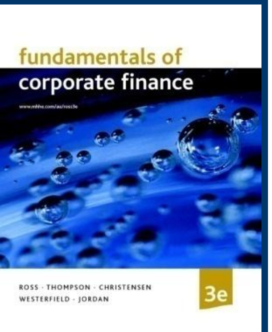
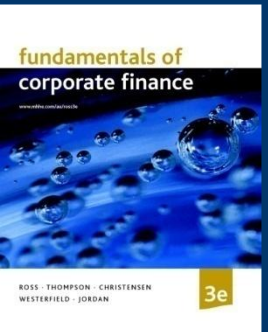


Chapter Five

Introduction to Valuation: The Time Value of Money



Chapter Outline

5.1 Future Value and Compounding

5.2 Present Value and Discounting

5.3 More on Present and Future Values

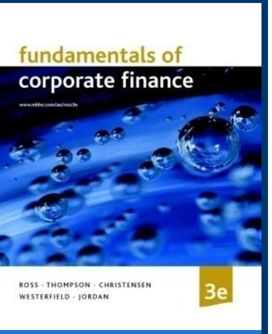
5.4 Present and Future Values of Multiple Cash Flows

5.5 Valuing Equal Cash Flows: Annuities and Perpetuities

5.6 Comparing Rates: The Effect of Compounding Periods

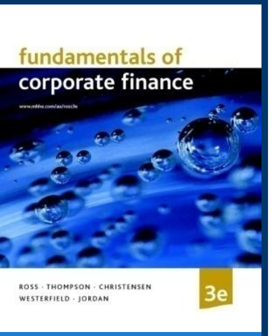
5.7 Loan Types and Loan Amortization

5.8 Summary and Conclusions



Chapter Objectives

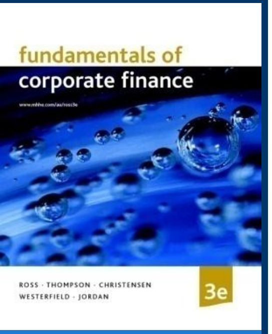
- Distinguish between simple and compound interest.
- Calculate the present value and future value of a single amount for both one period and multiple periods.
- Calculate the present value and future value of multiple cash flows.
- Calculate the present value and future value of annuities.
- Compare nominal interest rates (NIR) and effective annual interest rates (EAR).
- Distinguish between the different types of loans



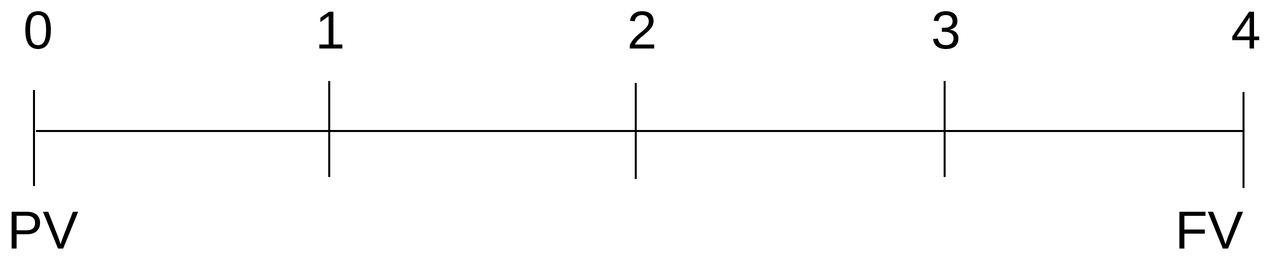
Time Value of money

In the most general sense, the phrase time value of money refers to the fact that a dollar in hand today is worth more than a dollar promised at some time in the future.

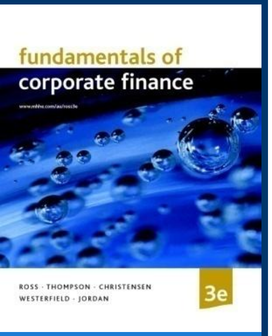
On a practical level, one reason for this is that you could earn interest while you waited; so a dollar today would grow to more than a dollar later. The trade-off between money now and money later thus depends on the rate you can earn by investing.



Time Value Terminology

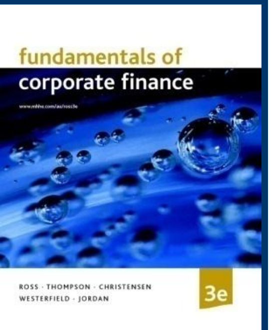


- **Future value (FV**) refers to the amount of money an investment will grow to over some period of time at some given interest rate. Put another way, future value is the cash value of an investment at some time in the future.
- **Present value (PV)** is the current value of one or more future cash flows from an investment.



Time Value Terminology

- The number of time periods between the present value and the future value is represented by '*t*'.
- The rate of interest for discounting or compounding is called '*r*'.
- All time value questions involve four values: PV, FV, *r* and *t*. Given three of them, it is always possible to calculate the fourth.



