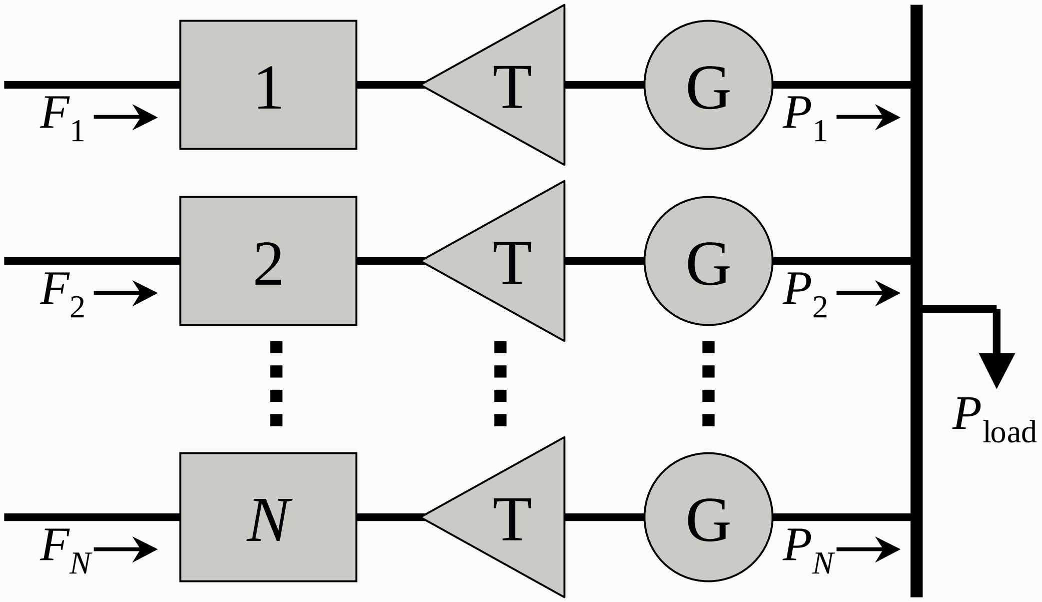
EEL 6266
Power System Operation and Control

Chapter 3

Economic Dispatch of Thermal Units

The Economic Dispatch Problem

- Consider a system that consists of N thermal-generating units serving an aggregated electrical load, P_{load}
  - input to each unit: cost rate of fuel consumed, F_i
  - output of each unit: electrical power generated, P_i
  - total cost rate, F_T, is the sum of the individual unit costs
  - essential constraint: the sum of the output powers must equal the load demand
  - the problem is to minimize F_T



The Economic Dispatch Problem

- The mathematical statement of the problem is a constrained optimization with the following functions:
  - objective function:

$$F_T = \sum_{i=1}^{N} F_i(P_i)$$

  - equality constraint:

$$\phi = 0 = P_{load} - \sum_{i=1}^{N} P_i$$

    - note that any transmission losses are neglected and any operating limits are not explicitly stated when formulating this problem
- Problem may be solved using the Lagrange function

$$L = F_T + \lambda\phi = \sum_{i=1}^{N} F_i(P_i) + \lambda\left(P_{load} - \sum_{i=1}^{N} P_i\right)$$

The Economic Dispatch Problem

- Principles
  - the Lagrange function establishes the necessary conditions for finding an extrema of an objective function with constraints
  - taking the first derivatives of the Lagrange function with respect to the independent variables allows us to find the extreme value when the derivatives are set to zero
    - there are $N_F + N_\lambda$ derivatives, one for each independent variable and one for each equality constraint
    - the derivatives of the Lagrange function with respect to the Lagrange multiplier λ merely gives back the constraint equation
    - the N_F partial derivatives result in

$$\frac{\partial L}{\partial P_i} = \frac{\mathrm{d}F_i\left(P_i\right)}{\mathrm{d}P_i} - \lambda = 0$$

