considered. In this example, Stations F 137 and J 231 are approximately 120 km apart in the north-south direction. As can be seen by example, the convergence of the equipotential surfaces is very modest over long distances. Thus, it is only considered in high-precision surveys involving long north-south extents.

After applying the orthometric correction, the resultant misclosure in the leveling circuit can be adjusted using least squares. However, for the most precise surveys, the gravity values at the intermediate benchmarks must also be considered. Readers, who wish to learn more on this topic, should consult the references at the end of this chapter.

19.15 GEODETIC POSITION COMPUTATIONS

Geodetic position computations involve two basic types of calculations, the *direct* and the *inverse* problems. In the direct problem, given the latitude and longitude of station A and the geodetic length and azimuth of line AB, the latitude and longitude of station B are computed. In the inverse problem, the geodetic length of AB and its forward and back azimuths are calculated, given the latitudes and longitudes of stations A and B.

For long lines it is necessary to account for the ellipsoidal shape of the Earth in these calculations to maintain suitable accuracy. Many formulas are available for making direct and inverse calculations, some of which are simplified approximations that only apply for shorter lines. This book will present those developed by Vincenty (1975). The procedures presented in the following subsections have a stated accuracy of a few centimeters for lines up to 20,000 km in length. These computations are demonstrated in the Excel® spreadsheet *vincenty.xls*, which is available on the companion website for this book at http://www.pearsonhighered.com/ghilani.

19.15.1 Direct Geodetic Problem

In the direct problem, ϕ_1 and λ_1 represent the latitude and longitude, respectively, s the geodetic length from station 1 to station 2, and α_1 the forward azimuth from station 1 to station 2. The variables a, b, and f are the defining parameters of the ellipsoid as presented in Section 19.2. The unknowns in the problem are ϕ_2 and λ_2, the geodetic latitude and longitude of the sighted station, and α_2 the azimuth of the line from station 2 to station 1. Note that the observations used in this computation must be corrected to the ellipsoid using procedures outlined in Section 19.14.

The computational steps are as follows:[15]

1. $\tan U_1 = (1 - f)\tan\phi_1$
2. $\tan\sigma_1 = \tan U_1/\cos\alpha_1$

[15]For the derivation of this formulation, and that of the inverse problem which follows, consult the publication by T. Vincenty cited in this chapter's bibliography.

3. $u = e' \cos \alpha$
4. $\sin \alpha = \cos U_1 \sin \alpha_1$
5. $A = 1 + \dfrac{u^2}{16{,}384}\{4096 + u^2[-768 + u^2(320 - 175u^2)]\}$
6. $B = \dfrac{u^2}{1024}\{256 + u^2[-128 + u^2(74 - 47u^2)]\}$
7. $2\sigma_m = 2\sigma_1 + \sigma$; where the first iteration uses $\sigma = \dfrac{s}{bA}$
8. $\Delta\sigma = B \sin \sigma \left\{\cos(2\sigma_m) + \dfrac{B}{4}\left[\cos \sigma\,(-1 + 2\cos^2 \sigma_m) - \dfrac{B}{6}\cos(2\sigma_m)(-3 + 4\sin^2 \sigma)(-3 + 4\cos^2 \sigma_m)\right]\right\}$
9. $\sigma = \dfrac{s}{bA} + \Delta\sigma$
10. Repeat steps 7, 8, and 9 until the $\Delta\sigma$ becomes negligible.
11. $\tan \phi_2 = \dfrac{\sin U_1 \cos \sigma + \cos U_1 \sin \sigma \cos \alpha_1}{(1 - f)\sqrt{\sin^2\alpha + (\sin U_1 \sin \sigma - \cos U_1 \cos \sigma \cos \alpha_1)^2}}$
12. $\tan \lambda = \dfrac{\sin \sigma \sin \alpha_1}{\cos U_1 \cos \sigma - \sin U_1 \sin \sigma \cos \alpha_1}$
13. $C = \dfrac{f}{16}\cos^2\alpha[4 + f(4 - 3\cos^2\alpha)]$
14. $L = \lambda - (1 - C)f \sin \alpha \{\sigma + C \sin \sigma [\cos 2\sigma_m + C \cos \sigma(-1 + 2\cos^2 2\sigma_m)]\}$
15. $\lambda_2 = \lambda_1 + L$
16. $\tan \alpha_2 = \dfrac{\sin \alpha}{-\sin U_1 \sin \sigma + \cos U_1 \cos \sigma \cos \alpha_1}$

19.15.2 Inverse Geodetic Problem

In the inverse geodetic problem, ϕ_1, λ_1, ϕ_2, and λ_2 represent the latitude and longitude of the first and second stations, respectively. In the solution, the geodetic length, s, between the two points, and the forward and back azimuths of the line, α_1 and α_2, respectively, are determined. The variables a, b, and f again are the defining parameters of the ellipsoid as presented in Section 19.2.