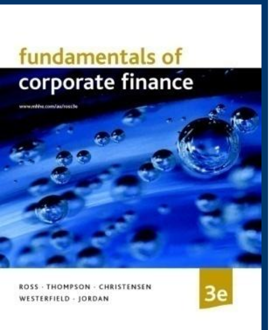
Interest Rate Terminology

- **Simple interest** refers to interest earned only on the original capital investment amount.

  With simple interest, the interest is not reinvested, so interest is earned each period only on the original principal.

- **Compound interest** refers to interest earned on both the initial capital investment and on the interest reinvested from prior periods.

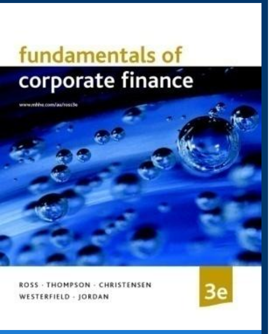
  Compounding the interest means earning interest on interest, so we call the result compound interest



Future Value of a Lump Sum

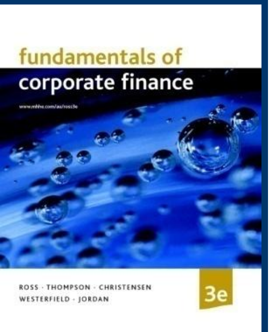
You invest \$100 in a savings account that earns 10 per cent interest per annum (compounded) for three years.

After one year:	$\$100 \times (1 + 0.10) = \$110$
After two years:	$\$110 \times (1 + 0.10) = \$121$
After three years:	$\$121 \times (1 + 0.10) = \$133.10$



Future Value of a Lump Sum

- The accumulated value of this investment at the end of three years can be split into two components:
  - original principal $100
  - interest earned $33.10
- Using simple interest, the total interest earned would only have been $30. The other $3.10 is from compounding.

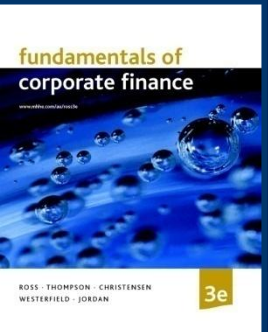


Future Value of a Lump Sum

- In general, the future value, FV_t, of $1 invested today at r per cent for t periods is:

$$FV_t = \$1 \times (1 + r)^t$$

- The expression $(1 + r)^t$ is the future value (interest) factor (FVIF)



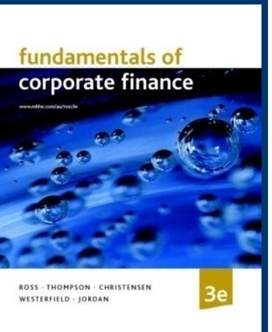
Example—Future Value of a Lump Sum

- What will \$1000 amount to in five years time if interest is 12 per cent per annum, compounded annually?

$$
\begin{aligned}
FV &= \$1000\,(1+0.12)^5 \\
&= \$1000 \times 1.7623 \\
&= \$1\,762.30
\end{aligned}
$$

- From the example, now assume interest is 12 per cent per annum, compounded monthly.
- Always remember that *t* is the number of compounding periods, not the number of years.

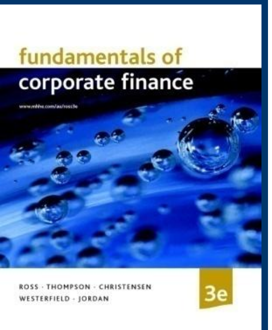
$$
\begin{aligned}
FV &= \$1000\,(1+0.01)^{60} \\
&= \$1000 \times 1.8167 \\
&= \$1816.70
\end{aligned}
$$



Interpretation

- The difference in values is due to the larger number of periods in which interest can compound.

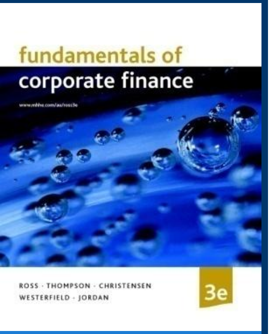
- Future values also depend critically on the assumed interest rate—the higher the interest rate, the greater the future value.

