# A Novel Fuzzy Model for Dynamic Systems

Sajjad Yazdani, Mohammad Bagher Menhaj, and Masoud Shahrabi
The Center of Excellence on Control and Robotics
Department of Electrical Engineering
Amirkabir University of Technology
Tehran, Iran
{sajjad.yazdani, menhaj ,masoudshahrabi}@aut.ac.ir

***Abstract*—This paper presents a fuzzy model for dynamic systems. The proposed fuzzy model takes the fuzzy sets as feedback signals in such a way that there is no need to put both the fuzzification and defuzzification blocks in the feedback loop, instead the pure fuzzy sets are used as the feedback signals. The experimental results easily illustrate that the proposed fuzzy model performs better than the ordinary one.**

***Keywords*— Fuzzy Modeling, Dynamic systems, Fuzzy system approximation**

## I. Introduction

The theory of fuzzy sets is based on uncertainty and vagueness. Fundamentally, it is a theory of graded concepts, where everything is a matter of degree, or to put it figuratively, everything has a degree of elasticity [1].As a universal function approximator, fuzzy systems can be used as a good tool for modeling. As it is making progress everyday and being more widely used, due to practical successes in consumer products and industrial process control, there has been an increasing amount of work on the rigorous theoretical studies of fuzzy systems and fuzzy control [2]. For example, more complex systems can be modeled especially when mathematical approaches failed in system modeling.

The objective of a plant modeling is to introduce a structure responding to an input as close as the original system [3]. There is always some uncertainty in modeling of plant and further that any smooth function can be approximated by fuzzy systems as shown by Wang in 1992[4]. This universal function approximation property of fuzzy systems makes them very suitable for modeling. The basic principles of fuzzy modeling were stated by Zade [5]. The Zade's proposal of modeling, the mechanism of human thinking with linguistic fuzzy values instead of numbers led to the introduction of fuzziness into systems theory. There are numerous successful applications of fuzzy systems in control and modeling [6]. Babuska proposed NARX model for discrete input-output modeling of dynamic systems [7]. There are a lot of researches on the fuzzy modeling and its properties [8, 9, 10, 11].

To make a proper background, the basic concept of using the fuzzy inference model for dynamic systems approximation is briefly explained in the next section followed by the proposed model which is fully described in section III. Implementation and simulation results are presented in section IV and finally the article is concluded in section V.



## II. Dynamic Fuzzy Inference Model

Generally speaking, a time invariant finite dimensional lumped dynamical system including dynamic control and dynamic models can be described in a state space form by differential equations as:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases} \tag{1}$$

where $u$ denotes the system input signal, $x$ is the state vector and $y$ is the output.

For discrete-time systems the dynamical equation can be expressed as:

$$\begin{cases} x^{t+1} = f(x^t, u^t) \\ y^t = g(x^t, u^t) \end{cases} \tag{2}$$

Fig. 1 depicts the block diagram of this system.

One of popular technique to handle uncertainty and vagueness is using fuzzy models [12]. Therefore, fuzzy approximators can be used to model $f$ and $g$.

This section discusses fuzzy approximation over a particular system. For a better understanding and ease of explanation we consider a simple system with two inputs and

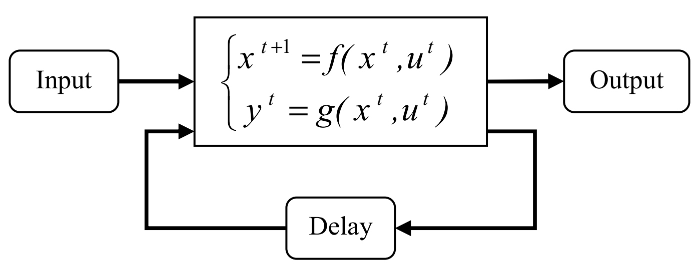


Fig.1. A dynamic system block diagram.

one output. The problem here is to estimate an unknown function (black box) like $y = h(u_1, u_2)$ , in which the output of function for each $u_1$ and $u_2$ in some bounded region is given in advance. To obtain an approximation of a function by fuzzy systems, we should go through the following three steps [4]:

- Step 1. Specify some fuzzy sets on each linguistic variable defined over some proper universal set $U_1$ containing $u_1$ and $\mathbf{U}_2$ containing $u_2$ so that a specific value for every input exists.
- Step 2. Construct a set of *M* fuzzy if-then rules that makes the knowledge database. Generally speaking, the if-then rules are in form of "if $u_1$ is $\mathbf{A}_1$ and $u_2$ is $\mathbf{A}_2$, then $y$ is **B**", where $\mathbf{A}_i$ and **B** are fuzzy sets.
- Step 3. Construct the fuzzy system from the fuzzy sets and rules obtained so far and by employing suitable methods for inference, fuzzification and defuzzification mechanisms.

