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International Journal of Systems Science

ISSN: 0020-7721 (Print) 1464-5319 (Online) Journal homepage: http://www.tandfonline.com/loi/tsys20

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To cite this article: Liang-Yuh Ouyang, Kun-Shan Wu, Chih-Te Yang & Hsiu-Feng Yen (2016) Optimal order policy in response to announced price increase for deteriorating items with limited special order quantity, International Journal of Systems Science, 47:3, 718-729, DOI: 10.1080/00207721.2014.902157

To link to this article: http://dx.doi.org/10.1080/00207721.2014.902157

Published online: 31 Mar 2014.

International Journal of Systems Science, 2015
Vol. 47, No. 3, 718–729, http://dx.doi.org/10.1080/00207721.2014.902157

Optimal order policy in response to announced price increase for deteriorating items with limited special order quantity

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(*Received 20 January 2012; accepted 8 September 2012*)

When a supplier announces an impending price increase due to take effect at a certain time in the future, it is important for each retailer to decide whether to purchase additional stock to take advantage of the present lower price. This study explores the possible effects of price increases on a retailer's replenishment policy when the special order quantity is limited and the rate of deterioration of the goods is assumed to be constant. The two situations discussed in this study are as follows: (1) when the special order time coincides with the retailer's replenishment time and (2) when the special order time occurs during the retailer's sales period. By analysing the total cost savings between special and regular orders during the depletion time of the special order quantity, the optimal order policy for each situation can be determined. We provide several numerical examples to illustrate the theories in practice. Additionally, we conduct a sensitivity analysis on the optimal solution with respect to the main parameters.

Keywords: inventory; deteriorating items; price increase; limited special order quantity

1. Introduction

Due to the recent increases in the prices of oil and raw materials, the prices of commodities have continued to increase worldwide. This has become a serious problem for enterprises. In particular, when managers make decisions relating to their inventory policy, it is essential for them to consider increases in commodity prices. When a supplier announces an impending price increase due to take effect at a certain time in the future, it is important for each retailer to decide whether to purchase additional stock before the price increase, to take advantage of the present lower price.

In many of the existing studies in this area, the authors have taken the announcement of a price increase problem into account and have proposed various analytical models to gain more insight into the inferences relating to inventory policy. Naddor (1966) was one of the early researchers who proposed an infinite horizon economic order quantity (EOQ) model where the supplier announces a price increase. Lev and Soyster (1979) developed a finite horizon inventory model and determined optimal ordering policies based on known information about an imminent price increase. Later, Goyal (1979) analysed Lev and Soyster's (1979) model and proposed an alternative method for determining the optimal policy. Taylor and Bradley (1985) extended Naddor's (1966) model and obtained the optimal ordering strategies for situations where the price increase does not coincide with the end of an EOQ cycle. Lev and Weiss (1990) subsequently developed a structure of optimal policies and procedures for computing the optimal policy. Goyal, Srinivasan, and Arcelus (1991) presented a review of a study on inventory policies under one-time-only incentives. Tersine (1996) proposed an economic production quantity (EPQ) model under an announced price increase. Ghosh (2003) and Huang and Kulkarni (2003) presented an infinite-horizon deterministic inventory model under an announced price increase. In contrast to single price change models, a small number of continuous price change models exist within inventory management literature. Erel (1992) and Khouja and Park (2003) considered EOQ models with continuous price changes (price increases or reductions). Recently, Sharma (2009a) has developed inventory models on price increases or temporary price reductions when shortages are allowed and partial backordering.

Most of the above research reveals that retailers are inclined to adopt a special order, with the special order quantity involved being unlimited, when suppliers announce a price increase. In practice, to avoid the retailer hoarding goods for later sale at a higher selling price, the supplier is willing to offer a limited quantity at the current price prior to the price increase. Therefore, the number of goods the retailer can order is also limited. This situation is widely seen in Taiwan. For example, when Taiwan joined the World

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Trade Organization (WTO), most people expected that the price of rice wine would subsequently increase. In this situation, the supplier limited the purchase quantity, while the retailer was inclined to store large quantities of the wine. Consequently, when the supplier announced the price increase, it was reasonable for the retailer to consider this fact when placing a special order.

The above inventory models account for the impact of price changes and focus on the determination of the optimal special order quantity for the retailer. A weakness with most of them is that they neglect the deterioration of goods, which is a common phenomenon. It is well known that certain products, such as medicine, volatile liquids, fruits, and vegetables, will deteriorate when kept in storage for a long period. For such products, losses due to deterioration cannot be ignored when determining the optimal order policy. Inventory problems relating to deteriorating items have been studied widely in previous research. Ghare and Schrader (1963) first developed an EOQ model for an exponentially decaying item for which there is constant demand. Later, Covert and Philip (1973) extended Ghare and Schrader's (1963) model and obtained an EOQ model for a variable deterioration rate, by assuming a two-parameter Weibull distribution. Philip (1974) then developed an inventory model with a three-parameter Weibull distribution deterioration rate. Goyal and Giri (2001) provided an excellent and detailed review of the literature on deteriorating inventory since the early 1990s. Moon, Giri, and Ko (2005) studied an effort to incorporate two opposite physical characteristics of stored items into inventory model ameliorating (value or utility increase with time) and deteriorating. Bakker, Riezebos, and Teunter (2012) have recently undertaken an up-to-date review of the advances made in the field of inventory control of perishable items (deteriorating inventory) since 2001. There is also a large body of literature on deteriorating inventory. These cover issues such as the type of demand (e.g., Begum, Sahoo, & Sahu, 2012; Deng, Lin, & Chu, 2007; Khanra, Sana, & Chaudhuri, 2010; Mishra & Shah, 2008; Skouri & Konstantaras, 2009); accounting for the time-value of money (e.g., Wee & Law, 1999, 2001); allowing shortages and backordering (e.g., Shah, 1998; Sharma, 2006; Yang, 2011); considering multiple items (e.g., Sharma, 2007a, 2007b, 2009b); the EPQ model (e.g., Min, Zhou, Liu, & Wang, 2012; Sharma, 2008a, 2008b, 2009c); and the two-warehouse problem (e.g., Pakkala & Achary, 1992; Sarma, 1987; Yang, 2006).

