# 5. Earth and Its Motions

5.1. Earth's annual motion
5.2. Earth's spin, precession, and nutation
5.3. Earth's free nutation
5.4. Observed polar motion and spin velocity variations

# Earth and its Motions

- It moves with our galaxy in respect to other galaxies
- It circulates with the solar system with in our galaxy
- It revolves around the sun , together with other planets
- It rotates around its instantaneous axis of rotation

# Earth's annual motion

- **Kepler's lows:**

1. The orbit of every planet is an ellipse with the Sun at one of the two foci.

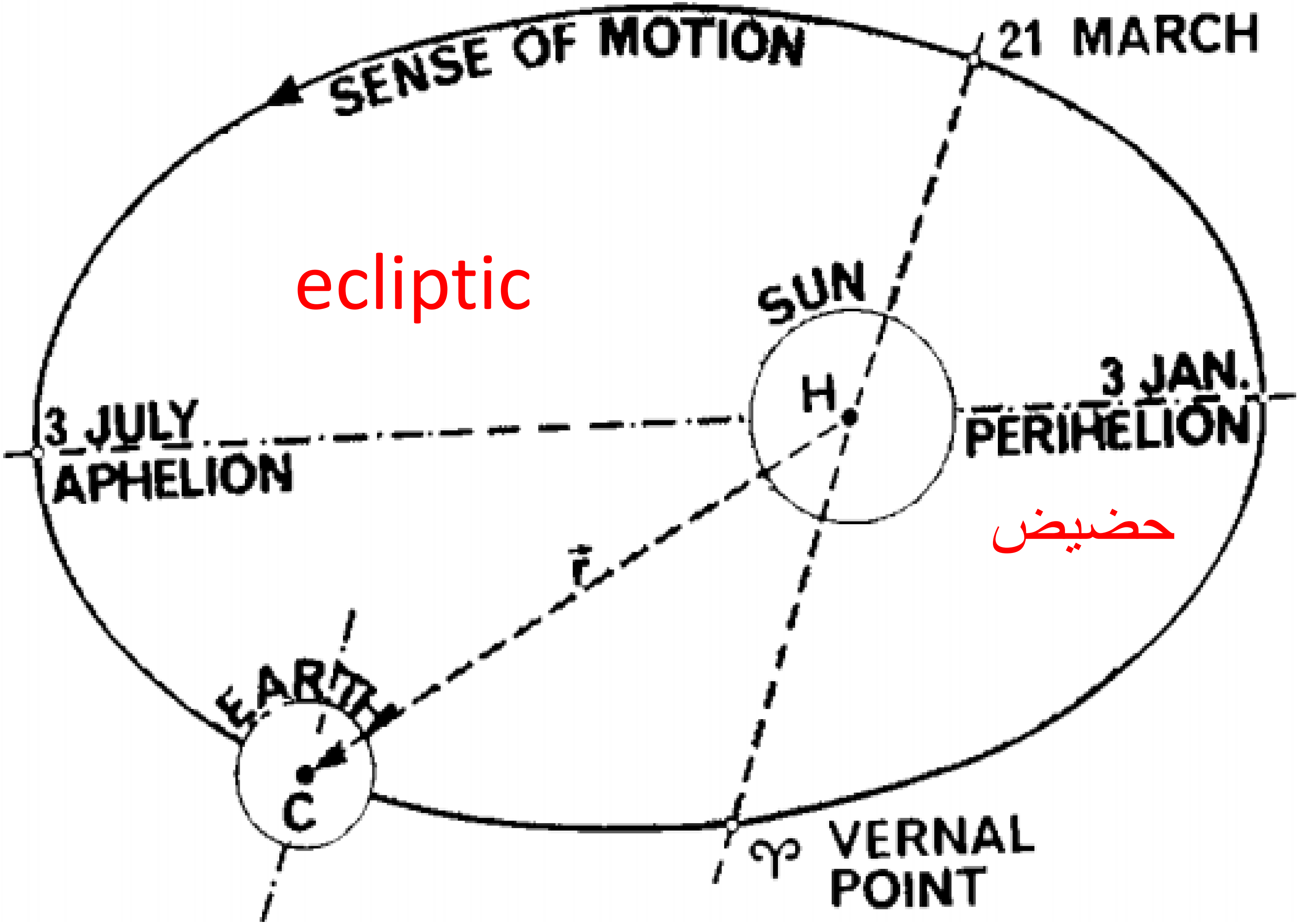
2. A line joining a planet and the Sun sweeps out equal areas during equal intervals of time

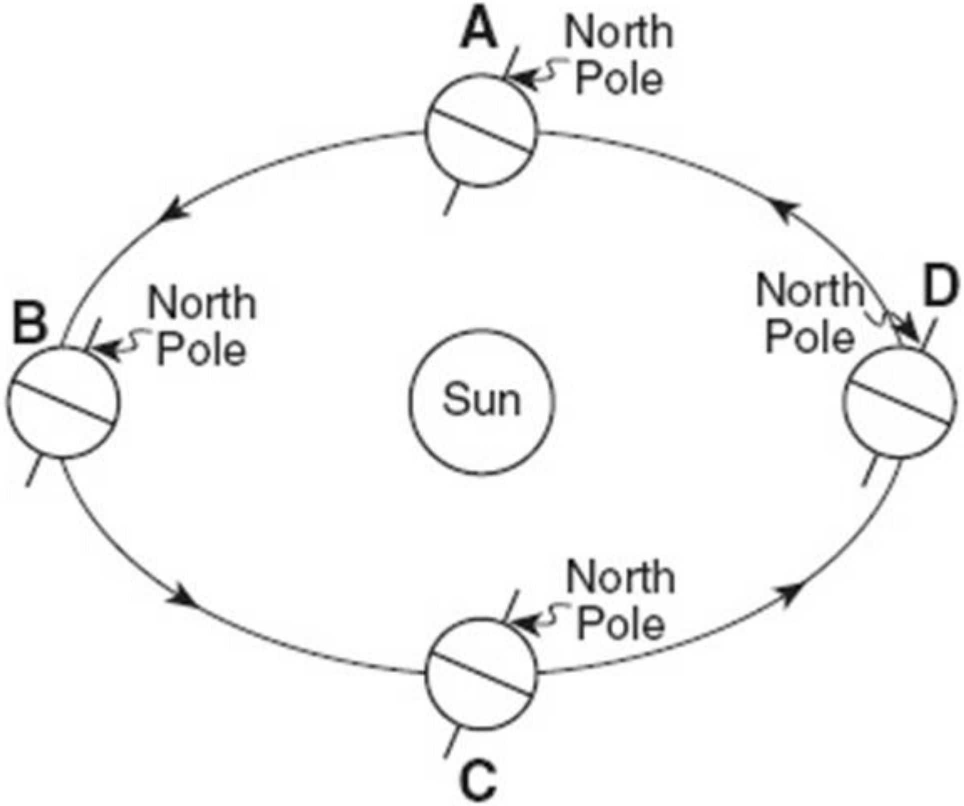
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

$$\frac{a^3}{T^2} = \frac{G^2}{4\pi^2}(M+m) = const.$$

$$G = (6.67259 \pm 0.00085) \times 10^{-11}\ \mathrm{m^3 s^{-2} kg^{-1}}\ (\mathrm{or\ Nm^2\ kg^{-2}})$$

که در آن G ثابت جهانی جاذبه، M جرم خورشید و m جرم زمین می باشد.


SENSE OF MOTION
21 MARCH
ecliptic
SUN
H
3 JAN.
PERIHELION
3 JULY
APHELION
حضيض
r
EARTH
C
♈ VERNAL POINT



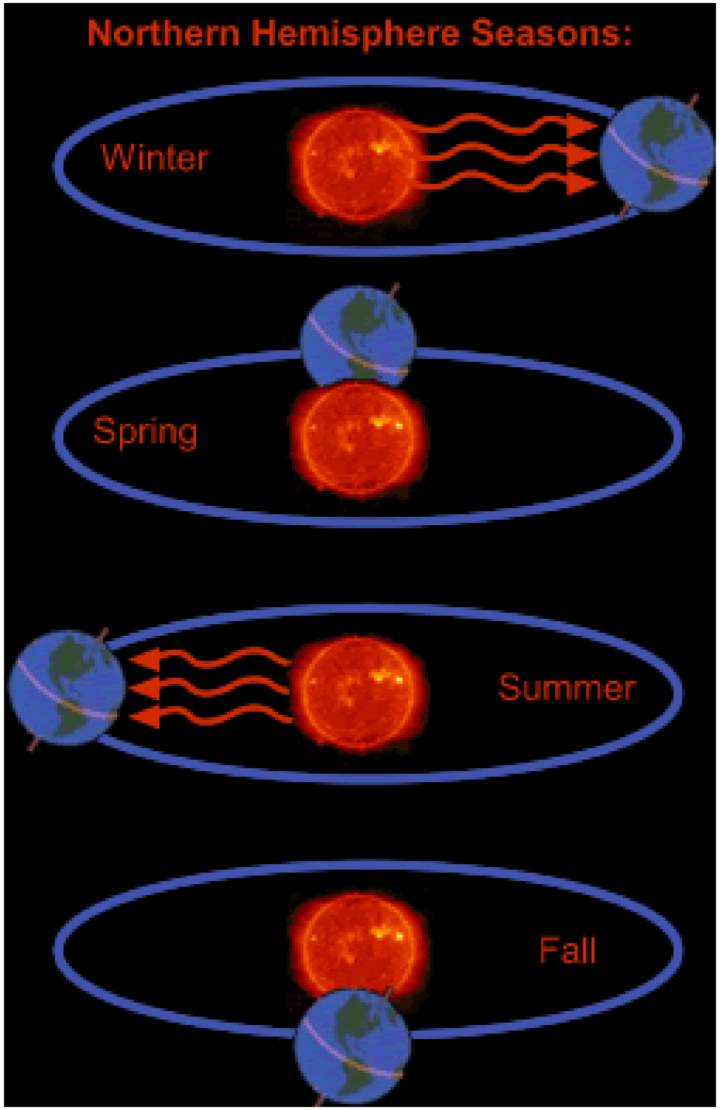
(Not drawn to scale)

Radius of the Earth's Orbit = 1 AU = 150,000,000 kilometers
Circumference of the Earth's Orbit = 2*pi*R = 942,000,000 kilometers
Time to complete one Orbit = 365.2422 days = 8766 hr

Speed of Revolution = Distance/Time = 942,000,000 km / 8766 hr = 107,000 km/hr = 30 km/sec

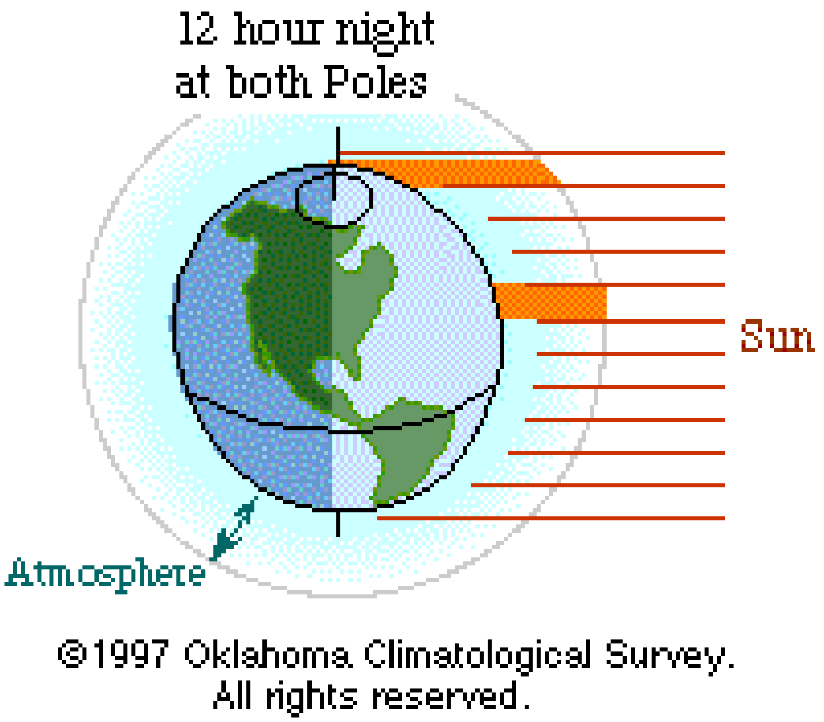
# Why do we have seasons?

