General instructions: Experimental Examination

July 18, 2017

The experimental examination lasts for 5 hours and is worth a total of 20 points.

Before the exam

- You must not open the envelopes containing the problems before the sound signal indicating the beginning of the examination.
- The beginning and end of the examination will be indicated by a sound signal. There will be announcements every hour indicating the elapsed time, as well as fifteen minutes before the end of the examination (before the final sound signal).

During the exam

- Dedicated answer sheets are provided for writing your answers. Enter the observations into the appropriate tables, boxes or graphs in the corresponding answer sheet. For every problem, there are work sheets for carrying out detailed work. Be sure to always use the work sheets that belong to the problem you are currently working on (check the problem number in the header). If you have written something on any sheet which you do not want to be graded, cross it out. Only use the front side of every page. Do not forget to put your student code numbers in each working sheets or answer sheets in the field provided. Answers or working results will not be graded if there is no student code numbers written.
- In your answers, try to be as concise as possible: use equations, logical operators and sketches to illustrate your thoughts whenever possible. Avoid the use of long sentences.
- Explicit error calculation is not required unless explicitly asked for. However, you are asked to give an appropriate number of significant digits when stating numbers. Also, you should decide on the appropriate number of data points or measurement repetitions unless specific instructions are given.
- You may often be able to solve later parts of a problem without having solved the previous ones.
- You are not allowed to leave your working place without permission. If you need any assistance (need to refill your drinking water bottle, broken calculator, need to visit a restroom, etc.), please draw the attention of a supervisor by lifting one of the three flags into the air ("Refill my water bottle, please", "I need to go to the toilet, please", or "I need help, please" in all other cases).

At the end of the exam

- At the end of the examination you must stop writing immediately.
- For every problem, sort the corresponding sheets in the following order: questions, answer sheets, work sheets.

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- For E1 (Experimental Problem No. 1) you need to **enclose the millimeter block paper** that has been used for making the deflactogram. Please **make sure that you have written** your **student code numbers** and **the concentration of the solutions** that you used on this millimeter block paper.
- You must write the **number and total number of pages used** on your **working sheets** at the bottom of each pages.
- Put all the sheets belonging to one problem into the same envelope. Also put the general instructions into the remaining separate envelope (if available or just put in outside the envelope if remaining separate envelope is not available). Also hand in empty sheets. You are not allowed to take any sheets of paper out of the examination area.
- Leave your writing equipment (provided by the committee, if any) as well as the provided calculator on the experiment table.
- Wait at your table until your envelopes and all the necessary equipment are collected. Once all envelopes and equipment are collected, your guide will escort you out of the examination area. Don't forget to take your water bottle with you or your remaining snack.

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Determination of Refractive Index Gradient and Diffusion Coefficient of Salt Solution from Laser Deflection Measurement

I. Introduction

Diffusion is a process involving random walk of atoms or molecules leading a system towards its thermodynamic equilibrium. For instance, if a vessel contains a mixture of water and a salt solution, there will be a diffusive flux of salt molecules from regions of high salt concentration towards regions of lower salt concentration. The diffusion rate is characterized by the diffusion coefficient D. Diffusion plays a major role in a wide range of processes, from biochemistry to astrophysics. In the following experimental problem, diffusion of salt molecules is studied. Salt molecules will move diffusively from a salt solution towards the region with distilled water, creating a transition layer of variable salt concentration. The refraction index of this solution depends on the salt concentration. Therefore, we can study diffusion process through optical experiments using laser beam deflection method.

II. Objectives

1. To determine the diffusion coefficient of salt-water solution in water by measuring the gradient of refractive index.
2. To determine the rate of change of the diffusion coefficient to the change of salt solution concentrations.

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III. List of Materials

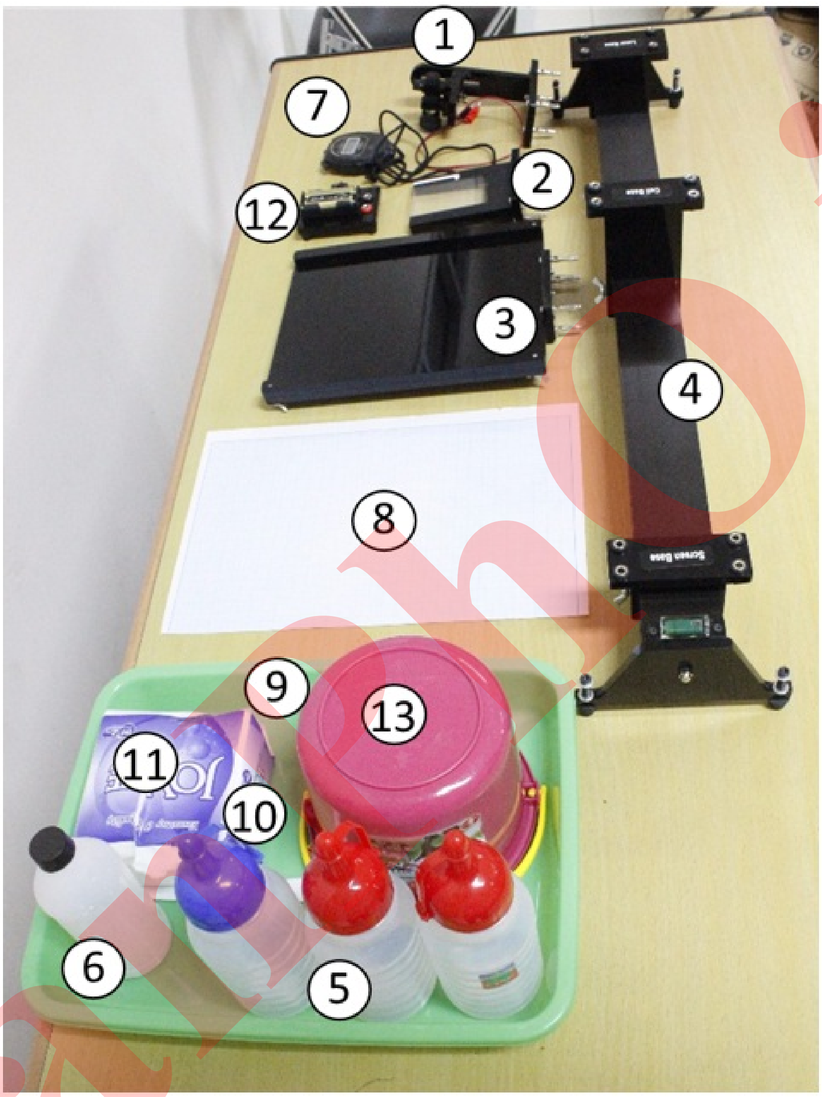

Figure 1. Materials for this experiment.

1. Line Laser Module (Diode laser with λ = 632 nm and cylindrical lens)
2. Diffusion cell (6.5 cm x 0.8 cm x 9.5 cm) with holder)
3. Screen with holder
4. Optical rail with length scale
5. Salt-water Solutions
6. Distilled Water (Aquadest)
7. Stopwatch
8. Paper with scale (block millimeter paper)

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9. Pipette (dropper)
10. Knife + Tissue for cleaner
11. Tissue
12. Battery
13. Buckets as waste container of salt and water solution

The schematic diagram of the experimental set-up can be seen in Figure 2.

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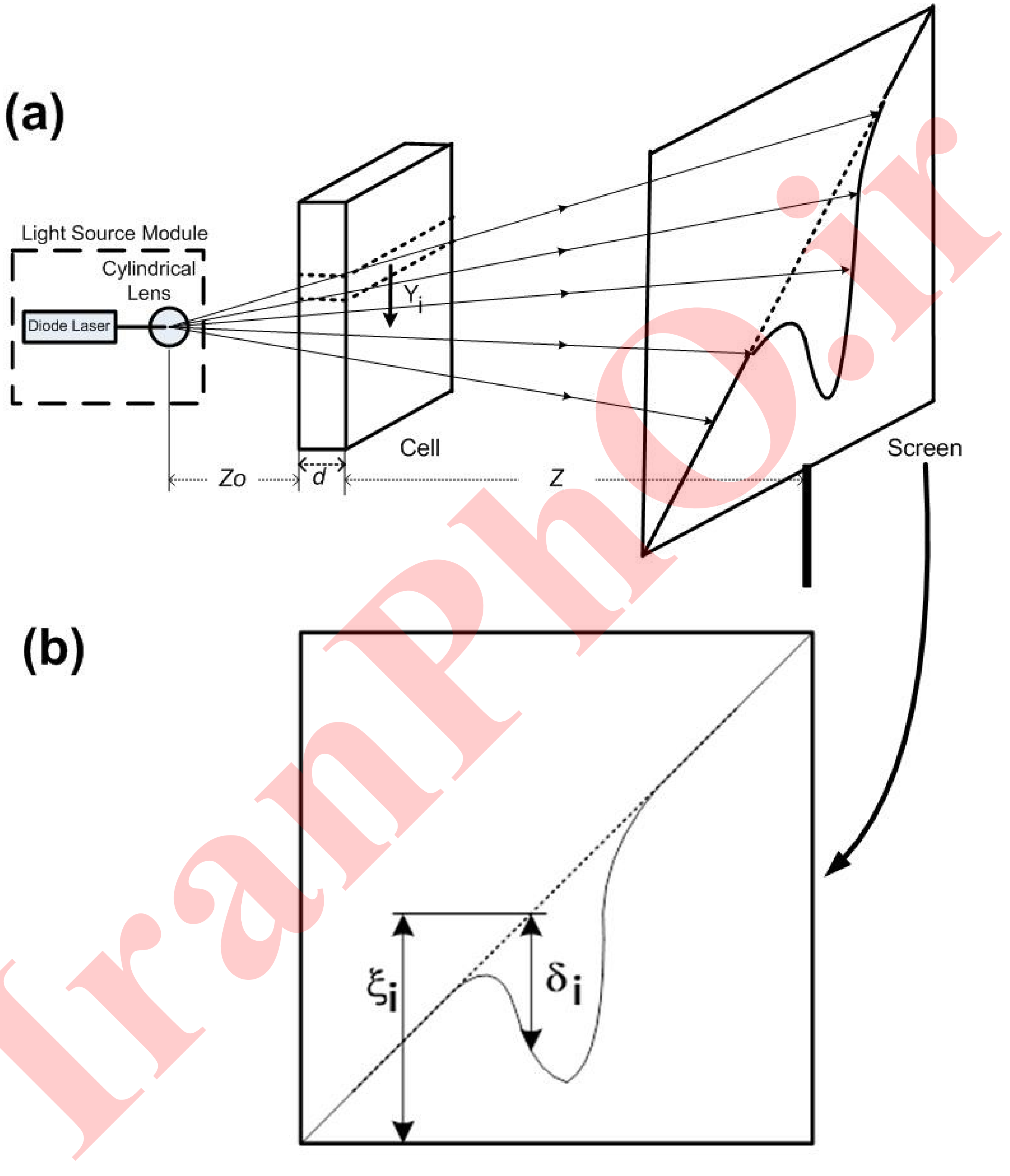

Figure 2. (a) Schematic diagram of the experiment. **The cell contains salt water solution with distilled water (aquadest) above it. (b)** Typical deflactogram, the deflected laser beam that appears on the screen when the diffusion between the solution 1 and solution 2 occurs.

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To obtain a profile of the refractive index gradient as a function of vertical position in the fluid we must relate the vertical position on the screen (ξ) to the vertical height in the cell (Y), and relate the vertical deflection (δ) to the gradient of refractive index (dn/dY). From the geometry of the experimental setup (see Figure 2) we have:

$$Y_i = \frac{\xi_i Z_0}{Z_0+d+Z} \qquad (1)$$

where Z_0, Z and d that are shown in Figure 2(a) denote the distance between the light source module and the diffusion cell, the distance between the diffusion cell and the screen, and the diffusion cell's thickness, respectively. **For measuring Z_0, the line marked on the laser module stand indicates the position of the cylindrical lens.**

The thickness of the cell (d) and the refractive index gradient are both small enough so that the refraction produces negligible vertical displacement of the ray within the cell. In this limit each ray travels at nearly constant vertical height within the cell and is deflected by a single refractive gradient associated with this height.

It can be shown that:

$$\left(\frac{dn}{dY}\right)_i = \frac{\delta_i}{Zd} \qquad (2)$$

Experimental Procedures:

- In order to obtain the deflected laser trace on the screen (as in Figure 2b) you must assemble all the components depicted in Figure 1 by following the schematic shown in Figure 2(a).