Thus, in order to address the above economic issues, this study investigates the possible effects of a price increase on a retailer's replenishment policies, where the special order quantity is limited. The contribution of this paper, relative to previous studies, is that we explore inventory decisions and the three issues of the traditional EOQ model simultaneously. These comprise of the following: (1) the retailer expects the price increase at a certain time in the future (as announced by the supplier) and decides whether to place a special order; (2) the goods deteriorate at a constant rate; and (3) the quantity of the special order is limited. Furthermore, because the time for placing the special order may or may not coincide with the replenishment time, we consider two cases: (1) when the special order time coincides with the retailer's replenishment time and (2) when the special order time occurs during the retailer's sales period. In Case 1, the retailer's optimal order policy is to decide whether to place a larger order which is always larger than regular EOQ. In Case 2, the retailer's optimal order policy is to decide whether to place an additional order which is not necessary larger than regular EOQ. The purpose of this study is to determine the retailer's optimal order policies in response to a price increase by maximising the total cost saving between special and regular orders during the depletion time of the special order quantity. We provide several numerical examples to illustrate the theories in practical use, and we conduct a sensitivity analysis of the optimal solution by examining the main parameters.

2. Notations and assumptions

The following notations and assumptions are used in this study.

2.1. Notations

- D market demand rate.
- v unit purchasing price.
- k unit price increase.
- A ordering cost per order.
- r holding cost rate per dollar.
- θ deterioration rate, where $0 \leq \theta < 1$ and is a constant.
- Q economic order quantity before the price increase.
- T the length of replenishment cycle time before price increase.
- Q_r economic order quantity after the price increase.
- T_r the length of replenishment cycle time after the price increase.
- Q_s special order quantity before the price increase, a decision variable.
- T_s depletion time for the special order quantity Q_s, a decision variable.
- W limited special order quantity at the present price.
- T_W depletion time of the limited special order quantity W.
- q residual inventory level when the special order is placed.
- t_q the length of time until the special order is placed during the retailer's regular replenishment period.

T_q depletion time for the inventory quantity $Q_s + q$.
$I(t)$ inventory level at time t before the price increase, $0 \le t \le T$.
$I_s(t)$ inventory level at time t when the special order is adopted, $0 \le t \le T_s$.
$I_q(t)$ inventory level at time t during the time interval $[0, T_q]$.
$TC(T)$ total cost per unit time during the replenishment period T.
$TC_r(T_r)$ total cost per unit time during the replenishment period T_r.
$g_i(T_s)$ total cost saving between the special order and regular order during the special cycle time for Case i, $i = 1, 2$.
$*$ the superscript represents optimal value.

2.2. Assumptions

(1) To reflect the increasing price of raw materials, the supplier announces that the unit price of an item will increase by a given amount k, at a certain future date.
(2) The retailer has only one opportunity to replenish its stock at the present price before the price increases. In addition, to avoid a decrease in profit, the supplier is willing to offer the retailer a limited quantity, W, prior to the price increase. This is a common industrial practice.
(3) The replenishment rate is infinite and the lead-time is zero.
(4) Shortages are not allowed.
(5) There is no replacement or repair of deteriorated units during the period under consideration.

3. Model formulation

This study explores the possible effects of price increases on a retailer's replenishment policy, when there is a limited quantity of a special order item that deteriorates in quantity over time. Depletion of the inventory occurs due to the combined effects of demand and physical deterioration. Hence, the change in inventory level before the price increase is illustrated by the following differential equation:

$$\frac{dI(t)}{dt} = -\theta\, I(t) - D, 0 < t < T. \tag{1}$$

Given the boundary condition $I(T) = 0$, the solution of Equation (1) is represented by

$$I(t) = \frac{D}{\theta}\left[e^{\theta(T-t)} - 1\right], 0 \le t \le T. \tag{2}$$

Thus, the order quantity is given by

$$Q = I(0) = \frac{D}{\theta}(e^{\theta T} - 1). \tag{3}$$

Prior to the price increase, the purchasing cost v follows that of the regular order, so the retailer follows the regular economic order policy with a unit purchasing cost, v, the total cost during the replenishment period T being the sum of the ordering cost, purchasing cost and holding cost, i.e.,

$$A + vQ + rv\int_0^T I(t)dt = A + \frac{vD}{\theta}\left(e^{\theta T} - 1\right) + \frac{rvD}{\theta^2}\left(e^{\theta T} - \theta T - 1\right). \tag{4}$$

Therefore, the total cost per unit time is

$$TC(T) = \frac{1}{T}\left[A + \frac{vD}{\theta}\left(e^{\theta T} - 1\right) + \frac{rvD}{\theta^2}\left(e^{\theta T} - \theta T - 1\right)\right]. \tag{5}$$

It can easily be shown that $TC(T)$ is a convex function of T. Hence, there is a unique value for T (say T^*) that minimises $TC(T)$. The value of T^* can be obtained by solving the equation $d\,TC(T)/d\,T = 0$, i.e.,

$$A - \frac{(\theta + r)vD}{\theta^2}(\theta T e^{\theta T} - e^{\theta T} + 1) = 0. \tag{6}$$

Once the optimal length of replenishment cycle time, T^*, is obtained, the optimal order quantity, Q^*, is obtained as follows:

$$Q^* = \frac{D}{\theta}\left(e^{\theta T^*} - 1\right). \tag{7}$$

Next, when the unit purchasing cost increases from v to $(v + k)$, the total cost per unit time becomes

$$TC_r(T_r) = \frac{1}{T_r}\left[A + \frac{(v+k)D}{\theta}\left(e^{\theta T_r} - 1\right) + \frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r} - \theta T_r - 1\right)\right]. \tag{8}$$

Similarly, there is a unique value for T_r (say T_r^*) that minimises $TC_r(T_r)$. The value of T_r^* is determined by solving the following equation:

$$A - \frac{(\theta + r)(v+k)D}{\theta^2}(\theta T_r e^{\theta T_r} - e^{\theta T_r} + 1) = 0. \tag{9}$$

The corresponding optimal order quantity, Q_r^*, is

$$Q_r^* = \frac{D}{\theta}\left(e^{\theta T_r^*} - 1\right). \tag{10}$$

Subsequently, when a supplier announces a price increase that is effective from a particular future date, the retailer may place a special order to take advantage of the current lower price, v, before the price increases. Alternatively, the retailer may ignore this notice and place a regular order. To avoid a decrease in profit, the supplier is only willing to offer the retailer a limited quantity, W, prior to the price increase. The purpose of this study is to determine the optimal special order quantity by maximising the total cost saving between special and regular orders during the depletion time of the special order quantity. As stated earlier, two specific situations arise, which we discuss in this study: when the special order time (1) coincides with the retailer's replenishment time or (2) occurs during the retailer's sales period. Next, we will formulate the corresponding total relevant inventory cost saving function for these two cases.

