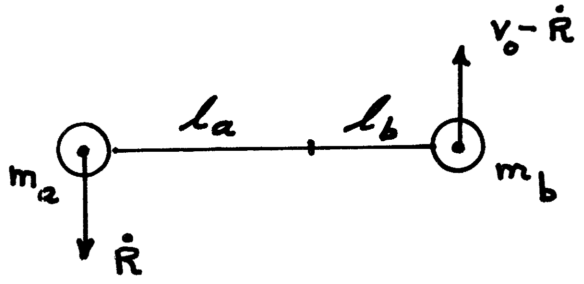
$$\dot{R} = \frac{m_a v_a(0) + m_b v_b(0)}{m_a + m_b} = \left(\frac{m_b}{m_a + m_b}\right) v_0$$

The system rotates and translates. Because $v_a(0) = 0$ angular velocity is correctly given by

$\omega = \frac{v_0}{l}$.




Alternate method: transfer to frame in which CM is at rest. System rotates about CM. Immediately after release,




$$\dot{R} = \left(\frac{m_b}{m_a+m_b}\right) v_0$$

$$v_0 - \dot{R} = \left(\frac{m_a}{m_a+m_b}\right) v_0$$

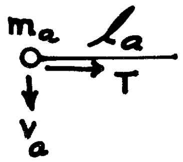
$$l_a = \left(\frac{m_b}{m_a+m_b}\right) l$$

$$l_b = \left(\frac{m_a}{m_a+m_b}\right) l$$

$$\omega = \left(\frac{m_b v_0}{m_a+m_b}\right) \Big/ l_a = \frac{v_0}{l}$$

To find T, do not use $m_a$ as an origin, because it is accelerating. Go to CM frame.

$$T = \frac{m_a v_a^2}{l_a}$$




$$v_a = \left(\frac{m_b}{m_a+m_b}\right) v_0$$

$$T = m_a \left(\frac{m_b}{m_a+m_b}\right)^2 v_0^2 \Big/ \left(\frac{m_b}{m_a+m_b}\right) l = \left(\frac{m_a m_b}{m_a+m_b}\right) \frac{v_0^2}{l}$$

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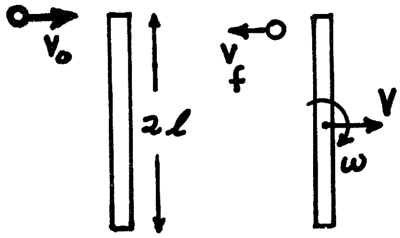
$\boxed{6.37}$ (a) $mv_0 = Mv - v_f$




$$\tfrac{1}{2} m v_0^2 = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} M v^2 + \tfrac{1}{2} I_0 \omega^2$$

About CM $\quad m v_0 l = -m v_f l + I_0 \omega$

$$\Rightarrow v = \frac{m}{M}(v_0 + v_f) \qquad \omega = m(v_0 + v_f) l / I_0$$

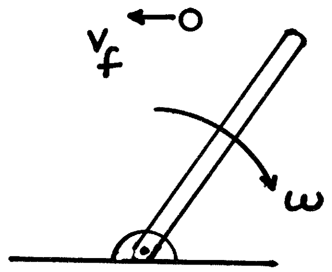
where $I_0 = \frac{1}{12} M (2\ell)^2 = \frac{1}{3} M\ell^2$

We find

$$\left(1 + \frac{4m}{M}\right) v_f^2 + \left(\frac{8m}{M} v_0\right) v_f - \left(1 - \frac{4m}{M}\right) v_0^2 = 0$$

which yields $\quad v_f = \left(\dfrac{1 - \frac{4m}{M}}{1 + \frac{4m}{M}}\right) v_0$

(b) Momentum need not be conserved because forces act at the pivot.




$$\frac{1}{2} m V_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I_p \omega^2$$

Take angular momentum about the pivot:

$$m v_0 (2\ell) = -m v_f (2\ell) + I_p \omega$$

$I_p = \frac{1}{3}(2\ell)^2 M = \frac{4}{3} M\ell^2$

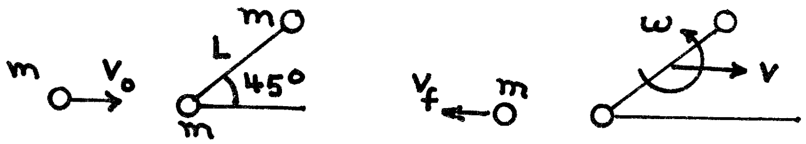
Hence $\quad \omega = \dfrac{2m\ell(v_0 + v_f)}{I_p}$

and $\quad \left(1 + \frac{3m}{M}\right) v_f^2 + \left(6\frac{m}{M} v_0\right) v_f - \left(1 - \frac{3m}{M}\right) v_0^2 = 0$

$$\Rightarrow \quad v_f = \left(\frac{1 - 3m/M}{1 + 3m/M}\right) v_0$$

---

$\boxed{6.38}$




$$m v_0 = 2mV - m v_f$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2}(2m) V^2 + \frac{1}{2} I_0 \omega^2 \qquad I_0 = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

Take angular momentum about CM

$$mv_0\left(\frac{L}{2}\right)\sin 45^\circ = -mv_f\left(\frac{L}{2}\right)\sin 45^\circ + I_0\omega$$

Hence $\quad v_0 = 2V - v_f$

$$v_0^2 = v_f^2 + 2V^2 + \frac{L^2}{2}\omega^2$$

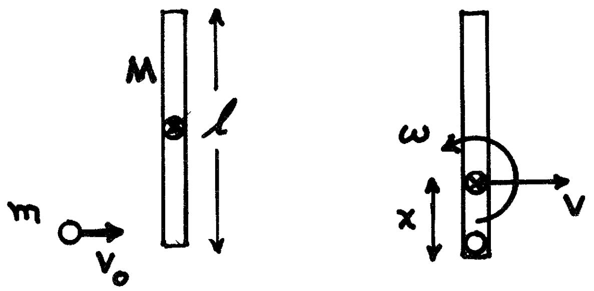
$$v_0 = -v_f + \sqrt{2}\,L\omega$$

Solving, $\quad \omega = \frac{4\sqrt{2}}{7}\frac{v_0}{L}$

---

$\boxed{6.39}$

(a) The boy plank system translates and rotates about its new CM.




$$mv_0 = (M+m)V$$

$$V = \frac{mv_0}{m+M}$$

Position of new CM is at

$$M\left(\frac{l}{2} - x\right) = mx$$

$$x = \frac{Ml/2}{M+m}$$

Take angular momentum about new CM

$$mv_0 x = [\,m x^2 + \underbrace{I_0 + M\left(\frac{l}{2} - x\right)^2}_{\text{by parallel axis theorem}}\,]\,\omega$$

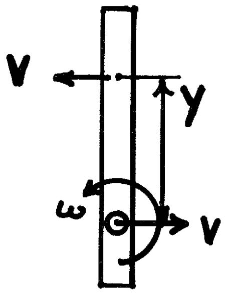
$$\left[m x^2 + I_0 + M\left(\frac{l}{2} - x\right)^2\right] = \frac{m\,M^2}{(m+M)^2}\frac{l^2}{4} + \frac{1}{12}Ml^2 + \frac{1}{4}Ml^2\left(\frac{m}{m+M}\right)^2$$

$$= \frac{1}{12}Ml^2 + \frac{mMl^2}{4(m+M)} = \frac{Ml^2}{12}\left(\frac{M+4m}{M+m}\right)$$

Hence $\quad \omega = \dfrac{m v_0 x}{\frac{M\ell^2}{12}\frac{(M+4m)}{(M+m)}} = \dfrac{m M \frac{\ell}{2} v_0}{\frac{M\ell^2}{12}(M+4m)}$

$$\omega = \left(\frac{m}{M+4m}\right)\frac{6V_0}{\ell}$$

(b) The point at rest has speed $-V$ due to rotation.




$$V = Y\omega$$

$$Y = \frac{V}{\omega} = \frac{\left(\frac{mV_0}{m+M}\right)}{\left(\frac{6V_0}{\ell}\right)\left(\frac{m}{M+4m}\right)} = \frac{\ell}{6}\left(\frac{M+4m}{M+m}\right)$$

$$Y + X = \frac{\left(\frac{M}{6} + \frac{2}{3}m + \frac{M}{2}\right)\ell}{M+m} = \frac{2}{3}\ell \text{ from boy}$$

---

$\boxed{6.40}$

(a) The motion for $x > \ell$ is given by

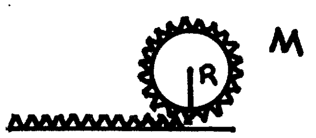
$x = \ell + b\cos\beta t \qquad$ where $\beta = \sqrt{k/m}$

$\dot{x} = -\beta b \sin(\beta t)$

Speed of CM at $x = \ell$ is $-\beta b$

Momentum and energy of wheel are not conserved in collision with the track. Take angular momentum about point on track. Torque of spring is zero at $x = \ell$, torque due to collision force doesn't enter.

$L_i = MRv = MR\beta b$




$L_f = MR^2\omega + MVR$

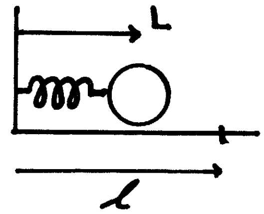
$= 2MR^2\omega$

Hence $\omega = \frac{\beta b}{2R}$ immediately after collision.

$V = R\omega = \frac{\beta b}{2}$ because $V = R\omega$ on gear tracks.

$E = \frac{1}{2}MV^2 + \frac{1}{2}MR^2\omega^2 = MR^2\omega^2 = \frac{M\beta^2 b^2}{4} = \frac{kb^2}{4}$

Spring is compressed to a length $L$




$\frac{1}{2}k(\ell - L)^2 = \frac{1}{4}kb^2$

$\ell - L = \frac{b}{\sqrt{2}}$, $L = \ell - \frac{b}{\sqrt{2}}$ from wall.

(b) There is no collision in the outward trip and energy is conserved. Speed at $x = \ell$ is given by

$\frac{1}{2}k(\ell - L)^2 = \frac{1}{4}kb^2 = \frac{1}{2}MV^2 + \frac{1}{2}(MR^2)\omega^2$

$= MV^2$

$V = \frac{1}{2}b\sqrt{k/M} = \frac{1}{2}\beta b$

$\omega = \frac{V}{R} = \frac{\frac{1}{2}\beta b}{R}$

For $x > \ell$, $\omega$ remains constant. Motion of wheel is given by $x = A\sin\beta t + B\cos\beta t$

$\dot{x} = \beta A\cos\beta t - \beta B\sin\beta t$

$x(0) = 0$

$\dot{x}(0) = \frac{1}{2}\beta b \implies B = 0,\ A = \frac{b}{2}$

$x = \frac{b}{2} \sin\beta t$

$\dot{x} = \frac{\beta b}{2} \cos\beta t$

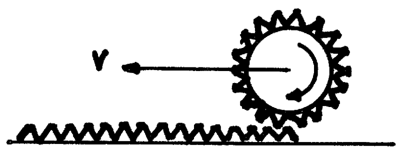
Wheel stops at $\beta t = \frac{\pi}{2}$ $\qquad x(\beta t = \frac{\pi}{2}) = \frac{b}{2}$

Hence wheel goes out to $x = \frac{b}{2}$

(c)

Wheel is spinning with $\omega = \frac{\frac{1}{2}\beta b}{R}$ as it hits track, and it has speed $-\frac{\beta b}{2}$.

$$L_i = MRV - MR^2\omega$$
$$= \frac{MR\beta b}{2} - \frac{MR\beta b}{2} = 0$$




Hence $L_f = 2MR^2\omega' = 0$,

$\omega' = 0$,

$v = R\omega' = 0$ and wheel comes to rest.

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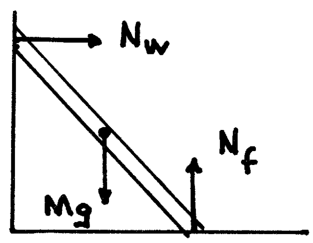
$\boxed{6.41}$

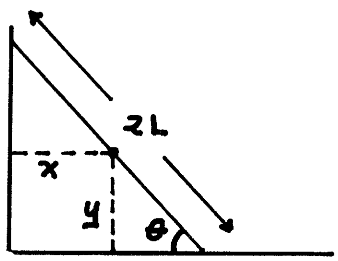
Only normal forces act at the surfaces. (No friction) The position of the CM and the angular position are given by a single coordinate, $\theta$.




Location of CM is given by

$x = L\cos\theta$

$y = L\sin\theta$



The horizontal force is given by

$$N_w = M\ddot{x} = ML(-\cos\theta\ \dot{\theta}^2 - \sin\theta\ \ddot{\theta})$$

Contact is lost when $N_w = 0$ or $\dot{\theta}^2 = -\tan\theta\ \ddot{\theta}$ (1)

Energy is conserved, so we can find $\dot{\theta}^2$ from the energy equation. A simple way to get $\ddot{\theta}$ is to differentiate the energy equation.

$$\begin{aligned} E = Mgy_0 &= \tfrac{1}{2}M(\dot{x}^2+\dot{y}^2) + \tfrac{1}{2}I_0\dot{\theta}^2 + Mgy \\ &= \tfrac{1}{2}ML^2\dot{\theta}^2 + \tfrac{1}{2}(\tfrac{1}{3})ML^2\dot{\theta}^2 + MgL\sin\theta \\ &= \tfrac{2}{3}ML^2\dot{\theta}^2 + MgL\sin\theta \qquad (2) \end{aligned}$$

Differentiating,

$$0 = \tfrac{4}{3}ML^2\dot{\theta}\ddot{\theta} + MgL\cos\theta\ \dot{\theta}$$

$$\ddot{\theta} = -\tfrac{3}{4}\tfrac{g}{L}\cos\theta \qquad (3)$$

Substituting (3) into (1) yields

$$\dot{\theta}^2 = \tfrac{3}{4}\tfrac{g}{L}\sin\theta$$

Substituting this into (2) we find

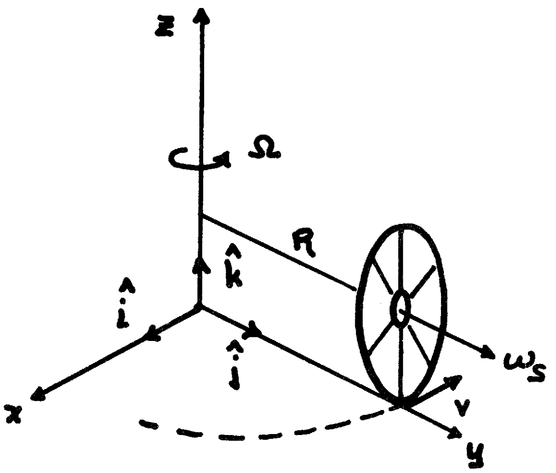
$$\begin{aligned} Mgy_0 &= \tfrac{1}{2}MgL\sin\theta + MgL\sin\theta \\ &= \tfrac{3}{2}MgL\sin\theta \end{aligned}$$

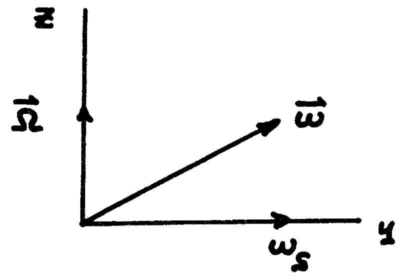
Hence $y = L\sin\theta = \tfrac{2}{3}y_0$.

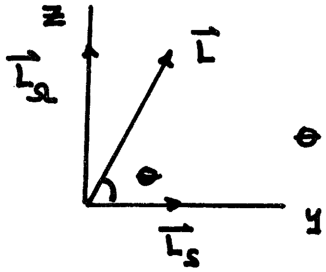
The board leaves the wall when the top has slipped 1/3 of the way down.

# Chapter 7

**7.1**




$\theta = \arctan \frac{3}{2}$

(a)

$$\omega_s = \frac{v}{R} = \frac{\Omega R}{R} = \Omega$$

$$\vec{\omega} = \vec{\omega}_s + \vec{\Omega} = \Omega(\hat{j} + \hat{k})$$

(b)

$$\vec{L} = \vec{L}_s + \vec{L}_\Omega = I_s \vec{\omega}_s + I_z \vec{\Omega} \qquad I_s = MR^2$$

By parallel axis theorem, $I_z = I_0 + MR^2 = \frac{3}{2} MR^2$

$$\vec{L} = MR^2 (\vec{\omega}_s + \tfrac{3}{2}\vec{\Omega}) = MR^2\Omega(\hat{j} + \tfrac{3}{2}\hat{k})$$

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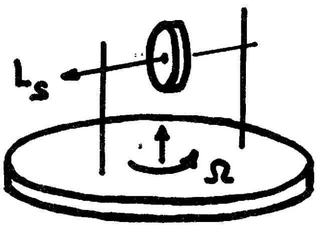
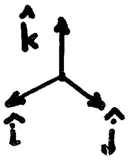
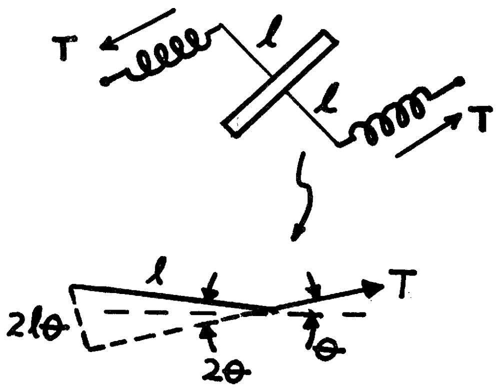
**7.2**




The spin angular momentum is $L_s = I_0 \omega_s$. Assuming that the axle is close to horizontal, $\left|\frac{dL}{dt}\right| = \Omega L_s$ along the $\hat{j}$ axis.

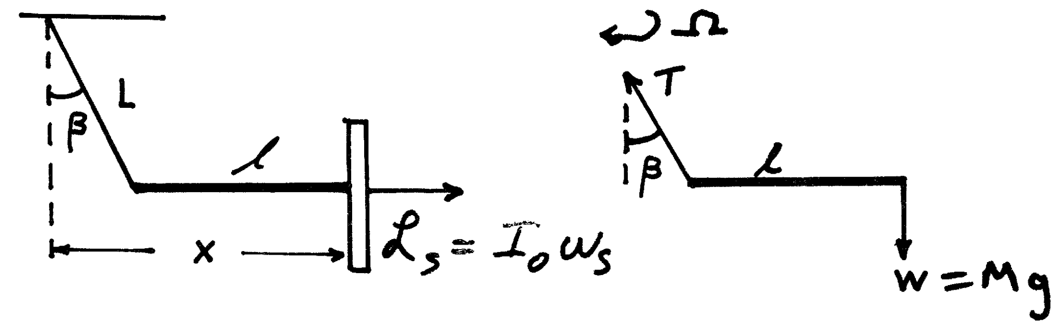
The torque due to each spring is $2\ell\theta T$.
The total torque is $4\ell\theta T$, hence

$$4\ell\theta T = \Omega L_s$$

$$\theta = \frac{\Omega L_s}{4\ell T}$$

---

**7.3**




The linear equations of motion are

$$T\cos\beta - Mg = 0$$

$$T\sin\beta = \frac{Mv^2}{x} = M\Omega^2 x$$

$$\text{Torque} = \ell T\cos\beta = \ell Mg = \frac{d\mathcal{L}_s}{dt} = \Omega\mathcal{L}_s = \Omega I_0\omega_s$$

with $\cos\beta \sim 1$ and $\sin\beta \sim \beta$ we have

$$\beta \sim \frac{M\Omega^2 x}{T} = \frac{\Omega^2 x}{g} = \frac{\left(\frac{Mg\ell}{I_0\omega_s}\right)^2 x}{g}$$

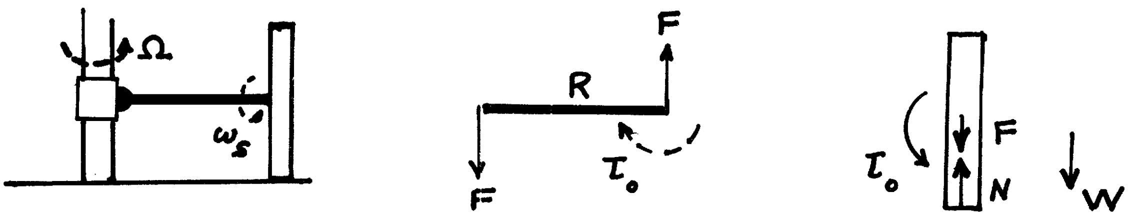
But $x = \ell + L\sin\beta \sim \ell + L\beta$

Let $\beta_0 = \dfrac{\left(\frac{Mg}{I_0\omega_s}\right)^2 \ell^3}{g}$

Then $\beta = \beta_0 + \frac{L}{\ell}\beta_0\beta$

or $\beta = \dfrac{\beta_0}{1 - \frac{L}{\ell}\beta_0}$

**7.4**




The axle exerts a torque $\tau_0$ on the millstone and a vertical downward force F. The millstone exerts opposite force and torque on the axle. The coupling at the vertical column must also exert a downward force F on the axle.