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- Ensure that the laser is on and its spot and its projection on the screen have diagonal form when it hits perpendicularly the diffusion cell. You may adjust Z, Z_0, and the focal length of the laser (by rotating the back of the laser) in order to obtain a bright and focussed line. You can also adjust the direction of the diagonal line on the screw by rotating the laser as a whole (release the laser using the screw on top). In the condition without any water or salt solutions you will see the straight diagonal laser beam.
- Deflected laser trace will appear when the two different solutions are diffusively mixed. Ensure that you pour the salt solution first into the diffusion cell container up to the limit which is marked by white line. Drop the water into the container slowly about 40 drops down the side channel using a pipette, after which you may turn on the stop watch to measure the evolution time of diffusion profile. If the set up of Z, Z_0 and the height of the laser are already optimized, the deflacted laser trace on the screen will be centered, clear and the depth of the dip as large as possible. You need to find this optimum set-up in order to minimize the error of the measurement.
- After the evolution time of 30 minutes you may draw the laser trace into millimeter block paper attached to the screen using a pencil. Note that in this experiment, you will be asked to do the measurement for 3 different salt solution concentrations (i.e. C_0 = 23 g/150 ml, C_0 = 28 g/150 ml and C_0 = 33 g/150 ml) so that you need to replace the millimeter block frequently. The millimeter block paper should be inserted to the screen and it can be loosen or tighten by rotating the screw attached in the corner of the screen.
- Please make sure that you have written your student code number and the concentration of the solution that you used on this millimeter block paper.

IV. Experiments and Tasks

A: Measurement of Refractive Index Gradient of Salt Water Solution (4.5 points)

You need to perform the steps below for all three salt concentration. Note that no error estimation is needed.

A.1	Perform the experiment to obtain the deflected laser trace on the screen. Duplicate the laser trace into the millimeter block paper attached to the screen using a pencil for evolution time of the diffusion (t) of 30 minutes.	1.2 pt.
A.2	Measure Z, d, Z_0, ξ_i and δ_i (with i = 1,...,20, are the data points corresponding to different horizontal position) from the laser trace in the millimeter block for evolution time of the diffusion (t) of 30 minutes. The parameters Z, d, Z_0, ξ_i and δ_i are stated in cm. Note that Z, d and Z_0 are the same for all measurement. Record the results in the Table 1.	1.5 pt.
	Calculate Y_i and $\left(\frac{dn}{dY}\right)_i$ (with *i* = 1, ..., 20, are the data points) for duration time of	

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A.3	diffusion (t) of 30 minutes. Note that Z, d and Z_0 are the same for all measurement. Record the results in Table 2. Plot $\left(\frac{dn}{dY}\right)_i$ vs. Y_i for t = 30 minutes.	1.5 pt.
A.4	Determine the Y_i at $\left(\frac{dn}{dY}\right)_i$ maximum obtained from question A3. Assign this Y_i as h.	0.3 pt.

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B: Determination of Diffusion Coefficient (4.2 points)

The curves found from question A.3 can be fitted using the following equations:

$$\left(\frac{dn}{dY}\right)_i = \left(\frac{dn}{dC}\right)\left(\frac{dC}{dY}\right)_i \qquad (3)$$

$$\left(\frac{dC}{dY}\right)_i \approx \frac{C_o}{2\sqrt{\pi D t}} e^{-\frac{(h-Y_i)^2}{4Dt}} \qquad (4)$$

where C, C_0, D, t, h denoted as concentration, initial salt solution concentration, diffusion coefficient, duration time of diffusion, and Y_i at maximum of refractive index gradient (dn/dY), respectively. Note that (dn/dC) is constant. The diffusion coefficient can be obtained by using equations (3) and (4) to form a linear relation between $(dn/dY)_i$ and Y_i.

B.1	Based on Eqns (3,4), find such functions $f\left(\frac{dn}{dY}\right)$ and $g(Y)$ that the dependence between $f\left(\frac{dn}{dY}\right)$ and $g(Y)$ would be linear.	0.9 pt.
B.2	Make a table (Table 3 in the answer sheet) that contains the data points, abscissa axis, ordinate axis of linear equation from B1 for dataset taken from Tasks A. Plot this table.	1.8 pt.
B.3	Determine the diffusion coefficient D from linear plot obtained from B2 for data set at t = 30 minutes. Note that linear dependence may hold only for a subrange of your data.	1.5 pt.

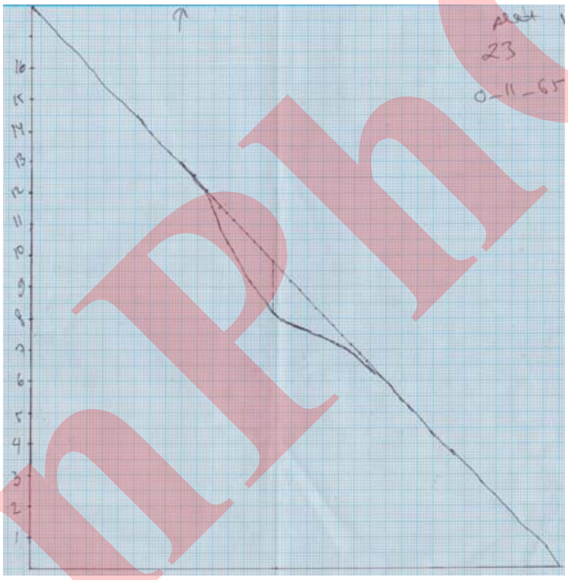
C. Nonlinear diffusion (1.3 points)

C.1	The analysis above is based on the assumption that D is independent of C. If this is not true, we have so-called nonlinear diffusion. However, near the maximum of $\frac{dn}{dY}$ we can consider this as an ordinary diffusion, with diffusion coefficient corresponding to the local value of concentration. Determine the rate of change of the diffusion coefficient with the change of salt solution concentration graphically using data from part B.	1.3 pt.

Determination of Refractive Index Gradient and Diffusion Coefficient of Salt Solution from Laser Deflection Measurement

(10 points)

A. Measurement of Refractive Index Gradient of Salt Water Solution (4.5 points)

Question	Answer	Marks
A1. **(1.2 pts)**		Deflectogram of C_0 = 23 g/150 mL Centred Depth of dip: 1.5 - 1.6 cm (0.4 pts)
	No dip	**-0.4**
	No reference line	**-0.05**
	Deflectogram (DL) not at the centre (+- 5mm) but the depth of dip still in 1.5 - 1.6 cm range	**-0.05**
	DL at the centre, the depth of dip <1.5 cm or >1.6 cm	**-0.05**
	DL not at the centre, the depth of dip <1.5 cm or >1.6 cm	**-0.1**
		Deflectogram

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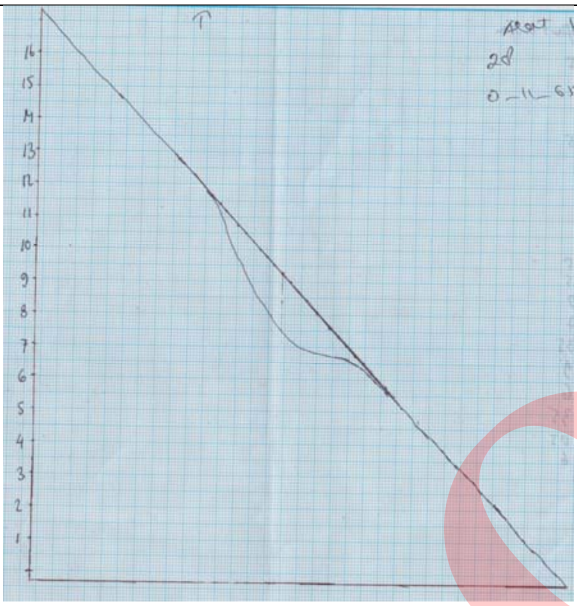
A1.		of C_0 = 28 gr/150 mL Centred Deep of dip: 1.7 - 1.9 cm (0.4 pts)
	No dip	**-0.4**
	No reference line	**-0.05**
	Deflectogram (DL) not at the centre (+- 5mm) but the depth of dip still in 1.7 cm - 1.9 cm range	**-0.05**
	DL at the centre, the depth of dip <1.7 cm or >1.9 cm	**-0.05**
	DL not at the centre, the depth of dip <1.7 cm or >1.9 cm	**-0.1**

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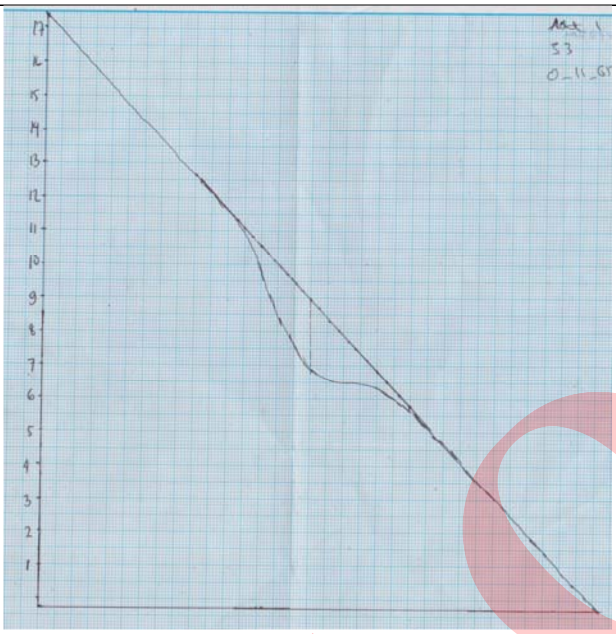
A1.



Deflectogram of

C_0 = 33 g/150 mL

Deep of dip:

1.9 - 2.3 cm

(0.4 pts)

No dip	**-0.4 pts**
No reference line	**-0.05 pts**
Deflectogram (DL) not at the centre (+-5mm) but the depth of dip still in 1.9 - 2.3 cm range	**- 0.05 pts**
DL at the centre, the depth of dip <1.9 cm or >2.3 cm	**- 0.05 pts**
DL not at the centre, the depth of dip <1.9 cm or >2.3 cm	**-0.1**

A2.

(1.5 pts)

i	δ_i (cm)	ξ_i (cm)	Z_0 (cm)	d(cm)	Z (cm)
1	0.05	11.55	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.35	11.3			
3	0.6	11.05			
4	0.9	10.85			
5	1	10.65			
6	1.1	10.35			
7	1.3	10.15			
8	1.4	9.85			
9	1.45	9.7			
10	1.5	9.45			
11	1.6	9.25			
12	1.5	8.95			
13	1.4	8.65			
14	1.2	8.35			
15	1	8.05			

Table 1 of

C_0 = 23 g/150 mL

Optimum Z and Z_0

data = 20

(0.5 pts)

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16	0.8	7.75			
17	0.7	7.55			
18	0.5	7.25			
19	0.3	6.95			
20	0.2	6.65			
21	0.05	6.4			

Correct data point must be extracted from deflectogram

# correct data points >= 20, but not all observable (Z, Z0, d) are written	-0.05 pts
Incorrect *d*	-0.05 pts
15<= # correct data points<20,	-0.15 pts
10<# correct data points<15	-0.3 pts
#correct data points<10	-0.45 pts

A2.

i	δ_i (cm)	ξ_i (cm)	Z_o (cm)	*d(cm)*	*Z (cm)*
1	0.05	11.65	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.25	11.4			
3	0.4	11.2			
4	0.8	11			
5	1	10.75			
6	1.2	10.4			
7	1.4	10.2			
8	1.5	10			
9	1.6	9.8			
10	1.7	9.5			
11	1.75	9.25			
12	1.7	8.95			
13	1.65	8.7			
14	1.5	8.4			
15	1.25	8.05			
16	0.9	7.6			
17	0.6	7.3			
18	0.4	7.05			
19	0.25	6.75			
20	0.05	6.3			

Table 1 of C_0 = 28 g/150 mL

Optimum *Z* and Z_0

data = 20

(0.5 pts)

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	Correct data point must be extracted from deflectogram	
	# correct data points >= 20, but not all observable (Z, Z0, d) are written	-0.05 pts
	Incorrect *d*	-0.05 pts
	15<= # correct data points<20,	-0.15 pts
	10<# correct data points<15	-0.3 pts
	#correct data points<10	-0.45 pts

A2.

i	δ_i (cm)	ξ_i (cm)	Z_o (cm)	*d(cm)*	*Z (cm)*
1	0.05	11.6	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.15	11.4			
3	0.35	11.1			
4	0.65	10.85			
5	1.1	10.6			
6	1.3	10.4			
7	1.5	10.2			
8	1.7	10			
9	1.85	9.7			
10	2	9.5			
11	2.1	9.25			
12	2	9			
13	1.8	8.6			
14	1.5	8.3			
15	1.25	8.05			
16	1	7.8			
17	0.75	7.45			
18	0.55	7.15			
19	0.4	6.8			
20	0.2	6.4			
21	0.05	6.1			

Table 1 of C_0 = 33 g/150 mL

data point >= 20

(0.5 pts)

Correct data point must be extracted from deflectogram	
# correct data points >= 20, but not all observable (Z, Z0, d) are written	-0.05 pts
Incorrect *d*	-0.05 pts
15<= # correct data points<20,	-0.15 pts