3.1. *Case 1: the special order time coincides with the retailer's replenishment time*

In this case, if the retailer decides to adopt a special order and orders Q_s units, then the inventory level at time t is

$$I_s(t) = \frac{D}{\theta}\left[e^{\theta(T_s - t)} - 1\right], 0 \le t \le T_s. \tag{11}$$

The special order quantity at the original unit purchasing price, v, is

$$Q_s = I_s(0) = \frac{D}{\theta}\left(e^{\theta T_s} - 1\right). \tag{12}$$

In order to ensure that the special order quantity Q_s is less than or equal to the limited quantity W, and is always larger than or equal to the optimal regular order quantity Q^* (i.e., $Q^* \le Q_s \le W$), we substitute Equations (7) and (12) into this inequality, and obtain

$$T^* \le T_s \le \frac{1}{\theta}\ln\left[\frac{\theta W + D}{D}\right] \equiv T_W. \tag{13}$$

The total cost of the special order during the time interval $[0, T_s]$ (denoted by $TCS_1(T_s)$) consists of the ordering cost, purchasing cost and holding cost, and is expressed by

$$TCS_1(T_s) = A + \frac{vD}{\theta}(e^{\theta T_s} - 1) + \frac{rvD}{\theta^2}(e^{\theta T_s} - \theta T_s - 1). \tag{14}$$

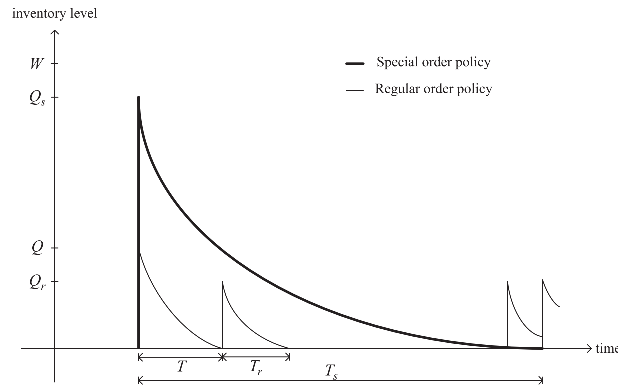

Figure 1. Special vs. regular order policies when the special order time coincides with the retailer's replenishment time.

If the retailer places its regular order, then the total cost of a regular order during the time interval $[0, T_s]$ will be divided into two periods (see Figure 1). In the first period, the retailer orders Q^* units at the unit purchasing price v. The corresponding total cost is similar to Equation (4), and is represented by

$$A + \frac{vD}{\theta}\left(e^{\theta T^*} - 1\right) + \frac{rvD}{\theta^2}\left(e^{\theta T^*} - \theta T^* - 1\right). \tag{15}$$

As to the rest period, the retailer follows regular EOQ policy for the unit purchasing price $v + k$. Thus, the total cost during the rest period is given by

$$\frac{T_s - T^*}{T_r^*}\left[A + (v+k)\frac{D}{\theta}(e^{\theta T_r^*} - 1) + \frac{r(v+k)D}{\theta^2}(e^{\theta T_r^*} - \theta T_r^* - 1)\right]. \tag{16}$$

Consequently, the total cost of a regular order during the time interval $[0, T_s]$ (denoted by $TCN_1(T_s)$) is

$$\begin{aligned} TCN_1(T_s) = {} & A + \frac{vD}{\theta}\left(e^{\theta T^*} - 1\right) + \frac{rvD}{\theta^2}\left(e^{\theta T^*} - \theta T^* - 1\right) \\ & + \frac{T_s - T^*}{T_r^*}\left[A + (v+k)\frac{D}{\theta}(e^{\theta T_r^*} - 1) \right. \\ & \left. + \frac{r(v+k)D}{\theta^2}(e^{\theta T_r^*} - \theta T_r^* - 1)\right]. \end{aligned} \tag{17}$$

Comparing Equation (14) with Equation (17), the total cost saving when the special order time coincides with the retailer's replenishment time (i.e., Case 1) can be formulated as follows:

$$\begin{aligned} g_1(T_s) &= TCN_1(T_s) - TCS_1(T_s) \\ &= \frac{T_s - T^*}{T_r^*}\left[A + \frac{(v+k)D}{\theta}(e^{\theta T_r^*} - 1)\right. \end{aligned}$$

$$+\frac{r(v+k)D}{\theta^2}(e^{\theta T_r^*}-\theta T_r^*-1)\Big]$$
$$-\frac{(\theta+r)vD}{\theta^2}(e^{\theta T_s}-e^{\theta T^*})+\frac{rvD}{\theta}(T_s-T^*). \quad (18)$$

3.2. Case 2: the special order time occurs during the retailer's sales period

Sometimes, the time of the price increase occurs during the retailer's sales period. In this situation, if the retailer decides to place a special order of quantity Q_s at the present price v, the inventory level will increase instantaneously from q to $Q_s + q$ when the special order quantity is delivered (see Figure 2). On the other hand, if the retailer ignores the notification about the price increase, that retailer will not place any orders until the next replenishment. We will formulate the total cost functions for the special and regular order policies and then compare the two.

When a special order is placed, the total cost during the time interval $[0, T_q]$ consists of the ordering cost A, the purchasing cost $v\,Q_s = [vD(e^{\theta T_s}-1)]/\theta$, and the holding cost, which is presented as follows.