The axle is in vertical equilibrium, and for rotational equilibrium $RF = \tau_0 = \mathcal{L}_0 \Omega$

The equations of motion for the millstone are

$$N = F + W$$

$$\tau_0 = \mathcal{L}_0 \Omega = \tfrac{1}{2} M b^2 \omega_s \Omega$$

Solving, $F = \tau_0 / R = \frac{1}{2} \dfrac{M b^2 \omega_s \Omega}{R}$

For contact at the rim, $\omega_s b = \Omega R$

Then $F = \dfrac{\frac{1}{2} M b^2 \omega_s \Omega}{R} = \frac{1}{2} M b \Omega^2 \rightarrow N = Mg + F = M\left(g + \frac{1}{2} b \Omega^2\right)$

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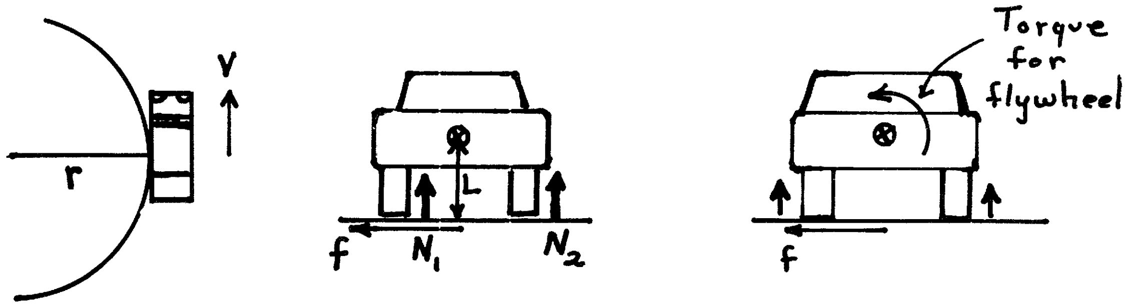
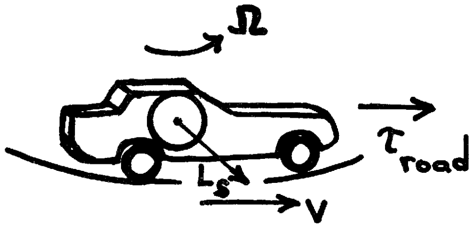
**7.5**

(a)

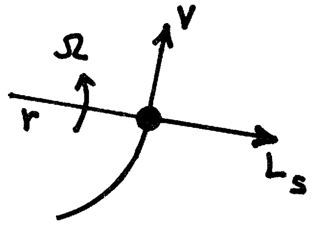


In the absence of the flywheel, the counterclockwise torque due to the normal forces at the wheels must oppose the clockwise torque due to the radial (transverse) friction force, $f = Mv^2/r$. With the flywheel, the normal forces exert no torque; so the flywheel must exert a clockwise torque $= Lf$, where $L$ is the distance from the road to the center of mass. The counterclockwise torque $Lf$ must be just sufficient to make the flywheel precess at the rate the car is turning.

If the flywheel is mounted with its angular momentum sideways, as shown, the torque it exerts on the car will tend to balance the torque due to the road. If the car turns to the right instead of the left, as illustrated, the torques due to the road and due to the flywheel will both change direction.




(b)




The torque due to the road is $\tau = Lf = Mv^2 R L$

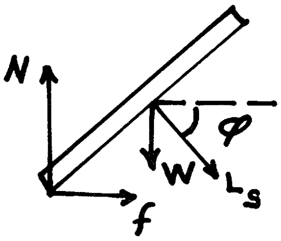
We require

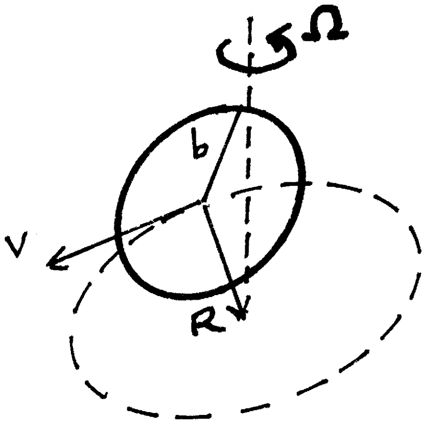
$\tau = L_s \Omega$ Using $\Omega = V/R$,

$$Mv^2 r L = L_s \Omega = \tfrac{1}{2} m R^2 \omega \frac{v}{r}$$

or $\omega = 2vML/mR^2$

7.6





Coin is accelerating, so we must use CM as origin for torque equation.

$N$ = normal force $= Mg$

$f$ = friction force $= Mv^2/R$

$L_s$ = spin angular momentum $= I\omega = \frac{1}{2}Mb^2\frac{v}{b} = \frac{1}{2}Mbv$

$\Omega$ = angular speed with which coin rotates about the vertical axis $= v/R$

$\Omega L_s \cos\phi$ = rate of change of angular momentum (in horizontal plane)

Torque equation

$$Nb\sin\phi - fb\cos\phi = \Omega L_s \cos\phi$$

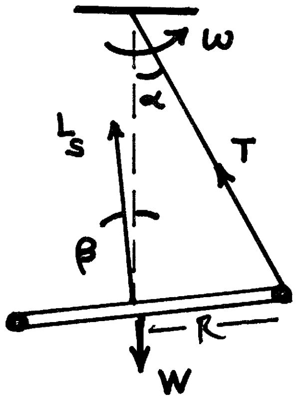
$$Mgb\sin\phi - \frac{Mv^2}{R}b\cos\phi = \frac{v}{R}\frac{1}{2}Mbv\cos\phi$$

Hence $\tan\phi = \dfrac{3v^2}{2gR}$

---

7.7

(a)



$$\Delta\vartheta = \omega\Delta t$$

$$\Delta L_s = (I\omega\sin\beta)\Delta\vartheta$$

$$= (I\omega\sin\beta)\omega\Delta t$$

$$\tau_H = T\cos\alpha\, R\cos\beta = MgR\cos\beta$$

$$\tau_H = \frac{\Delta L_s}{\Delta t} \rightarrow MgR\cos\beta = I\omega^2 \sin\beta$$

$$\tan\beta \sim \beta = \frac{MgR}{I\omega^2} = \frac{g}{R\omega^2}$$

The gyroscope approximation, although not justified here, gives the same result.

(b) The radial force is $T\sin\alpha$, so

$$T\sin\alpha = Mr\omega^2,$$

where $r$ is the radius of the path of the CM

Using $T\cos\alpha = Mg$, we find $r = \frac{g}{\omega^2}\tan\alpha$.

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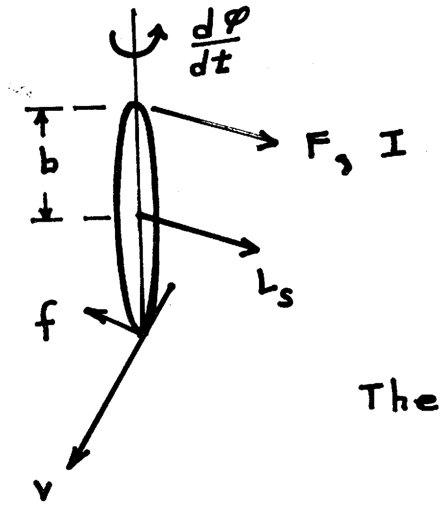
$\boxed{7.8}$




During the blow, the forces acting are the impulsive force $F$ and the frictional force $f$. We assume $F >> f$ and neglect $f$.

The torque about CM is $\tau = Fb$.

(a) The spin angular momentum is $L_s = I_H\omega = I_H \frac{v}{b}$

where $I_H$ is the moment of inertia about the horizontal axis $= Mb^2$

$$Fb = L_s \frac{d\varphi}{dt} = Mb^2 \frac{v}{b} \frac{d\varphi}{dt}$$

$$\frac{d\varphi}{dt} = F/Mv \Rightarrow \varphi = \frac{1}{Mv}\int F\,dt = \frac{I}{Mv}$$

(b) The gyroscope approximation asserts that the spin angular momentum is large compared to any other components of angular momentum.

The moment of inertia about the vertic axis is $I_v = Mb^2/2$ and the vertic component of angular momentum is $I_v \frac{d\varphi}{dt}$.

We require $L_s >> I_v \frac{d\varphi}{dt}$

OR $\qquad Mb^2 \frac{V}{b} >> \frac{1}{2} Mb^2 \frac{d\varphi}{dt} = \frac{1}{2} Mb^2 \frac{F}{MV}$

$\Rightarrow \qquad F << 2V^2/b$

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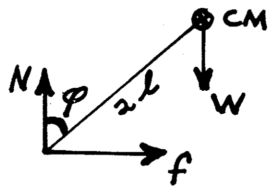
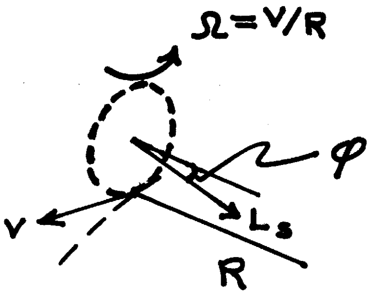
$\boxed{7.9}$




The torque about CM is

$\tau_0 = 2lN\sin\varphi - 2lf\cos\varphi$

For vertical equilibrium, $N = W$ and the horizontal acceleration is given by

$f = Mv^2/R$

$\tau_0 = 2lM\,(g\sin\varphi - v^2/R\,\cos\varphi)$

The angular momentum of the two wheels is

$$L_s = 2I\omega = 2m\ell^2 v/\ell = 2m\ell v$$

The rate of change of horizontal angular momentum as the bike turns is

$$\frac{dL_s}{dt} = \Omega L_s \cos\varphi = \frac{v}{R}(2m\ell v)\cos\varphi$$

$$\mathcal{T}_o = \frac{dL_s}{dt} \Rightarrow 2\ell M\left(g\sin\varphi - \frac{v^2}{R}\cos\varphi\right) = 2m\ell\frac{v^2}{R}\cos\varphi$$

$$g\sin\varphi - \frac{v^2}{R}\cos\varphi = \frac{m}{M}\frac{v^2}{R}\cos\varphi$$

$$\tan\varphi = \frac{v^2}{Rg}\left(1 + \frac{m}{M}\right)$$

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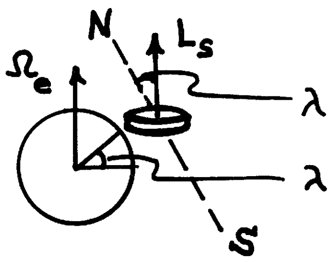
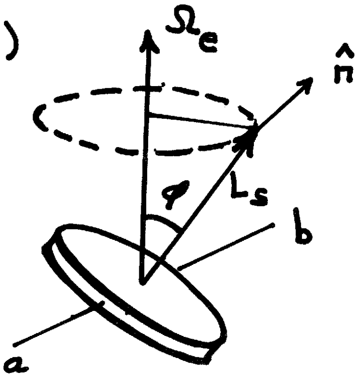
**7.10**




(a) If the spin angular momentum, $\vec{L}_s$, is parallel to the earth's angular velocity, $\vec{\Omega}_e$, then $\vec{L}_s$ does not change direction as the earth rotates, and the gyro will remain stationary.

The spin axis is then at the latitude angle $\lambda$ with respect to the local horizontal in the N-S direction (line N-S)

(b)




Let the gyro spin axis be at angle $\varphi$ with respect to $\vec{\Omega}_e$. The rate of change of spin angular momentum in the direction $\hat{n}$ is $L_s \sin\varphi\, \Omega_e \cong L_s \varphi \Omega_e$

In addition, if $I_\perp$ is the moment of inertia along the axis a-b, the angular momentum in direction $\hat{n}$ due to a change in $\varphi$ is $I_\perp \dot{\varphi}$.

There is no torque along the a-b axis, so

$$\frac{d}{dt}(I_\perp \dot{\varphi}) + L_s \Omega_e \varphi = 0$$

$$I_\perp \ddot{\varphi} + L_s \Omega_e \varphi = 0$$

This is the equation for simple harmonic motion. The oscillation frequency is

$$\omega_{osc} = \sqrt{\frac{L_s \Omega_e}{I_\perp}}$$

For $L_s = I_o \omega_s$, $\quad \omega_{osc} = \sqrt{\frac{I_o \omega_s \Omega_e}{I_\perp}}$

For a thin disk, $I_\perp = \frac{1}{2} I_o$

$$\Omega_e = \frac{2\pi \text{ rad/day}}{8.64 \times 10^4 \text{ s/day}} = 7.28 \times 10^{-5} \text{ rad/s}$$

$$\omega_s = 4 \times 10^4 \text{ rotations/min} \times \frac{2\pi}{60} = 4.18 \times 10^3 \text{ rad/s}$$

Hence $\omega_{osc} = \sqrt{2 \times 4.18 \times 10^{3} \times 7.28 \times 10^{-5}} = 0.77$ rad/s

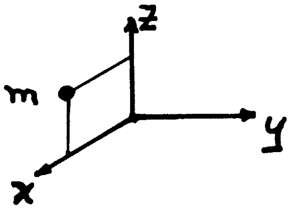
The period of oscillation is $T = \frac{2\pi}{\omega} = 8.2$ s

---

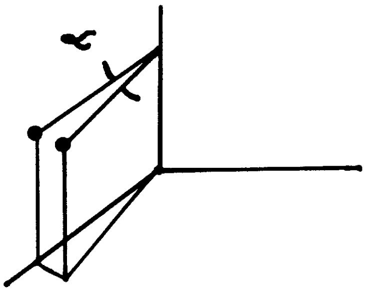
$\boxed{7.11}$

(a)
$$I_{xx} = m(y^2+z^2) = 9m$$
$$I_{xy} = -mxy = 0$$
$$I_{xz} = -mxz = -6m \quad \text{etc.}$$
$$\left\{ \begin{array}{l} x = 2 \\ y = 0 \\ z = 3 \end{array} \right.$$

$$I = m \begin{pmatrix} 9 & 0 & -6 \\ 0 & 13 & 0 \\ -6 & 0 & 4 \end{pmatrix}$$




(b)




$$\left\{ \begin{array}{l} x = 2(1-\cos\alpha) \\ y = 2\sin\alpha \\ z = 3 \end{array} \right.$$

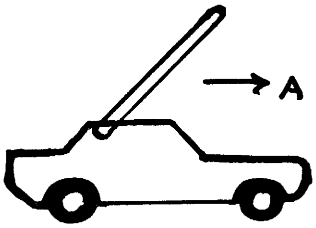
To first order in $\alpha$
$$\left\{ \begin{array}{l} x = 2 \\ y = 2\alpha \\ z = 3 \end{array} \right.$$

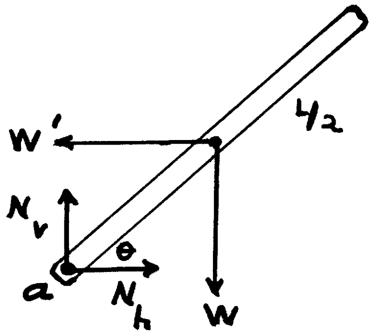
$$I' = m \begin{pmatrix} 9+4\alpha^2 & -4\alpha & -6 \\ -4\alpha & 13 & -6\alpha \\ -6 & -6\alpha & 4+4\alpha \end{pmatrix}$$

Comparing with part (a) we see that the moments of inertia vary only by terms of $\alpha^2$. The products vary linearly.

# Chapter 8

**8.1**




The force diagram is in the accelerating system. $W'$ is the fictitious force: $\vec{W}' = -M\vec{A}$. The torque about the pivot point is

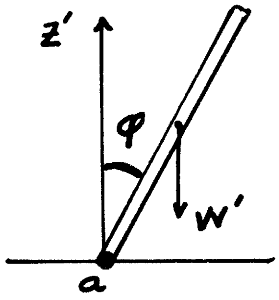
$$\tau_a = -\frac{l}{2}\sin\theta\, W' + \frac{l}{2}\cos\theta\, W$$

(a) For equilibrium, $\tau_a = 0$.

$\cos\theta\, Mg = \sin\theta\, MA$

$\Rightarrow \tan\theta = g/A$

(b) To analyze motion about equilibrium, introduce a coordinate system with the $z'$ axis along the board in equilibrium. The effective gravitational field is $g' = \sqrt{g^2 + A^2}$.




For small displacement, torque is

$\tau = \frac{l}{2} M g' \varphi$

Hence $I_a \ddot{\varphi} = \frac{l}{2} M g' \varphi$

Using $I_a = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$

$$\frac{1}{3}ML^2\ddot{\varphi} - \frac{1}{2}Mg'\varphi = 0$$
$$\ddot{\varphi} - \frac{3}{2}g'\varphi/L = 0$$

The motion is hyperbolic: $\varphi = \varphi_0 e^{\pm\gamma t}$

where $\gamma = \sqrt{\frac{3/2\, g'}{L}}$

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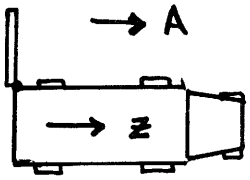
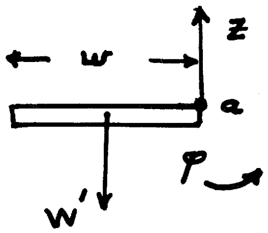
**8.2**




Consider the motion of the truck door in a system fixed to the truck. The door appears to "fall" from rest in a gravitational field $g' = A$.