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<table>
<tr><td></td><td>10<# correct data points<15
#correct data points<10</td><td>-0.3 pts
-0.45 pts</td></tr>
<tr><td>A3.
(1.5 pts)</td><td>
<table>
<tr><th>i</th><th>Y_i (cm)</th><th>dn/dY</th></tr>
<tr><td>1</td><td>1.85944</td><td>0.00117</td></tr>
<tr><td>2</td><td>1.81919</td><td>0.00819</td></tr>
<tr><td>3</td><td>1.77894</td><td>0.01404</td></tr>
<tr><td>4</td><td>1.74674</td><td>0.02106</td></tr>
<tr><td>5</td><td>1.71455</td><td>0.02340</td></tr>
<tr><td>6</td><td>1.66625</td><td>0.02574</td></tr>
<tr><td>7</td><td>1.63405</td><td>0.03043</td></tr>
<tr><td>8</td><td>1.58575</td><td>0.03277</td></tr>
<tr><td>9</td><td>1.56161</td><td>0.03394</td></tr>
<tr><td>10</td><td>1.52136</td><td>0.03511</td></tr>
<tr><td>11</td><td>1.48916</td><td>0.03745</td></tr>
<tr><td>12</td><td>1.44086</td><td>0.03511</td></tr>
<tr><td>13</td><td>1.39257</td><td>0.03277</td></tr>
<tr><td>14</td><td>1.34427</td><td>0.02809</td></tr>
<tr><td>15</td><td>1.29597</td><td>0.02340</td></tr>
<tr><td>16</td><td>1.24767</td><td>0.01872</td></tr>
<tr><td>17</td><td>1.21548</td><td>0.01638</td></tr>
<tr><td>18</td><td>1.16718</td><td>0.01170</td></tr>
<tr><td>19</td><td>1.11888</td><td>0.00702</td></tr>
<tr><td>20</td><td>1.07058</td><td>0.00468</td></tr>
<tr><td>21</td><td>1.03034</td><td>0.00117</td></tr>
</table>
Jury must check the data in table
wrong data point < 3
3<# wrong data point < 6
wrong data point > 6</td><td>Table 2 of
C_0 = 23 g/150 mL.
data = 20
(0.25 pts)
- 0
- 0.05 pts
- 0.25pts</td></tr>
</table>

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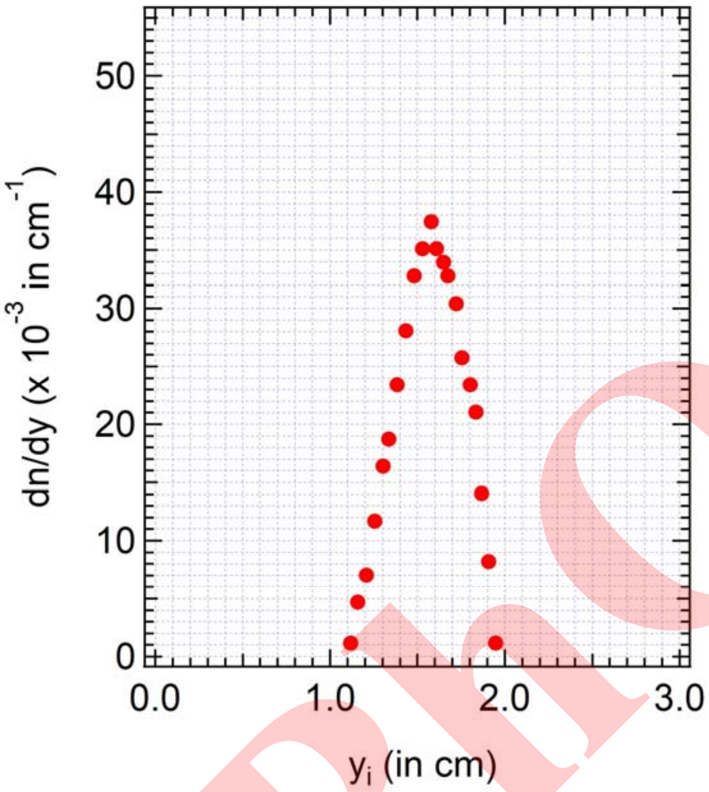
<table>
<tr>
<td>A3.</td>
<td>

No x-axis label
No x-axis unit
Without x-axis unit
No y-axis label
No y-axis unit
Without y-axis unit
Ordinate axis represented in 2 digid behind point
Ordinate axis represented in 3 digid behind point
Random shape</td>
<td>Plot dn/dY vs Y
C$_0$ = 23 g/150 mL.
“Gaussian-Like” shape
(0.25 pts)
-0.01 pts
-0.01 pts
-0.01 pts
-0.01 pts
-0.01 pts
-0.01 pts
-0.05 pts
-0 pts
-0.25 pts</td>
</tr>
<tr>
<td>A3.</td>
<td><table><tr><th>i</th><th>Y$_i$ (cm)</th><th>dn/dY</th></tr><tr><td>1</td><td>1.87554</td><td>0.00117</td></tr><tr><td>2</td><td>1.83529</td><td>0.00585</td></tr><tr><td>3</td><td>1.80309</td><td>0.00936</td></tr><tr><td>4</td><td>1.77089</td><td>0.01872</td></tr></table></td>
<td>Table 2 of
C$_0$ = 28 g/150 mL.
data = 20</td>
</tr>
</table>

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5	1.73065	0.02340
6	1.67430	0.02809
7	1.64210	0.03277
8	1.60990	0.03511
9	1.57770	0.03745
10	1.52941	0.03979
11	1.48916	0.04096
12	1.44086	0.03979
13	1.40061	0.03862
14	1.35232	0.03511
15	1.29597	0.02926
16	1.22352	0.02106
17	1.17523	0.01404
18	1.13498	0.00936
19	1.08668	0.00585
20	1.01424	0.00117

(0.25 pts)

Jury must check the data in table

# wrong data point < 3	- 0
3<# wrong data point < 6	- 0.05 pts
# wrong data point > 6	- 0.25pts

A3.

Plot dn/dY vs Y

C_0 = 28 g/150 mL.

“Gaussian-Like” shape

(0.25 pts)

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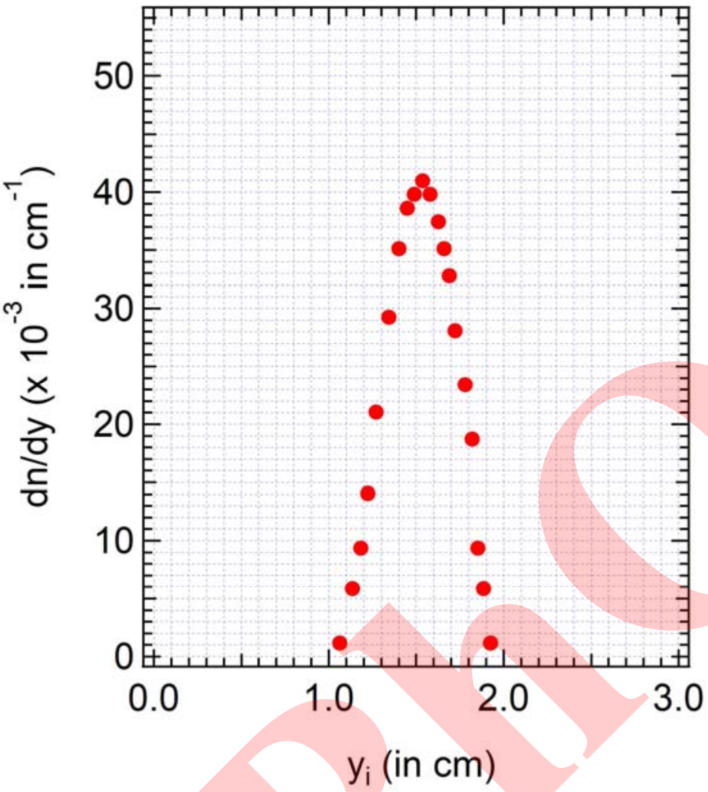

without x-axis label	-0.01 pts
without x-axis unit	-0.01 pts
wrong x-axis unit	-0.01 pts
without y-axis label	-0.01 pts
without y-axis unit	-0.01 pts
wrong y-axis unit	-0.01 pts
Ordinate axis represented in 2 digid behind point	-0.05 pts
Ordinate axis represented in 3 digid behind point	-0 pts
Random shape of the curve	-0.25 pts

A3.

i	Y_i (cm)	dn/dY
1	1.86749	0.00117
2	1.83529	0.00351
3	1.78699	0.00819
4	1.74674	0.01521
5	1.70650	0.02574

Table 2 of

C_0 = 33 g/150 mL.

data = 20

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<table>
<tr>
<td></td>
<td>
<table>
<tr><td>6</td><td>1.67430</td><td>0.03043</td></tr>
<tr><td>7</td><td>1.64210</td><td>0.03511</td></tr>
<tr><td>8</td><td>1.60990</td><td>0.03979</td></tr>
<tr><td>9</td><td>1.56161</td><td>0.04330</td></tr>
<tr><td>10</td><td>1.52941</td><td>0.04681</td></tr>
<tr><td>11</td><td>1.48916</td><td>0.04915</td></tr>
<tr><td>12</td><td>1.44891</td><td>0.04681</td></tr>
<tr><td>13</td><td>1.38452</td><td>0.04213</td></tr>
<tr><td>14</td><td>1.33622</td><td>0.03511</td></tr>
<tr><td>15</td><td>1.29597</td><td>0.02926</td></tr>
<tr><td>16</td><td>1.25572</td><td>0.02340</td></tr>
<tr><td>17</td><td>1.19938</td><td>0.01755</td></tr>
<tr><td>18</td><td>1.15108</td><td>0.01287</td></tr>
<tr><td>19</td><td>1.09473</td><td>0.00936</td></tr>
<tr><td>20</td><td>1.03034</td><td>0.00468</td></tr>
<tr><td>21</td><td>0.98204</td><td>0.00117</td></tr>
</table>

Jury must check the data in table

wrong data point < 3

3<# wrong data point < 6

wrong data point > 6
</td>
<td>(0.25 pts)
- 0
- 0.05 pts
- 0.25pts</td>
</tr>
<tr>
<td>A3.</td>
<td></td>
<td>Plot dn/dY vs Y
C_0 = 33 g/150 mL.
(0.25 pts)</td>
</tr>
</table>

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	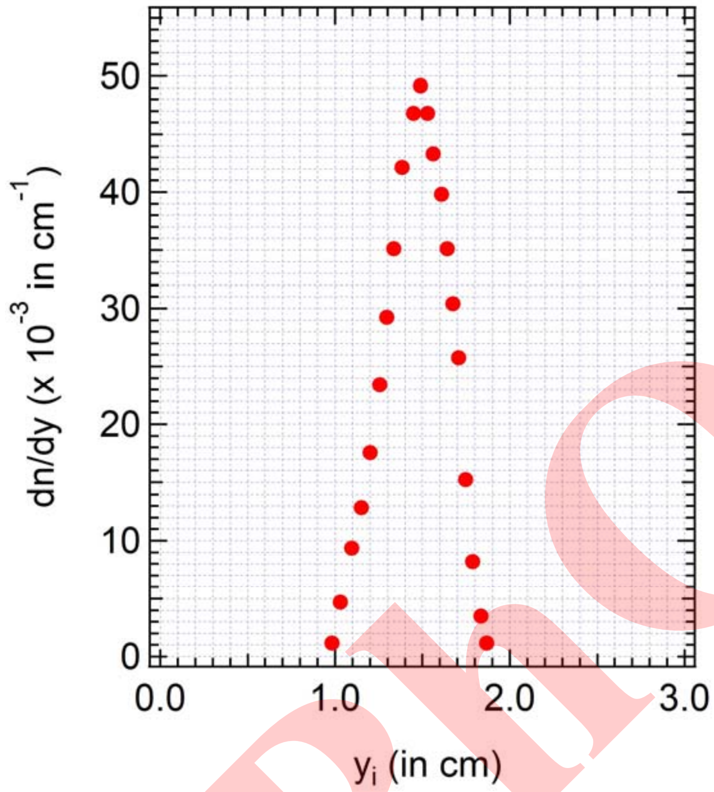 without x-axis label without x-axis unit wrong x-axis unit without y-axis label without y-axis unit wrong y-axis unit Ordinate axis represented in 2 digid behind point Ordinate axis represented in 3 digid behind point Random shape of the curve	-0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.05 pts -0 pts -0.25 pts
A4. **(0.3 pts)**	*h* for 23 g/ 150 mL = (1.5 ± 0.1) cm	0.1 pts

	h for 28 g/ 150 mL = (1.5 ± 0.1) cm	0.1 pts
	h for 33 g/ 150 mL = (1.5 ± 0.1) cm	0.1 pts
	If *h* is correctly determined from graph A3 for each concentration	- 0
	If *h* is not correctly determined from graph A3 for each concentration	-0.1

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B : Determination of Diffusion Coefficient (4.2 points)