As the special order quantity arrives, the maximum inventory is given by

$$Q_s+q=\frac{D}{\theta}\left(e^{\theta T_s}-1\right)+\frac{D}{\theta}\left[e^{\theta(T^*-t_q)}-1\right]$$
$$=\frac{D}{\theta}\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-2\right]. \quad (19)$$

Furthermore, the inventory level at time t during the time interval $[0, T_q]$ can be obtained by

$$I_q(t)=\frac{D}{\theta}\left[e^{\theta(T_q-t)}-1\right], 0\le t\le T_q. \quad (20)$$

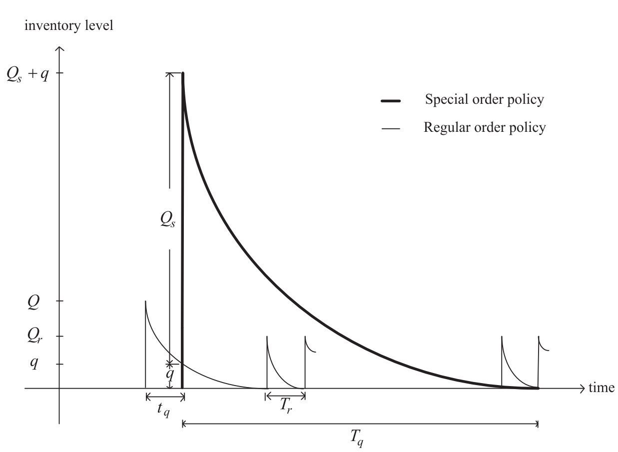

Figure 2. Special vs. regular order policies when the special order time occurs during the retailer's sales period.

Because $I_q(0) = Q_s + q$, from Equations (19) and (20), we have

$$\frac{D}{\theta}\left(e^{\theta T_q}-1\right)=\frac{D}{\theta}\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-2\right]. \quad (21)$$

Thus,

$$T_q=\frac{1}{\theta}\ln\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1\right]. \quad (22)$$

Therefore, the total holding cost of the special order is

$$r\,v\int_0^{T_q}I_q(t)dt=\frac{rvD}{\theta^2}\left(e^{\theta T_q}-\theta T_q-1\right)$$
$$=\frac{rvD}{\theta^2}\left\{e^{\theta T_s}+e^{\theta(T^*-t_q)}-2-\ln\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1\right]\right\}. \quad (23)$$

Consequently, the total cost of the special order during the time interval $[0, T_q]$ (denoted by $TCS_2(T_s)$) can be formulated as follows:

$$TCS_2(T_s)=A+\frac{vD}{\theta}(e^{\theta T_s}-1)+\frac{r\,vD}{\theta^2}\left\{e^{\theta T_s}+e^{\theta(T^*-t_q)}-2-\ln\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1\right]\right\}. \quad (24)$$

On the other hand, if the retailer ignores notification of the price increase and places its regular order, the total cost during the time interval $[0, T_q]$ will also be divided into two periods. In the first period, the retailer only has the cost during the depletion time of residual q, T^*-t_q. We use the average cost analysis approach, which gives us the following:

$$\frac{T^*-t_q}{T^*}\left[A+\frac{vD}{\theta}\left(e^{\theta T^*}-1\right)+\frac{rvD}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right].$$

Next, the retailer places the regular order with the unit purchase cost $v + k$ during the rest period. To obtain the total cost in this period, we use the average cost analysis approach, which is given by

$$\frac{T_q-(T^*-t_q)}{T_r^*}\Big[A+\frac{(v+k)D}{\theta}\left(e^{\theta T_r^*}-1\right)+\frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\Big]$$
$$=\frac{\ln[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1]-\theta(T^*-t_q)}{\theta T_r^*}$$

$$\times\left[A+\frac{(v+k)D}{\theta}\left(e^{\theta T_r^*}-1\right)+\frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right].$$

As a result, when the retailer ignores the notification and places its regular order during the time interval [0, T_q], the total cost (denoted by $TCN_2(T_s)$) is

$$TCN_2(T_s)=\frac{T^*-t_q}{T^*}\left[A+\frac{vD}{\theta}\left(e^{\theta T^*}-1\right)+\frac{rvD}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]+\left\{\frac{\ln[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1]}{\theta T_r^*}-\frac{(T^*-t_q)}{T_r^*}\right\}\times\left[A+\frac{(v+k)D}{\theta}\left(e^{\theta T_r^*}-1\right)+\frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]. \quad (25)$$

Therefore, the total cost saving when the special order time occurs during the retailer's sales period can be formulated as follows:

$$\begin{aligned} g_2(T_s) &= TCN_2(T_s)-TCS_2(T_s)\\ &=\frac{T^*-t_q}{T^*}\left[A+\frac{vD}{\theta}\left(e^{\theta T^*}-1\right)+\frac{rvD}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]\\ &\quad+\left\{\frac{\ln[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1]}{\theta T_r^*}-\frac{(T^*-t_q)}{T_r^*}\right\}\\ &\quad\times\left[A+\frac{(v+k)D}{\theta}\left(e^{\theta T_r^*}-1\right)+\frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]\\ &\quad-\left\{A+\frac{vD}{\theta}(e^{\theta T_s}-1)+\frac{r\,vD}{\theta^2}\{e^{\theta T_s}+e^{\theta(T^*-t_q)}-2-\ln\left[e^{\theta T_s}+e^{\theta(T^*-t_q)}-1\right]\}\right\}. \end{aligned} \quad (26)$$

Remark 1: Note that it is worth placing a special order only when the total cost saving is positive in the above two cases. Otherwise, the retailer will ignore the opportunity to place the special order.

4. Theoretical results

In this section, the optimal value of T_s, representing the maximisation of the total cost saving is determined.

4.1. *Case 1: the special order time coincides with the retailer's replenishment time*

Taking the first and second order derivatives of $g_1(T_s)$ in Equation (18) with respect to T_s leads to

$$\frac{dg_1(T_s)}{dT_s}=\frac{1}{T_r^*}\left[A+\frac{(v+k)D}{\theta}(e^{\theta T_r^*}-1)+\frac{r(v+k)D}{\theta^2}(e^{\theta T_r^*}-\theta T_r^*-1)\right]-\frac{(\theta+r)vD}{\theta}e^{\theta T_s}+\frac{rvD}{\theta} \quad (27)$$

and

$$\frac{d^2g_1(T_s)}{dT_s^2}=-(\theta+r)vDe^{\theta T_s}<0. \quad (28)$$

Consequently, $g_1(T_s)$ is a concave function of T_s; hence, there exists a unique value of T_s (say T_{s_1}) that maximises $g_1(T_s)$. T_{s_1} can be obtained by computing $dg_1(T_s)/dT_s=0$ and is given by

$$T_{s_1}=\frac{1}{\theta}\ln\left[\frac{rvD+\theta y}{vD(\theta+r)}\right], \quad (29)$$

where $y=\frac{1}{T_r^*}\left[A+\frac{(v+k)D}{\theta}(e^{\theta T_r^*}-1)+\frac{r(v+k)D}{\theta^2}(e^{\theta T_r^*}-\theta T_r^*-1)\right]>0$.