The Economic Dispatch Problem

- Example
  - determine the economic operating point for the three generating units when delivering a total of 850 MW
    - input-output curves
      - unit 1: coal-fired steam unit: $H_1 = 510 + 7.2P_1 + 0.00142P_1^2$
      - unit 2: oil-fired steam unit: $H_2 = 310 + 7.85P_2 + 0.00194P_2^2$
      - unit 3: oil-fired steam unit: $H_3 = 78 + 7.97P_3 + 0.00482P_3^2$
    - fuel costs
      - coal: $ 3.30 / MBtu
      - oil: $ 3.00 / MBtu
    - the individual unit cost rate functions

$$F_1(P_1) = H_1(P_1) \times 3.3 = 1683 + 23.76P_1 + 0.004686P_1^2$$

$$F_2(P_2) = H_2(P_2) \times 3.0 = 930 + 23.55P_2 + 0.00582P_2^2$$

$$F_3(P_3) = H_3(P_3) \times 3.0 = 234 + 23.70P_3 + 0.01446P_3^2$$

The Economic Dispatch Problem

- Example
  - the conditions for an optimal dispatch

$$\mathrm{d}F_1/\mathrm{d}P_1 = 23.76 + 0.009372 P_1 = \lambda$$

$$\mathrm{d}F_2/\mathrm{d}P_2 = 23.55 + 0.01164 P_2 = \lambda$$

$$\mathrm{d}F_3/\mathrm{d}P_3 = 23.70 + 0.02892 P_3 = \lambda$$

$$P_1 + P_2 + P_3 = 850$$

- solving for λ yields

$$\frac{\lambda - 23.76}{0.009372} + \frac{\lambda - 23.55}{0.01164} + \frac{\lambda - 23.70}{0.02892} = 850$$

$$\lambda = 27.41$$

- then solving for the generator power values

$$P_1 = (27.41 - 23.76)/0.009372 = 389.8$$

$$P_2 = (27.41 - 23.55)/0.01164 = 331.8$$

$$P_3 = (27.41 - 23.70)/0.02892 = 128.4$$

The Economic Dispatch Problem

- In addition to the cost function and the equality constraint
  - each generation unit must satisfy two inequalities
    - the power output must be greater than or equal to the minimum power permitted: $P_i \geq P_{i,\min}$
      - minimum heat generation for stable fuel burning and temperature
    - the power output must be less than or equal to the maximum power permitted: $P_i \leq P_{i,\max}$
      - maximum shaft torque without permanent deformation
      - maximum stator currents without overheating the conductor
  - then the necessary conditions are expanded slightly

$$\mathrm{d}F_i / \mathrm{d}P_i = \lambda \quad \forall P_i : P_{i,\min} \leq P_i \leq P_{i,\max}$$

$$\mathrm{d}F_i / \mathrm{d}P_i \geq \lambda \quad \forall P_i = P_{i,\min}$$

$$\mathrm{d}F_i / \mathrm{d}P_i \leq \lambda \quad \forall P_i = P_{i,\max}$$

The Economic Dispatch Problem

- Example
  - reconsider the previous example with the following generator limits and the price of coal decreased to \$2.70 / MBtu
    - generator limits
      - unit 1: $150 \le P_1 \le 600$ MW
      - unit 2: $100 \le P_2 \le 400$ MW
      - unit 3: $50 \le P_3 \le 200$ MW
    - new fuel cost rate function for unit 1:
      $$F_1(P_1) = H_1(P_1) \times 2.7 = 1377 + 19.44P_1 + 0.003834P_1^2$$
    - solving for λ yields
      $$\frac{\lambda - 19.44}{0.007668} + \frac{\lambda - 23.55}{0.01164} + \frac{\lambda - 23.70}{0.02892} = 850$$
      $$\lambda = 24.82$$
      $$P_1 = 701.9 \quad P_2 = 109.3 \quad P_3 = 38.8$$

The Economic Dispatch Problem

- Example
  - this solution meets the constraint of generation meeting the 850 MW load demand, but units 1 and 3 are not within limit
    - let unit 1 be set to its maximum output and unit 3 to its minimum output. The dispatch becomes:

$$P_1 = 600 \text{ MW} \qquad P_2 = 200 \text{ MW} \qquad P_3 = 50 \text{ MW}$$

      - hence, λ must equal the incremental cost of unit 2 since it is the only unit not at either limit

$$\lambda = \left.\frac{\mathrm{d}F_2}{\mathrm{d}P_2}\right|_{P_2=200} = 23.55 + 0.01164(200) = 25.88$$

      - next compute the incremental costs for units 1 and 3

$$\left.\frac{\mathrm{d}F_1}{\mathrm{d}P_1}\right|_{P_1=600} = 19.44 + 0.007668(600) = 24.04$$

$$\left.\frac{\mathrm{d}F_3}{\mathrm{d}P_3}\right|_{P_3=50} = 23.7 + 0.02892(50) = 25.15$$