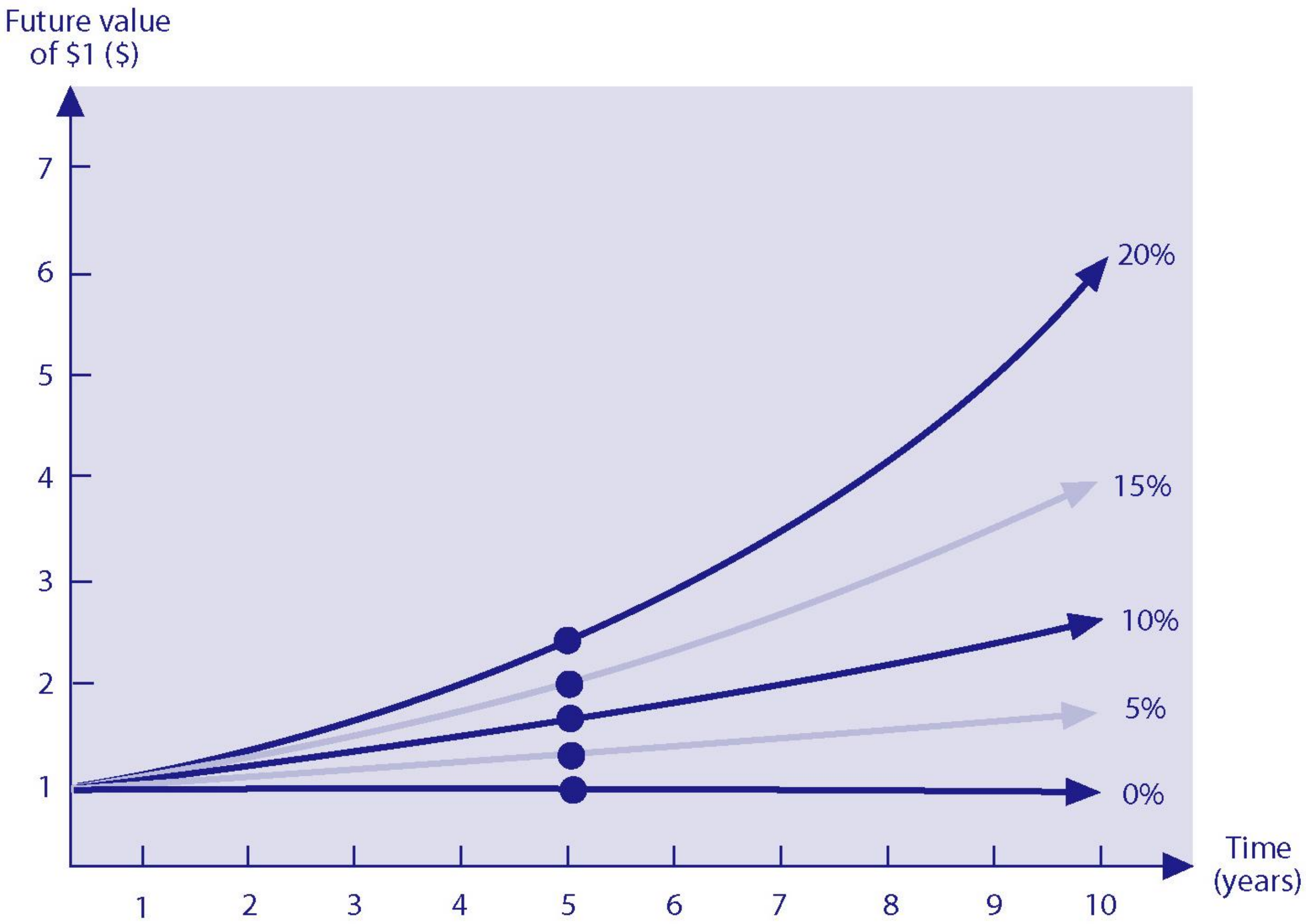


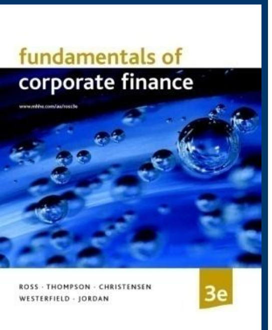
Future Values at Different Interest Rates

Number of periods	Future value of $100 at various interest rates			
	5%	10%	15%	20%
1	$105.00	$110.00	$115.00	$120.00
2	$110.25	$121.00	$132.25	$144.00
3	$115.76	$133.10	$152.09	$172.80
4	$121.55	$146.41	$174.90	$207.36
5	$127.63	$161.05	$201.14	$248.83

Future Value of $1 for Different Periods and Rates



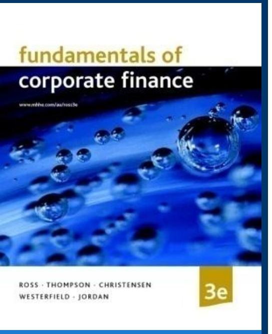




Present Value of a Lump Sum

You need \$1000 in three years time. If you can earn 10 percent per annum, how much do you need to invest now?

Discount one year:	$\$1000\ (1 + 0.10)^{-1}$	$= \$909.09$
Discount two years:	$\$909.09\ (1 + 0.10)^{-1}$	$= \$826.45$
Discount three years:	$\$826.45\ (1 + 0.10)^{-1}$	$= \$751.32$

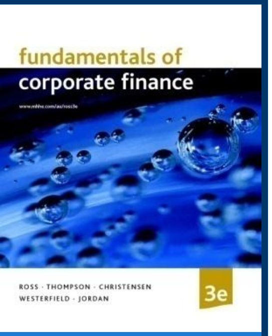


Interpretation

- In general, the present value of $1 received in t periods of time, earning r percent interest is:

$$\text{PV} = \$1 \times (1+r)^{-t} = \frac{\$1}{(1+r)^t}$$

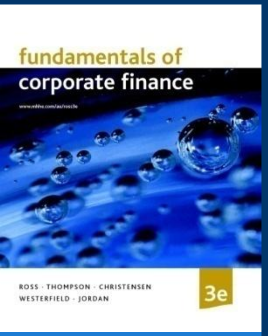
- The expression $\frac{1}{(1+\text{r})^{\text{t}}}$ is the *present value interest factor* (PVIF).