(a) By conservation of energy, the kinetic energy of the door after rotation through angle $\varphi = \pi/2$ is

$$\frac{1}{2} I_a \dot{\varphi}^2 = W' \frac{w}{L}$$

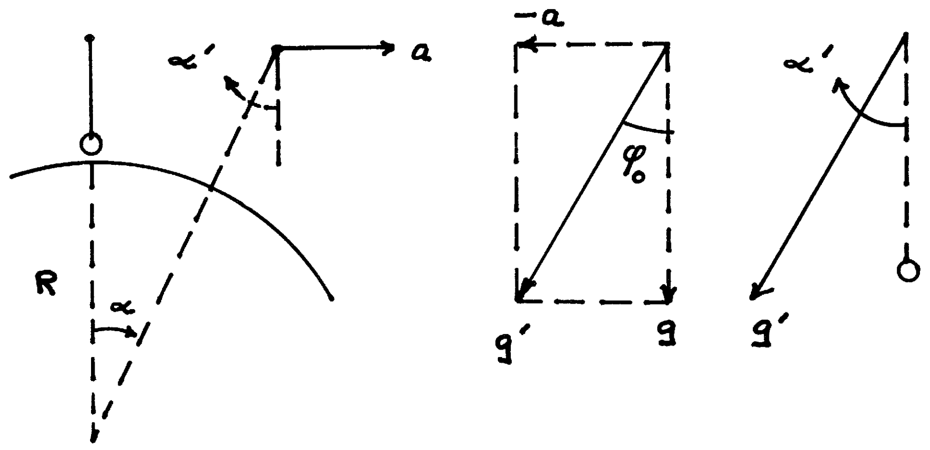
Using $I_a = \frac{1}{3}Mw^2$ we have

$$\frac{1}{6}Mw^2\dot{\varphi}^2 = MAw/2$$
$$\dot{\varphi} = \sqrt{3A/w}$$

(b) $F = MR\dot{\varphi}^2$

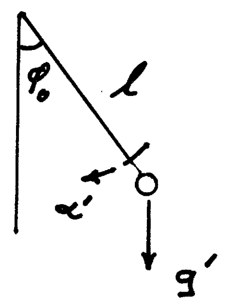
$$= M\frac{w}{2}\left(\frac{3A}{w}\right)$$
$$F = \frac{3}{2}MA$$

**8.3**




For the pendulum to point continually toward the earth, its angular acceleration $\alpha'$ must be equal and opposite to the angular acceleration of its point of support with respect to the center of the earth, $\alpha = \frac{a}{R}$.

In the accelerating system, the pendulum starts to swing in an effective field $g' = \sqrt{g^2 + a^2}$ from an initial angular displacement $\varphi_0 = \arctan \frac{a}{g}$. If the effective length of the equivalent simple pendulum is $l$ and its mass $m$, then the torque is




$\tau' = mg'l \sin \varphi_0$ and

$$\alpha' = \tau'/I_0 = mg'l \sin\varphi_0 / ml^2$$
$$= g' \sin\varphi_0 / l$$

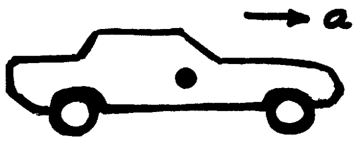
But $g' \sin\varphi_0 = a$, so

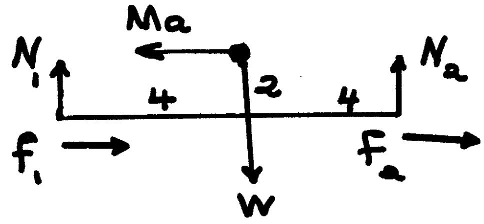
$\alpha' = a/l$

In order for the pendulum to point to the center of the earth we require $\alpha' = \alpha$ or $\frac{a}{l} = \frac{a}{R}$. Thus $l = R$.

$$\text{Period} = T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{R_e}{g}} = 88 \text{ minutes}.$$

8.4




$N_1, N_2$ are the normal forces on the wheels, $f_1$ and $f_2$ are the frictional forces, and $Ma$ is the fictitious force.

(a) Taking torque about the back wheels at the point of contact with the road, we have

$$2Ma + 8N_2 - 4Mg = 0$$
$$N_2 = \tfrac{1}{2}Mg - \tfrac{1}{4}Ma$$

For $N_2 = 0$, $a = 2g = 64\ \text{ft/sec}^2$

(b) If $a = -g$ (with direction as shown) then
$N_2 = \tfrac{1}{2}Mg - \tfrac{1}{4}M(-g) = 3/4\ Mg = 2400$ lbs.
Since $N_1 + N_2 = Mg$, $N_1 = 800$ lbs.

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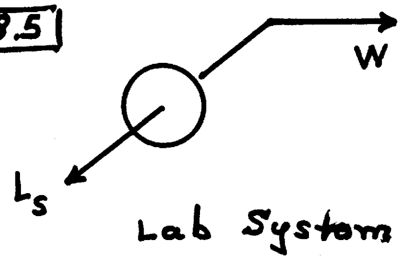
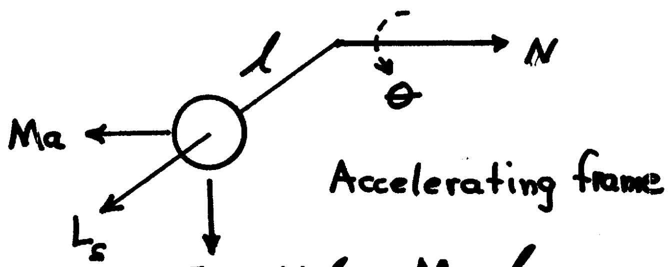
8.5




$\tau_0 = Nl = Mal$

In the accelerating system, gyroscope sees an effective gravitational field $-\vec{a}$ and precesses under the torque

$$\tau_0 = N\ell = Ma\ell$$

$$|\vec{\tau}_0| = |d\vec{L}/dt| = L_s\dot{\theta} = I_0\omega_s\dot{\theta}$$

$$\frac{d\theta}{dt} = \frac{Ma\ell}{I_0\omega_s} = \frac{M\ell}{I_0\omega_s}\frac{dv}{dt}$$

Integrating from rest, we obtain

$$v = \frac{I_0\omega_s}{M\ell}\theta$$

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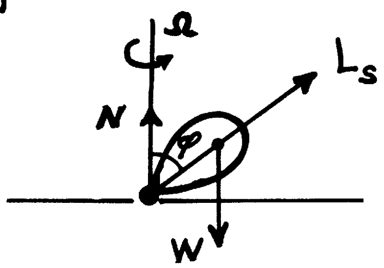
$\boxed{8.6}$




Torque equation about CM:

$$\ell N\cos\varphi = \left(\frac{d}{dt}L_s\right)_{Horizontal}$$

$$(L_s)_{Horizontal} = L_s\cos\varphi$$

$$N = W$$

Hence $\ell W\cos\varphi = L_s\cos\varphi\,\Omega$

(a) precession rate is $\Omega = \dfrac{\ell W}{L_s}$

(b) If the elevator is accelerating down at rate $2g$, then $g_{effective} = g - 2g = -g$ (ie it is up). Then

$$\Omega' = -\frac{\ell W}{L_s}$$

Top reverses its direction of precession

$\boxed{8.7}$ $g_{equator} = g - R_e\Omega_e^2$

$g_{pole} = g$

$\Delta g = R_e\Omega_e^2$ where $\Omega_e = \dfrac{2\pi \text{ rad/day}}{8.64\times10^4 \text{ s/day}} = 7.26\times10^{-5}$ rad/s

$R_e = 6.37\times10^6$ m

$\Rightarrow \Delta g = 3.45\times10^{-2}$ m/s$^2$ or $\dfrac{\Delta g}{g} = 3.52\times10^{-3}$

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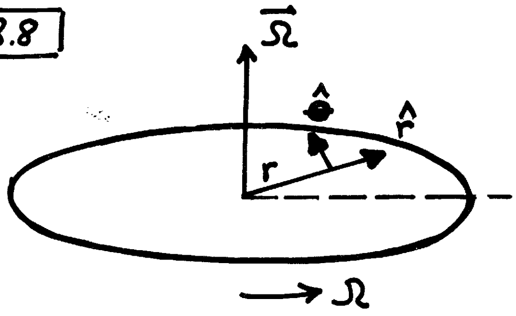
$\boxed{8.8}$




$\vec{V}_{inertial} = \vec{V}_{rot} + \vec{\Omega}\times\vec{r}$

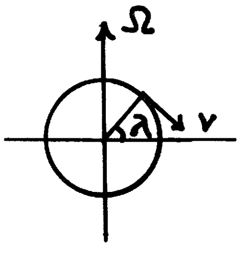
In rotating system, velocity instantaneously lies along $\vec{r}$. Hence $\vec{V}_{rot} = \dot{r}\,\hat{r}$

Also, $\vec{\Omega}\times\vec{r} = \Omega r\,\hat{\theta} = \dot{\theta}\,r\,\hat{\theta}$

Therefore $\vec{V}_{inertial} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$

---

$\boxed{8.9}$ The Coriolis force is




$F_{cor} = -2M\vec{\Omega}\times\vec{V}$

(a) $F_{cor} = 2M\Omega v \sin\lambda$

$\Omega = \dfrac{2\pi \text{ rad/day}}{8.64\times10^4 \text{ s/day}} = 7.26\times10^{-5}$ rad/s

$v = 60\ \text{mi/h} = 88\ \text{ft/sec}$

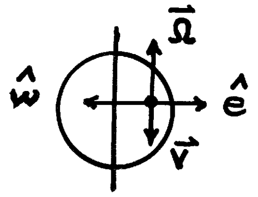
$\sin\lambda = \sin 60^\circ = 0.866$

$$\frac{F_{cor}}{W} = \frac{2M\Omega v \sin\lambda}{Mg} = \frac{2\Omega v\sin\lambda}{g}$$

$$F_{cor} = \frac{2\times 7.26\times 10^{-5}\ \text{r/s} \times 88\ \text{ft/s} \times 0.866 \times (400\times 2000)\,\text{lbs}}{32\ \text{ft/s}^2}$$

$$= 276\ \text{lbs.}$$

(b) $\vec{\Omega}\times\vec{v}$ is directed along the unit vector $\hat{e}$, for east. The coriolis force on the train is directed along $-\hat{e}$, ie toward the west. The reaction force on the tracks is directed oppositely – the tracks are pushed east as the train is pushed west:

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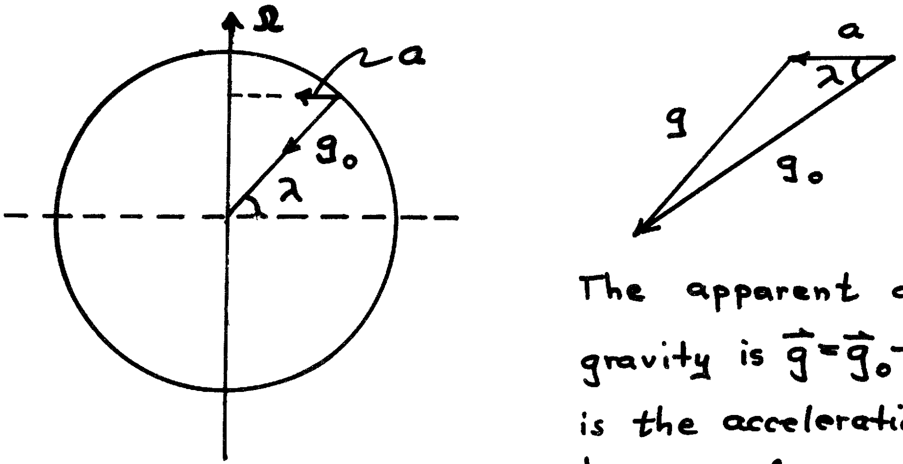
$\boxed{8.10}$




The apparent acceleration of gravity is $\vec{g} = \vec{g}_o - \vec{a}$, where $\vec{a}$ is the acceleration of the local reference frame.

Using the law of cosines,

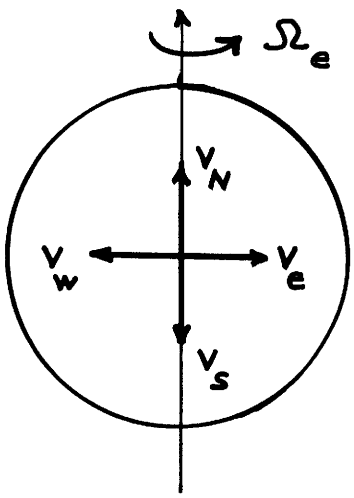
$$g^2 = g_o^2 + a^2 - 2ag_o\cos\lambda$$
$$a = \Omega^2 R_e \cos\lambda$$
$$g^2 = g_o^2 + (\Omega^2 R_e)^2\cos^2\lambda - 2\Omega^2 R_e g_o \cos^2\lambda$$

Let $x = \Omega^2 R_e/g_o$

$$g^2 = g_o\left(1 + x^2\cos^2\lambda - 2x\cos^2\lambda\right)^{1/2} = g_o\left(1 - (2x - x^2)\cos^2\lambda\right)^{\frac{1}{2}}$$

---

$\boxed{8.11}$




The velocity dependent fictitious force is $\vec{F}_{fict} = -2m\vec{\Omega}\times\vec{v}$

The apparent change in gravity is the normal component of $F_{fict}/m$

(a) East : $\vec{\Omega}\times\vec{v}$ points radially out

$\Rightarrow$ $\dfrac{F_{fict}}{m} = 2\Omega v$ points radially in

Therefore $g_{apparent}$ is decreased.

$$\frac{\Delta g}{g} = -\frac{F_{fict}}{mg} = -\frac{2\Omega v}{g}$$

$$\Omega = \frac{2\pi \text{ rad}}{24 \text{ hrs}} = 7.26\times10^{-5} \text{ rad/s}$$

$v = 200$ miles/hr $= 294$ ft/s

$$\frac{\Delta g}{g} = \frac{-2 \times 7.26 \times 10^{-5}\ \text{rad/s} \times 294\ \text{ft/s}}{32\ \text{ft/s}^2}$$

$$= -1.33 \times 10^{-3}$$

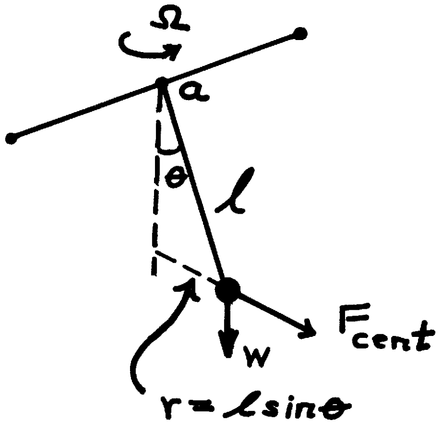
(b) West: sign is reversed. $\frac{\Delta g}{g} = +1.33 \times 10^{-3}$

(c) South: $\vec{\Omega}$ and $\vec{v}$ are parallel. No change in $g$

(d) North: Again, $\Delta g = 0$

---

$\boxed{8.12}$ In the rotating system there is a radial force outward.




$F_{cent} = Mr\Omega^2$

The torque equation about the pivot point $\underline{a}$ is

$$W\ell\sin\theta - F_{cent}\,\ell\cos\theta = M\ell^2\ddot{\theta}$$

$$M\ell^2\ddot{\theta} + Mg\ell\sin\theta - M\ell^2\sin\theta\,\Omega^2 = 0$$

where we have used $r = \ell\sin\theta$

For small angles $\sin\theta \cong \theta$, and

$\ddot{\theta} + \left(\frac{g}{\ell} - \Omega^2\right)\theta = 0$ which is the equation for SHM

The solution is

$$\theta = \theta_0 \cos(\omega t + \varphi_0)$$

where $\omega = \sqrt{\frac{g}{l} - \Omega^2}$

Note that if $\Omega^2 > \frac{g}{l}$ the motion is no longer harmonic. The pendulum flies out and oscillates about some equilibrium angle. (The situation is reminiscent of the conical pendulum, example 2.8.)

# Chapter 9

$\boxed{9.1}$

$$E = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) + U(r)$$

$$\frac{dE}{dt} = \frac{1}{2}\mu(2\dot{r}\ddot{r} + 2r\dot{r}\dot{\theta}^2) + \frac{dU}{dr}\dot{r} = 0$$

$$\mu(\dot{r}\ddot{r} + r\dot{r}\dot{\theta}^2) = -\frac{dU}{dr}\dot{r}$$

$$\frac{dU}{dr} = -f(r)$$

$$\mu(\ddot{r} + r\dot{\theta}^2) = f(r) \qquad (\text{Eqn. } 9.7a)$$

$$l = \mu r^2\dot{\theta}$$

$$\frac{dl}{dt} = \mu(2r\dot{r}\dot{\theta} + r^2\ddot{\theta}) = 0$$

$$\mu(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \qquad (\text{Eqn } 9.7b)$$

---

$\boxed{9.2}$

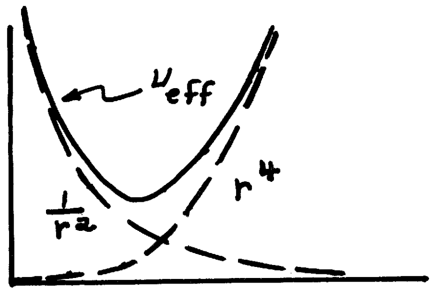
$$\vec{f}(r) = -Ar^3\hat{r}$$

$$U(r) = \frac{1}{4}Ar^4$$

$$U_{eff} = U + \frac{l^2}{2mr^2}$$

$A = 4$ dynes, $\quad l = 10^3$ g cm$^2$/s , $\quad m = 50$g

(a) $U_{eff} = 1r^4 + \dfrac{10^6}{2\times 50 r^2} = r^4 + \dfrac{10^4}{r^2}$

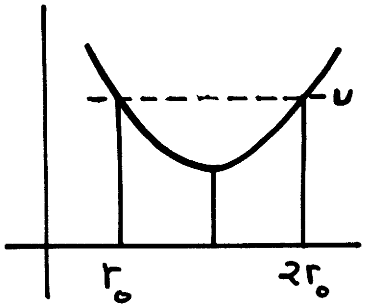
(b) 


Minimum at $4r^3 = \dfrac{2\times 10^4}{r^3}$

$r_{min}^6 = \frac{1}{2}\times 10^4 = 5\times 10^3$

$r_{min} = 4.1$ cm

(c)




$$U(r_o) = U(2r_o)$$
$$r_o^4 + \frac{10^4}{r_o^2} = 16 r_o^4 + \frac{1}{4}\times 10^4 \frac{1}{r_o^2}$$
$$15 r_o^4 = \frac{3}{4}\frac{10^4}{r_o^2}$$
$$r_o^6 = \frac{3}{60}\times 10^4 = 500$$
$$r_o = 2.8 \text{ cm}$$

---

$\boxed{9.3}$ $f(r) = -\frac{2A}{r^3}\hat{r}$ (attractive inverse cube force)

$$U(r) = -\frac{A}{r^2}$$
$$U_{eff} = -\frac{A}{r^2} + \frac{\ell^2}{2\mu r^2}$$

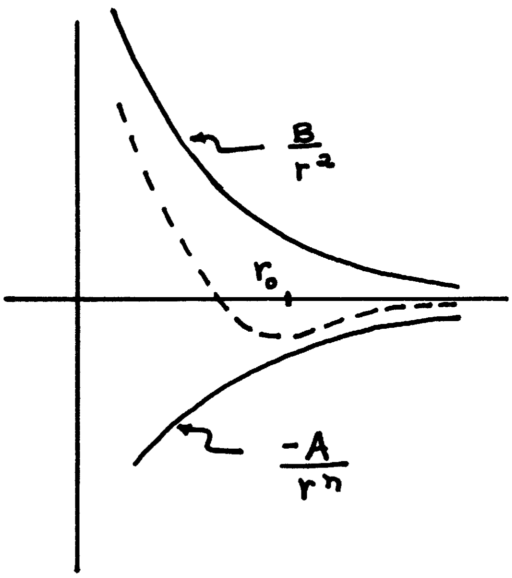
If $\ell^2 = 2\mu A$, $U_{eff} = 0$ and radial motion is like that of a free particle, $r = r_o + vt$

$$\ell^2 = \mu^2 r^4 \dot\theta^2$$
$$\mu^2 r^4 \dot\theta^2 = 2\mu A$$
$$\dot\theta = \left(\frac{2A}{\mu}\right)^{\frac{1}{2}} \frac{1}{r^2}$$
$$\frac{d\theta}{dt} = \left(\frac{2A}{\mu}\right)^{1/2} \frac{1}{r^2}$$
$$\int_{\theta_o}^{\theta} d\theta = \left(\frac{2A}{\mu}\right)^{\frac{1}{2}} \int \frac{dt}{r^2} = \left(\frac{2A}{\mu}\right)^{\frac{1}{2}} \int_{r_o}^{r} \left(\frac{dt}{dr}\right)\frac{dr}{r^2}$$

For free radial motion $\frac{dt}{dr} = \frac{1}{v} = \text{const.}$

$$\theta - \theta_o = \left(\frac{2A}{\mu}\right)^{1/2} \frac{1}{v}\left(\frac{1}{r_o} - \frac{1}{r}\right)$$

**9.4**




$$U_{eff} = U + \frac{\ell^2}{2\mu r^2}$$

$$= -\frac{A}{r^n} + \frac{B}{r^2}, \quad B = \frac{\ell^2}{2\mu}$$

For a stable circular orbit, potential must have a minimum at $r_0$. Hence $U''(r_0) > 0$

$$U' = \frac{nA}{r^{n+1}} - \frac{2B}{r^3}$$

$$U'(r_0) = 0 \Rightarrow \frac{nA}{r_0^{n+1}} = \frac{2B}{r_0^3}$$

$$U''(r_0) = \frac{-n(n+1)A}{r_0^{n+2}} + \frac{6B}{r_0^4} = -\frac{n+1}{r_0}\left(\frac{2B}{r_0^3}\right) + \frac{6B}{r_0^4} > 0$$

$\Rightarrow n+1 < 3$ which requires $n < 2$.