- Seasons are the result of the tilt of the Earth's axis.
- Earth’s axis is tilted 23.5°.
- This tilting is why we have SEASONS like fall, winter, spring, summer.
- The number of daylight hours is greater for the hemisphere, or half of Earth, that is tilted toward the Sun.



# Equinoxes

- A day lasts 12 hours and a night lasts 12 hours at all latitudes.
- Equinox literally means "equal night".
- Sunlight strikes the earth most directly at the equator.
- This occurs twice a year.




http://k12.ocs.ou.edu/teachers/reference/equinox.gif

# Equinox

- The vernal (spring) equinox occurs March 21.
- The autumnal (fall) equinox occurs September 21.

| UT date and time of equinoxes and solstices on Earth[1] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| event | equinox | | solstice | | equinox | | solstice | |
| month | March | | June | | September | | December | |
| year | day | time | day | time | day | time | day | time |
| 2010 | 20 | 17:32 | 21 | 11:28 | 23 | 03:09 | 21 | 23:38 |
| 2011 | 20 | 23:21 | 21 | 17:16 | 23 | 09:04 | 22 | 05:30 |
| 2012 | 20 | 05:14 | 20 | 23:09 | 22 | 14:49 | 21 | 11:12 |
| 2013 | 20 | 11:02 | 21 | 05:04 | 22 | 20:44 | 21 | 17:11 |
| 2014 | 20 | 16:57 | 21 | 10:51 | 23 | 02:29 | 21 | 23:03 |
| 2015 | 20 | 22:45 | 21 | 16:38 | 23 | 08:20 | 22 | 04:48 |
| 2016 | 20 | 04:30 | 20 | 22:34 | 22 | 14:21 | 21 | 10:44 |
| 2017 | 20 | 10:28 | 21 | 04:24 | 22 | 20:02 | 21 | 16:28 |
| 2018 | 20 | 16:15 | 21 | 10:07 | 23 | 01:54 | 21 | 22:23 |
| 2019 | 20 | 21:58 | 21 | 15:54 | 23 | 07:50 | 22 | 04:19 |
| 2020 | 20 | 03:50 | 20 | 21:44 | 22 | 13:31 | 21 | 10:02 |

- *The 2013* **perihelion** *is around 05:00 UTC on January 2(دوازدهم دیماه), 2013 and the aphelion is around July 5(چهاردهم تیر ماه), 2013 at 15:00 UTC. In 2014 the perihelion is around 12:00 UTC on January 4, 2014 and the aphelion is around 00:00 on July 4, 2014*

- The earth is farthest away from the sun around July 4 when it is 152,171,522 km from the sun. This point in the earth's orbit is called **aphelion**.

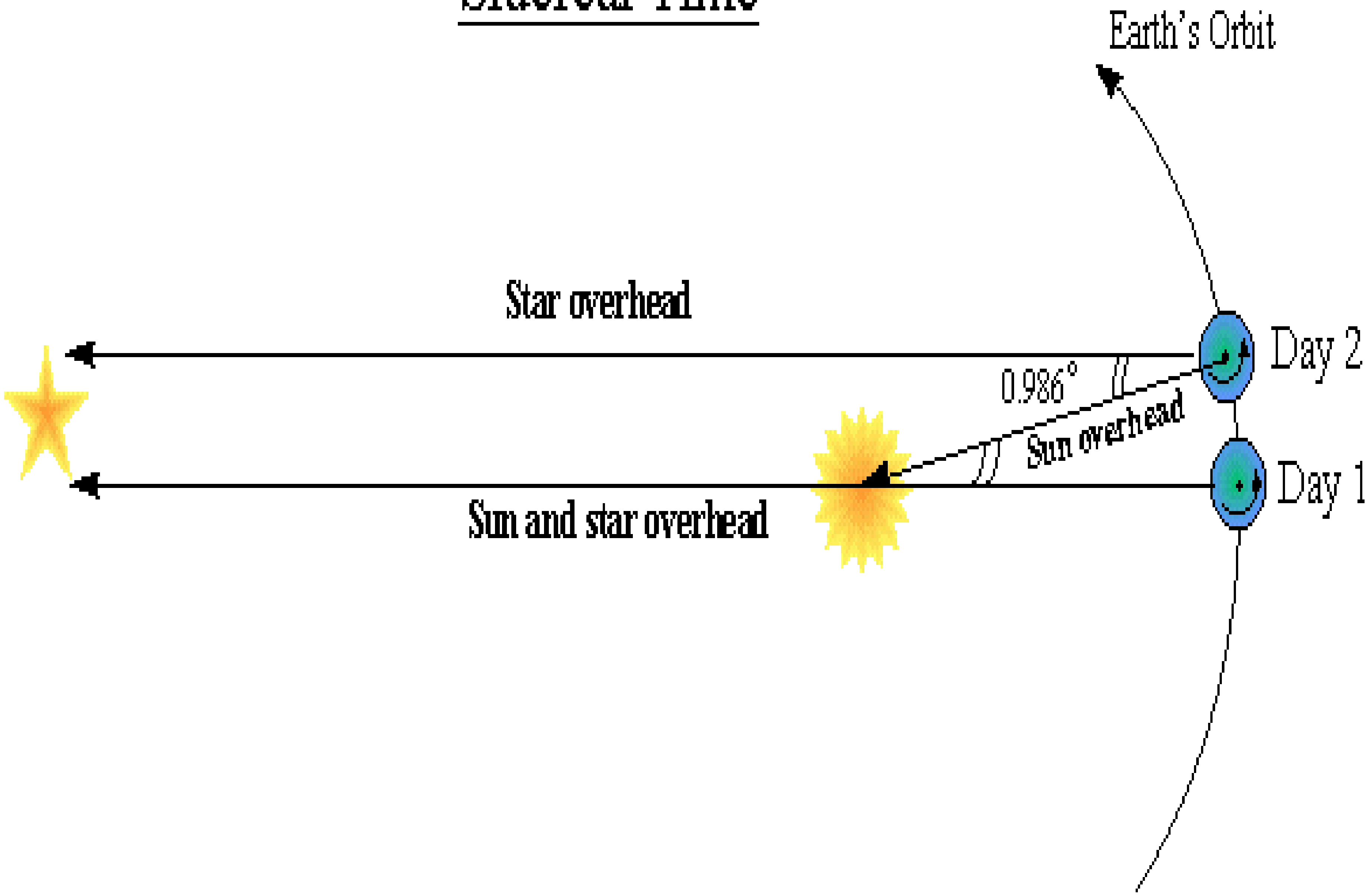
- The earth takes 365 days, 5 hours, 48 minutes, and 46 seconds (365.242199 days) to make a full revolution around the sun.

- Scientists utilize the average distance from the earth to the sun as the standard for one astronomical unit (1 AU). This average distance from the earth to the sun is 149,597,870.691 km. It takes light from the sun about 8.317 minutes to reach the earth.