The modeling procedure in classical fuzzy modeling of dynamic systems is almost the same as the above, with the only difference that some inputs and outputs should be replaced by states of the system.

Fig.2 presents a block diagram representation of the common fuzzy modeling of dynamic systems. In fact, $f$ and $g$ are approximated by some appropriately developed fuzzy inference system (FIS). In this ordinary fuzzy modeling of dynamic systems the state space variables are considered to be crisp.

## III. The Proposed Dynamic Fuzzy Inference Model

In this study, we propose a new model given in Fig.3. As shown in this figure, some FIS output variables are given directly to the input with one unit delay. The feedback signals are not crisp, instead they are fuzzy sets. In other words, the fuzzification and defuzzification parts have been omitted in feedback loop and the pure fuzzy sets are instead used as the feedback signals.

Consequently, the state values are not required to be crisp, while in an ordinary fuzzy modeling, they are forced to be crisp through the feedback defuzzification process. In the ordinary fuzzy models, we may lose information due to the defuzzification process, while in the proposed model, all of the information exists in the feedback signals.

The main idea in the proposed method is using the pure fuzzy sets as feedback signals. The fuzzy rules knowledge base of the proposed model and the ordinary model both have the same expression but have different sense. In ordinary model, antecedent part of rules is in the form of "$x$ is $A$ " where $x$ is crisp and $A$ is a fuzzy set. The truth-value of the rule depends on membership function value of $A$ at $x$ (i.e. $A(x)$ ). In the proposed model, the system input is processed in the same way through the "Fuzzifier" but the states of system are processed differently by feeding back directly from the inference mechanism block, which is indeed fuzzy variable.

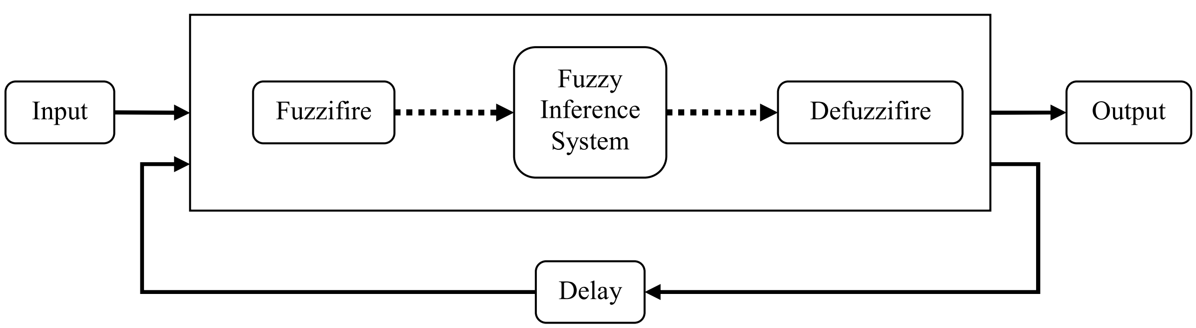


Fig.2. Ordinary fuzzy dynamic (numerical feedback) model.
solid lines represent the crisp values and dotted line represent fuzzy values.

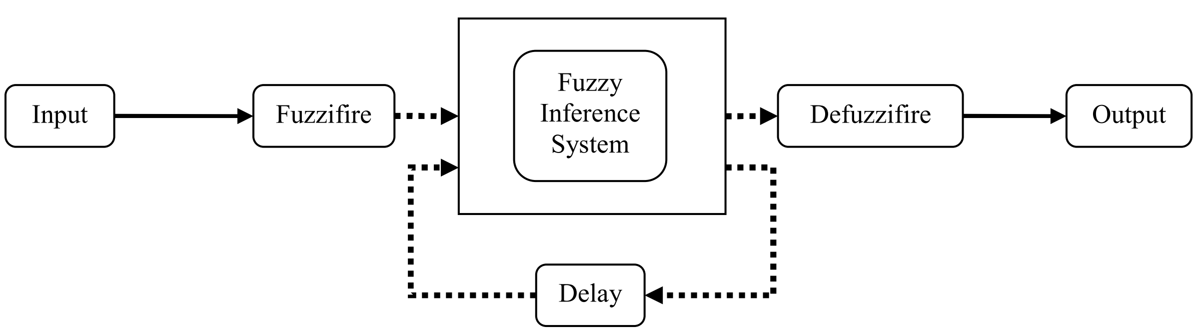


Fig.3. Proposed fuzzy dynamic (fuzzy feedback) model
solid lines represent the crisp values and dotted line represent fuzzy values.