---

**9.5**

(a) For motion in a circular orbit the energy is
$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2$

For equilibrium, $\frac{mv^2}{r} = kr \Rightarrow E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = kr^2$

For $E = 12$ J and $k = 3$ N/m Then $r = \sqrt{E/k} = 2$ m
and $v = \sqrt{\frac{kr^2}{m}} = \sqrt{6}$ m/s

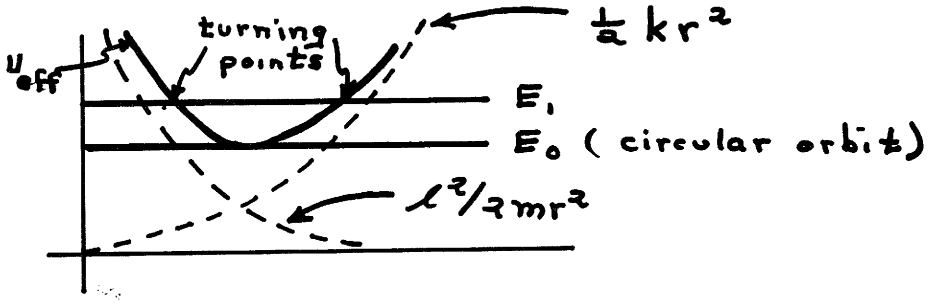
(b) The blow changes the total energy but not the angular momentum or the effective potential.

The initial energy is

$$E_0 = \frac{1}{2}kr^2 + \frac{1}{2}mv^2 = kr^2 = 12\ \mathrm{J}$$

The final energy is

$$E_1 = E_0 + \frac{1}{2}m\dot{r}^2 = 12 + \frac{1}{2} \times 2 \times 1^2 = 13\ \mathrm{J}$$




(c)

$$U_{eff} = \frac{1}{2}kr^2 + \frac{\ell^2}{2mr^2}, \quad \ell = mvr \quad \text{(initial)}$$

$$\ell = mv_0r_0$$

$$U_{eff} = \frac{1}{2}kr^2 + \frac{1}{2}mv_0^2\frac{r_0^2}{r^2}$$

At turning point, $U_{eff} = E_1$

$$\frac{1}{2}kr^2 + \frac{1}{2}mv_0^2\frac{r_0^2}{r^2} = E_1$$

$$\frac{1}{2} \times 3r^2 + \frac{1}{2} \times 2 \times 6 \times \frac{4}{r^2} = 13$$

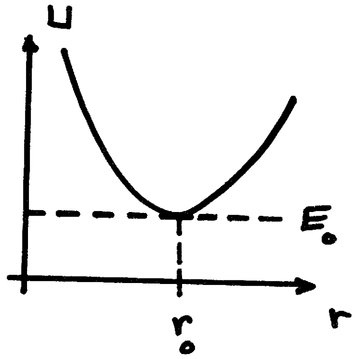
$$\frac{3}{2}r^2 + \frac{24}{r^2} = 13$$

$$r^4 - \frac{26}{3}r^2 + 16 = 0$$

$$r^2 = \frac{1}{2}\left[\frac{26}{3} \pm \sqrt{\left(\frac{26}{3}\right)^2 - 4(16)}\right] = \frac{1}{2}\left[8.65 \pm \sqrt{11.1}\right]$$

$r_1 = 1.63$ m and $r_2 = 2.45$ m

$\boxed{9.6}$




$$F = -Kr^4 \hat{r}$$
$$U_{eff} = \frac{1}{5}Kr^5 + \frac{\ell^2}{2\mu r^2}$$

For circular motion, $E = U_{eff}(r_0)$

$$U'_{eff} = Kr^4 - \frac{\ell^2}{\mu r^3}$$

$$U'_{eff}(r_0) = 0$$
$$Kr_0^4 = \frac{\ell^2}{\mu r_0^3} \quad \Rightarrow \quad r_0 = \left(\frac{\ell^2}{K\mu}\right)^{1/7}$$

$$E_0 = U_{eff}(r_0) = \frac{1}{5}Kr_0^5 + \frac{\ell^2}{2\mu}\frac{1}{r_0^2} = \frac{7}{10}\left(\frac{\ell^2 K^6}{\mu}\right)^{5/7}$$

Frequency of oscillation is $\omega = \sqrt{\dfrac{U''(r_0)}{\mu}}$

$$U'' = 4Kr^3 + \frac{3\ell^2}{\mu r^4}$$
$$U''(r_0) = 4Kr_0^3 + \frac{3\ell^2}{\mu r_0^4} \qquad \text{But } Kr_0^4 = \frac{\ell^2}{\mu r_0^3}, \text{ so}$$
$$U''(r_0) = \frac{7\ell^2}{\mu r_0^4} = 7\frac{\ell^2}{\mu}\left(\frac{K\mu}{\ell^2}\right)^{4/7} = \frac{7K^{4/7}\ell^{6/7}}{\mu^{3/7}}$$
$$\omega = \left(\frac{7K^{4/7}\ell^{6/7}}{\mu}\right)^{\frac{1}{2}}$$

---

$\boxed{9.7}$

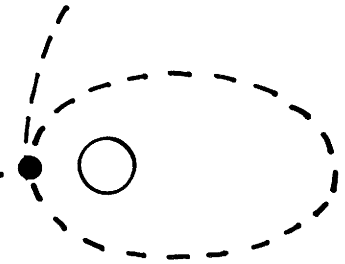
The increase in total energy is $\Delta E = (K+U)_{final} - (K+U)_{initial}$

The potential energy is constant during the firing, assuming that the firing occurs quickly. (This is true for the actual, but not the effective, potential.)

Using $\vec{V}_f = \vec{V}_i + \vec{\Delta V}$

$$\Delta E = K_f - K_i = \frac{1}{2}m(\vec{V}_i + \Delta\vec{V})^2 - \frac{1}{2}mV_i^2 = m\vec{V}_i \cdot \vec{\Delta V} + \frac{1}{2}m\Delta V^2$$

$\Delta E$ is a maximum when $\Delta V$ is parallel to $V_i$ and when $V_i$ is a maximum. The speed is greatest at perigee.

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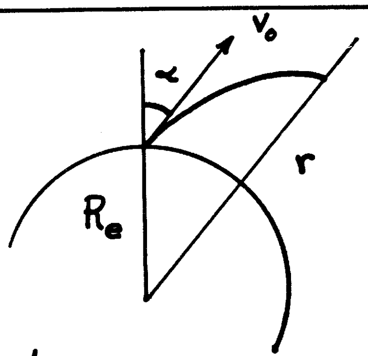
$\boxed{9.8}$ The initial energy is




$$E = U(R_e) + \frac{1}{2}m\dot{r}^2 + \frac{\ell^2}{2mR_e^2}$$
$$= -\frac{GMm}{R_e} + \frac{1}{2}mV_0^2$$

At the top of the trajectory, $\dot{r} = 0$ and

$$E = -GMm\frac{1}{r} + \frac{\ell^2}{2mr^2}$$
$$E = -GMm\frac{1}{r} + \frac{1}{2}mV_0^2\sin^2\alpha\left(\frac{R_e}{r}\right)^2$$

Equating the initial and final energies, we have

$$-\frac{GMm}{R_e} + \frac{1}{2}mV_0^2 = -GMm\frac{1}{r} + \frac{1}{2}mV_0^2\sin^2\alpha\left(\frac{R_e}{r}\right)^2$$

$$-1 + \frac{1}{2}\frac{V_0^2 R_e}{GM} = -\frac{R_e}{r} + \frac{1}{2}\frac{R_e V_0^2}{GM}\sin^2\alpha\left(\frac{R_e}{r}\right)^2$$

we have $\frac{V_0^2 R_e}{GM} = 1$ so that

$$-\frac{1}{a} = -\frac{R_e}{r} + \frac{1}{2}\sin^2\alpha \left(\frac{R_e}{r}\right)^2$$

Let $x = \frac{r}{R_e}$

$$-\frac{1}{2}x^2 = -x + \frac{1}{2}\sin^2\alpha$$

$$x^2 - 2x + \sin^2\alpha = 0$$

$$x = \frac{1}{2}\left[2 \pm \sqrt{4 - 4\sin^2\alpha}\,\right] \qquad (x > 0 \Rightarrow \text{positive root})$$

$$x = 1 + \sqrt{1 - \sin^2\alpha} = 1 + \cos\alpha$$

Hence $r = R_e\,(1 + \cos\alpha)$

---

$\boxed{9.9}$ The period of a satellite depends only on its major axis A. From Eqn. 9.30,

$$A = \left[2\left(\frac{T}{\pi}\right)^2 (M+m)G\right]^{1/3}$$

$$T = 76\text{ years} = 76 \times 3.16 \times 10^7 = 2.4 \times 10^9\text{ s}$$

$$M + m = M_{sun} = 2 \times 10^{30}\text{ kg}$$

$$A = \left[2 \times \left(\frac{2.4 \times 10^9}{\pi}\right)^2 \times 2 \times 10^{30} \times 6.67 \times 10^{-11}\right]^{1/3} = 5.4 \times 10^{12}\text{ m}$$

From Eqn. 9.25

$$r_0 = \frac{1}{2}A\,(1 - \epsilon^2)$$

$$r_0 = \frac{1}{2}(5.4 \times 10^{12}) \times \left[(1 - (0.967)^2\right]$$

Using $1 - \epsilon^2 = (1-\epsilon)(1+\epsilon) \cong 2(1\ \ \epsilon)$ for $\epsilon \cong 1$

$$r_0 = \frac{1}{2}(2 \times 5.4 \times 10^{12} \times .033) = 1.8 \times 10^{11}\text{ m}$$

(a) perihelion $= \frac{r_0}{1+\epsilon} \cong \frac{r_0}{2} = 0.9 \times 10^{11}\text{ m}$

$$\text{aphelion} = \frac{r_0}{1-\epsilon} = \frac{r_0}{.033} = 5.5 \times 10^{12}\, m$$

(b) From Eqn. 9.19 $\ell^2 = r_0 \mu G M m$
where $\mu$ is the reduced mass $\cong m_{comet}$

At perihelion, $\ell = m v r_{min}$

$$m^2 v^2 r_{min}^2 = r_0 m^2 GM$$

$$v = \left(\frac{r_0 G M}{r_{min}^2}\right)^{1/2}$$

Since $r_{min} = \frac{1}{2} r_0$

$$v = \left(\frac{4 G M}{r_0}\right)^{1/2} = \left(\frac{4 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.8 \times 10^{11}}\right)^{1/2} = 5.4 \times 10^{4}\, m/s$$

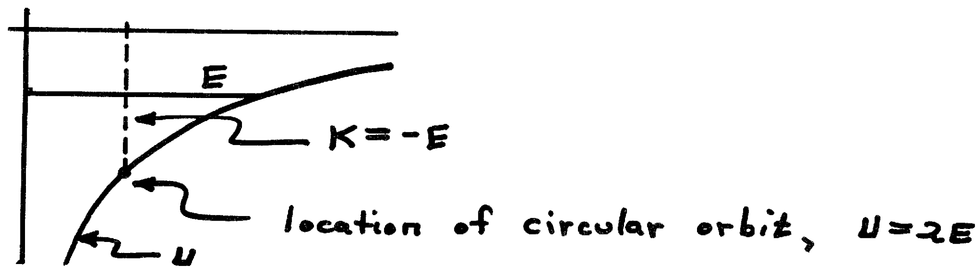
---

$\boxed{9.10}$ In a circular orbit under an attractive force $F = c/r^2$, we have

$$\frac{mv^2}{r} = \frac{c}{r^2} \qquad \text{OR} \qquad \frac{1}{2} m v^2 = \frac{1}{2}\frac{c}{r} = -\frac{1}{2} U(r)$$

(a) The total energy is

$$E = K + U = -\tfrac{1}{2}U + U = \tfrac{1}{2}U = -K = -\frac{1}{2}\frac{c}{r}$$



(b) Work done by friction is $\Delta W = -2\pi r f = \Delta E$

$$\Delta E = \left(\frac{dE}{dr}\right)\Delta r$$
$$\Delta r = \Delta E \Big/ \frac{dE}{dr} = \frac{-2\pi r f}{\frac{1}{2}\frac{C}{r^2}} = \frac{4\pi r^4 f}{C}$$

(c) In circular orbit, $E = -K$

$$\Delta K = -\Delta E = +2\pi r f$$
Friction results in the satellite speeding up!

---

$\boxed{9.11}$ Kepler's third law states $T^2 = \frac{\pi^2}{2(M+m)}\frac{A^3}{G}$

$$M = \frac{\pi^2}{2}\frac{A^3}{T^2}\frac{1}{G}$$

$A =$ major axis $= r_{max} + r_{min} = 1861 + 1838 = 3.699 \times 10^6\ m$
$T = 119$ min $= 7.14 \times 10^3$ seconds
$G = 6.67 \times 10^{-11}$

$\rightarrow M = 7.3 \times 10^{22}$ kg

---

$\boxed{9.12}$ In general, $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$

For a circular orbit, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

and $E = -\frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}\frac{GM}{R_e^2}m\,\frac{R_e^2}{r} = -\frac{1}{2}R_e W \frac{R_e}{r}$

where $W = mg$ = weight at earth's surface.

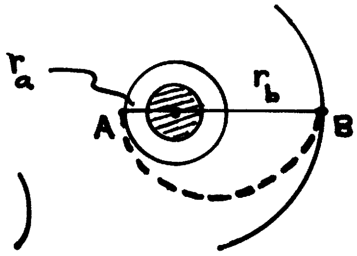
(a) $E_A = -\frac{1}{2} R_e W \left( \frac{R_e}{2 R_e} \right) = -\frac{1}{4} R_e W$

$E_B = -\frac{1}{8} R_e W$

$\Delta E = \left(-\frac{1}{8} + \frac{1}{4}\right) R_e W = \frac{1}{8} R_e W = \frac{1}{8} \times 6.4 \times 10^6 \, m \times 3 \times 10^3 \times 9.8$

$= 2.35 \times 10^{10}$ Joules

(b) The transfer orbit is an ellipse with perigee at A and apogee at B. The semimajor axis is $A = r_a + r_b$.




The energy is

$$E' = -\frac{GMm}{A} = -\frac{GMm}{R_e^2} R_e \left( \frac{1}{\frac{r_a}{R_e} + \frac{r_b}{R_e}} \right)$$

$$E' = -WR_e \left( \frac{1}{\frac{r_a}{R_e} + \frac{r_b}{R_e}} \right) = -WR_e \left( \frac{1}{2+4} \right) = -\frac{1}{6} W R_e$$

The initial speed at A is given by

$\frac{1}{2} m v_0^2 = K_A = -E_A = +\frac{1}{4} R_e mg$

$v_0 = \sqrt{1/2} \sqrt{R_e g}$

The final speed at A is given by

$K_A' + U_A = E'$

$\frac{1}{2} m v'^2 = -\frac{1}{6} mg R_e + \frac{mMG}{2 R_e} = \left(\frac{1}{2} - \frac{1}{6}\right) mg R_e$

$v' = \sqrt{2/3} \sqrt{R_e g}$

Hence

$$\Delta v_A = \left(\sqrt{2/3} - \sqrt{1/2}\right)\sqrt{R_e g} = .108 \times 7.9 \times 10^3\ m/s = 850\ m/s$$

At B the final speed is

$$v_f = \sqrt{2/8}\sqrt{R_e g} = \tfrac{1}{2}\sqrt{R_e g}$$

The initial speed at B is

$$\tfrac{1}{2} m v_B'^2 = E' - U_B = -\tfrac{1}{6} mgR_e + \frac{mMG}{4R_e} = \left(\tfrac{1}{4} - \tfrac{1}{6}\right) mgR_e = \tfrac{1}{12} mgR_e.$$

$$v_B' = \sqrt{1/6}\sqrt{R_e g}$$

Hence

$$\Delta v_B = \left(\tfrac{1}{2} - \sqrt{\tfrac{1}{6}}\right)\sqrt{R_e g} = (0.5 - 0.408) \times 7.9 \times 10^3 = 730\ m/s.$$

# Chapter 10

$\boxed{10.1}$

$$\langle \sin^2(\omega t)\rangle = \frac{1}{T}\int_{t_1}^{t_2=t_1+T} \sin^2\omega t\, dt$$

Using $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$\langle \sin^2(\omega t)\rangle = \frac{1}{2T}\int_{t_1}^{t_1+T}(1-\cos 2\omega t)dt = \frac{1}{2T}\left(t+\frac{1}{2\omega}\sin 2\omega t\right)\Bigg|_{t_1}^{t_1+T}$$

For $T=\frac{2\pi}{\omega}$, $\sin 2\omega(t_1+T) = \sin(2\omega t_1 + 2\pi) = \sin 2\omega t_1$

$$\langle \sin^2(\omega t)\rangle = \frac{1}{2T}\left[t_1+T-t_1+\frac{1}{2\omega}\left(\sin 2\omega t_1 - \sin(2\omega t_1+2\pi)\right)\right] = \frac{1}{2}$$

$$\langle \sin\omega t\cos\omega t\rangle = \frac{1}{T}\int_{t_1}^{t_1+T}\sin\omega t\cos\omega t\, dt = \frac{1}{\omega T}\int_{t_1}^{t_1+T}\sin\omega t\, d(\sin\omega t)$$

$$= \frac{1}{\omega T}\frac{1}{2}\sin^2\omega t\Big|_{t_1}^{t_1+T} = 0$$

---

$\boxed{10.2}$ $\quad Q = \frac{\omega}{\gamma}$, $\quad \omega = 2\pi\,\text{rad/cycle} \times 2\,\text{cycles/sec} = 4\pi\,\text{rad/s}$

$$\gamma = \frac{4\pi\ \text{rad/s}}{60} = 0.21\ \text{s}^{-1}$$

$$b = \gamma m = 0.063\ \text{kg s}^{-1}$$

$$\omega = \sqrt{k/m}$$

$$k = m\omega^2 = 0.3\times(4\pi)^2 = 47.5\ \text{N/m}$$

---

$\boxed{10.3}$

For a damped oscillator

$$x = x_0 e^{-\frac{\gamma}{2}t}\sin\omega_0 t$$

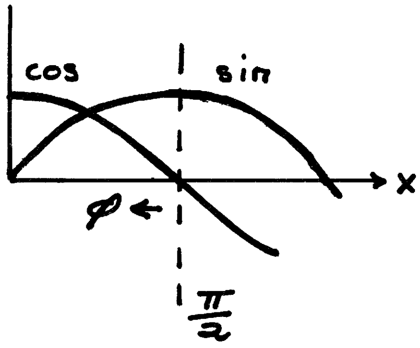
Zero crossings occur at $\omega_0 t = 0, \pi, 2\pi, \ldots n\pi$