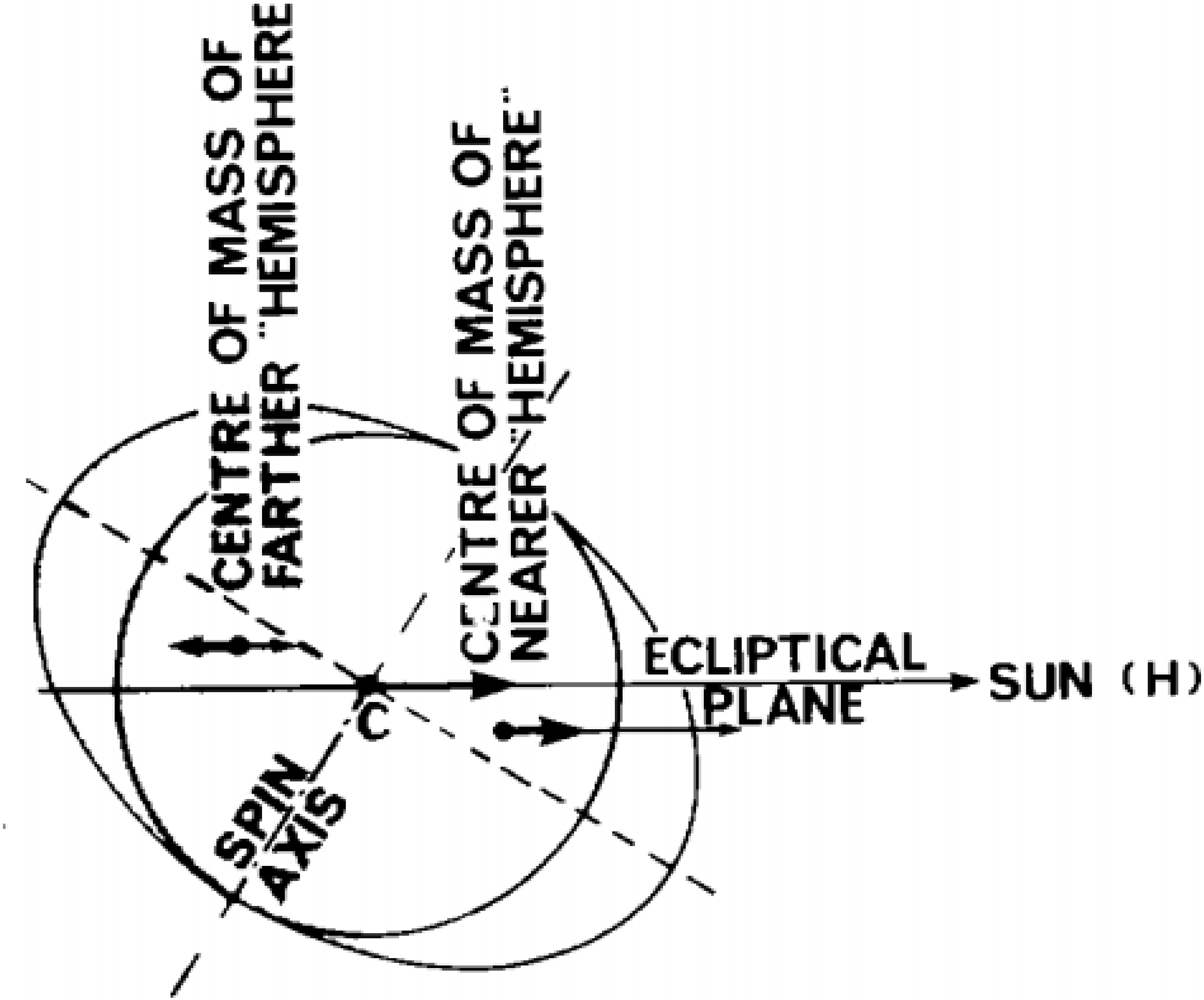
# Earth's spin ,precession and nutation

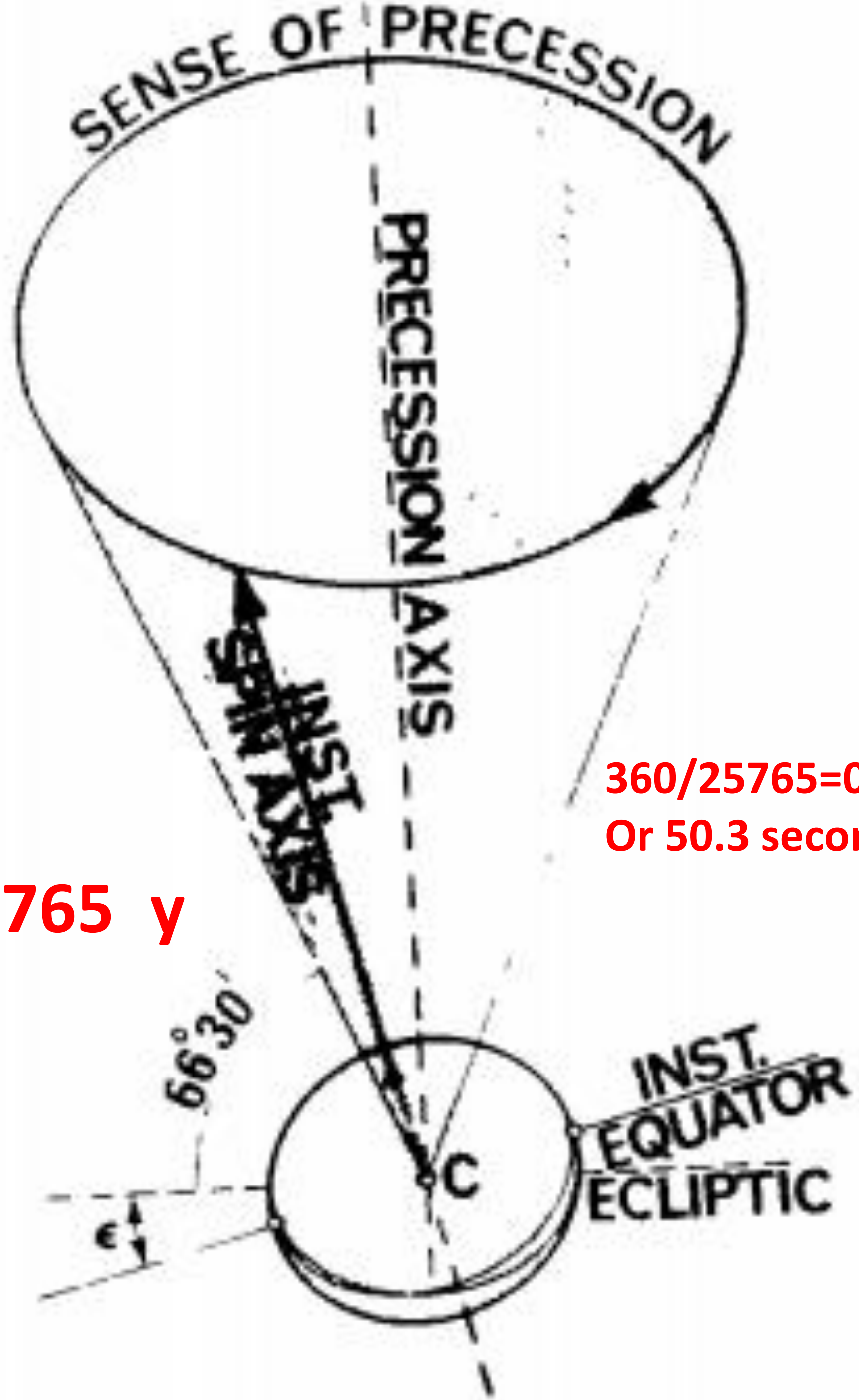
- Earth's rotation is the rotation of the solid Earth around its own axis. The Earth rotates from the west towards the east. As viewed from the North Star or polestar Polaris, the Earth turns counter-clockwise.
- **solar day**
- **sidereal day**

# Sidereal Time



- **A sidereal year** (365.256363 days) **is the time taken by the Earth to orbit the Sun once with respect to the fixed stars. Hence it is also the time taken for the Sun to return to the same position with respect to the fixed stars after apparently travelling once around the ecliptic. This differs from the solar or tropical year** (365.242199 days) **which has length equal to the time interval between vernal equinoxes in successive years**