Next, substituting Equation (29) into Equation (18) results in

$$g_1(T_{s_1})=\frac{(\theta+r)vD}{\theta^2}\left[\theta e^{\theta T_{s_1}}(T_{s_1}-T^*)-e^{\theta T_{s_1}}+e^{\theta T^*}\right]. \quad (30)$$

We can easily show that $g_1(T_{s_1})>0$ for $T_{s_1}\geq T^*$. It means that the total cost saving function $g_1(T)$ has a positive value at the point $T_s=T_{s_1}$. Furthermore, if we let T_s^* denote the optimal solution of Case 1, we can obtain the following result (details of the proof are shown in Appendix A):

$$T_s^*=\begin{cases} T^*, & \text{if} \quad \Delta_2<0,\\ T_{s_1}, & \text{if} \quad \Delta_1\leq 0\leq\Delta_2,\\ T_W, & \text{if} \quad \Delta_1>0,\end{cases}$$

where $\Delta_1\equiv y-vDe^{\theta T_W}-\frac{rvD}{\theta}\left(e^{\theta T_W}-1\right)$ and $\Delta_2\equiv y-vDe^{\theta T^*}-\frac{rvD}{\theta}\left(e^{\theta T^*}-1\right)$.

Remark 2: When the optimal depletion time for the special order quantity $T_s^*=T^*$, it is not worthwhile for the retailer to place a special order; instead, the retailer should place its regular order.

4.2. Case 2: the special order time occurs during the retailer's sales period

Taking the first order derivative of $g_2(T_s)$ in Equation (26) with respect to T_s, we obtain

$$\frac{dg_2(T_s)}{dT_s} = \left(y + \frac{rvD}{\theta}\right)\frac{e^{\theta T_s}}{e^{\theta T_s} + e^{\theta(T^*-t_q)} - 1} - \frac{(\theta+r)vDe^{\theta T_s}}{\theta}. \tag{31}$$

If we let Equation (31) be equal to 0 and solve this equation, we can obtain a unique solution for T (say T_{s_2}) as

$$T_{s_2} = \frac{1}{\theta}\ln\left[\frac{\theta y + rvD - (\theta+r)vD[e^{\theta(T^*-t_q)} - 1]}{(\theta+r)vD}\right]. \tag{32}$$

Substituting Equation (32) into Equation (26), we obtain

$$\begin{aligned} g_2(T_{s_2}) = {} & \frac{T^*-t_q}{T^*}\left[A + \frac{vD}{\theta}\left(e^{\theta T^*}-1\right) + \frac{rvD}{\theta^2}\left(e^{\theta T^*} - \theta T^* - 1\right)\right] \\ & + \left\{\frac{\ln[e^{\theta T_{s_2}} + e^{\theta(T^*-t_q)} - 1]}{\theta T_r^*} - \frac{(T^*-t_q)}{T_r^*}\right\} \\ & \times\left[A + \frac{(v+k)D}{\theta}\left(e^{\theta T_r^*}-1\right) + \frac{r(v+k)D}{\theta^2}\left(e^{\theta T_r^*} - \theta T_r^* - 1\right)\right] \\ & - \left\{A + \frac{vD}{\theta}(e^{\theta T_{s_2}} - 1) + \frac{rvD}{\theta^2}\{e^{\theta T_{s_2}} + e^{\theta(T^*-t_q)} \right. \\ & \left. - 2 - \ln\left[e^{\theta T_{s_2}} + e^{\theta(T^*-t_q)} - 1\right]\}\right\}. \end{aligned} \tag{33}$$

It is worth noting that the value T_{s_2} in Equation (32) must satisfy $0 \le Q_s \le W$(i.e., $0 \le T_s \le T_W$) and $g_2(T_{s_2}) > 0$. Now, let T_s^* denote the optimal solution of Case 2, we can obtain the following result (details of the proof are shown in Appendix B):

$$T_s^* = \begin{cases} T_{s_2}, & \text{if} \quad \Delta_3 \le 0 \le \Delta_4 \text{ and } \Delta_5 > 0, \\ T_W, & \text{if} \quad \Delta_3 > 0 \text{ and } \Delta_6 > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\Delta_3 \equiv y - vD[e^{\theta T_W} + e^{\theta(T^*-t_q)} - 1] - \frac{rvD}{\theta}[e^{\theta T_W} + e^{\theta(T^*-t_q)} - 2]$, $\Delta_4 \equiv y - vD\, e^{\theta(T^*-t_q)} - \frac{rvD}{\theta}[e^{\theta(T^*-t_q)} - 1]$, $\Delta_5 \equiv g_2(T_{s_2})$ and $\Delta_6 \equiv g_2(T_W)$.

Remark 3: When the optimal depletion time for special order quantity $T_s^* = 0$, the retailer should not order until the next replenishment time.

Summarising the above results, we can develop an algorithm to obtain the optimal solution, T_s^*, for the two situations.

Algorithm:

Step 1. Determine T^*, T_r^* and T_W from Equations (6), (9) and (13), respectively. If $q = 0$, go to Step 2. Otherwise, go to Step 3.

Step 2. Calculate $\Delta_1 = y - vDe^{\theta T_W} - \frac{rvD}{\theta}\left(e^{\theta T_W} - 1\right)$ and

$$\Delta_2 = y - vDe^{\theta T^*} - \frac{rvD}{\theta}\left(e^{\theta T^*} - 1\right).$$

(1) If $\Delta_1 \le 0 \le \Delta_2$, then find T_{s_1} from Equation (29), and the optimal length of replenishment cycle time $T_s^* = T_{s_1}$. Go to Step 4.
(2) If $\Delta_2 < 0$, then the optimal length of replenishment cycle time $T_s^* = T^*$. Go to Step 4.
(3) If $\Delta_1 > 0$, then the optimal length of replenishment cycle time $T_s^* = T_W$. Go to Step 4.