To handle "$X$ is $A$" where both $X$ and $A$ are fuzzy sets, we use the concept of degree of subsethood. In the literature, there are several well-known measures of subsethood [13]. This concept rooted in the crisp subsethood and can be calculated by (2).

$$\deg(X \subseteq A) = \frac{1}{|X|}\left(|X| - \sum_{s\in U} \max\left(0, X(s) - A(s)\right)\right) \quad (2)$$

where $|X|$ is cardinality of $X$ which is defined on a finite universal set **U.** Cardinality of $X$ can be calculated by (3).

$$|X| = \sum_{s\in U} X(s) \quad (3)$$

In order to understand the degree of subsethood concept for fuzzy sets, ssuppose we have a linguistic variable with the name of temperature. One of its values is *hot*. Suppose "air with the temperature of 30° is *hot* " is true by 0.6 degree of truth value, which is easily understandable from membership function of *hot* (i.e. **hot**(30°) ). Now assume that we do not have the exact value of the temperature. As a solution, we might use fuzzy sets, i.e., define the temperature by a fuzzy set. Now the question is that how much the weather is hot?

Assume the linguistic variable value "hot" is a fuzzy set **B**(t) where t is temperature. Also the temperature of air is a fuzzy set **A**(t). The question "how much the air is hot?" is answered by a value in [0,1], which is the degree of subsethood of **A** in **B**.

For implementation of this model we have to use the finite universal sets which are obtained by sampling the continuous universal set with finite number of samples (e.g. 100 points distributed in linguistic variable range). In fact, the membership function values of samples are fed back to the system.

Other calculations in the proposed model such as input signal fuzzification, output signal defuzzification and remaining inference processes are the same as the those given in the ordinary one. It should be noted that the input signal and also the output signal of the system are obtained by the same sampling rate of the fuzzy sets.

Although in the proposed model, the degree of subsethood computation increases the amount of calculation burden of FIS though those of fuzzifier and defuzifier are decreased. Also, the amount of memory usage is increased due to feeding back system's states via fuzzy sets in the comparison with the classic fuzzy model.

## IV. SIMULATION RESULTS

For evaluation of the proposed model, we first assume a system with the following dynamic equations:

$$\begin{cases} x_1^{\,t} = 0.7x_1^{\,t-1} + 0.3x_2^{\,t-1} + u^t \\ x_2^{\,t} = 0.4x_1^{\,t-1} + 0.2x_2^{\,t-1} \\ y^t = x_1^{\,t-1} + 2x_2^{\,t-1} \end{cases} \quad (4)$$

where $x_1$ and $x_2$ are the states of the system. For a better understanding and ease of explanation we go through of a simple system with two states.

### *A. Develop a Fuzzy Inferience System*

A fuzzy model for this system is developed based on input-output data pairs which were properly collected from (4). For this system, we can use different number of fuzzy sets for each state, input and output of the system. For easy explanation of the system, let us assume five fuzzy sets for each state. The set of linguistic terms {BN, SN, Z, SP, BP} used for $x_1$ and $x_2$. These terms are abbreviation of "Big Negative", "Small Negative", "Zero", "Small Positive" and "Big Positive", respectively. The system input and output signals have been represented by 3 fuzzy sets and linguistic terms N, Z and P have been assigned to them which are the abbreviations of "Negative", "Zero", and "Positive", respectively. Table I represents the fuzzy if-then rules which are designed based on (4).

Moreover, it is possible to use more fuzzy sets to have more accurate approximation of the real system. In the case of having numerous fuzzy sets in antecedent part, it is so difficult to use the supervisor knowledge for making proper fuzzy rules. Instead of using this method, we increase the quantity of the consequent part fuzzy sets up to the rules number and fuzzy rules are derived from the real system.

For each combination of antecedent part fuzzy sets, a rule will be assigned with specified fuzzy sets for each output in consequent part. The rules are in the form of "if $\mathbf{X_1}$ is $\mathbf{St^{1i}}_i$ and $\mathbf{X_2}$ is $\mathbf{St^{2i}}_j$ and u is $\mathbf{U}_k$ , then $\mathbf{X_1}$ is $\mathbf{St^{1o}}_r$ and $\mathbf{X_2}$ is $\mathbf{St^{2o}}_r$ and y is $\mathbf{Out}_r$" where $\mathbf{St^{1i}}$, $\mathbf{St^{2i}}$ and **U** are input linguistic terms for the 1st state, the 2nd state and the system input, respectively. Moreover, $\mathbf{St^{1o}_r}$, $\mathbf{St^{2o}_r}$ and $\mathbf{Out_r}$ are linguistic terms for the first state, the second state and the system output in the consequent part of $r^{th}$ rule, respectively.