The Economic Dispatch Problem

- Example
  - note that the incremental cost for unit 1 is less than λ indicating that it is at its maximum
  - however, the incremental cost for unit 3 is not greater than λ so it should not be forced to its minimum
- rework with units 2 and 3 incremental cost equal to λ

$$\frac{\lambda - 23.55}{0.01164} + \frac{\lambda - 23.70}{0.02892} = 850 - P_1 = 250$$

$$\lambda = 25.67$$

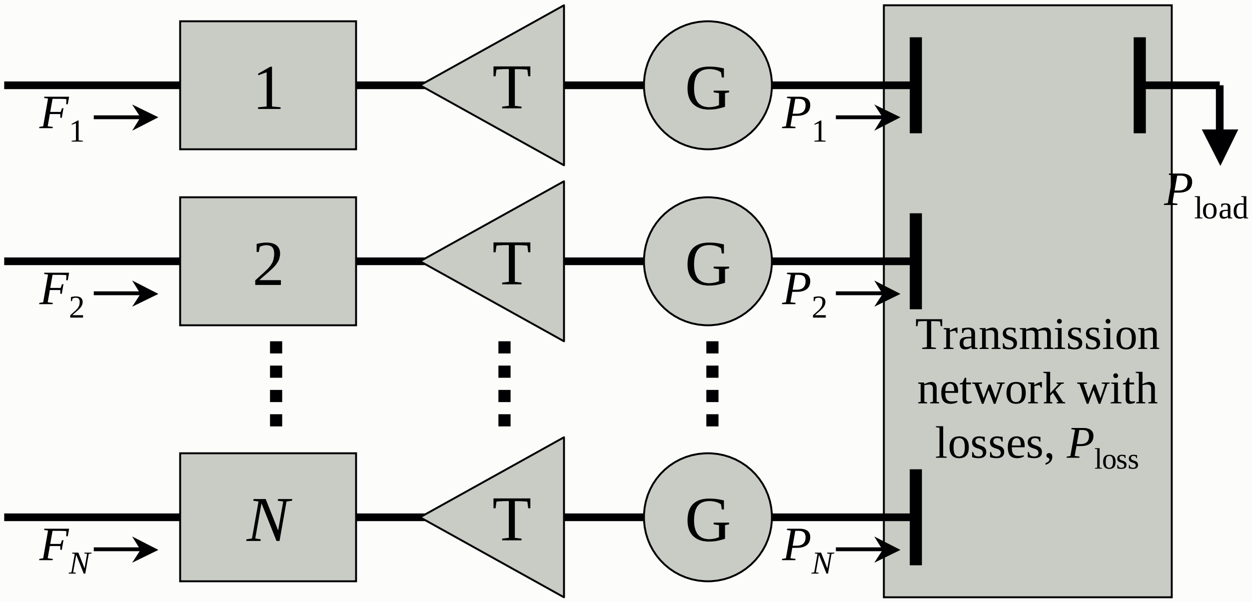
$$P_1 = 600 \quad P_2 = 182.0 \quad P_3 = 68.0$$

  - note that this dispatch meets the necessary conditions
  - incremental cost of electricity = 2.567 cents / kilowatt-hour

Network Losses

- Consider a similar system, which now has a transmission network that connects the generating units to the load
  - the economic dispatch problem is slightly more complicated
  - the constraint equation must include the network losses, P_{loss}
  - the objective function, F_T is the same as before
  - the constraint equation must be expanded as:

$$\phi = 0 = P_{load} + P_{loss} - \sum_{i=1}^{N} P_i$$



Network Losses

- The same math procedure is followed to establish the necessary conditions for a minimum-cost operating solution
  - Lagrange function and its derivatives w.r.t. the input power:

$$L = F_T + \lambda\phi = \sum_{i=1}^{N} F_i(P_i) + \lambda\left(P_{load} + P_{loss} - \sum_{i=1}^{N} P_i\right)$$

$$\frac{\partial L}{\partial P_i} = \frac{\mathrm{d}F_i}{\mathrm{d}P_i} - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_i}\right) = 0 \quad \rightarrow \quad \frac{\mathrm{d}F_i}{\mathrm{d}P_i} + \lambda\frac{\partial P_{loss}}{\partial P_i} = \lambda$$

  - the transmission network loss is a function of the impedances and the currents flowing in the network
    - for convenience, the currents may be considered functions of the input and load powers
  - it is more difficult to solve this set of equations