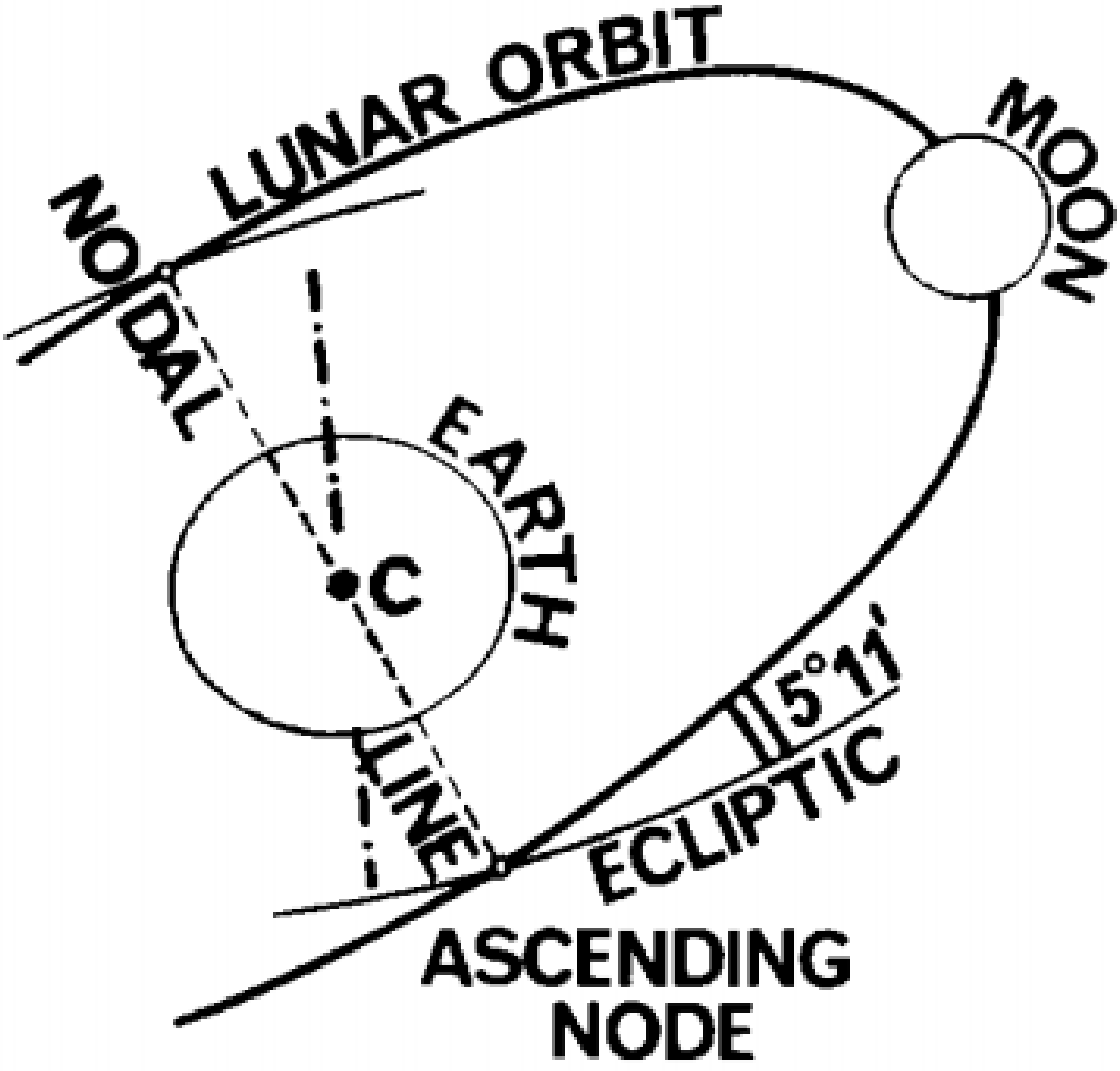

CENTRE OF MASS OF
FARTHER "HEMISPHERE"
CENTRE OF MASS OF
NEARER "HEMISPHERE"
ECLIPTICAL
PLANE
SUN (H)
C
SPIN
AXIS

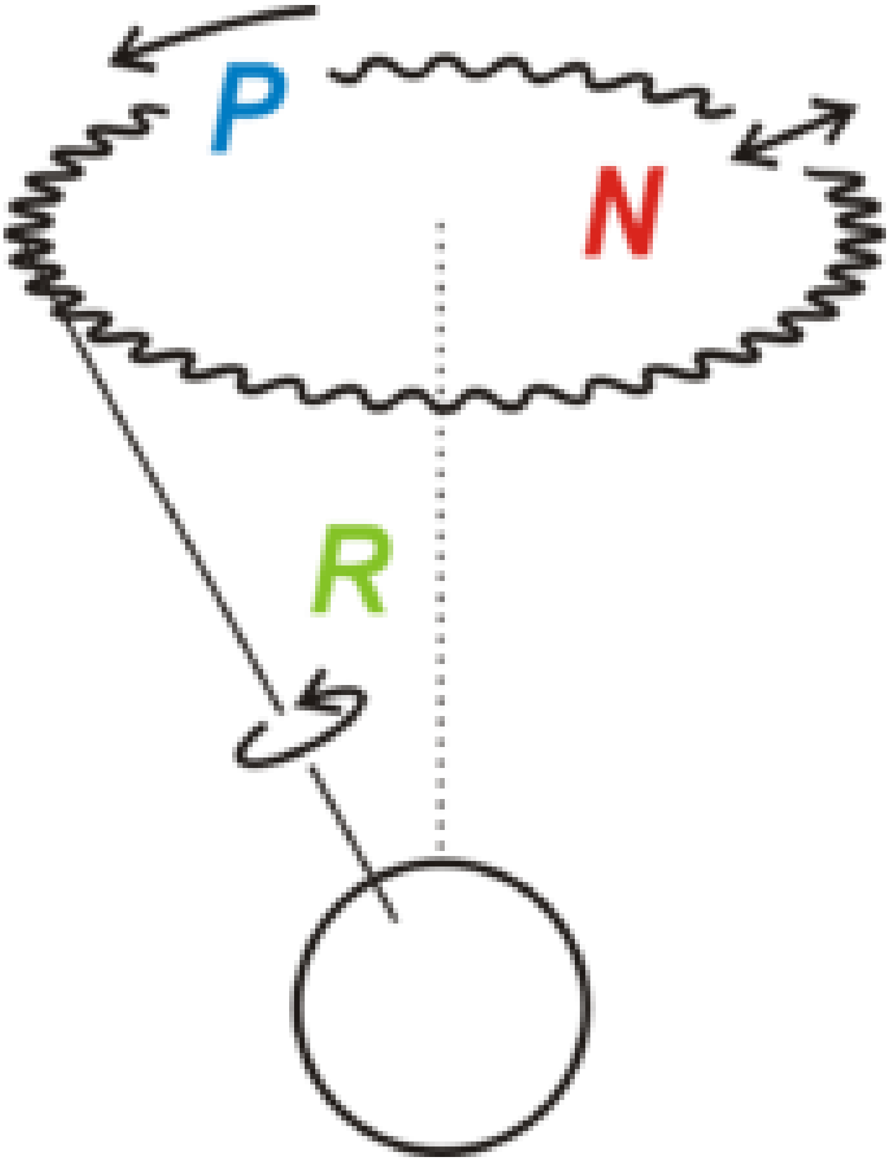


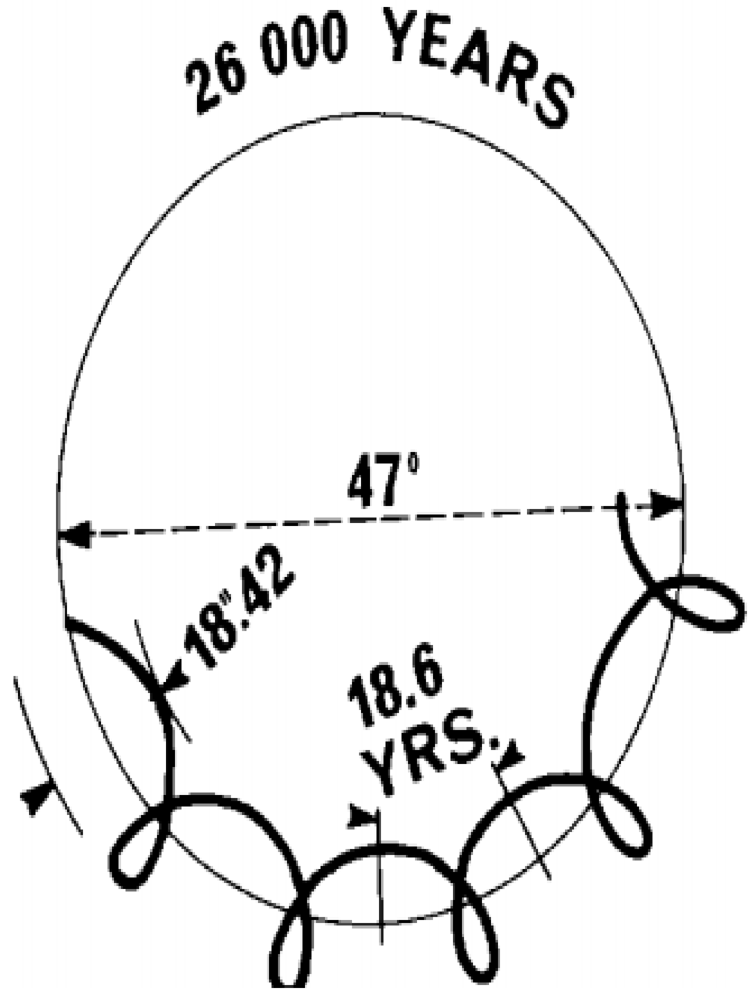

360/25765=0.014 degree
Or 50.3 second+/- 0.002 second

Platonic year=25765 y

- **The motion of the vernal point along the ecliptic is clockwise , i.e. , against the annual motion of the earth.**


LUNAR ORBIT
MOON
NODAL LINE
EARTH
C
5°11'
ECLIPTIC
ASCENDING NODE


P
N
R


26 000 YEARS
47°
18".42
18.6
YRS.

# Free nutation

- In addition to precession and nutation, the spin axis also undergoes a torque-free nutation, also called a wobble, with respect to the earth. More accurately, the wobble should be viewed as the motion of the earth with respect to the instantaneous spin axis. It is governed by famous Euler’s gyroscopic equation:

# Kinetic Energy of Rotation

The total kinetic energy $K$ can be expressed

$$K = \frac{1}{2} m_1 \cdot v_1^{\ 2} + \frac{1}{2} m_2 \cdot v_2^{\ 2} + \frac{1}{2} m_3 \cdot v_3^{\ 2} + \ldots + \frac{1}{2} m_n \cdot v_n^{\ 2}$$

$$K = \sum_{i=1}^{n} \frac{1}{2} m_i \cdot v_i^2$$

where, $m_i$ is the mass of the $i^{th}$ particle and $v_i$ is the speed of the $i^{th}$ particle. Since $v_i = \omega \cdot r_i$, this equation can be further written as

$$K = \sum_{i=1}^{n} \frac{1}{2} m_i \cdot \omega^2 \cdot r_i^2$$

$$K = \frac{1}{2} \left( \sum_{i=1}^{n} m_i \cdot r_i^2 \right) \cdot \omega^2$$

moment of inertia.

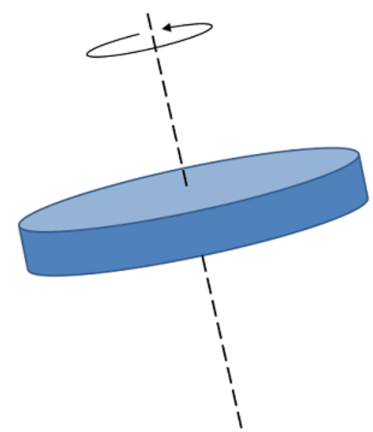
$$I = \sum_{i=1}^{n} m_i \cdot r_i^2$$

$$K = \frac{1}{2} \cdot I \cdot \omega^2$$

$$I = \sum_{i=1}^{n} m_i \cdot r_i^2 = \int r^2 \, \mathrm{d}m$$

**Problem Statement:** Find the moment of inertia of a disk of radius $R$, thickness $t$, total mass $M$, and total volume $V$ about its central axis as shown in the image below.

$$I_{disk} = \frac{M}{2} \cdot R^2$$



# Parallel Axis Theorem

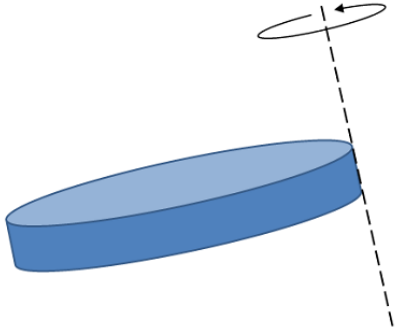
$$I = I_{COM} + Mh^2$$

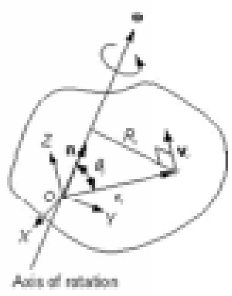
center of mass (COM)

where, $h$ is the distance between the two axes and $M$ is the total mass of the object.

**Proof** ? ?

**Problem Statement:** Find the moment of inertia of a disk rotating about an axis passing through the disk's circumference and parallel to its central axis, as shown below. The radius of the disk is R, and the mass of the disk is M.






## تنسور اینرسی و محورهای اصلی

فرض کنید یک جسم در یک سیستم مختصات سه بعدی، حول یک محور(یک بردار سه بعدی) گذرنده از مرکز سیستم مختصات با بردار سرعت زاویه ای $\vec{\omega}=(\omega_x,\omega_y,\omega_z)$ در حال دوران باشد. در این صورت اندازه حرکت زاویه ای $L$ (angular momentum) برابر است با:

$$L=\sum m_i\vec{r}_i\times\vec{v}_i$$
$$=\sum m_i\vec{r}_i\times(\vec{\omega}\times\vec{r}_i)$$

که در آن $\vec{r}_i$ بردار مکان هر کدام از ذرات این جسم از محور دوران می باشد. معادله فوق به صورت زیر قابل باز نویسی است.

$$\mathbf{r}\times(\boldsymbol{\omega}\times\mathbf{r})=\big((y^2+z^2)\omega_x-xy\omega_y-xz\omega_z,$$
$$-yx\omega_x+(z^2+x^2)\omega_y-yz\omega_z,$$
$$-zx\omega_x-zy\omega_y+(x^2+y^2)\omega_z\big).$$

$$\mathbf{L} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} \quad و \quad \left. \begin{array}{rcl} L_x & = & I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z \\ L_y & = & I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z \\ L_z & = & I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z \end{array} \right\}$$

که در آن $I_{xx}$، $I_{yy}$، و $I_{zz}$ گشتاور اینرسی و بقیه شش عنصر $I_{xy}$ ... حاصلضربهای اینرسی (Products of Inertia) می باشند:

$$I_{xx} = -\sum m_i (y_i^2 + z_i^2) \quad , \quad I_{yy} = -\sum m_i (x_i^2 + z_i^2) \quad , \quad I_{zz} = -\sum m_i (x_i^2 + y_i^2)$$

$$I_{xz} = -\sum m_i x_i z_i \quad , \quad I_{yz} = -\sum m_i y_i z_i \quad , \quad I_{xy} = -\sum m_i x_i y_i$$

$$I_{xz} = I_{zx} \quad , \quad I_{xy} = I_{yx} \quad , \quad I_{yz} = I_{zy}$$

در حالت جسم صلب سیگما به انتگرال تبدیل می شود.

در این حالت مقدار اندازه حرکت زاویه ای به صورت حاصلضرب ماتریس تنسور اینرشیای جسم، $I$، در بردار سرعت زاویه ای محاسبه می شود.

$$L = I\vec{\omega} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

که در آن $I$، ماتریس تنسور اینرشیا و یک ماتریس متقارن می باشد.

**گشتاور اینرسی $I$ (moment of inertia)**: یا اینرسی دورانی یک جسم در حال دوران، یک سنجه (مقیاس) برای بیان مقاومت یک شیء در مقابل تغییرات دوران آن جسم می باشد.

## محورهای اصلی جسم

در حالت کلی بردار اندازه حرکت زاویه ای $\mathbf{L}$ برای جسمی که در حول یک نقطه مانند O دوران می کند با بردار محور دوران هم راستا نمی باشند یعنی بردار $\mathbf{L}$ با بردار $\boldsymbol{\omega}$ موازی نیست. ولی برای هر جسم صلب که در حول یک نقطه مانند O دوران می کند سه محور دو به دو عمود برهم و گذرنده از نقطه O وجود دارد که در آن بردار $\mathbf{L}$ با بردار $\boldsymbol{\omega}$ موازی است که به این محورها محورهای اصلی جسم (Principle Axes) گفته می شود. این محورها منطبق بر بردارهای ویژه ماتریس تنسور اینرشیا می باشند و ماتریس تنسور اینرشیا در این حالت به صورت قطری خواهد بود.

$$\mathbf{L} = \lambda \boldsymbol{\omega}$$

$$\mathbf{I} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad , \quad \mathbf{L} = \mathbf{I}\boldsymbol{\omega} = (\lambda_1\omega_x, \lambda_2\omega_y, \lambda_3\omega_z)$$

# Earth's free nutation

$$\boldsymbol{J}\dot{\vec{\omega}} + \vec{\omega} \times \boldsymbol{J}\vec{\omega} = \vec{0},$$

**Euler's equation**

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \vec{0},$$

**J tensor of inertia**

**Observational evidence shows that the earth's two equatorial moments of inertia I1 and I2 are to high degree of accuracy , equal to each other but significantly different from the polar moment of inertia I3.**

$$\dot{\omega}_1 + \frac{I_3 - I_1}{I_1}\omega_2\omega_3 = 0, \quad \dot{\omega}_2 - \frac{I_3 - I_1}{I_1}\omega_1\omega_3 = 0,$$

$$\dot{\omega}_3 \doteq 0. \quad \Longrightarrow \quad \omega_3(\tau) \doteq \text{const.} = \mu$$