Step 3. Calculate $\Delta_3 = y - vD[e^{\theta T_W} + e^{\theta(T^*-t_q)} - 1] - \frac{rvD}{\theta}[e^{\theta T_W} - e^{\theta(T^*-t_q)} - 2]$, and $\Delta_4 = y - vDe^{\theta(T^*-t_q)} - \frac{rvD}{\theta}[e^{\theta(T^*-t_q)} - 1]$.

(1) For $\Delta_3 \le 0 \le \Delta_4$, find T_{s_2} from Equation (32), and determine $g_2(T_{s_2})$ from Equation (33). If $\Delta_5 = g_2(T_{s_2}) > 0$, the optimal length of replenishment cycle time $T_s^* = T_{s_2}$.
(2) For $\Delta_3 > 0$, substitute $T_s = T_W$ into Equation (26) and obtain $g_2(T_W)$. If $\Delta_6 = g_2(T_W) > 0$, then the optimal length of replenishment cycle time $T_s^* = T_W$.
(3) Otherwise, the optimal length of the replenishment cycle time $T_s^* = 0$.

Step 4. Stop.

Once the optimal solution T_s^* is obtained, the optimal special order quantity $Q_s^* = D(e^{\theta T_s^*} - 1)/\theta$ and the maximum total cost saving, $g_1(T_s^*)$ or $g_2(T_s^*)$, follow.

5. Numerical examples and sensitivity analysis

We present the following examples to illustrate the optimal ordering policy for each of the two situations:

Example 1: We first consider that the special order time coincides with the retailer's replenishment time. Given an inventory system with the following parameters, $D = 1000$, $v = 10$, $A = 30$, $\theta = 0.1$, $r = 0.3$, in appropriate units, it can be found for the regular order that $T^* = 0.12198$ (i.e., $Q^* = 122.72$). Furthermore, we set $W = \{500, 1000, 1500\}$ and $k = \{1, 2, 3, 4, 5\}$. From the algorithm, the optimal ordering policies depend upon whether the different above-noted parameters can be obtained. The computational results are shown in Table 1.

Table 1. Optimal solutions of Example 1 under the different values of W and k.

W	k	T_s^*	Q_s^*	TCN_1^*	TCS_1^*	g_1^*
500	1	$T_{s_1} = 0.372$	378.70	4155.25	4026.86	128.39
	2	$T_W = 0.488$	500	5867.48	5392.95	474.53
	3	$T_W = 0.488$	500	6241.43	5392.95	848.48
	4	$T_W = 0.488$	500	6615.07	5392.95	1222.12
	5	$T_W = 0.488$	500	6988.44	5392.95	1595.49
1000	1	$T_{s_1} = 0.372$	378.70	4155.25	4026.86	128.39
	2	$T_{s_1} = 0.615$	634.41	7462.28	6953.48	508.80
	3	$T_{s_1} = 0.852$	889.89	11,185.20	10,050.70	1134.50
	4	$T_W = 0.953$	1000	13,398.00	11,436.90	1961.10
	5	$T_W = 0.953$	1000	14,246.00	11,436.90	2809.10
1500	1	$T_{s_1} = 0.372$	378.70	4155.25	4026.86	128.39
	2	$T_{s_1} = 0.615$	634.41	7462.28	6953.48	508.80
	3	$T_{s_1} = 0.852$	889.89	11,185.20	10,050.70	1134.51
	4	$T_{s_1} = 1.084$	1145.16	15,309.60	13,310.40	1999.19
	5	$T_{s_1} = 1.311$	1400.25	19,821.80	16,724.90	3096.90

Example 2: In this example, we consider another situation in which the special order time occurs during the retailer's sales period. The data used are the same as those in Example 1, except that $q = 50$. Similarly, by using the algorithm, the optimal ordering policies under various limited quantities $W = \{500,\ 1000,\ 1500\}$ and price increases $k = \{1,\ 2,\ 3,\ 4,\ 5\}$ are listed in Table 2.

From Tables 1 and 2, some observations may be made. First, with the limited special order quantity, regardless of when the special order time occurs, the retailer will place a special order before the price increase to take advantage of the lower price. Second, as the optimal value of T_s is equal to T_W, i.e., $Q_s^* = W$, it implies that the retailer will place a special order for the maximum quantity the supplier can provide. Finally, when the value of k increases, the optimal special order quantity Q_s^* and the maximum total cost saving between special and regular orders during the depletion time period of the special order quantity g_i^*, $i = 1, 2$ will increase. This means that the higher the unit price increases, the more likely the retailer is to place a special order to take advantage of the present lower price.

Example 3: In this example, sensitivity analysis demonstrates how the robustness of the model will be established. That is, the effects of changes in the system parameters like v, D, A, θ, r and q will be discussed, in terms of the optimal special ordering quantity Q_s^*, TCN_2^*, TCS_2^* and maximum total cost saving g_2^*. The data used are the same as in Example 2. For convenience, the case in which the fixed $W = 1000$ and $k = 3$ is taken into account. Sensitivity analysis is performed by changing each of the parameters by -50%, -25%, $+25\%$ and $+50\%$, one parameter at a time while keeping all the remaining parameters unchanged. The results are shown in Table 3.

Table 2. Optimal solutions of Example 2 under the different values of W and k.

W	k	T_s^*	Q_s^*	TCN_2^*	TCS_2^*	g_2^*
500	1	$T_{s_2} = 0.323$	328.70	4229.07	3526.86	702.21
	2	$T_W = 0.488$	500	6610.67	5467.77	1142.90
	3	$T_W = 0.488$	500	7106.85	5467.77	1639.08
	4	$T_W = 0.488$	500	7602.62	5467.77	2134.85
	5	$T_W = 0.488$	500	8098.03	5467.77	2630.26
1000	1	$T_{s_2} = 0.323$	328.70	4229.07	3526.86	702.21
	2	$T_{s_2} = 0.568$	584.41	7609.86	6453.48	1156.38
	3	$T_{s_2} = 0.806$	839.89	10,406.5	9550.70	1855.77
	4	$T_W = 0.953$	1000	14,354.1	11,576.40	2777.73
	5	$T_W = 0.953$	1000	15,322.0	11,576.40	3745.60
1500	1	$T_{s_2} = 0.323$	328.70	4229.07	3526.86	702.21
	2	$T_{s_2} = 0.568$	584.41	7609.86	6453.48	1156.38
	3	$T_{s_2} = 0.806$	839.89	11,406.5	9550.70	1855.77
	4	$T_{s_2} = 1.039$	1095.16	15,604.4	12,810.40	2794.07
	5	$T_{s_2} = 1.267$	1350.25	20,190.3	16,224.90	3965.33

Table 3. Effect of changes in various parameters of Example 3.