The parameters of the fuzzy sets corresponding to the consequents part are calculated directly from (4) and the parameters of the fuzzy sets corresponding to the antecedent part.

Triangular-shaped membership functions have been used for all fuzzy sets. Fig. 4 illustrates the linguistic term sets for the input signal as an example.

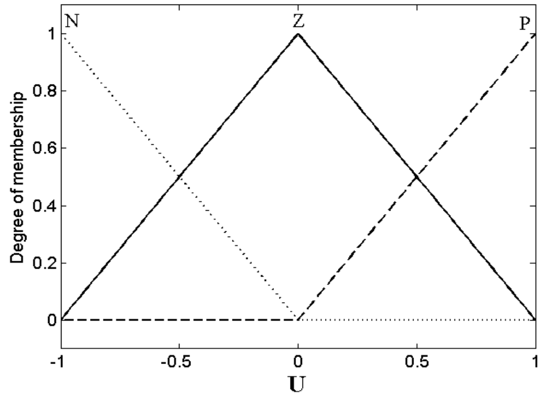


Fig.4. Linguistic term sets for the input signal.

TABLE I. EXAMPLE OF IF-THEN RULE DATA BASE

| $X_1 \backslash X_2$ | | BP | SP | Z | SN | BN |
|---|---|---|---|---|---|---|
| **BP** | $X_1$ (U is **P**) | BP | BP | BP | BP | BP |
| | $X_1$ (U is **Z**) | BP | SP | SP | SP | SN |
| | $X_1$ (U is **N**) | SN | SN | SN | BN | BN |
| | $X_2$ | SN | Z | SP | SP | BP |
| | Y | P | P | P | P | Z |
| **SP** | $X_1$ (U is **P**) | BP | BP | BP | BP | SP |
| | $X_1$ (U is **Z**) | BP | SP | Z | Z | SN |
| | $X_1$ (U is **N**) | SN | BN | BN | BN | BN |
| | $X_2$ | SN | Z | Z | SP | SP |
| | Y | P | P | P | Z | N |
| **Z** | $X_1$ (U is **P**) | BP | BP | BP | BP | SP |
| | $X_1$ (U is **Z**) | BP | SP | Z | SN | BN |
| | $X_1$ (U is **N**) | SN | BN | BN | BN | BN |
| | $X_2$ | SN | SN | Z | SP | SP |
| | Y | P | P | Z | N | N |
| **SN** | $X_1$ (U is **P**) | BP | BP | BP | BP | SP |
| | $X_1$ (U is **Z**) | SP | Z | Z | SN | BN |
| | $X_1$ (U is **N**) | SN | BN | BN | BN | BN |
| | $X_2$ | SN | SN | Z | Z | SP |
| | Y | P | Z | N | N | N |
| **BN** | $X_1$ (U is **P**) | BP | BP | SP | SP | SP |
| | $X_1$ (U is **Z**) | SP | SN | SN | SN | BN |
| | $X_1$ (U is **N**) | SN | BN | BN | BN | BN |
| | $X_2$ | BN | SN | SN | Z | SP |
| | Y | Z | N | N | N | N |

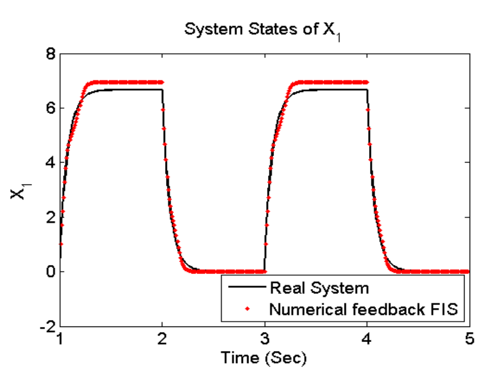


Fig.6 . $x_1$ value in the ordinary fuzzy modeling versus the real value of $x_1$.

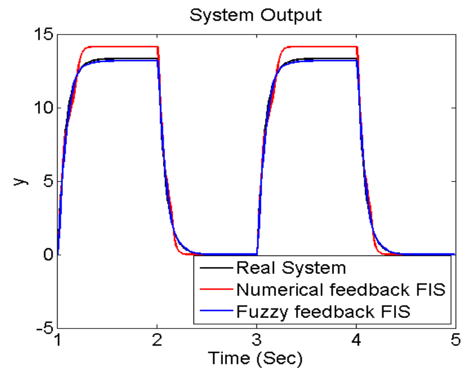


Fig.5. system output for real system, ordinary model and Proposed model .