**By differentiating with respect to time ,the first two equation we have:**

$$\ddot{\omega}_1 + \left(\frac{I_3 - I_1}{I_1}\right)^2 \mu^2 \omega_1 = 0, \qquad \ddot{\omega}_2 + \left(\frac{I_3 - I_1}{I_1}\right)^2 \mu^2 \omega_2 = 0.$$

$$\omega_1(\tau) = \beta \cos\left(\frac{I_3 - I_1}{I_1}\mu\tau + \psi\right),$$

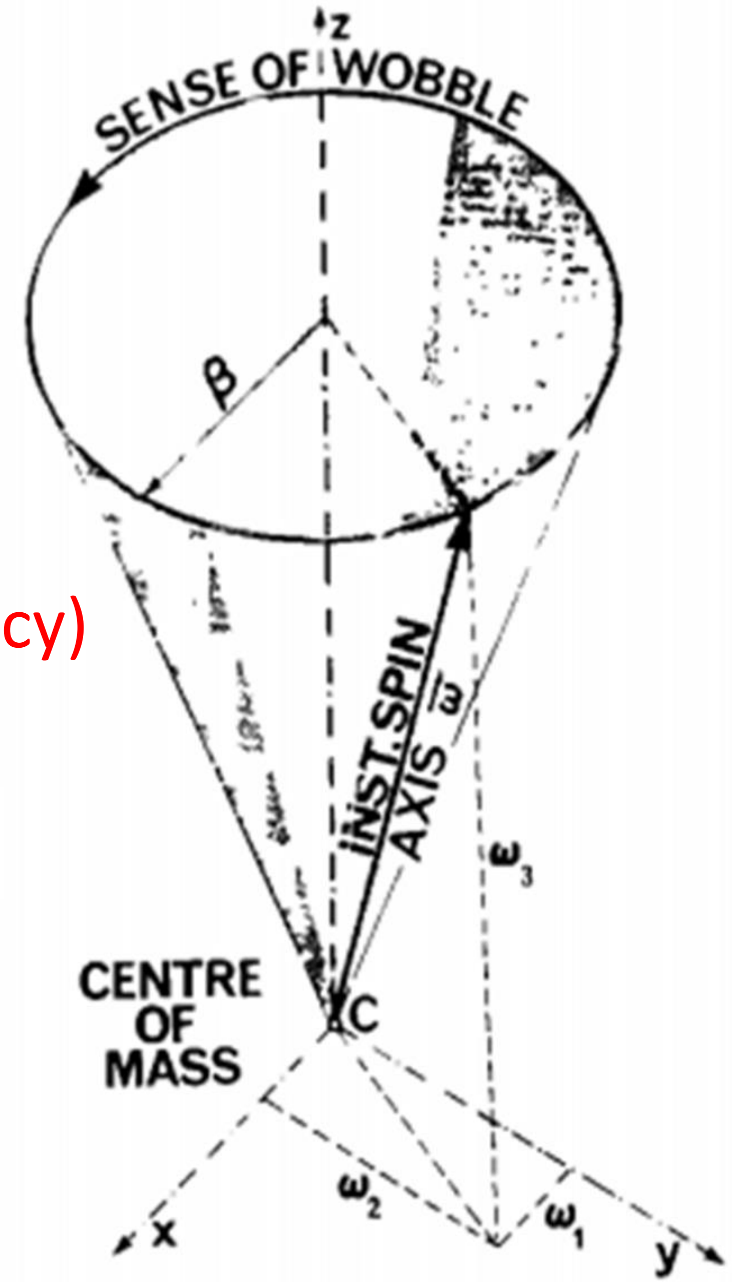
$$\omega_2(\tau) = \beta \sin\left(\frac{I_3 - I_1}{I_1}\mu\tau + \psi\right),$$

Earth's angular velocity(frequency)

$$\omega_3 \simeq \mu \doteq \omega = 2\pi / 1 \text{ sidereal day}.$$

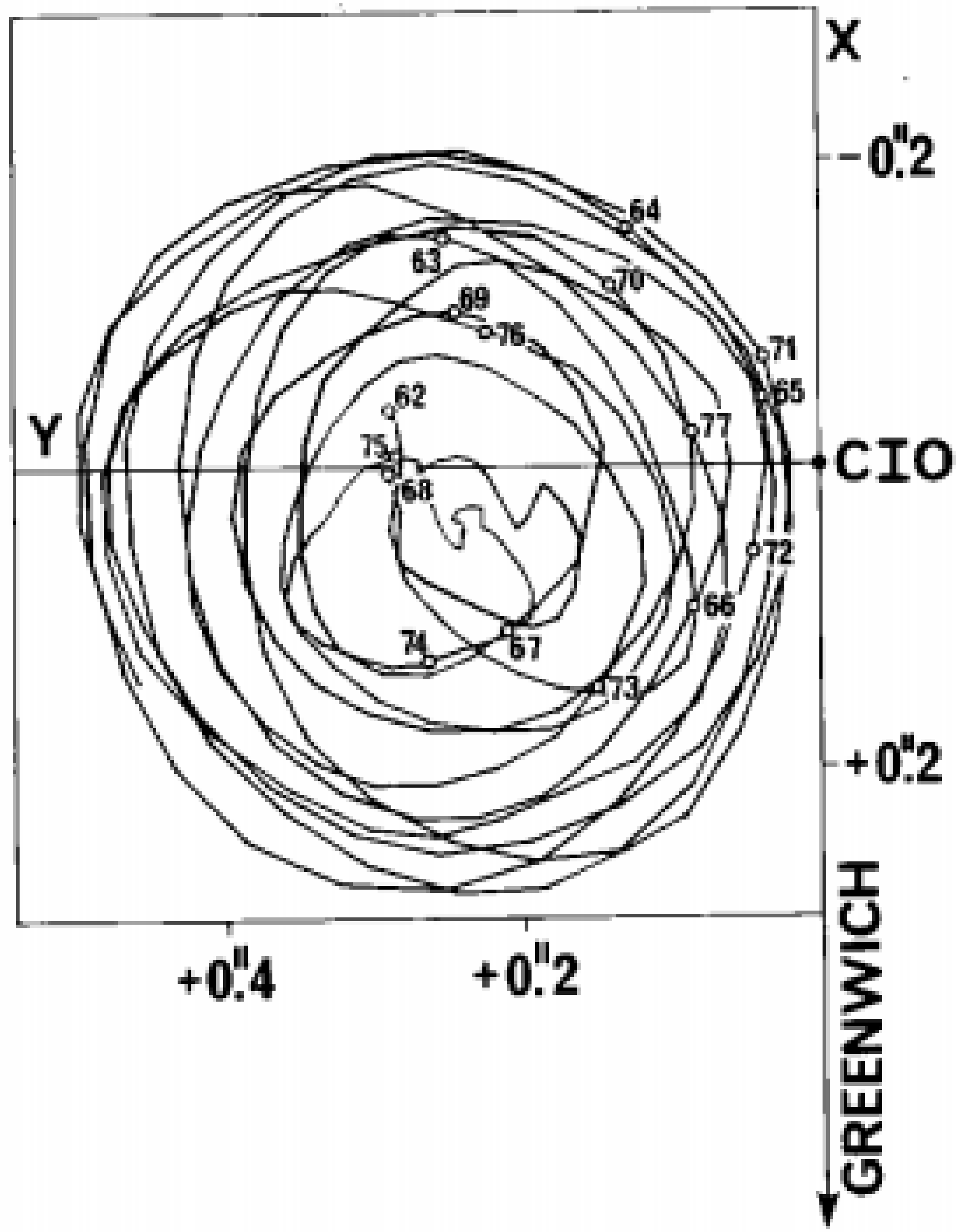
Since the free nutation period P is equal 2π/frequency

$$P = 2\pi \frac{I_1}{(I_3 - I_1)\mu}.$$

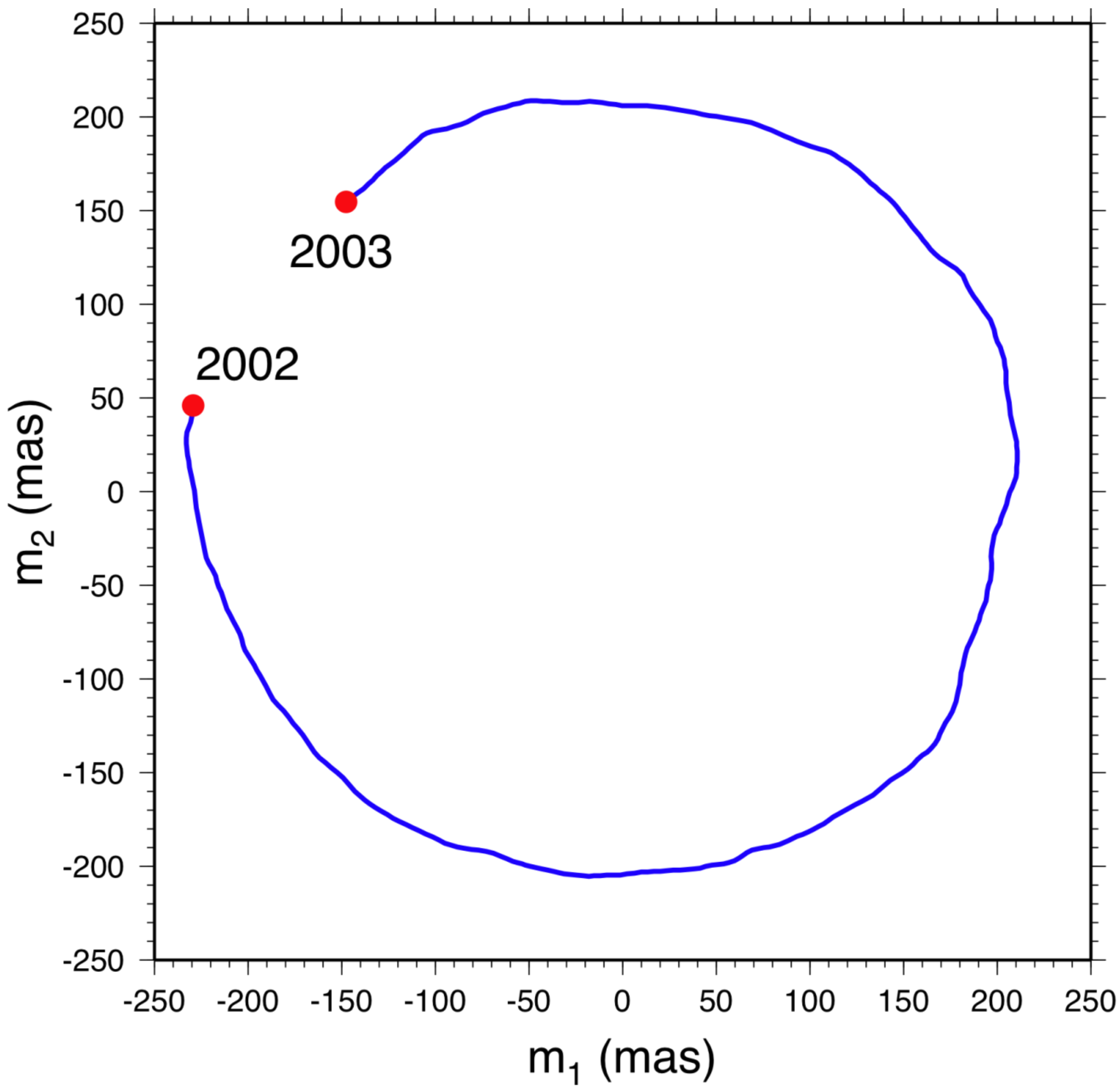


This value equal 305 sidereal day. This value is usually referred to as Euler period. But the actual value is about 40% longer than the Euler period.