Example—Present Value of a Lump Sum

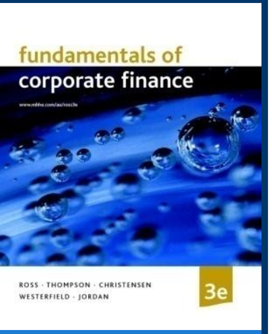
Your rich grandmother promises to give you $10 000 in 10 years time. If interest rates are 12 per cent per annum, how much is that gift worth today?

$$PV = \$10\ 000 \times (1 + 0.12)^{-10}$$
$$= \$10\ 000 \times 0.3220$$
$$= \$\ 3220$$

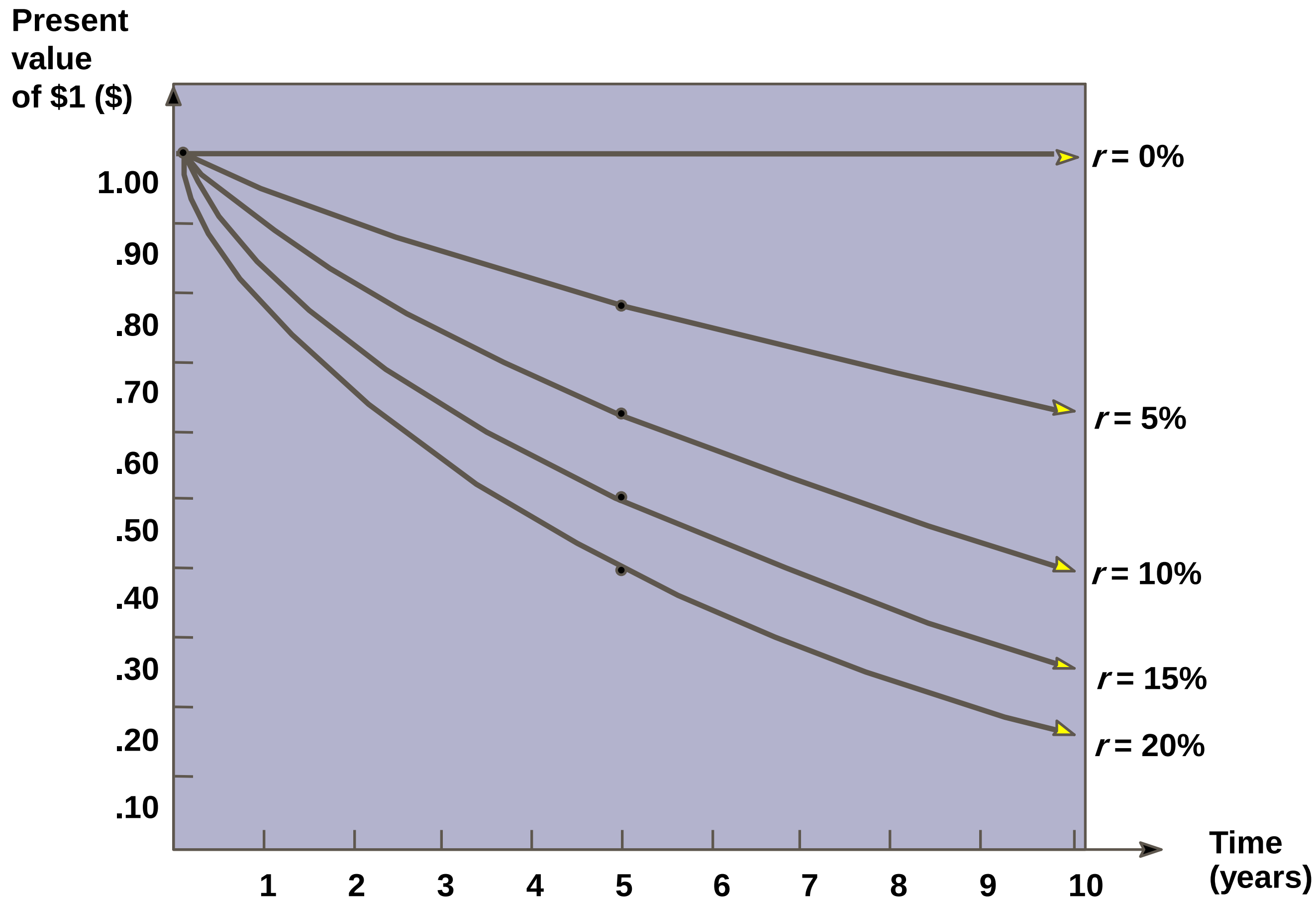


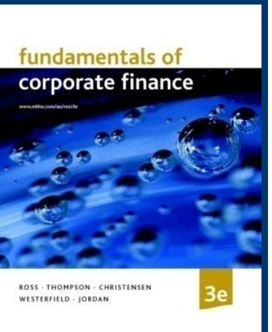
Present Values at Different Interest Rates