Maxima occur when $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = x_0 \left[ \omega_0 \cos\omega_0 t - \frac{\gamma}{2} \sin\omega_0 t \right] e^{-\frac{\gamma}{2}t}$$

Zeros in $\frac{dx}{dt}$ occur when

$$\frac{\cos\omega_0 t}{\sin\omega_0 t} = \frac{\gamma}{2\omega_0} << 1$$




Let $x = \omega_0 t$ and let $\varphi = \frac{\pi}{2} - x$

Then $\cos\omega_0 t = \sin\varphi \cong \varphi$

$\sin\omega_0 t = \cos\varphi \cong 1$

and $\frac{\cos\omega_0 t}{\sin\omega_0 t} \cong \varphi = \frac{\gamma}{2\omega_0} = \frac{1}{2Q}$

---

$\boxed{10.4}$

The motion of a damped harmonic oscillator is given by $x = x_0 e^{-\frac{\gamma}{2}t} \sin\omega_0 t$

Ratio of two maxima is $R = \frac{x_1}{x_2} = \frac{x_0 e^{-\frac{\gamma}{2}t_1} \sin\omega_0 t_1}{x_0 e^{-\frac{\gamma}{2}t_2} \sin\omega_0 t_2}$

For $\omega_0 t_2 = \omega_0 t_1 + 2\pi$, $\sin\omega_0 t_2 = \sin\omega_0 t_1$ and

$$R = e^{-\frac{\gamma}{2}(t_1 - t_2)} = e^{\frac{\gamma}{2}\left(\frac{2\pi}{\omega_0}\right)} = e^{\frac{\gamma\pi}{\omega_0}} = e^{\frac{\pi}{Q}}$$

$\delta = \ln R = \frac{\pi}{Q}$

$\delta = \pi\gamma/\omega_0 \Rightarrow \gamma = \frac{\delta\omega_0}{\pi} = \frac{.02 \times \pi}{\pi} = .02\ s^{-1}$

$b = \gamma m = .02\ s^{-1} \times 5\ kg = 0.1\ kg/s = 0.1\ Ns/m$

$k = \omega_0^2/m = (\pi\ rad/sec)^2/5\ kg = 1.97\ N/m$

$\boxed{10.5}$

Equation of motion is $\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$

For $\gamma = 2\omega_0$, $\ddot{x} + 2\omega_0\dot{x} + \omega_0^2 x = 0$

Take $x = (A+Bt)e^{-\gamma/2\, t} = (A+Bt)e^{-\omega_0 t}$

$$\dot{x} = -\omega_0(A+Bt)e^{-\omega_0 t} + Be^{-\omega_0 t}$$
$$\ddot{x} = \omega_0^2(A+Bt)e^{-\omega_0 t} - 2B\omega_0 e^{-\omega_0 t}$$

$$\ddot{x} + 2\omega_0\dot{x} + \omega_0^2 x = [\omega_0^2(A+Bt) - 2B\omega_0 - 2\omega_0^2(A+Bt) + 2B\omega_0 + \omega_0^2(A+Bt)]e^{-\omega_0 t} = 0$$

Initial conditions are

$x(0) = 0$

$\dot{x}(0) = I/m$

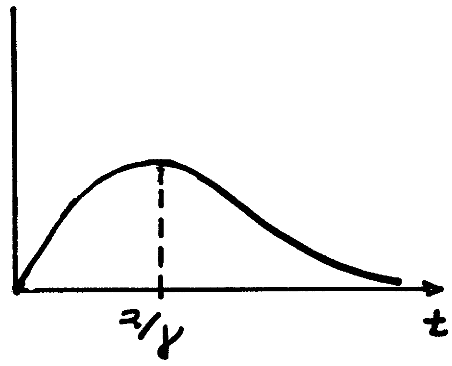
Using $x = (A+Bt)e^{-\frac{\gamma}{2}t}$ we have

$x(0) = 0 \Rightarrow A = 0$

$\dot{x}(0) = I/m \Rightarrow B = I/m$

Hence $x = \frac{I}{m}t\,e^{-\frac{\gamma}{2}t}$

which is maximum at $t = \frac{2}{\gamma}$

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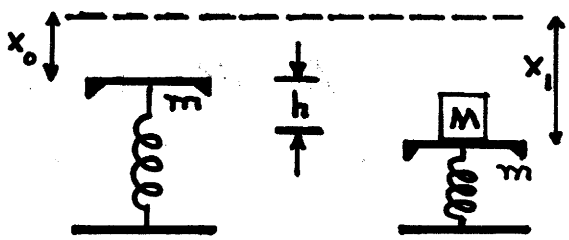
$\boxed{10.6}$ (a)

$x$ = displacement of pan from equilibrium position of spring

$m$ = mass of pan



$M$ = mass of block

$$kx_0 = mg$$
$$kx_1 = (m+M)g$$
$$k(x_1 - x_0) = kh = Mg$$
$$k = \frac{Mg}{h} = \frac{10 \times 9.8}{0.1} = 980 \text{ N/m}$$

(b) The equation of motion for a critically damped oscillator is, from Note 10.1,

$$x = (A+Bt)e^{-\frac{\gamma}{2}t} \qquad \text{where } \gamma = 2\omega_0$$

Now measuring $x$ from the final resting place,

$$x(0) = h = A$$
$$\dot{x}(0) = v_0 = -\frac{\gamma}{2}(A+Bt)e^{-\frac{\gamma}{2}t} + Be^{-\frac{\gamma}{2}t}\Big|_{t=0}$$
Hence $v_0 = -\frac{\gamma}{2}A + B = -\frac{\gamma h}{2} + B$

Numerical values:

$$\gamma = 2\omega_0 = 2\sqrt{\frac{k}{M+m}} = 2\sqrt{\frac{980}{12}} = 18 \text{ s}^{-1}$$
$$A = h = 0.1 \text{ m}$$

For $v_0$, use conservation of momentum when block hits. (Mech energy not conserved here.)

$$MV = (m+M)v_0$$
$$\frac{1}{2}MV^2 = MgH \qquad \text{where } H = \text{distance block falls.}$$

$$v_o = \frac{M}{M+m} V = \frac{M}{m+M}\sqrt{2gH} = \frac{10}{12}\sqrt{2\times 9.8\times\tfrac{1}{2}} = 2.6\ m/s$$

Since motion is down, $v_o = -2.6\ m/s$

$$B = v_o + \tfrac{1}{2}\gamma h = -2.6 + \tfrac{1}{2}\times 0.1\times 18 = -1.7\ m/s$$

Hence $x = [0.1 - 1.7t]\, e^{-9t}$ meters

---

$\boxed{10.7}$ From Eqn. 10.25, the motion of an oscillator under driving force $F = F_o \cos\omega t$ is

$$x = A\cos(\omega t + \varphi) \quad \text{where} \quad \varphi = \arctan\left(\frac{\gamma\omega}{\omega_o^2 - \omega^2}\right)$$

The velocity is

$$v = -A\omega \sin(\omega t + \varphi)$$

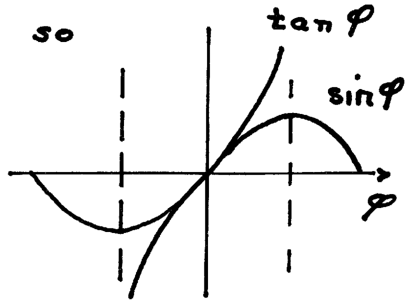
For the velocity to be in phase with the driving force, we require $-\sin(\omega t + \varphi) = \cos\omega t$

$\sin(\omega t + \varphi) = \sin\omega t\cos\varphi + \cos\omega t \sin\varphi$, so

$\sin\varphi = -1$

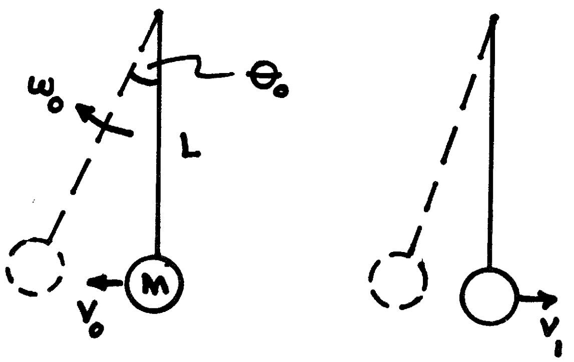
$\varphi = -\frac{\pi}{2}$

$\tan\varphi = -\infty$




This occurs when $\omega = \omega_o$, ie at the resonance frequency of the undamped oscillator. This result is not completely expected, for the amplitude of the driven oscillator is maximum for $\omega$ slightly less than $\omega_o$.

$\boxed{10.8}$ Let the pendulum have speed $v_0$ as it starts to swing up and speed $v_1$ as it returns.




The loss in energy is

$$\Delta E = \frac{1}{2} M (v_0^2 - v_1^2)$$

The motion takes half a cycle ($\pi$ rad.) By definition, $Q$ = fraction of energy lost per radian. Hence $\Delta E = \frac{\pi E}{Q} = \frac{\pi}{Q} \frac{1}{2} M v_0^2$

$$\frac{1}{2} M (v_0^2 - v_1^2) = \frac{\pi}{Q} \frac{1}{2} M v_0^2$$

$$(v_0 - v_1)(v_0 + v_1) = \frac{\pi}{Q} v_0^2$$

Since $v_1$ is close to $v_0$, we have

$$\Delta v = v_0 - v_1 \cong \frac{\pi}{2Q} v_0$$

The required impulse is

$$I = M \Delta v = \frac{\pi}{2Q} M v_0$$

If the motion is given by $\theta = \theta_0 \sin \omega t$, $\omega = \sqrt{\frac{g}{L}}$

$v_0 = L\dot{\theta} = \omega \theta_0 L = \sqrt{gL}\, \theta_0$ and

(a) $$I = \frac{\pi \theta_0}{2Q} M \sqrt{gL}$$

(b) The change in velocity for a given impulse is $\Delta v = I/M$. However, the change in energy is

$$\Delta E = \tfrac{1}{2}M(v+\Delta v)^2 - \tfrac{1}{2}Mv^2 = Mv\Delta v + \tfrac{1}{2}M\Delta v^2$$

The point in the cycle where the impulse occurs can vary due to mechanical imperfections such as play in the mechanism or wear. To minimize changes in the energy transferred, the impulse should occur when v is constant, ie at the bottom of the swing.

Proof:

The energy equation is

$$\tfrac{1}{2}Mv^2 + gL(1-\cos\theta) \simeq \tfrac{1}{2}mv^2 + \tfrac{1}{2}gL\theta^2 = \text{const.}$$

$$vMdv + gL\theta d\theta = 0$$

$$\frac{dv}{d\theta} = \frac{gL}{M}\frac{\theta}{v}$$

$$\frac{dv}{d\theta} = 0 \text{ when } \theta = 0.$$

---

$\boxed{10.9}$

The energy dissipated by a damped oscillator is the energy converted to heat by the viscous retarding force bv.

Take the motion to be given by $x = A\cos(\omega t + \varphi)$

The velocity is then
$v = -A\omega \sin(\omega t + \varphi)$

The power dissipated is $P = v \times bv$
$P = bv^2 = bA^2\omega^2 \sin^2(\omega t + \varphi)$

The average power dissipated $= \overline{P} = \frac{1}{2} bA^2\omega^2$
Hence the average energy dissipated during 1 radian is
$\Delta E = \overline{P} \times \text{time} = P/\omega = \frac{1}{2}\omega b A^2$

For a lightly damped oscillator, $\omega^2 \cong \omega_0^2 = \frac{k}{m}$
The average energy of the oscillator is
$\overline{E} = \frac{1}{2} k \overline{x^2} + \frac{1}{2} m \overline{v^2} = \frac{1}{2}k(\frac{1}{2}A^2) + \frac{1}{2}m(\frac{1}{2}\omega^2 A^2)$
$\overline{E} = \frac{1}{2} k A^2$

$$\text{Hence} \quad \frac{\overline{E}}{\Delta E} = \frac{\frac{1}{2}kA^2}{\frac{1}{2}\omega b A^2} = \frac{k}{\omega b} = \sqrt{\frac{m}{k}}\,\frac{k}{m\gamma} = \frac{\omega_0}{\gamma} = Q.$$

---

$\boxed{10.10}$ The power to the clock from the descending weight equals the power dissipated by friction. If the weight descends distance $L$ in time $T$, then
$\overline{P} = MgL/T$

The energy lost per radian is
$\overline{\Delta E} = \overline{P}/\omega$

The average stored energy is $\overline{E} = \frac{1}{2} m\overline{v^2} + \frac{1}{2} mg\ell \overline{\theta^2}$

The kinetic and potential energies are equal, on the average, and $\bar{E} = \frac{1}{2} mg\ell\theta_0^2$ where $\theta_0$ is the angular amplitude.

$$Q = \frac{\bar{E}}{\Delta E} = \frac{\frac{1}{2} mg\ell\theta_0^2}{\bar{P}}\omega = \frac{\frac{1}{2} mg\ell\theta_0^2}{MgL}\omega T$$

$$Q = \frac{1}{2}\frac{m}{M}\frac{\ell}{L}\theta_0^2\omega T$$

we have the numerical values

$\ell = 0.25 m$

$L = 2 m$

$m = 10^{-2} kg$

$M = 0.2 kg$

$\theta_0 = 0.2$

$\omega = 2\pi$ rad/s

$T = 8.64 \times 10^4$ sec

Hence $Q = \frac{1}{2} \times \frac{10^{-2}}{0.2} \times \frac{0.25}{2} \times (0.2)^2 \times 2\pi \times 8.64 \times 10^4$

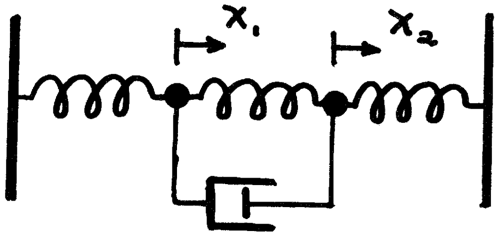
$Q = 68$

The energy to drive the clock one day is

$E = MgL = 0.2 kg \times 9.8 m/s^2 \times 2 m = 4$ joules.

Therefore, the clock would run only 6 hours on a 1 joule battery.

$\boxed{10.11}$ (a)




The force on mass 1 is

$$F_1 = -kx_1 - k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2)$$

The equation of motion is

$$M\ddot{x}_1 = -kx_1 - k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2)$$

Similarly, for mass 2,

$$M\ddot{x}_2 = -kx_2 - k(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$$

(b) Subtracting, we obtain

$$M(\ddot{x}_1 - \ddot{x}_2) = -k(x_1 - x_2) - 2k(x_1 - x_2) - 2b(\dot{x}_1 - \dot{x}_2)$$

Let $y_1 = x_1 - x_2$

$$M\ddot{y}_1 = -3ky_1 - 2b\dot{y}_1 \qquad (1)$$

Adding, we obtain

$$M(\ddot{x}_1 + \ddot{x}_2) = -k(x_1 + x_2)$$

Let $y_2 = x_1 + x_2$

$$M\ddot{y}_2 = -ky_2 \qquad (2)$$

(c) The motion represented by $y_1$ is damped and eventually vanishes. $y_2$ represents free motion which continues undiminished.

To evaluate the initial conditions, we need to express $x_1$ and $x_2$ in terms of $y_1$ and $y_2$.

$x_1 = \frac{1}{2}(y_1 + y_2)$
$x_2 = \frac{1}{2}(y_1 - y_2)$

At $t=0$, $x_1 = x_2 = 0 \Rightarrow y_1(0) = y_2(0) = 0$
At $t=0$, $\dot{x}_1 = v_0$, $\dot{x}_2 = 0$
Hence
$v_0 = \frac{1}{2}(\dot{y}_1 + \dot{y}_2)$
$0 = \frac{1}{2}(\dot{y}_1 - \dot{y}_2)$

We have $\dot{y}_1(0) = \dot{y}_2(0) = v_0$
From Eqn. (2) we find
$y_2 = A\sin\omega t + B\cos\omega t$ where $\omega = \sqrt{\frac{k}{m}}$

Since $y_2(0) = 0$, $B = 0$, and $v_0 = A\omega$

Hence $y_2 = \frac{v_0}{\omega}\sin\omega t$, $x_1 = x_2 = \frac{1}{2}y_2 = \frac{v_0}{2\omega}\sin\omega t$
$\ddot{y}_2 = -\omega v_0 \sin\omega t$ From $M\ddot{y}_2 = -ky_2$, $\omega = \sqrt{k/M}$

$\boxed{10.12}$ (a)
Equation of a free damped oscillator is
$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \qquad (1)$$

For a forced damped oscillator
$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t \qquad (2)$$

Let $x_b$ be a solution to (1) and $x_a$ a solution to (2).

Substituting in (2) and defining $x_c = x_a + x_b$

$$\ddot{x}_c + \gamma\dot{x}_c + \omega_0^2 x_c = \ddot{x}_a + \gamma\dot{x}_a + \omega_0^2 x_a + \ddot{x}_b + \gamma\dot{x}_b + \omega_0^2 x_b$$
$$= \frac{F_0}{m}\cos\omega t + 0$$

Hence $x_c$ also satisfies Eqn (2).

(b) We have $x_a = A\cos(\omega t + \varphi)$ where $A$ and $\varphi$ are given by Eqn. 10.25.