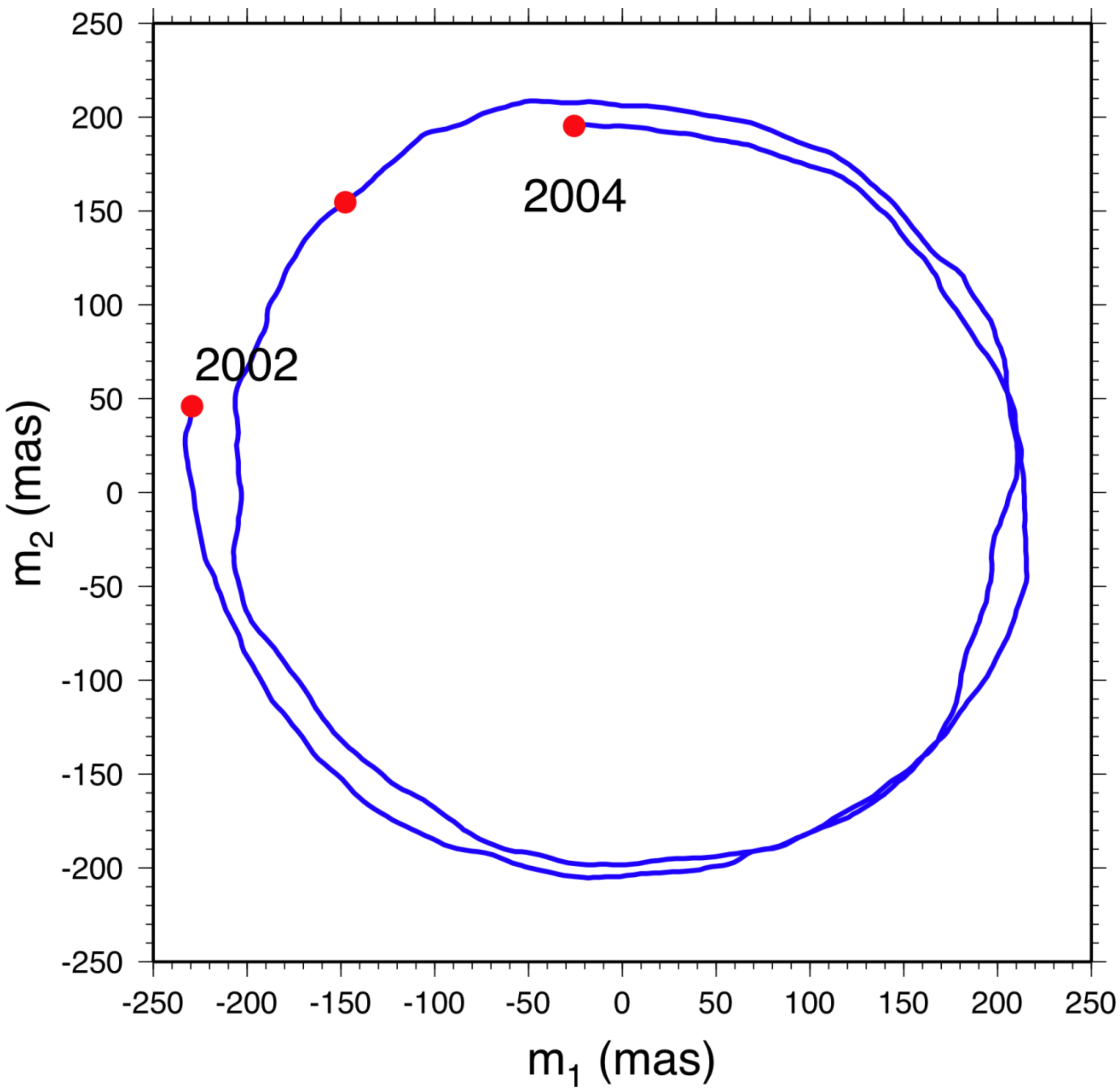
The non-rigidity of the earth tends to increase the wobble period. Which is now known to be about 435 days and is called the Chandler period.