Steps:

1. $L = \lambda = \lambda_2 - \lambda_1$
2. $\tan U_1 = (1 - f) \tan \phi_1$
3. $\tan U_2 = (1 - f) \tan \phi_2$
4. $\sin^2 \sigma = (\cos U_2 \sin \lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos \lambda)^2$

5. $\cos \sigma = \sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos \lambda$
6. $\sin \alpha = \cos U_1 \cos U_2 \sin \lambda / \sin \sigma$
7. $\cos 2\sigma_m = \cos \sigma - 2 \sin U_1 \sin U_2 / \cos^2 \alpha$
8. $C = \dfrac{f}{16} \cos^2 \alpha \, [4 + f(4 - 3 \cos^2 \alpha)]$
9. $\lambda = L - (1 - C) f \sin \alpha \, \{\sigma + C \sin \sigma [\cos 2\sigma_m$
   $+ C \cos \sigma (-1 + 2 \cos^2 2\sigma_m)]\}$
10. Repeat steps 8 and 9 until changes in λ become negligible.
11. $s = bA(\sigma - \Delta\sigma)$ where $\Delta\sigma$ comes from the steps 12 to 15 below.
12. $u = e' \cos \alpha$
13. $A = 1 + \dfrac{u^2}{16,384} \{4096 + u^2[-768 + u^2(320 - 175u^2)]\}$
14. $B = \dfrac{u^2}{1024} \{256 + u^2[-128 + u^2(74 - 47u^2)]\}$
15. $\Delta\sigma = B \sin \sigma \left\{ \cos(2\sigma_m) + \dfrac{1}{4} B \left[\cos \sigma (-1 + 2 \cos^2 \sigma_m) \right.\right.$
    $\left.\left. - \dfrac{1}{6} B \cos(2\sigma_m)\,(-3 + 4 \sin^2 \sigma)(-3 + 4 \cos^2 \sigma_m) \right] \right\}$
16. $\tan \alpha_1 = \dfrac{\cos U_2 \sin \lambda}{\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos \lambda}$
17. $\tan \sigma_2 = \dfrac{\cos U_1 \sin \lambda}{-\sin U_1 \cos U_2 + \cos U_1 \sin U_2 \cos \lambda}$

The software WOLFPACK on the companion website for this book at http://www.pearsonhighered.com/ghilani can be used to do both of these computations. Figure 19.23 shows the data entry screen for the direct geodetic problem. A similar

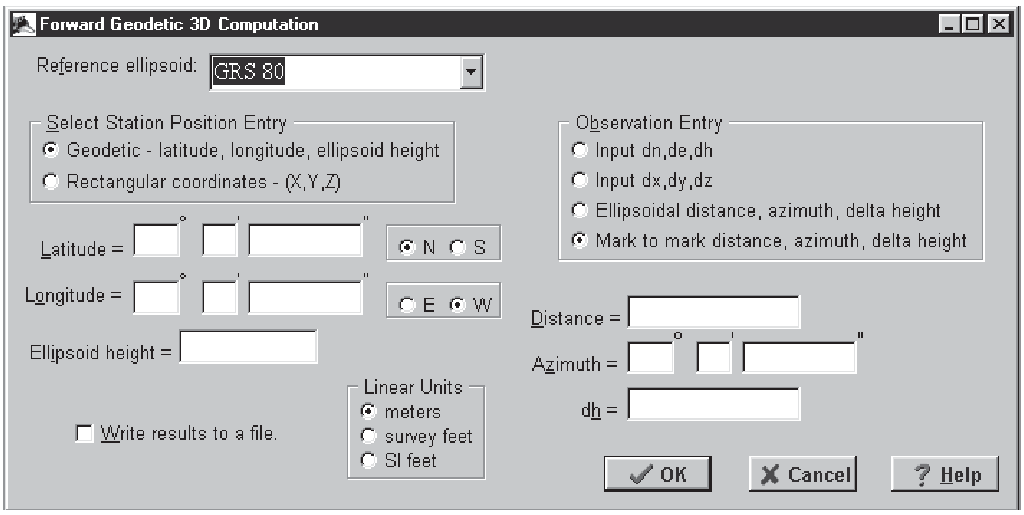

Figure 19.23 Data entry screen for forward computation from WOLFPACK.