		% change in			
Parameter value	% change	Q_s^*	TCN_2^*	TCS_2^*	g_2^*
v	−50	19.1	−27.5	−39.2	33.0
	−25	19.1	−5.0	−9.0	15.5
	+25	−20.0	−2.9	−1.9	−7.5
	+50	−33.4	−4.5	−3.4	−10.5
D	−50	−49.5	−46.6	−48.7	−36.0
	−25	−24.6	−23.1	−24.1	−17.7
	+25	19.1	18.0	18.1	17.2
	+50	19.1	18.4	16.1	30.4
A	−50	−4.9	−5.7	−5.5	−6.5
	−25	−2.2	−2.6	−2.5	−3.0
	+25	2.0	2.3	2.2	2.6
	+50	3.8	4.4	4.3	5.0
θ	−50	13.9	15.1	15.9	11.1
	−25	6.5	7.0	7.3	5.1
	+25	−5.7	−6.1	−6.4	−4.5
	+50	−10.9	−11.4	−12.0	−8.5
r	−50	19.1	16.3	13.1	32.9
	−25	19.1	16.9	17.2	15.4
	+25	−15.5	−14.0	−14.6	−10.8
	+50	−26.8	−24.3	−25.4	−18.9
q	−50	3.0	0.7	2.6	-9.4
	−25	1.5	0.3	1.3	-4.7
	+25	−1.5	−0.3	−1.3	4.7
	+50	−3.0	−0.6	−2.6	9.4

From the results shown in Table 3, the following observations can be made:

(1) When the purchase cost v decreases, both the optimal special order quantity Q_s^* and maximum total cost saving g_2^* increases. The simple economic explanation for this is that if the purchase cost is lower, the retailer will place a special order for a larger quantity, resulting in a higher total cost saving. Moreover, the optimal special order quantity and total cost saving are sensitive to the purchase cost.

(2) It is obvious that all the values of Q_s^*, TCN_2^*, TCS_2^* and g_2^* increase as the parameter D or A increases. That is, both demand rate and order cost have positive effects on special order quantity and the total cost, whether the retailer makes an additional special order or not. Furthermore, it is a benefit to the retailer's total cost saving when both the market demand rate and order cost increase.

(3) It is evident that when θ or r increases, the values of Q_s^*, TCN_2^*, TCS_2^* and g_2^* all decrease. This implies that the higher the deterioration rate or holding cost rate, the lower the special order quantity, total costs (whether the retailer places an additional special order or not) and total cost saving will be. If the deterioration rate or holding cost rate can be reduced by improving storage, it will contribute to the special order quantity, total costs and the total cost saving. In addition, it can be found that the deteriorating rate has few significant impacts on the results. The reason is that the value of the deteriorating rate is very small which implies the deterioration cost has a little weight on the retailer's total cost saving in general. Still it is our belief that taking the characteristic of the deterioration into the model is reasonable and necessary.

(4) It can also be shown that the larger the level of residual inventory (prior to the delivery of the special order quantity q), the smaller the values of Q_s^*, TCN_2^*, TCS_2^* and the larger the values for g_2^* will be. This implies that it will be beneficial for total costs and total cost saving if the residual inventory is as large as possible when the special order time occurs during the retailer's sales period.

6. Conclusions

This study investigates the possible effects of a price increase announced by a supplier on a retailer's replenishment policy, when the quantity of the item available by special order is limited. The purpose of the study is to determine the retailer's optimal order policies when such a special order item is limited in quantity. By analysing the total cost savings between special and regular order policies, we develop several results that may be useful for characterising the optimal solution, and providing an algorithm with which to identify the optimal order quantity. In addition, we present numerical examples to illustrate the solution procedure and perform a sensitivity analysis of the optimal solution. The results reveal that (1) given a limited special order quantity, the retailer will place a special order to take advantage of the lower price regardless of when the special order time occurs; (2) the higher the unit price, the more the retailer is likely to place a special order in order to take advantage of the current lower price; (3) the higher the deterioration rate or holding cost rate, the lower the optimal special order, total costs (whether or not the retailer places a special order) and total cost saving will be; and (4) when the retailer's special order time occurs during the sales period, it will be beneficial for the total costs and total cost saving if the residual inventory is as large as possible.

The proposed model could be extended in several ways. For example, the proposed inventory model may deal with the demand rate as a function of selling price, time, stock, and so on. Furthermore, the special order time could be treated as a decision variable. Finally, the model could be generalised to allow for shortages, quantity discounts, inflation, and other circumstances.

Acknowledgements

The authors greatly appreciate the anonymous referees for their valuable and helpful suggestions regarding the earlier version of the paper.

Funding

This research was partially supported by the National Science Council of the Republic of China [grant number NSC 100-2221-E-231-014].

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Appendix A

To ensure $Q^* \le Q_s \le W$ (i.e., $T^* \le T_s \le T_W$), we substitute Equation (29) into this inequality, and it results in

$$\text{if } \Delta_1 \le 0 \le \Delta_2, \text{ then } T^* \le T_{s_1} \le T_W, \tag{A1}$$

where $\Delta_1 \equiv y - vDe^{\theta T_W} - \frac{rvD}{\theta}\left(e^{\theta T_W} - 1\right)$ and $\Delta_2 \equiv y - vDe^{\theta T^*} - \frac{rvD}{\theta}\left(e^{\theta T^*} - 1\right)$.

For the completeness of our theoretical results, we further discuss two situations where $\Delta_2 < 0$ and $\Delta_1 > 0$ as follows. When $\Delta_2 < 0$, we have $y < [(\theta + r)vDe^{\theta T^*}/\theta] - (rvD/\theta)$, which implies

$$\begin{aligned} \frac{dg_1(T_s)}{dT_s} &= y - \frac{(\theta + r)vD}{\theta}e^{\theta T_s} + \frac{rvD}{\theta} \\ &< \frac{(\theta + r)vD}{\theta}\left(e^{\theta T^*} - e^{\theta T_s}\right) \\ &\le 0 \text{ for } T_s \ge T^*. \end{aligned} \tag{A2}$$

Hence, $g_1(T_s)$ is a strictly decreasing function of $T_s \in [T^*, T_W]$, and therefore, $g_1(T_s)$ has a maximum value at the lower boundary point $T_s = T^*$.