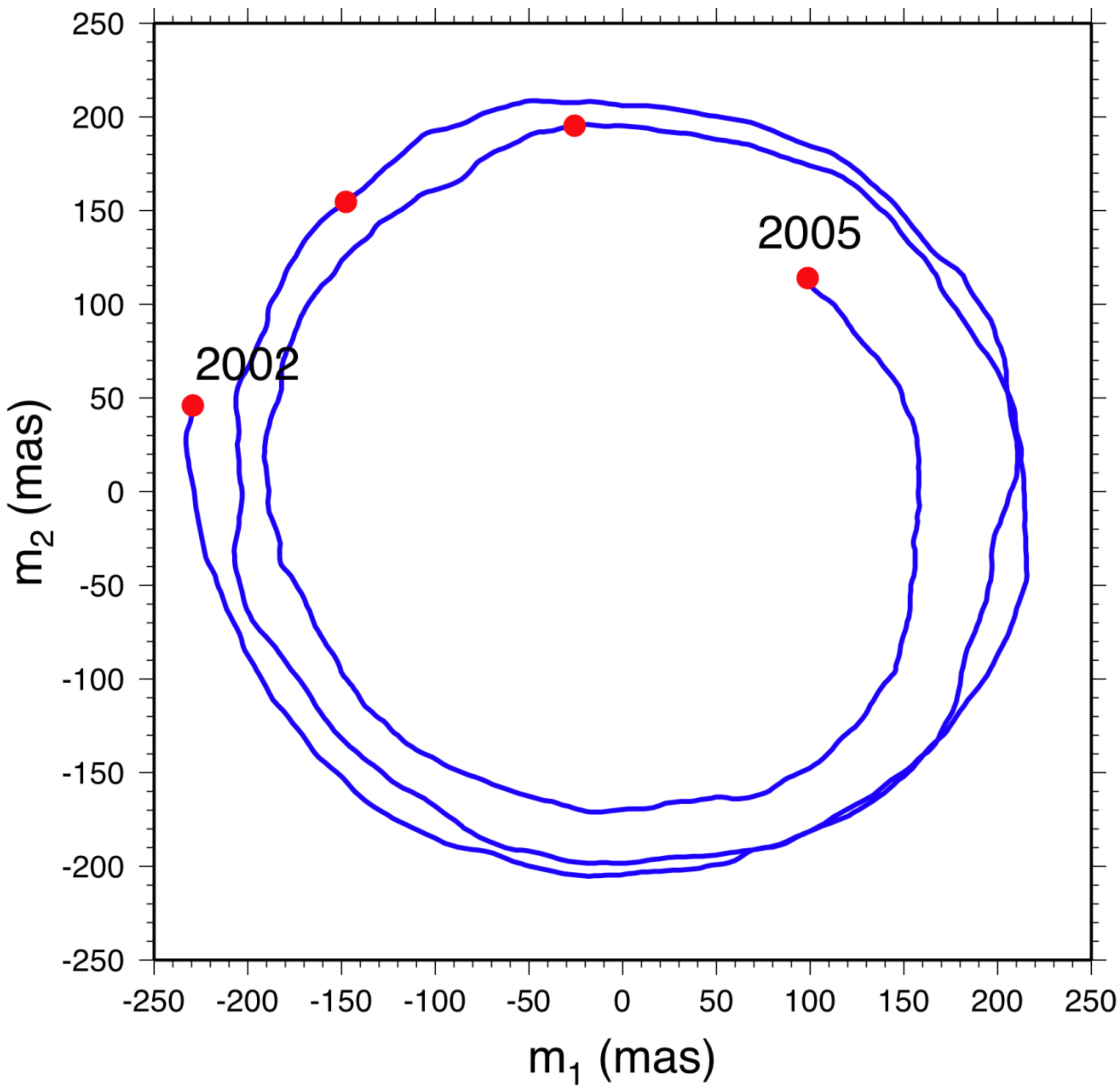
# Polar motion



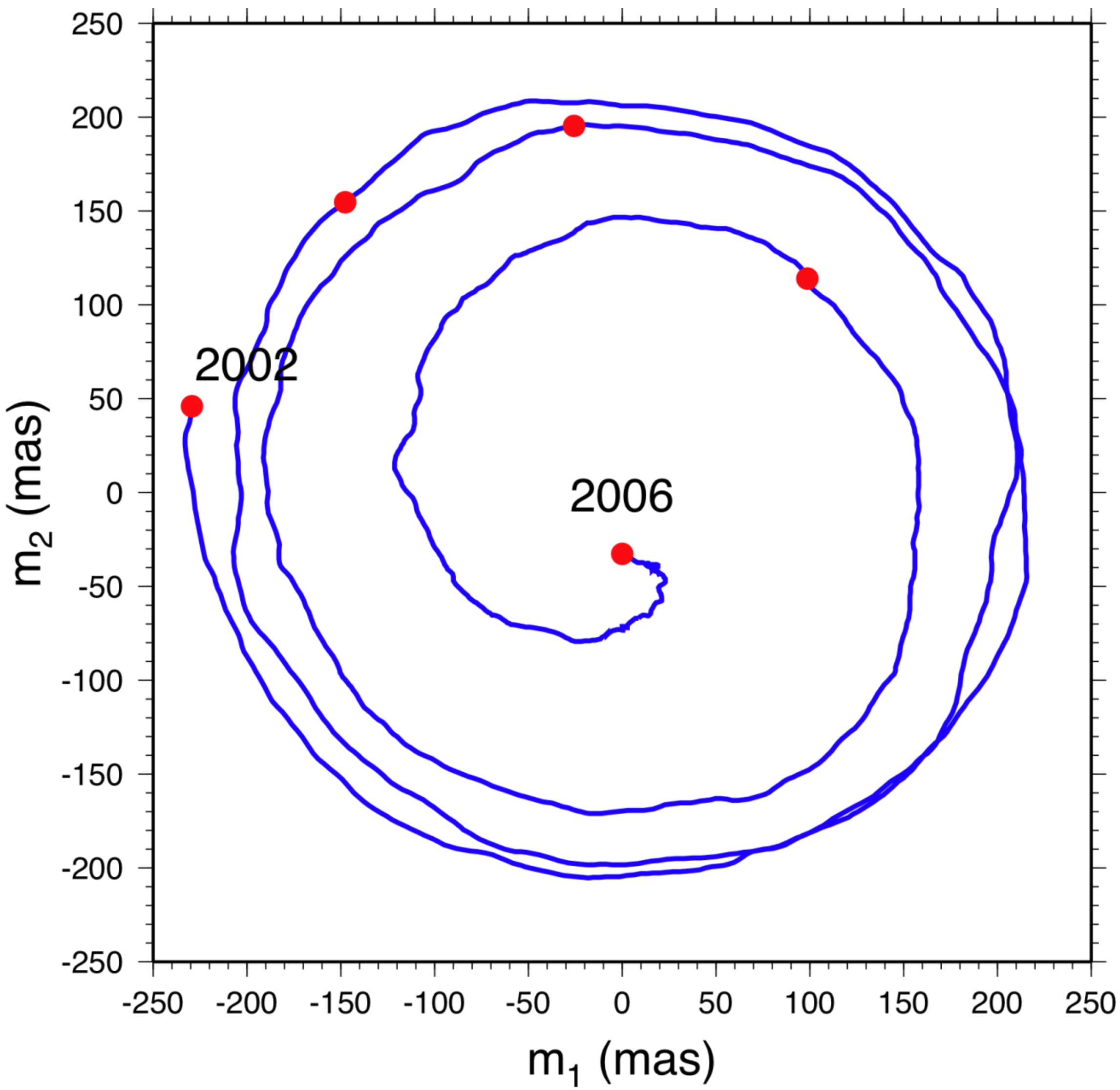
# Polar motion



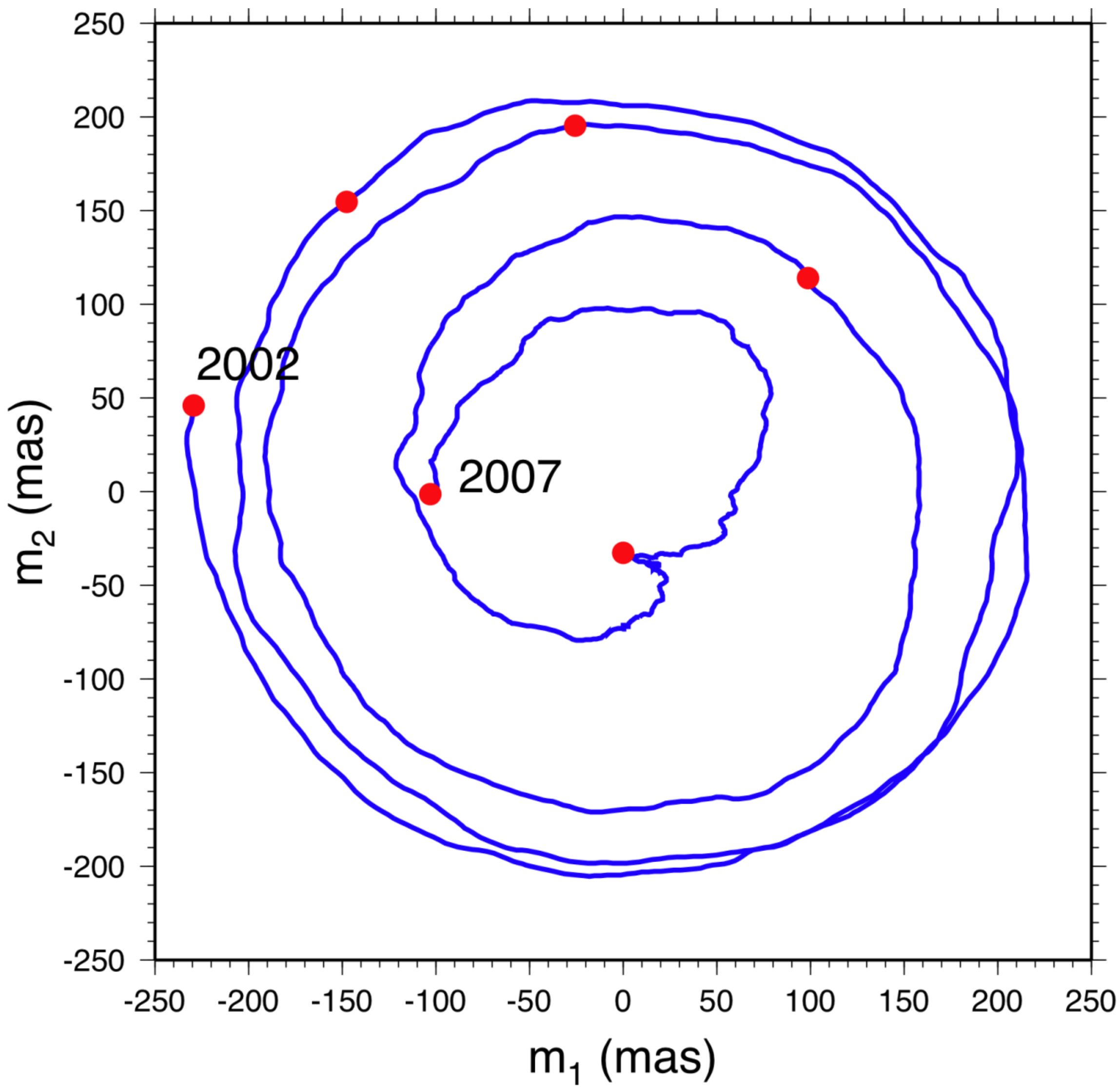
# Polar motion



# Polar motion



# Polar motion



# Polar motion

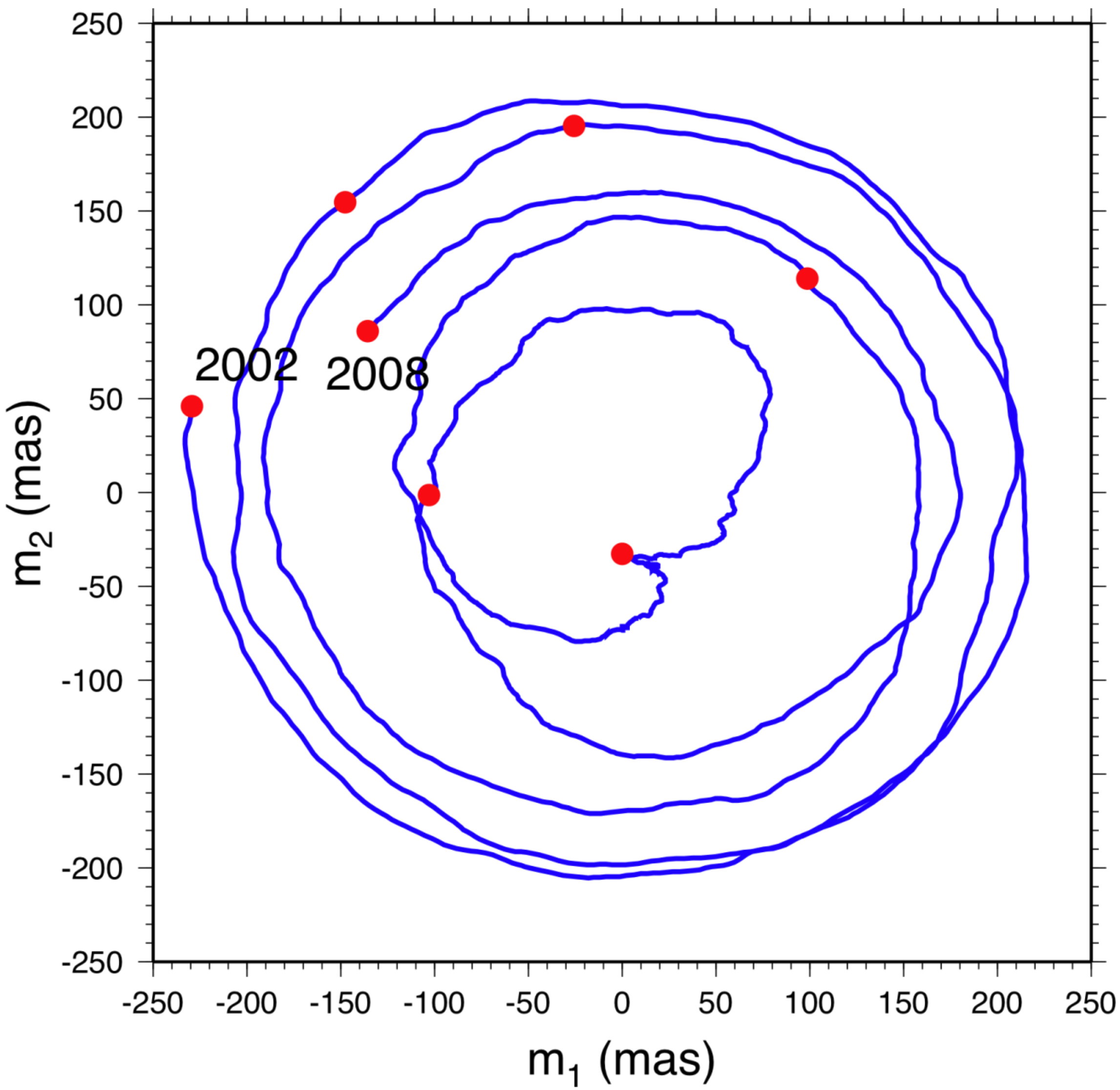
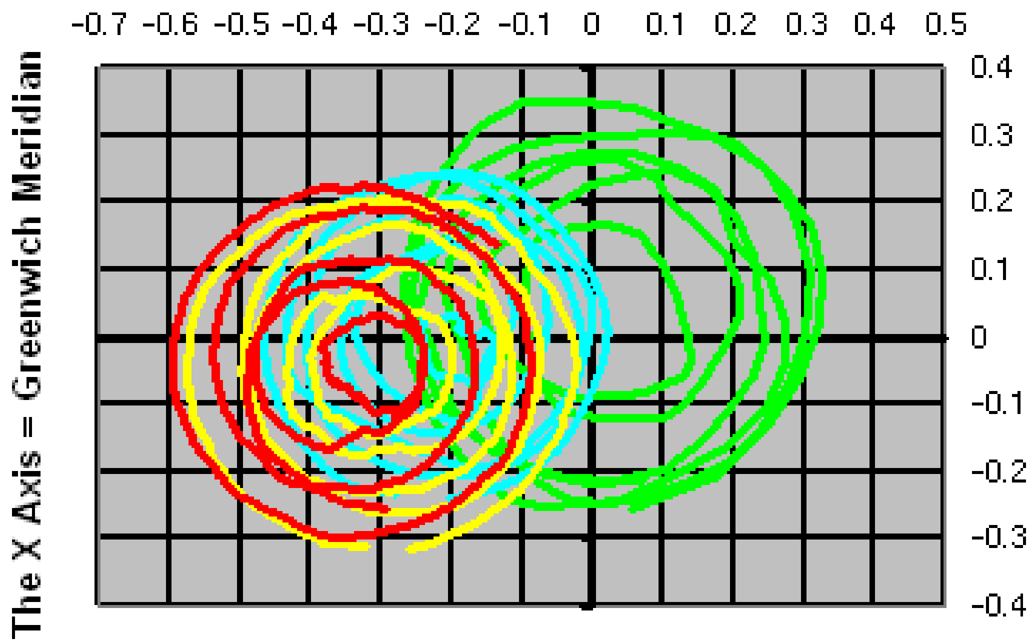


Table 5.1
Some results of determination of the Chandler period

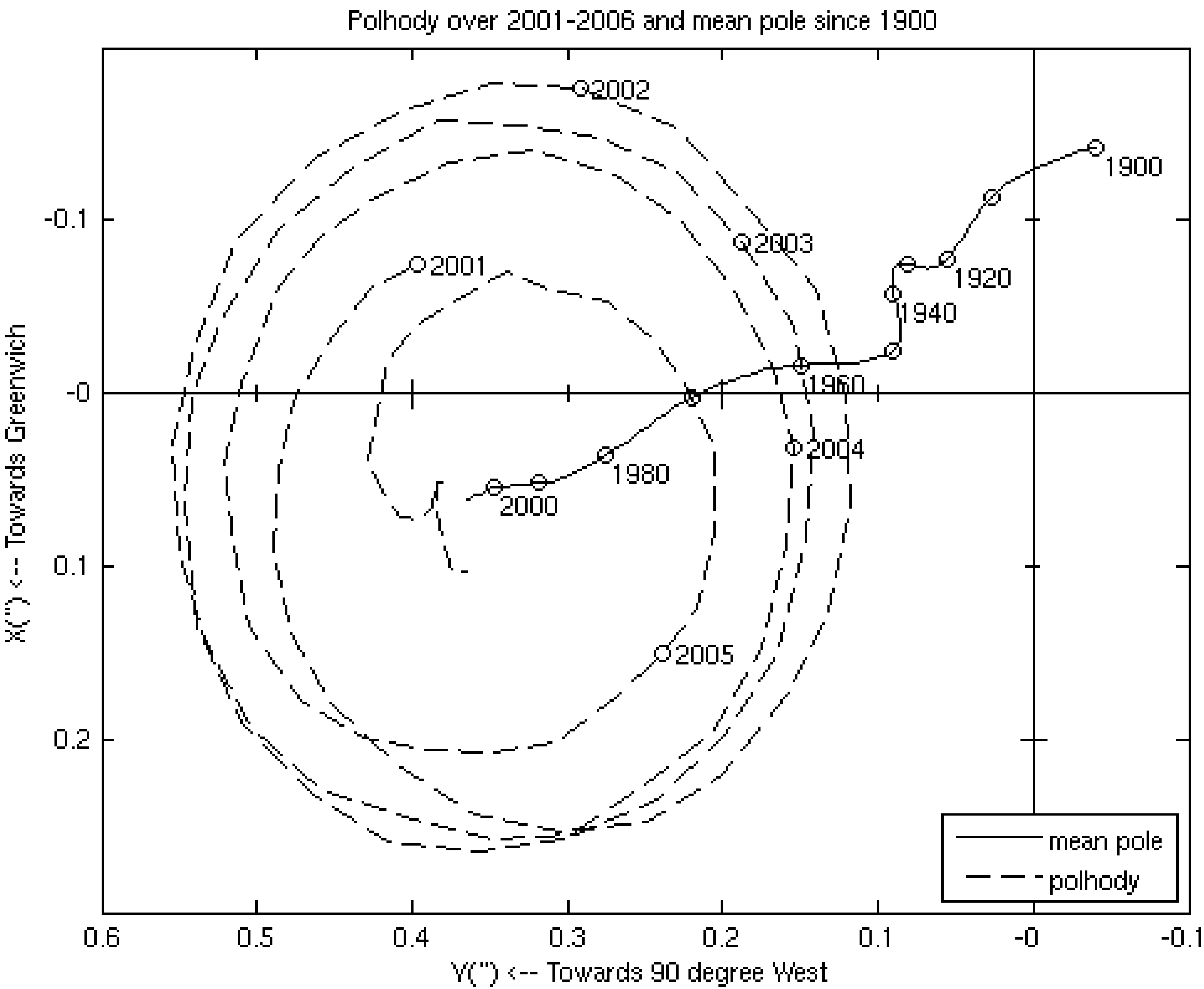
| Solution | Chandler period (Solar days) | Span of data | Source of data |
|---|---|---|---|
| Jeffreys [1968] | 433.15 | 1899–1967 | ILS |
| Vaníček [1969] | 435.1 | 1951–1966 | BIH |
| Yumi [1970] | 429.9 | 1890–1969 | ILS |
| | 439.4 | 1963–1969 | ILS |
| Anderle [1970] | 416.6 | 1967–1970 | DPMS |
| Currie [1974] | 432.95 | 1900–1973 | ILS |
| Graber [1976] | 430.8 | 1960–1974 | IPMS |

# Polar Motion 1909 - 2001

The track made by the location of the North Spin Axis through the 6.5 year spirals of Chandler's Wobble. Each color shows one 6.5 year spiral cycle. The shift towards the left shows the drift of the average location of the pole.