Network Losses

- Example
  - repeat the first example, but include a simplified loss expression for the transmission network

$$P_{loss} = 0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2$$

    - the incremental cost functions and the constraint function are formed as:

$$\frac{\mathrm{d}F_i}{\mathrm{d}P_i} = \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_i}\right)$$

$$\mathrm{d}F_1/\mathrm{d}P_1 = 23.76 + 0.009372P_1 = \lambda\left[1 - 2(0.00003)P_1\right]$$

$$\mathrm{d}F_2/\mathrm{d}P_2 = 23.55 + 0.01164P_2 = \lambda\left[1 - 2(0.00009)P_2\right]$$

$$\mathrm{d}F_3/\mathrm{d}P_3 = 23.70 + 0.02892P_3 = \lambda\left[1 - 2(0.00012)P_3\right]$$

$$P_1 + P_2 + P_3 - 850 = P_{loss} = 0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2$$

      - this is no longer a set of linear equations as before

Network Losses

- Example
  - a new iterative solution procedure

step 1 pick starting values for P_1, P_2, and P_3 that sum to the load

step 2 calculate $\partial P_{\text{loss}}/\partial P_{\text{i}}$ and the total losses P_{loss}

step 3 calculate λ that causes P_1, P_2, & P_3 to sum to P_{load} & P_{loss}

step 4 compare P_1, P_2, & P_3 of step 3 to the values used in step 2; if there is significant change to any value, go back to step 2, otherwise, the procedure is done

- pick generation values

$$P_1 = 400 \text{ MW} \quad P_2 = 300 \text{ MW} \quad P_3 = 150 \text{ MW}$$

- find the incremental losses

$$\partial P_{loss}/\partial P_1 = 2(0.00003)(400) = 0.0240$$

$$\partial P_{loss}/\partial P_2 = 2(0.00009)(300) = 0.0540$$

$$\partial P_{loss}/\partial P_3 = 2(0.00012)(150) = 0.0360$$

Network Losses

- Example
  - total losses

$$P_{loss} = 3\times10^{-5}(400)^2 + 9\times10^{-5}(300)^2 + 12\times10^{-5}(150)^2 = 15.6\text{ MW}$$

  - solve for λ

$$23.76 + 0.009372P_1 = \lambda(1-0.024)$$

$$23.55 + 0.01164P_2 = \lambda(1-0.054)$$

$$23.70 + 0.02892P_3 = \lambda(1-0.036)$$

$$P_1 + P_2 + P_3 - (850+15.6) = 0$$

  - in matrix form

$$\begin{bmatrix} -0.009372 & 0 & 0 & 0.976 \\ 0 & -0.01164 & 0 & 0.946 \\ 0 & 0 & -0.02892 & 0.964 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 23.76 \\ 23.55 \\ 23.70 \\ 865.6 \end{bmatrix}$$

$$P_1 = 437.20 \quad P_2 = 296.49 \quad P_3 = 131.91 \quad \lambda = 28.54$$

Network Losses

- Example
  - since the values of P_1, P_2, and P_3 are quite different from the starting values, we return to step 2
    - find the incremental losses and total losses

$$\partial P_{loss}/\partial P_1 = 2(0.00003)(437.2) = 0.0262$$

$$\partial P_{loss}/\partial P_2 = 2(0.00009)(296.5) = 0.0534$$

$$\partial P_{loss}/\partial P_3 = 2(0.00012)(131.9) = 0.0317$$

$$P_{loss} = 3\times 10^{-5}(437.2)^2 + 9\times 10^{-5}(296.5)^2 + 12\times 10^{-5}(131.9)^2 = 15.73\text{ MW}$$

    - solve for λ in matrix form

$$\begin{bmatrix} -0.009372 & 0 & 0 & 0.9738 \\ 0 & -0.01164 & 0 & 0.9466 \\ 0 & 0 & -0.02892 & 0.9683 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 23.76 \\ 23.55 \\ 23.70 \\ 865.73 \end{bmatrix}$$

$$P_1 = 431.03 \quad P_2 = 298.38 \quad P_3 = 136.33 \quad \lambda = 28.55$$

Network Losses

- Example
  - summarization of the iteration process

iteration count	P_1 (MW)	P_2 (MW)	P_3 (MW)	losses (MW)	λ (\$/MWh)
1	400.00	300.00	150.00	15.60	28.54
2	437.20	296.49	131.91	15.73	28.55
3	431.03	298.38	136.33	15.82	28.55
4	432.45	297.92	135.45	15.80	28.55
5	432.11	298.06	135.63	15.81	28.55
6	432.19	298.02	135.59	15.80	28.55
7	432.17	298.03	135.60	15.80	28.55