Bessel's Solution of the Direct Geodetic Problem

1 $\tan\beta_1 = (1-f)\tan\varphi_1$

2 $\tan\sigma_1 = \dfrac{\tan\beta_1}{\cos\alpha_1}$

3 $\cos\beta_n = \cos\beta_1 \sin\alpha_1$

4 (1) $t = \frac{1}{4}e'^2 \sin^2\beta_n$

(2) $K_1 = 1 + t\Big(1 - \frac{1}{4}t(3 - t(5 - 11t))\Big)$

(3) $K_2 = t\Big(1 - t(2 - \frac{1}{8}t(37 - 94t))\Big)$

5 (1) $v = \frac{1}{4}f\sin^2\beta_n$

(2) $K_3 = v\Big(1 + f + f^2 - v(3 + 7f - 13v)\Big)$

6 (1) $\sigma = \frac{S}{K_1 b} + \Delta\sigma$

(2) $\sigma_m = 2\sigma_1 + \sigma$

7 $\Delta\sigma = K_2 \sin\sigma\Big(\cos\sigma_m + \frac{1}{4}K_2(\cos\sigma\cos 2\sigma_m + \frac{1}{6}K_2(1+2\cos 2\sigma)\cos 3\sigma_m)\Big)$

Steps **6** and **7** are iterated until the change in $\Delta\sigma$ becomes less than the limit value given in advance. Initially we set $\Delta\sigma = 0$.

8 (1) $\tan\beta_2 = \dfrac{\sin\beta_1\cos\sigma + \cos\beta_1\sin\sigma\cos\alpha_1}{\sqrt{1-\sin^2\beta_n\sin^2(\sigma_1+\sigma)}}$

(2) $\tan\varphi_2 = \dfrac{\tan\beta_2}{1-f}$

9 $\Delta\omega = (1-K_3)f\cos\beta_n\Big(\sigma + K_3\sin\sigma(\cos\sigma_m + K_3\cos\sigma\cos 2\sigma_m)\Big)$

10 (1) $$\tan\omega = \frac{\sin\sigma \sin\alpha_1}{\cos\beta_1 \cos\sigma - \sin\beta_1 \sin\sigma \cos\alpha_1}$$

(2) $$\lambda_2 = \lambda_1 + \omega - \Delta\omega$$

11 $$\tan\alpha_2 = \frac{\cos\beta_1 \sin\alpha_1}{\cos\beta_1 \cos\sigma \cos\alpha_1 - \sin\beta_1 \sin\sigma}$$

Bessel's Solution of the Inverse Geodetic Problem

1 (1) $\tan\beta_1 = (1-f)\tan\varphi_1$

(2) $\tan\beta_2 = (1-f)\tan\varphi_2$

$\Delta\omega = 0$

2 $\omega = \lambda_2 - \lambda_1 + \Delta\omega$

3 $\tan\sigma =$
$$\frac{\sqrt{(\cos\beta_2\sin\omega)^2+(\cos\beta_1\sin\beta_2-\sin\beta_1\cos\beta_2\cos\omega)^2}}{\sin\beta_1\sin\beta_2+\cos\beta_1\cos\beta_2\cos\omega}$$

4 $\cos\beta_n = \dfrac{\cos\beta_1\cos\beta_2\sin\omega}{\sin\sigma}$

5 $\cos\sigma_m = \cos\sigma - \dfrac{2\sin\beta_1\sin\beta_2}{\sin^2\beta_n}$

6 Two equations are the same as step 5 in direct.

7 The same as step 9 in direct.

The procedure is iterated starting with step 2 and ending with step 7 repeatedly until the change in $\Delta\omega$ is negligible compared with the limit value given in advance. Initially we set $\Delta\omega = 0$.

8 All equations are the same as step 4 in direct.

9 The same as step 7 in direct.

10 $S = K_1 b(\sigma - \Delta\sigma)$

11 (1) $\tan\alpha_1 = \dfrac{\cos\beta_2 \sin\omega}{\cos\beta_1 \sin\beta_2 - \sin\beta_1 \cos\beta_2 \cos\omega}$

(2) $\tan\alpha_2 = \dfrac{\cos\beta_1 \sin\omega}{\cos\beta_1 \sin\beta_2 \cos\omega - \sin\beta_1 \cos\beta_2}$