Number of periods	Present value of $100 at various interest rates			
	5%	10%	15%	20%
1	$95.24	$90.91	$86.96	$83.33
2	$90.70	$82.64	$75.61	$69.44
3	$86.38	$75.13	$65.75	$57.87
4	$82.27	$68.30	$57.18	$48.23
5	$78.35	$62.09	$49.72	$40.19



Present Value of $1 for Different Periods and Rates



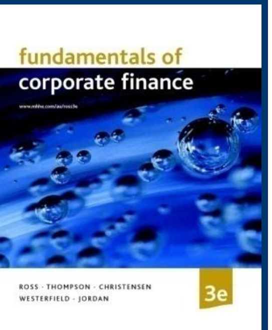


The Rule of 72

- The ‘Rule of 72’ is a handy rule of thumb that states:

  If you earn *r* per cent per year, your money will double in about 72/*r* percent years.

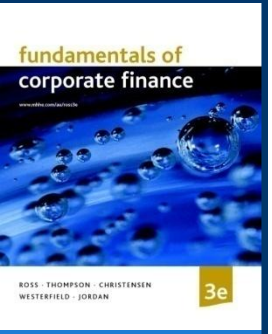
- For example, if you invest at 6 per cent, your money will double in about 12 years.
- This rule is only an approximate rule.



Future Value of Multiple Cash Flows

You deposit $1000 now, $1500 in one year, $2000 in two years and $2500 in three years in an account paying 10 per cent interest per annum. How much do you have in the account at the end of the third year?

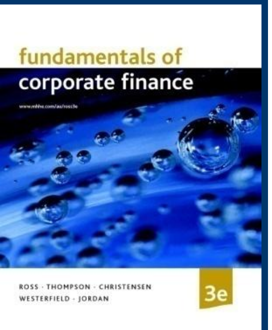
$\$1000 \times (1.10)^3$	=	\$1331
$\$1500 \times (1.10)^2$	=	\$1815
$\$2000 \times (1.10)^1$	=	\$2200
$\$2500 \times 1.00$	=	\$2500
Total	=	\$7846



Present Value of Multiple Cash Flows

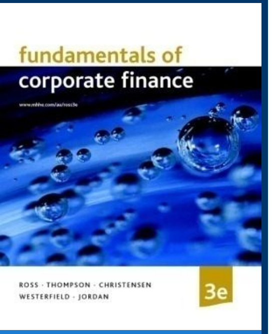
You will deposit $1500 in one year's time, $2000 in two years time and $2500 in three years time in an account paying 10 per cent interest per annum. What is the present value of these cash flows?

$\$2500 \times (1.10)^{-3}$	=	$1878
$\$2000 \times (1.10)^{-2}$	=	$1653
$\$1500 \times (1.10)^{-1}$	=	<u>$1364</u>
Total	=	<u>$4895</u>



Annuities

- An **ordinary annuity** is a series of equal cash flows that occur at the end of each period for some fixed number of periods.
- Examples include consumer loans and home mortgages.
- A **perpetuity** is an annuity in which the cash flows continue forever. the cash flows are perpetual. Perpetuities are also called **consol.**



Present Value of an Annuity

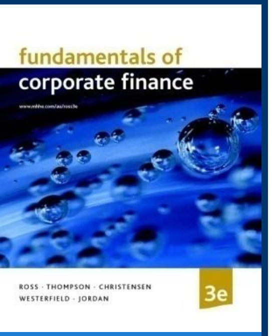
$$\text{Annuity present value} = C \times \left(\frac{1 - \text{Present value factor}}{r}\right)$$

$$= C \times \left(\frac{1 - [1/(1 + r)^t]}{r}\right)$$

C = equal cash flow

- The discounting term is called the present value interest factor for annuities (PVIFA)

$$\text{PVIFA} = \frac{1 - \text{Present value factor}}{r}$$



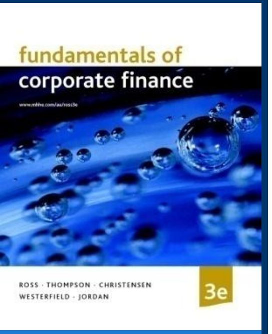
Example 1

You will receive $500 at the end of each of the next five years. The current interest rate is 9 per cent per annum. What is the present value of this series of cash flows?

$$PV = \$500 \times \left[\frac{1 - \left\{ 1/(1.09)^5 \right\}}{0.09} \right]$$

$$= \$500 \times 3.8897$$

$$= \$1\,944.85$$



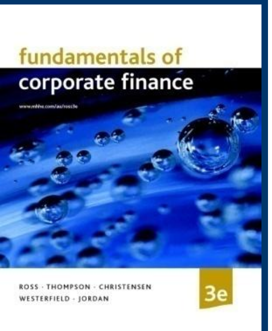
- Example 2

You borrow $7500 to buy a car and agree to repay the loan by way of equal monthly repayments over five years. The current interest rate is 12 percent per annum, compounded monthly. What is the amount of each monthly repayment?