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>B1.
(0.9 pts)</td><td>**Linear form of eq.(3)**
$\ln\left(\frac{dn}{dY}\right) \approx m(h-Y)^2 + C$ (b1)
$m = -\frac{1}{4D_e t}$
Constant : $C = \ln\left(\left(\frac{dn}{dc}\right)\left(\frac{C_0}{2\sqrt{\pi D_e t}}\right)\right)$
Other than (b1)</td><td>0.9 pt
-0.9 pts</td></tr>
<tr><td>B2.
(1.8 pts)</td><td><table>
<tr><th>i</th><th>$(h-yi)^2$</th><th>ln(dn/dy)</th></tr>
<tr><td>1</td><td>0.06592</td><td>-3.86003</td></tr>
<tr><td>2</td><td>0.050423</td><td>-3.75467</td></tr>
<tr><td>3</td><td>0.031065</td><td>-3.65936</td></tr>
</table></td><td>Table 3 of
C_0 = 23 g /150 mL.</td></tr>
</table>

4	0.020752	-3.4923
5	0.00917	-3.41819
6	0.005128	-3.3831
7	0.000984	-3.3492
8	6.99E-07	-3.28466
9	0.002414	-3.3492
10	0.009493	-3.41819
11	0.021237	-3.57235
12	0.037646	-3.75467
13	0.05872	-3.97781

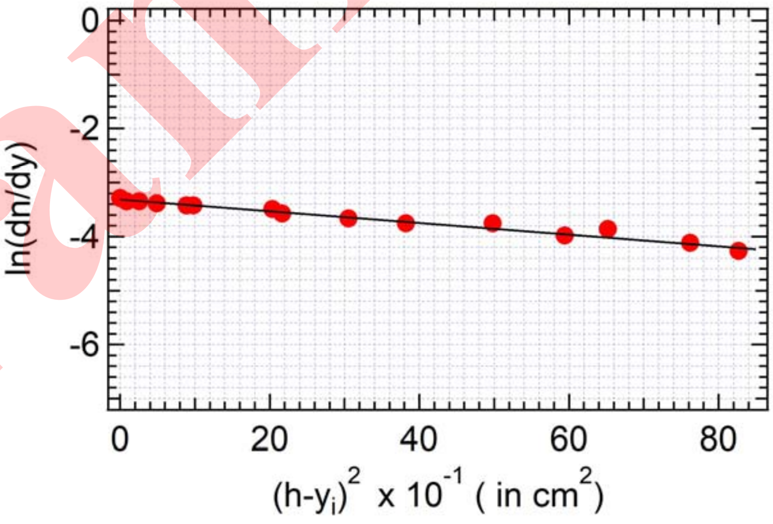
data = 10

(0.3 pts)

Jury must check the data in table

# of data point > 10	-0 pts
3 <= # of data point < 10	-0.05 pts
# of data point < 3	-0.3 pts
# wrong data point < 3	- 0
3<# wrong data point < 6	- 0.05 pts
# wrong data point >6	- 0.25 pts

B2



Using linear regression of eq. (B1.1), we obtain

m (slope) = -10 cm^{-2} till -8.8 cm^{-2}

Plot of Table 3

C_0 = 23 g/150 mL

data = 10

(0.3pts)

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	without x-axis label without x-axis unit wrong x-axis unit without y-axis label without y-axis unit wrong y-axis unit # of data point in linear range > 10 3 <= # of data point in linear range < 10 # of data point in linear range < 3 or random shape of curve m is out of range	-0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts - 0 - 0.05 pts - 0.25 pts -0.3 pts

B2.

i	$(h-y_i)^2$	$\ln(dn/dy)$
1	0.057912	-3.75467
2	0.033968	-3.57235
3	0.023136	-3.41819
4	0.014378	-3.3492
5	0.007693	-3.28466
6	0.001553	-3.22404
7	6.99E-07	-3.19505
8	0.002414	-3.22404
9	0.007989	-3.25389
10	0.018955	-3.3492
11	0.037646	-3.53152
12	0.071007	-3.86003
13	0.099079	-4.26549

Jury must check the data in table

Table 3 of

C_0 = 28 g /150 mL

data = 10

(0.3 pts)

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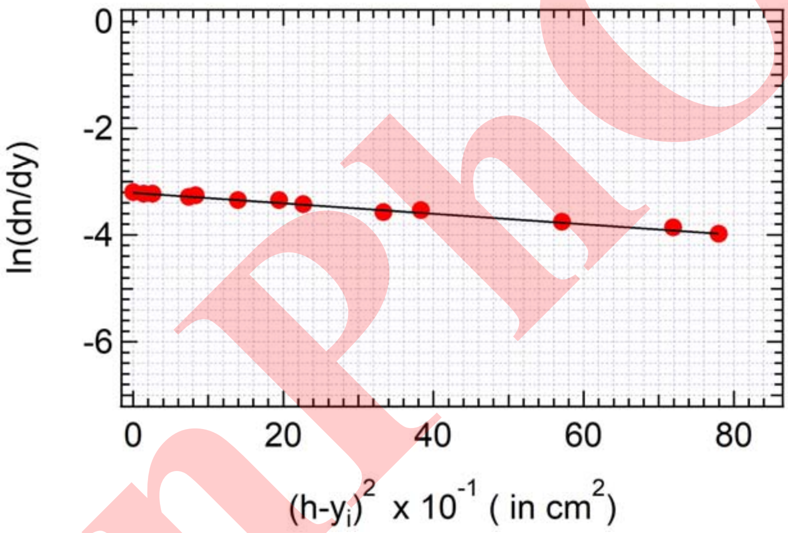
	# of data point > 10	-0 pts
	3 <= # of data point < 10	-0.05 pts
	# of data point < 3	-0.3 pts
	# wrong data point < 3	- 0
	3<# wrong data point < 6	- 0.05 pts
	# wrong data point >6	- 0.25 pts
B2.		Plot of Table 3 C_0 = 28 g/150 mL # data = 10
	Using linear regression of eq. (B1.1), we obtain	
	m (slope) = -10.3 cm^{-2} till -11 cm^{-2}	(0.3pts)
	without x-axis label	-0.01 pts
	without x-axis unit	-0.01 pts
	wrong x-axis unit	-0.01 pts
	without y-axis label	-0.01 pts
	without y-axis unit	-0.01 pts
	wrong y-axis unit	-0.01 pts
		-0 pts

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<table>
<tr>
<td></td>
<td># of data point in linear range > 10
3 <= # of data point in linear range < 10
# of data point in linear range < 3 or random shape of curve
m is out of range</td>
<td>-0.05 pts
-0.3 pts
-0.3 pts</td>
</tr>
<tr>
<td>B2.</td>
<td>
<table>
<tr><th>i</th><th>$(h\text{-}yi)^2$</th><th>ln(dn/dy)</th></tr>
<tr><td>1</td><td>0.046873</td><td>-3.65936</td></tr>
<tr><td>2</td><td>0.033968</td><td>-3.4923</td></tr>
<tr><td>3</td><td>0.023136</td><td>-3.3492</td></tr>
<tr><td>4</td><td>0.014378</td><td>-3.22404</td></tr>
<tr><td>5</td><td>0.005128</td><td>-3.13948</td></tr>
<tr><td>6</td><td>0.001553</td><td>-3.06152</td></tr>
<tr><td>7</td><td>6.99E-07</td><td>-3.01273</td></tr>
<tr><td>8</td><td>0.001688</td><td>-3.06152</td></tr>
<tr><td>9</td><td>0.011126</td><td>-3.16688</td></tr>
<tr><td>10</td><td>0.023647</td><td>-3.3492</td></tr>
<tr><td>11</td><td>0.037646</td><td>-3.53152</td></tr>
<tr><td>12</td><td>0.054884</td><td>-3.75467</td></tr>
<tr><td>13</td><td>0.08446</td><td>-4.04235</td></tr>
</table>

Jury must check the data in table
# of data point > 10
3 <= # of data point < 10
# of data point < 3
wrong data point < 3
3<# wrong data point < 6
wrong data point >6
</td>
<td>Table 3 of
C_0 = 33 g /150 mL
data = 10
(0.3 pts)
-0 pts
-0.05 pts
-0.3 pts
- 0
- 0.05 pts
- 0.25</td>
</tr>
<tr>
<td>B2.</td>
<td></td>
<td>Plot of Table 3
C_0 = 33 g/150 mL</td>
</tr>
</table>

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	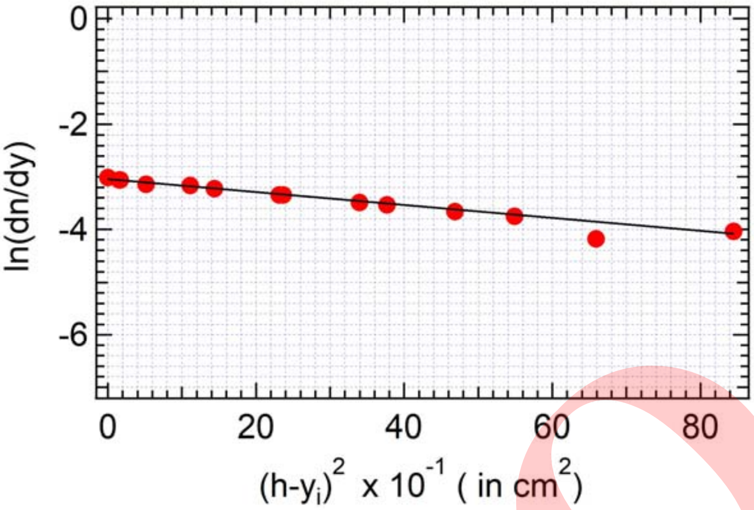 Using linear regression of eq. (B1.1), we obtain *m* (slope) = −11.3 cm^{-2} till −12.8 cm^{-2} without x-axis label without x-axis unit wrong x-axis unit without y-axis label without y-axis unit wrong y-axis unit m is out of range # of data point in linear range > 10 3 <= # of data point in linear range < 10 # of data point in linear range < 3 or random shape of curve	# data = 10 (0.3pts) -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.01 pts -0.3 pts -0 pts -0.05 pts -0.3
B3 **(1.5 pts)**	*D* of 23 g/ 150 mL = (1.38 till 1.58) x 10^{-5} cm^2/s *D* of 28 g/ 150 mL = (1.26 till 1.46) x 10^{-5} cm^2/s	0.5 pts 0.5 pts

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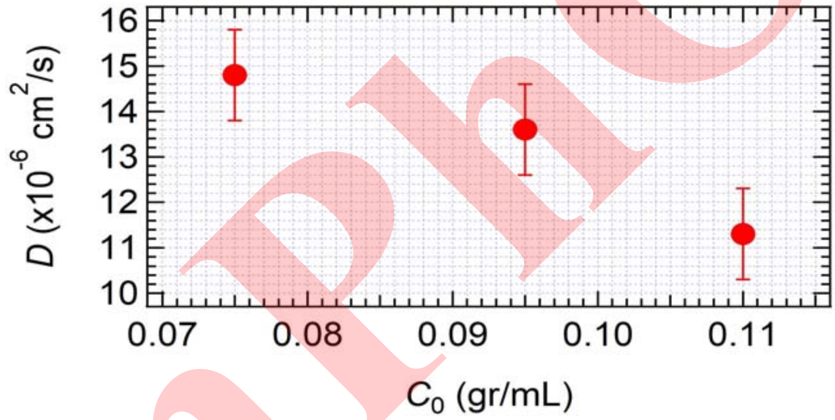
	D of 33 g/ 150 mL = (1.03 till 1.23) x 10^{-5} cm^2/s *D* is out of range for each concentration	0.5 pts- -0.5 pts

C. Nonlinear diffusion (1.3 points)

Question	**Answer**	**Marks**
C1. **(1.3 pts)**	 Without error bars **Value of C not stated in $C_0/2$**	Plot *D* vs. C_0 0.8 pts -0 -0.4 pts
C1.	$\frac{d}{dC}D = -4.2 \times 10^{-5} cm^2\ mL\ g^{-1} s^{-1}$ till $-15.8 \times 10^{-5} cm^2\ mL\ g^{-1} s^{-1}$ Without or wrong unit Out of range	0.5 pts -0.01 pts -0.5 pts

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Parallel Dipole Line Magnetic Trap for Earthquake and Volcanic Sensing