$x_b = Be^{-\gamma t}\cos\omega_1 t + Ce^{-\gamma t}\sin\omega_1 t$ where
$\omega_1^2 = \omega_0^2 - \left(\frac{\gamma}{2}\right)^2$

We require $x_c(0) = \dot{x}_c(0) = 0$

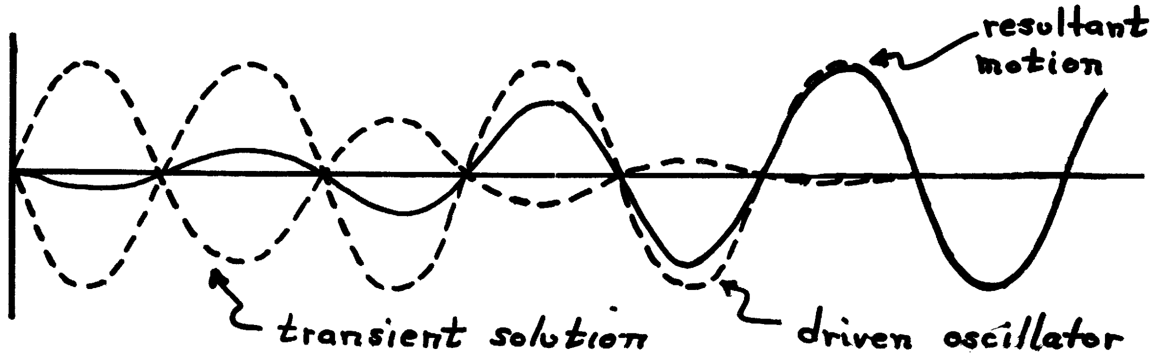
$x_c(0) = 0 \implies A\cos\varphi + B = 0$

$\dot{x}_c(0) = 0 \implies -A\omega\sin\varphi - B\gamma + C\omega_1 = 0$

(c) At resonance $(\omega = \omega_0)$ $A = \dfrac{F_0}{m\omega_0\gamma}$

$\varphi = \arctan\infty = \frac{\pi}{2}$

Hence $B = 0$, $C = \dfrac{A\omega}{\omega_1} = \dfrac{F_0}{m\omega_1\gamma} \approx \dfrac{F_0}{m\omega_0\gamma}$ $\left\{ \begin{array}{l} \text{if } \gamma \\ \ll 2\omega_0 \end{array} \right.$



# Chapter 11

$\boxed{11.1}$

From the analysis on p. 450, the shift in fringes is

$$N = \frac{2l}{\lambda}\frac{v^2}{c^2} \qquad \text{OR} \qquad v = c\sqrt{\frac{N\lambda}{2l}}$$

If $N$ is an upper limit,

$$v \le c\sqrt{\frac{N\lambda}{2l}}$$

Using $N = .01$, $\lambda = 5.9 \times 10^{-7}$ m, $l = 11$ m

$$v \le c\sqrt{\frac{5.9\times10^{-9}}{2\times 11}} = 1.6\times10^{-5}c = 4.8\times10^{3} \text{ m/s}$$

The speed of the earth in orbit around the sun is

$$v = \frac{2\pi \times 1.5\times10^{11} \text{ m/year}}{3.16\times10^{7} \text{ s/year}} = 3.0\times10^{4} \text{ m/s}$$

Michelson and Morley's upper limit was smaller than the speed of the earth around the sun by a factor of $3.0\times10^4 / 4.8\times10^3 = 6.3$

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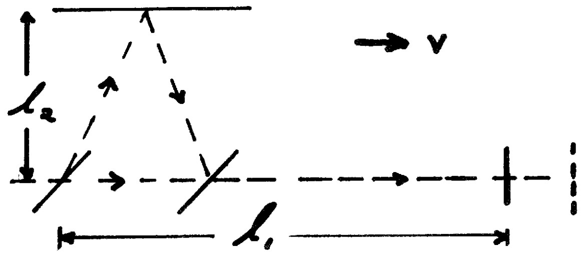
$\boxed{11.2}$ The time for light to complete the round trip along the arm parallel to v, assuming that the velocity of light is added vectorally to the velocity through the



ether, is given by

$$T_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2\ell_1 c}{c^2-v^2} = \frac{2\ell_1}{c}\frac{1}{1-\frac{v^2}{c^2}} \cong \frac{2\ell_1}{c}\left(1+\frac{v^2}{c^2}\right)$$

The time to complete the trip along the arm perpendicular to v is

$$T_2 = 2\tau = \frac{2}{c}\left[\ell_2^2 + (v\tau)^2\right]^{\frac{1}{2}}$$

$$\Rightarrow T_2 = 2\frac{\ell_2}{c}\frac{1}{\sqrt{1-v^2/c^2}} \cong 2\frac{\ell_2}{c}\left(1+\frac{1}{2}v^2/c^2\right)$$

The difference in time is

$$T_1 - T_2 = 2\frac{\ell_1}{c}\left(1+\frac{v^2}{c^2}\right) - 2\frac{\ell_2}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$

If the apparatus is rotated $90°$, the arms are reversed. The change in delay is

$$\Delta T = (T_1' - T_2') - (T_1 - T_2) = T_1' - T_1 - (T_2' - T_2)$$

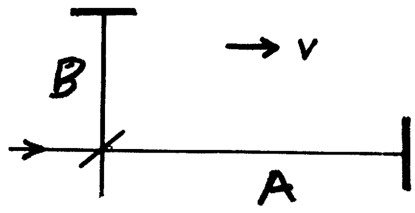
$$= 2\frac{\ell_1}{c}\left[\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) - \left(1+\frac{v^2}{c^2}\right)\right] - 2\frac{\ell_2}{c}\left[\left(1+\frac{v^2}{c^2}\right) - \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\right]$$

$$\Delta T = -\left(\frac{\ell_1+\ell_2}{c}\right)\frac{v^2}{c^2}$$

The fringe shift is therefore

$$N = |\nu \Delta T| = \frac{c}{\lambda}|\Delta T| = \left(\frac{\ell_1+\ell_2}{\lambda}\right)\frac{v^2}{c^2}$$

$\boxed{11.3}$ Suppose that both arms have the same rest length, $\ell_o$, and that in the experiment the lengths are $\ell_A$ and $\ell_B$.




The times to traverse the two arms are

$$T_A \cong \frac{2\ell_A}{c}\left(1+\frac{v^2}{c^2}\right)$$

$$T_B = 2\frac{\ell_B}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$

The difference in the times is

$$\Delta T = T_A - T_B = \frac{2}{c}(\ell_A - \ell_B) + \frac{2}{c}\frac{v^2}{c^2}\left(\ell_A - \frac{1}{2}\ell_B\right)$$

If we take $\ell_A = \ell_o\left(1-\frac{v^2}{2c^2}\right)$ and $\ell_B = \ell_o$,

$$\Delta T = \frac{2}{c}\ell_o\left(-\frac{v^2}{2c^2}\right) + \frac{2}{c}\frac{v^2}{c^2}\left[\ell_o\left(1-\frac{1}{2}\frac{v^2}{c^2}\right) - \frac{1}{2}\ell_o\right]$$

$$\Delta T = 0 + \text{order}\left(\frac{v}{c}\right)^4$$

So to order $\left(\frac{v}{c}\right)^2$ there is no fringe shift. Since $\left(\frac{v}{c}\right)^2 \cong 10^{-8}$ the next terms, $\left(\frac{v}{c}\right)^4 \sim 10^{-16}$, were too small to be detected by the Michelson Morley experiment.

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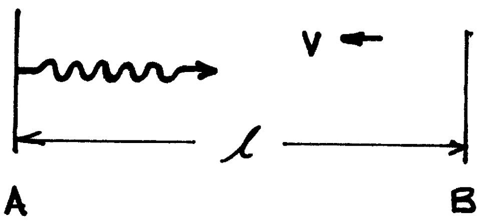
$\boxed{11.4}$ The time of flight to make the round trip "upstream" is $T_{up} = \frac{\ell}{c-v}$



The time to go downstream is

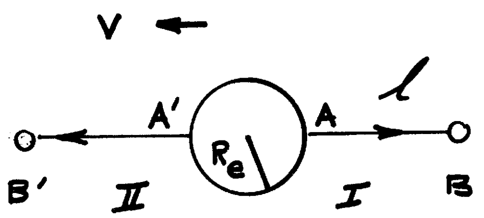
$$T_{Down} = \frac{\ell}{c+V}$$

(a) If the apparatus is rotated, the signal from A to B will show a change in arrival time given by

$$T_{Up} - T_{Down} = \frac{\ell}{c-V} - \frac{\ell}{c+V} \cong \frac{\ell}{c}\left(1 + \frac{V}{c} + \left(\frac{V}{c}\right)^2 + \dots\right) - \frac{\ell}{c}\left(1 - \frac{V}{c} + \left(\frac{V}{c}\right)^2 + \dots\right)$$

$$= 2\frac{\ell}{c}\left(\frac{V}{c}\right) + \text{order } \left(\frac{V}{c}\right)^3$$

(b)




Suppose the ground station sends a signal to the satellite at time $T_I$, in position $I$, and at time $T_{II}$, twelve hours later, in position $II$.

The elapsed time on the ground is $T_A = T_{II} - T_I$.

The signals arrive at the satellite at

$$T_I' = T_I + \frac{\ell}{c-V}$$

$$T_{II}' = T_{II} + \frac{\ell}{c+V}$$

The elapsed time on the satellite is $T_B = T_{II}' - T_I'$

$$T_B = T_{II} - T_I - 2\frac{\ell}{c}\frac{V}{c} = T_A - 2\frac{\ell}{c}\frac{V}{c}$$

The second signal arrives early by the amount

$$\Delta T = 2\frac{l}{c}\frac{v}{c}$$

If the smallest detectable time delay is $\delta T$, the the upper limit to the velocity is

$$v \leq \frac{c^2 \delta T}{2l}$$

Assuming that $\delta T$ is limited by the stability of the clocks, 1 part in $10^{14}$, and that the elapsed time between the signals is 12 hours $= 4.32 \times 10^4$ s, then

$\delta T = 4.3 \times 10^4 / 10^{14} = 4.3 \times 10^{-10}$ s and

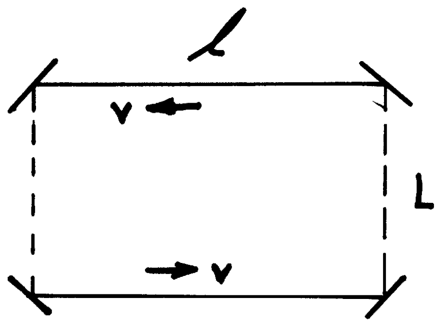
with $l = 5.6\, R_e - R_e = 2.9 \times 10^7$ m we have

$$v \leq \frac{(3 \times 10^8)^2 \times 4.3 \times 10^{-10}}{2 \times 2.9 \times 10^7} = 0.67 \text{ m/s}$$

(In order to achieve this precision, it is necessary to assume that the distance to the spacecraft has not changed appreciably. This requires that the time delay due to change in the spacecraft's radial position be less than $\delta T = 4 \times 10^{-10}$ s, or that its position be known to a precision of $\delta l = c\,\delta T = 0.1$ m. Such precision is possible by the use of continuous Doppler tracking techniques.)

$\boxed{11.5}$

The time to go around the interferometer counterclockwise is




$$T_1 = \frac{2\ell}{c_+} + \frac{2L}{c}$$

where $c_+$ is the velocity of light in the direction of water flow. If the light is carried along with the water, then

$$c_+ = \frac{c}{n} + v$$

The time to go clockwise is

$$T_2 = \frac{2\ell}{c_-} + \frac{2L}{c} \quad \text{where} \quad c_- = \frac{c}{n} - v$$

The difference in time is given by

$$\Delta T = T_2 - T_1 = \frac{2\ell}{\frac{c}{n} - v} - \frac{2\ell}{\frac{c}{n} + v} = \frac{2\ell n}{c}\left(1 + \frac{nv}{c} - \left(1 - \frac{nv}{c}\right)..\right)$$

$$\Delta T = \frac{4\ell n^2 v}{c^2}$$

Hence the fringe shift is

$$N = \nu\,\Delta T = \frac{c}{\lambda}\,\Delta T = 4n^2 \frac{\ell}{\lambda c}\, v$$

# Chapter 12

$\boxed{12.1}$

$\gamma = (1-v^2/c^2)^{-\frac{1}{2}} = (1-.36)^{-\frac{1}{2}} = (.64)^{-\frac{1}{2}} = 1.25$

$x' = \gamma(x - vt)$

$t' = \gamma(t - \frac{xv}{c^2})$

$\frac{v}{c} = 0.6$

(a) $x = 4\,m \qquad t = 0\,s$

$x' = 1.25 \times 4 = 5\,m$

$t' = 1.25\ \frac{(-4)(.6)}{3\times10^8} = -1\times10^{-8}\,s$

(b) $x = 4\,m \qquad t = 1\,s$

$x' = 1.25\,(4 - .6\times3\times10^8) \cong -2.25\times10^8\,m$

$t' = 1.25\,(1 - \frac{4\times.6}{3\times10^8}) \cong 1.25\,s$

(c) $x = 1.8\times10^8\,m \qquad t = 1\,s$

$x' = 1.25\times\ (1.8\times10^8 - 1.8\times10^8) = 0$

$t' = 1.25\times(1 - \frac{1.8\times10^8\times.6}{3\times10^8}) = 1.25\,(1-.36) = 0.80\,s$

(d) $x = 10^9\,m \qquad t = 2\,s$

$x' = 1.25\,(10^9 - 2\times1.8\times10^8) = 8\times10^8\,m$

$t' = 1.25\,(2 - \frac{10^9\times.6}{3\times10^8}) = 0$

---

$\boxed{12.2}$

$x = \gamma(x' + vt')$

$6\times10^8 = \gamma\,(6\times10^8 + 4\,\frac{v}{c}\times3\times10^8)$

$$1 = \gamma\left(1 + 2\frac{v}{c}\right)$$

$$1 + 2\frac{v}{c} = \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

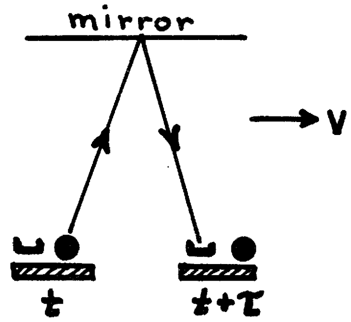
$$1 + 4\frac{v}{c} + 4\frac{v^2}{c^2} = 1 - \frac{v^2}{c^2}$$

$$5\frac{v^2}{c^2} + 4\frac{v}{c} = 0$$

$$\frac{v}{c} = -\frac{4}{5}$$

---

**12.3** The clock moves to the right at velocity $V$, and the light travels the path shown.

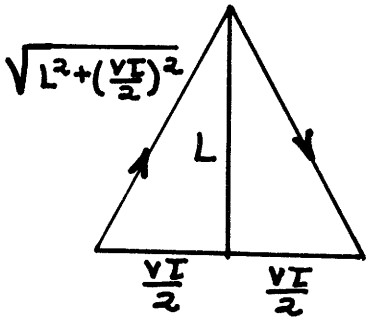



$$\text{Time of travel} = T = \frac{2\sqrt{L^2 + \left(\frac{VT}{2}\right)^2}}{c}$$

$$c^2T^2 = 4\left(L^2 + \left(\frac{VT}{2}\right)^2\right)$$

$$T^2(c^2 - V^2) = 4L^2$$

$$T = \frac{2L}{\sqrt{c^2 - V^2}} = \frac{2L}{c}\frac{1}{\sqrt{1 - V^2/c^2}}$$




Hence

$$T = \frac{T_0}{\sqrt{1 - V^2/c^2}}, \quad \text{since } T_0 = \frac{2L}{c}.$$

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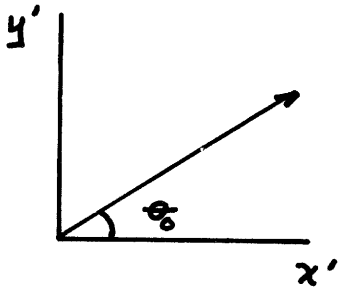
**12.4**

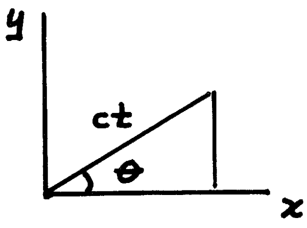
(a) Consider the pulse sent out in $S'$. After 1 second, the coordinates are




$$x' = c\cos\theta_0$$

$$y' = c\sin\theta_0$$

In $S$, the coordinates are



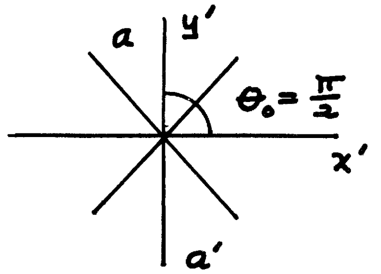

$x = \gamma(x' + vt') = \gamma(c\cos\theta_0 + v)$

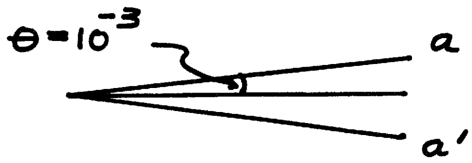
$y = y'$

$t = \gamma(t' + \frac{vx'}{c^2}) = \gamma(1 + \frac{v}{c}\cos\theta_0)$

$$\cos\theta = \frac{x}{ct} = \frac{\gamma(c\cos\theta_0 + v)}{\gamma(c + v\cos\theta_0)} = \frac{\cos\theta_0 + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta_0}$$

(b)




In $S'$, half of radiation is emitted in the region to the right of $y'$ axis, ie. limited by $\theta_0 = \pm\frac{\pi}{2}$. Since $\cos\theta_0 = 0$, we have, from part (a),

$\cos\theta = \frac{0 + v/c}{1 + 0} = \frac{v}{c}$ or $v = c\cos\theta$

For $\theta \ll 1$, $\cos\theta \cong 1 - \frac{1}{2}\theta^2 = 1 - \frac{1}{2}(10^{-3})^2$

Hence $v = c\,(1 - 5\times10^{-7})$

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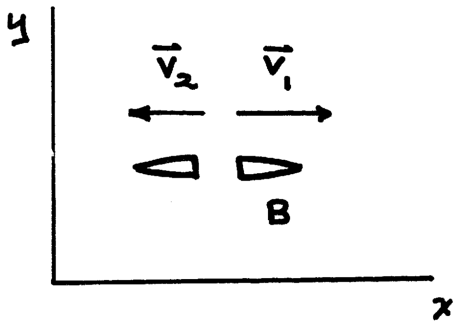
$\boxed{12.5}$




In $S$

$v_1 = v_2 = 0.99c$

In $S'$, moving with B,

$$v_2' = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{2v}{1 + \frac{v^2}{c^2}}$$

Let $v = c(1-x)$ Then

$$v_2' = \frac{2c(1-x)}{1+(1-x)^2} = \frac{2c(1-x)}{2-2x+x^2} = \frac{c(1-x)}{1-x+x^2/2}$$

$$= c(1-x)\left[1+(x-x^2/2)+(x-x^2/2)^2\right]$$

$$= c(1-x)\left[1+x-x^2/2+x^2+\mathcal{O}(x^3)\right]$$

$$= c(1-x^2+x^2/2)$$

$$= c(1-x^2/2)$$

For $x = 0.01$

$$\frac{x^2}{2} = 5\times 10^{-5}$$

$$v_2' = c(1-5\times 10^{-5}) = 0.99995\,c$$

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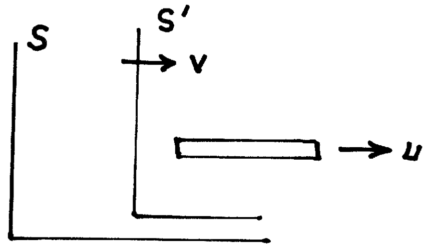
$\boxed{12.6}$




Speed of rod in $S'$ is

$$V = \frac{u-v}{1-\frac{uv}{c^2}}$$

From the discussion of the Lorentz contraction we have

$$l' = l_0\sqrt{1-V^2/c^2} = \frac{l_0}{c}\sqrt{c^2-\left(\frac{u-v}{1-\frac{uv}{c^2}}\right)^2}$$

$$l' = \frac{l_0}{c}\,\frac{1}{c^2-uv}\sqrt{c^2-2uv+(uv)^2/c^2-u^2+2uv-v^2}$$

$$l' = l_0\,\frac{\sqrt{(c^2-u^2)(c^2-v^2)}}{c^2-uv}$$

$\boxed{12.7}$

(a) $\nu' = \nu \sqrt{\dfrac{1-v/c}{1+v/c}}$

$\dfrac{v}{c} = \dfrac{3\times10^6}{3\times10^8} = 10^{-2}$

Since $\lambda = c/\nu$ we have

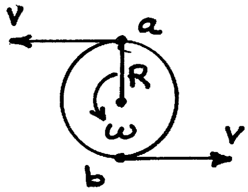
$\lambda' = \lambda \sqrt{\dfrac{1+v/c}{1-v/c}} \cong \lambda\sqrt{1+2v/c} \cong \lambda(1+v/c) = 1.01\lambda$

Hence

$\lambda' - \lambda = \lambda\dfrac{v}{c} = .01\lambda = 6.6\times10^{-9}\,m$

$\lambda' = 662.7\times10^{-9}\,m$

(b) $\lambda_a - \lambda_b = 2\dfrac{v}{c}\lambda$




$\dfrac{v}{c} = \dfrac{\lambda_a - \lambda_b}{2\lambda} = \dfrac{9\times10^{-12}}{2\times6.56\times10^{-7}} = 6.85\times10^{-6}$

$v = 6.85\times10^{-6}\times3\times10^{8} = 2.06\times10^{3}\ m/s$

$V = \omega R = \omega D/2$

$\omega = \dfrac{2V}{D} = \dfrac{4.12\times10^3}{1.4\times10^9} = 2.95\times10^{-6}$

$T = \dfrac{2\pi}{\omega} = 2.13\times10^6\ s \cong 25\ days$

---

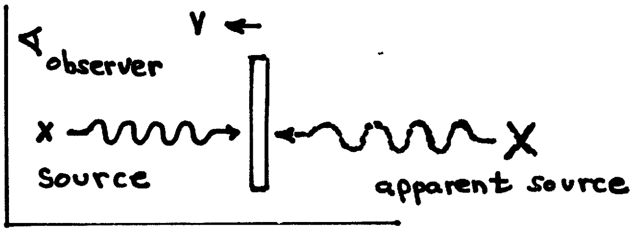
$\boxed{12.8}$


Let frequency in lab system be $\nu_0$. An observer on the mirror sees light arrive at frequency $\nu_0' = \nu_0\sqrt{\dfrac{1+v/c}{1-v/c}}$.