$$\$7\,500 = C \times \left[\frac{1 - \left\{ 1/(1.01)^{60} \right\}}{0.01} \right]$$

$$C = \$7\,500 \div 39.1961$$

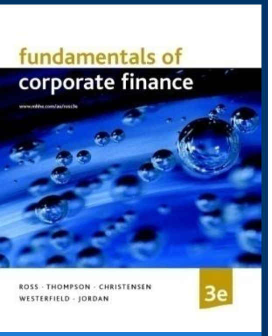
$$= \$191.35$$



Future Value of an Annuity

$$\text{FV} = C \times \frac{\left[(1+r)^t - 1\right]}{r}$$

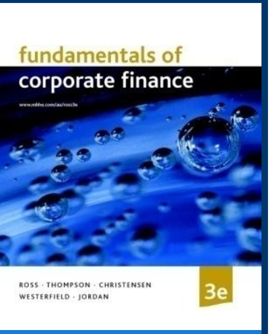
- The compounding term is called *the future value interest factor for annuities* (FVIFA).



Example—Future Value of an Annuity

What is the future value of $200 deposited at the end of every year for 10 years if the interest rate is 6 per cent per annum?

$$
\begin{aligned}
FV &= \$200 \times \frac{\left[(1.06)^{10} - 1\right]}{0.06} \\
&= \$200 \times 13.181 \\
&= \$2636.20
\end{aligned}
$$

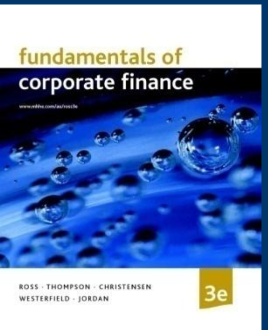


Perpetuity

- The future value of a perpetuity cannot be calculated as the cash flows are infinite.

- The present value of a perpetuity is calculated as follows:

$$\text{PV} = \frac{C}{r}$$



Comparing Rates

If a rate is quoted as 10 percent compounded semiannually, then what this means is that the investment actually pays 5 percent every six months.

A natural question then arises: Is 5 percent every six months the same thing as 10 percent per year?

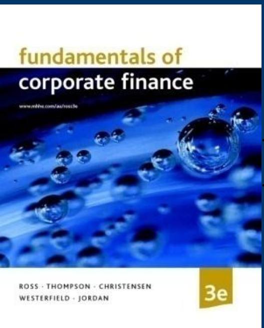
If you invest $1 at 10 percent per year, you will have $1.10 at the end of the year.

$$1 \times 1.1 = 1.1$$

If you invest at 5 percent every six months, then you'll have the future value of $1 at 5 percent for two periods, or:

$$\$1 \times 1.05^2 = \$1.1025$$

Comparing Rates

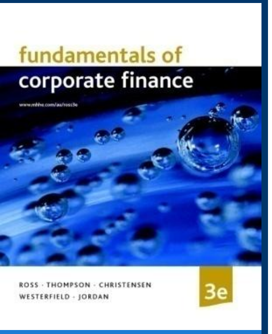


In our example, the 10 percent is called a **stated or quoted interest rate**.

The 10.25 percent, which is actually the rate that you will earn, is called the **effective annual rate (EAR).**

To compare different investments or interest rates, we will always need to convert to effective rates

Comparing Rates

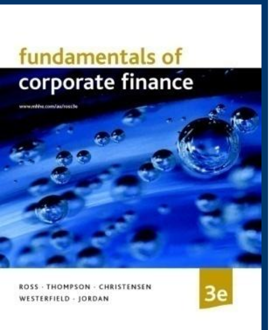


- **The nominal interest rate (NIR)** is the interest rate expressed in terms of the interest payment made each period.

  The stated interest rate, quoted interest rate, and annual percentage rate (APR) are names that refer to the Nominal interest rate (NIR).

- **The effective annual interest rate (EAR)** is the interest rate expressed as if it was compounded once per year.

  When interest is compounded more frequently than annually, **the EAR will be greater than the NIR**.