A. Introduction

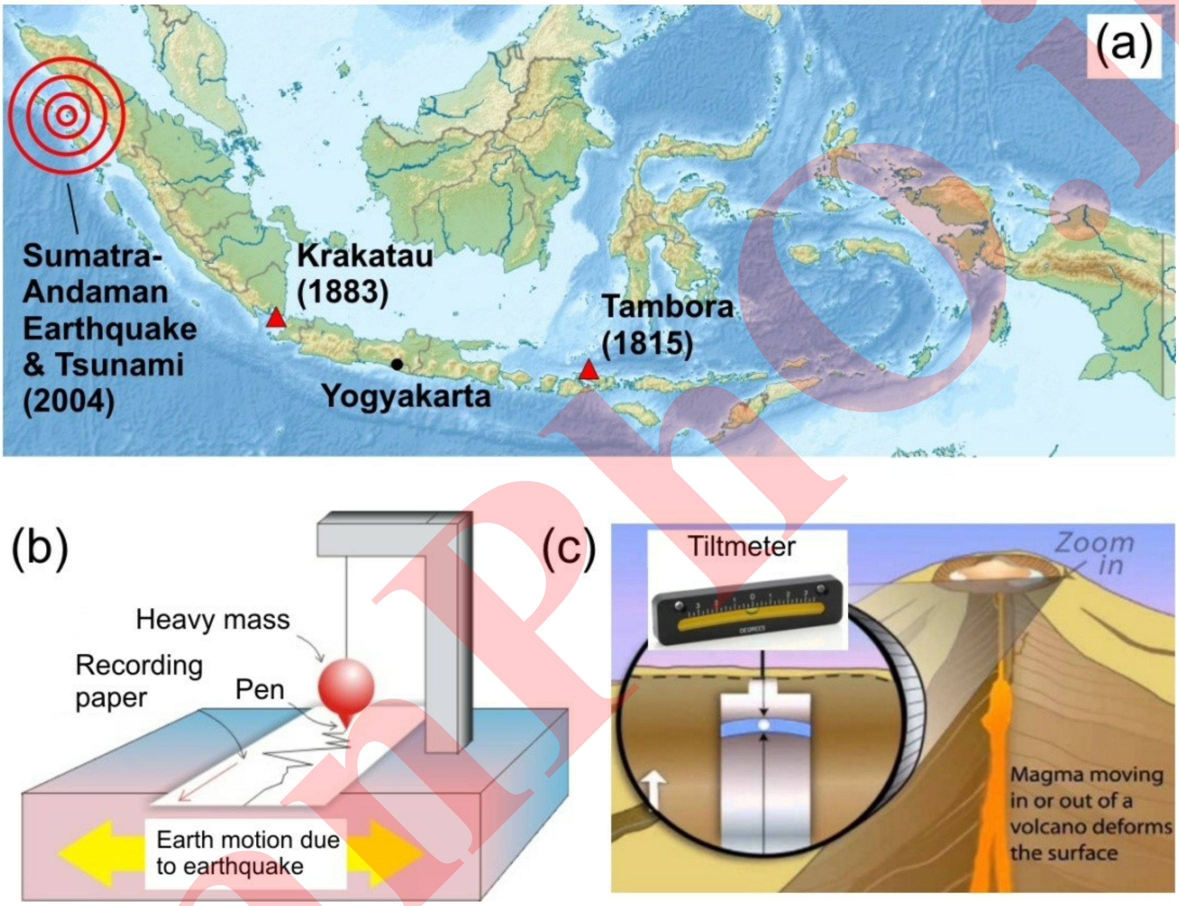

Figure. 1 (a) Map of Indonesia showing its well-known disasters. (b) Basic seismometer to detect earthquake. (c) Tiltmeter to monitor volcano.

Indonesia is the world's largest archipelago, with about 17,000 islands sprawling in the tropics and thus often called "the jewel of the equator". Unfortunately, it has plenty of natural threats such as earthquake and volcanic eruptions. Colossal catastrophic events (Fig. 1a) such as the Sumatra-Andaman earthquake and tsunami (2004), Krakatau (1883) and Tambora (1815) volcanic eruptions are among the deadliest disasters in the recorded history of the world. To detect earthquake, we use a *seismometer*, usually a pendulum-based system to *measure the ground displacement or acceleration* (Fig. 1b). To monitor volcano, we use *tiltmeter* to detect a change in *ground inclination* due to underground magma movement (Fig. 1c). In this problem we will explore the physics and applications of a new kind of magnetic trap and sensor - called *Parallel Dipole Line* (PDL) trap system - for sensing earthquake and to monitor volcano.

Parallel dipole line system is an arrangement of two linear distribution of magnetic dipole (also called *a*

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dipole line) as shown in Fig. 2. Recently two Indonesian physicists discovered a very interesting effect in this system: if the length of the dipole line is longer than certain critical length the magnetic field becomes stronger on the edges which produces a "*camelback* potential" as shown in Fig. 2a.* This "*camelback effect*" is important as it enables this system to serve as *a new type of magnetic trap called Parallel Dipole Line (PDL) trap*. Experimentally we can realize this PDL trap using a pair of diametric magnets *i.e.* a cylinder magnets with magnetization along diameter as shown in Fig. 2c where the north and south poles are on the curved sides instead of the flat faces.

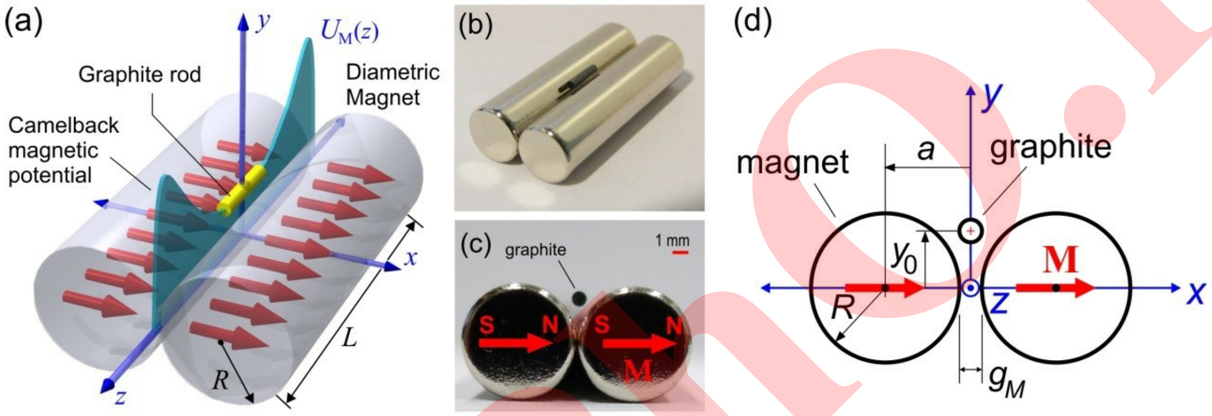

Figure 2. (a) Parallel dipole line trap model with the camelback potential along z-direction. (b) Experimental setup using "diametric" magnets. (c) Cross section view. (d) Schematics of the PDL trap. [* Gunawan and Virgus, J. Appl. Phys. 121, 133902 (2017)].

If we drop a graphite rod (ordinary pencil lead) into the trap it will levitate or gets trapped in a stable condition. This occurs because in x-direction the graphite is repelled by the magnets from both sides and in vertical (y) direction the magnetic repulsion force balance the gravity making it levitates at height y_0 (Fig. 2d). In longitudinal direction (z) the camelback potential holds the graphite stable.

The *camelback potential* of the magnetic trap serves as a *one-dimensional oscillator*. If you give a little perturbation along the z-axis to the graphite rod, it will exhibit underdamped oscillation as shown in Fig. 3a. This PDL trap can be used as a sensitive seismometer. If the ground underneath shakes, the graphite rod tends to remain stable and its relative displacement (Fig. 3b) is the "earthquake" signal. Similarly, it can also be used as sensitive tiltmeter: if you tilt the trap slightly, the graphite rod will move significantly without any friction.

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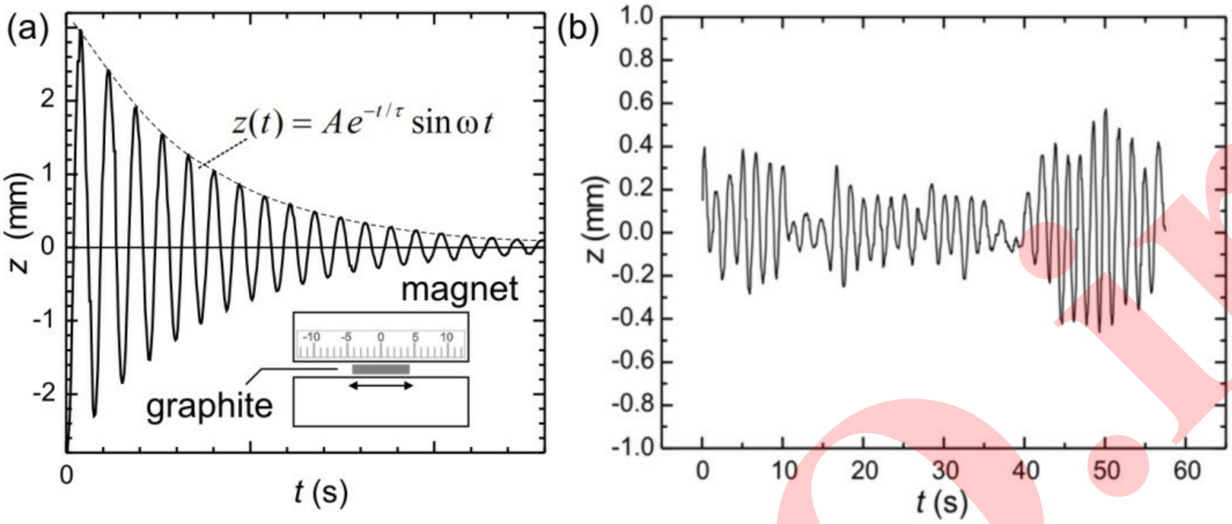

Figure 3. (a) Underdamped oscillation of a graphite rod along the camelback potential. (b) Seismometer application: Ground vibration detection by PDL trap.

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We will now investigate the physics and application of this PDL trap in two sections.

Section A: Basic characteristics

(1) Determination of the magnet's magnetization M (2.5 pt.)

(2) Magnetic levitation and magnetic susceptibility χ (1.0 pt.)

(3) The camelback potential oscillation and magnetic susceptibility χ (1.0 pt.)

(4) Oscillator quality factor Q and determination of air viscosity μ_A (3.0 pt.)

Section B: Applications

(5) *PDL Trap Seismometer* (0.5 pt.)

(6) *PDL Trap Tiltmeter* (2.0 pt.)

B. Apparatus

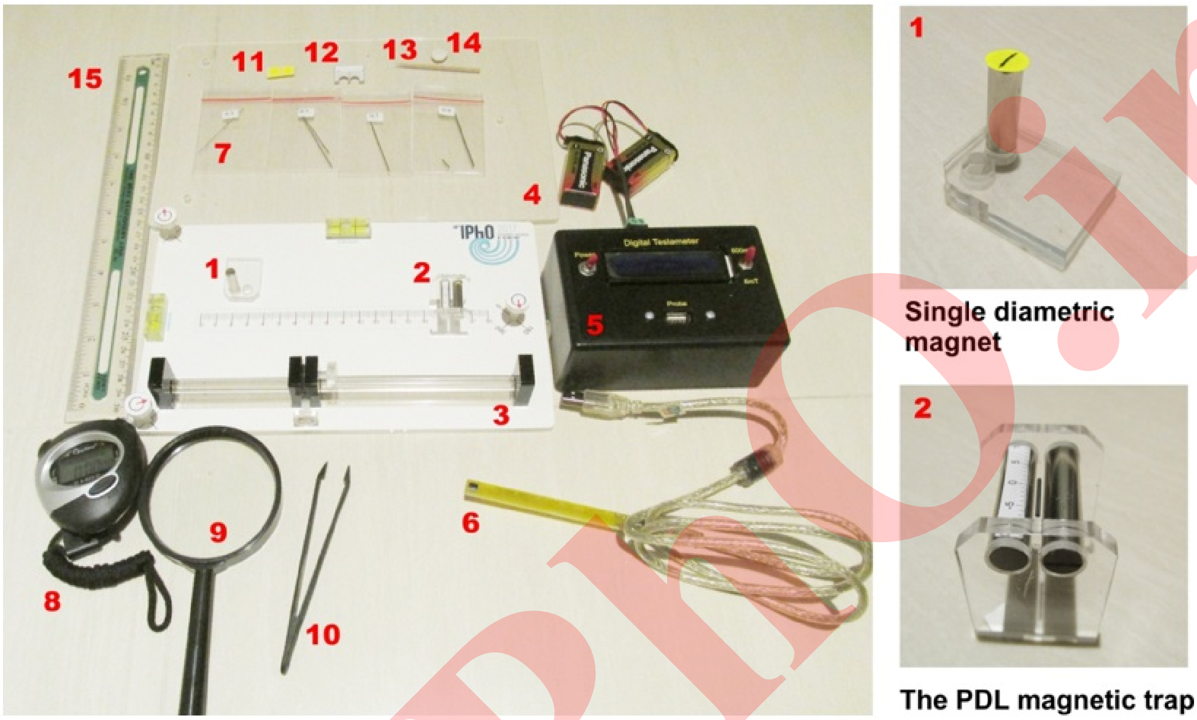

Figure 4. The experimental setup.