The direction of light is reversed. Observer in lab now sees it at frequency $\nu = \nu_o' \sqrt{\frac{1+v/c}{1-v/c}}$

Hence $\nu = \nu_o \left( \frac{1+v/c}{1-v/c} \right)$.

The apparent source is located behind mirror and is moving toward mirror with velocity v (relative to mirror)




The velocity of the apparent source as seen by the observer is, by the velocity addition formula,

$$V = \frac{2v}{1+\frac{v^2}{c^2}}$$

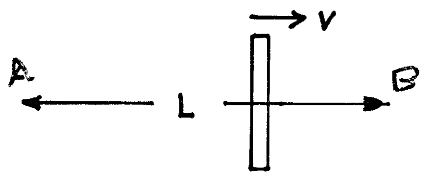
Hence the doppler shift of the apparent source is given by

$$\nu' = \nu_o \sqrt{\frac{1+V/c}{1-V/c}}$$

$$\nu' = \nu_o \sqrt{\frac{1+\frac{2v/c}{1+v^2/c^2}}{1-\frac{2v/c}{1+v^2/c^2}}} = \nu_o \sqrt{\frac{1+\frac{v^2}{c^2}+2\frac{v}{c}}{1+\frac{v^2}{c^2}-\frac{2v}{c}}}$$

$\nu' = \nu_o \left( \frac{1+v/c}{1-v/c} \right)$ as above.

$\boxed{12.9}$




Let

$t_0$ = time spent outside glass

$t_1$ = time spent passing through glass

To find the time for the passage of light through glass, consider two events in system moving with glass.

(a) light enters at $t_a' = 0 \quad x_a' = 0$

(b) light leaves at $t_b' = \frac{nD}{c} \quad x_b' = D$

In lab system, light enters at $t_a = x_a = 0$ and leaves at

$$t_1 = t_b = \gamma\left(t_b' + x_b'\frac{v}{c^2}\right) = \gamma\left(\frac{nD}{c} + \frac{vD}{c^2}\right) = \gamma\frac{D}{c}\left(n + \frac{v}{c}\right)$$

The distance light travels while passing through glass in lab system is

$$x_b = \gamma\left(x_b' + v t_b'\right) = \gamma\left(D + n\frac{v}{c}D\right) = \gamma D\left(1 + n\frac{v}{c}\right)$$

Since the distance traveled in free space is $L - x_b$,

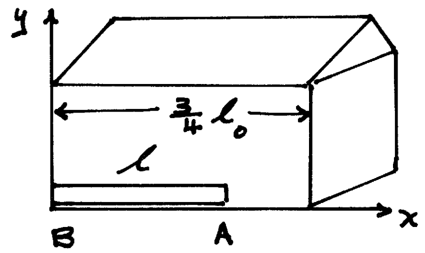
$t_0 = (L - x_b)/c$

$$T = (L - x_b)/c + t_1 = \frac{L}{c} - \gamma\frac{D}{c}\left(1 + n\frac{v}{c}\right) + \gamma\frac{D}{c}\left(n + \frac{v}{c}\right)$$

$$= \frac{L}{c} + \gamma\frac{D}{c}\left(n - 1 + \frac{v}{c}(1 - n)\right) = \frac{L}{c} + \frac{D}{c}\frac{1}{\sqrt{1 - v^2/c^2}}(n-1)\left(1 - \frac{v}{c}\right)$$

$$T = \frac{L}{c} + \frac{D}{c}(n-1)\sqrt{\frac{1 - v/c}{1 + v/c}}$$

$\boxed{12.10}$ Let the farmer shut the door the instant the pole is inside. The end points of the pole are described by two events,




$(x_A = \ell,\ t_A = 0)$
$(x_B = 0,\ t_B = 0)$

From the discussion of Lorentz contraction we have
$x_A - x_B = \ell = \ell_0/\gamma$

Since $\gamma^{-1} = \sqrt{1 - v^2/c^2} = \sqrt{1 - 3/4} = \frac{1}{2}$, it follows that $x_A - x_B = \ell_0/2$ and the pole fits in the barn.

Now look at the same events from a coordinate system moving with the pole:

$$\left.\begin{aligned} x_B' &= \gamma(x_B - vt_B) = 0 \\ t_B' &= \gamma\left(t_B - v\frac{x_B}{c^2}\right) = 0 \end{aligned}\right\} \text{ and } \left\{\begin{aligned} x_A' &= \gamma(x_A - vt_A) = \gamma x_A = \gamma\ell = \ell_0 \\ t_A' &= \gamma\left(t_A - v\frac{x_A}{c^2}\right) = -\frac{\gamma v}{c^2}\ell = -\frac{\ell_0 v}{c^2} \end{aligned}\right.$$

In the runner's system the two ends of the pole do not lie inside at the same instant; from his point of view event A occurs before the farmer shuts the door. Let us examine the situation more closely. From the runner's point of view, at the instant the door is closed, $t_B = t_B' = 0$; the runner observes that the front end of his pole is at $x_A' = \ell_0$, $t_{A'} = 0$, so that it is projecting through the barn wall. How does this event, call it C, look to the farmer? To him, coordinates are

$$x_c = \gamma(x_A' + v t_A') = \gamma \ell_0 = 2\ell_0$$
$$t_c = \gamma\left(t_A' + \frac{v}{c^2} x_A'\right) = 2\ell_0 \frac{v}{c^2}$$

Since $t_c > 0$, the farmer observes that event C occurs after the door is shut. So the farmer and the pole vaulter are both right; the bet can't be settled until they agree on whose coordinate system is to be used.

---

$\boxed{12.11}$ By the velocity addition formula,

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$$

Taking differentials, we obtain (dropping subscripts)

$$du = \frac{du'}{1 + \frac{v}{c^2} u'} - \frac{u' + v}{\left(1 + \frac{v}{c^2} u'\right)^2} \frac{v}{c^2} du'$$

We also have

$$dt = \gamma\left(dt' + v \frac{dx'}{c^2}\right) = \gamma dt' \left(1 + \frac{u' v}{c^2}\right)$$

Dividing,

$$\frac{du}{dt} = \frac{du'}{dt'} \frac{1}{\gamma} \frac{1}{\left(1 + \frac{v}{c^2} u'\right)^2} \left[1 - \frac{u' + v^2/c^2}{\left(1 + \frac{v}{c^2} u'\right)^2}\right]$$

In rest frame, $\frac{du'}{dt'} = a'$ and $u' = 0$ .

Hence $a = a' \frac{1}{\gamma}\left(1 - \frac{v^2}{c^2}\right) = \frac{a'}{\gamma^3}$

$\boxed{12.12}$ Using the results of the last problem, with $a' = a_0$, and denoting the instantaneous velocity of the ship in the initial rest system by $v$, we have

$$\frac{dv}{dt} = a_0\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

$$\int_0^{v_f} \frac{dv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} = \int_0^t a_0 dt = a_0 t$$

Using the result $\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}}$ we obtain

$$c\frac{v/c}{\sqrt{1-v^2/c^2}}\Big|_0^v = \frac{v}{\sqrt{1-v^2/c^2}} = a_0 t$$

$$v = \frac{a_0 t}{\gamma}$$

$$v^2 = (a_0 t)^2\left(1 - \frac{v^2}{c^2}\right) = \frac{(a_0 t)^2}{\left(1 + \left(\frac{a_0 t}{c}\right)^2\right)}$$

---

$\boxed{12.13}$

The time for the trip is

$$T = 2 \times \frac{D}{v} = 2 \times 4.3 \times \frac{c}{v} = 43 \text{ years}$$

$$\Delta T = T\left(\sqrt{1 - v^2/c^2} - 1\right) = T\left(\sqrt{1 - \frac{1}{25}} - 1\right) \cong \frac{T}{50} \cong 10 \text{months}$$

---

$\boxed{12.14}$

$$(\Delta x')^2 = \gamma^2(\Delta x - v\Delta t)^2$$

$$(\Delta y')^2 = \Delta y^2$$

$$(\Delta z')^2 = \Delta z^2$$

$$(\Delta t')^2 = \gamma^2\left(\Delta t - \frac{v}{c^2}\Delta x\right)^2$$

$$\Delta s^2 = -\Delta y^2 - \Delta z^2 - \gamma^2 \Big[ \Delta x^2 - 2v\cancel{\Delta x}\Delta t + v^2\Delta t^2 - c^2\Delta t^2 + 2v\cancel{\Delta x}\Delta t - \frac{v^2}{c^2}\Delta x^2 \Big]$$

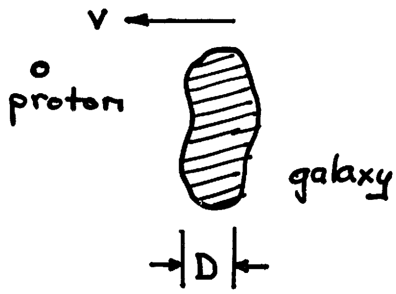
$$\Delta s^2 = -\Delta y^2 - \Delta z^2 - \gamma^2 \Big[ \Delta x^2 \Big(1 - \frac{v^2}{c^2}\Big) - \Delta t^2 (c^2 - v^2) \Big]$$

$$\Delta s^2 = -\Delta y^2 - \Delta z^2 - \Delta x^2 - c^2\Delta t^2$$

# Chapter 13

$\boxed{13.1}$

On a system moving with the proton, the galaxy is moving at speed $V$ and has thickness $D = \frac{D_0}{\gamma}$.




The time to traverse the galaxy is

$$T = \frac{D}{V} = \frac{D_0}{\gamma V} \cong \frac{D_0}{\gamma c}$$

$$E = \gamma m_0 c^2 \quad \text{or} \quad \gamma = \frac{E}{m_0 c^2}$$

$$m_0 c^2 = 1.67\times10^{-27} \times (3\times10^8)^2 / 1.6\times10^{-19} = 9.4\times10^8\,\text{ev} = 940\,\text{MeV}$$

$$\gamma = \frac{10^{13}}{0.94\times10^3} \cong 10^{10}$$

$$D = 10^5 \text{ light years} = 10^5 \times 3\times10^7 \times 3\times10^8 = 9\times10^{20}\,\text{m}$$

$$T = \frac{9\times10^{20}}{10^{10}\times3\times10^8} = 3\times10^2\,\text{s} = 5 \text{ minutes}$$

---

$\boxed{13.2}$

$$K = (\gamma-1)m_0c^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) m_0c^2$$

$$= \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\left(\frac{v^2}{c^2}\right)^2 + \dots - 1\right) m_0c^2$$

$$K \cong \frac{1}{2}m_0v^2 - \frac{1}{2}m_0v^2\left(\frac{3}{4}\frac{v^2}{c^2}\right)$$

(we have used $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x - (-\frac{1}{2})(-\frac{3}{2})\frac{x^2}{2} + \dots$)

Hence

$$\frac{K}{K_{cl}} = 1 - \frac{3}{4}\frac{v^2}{c^2}$$

(a) For $\frac{K}{K_{cl}} = 0.9$, $\frac{3}{4}\frac{v^2}{c^2} = 0.1$ and $\frac{v^2}{c^2} = 0.133$

(b)

K (electron) = $0.9 \times 0.51$ MeV = $0.450$ MeV

K (proton) = $0.9 \times 930$ MeV = $840$ MeV

---

$\boxed{13.3}$

$$K = (m - m_0)c^2 = m_0 c^2 (\gamma - 1)$$

$$\vec{p} = m_0 \vec{v}\, \gamma$$

$$dK = m_0 c^2 d\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = m_0 c^2 \left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \frac{v\,dv}{c^2}\right]$$

$$= m_0 v \gamma^3 dv$$

$$d\vec{p} = m_0 \gamma d\vec{v} + m_0 \vec{v}\, d\left(\frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

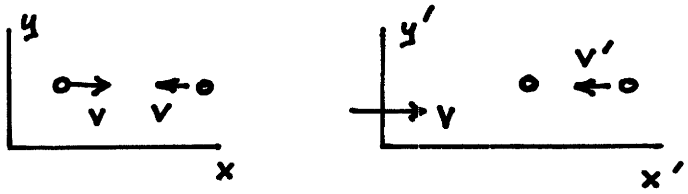
$$= m_0 \gamma d\vec{v} + m_0 \vec{v}\, (\vec{v} \cdot d\vec{v})\, \gamma^3 / c^2$$

$$\vec{v} \cdot d\vec{p} = m_0 \gamma v\,dv + m_0 v^2 (\vec{v} \cdot d\vec{v}) \frac{\gamma^3}{c^2} = m_0 v\,dv \left(\gamma + \frac{v^2}{c^2}\gamma^3\right)$$

$$= m_0 v\,dv\, \gamma \left(1 + \frac{v^2}{c^2}\frac{1}{1 - \frac{v^2}{c^2}}\right)$$

$$= m_0 v\,dv\, \gamma \left(\frac{1}{1 - \frac{v^2}{c^2}}\right) = m_0 v\,dv\, \gamma^3 = dK$$

$\boxed{13.4}$




$$v' = \frac{2v}{1+v^2/c^2}$$

$$E' = m_0 c^2 \gamma$$

$$\gamma = 1 \Big/ \sqrt{1 - v'^2/c^2} = \frac{1}{\sqrt{1 - \frac{4v^2/c^2}{(1+v^2/c^2)^2}}} = \frac{1 + v^2/c^2}{\sqrt{1 + \frac{2v^2}{c^2} + \left(\frac{v^2}{c^2}\right)^2 - \frac{4v^2}{c^2}}}$$

$$\gamma = \frac{1+v^2/c^2}{1-v^2/c^2} \qquad \text{hence} \quad E' = m_0 c^2 \left(\frac{1+v^2/c^2}{1-v^2/c^2}\right)$$

---

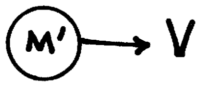
$\boxed{13.5}$

<u>Initially</u>


$$p = m v \gamma$$

$$E = mc^2\gamma + Mc^2$$

<u>Finally</u>




$$p' = M' V \Gamma$$

$$E' = M' c^2 \Gamma$$

Equating, $\quad m v \gamma = M' V \Gamma$

$$mc^2\gamma + Mc^2 = M'c^2\Gamma$$

Dividing, $\quad \dfrac{m v \gamma}{mc^2\gamma + Mc^2} = \dfrac{V}{c^2} \;\Rightarrow\; V = \dfrac{\gamma v m}{\gamma m + M}$

$\boxed{13.6}$ Initial Final

$$\underset{m_o}{\circ}\longrightarrow v \qquad \underset{m_o}{\circ} \qquad\qquad\qquad \underset{m'}{\circ}\longrightarrow V$$

$K = x m_o c^2$

$E_i = mc^2 + m_o c^2 = m_o c^2(\gamma + 1)$ $\qquad E_f = m'c^2 = m_o' c^2\, \Gamma$

$P_i = m_o v \gamma$ $\qquad P_f = m_o' V \Gamma$

$K = x m_o c^2 = (\gamma - 1) m_o c^2 \Rightarrow x = \gamma - 1$

$\gamma = x + 1$

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} = \frac{1}{(x+1)^2}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{(x+1)^2}} = \frac{\sqrt{x^2+2x+1-1}}{x+1} = \frac{\sqrt{x^2+2x}}{x+1}$$

$P_i = P_f \Rightarrow$ $$\frac{m_o c\sqrt{x^2+2x}}{(x+1)}(x+1) = m_o' V \Gamma$$

$E_i = E_f \Rightarrow$ $$m_o c^2 (x+2) = m_o' c^2 \Gamma$$

Dividing, $$\frac{\sqrt{x^2+2x}}{x+2} = \frac{V}{c} \equiv r$$

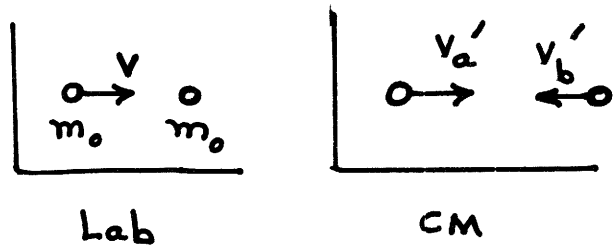
$$V\Gamma = rc\frac{1}{\sqrt{1-r^2}} = \frac{\sqrt{x^2+2x}}{x+2}\,\frac{c}{\sqrt{1-\frac{x^2+2x}{(x+2)^2}}} = \frac{c\sqrt{x^2+2x}}{\sqrt{x^2+4x+4-x^2-2x}}$$

$$V\Gamma = c\frac{\sqrt{x^2+2x}}{\sqrt{2x+4}} = c\sqrt{\frac{x}{2}}$$

From $P_i = P_f$

$$m_o' = m_o c\sqrt{x^2+2x}\,/V\Gamma = m_o\sqrt{\frac{2}{x}}\sqrt{x^2+2x} = m_o\sqrt{2}\sqrt{x+2}$$

13.7




The CM frames moves to the right with speed $V$. The speeds of the particles are

$$v_a' = \frac{v - V}{1 - \frac{vV}{c^2}}$$

$$v_b' = -V$$

For zero momentum in CM frame, $v_a' = -v_b'$

$$\frac{v - V}{1 - \frac{vV}{c^2}} = V$$

$$v - V = V - \frac{vV^2}{c^2}$$

$$V^2 - 2\frac{c^2}{v}V + c^2 = 0$$

$$\Rightarrow V = \frac{c^2}{v} \pm \sqrt{\left(\frac{c^2}{v}\right)^2 - c^2} = \frac{c^2}{v}\left(1 \pm \sqrt{1 - \frac{v^2}{c^2}}\right)$$