On the other hand, if $\Delta_1 > 0$, we have $y > [(\theta + r)vDe^{\theta T_W}/\theta] - (rvD/\theta)$, which implies

$$\frac{dg_1(T_s)}{dT_s} = y - \frac{(\theta + r)vD}{\theta} e^{\theta T_s} + \frac{rvD}{\theta} > \frac{(\theta + r)vD}{\theta}\left(e^{\theta T_W} - e^{\theta T_s}\right) \geq 0 \text{ for } T_s \leq T_W. \tag{A3}$$

Thus, $g_1(T_s)$ is a strictly increasing function of $T_s \in [T^*, T_W]$, and therefore, $g_1(T_s)$ has a maximum value at the upper boundary point $T_s = T_W$. The results summarised above offer sufficient evidence for the stated argument.

Appendix B

First, to ensure that $0 \leq Q_s \leq W$ (i.e., $0 \leq T_s \leq T_W$), we substitute Equation (32) into this inequality, which results in

$$\text{if } \Delta_3 \leq 0 \leq \Delta_4, \text{ then } 0 \leq T_{s_2} \leq T_W, \tag{B1}$$

where

$$\Delta_3 \equiv y - vD[e^{\theta T_W} + e^{\theta(T^* - t_q)} - 1] - \frac{rvD}{\theta}[e^{\theta T_W} + e^{\theta(T^* - t_q)} - 2]$$

and

$$\Delta_4 \equiv y - vD\, e^{\theta(T^* - t_q)} - \frac{rvD}{\theta}[e^{\theta(T^* - t_q)} - 1].$$

It is noted that when $\Delta_4 \geq 0$, we have $y \geq vDe^{\theta(T^* - t_q)} + \frac{rvD}{\theta}[e^{\theta(T^* - t_q)} - 1]$, which implies

$$\frac{\theta y + rvD - (\theta + r)vD[e^{\theta(T^* - t_q)} - 1]}{(\theta + r)vD} \geq 1,$$

or equivalently,

$$\ln\left[\frac{\theta y + rvD - (\theta + r)vD[e^{\theta(T^* - t_q)} - 1]}{(\theta + r)vD}\right] \geq 0.$$

Thus, T_{s_2} in Equation (32) is well defined. Furthermore, we can show that

$$\left.\frac{d^2 g_2(T_s)}{dT_s^2}\right|_{T_s = T_{s_2}} = \left(y + \frac{rvD}{\theta}\right)\frac{\theta\, e^{\theta T_{s_2}}[e^{\theta(T^* - t_q)} - 1]}{[e^{\theta T_{s_2}} + e^{\theta(T^* - t_q)} - 1]^2} - (\theta + r)vDe^{\theta T_{s_2}} = -\frac{(\theta + r)vDe^{2\theta T_{s_2}}}{e^{\theta T_{s_2}} + e^{\theta(T^* - t_q)} - 1} < 0.$$

Next, let $\Delta_5 \equiv g_2(T_{s_2})$, and it is obvious that when $\Delta_5 > 0$, T_{s_2} in Equation (32) is the optimal solution for T_s, which maximises $g_2(T_s)$.

Furthermore, we discuss two situations where $\Delta_4 < 0$ and $\Delta_3 > 0$ as follows. If $\Delta_4 < 0$, then we have

$$\begin{aligned}\frac{dg_2(T_s)}{dT_s} &= \left(y + \frac{rvD}{\theta}\right)\frac{e^{\theta T_s}}{e^{\theta T_s} + e^{\theta(T^* - t_q)} - 1} - \frac{(\theta + r)vDe^{\theta T_s}}{\theta} \\ &< \left\{vDe^{\theta(T^* - t_q)} + \frac{rvD}{\theta}[e^{\theta(T^* - t_q)} - 1] + \frac{rvD}{\theta}\right\} \times \frac{e^{\theta T_s}}{e^{\theta T_s} + e^{\theta(T^* - t_q)} - 1} - \frac{(\theta + r)vDe^{\theta T_s}}{\theta} \\ &= -\frac{(\theta + r)vDe^{\theta T_s}}{\theta(e^{\theta T_s} + e^{\theta(T^* - t_q)} - 1)}\left(e^{\theta T_s} - 1\right) < 0.\end{aligned} \tag{B2}$$

Thus, $g_2(T_s)$ is a strictly decreasing function of $T_s \in [0, T_W]$, which implies that $g_2(T_s)$ has a maximum value at the lower boundary point $T_s = 0$.

On the other hand, if $\Delta_3 > 0$, then we have

$$\begin{aligned}\frac{dg_2(T_s)}{dT_s} &> \left\{vD[e^{\theta T_W} + e^{\theta(T^* - t_q)} - 1] + \frac{rvD}{\theta}[e^{\theta T_W} + e^{\theta(T^* - t_q)} - 2] + \frac{rvD}{\theta}\right\} \\ &\quad \times \frac{e^{\theta T_s}}{e^{\theta T_s} + e^{\theta(T^* - t_q)} - 1} - \frac{(\theta + r)vDe^{\theta T_s}}{\theta} \\ &= \frac{(\theta + r)vDe^{\theta T_s}}{\theta[e^{\theta T_s} + e^{\theta(T^* - t_q)} - 1]}\left(e^{\theta T_W} - e^{\theta T_s}\right) \\ &\geq 0, \text{ for } T_s \leq T_W.\end{aligned} \tag{B3}$$

Thus, $g_2(T_s)$ is a strictly increasing function of $T_s \in [0, T_W]$, which implies that $g_2(T_s)$ has a maximum value at the upper boundary point $T_s = T_W$. Furthermore, substituting $T_s = T_W$ into Equation (26), and let $\Delta_6 \equiv g_2(T_W)$. If $\Delta_6 > 0$, then $T_s = T_W$ is the optimal solution which maximises $g_2(T_s)$. The results summarised above offer sufficient evidence for the stated argument.