1. Single diametric magnet assembly. Yellow sticker is attached to mark magnetization.
2. The PDL magnetic trap assembly, shown with levitating graphite. Please do not remove the magnets from the assembly.
3. Top platform with 3 screws
4. Bottom platform
5. Tesla meter to measure magnetic field. Batteries are provided to power up the Tesla meter and a cable to connect the Hall probe to Tesla meter.
6. Hall sensor probe of the Tesla meter
7. Graphite rods (pencil leads) with 4 diameter sizes HB/0.3, HB/0.5, HB/0.7, and HB/0.9. See the constants and data for exact diameters. You may need to break these graphite rods to specific lengths as required.
8. Stopwatch
9. Magnifying glass
10. Tweezer, anti-static

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11. Round yellow sticker—to mark the magnetization direction (north-south pole) of single magnet
12. "Insert-ruler" to measure graphite levitation height
13. Toothpick to move around the graphite rod
14. Silly putty to stick magnet assemblies to platform
15. Ruler

INSTRUCTIONS & WARNING:

1. **Keep the single magnet and the PDL trap (double magnet) assemblies away from each other. They can hit each other and crack!**
2. **Turn off the Tesla meter if not in use to save battery!**
3. Please detach items 7, 11-14 carefully from bottom platform (item 4) and then place the top platform (item 3) on the bottom platform.
4. You can use the three screws to adjust the level of the top platform.

CONSTANTS AND DATA:

Radius of the diametric magnet : R = 3.2 mm
Length of the diametric magnet : L = 25.4 mm
Gap of the PDL trap : g_M = 1.5 mm

Mass density of graphite : ρ = 1680 kg/m^3
Graphite rod "HB/0.3" diameter : d = 0.38 mm
Graphite rod "HB/0.5" diameter : d = 0.56 mm
Graphite rod "HB/0.7" diameter : d = 0.70 mm
Graphite rod "HB/0.9" diameter : d = 0.90 mm

Room temperature : T = 298 K
Magnetic permeability in vacuum : μ_0 = 1.257$\times 10^{-6}$ H/m
Boltzman constant : k_B = 1.38064852 $\times$ 10^{-23} m^2 kg/s^2 K
Acceleration of gravity : g = 9.8 m/s^2

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C. Experiment & Questions

SECTION A. BASIC CHARACTERISTICS OF THE PDL TRAP

[1] Determination of the magnet's magnetization (M) (2.5 pt.)

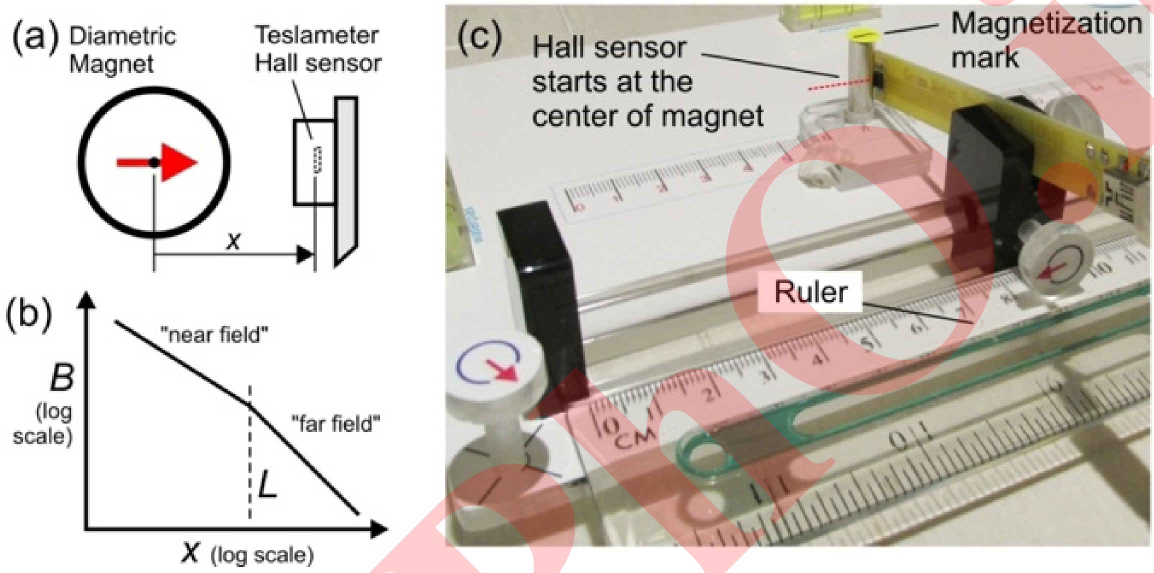

Figure 5. (a) Measuring the magnetic field. (b) Magnetic field profile (c) Setup.

The strength of the magnetic trap depends on the total magnetic dipole moment of the magnet m. It depends on the magnetization M which is magnetic dipole per unit volume and a characteristics of the magnetic material. For our cylindrical magnet:

$$M = \frac{m}{\pi R^2 L} \qquad (1)$$

where R is the radius and L is the length of the magnet (see Constants and Data). The value of M is considered the same for all magnets in this experiment. We will study the magnetic field profile and the determine M of the diametric magnet used in our PDL trap.

Take the single diametric magnet assembly and setup the experiment as shown in Fig. 5c. Align the magnetization (as shown in Fig. 6a) pointing towards the Hall (magnetic field) sensor. Measure the strength of the magnetic field along the x-axis using Tesla meter. The magnetic field profile B in near field or "Dipole Line" limit for approximately $x \leq 16$ mm:

$$B_I(x) = \frac{\mu_0 m}{2\pi x^P L} \qquad (2)$$

The x-axis is along the magnetization axis of the diametric magnet as shown in Figure 6a and x refers to the distance from the center of diametric magnet to hall probe sensor inside the sensor chip. Please refer to issue of offset shown in Figure 6b.

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We will perform measurements only in the "Near field" region:

A.1	Record zero offset (B_0) of the Teslameter without any magnet nearby. Subtract subsequent field measurement with this value.	0.1 pt.
A.2	Measure magnetic field B vs. x *in the near field region* (7 $\leq x \leq$ 16 mm)! Where x is the position measured from the center of the magnet. Record and plot your result on the answer sheet. Follow the "HINT & DIRECTIONS" below.	1.15 pt.
A.3	Use your experimental data to determine the value of the exponent p.	0.75 pt.
A.4	Determine the magnet's magnetization M.	0.5 pt.

HINTS & DIRECTIONS:

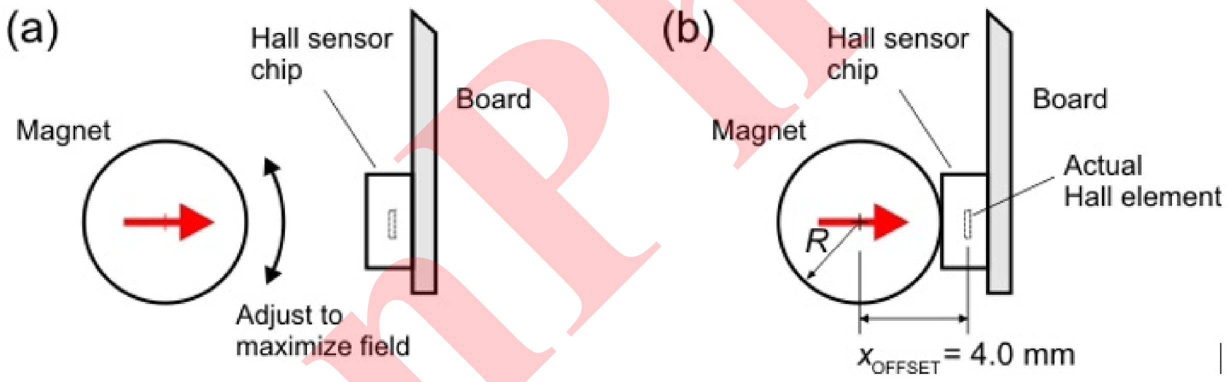

Figure 6. Magnetic field measurement (a) Adjustment (b) Offset issue

1. **Turn off the Tesla meter if not in use to save battery!**
2. For Teslameter, wait approximately ~2 sec for each data point before taking reading.
3. Note that x is measured from the center of magnet. Magnet radius is R = 3.2 mm.
4. Use recommended measurement setup in Fig. 5c.
5. See Fig. 6a, adjust the rotation of the magnet so its magnetization is pointing to the Hall sensor thus maximizing the field. You can use the yellow round sticker to mark the magnetization direction on the magnet.
6. When the Hall sensor touches the magnet the actual distance between the center of the magnet and the actual Hall sensor element is the offset value given as: x_{OFFSET} = 4 mm
7. Start your measurement with Hall sensor at x = 5 mm! Don't use the data when the sensor touches the magnet (x = 4 mm) as the sensor is saturated or the probe is flexing during touch.

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[2] The Magnetic Levitation effect and Magnetic Susceptibility (χ) (1 pt.)

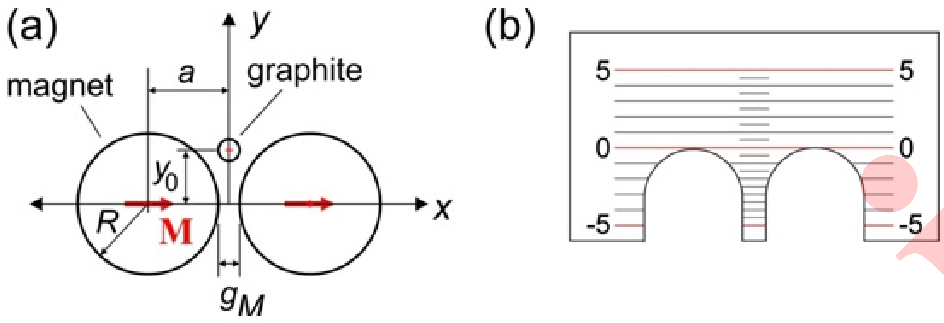

Figure 7. (a) The magnetic levitation effect in the PDL trap. (b) Insert ruler for measuring the levitation height y_0.

The PDL trap also demonstrates magnetic levitation effect. The graphite levitates at the center of the trap at height y_0 as shown in Fig. 7(a). The graphite is repelled by the magnet with a force $F_M(y_0)$ that depends on the magnetic susceptibility χ and the rod position y_0. Magnetic susceptibility describes how much a material gets magnetized in response to an applied field. It appears in relation: $\mu = (1+\chi)\mu_0$ where μ is the magnetic permeability of the material. This magnetic repulsion force on a graphite rod in the PDL trap is given as:

$$F_M(y_0) = -\frac{\mu_0 M^2 \chi V_r}{2}\frac{R^4}{a^5} f\left(\frac{y_0}{a}\right) \qquad (3)$$

Note that when $F_M(y_0)$ is positive the force is directed upwards and there is a negative sign in the formula. Here V_r is the volume of the graphite rod, M is the volume magnetization of the magnet (obtained from Question 1), a is the position of the magnet center given as: $a = R + g_M/2$ (see Fig. 7a) where g_M is the gap between magnets: g_M = 1.5 mm. $f(u)$ is the dimensionless function for the magnetic repulsion force in this trap given as:

$$f(u) = \frac{4u(3-u^2)(1-u^2)}{(1+u^2)^5} \qquad (4)$$

A.5	Place gently a graphite rod HB/0.5 and length = 8 mm. Measure the levitation height y_0 of the rod (see Fig. 7a)! Hint: Use the insert ruler provided as shown in Fig. 7b. Press the ruler on the magnets to read the position of the graphite rod.	0.1 pt.
A.6	Use the result from part A.5 to determine the magnetic susceptibility χ of the graphite rod.	0.8 pt.
A.7	What kind of magnetic material is graphite? Choose one: (i) Ferromagnetic; (ii) Paramagnetic; or (iii) Diamagnetic?	0.1 pt.

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[3] The camelback potential oscillation and magnetic susceptibility (χ) (1 pts)

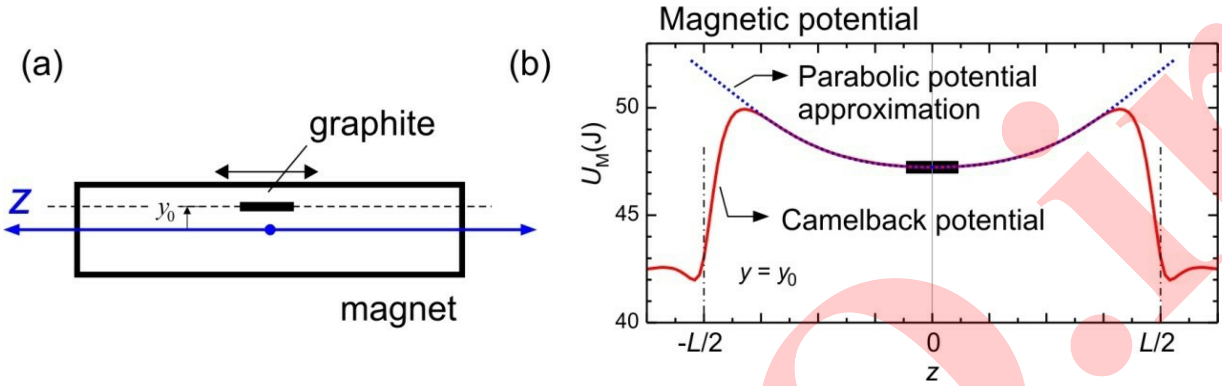

Figure 8. (a) Graphite oscillation at height y_0. (b) The camelback potential of the PDL trap and its parabolic approximation.