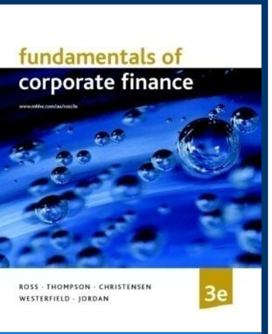


Calculation of EAR

$$\text{EAR} = \left[1 + \frac{NIR}{m}\right]^{m} - 1$$

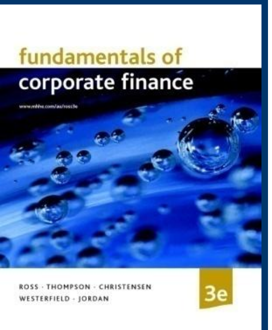
m = number of times the interest is compounded

NIR = The nominal interest



Comparing EARS

- Consider the following interest rates quoted by three banks:
  - Bank A: 15%, compounded daily
  - Bank B: 15.5%, compounded quarterly
  - Bank C: 16%, compounded annually

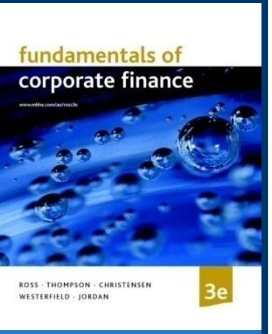


Comparing EARS

$$EAR_{Bank\,A} = \left[1 + \frac{0.15}{365}\right]^{365} - 1 = 16.18\%$$

$$EAR_{Bank\,B} = \left[1 + \frac{0.155}{4}\right]^{4} - 1 = 16.42\%$$

$$EAR_{Bank\,C} = \left[1 + \frac{0.16}{1}\right]^{1} - 1 = 16\%$$

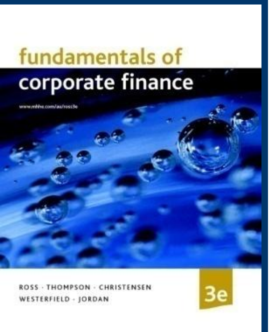


Comparing EARS

- Which is the best rate? For a saver, Bank B offers the best (highest) interest rate. For a borrower, Bank C offers the best (lowest) interest rate.

- The highest NIR is not necessarily the best.

- Compounding during the year can lead to a significant difference between the NIR and the EAR.

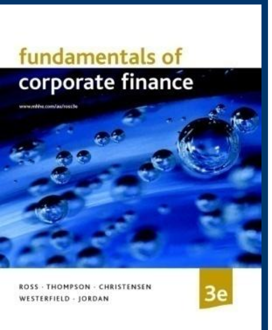


Types of Loans

- A **pure discount loan** is a loan where the borrower receives money today and repays a single lump sum in the future.

- An **interest-only loan** requires the borrower to only pay interest each period and to repay the entire principal at some point in the future.

- **An amortized loan** requires the borrower to repay parts of both the principal and interest over time.



Loan amortization schedule

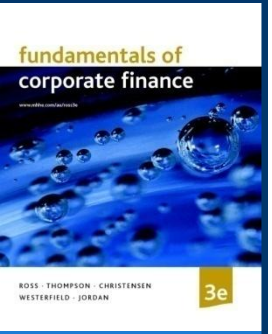
Suppose:

loan= $5,000

t= five-year

r = 9 percent

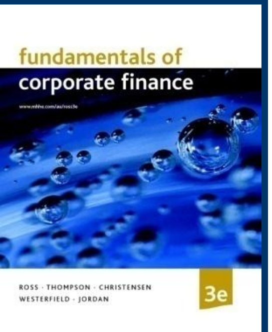
Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000.00	$1,285.46	$ 450.00	$ 835.46	$4,164.54
2	4,164.54	1,285.46	374.81	910.65	3,253.88
3	3,253.88	1,285.46	292.85	992.61	2,261.27
4	2,261.27	1,285.46	203.51	1,081.95	1,179.32
5	1,179.32	1,285.46	106.14	1,179.32	0.00
Totals		$6,427.30	$1,427.31	$5,000.00	



Test Questions

1. In a typical loan amortization schedule, the total dollar amount of money paid each:

1. increases with each payment
2. decreases with each payment
3. remains constant with each payment
4. None of the above



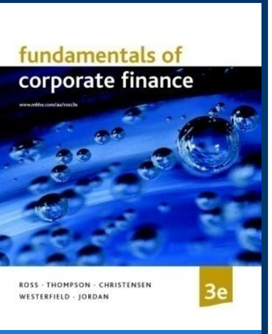
Test Questions

2. You can use to roughly estimate how many years a given sum of money must earn at a given compound annual interest rate in order to double that initial amount :

1.Rule 415   2. the Rule of 72   3.the Rule of 78   4. Rule 144

3. The expression $(1 + r)^t$ is the:

1. PVIF   2. FVIF   3. PVIFA   4.FVIFA



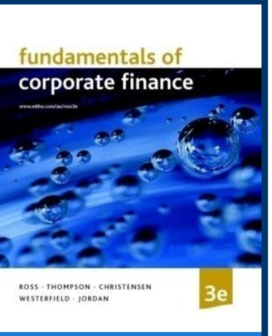
Test Questions

4. To increase a given present value, the discount rate should be adjusted:

1. Upward 2.Downward 3.True 4.Nominal

5……….. is a constant series of cash flows, occurring at regular intervals that continues forever.