To have an accurate response, we use 11 fuzzy sets for input states (**St**$^{1i}$ and **St**$^{2i}$) and 3 fuzzy sets for the system input (**U**). By this number of fuzzy sets, 363 rules can be obtained leading in turn to 363 different fuzzy sets (**St**$^{1o}$, **St**$^{2o}$ and **Out**) of the out.

By this method, the rules and membership functions are appropriately designed in order to minimize the modeling errors. Moreover, the centroid method has been used for defuzzification. and the maximum and the product methods are used for "or" and "and" operations, respectively.

### *B. Implimentation of the proposed FIS*

The developed FIS is used in both ordinary and proposed models. Fig.5 presents the output of fuzzy models and system for an arbitrary input signal. It is obvious that the proposed model has a better response in both transient and steady state parts.

The defuzzifier in ordinary model causes the states to tend to a wrong value but the proposed model follows the real system value. As it is clearly seen in Fig.4, in steady state, the output should be near 13, but in the ordinary model the output is almost 14. This error is a simple consequence of the error in states values.

Fig.6 and Fig.7 respectively demonstrate the first system

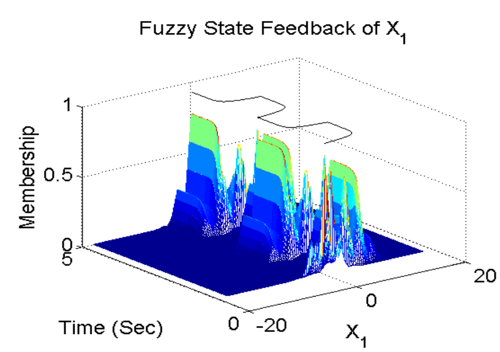


Fig.7. The 1st fuzzy set feedback signal ($x_1$) in the proposed fuzzy model.

state ($x_1$) of the ordinary fuzzy model and the proposed fuzzy model. It should be noted that a feedback signal in the form of fuzzy set has more information about state versus a numerical feedback signal.

## V. CONCLUSION

This paper introduced a fuzzy model for dynamic systems. In the proposed model, feedback signals were indeed fuzzy sets. Also, simulation results indicated that the proposed model was more accurate. However, in the proposed fuzzy model, feedback signal was fuzzy sets and amount of memory usage were increased (in proportion of classic fuzzy model).

## REFERENCES


[1] H. J. Zimmermann, Fuzzy Set Theory and Its Applications.: Kluwer Academic Publishers, 1996.

[2] Li-Xin Wang, *A Course in Fuzzy Systems and Control*., 1997.

[3] Mahdi Karrary*, System Identification*. Tehran: Amirkabir University of Tehran (In Farsi), 1388.

[4] Li-Xin Wang, "Fuzzy systems are universal approximators," in Proc. Of the *IEEE Int. Conf. on Fuzzy Systems*, pp. 1163–1170, San Diego, CA, 8-12 Mar 1992.

[5] L.Zadeh, "Outline of new approach to the analysis of complex systems and decision processes," *IEEE Transactions Systems*, pp. 28-44, 1973.

[6] Ronold R.yager and Dimitar P.Filev, *Essentials of Fuzzy Modeling and Control*. USA, 1994.

[7] Robert Babuska, "Fuzzy Systems, Modeling and Identification,".

[8] Robert Babuska, "Fuzzy modeling-a control engineering perspective," *Proceedings of 1995 IEEE International Conference on Fuzzy Systems, International Joint Conference of the Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium.,* vol. 4, pp. 1897--1902, 1995.

[9] AF Gomez-Skarmeta and F Jimenez, "Fuzzy modeling with hybrid systems," *Fuzzy Sets and Systems*, vol. 104, no. 2, pp. 199--208, 1999.

[10] A Nurnberger and R Kruse, "A neuro-fuzzy approach to optimize hierarchical recurrent fuzzy systems," *Fuzzy Optimization and Decision Making*, vol. 1, no. 2, pp. 221--248, 2002.

[11] Andreas Schwung and Jurgen Adamy, "Modeling with discrete-time recurrent fuzzy systems via mixed-integer optimization," *Fuzzy Sets and Systems*, vol. 203, pp. 1--16, 2012.

[12] Mohammad Bagher Menhaj, *Fuzzy Computation*, 3rd ed. Tehran: Daneshnegar, (In Farsi) 1391.

[13] Dhar, Mamoni., "A Note on Subsethood Measure of Fuzzy Sets." s.l. : International Journal of Energy, Information and Communications, August 2012, no 3, vol. 3.