We will determine χ independently using the oscillation in the magnetic "camelback potential" of the PDL trap as shown in Fig. 8. For small amplitude (z < 4 mm) the magnetic potential can be approximated as a parabolic (shown as dotted curve in Fig. 8b):

$$U_M = \frac{1}{2}k_z z^2 \qquad (5)$$

where k_z is the spring constant of the potential and z is center of mass displacement of the rod. This spring constant k_z depends on the magnet magnetization M (from Question 1) and χ:

$$k_z = -C_1 \mu_0 \chi M^2 V_r \qquad (6)$$

where μ_0 is the magnetic permeability, V_r is the volume of the graphite rod, C_1 = 198.6 /m^2 is a constant for this magnetic trap setup.

Drop the graphite rod at the center of the magnetic trap. Adjust the platform level with the screw knobs so that the rod stays at the center of the trap. Displace the rod with a toothpick to induce oscillation along the camelback potential.

A.8	Perform an oscillation for the "HB/0.5" graphite and l = 8 mm. Limit to small oscillation amplitude i.e. A < 4mm. Determine the oscillation period. (The oscillation will decay over time due to damping, ignore this damping effect).	0.2 pt.
A.9	Calculate the magnetic susceptibility (χ) of the graphite using this oscillation.	0.8 pt.

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[4] Oscillator quality factor (Q) and estimate of air viscosity (3 pt.)

We observe that the graphite rod oscillation is damped due to the air friction and we want to understand how the friction depends on the size of the graphite rod (diameter and length) and estimate the air viscosity μ_A. The motion of the rod can be modeled as underdamped oscillation: $z(t) = Ae^{-t/\tau} sin(\omega t)$ as shown in Fig. 3(a) where A is the initial amplitude and $\omega = 2\pi f$ is the angular frequency and t is time. The amplitude decays with time by a factor of $\exp(-t/\tau)$ where τ is the damping time constant. This determines the oscillator "quality factor" defined as: $Q = \omega\tau/2$. If $Q > 0.5$ the oscillation is underdamped, $Q = 0.5$ is critically damped and $Q < 0.5$ is overdamped. *This quality factor is important for designing PDL trap as seismometer or tiltmeter sensors.*

We can calculate the damping time constant τ by approximating the cylinder rods as long ellipsoid and calculate the Stokes drag force. The damping time constant is given as:

$$\tau = \frac{2}{3}\frac{\rho r^2}{\mu_A} ln\left(0.607 \times \frac{l}{r}\right) \qquad (7)$$

where ρ, r and l are the mass density, radius and length of the graphite rod and μ_A is the viscosity of the air. We want to estimate the air viscosity using this model.

A.10	We need to determine the damping time constant of the oscillation τ. Sketch how you measure τ in *a simple way.*	0.5 pt.
A.11	Perform oscillation damping experiments with a group of rods with various diameters and fixed length of 8 mm. Determine the damping time constant τ for each rods.	1.5 pt.
A.12	Determine the air viscosity μ_A.	1.0 pt.

SECTION B. SENSOR APPLICATIONS

[5] PDL Trap Seismometer (0.5 pt.)

Imagine you are designing seismometer using this PDL magnetic trap. For seismometer application we want very high sensitivity or *very low acceleration "noise floor"* i.e. the lowest acceleration that it can detect. This acceleration noise floor is given as (in unit of m/(s^{2}Hz$^{0.5}$)):

$$a_n = \sqrt{\frac{4k_B T\omega}{Qm_R}} \qquad (8)$$

where k_B is the Boltzmann constant, T is the temperature (see Constants and Data), and m_R is the mass of the rod, all are in SI units. In Question 4 in you have measured τ of several graphite diameters. Pick one that you think will serve as the best seismometer.

B.1	Which diameter of rod do you choose?	0.2 pt.
B.2	Calculate the seismometer acceleration noise floor (a_n) for the rod of your choice.	0.3 pt.

[6] PDL Trap Tiltmeter (2 pt.)

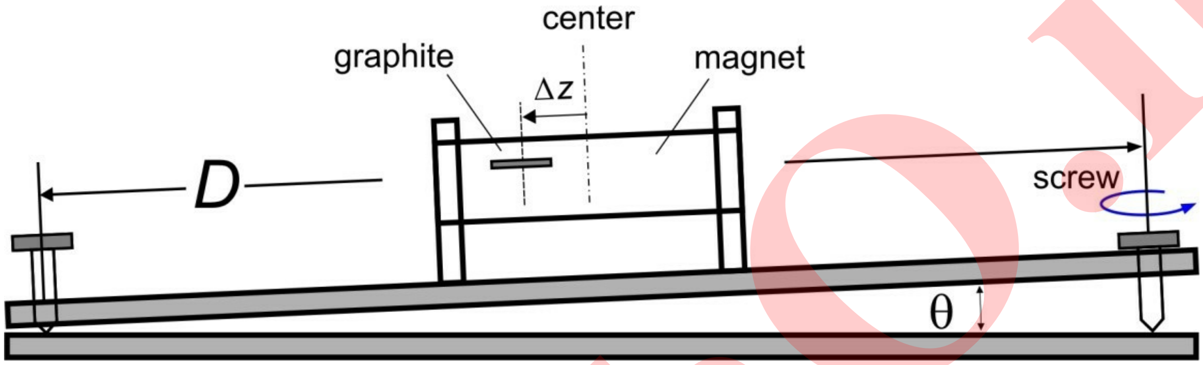

Figure 9. PDL trap system as tiltmeter

We will investigate the PDL trap as a very sensitive tiltmeter to monitor volcano. The change in the ground inclination is simulated by turning the screw and we want to determine the screw thread size S where S is the change of height per unit turn. We show that by measuring displacement of the graphite rod in the trap we can measure the inclination (tilt) precisely.

Use pencil rod HB/0.5 and length l = 8 mm in this experiment. Start from the center position. Assume the camelback potential can be approximated as harmonic potential like in problem 3:

B.3	Derive the relation theoretically between displacement Δz with the screw thread size S and the number of turns (N).	0.5 pt.
B.4	By turning the screw slowly, determine the rod displacement Δz vs. the number of screw turns (N). Determine the thread size S.	1.25 pt.
B.5	When the ground tilt changes we want the graphite rod to go to equilibrium as fast as possible (instead of sustaining very long oscillation) to allow easy reading. What is the ideal Q factor for a tiltmeter?	0.25 pt.

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Parallel Dipole Line Magnetic Trap for Earthquake & Volcanic Sensing (10 points)

A. BASIC CHARACTERISTICS OF PDL TRAP

1. Determination of the magnet's magnetization (M) (2.5 pts)

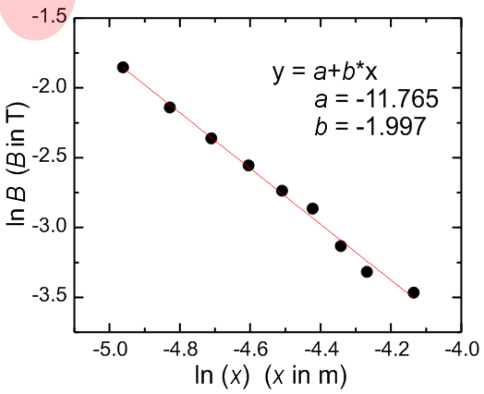
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>A.1
0.1
pts</td><td><i>Record zero offset (B_0) of the Teslameter without any magnet nearby. Subtract subsequent field measurement with this value</i>

Example from a Teslameter unit: B_0 = 0.86 mT</td><td>0.08 pts range (-10 mT to 10 mT)

Correct unit: 0.02 pts</td></tr>
<tr><td>A.2
1.15
pts</td><td><i>Measure magnetic field B vs. x in the near field region ($7 \leq x \leq 16$ mm). Where x is the position measured from the center of the magnet. Record and plot your result on the answer sheet.</i>

x_0 = 4 mm, B_0=0.86 mT. Δx is measured from surface. B = Braw - B_0

<table>
<tr><th>Δx (mm)</th><th>X (mm)</th><th>Braw (T)</th><th>B (T)</th><th>ln(x) x in m</th><th>ln(B) B in T</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>7</td><td>0.1576</td><td>0.1567</td><td>-4.962</td><td>-1.853</td></tr>
<tr><td>4</td><td>8</td><td>0.1186</td><td>0.1177</td><td>-4.828</td><td>-2.139</td></tr>
<tr><td>5</td><td>9</td><td>0.0951</td><td>0.0942</td><td>-4.710</td><td>-2.362</td></tr>
<tr><td>6</td><td>10</td><td>0.0785</td><td>0.0776</td><td>-4.605</td><td>-2.556</td></tr>
<tr><td>7</td><td>11</td><td>0.0657</td><td>0.0648</td><td>-4.510</td><td>-2.736</td></tr>
<tr><td>8</td><td>12</td><td>0.0579</td><td>0.0570</td><td>-4.423</td><td>-2.864</td></tr>
<tr><td>9</td><td>13</td><td>0.0445</td><td>0.0436</td><td>-4.343</td><td>-3.132</td></tr>
<tr><td>10</td><td>14</td><td>0.0371</td><td>0.0362</td><td>-4.269</td><td>-3.318</td></tr>
<tr><td>12</td><td>16</td><td>0.0321</td><td>0.0312</td><td>-4.135</td><td>-3.466</td></tr>
</table>

Plot:


</td><td>Correct label and unit for data: 0.1 pts

Number of correct data for x <= 16 mm:
0.05 pts for each correct data, max 0.45 pts

Plot:
-Correct axis label and unit: 0.05 pts

- Using around 75% of plot area: 0.05 pts

-For each correct data point: 0.05 pts, max. 0.4 pts

-Adding trendline: 0.1 pts</td></tr>
</table>

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A.3 0.75 pts	*Use your experimental data to determine the value of the exponent p.* Linear regression (LR) $y = a + b\,x$: $B = \frac{\mu_0 m}{2\pi\ L}\frac{1}{x^p}$ $\ln(B) = a - p\ln x$ where $a = \ln\left(\frac{\mu_0 m}{2\pi L}\right)$. LR yields : *a* = *-11.765* and *b* = *-1.997* The power exponent: *p = -b = 2.0* Note that this is in very good agreement with the exact result: at short distance ($x < L$) a diametric (or a dipole line) magnet has $B \sim 1/r^2$ dependence. See Ref. [1] , Fig. 2c.	Obtaining *p* from graph: 0.05 pts Obtaining *p* from linear regression: 0.1 pts Result: *p* = 1.8 - 2.2 : 0.65 pts *p* = 1.6 - 2.4 : 0.35 pts Result with wrong sign: *p* = (-1.8) – (-2.2) : 0.4pts *p* = (-1.6) – (-2.4) :0.1pts More than two sig. figs.: minus 0.05 pts
A.4 0.5 pts	*Determine the magnet's magnetization M.* $m = \frac{2\pi L}{\mu_0}\exp(a) = 0.987\,\mathrm{Am}^2$ $M = \frac{m}{\pi R^2\ L} = 1.2\mathrm{x}10^6\,\mathrm{A/m}$ This is close to the more accurate results from more extensive measurements to far field (see Ref. [1], Fig. 2c) and *we use this value for subsequent questions:* $M = 1.1\mathrm{x}10^6\,\mathrm{A/m}$	Correct unit: 0.05 pts Obtaining intercept (*a*) from graph: 0.025 pts Obtaining intercept from LR: 0.05 pts Correct formula for *m* and/or *M* : 0.1 pts Result for *M* ($\mathrm{x}10^6$ A/m): 0.9 - 1.4 : 0.3 pts 0.1 - 2.5 : 0.15 pts More than 2 sig. figs.: minus 0.05 pts

2. The Magnetic Levitation Effect and Magnetic Susceptibility (χ) (1 pts)

Question	Answer	Marks
A.5 0.1 pts	*Place gently a graphite rod HB/0.5 and length = 8 mm. Measure the levitation height y_0 of the rod (see Fig. 7a). Hint: Use the insert ruler provided as shown in Fig. 7b. Press the ruler on the magnets to read the position of the graphite rod* We levitate graphite HB/0.5, l = 8mm. Using the insert-ruler, we measure approximately Δy = 1 mm from the top of the magnet surface. Thus: $y_0 = R - \Delta y$ = (3.2 - 1) mm = 2.2 mm	correct unit: 0.02 y_0 = (1.7 - 2.2) mm: 0.08 pts partial credit: Only Δy = (1 - 1.5) mm: 0.03 pts

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<table>
<tr>
<td>A.6
0.8
pts</td>
<td><i>Use the result from part A.5 to determine the magnetic susceptibility χ of the graphite rod.</i>

Solving for χ: $mg = F_y = -\frac{\mu_0 M^2 \chi V_R}{2}\frac{R^4}{a^5} f_Y(y_0/a)$
$\chi = -\frac{2\rho g a^5}{\mu_0 M^2 R^4 f_Y(y_0/a)}$

We calculate: $a = R + g_M/2$ =(3.2+1.5/2) mm=3.95 mm.
Using y0 = 2.2 mm: $f_Y(u) = \frac{4u(3-u^2)(1-u^2)}{(1+u^2)^5}$,
$f_Y(y_0/a) = f_Y(2.2/3.95)$=1.07
Using the correct M = 1.1x10^{6}A/m; and R = 3.2 mm, ρ = 1680 kg/m^3 we have: $\chi = -1.85\text{x}10^{-4}$.