1.Continuity 2. Perpetuity 3. Pension 4. Annuity



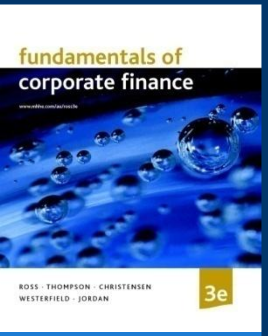
Test Questions

6. If you invest 100 for two years at an annual interest rate of 4% compounded semi annually, the end value is:

1. 104.25 2. 108 3.108.24 4. 116

7. the effective annual rate

1. Increases with the compounding frequency
2. Decreases with the compounding frequency
3. Does not depend on the compounding frequency
4. none



Test Questions

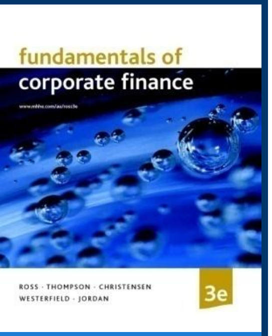
8. We need to calculate the future value of $10,000 at 6 percent for five years. The future value factor is:

a: 1.06^5

b: 1/(1.06^5)

c: 1.06

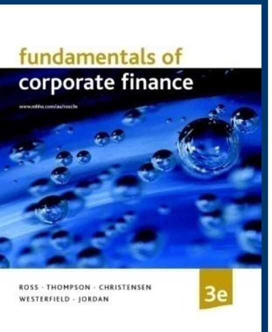
d: (1- (1/1.06))



Test Questions

9. If the compound period is greater than one?

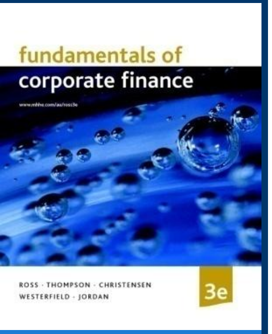
1. the effective annual interest rate is always equal to the annual percentage rate.
2. the effective annual interest rate is always less than the annual percentage rate.
3. the effective annual interest rate is always greater than the annual percentage rate.
4. None of the above



Test Questions

10. The time value of money concept can be defined as:

1. The time in your life when you receive an inheritance
2. The relationship between money spent versus money received.
3. The relationship between a dollar to be received in the future and a dollar today
4. non of above



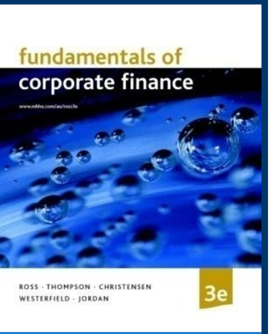
Test Questions

11. you are considering a one-year investment. If you put up $ 100, you would get back $ 112 what rate is this investment paying?

1. 12% 2. 11.2% 3. 1.12% 4.Non of the above

12. Comparisons of investment alternatives with different compounding periods should be made based on the:

1. nominal interest rates (NIR)
2. quoted interest rates
3. annual percentage rates (APR)
4. effective annual interest rate (EAR)



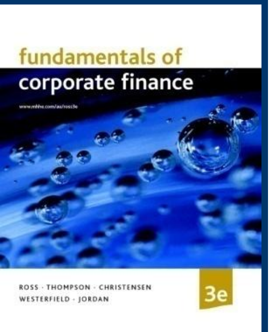
Test Questions

13. The process of finding the present value of a cash flow or a series of cash flows is called:

1. Compounding
2. time lines
3. the opportunity cost rate
4. discounting

14. An annuity whose payments occur at the end of each period for fixed number of periods is called:

1. an ordinary annuity
2. A perpetuity
3. an opportunity cost annuity
4. Non of the above



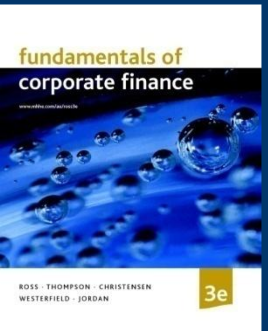
Test Questions

15. A loan that is repaid in equal payments over its life with each payment including a portion of interest and principal is:

1. A pure discount loan
2. an interest only loan
3. An amortized loan
4. all of the above

16. The future value of a lump sum will:

1. increase if the interest rate increases
2. be unchanged if the interest rate changes
3. decrease if the interest rate increases
4. Non of the above



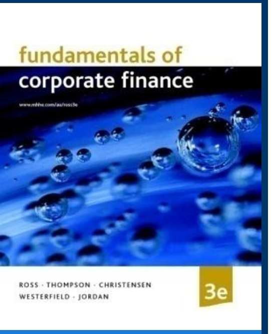
Test Questions

17. The present value of a lump sum to be received in the future:

1. increase if the interest rate increases
2. be unchanged if the interest rate changes
3. decrease if the interest (discount) rate increases
4. Non of the above

18. The effective annual rate of interest is:

1. greater than or equal to the nominal interest rate
2. always equal to the nominal interest rate
3. less than or equal to the nominal interest rate



Concept Questions

1. In general, what is the present value of $1 to be received in t periods, assuming a discount rate of r per period?
2. What do we mean by the present value of an investment?
3. In general, what is the present value of a perpetuity?
4. What is an APR? What is an EAR? Are they the same thing?
5. What is a pure discount loan? An interest-only loan?
6. What does it mean to amortize a loan?