The "Y" Axis = Longitude West 90; Shown in Arcseconds; 0.1 Arcsecond = 3.087 meters or 10.127 feet; NOTE: Algebraic signs on margins must be reversed to compare with IERS positions. The graph plots as shown are 100% comparable. 



Polhody over 2001-2006 and mean pole since 1900
X('') <-- Towards Greenwich
Y('') <-- Towards 90 degree West
-0.1
-0
0.1
0.2
0.6
0.5
0.4
0.3
0.2
0.1
-0
-0.1
2002
2003
2001
1900
1920
1940
1960
2004
1980
2000
2005
mean pole
polhody

# Fluctuations in the length of day

- There are various factors which cause the orientation of the Earth to change with time.
- - Polar motion
- - The secular variation (linear increase in the length of the day , about 0.0015 to 0.0020 seconds per day per century.
- - The irregular changes in speed appear to be the result of random accelerations,
- - Periodic variations are associated with periodically repeatable physical processes affecting the Earth.

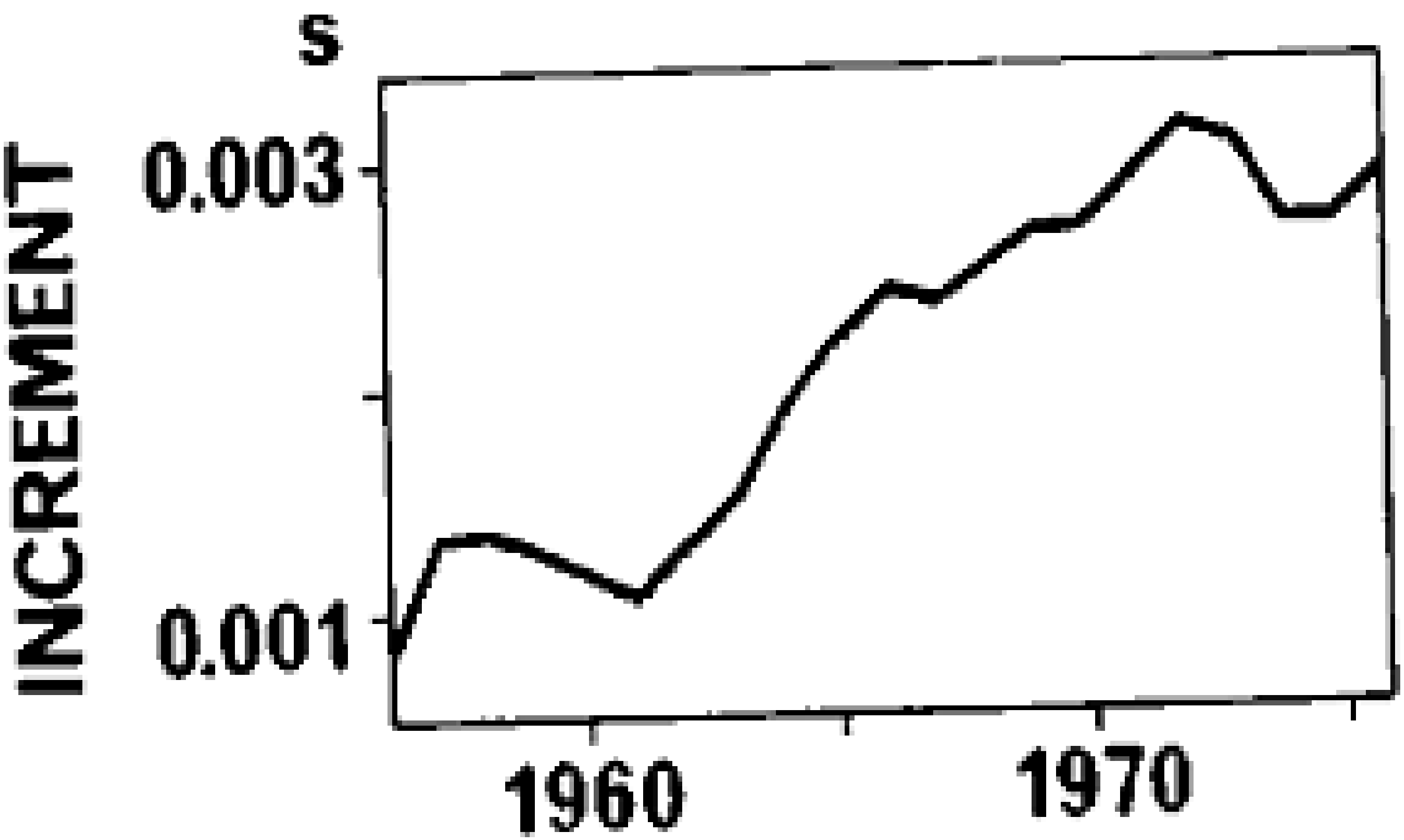
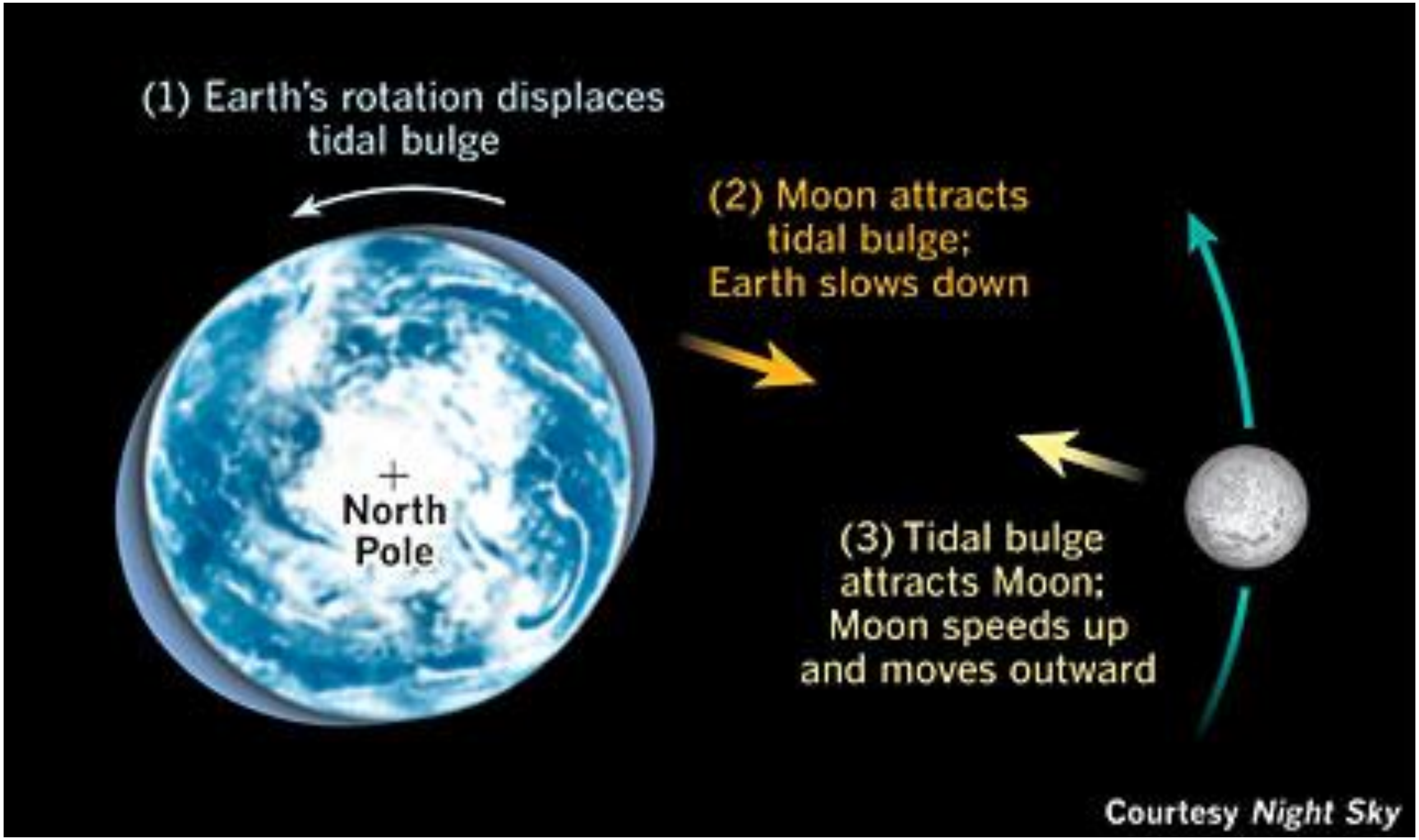



FIG. 5.9. Variation in the earth's spin velocity in terms of the length of the day (86 400 s plus increment) with periodic terms removed. (Courtesy of Dr. B. Guinot [1977], Director of BIH.)

# Secular change: Tidal friction

Earth's rotation is indeed slowing down at a present rate of 0.0018 seconds per century



| Orbital characteristics | |
|---|---|
| Epoch J2000.0[note 1] | |
| Aphelion | 152,098,232 km<br>1.01671388 AU[note 2] |
| Perihelion | 147,098,290 km<br>0.98329134 AU[note 2] |
| Semi-major axis | 149,598,261 km<br>1.00000261 AU[1] |
| Eccentricity | 0.01671123[1] |
| Orbital period | 365.256363004 days[2]<br>1.000017421 yr |
| Average orbital speed | 29.78 km/s[3]<br>107,200 km/h |
| Mean anomaly | 357.51716°[3] |
| Inclination | 7.155° to Sun's equator<br>1.57869°[4] to invariable plane |
| Longitude of ascending node | 348.73936°[3][note 3] |
| Argument of perihelion | 114.20783°[3][note 4] |
| Satellites | 1 natural (The Moon)<br>8681 artificial[5] |

| Physical characteristics | | | |
|---|---|---|---|
| Mean radius | 6,371.0 km[6] | | |
| Equatorial radius | 6,378.1 km[7][8] | | |
| Polar radius | 6,356.8 km[9] | | |
| Flattening | 0.0033528[10] | | |
| Circumference | 40,075.017 km (equatorial)[8]<br>40,008.00 km (meridional)[11] | | |
| Surface area | 510,072,000 $km^2$[12][13][note 5]<br>148,940,000 $km^2$ land (29.2 %)<br>361,132,000 $km^2$ water (70.8 %) | | |
| Volume | $1.08321 \times 10^{12}$ $km^3$[3] | | |
| Mass | $5.9736 \times 10^{24}$ kg[3] | | |
| Mean density | 5.515 $g/cm^3$[3] | | |
| Equatorial surface gravity | 9.780327 $m/s^2$[14]<br>0.99732 *g* | | |
| Escape velocity | 11.186 km/s[3] | | |
| Sidereal rotation period | 0.99726968 d[15]<br>23h 56m 4.100s | | |
| Equatorial rotation velocity | 1,674.4 km/h (465.1 m/s)[16] | | |
| Axial tilt | 23°26'21".4119[2] | | |
| Albedo | 0.367 (geometric)[3]<br>0.306 (Bond)[3] | | |
| Surface temp. | min | mean | max |
| Kelvin | 184 K[17] | 287.2 K[18] | 331 K[19] |
| Celsius | -89.2 °C | 14 °C | 57.8 °C |