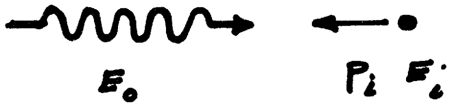
To determine the correct root, take the limit $v \to 0$

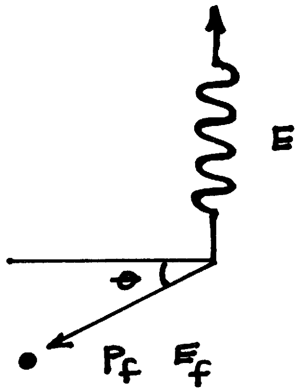
$$V \cong \frac{c^2}{v}\left(1 \pm \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)\right)$$

The negative root gives $V \cong \frac{c^2}{v}\left(\frac{1}{2}\frac{v^2}{c^2}\right) = \frac{v}{2}$ as required

Hence $V = \frac{c^2}{v}\left(1 - \sqrt{1 - v^2/c^2}\right)$

$\boxed{13.8}$ **Initial** **Final**




Conservation of Momentum,

$$E_0/c - p_i = -p_f \cos\theta \qquad \text{(a)}$$

$$E/c = p_f \sin\theta \qquad \text{(b)}$$

Conservation of Energy,

$$E_0 + E_i = E + E_f \qquad \text{(c)}$$

Squaring (a) and (b) and adding yields

$$\frac{E_0^2}{c^2} - \frac{2E_0}{c} p_i + p_i^2 + \frac{E^2}{c^2} = p_f^2 \qquad \text{(d)}$$

From (c)

$$(E_0 + E_i - E)^2 = E_f^2 \qquad \text{(e)}$$

Since $c^2p^2 = E^2 - (m_0c^2)^2$, (d) and (e) give

$$\cancel{E_0^2} - 2E_0 p_i c + \cancel{E_i^2} - \cancel{(m_0c^2)^2} + \cancel{E^2} = \cancel{E_0^2} + \cancel{E_i^2} + \cancel{E^2} + 2E_0E_i - 2E_0E - 2E_iE - \cancel{(m_0c^2)^2}$$

$$2E(E_0 + E_i) = 2E_0(p_ic + E_i)$$

$$E = E_0 \frac{cp_i + E_i}{E_0 + E_i} = E_0 \frac{cp_i/E_i + 1}{1 + E_0/E_i}$$

Using $p_i = m_0 v\gamma = \frac{v}{c^2}E$, we obtain $E = E_0 \dfrac{\frac{v}{c} + 1}{1 + E_0/E_i}$

(b) $E = \frac{hc}{\lambda}$, $\lambda = \frac{hc}{E} \Rightarrow \Delta\lambda = -\frac{hc}{E^2}\Delta E$

$$\frac{\Delta\lambda}{\lambda} = -\frac{\Delta E}{E} = -\left(\frac{E - E_0}{E}\right) = -1 + \frac{E_0}{E}$$

$$= -1 + \frac{\left(1 + \frac{E_0}{E_i}\right)}{\left(V/c + 1\right)} = \frac{-\frac{V}{c} + \frac{E_0}{E_i}}{V/c + 1}$$

$$V/c = 6 \times 10^{-3}, \quad \frac{E_0}{E_i} = \frac{hc}{\lambda_0}\frac{1}{m_0 c^2} = \frac{2.426 \times 10^{-12}}{0.711 \times 10^{-10}} = 34 \times 10^{-3}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{-6 \times 10^{-3} + 34 \times 10^{-3}}{1.006} = 28 \times 10^{-3}$$

$$\Delta\lambda = (0.711 \times 10^{-10})(28 \times 10^{-3}) = 20 \times 10^{-12}\ \text{m}$$

$$= 0.020\ \text{Å}$$

---

$\boxed{13.9}$ $F_{solar} = (\text{solar constant})\,\pi R_e^2 / c$

$$= \frac{1.4 \times 10^3\ \text{W/m}^2 \times \pi \times (6.4 \times 10^6\ \text{m})^2}{3 \times 10^8\ \text{m/s}} = 6.0 \times 10^8\ \text{N}$$

$$F_{grav} = \frac{G M_{sun} m_e}{R^2} = \frac{(6.7 \times 10^{-11})(1.96 \times 10^{30})(6.0 \times 10^{24})}{(1.5 \times 10^{11})^2}$$

$$= 3.5 \times 10^{22}\ \text{N} \sim 6 \times 10^{13}\ F_{solar}$$

---

$$\rho = 5 \times 10^3\ \text{kg/m}^3, \quad m = \rho \tfrac{4}{3}\pi r^3$$

$$F_{sol} = \frac{(1.4 \times 10^3)\pi r^2 R_{eorb}^2}{c R^2} \geq F_{grav} = \frac{\rho G M_s \frac{4}{3}\pi r^3}{R^2}$$

$$r \leq \frac{3}{4}\frac{(1.4 \times 10^3) R_{eorb}^2}{\rho c G M_s} = 1.2 \times 10^{-7}\ \text{m}$$

$\boxed{13.10}$ Assume that the light is reflected from the sphere so that

$$F = 2\ \frac{\text{Power}}{c} = \frac{2\times10^3}{3\times10^8} = 6.7\times10^{-6}\,N$$

At equilibrium, $F = mg$

$$m = \frac{6.7\times10^{-6}}{9.8} = 6.8\times10^{-7}\,kg = 6.8\times10^{-4}\,g$$

$$m = \frac{4}{3}\pi r^3 \rho\ , \quad \rho = \text{dencity} = 2.7\,g/cm^3$$

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3\times6.8\times10^{-4}}{4\pi\times2.7}\right)^{1/3} = \left(6.0\times10^{-5}\right)^{1/3}$$

$$\Rightarrow r = 3.9\times10^{-3}\,cm$$

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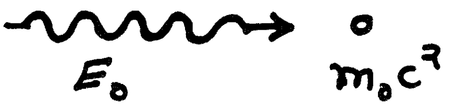
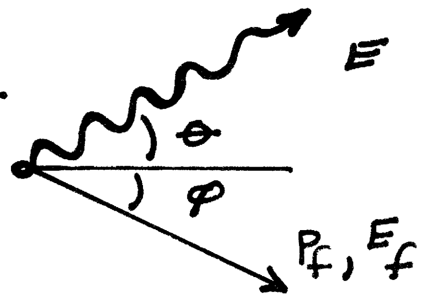
$\boxed{13.11}$





$$p_f c \cos\varphi + E\cos\theta = E_0 \quad (a)$$
$$p_f c \sin\varphi = E\sin\theta \quad (b)$$
$$E_0 + m_0c^2 = E + E_f \quad (c)$$

(a), (b) give $\cot\varphi = \frac{E_0}{E\sin\theta} - \cot\theta$ (d)

and $(p_f c)^2 = (E_0 - E\cos\theta)^2 + E^2\sin^2\theta$ (e)

(c) gives

$$(E_0 - E + m_0c^2)^2 = E_f^2 = (p_f c)^2 + (m_0c^2)^2 \quad (f)$$

Inserting (e) into (f) yields:

$$\cancel{E_0^2} - 2E_0E + \cancel{E^2} + 2(E_0 - E)m_0c^2 + \cancel{(m_0c^2)^2} = \cancel{E_0^2} - 2E_0E\cos\theta + \cancel{E^2} + \cancel{(m_0c^2)^2}$$

$$E_0E(1-\cos\theta) = (E_0 - E)m_0c^2$$

$$E_0(1-\cos\theta) = \left(\frac{E_0}{E} - 1\right)m_0c^2$$

$$\frac{E_0}{E} = \frac{E_0}{m_0c^2}(1-\cos\theta) + 1 \qquad (g)$$

Inserting (g) into (d) yields

$$\cot\varphi = \frac{E_0}{m_0c^2}\,\frac{1-\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - \cot\theta$$

$$\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

$$\frac{1}{\sin\theta} - \cot\theta = \tan\frac{\theta}{2}$$

$$\cot\varphi = \frac{E_0}{m_0c^2}\tan\frac{\theta}{2} + \tan\frac{\theta}{2} = \tan\frac{\theta}{2}\left(1 + \frac{E_0}{m_0c^2}\right)$$

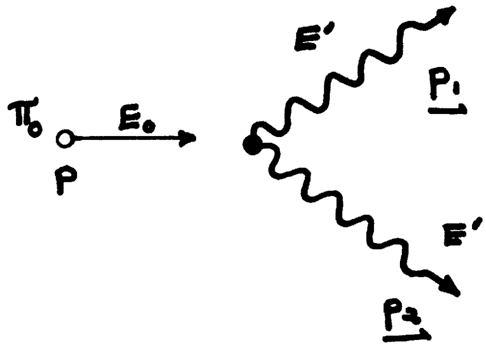
# Chapter 14

$\boxed{14.1}$

$\underline{P} = \Gamma M_0 c\ (\frac{v}{c}, 0, 0, i)$

$\underline{P_1} = \frac{E'}{c}\ (\cos\theta, \sin\theta, 0, i)$

$\underline{P_2} = \frac{E'}{c}\ (\cos\theta, -\sin\theta, 0, i)$




(a) $\underline{P} = \underline{P_1} + \underline{P_2}$

$\Gamma M_0 V = \frac{2E'}{c} \cos\theta$

$\Gamma \frac{v}{c} = \frac{2E'}{M_0 c^2} \cos\theta$

(b) $\Gamma M_0 c = 2 \frac{E'}{c}$

$\Gamma = \frac{2E'}{M_0 c^2}$

Let

$x = \frac{E'}{M_0 c^2} = \frac{1}{1.35}$

From (b)

$\Gamma = 2x$

$1 - \frac{v^2}{c^2} = (\frac{1}{2x})^2$

$\frac{v^2}{c^2} = 1 - (\frac{1.35}{2})^2 = .546$

$\frac{v}{c} = 0.74$

From (a)

$\cos\theta = \Gamma \frac{v}{c} \frac{1}{2x} = \frac{v}{c} = 0.74$

$\theta = 42°$

---

$\boxed{14.2}$ $\gamma + p \rightarrow p + \pi_0$

$\underline{P_1} + \underline{P_2} = \underline{P_3} + \underline{P_4}$

$P_1^2 + P_2^2 + 2\underline{P_1} \cdot \underline{P_2} = (\underline{P_3} + \underline{P_4})^2$

In lab frame $\underline{P_1} = \frac{E_\gamma}{c}(1, 0, 0, i)$ and $\underline{P_2} = M_p c\ (0, 0, 0, i)$

$$\text{l.h.s.} = 0 - (M_p c)^2 - 2 M_p E_\gamma$$

Evaluate r.h.s. in c of m frame :

$$(\underline{p_3} + \underline{p_4})^2 = -(M_p + M_{\pi_0})^2 c^2$$

so $(M_p c^2)^2 + 2 M_p c^2 E_\gamma = (M_p c^2)^2 + (M_{\pi_0} c^2)^2 + 2 M_p c^2 M_{\pi_0} c^2$

$$E_\gamma = M_{\pi_0} c^2 \left(1 + \frac{M_{\pi_0} c^2}{2 M_p c^2}\right) = 135\,\text{MeV}\left(1 + \frac{135}{2 \times 938}\right)$$

$$= 144\ \text{MeV}$$

---

$\boxed{14.3}$ $\quad \gamma + e^- \longrightarrow e^- + (e^- + e^+)$

$$(\underline{p_1} + \underline{p_2})^2 = (\underline{p_3} + \underline{p_4} + \underline{p_5})^2$$

$$(\underline{p_1} + \underline{p_2})^2 = \left[ \frac{E_0}{c}(1, 0, 0, i) + m_e c\,(0, 0, 0, i) \right]^2$$

$$= \left(\frac{E_0}{c}\right)^2 - \left(\frac{E_0}{c} + m_e c\right)^2 = -2 m_e E_0 - (m_e c)^2$$

evaluate r.h.s. in c of m frame :

$$(p_3 + p_4 + p_5)^2 = -(3 m_e c)^2$$

Hence $\quad 2 m_e E_0 + (m_e c)^2 = 9 (m_e c)^2$

$$\Rightarrow E_0 = 4 m_e c^2 = 4 \times 0.51\ \text{MeV} = 2.04\ \text{MeV}$$

---

$\boxed{14.4}$ $\quad \underline{P} = \underline{p_1} + \underline{p_2}$

$\underline{P} = Mc\,(0, 0, 0, i) \qquad \underline{p_1} = \left(\vec{p}, 0, 0, i\frac{E_1}{c}\right)$

$\underline{p_2} = \left(-\vec{p}, 0, 0, i\frac{E_2}{c}\right)$

$$Mc = (E_1 + E_2)/c$$

$$p^2 = p_1^2 + p_2^2 + 2\underrightarrow{p_1} \cdot \underrightarrow{p_2}$$

$$-(Mc)^2 = -(m_1 c)^2 - (m_2 c)^2 - 2\left(p^2 + \frac{E_1 E_2}{c^2}\right)$$

Use $p_1^2 = \left(\frac{E_1}{c}\right)^2 - (m_1 c)^2$

$E_2$

$$(Mc)^2 = (m_1 c)^2 + (m_2 c)^2 + 2\left(\frac{E_1}{c}\right)^2 - 2(m_1 c)^2 + 2\frac{E_1}{c^2}(Mc^2 - E_1)$$

$$= -(m_1 c)^2 + (m_2 c)^2 + 2\frac{E_1}{c^2} M c^2$$

$$E_1 = \frac{(M^2 + m_1^2 - m_2^2)c^2}{2M}$$ and, by interchanging subscripts,

$$E_2 = \frac{(M^2 - m_1^2 + m_2^2)c^2}{2M}$$

---

$\boxed{14.5}$

$$\underset{M_1}{\longrightarrow} \qquad \overset{\circ}{M_2} \qquad\qquad \underset{M_3}{\longrightarrow} \qquad \underset{M_4}{\longrightarrow}$$

$$\underrightarrow{p_1} + \underrightarrow{p_2} = \underrightarrow{p_3} + \underrightarrow{p_4}$$

$$\underrightarrow{p_1} = M_1 \gamma (v, 0, 0, ic) \qquad p_1^2 = -(M_1 c)^2$$

$$\underrightarrow{p_2} = M_2(0, 0, 0, ic) \qquad p_2^2 = -(M_2 c)^2$$

At threshold, $(\underrightarrow{p_3} + \underrightarrow{p_4})^2 = -(M_3 + M_4)^2 c^2$

$$p_1^2 + p_2^2 + 2\underrightarrow{p_1} \cdot \underrightarrow{p_2} = (p_3 + p_4)^2$$

$$-(M_1 c)^2 - (M_2 c)^2 - 2M_1 M_2 \gamma c^2 = -(M_3 + M_4)^2 c^2$$

$$M_3 + M_4 = M_1 + M_2 + Q/c^2$$

$$M_1^2 + M_2^2 + 2M_1M_2\gamma = (M_1 + M_2 + Q/c^2)^2$$
$$= M_1^2 + M_2^2 + 2M_1M_2 + 2(M_1+M_2)\frac{Q}{c^2} + \left(\frac{Q}{c^2}\right)^2$$

$$2M_1M_2(\gamma - 1) = 2(M_1+M_2)\frac{Q}{c^2} + \left(\frac{Q}{c^2}\right)^2$$

$$K_1 = M_1(\gamma-1)c^2 = \frac{M_1+M_2}{M_2}Q + \frac{1}{2M_2c^2}Q^2$$

---

$\boxed{14.6}$

The initial four momentum is

$\underline{P_0} = M_0(0, 0, 0, ic)$

The final four momentum is

$\underline{P_r} = \gamma M_f(v, 0, 0, ic)$

(a) By conservation of momentum, the exhaust carries away total momentum $\vec{P}_e = -\gamma M_f \vec{v}$

The energy of the exhaust is given by

$E_e = P_e c$

Hence

$\underline{P_e} = \gamma M_f v(-1, 0, 0, i)$

(b) Conservation of four momentum yields

$\underline{P_0} = \underline{P_r} + \underline{P_e}$

$M_0(0, 0, 0, ic) = (0, 0, 0, i[\gamma M_f(c + v)])$

Hence

$M_o c = \gamma M_f (c+v)$

$\frac{M_o}{M_f} = \chi = \frac{1}{c}\frac{c+v}{\sqrt{1-v^2/c^2}}$

$\chi^2 = \frac{(c+v)^2}{c^2-v^2} = \frac{c+v}{c-v}$ and

$v = \frac{\chi^2-1}{\chi^2+1} c$

---

$\boxed{14.7}$

For motion along the x axis, the four velocity is

$\underline{u} = \gamma(u, 0, 0, ic)$

The four acceleration is given by

$\underline{a} = \frac{d\underline{u}}{d\tau} = \gamma \frac{d\underline{u}}{dt} = \gamma \frac{d}{dt} \gamma [u, 0, 0, ic]$

$= \gamma^2 [a, 0, 0, 0] + \gamma \frac{d\gamma}{dt} [u, 0, 0, ic]$ where $a = \frac{du}{dt}$

Using $\frac{d\gamma}{dt} = \frac{d}{dt}(1 - u^2/c^2)^{-\frac{1}{2}} = \gamma^3 a \frac{u}{c^2}$ ,

$\underline{a} = \gamma^2 [a(1+\gamma^2 u^2/c^2), 0, 0, i\gamma^2 au/c]$

$1 + \frac{\gamma^2 u^2}{c^2} = 1 + \frac{u^2}{c^2-u^2} = \frac{c^2}{c^2-u^2} = \gamma^2$

Hence $\underline{a} = \gamma^4 a [1, 0, 0, i\frac{u}{c}]$

Norm of $\underline{a} = (\underline{a})^2 = \gamma^8 a^2 (1 - \frac{u^2}{c^2}) = \gamma^6 a^2$

$\boxed{14.8}$ The phase of the wave is given by $\varphi = 2\pi(\frac{x}{\lambda} - \nu t)$

In terms of a four vector $\underline{k} = 2\pi(1/\lambda, 0, 0, i\,\nu/c)$ the phase can be written as $\varphi = \underline{k}\cdot\underline{r}$ where $\underline{r}$ is the displacement vector $(x, 0, 0, ict)$

(a) Since $\varphi$ is a Lorentz invariant (scalar) it follows that $\varphi^2 = (2\pi)^2(x^2/\lambda^2 - \nu^2 t^2)$ is an invariant. Hence $(\varphi/2\pi)^2 = \frac{1}{\lambda^2}(x^2 - \lambda^2\nu^2 t^2)$ has the same value in all frames. For light, $x^2 = c^2t^2$ in all frames. In one frame, if $\lambda\nu = c$, $(\varphi/2\pi)^2 = 0$. It follows that $x'^2 - (\lambda'\nu')^2 t'^2 = t'^2(c^2 - \lambda'^2\nu'^2) = 0$ and $\lambda'\nu' = c$ in all frames, as required for a light wave. This confirms that $\underline{k}$ is a four vector.

(b) We can immediately find the transformation properties of its components. From Eqn. 14.4 we have

$$2\pi i\,\nu'/c = \gamma\left(2\pi i\,\nu/c - i\,\frac{V}{c}\,\frac{2\pi}{\lambda}\right)$$
$$\nu' = \gamma\left(\nu - \frac{V}{\lambda}\right)$$

Using $\nu = c/\lambda$ we obtain $\nu' = \gamma\left(\nu - \frac{V}{c}\right)$

(c) For a wave moving along the y axis, $\underline{k} = 2\pi(0, 1/\lambda, 0, i\,\nu/c)$ In the $x'y'z'$ system, the fourth component is $2\pi i\frac{\nu'}{c} = 2\pi i\,\gamma\frac{\nu}{c} \rightarrow \nu' = \gamma\nu$