Note that this is very good agreement with the literature value for graphite pencil lead: $\chi = -2\times10^{-4}$ (see Ref.[1], pg. 2 & Ref.[2]). The sign is negative indicating a diamagnetic material.</td>
<td>Correct expression for χ: 0.4 pts

Result for χ (x10^{-4})
-(1.4 to 2.6) : 0.4 pts
-(0.5 to 4) : 0.2 pts

Wrong sign: minus 0.1 pts</td>
</tr>
<tr>
<td>A.7
0.1
pts</td>
<td><i>What kind of magnetic material is graphite? Choose one: (i) Ferromagnetic; (ii) Paramagnetic; or (iii) Diamagnetic?</i>

(iii) Diamagnetic. Because:
(1) Graphite is repelled by magnetic field
(2) The sign of χ is negative.</td>
<td>Correct choice: 0.1 pts</td>
</tr>
</table>

3. The camelback potential oscillation and magnetic susceptibility (χ) (1 points)

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr>
<td>A.8
0.2
pts</td>
<td><i><u>Perform an oscillation for the "HB/0.5" graphite and l =8 mm. Limit to small oscillation amplitude i.e. A < 4mm. Determine the oscillation period. (The oscillation will decay over time due to damping, ignore this damping effect).</u></i>

Example, we measured 5 oscillations of HB/0.5 with length l = 8 mm. We displaced it by ~3 mm and let it oscillates. We measured 5 oscillation periods:

<table>
<tr><td>Trial</td><td>5 Tz
(s)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>6.12</td><td></td><td></td><td></td><td></td></tr>
</table>
</td>
<td>Correct label and unit: 0.02 pts

Number of correct data each 0.01 pts, max 0.03 pts

Number of oscillation
< 3 : 0 pts
>= 3 : 0.05 pts

T_z = (1.2 - 1.5) s: 0.1 pts</td>
</tr>
</table>

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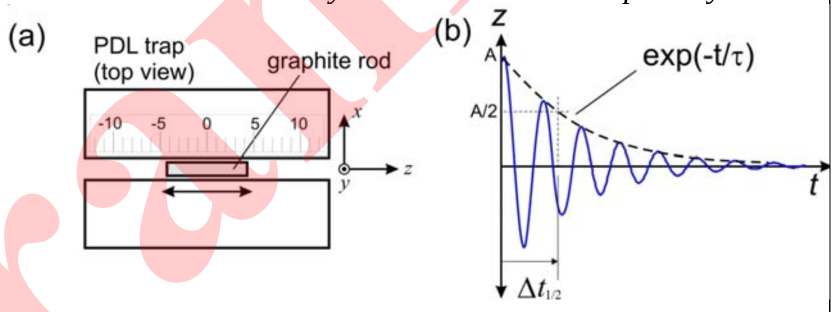
<table>
<tr><td></td><td><table><tr><td>2</td><td>6.13</td><td></td><td></td><td></td><td></td></tr><tr><td>3</td><td>6.14</td><td></td><td></td><td></td><td></td></tr></table>
Average : T_z = 1.23 s</td><td></td></tr>
<tr><td>A.9
0.8
pts</td><td><i>Calculate the magnetic susceptibility (χ) of the graphite using this oscillation</i>
For harmonic oscillator : $k_z = m_R\,\omega^2$, solving for χ:
$$\chi = -\frac{k_z}{C_1\mu_0 M^2 V_r} = \frac{\omega^2\rho}{C_1\mu_0 M^2}$$
Using the correct $M = 1.1\text{x}10^6$A/m.
Using $C_1 = 198.6/\text{m}^2$, and T_z = 1.23 s, we obtain
$\chi = -1.5\times10^{-4}$.
Note that this is in good agreement with the literature value of the graphite pencil lead: $\chi = -2\times10^{-4}$ (Ref.[1], pg. 2); and the sign is negative indicating a diamagnetic material.</td><td>Correct expression for χ: 0.4 pts
Result for χ ($\text{x}10^{-4}$)
-(1.4 to 2.6) : 0.4 pts
-(0.5 to 4) : 0.2 pts
Wrong sign: minus 0.1 pts</td></tr>
</table>

4. Oscillator quality factor (Q) and estimate of air viscosity μ_A (3.0 points)

Question	Answer	Marks
A.10 0.5 pts	*We need to determine the damping time constant of the oscillation τ. Sketch how you measure τ in a simple way .*  The trick is to use "half-time" concept of exponential decay. We set the oscillation and measure the time taken for the amplitude to halve. The lifetime is: $$\tau = \frac{\Delta t_{1/2}}{\ln 2}$$	Correct idea: 0.3 pts Correct expression for τ: 0.2 pts
A.11 1.5 pts	*Perform oscillation damping experiments with a group of rods with various diameters and fixed length of 8 mm. Determine the damping time constant τ for each rods*	Correct label and unit 0.1 Number of correct data

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We displaced the graphite by ~4 mm, started the stopwatch and then waited until it decays to half.

Trial	Diam.	Actual Radius	$\Delta t_{1/2}$	Mean $\Delta t_{1/2}$	τ	r^2xln(0.607 *l/r*)
	(mm)	(mm)	(s)	(s)	(s)	(mm^2)
1	0.3	0.19	3.89	3.913	5.646	0.117
			3.97			
			3.88			
2	0.5	0.28	7.69	7.617	10.989	0.224
			7.57			
			7.59			
3	0.7	0.35	8.77	8.82	12.73	0.322
			8.81			
			8.88			
4	0.9	0.45	12.4	11.70	16.88	0.482
			11.33			
			11.38			

for each diameter (4):
< 3 : 0.1 pts
>=3 : 0.25 pts
(max 1.0 pts)

Positive monotonic trend for τ vs. diameter from 0.3 to 0.9 mm with τ = 5 to 20 sec : 0.4 pts

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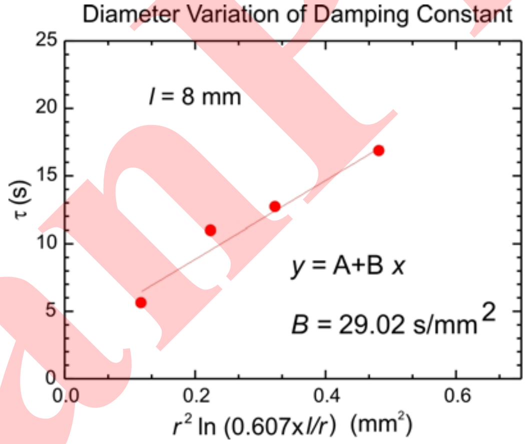
A.12
1 pts

Determine the air viscosity μ_A



We have: $\tau = b r^2 \ln\left(0.607 \times \frac{l}{r}\right)$, where: $b = \frac{2}{3}\frac{\rho}{\mu_A}$. We performed linear regression *y*= *a*+*b x* , with *y* = τ and $x = r^2 \ln\left(0.607 \times \frac{l}{r}\right)$. We obtain: *b* = 29.02 s/mm^2.

$$\mu_A = \frac{2}{3}\frac{\rho}{b} = 38.6\ 10^{-6}\ \text{Pa.s} \quad (1\ \text{Pa.s} = 1\ \text{kg /m s})$$

Note that this is about 2.1x the actual viscosity of air of 18.2μ.Pa.s. The discrepancy is due to the ellipsoidal

Correct unit: 0.05

Obtaining result with linear regression or plot: 0.25 pts

Result μ_A (x10^{-6} Pa.s):
20 - 60 : 0.7 pts
10 - 80 : 0.4 pts
1 - 100 : 0.1 pts

	approximation of the Stokes drag (vs. the actual cylindrical shape of the rod) and the proximity effect of the rod to the magnet (wall effect). Another factor is the crude nature of our manual τ determination. See Ref. [1], pg. 8.	

B. SENSOR APPLICATION OF THE PDL TRAP

5. PDL Trap Seismometer (0.5 pts)

Question	Answer	Marks
B.1 0.2 pts	*Which diameter of rod do you choose?* To obtain the lowest acceleration noise floor "a_n" we should choose the largest diameter graphite i.e. 0.9 mm, because their damping time is the longest and the mass is the largest.	Correct answer: 0.2 pts
B.2 0.3 pts	*Calculate the seismometer acceleration noise floor (a_n) for the rod of your choice!* For HB/0.9 and length l = 8 mm: We use τ = 16.9 s; and T = 298 K, we have: $m_R = \rho \pi r^2 l$ = $8.55\text{x}10^{-6}$ kg : $a_n = \sqrt{\dfrac{4k_B T \omega_0}{Q m_R}} = \sqrt{\dfrac{8k_B T}{\tau m_R}} = 1.5\text{x}10^{-8} m/(s^2 Hz^{0.5})$	Correct unit: 0.1 Correct answer: 0.2 pts

6. PDL Trap Tiltmeter (2 pts)

Question	Answer	Marks
B.3 0.5 pts	*Derive the relation theoretically between displacement Δz with the screw thread size S and the number of turns (N).* $k_z \Delta z = m\, g \sin\theta = m\, g\, N\, S / D \qquad \Delta z = \dfrac{m\, g\, S\, N}{k_z D}$ From Question 3, we also have $k_z = m\omega^2$: $\Delta z = \dfrac{g\ S}{\omega^2\ D} N$	Correct expression: 0.5 pts Partial credit $k_z \Delta z = m\, g \sin\theta$: 0.2
B.4 1.25 pts	*By turning the screw slowly, determine the rod displacement Δz vs. the number of screw turns (N). Determine the thread size S*	Correct label and unit: 0.1 pts

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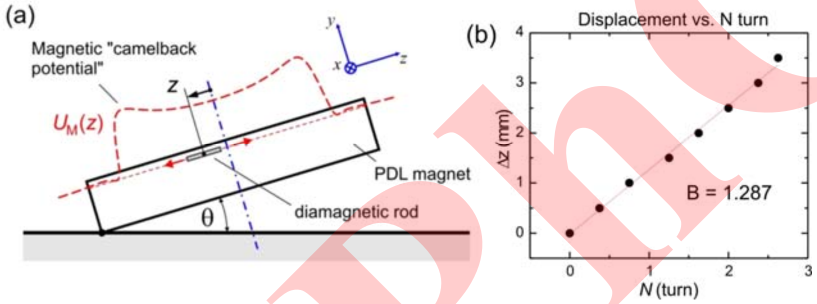
We measured the distance between screws: D = 22 cm, and we used the period from Q3: $T_z = 1.23$ s

Δz (mm)	ϕ	N (turn)			
0	0	0			
0.5	135	0.375			
1	270	0.75			
1.5	450	1.25			
2	585	1.625			
2.5	720	2.0			
3	855	2.375			
3.5	945	2.625			



By performing linear regression: $y = a + b\,x$

We have b = 1.287 mm/turns : $S = \dfrac{b\omega^2 D}{g}$ = 0.75 mm/turn.

This is reasonably close to the actual value of the thread size: S = (0.8±0.1) mm/turn.

Distance between screws:
$22.8 < D < 22.2$ cm : 0.1 pts

Number of correct data:
< 3 sets : 0 pts
3-5 sets: 0.15 pts
\>5 sets : 0.25 pts

Obtaining result with linear regression or plot: 0.2 pts

Result:
$0.7 < S < 0.9$: 0.55 pts
$0.5 < S < 1.1$: 0.15 pts

Correct unit for S : 0.05

B.5
0.25 pts

When the ground tilt changes we want the graphite rod to go to equilibrium as fast as possible (instead of sustaining very long oscillation) to allow easy reading. What is the ideal Q factor for a tiltmeter?

We need critical damping thus: $Q = 0.5$

Correct Q : 0.25 pts

REFERENCES:

[1] Gunawan, O. & Virgus, Y. *The one-dimensional camelback potential in the parallel dipole line trap: Stability conditions and finite size effect.* J. Appl. Phys. 121, 133902, (2017). DOI:10.1063/1.4978876.
[2] Gunawan, O., Virgus, Y. & Fai Tai, K. *A parallel dipole line system.* Appl. Phys. Lett. 106, 062407, (2015). DOI: 10.1063/1.4907931.

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