In The Name of
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www.jphil.ir
telegram.me/tmuphil
tmu.philosophy@gmail.com

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Skolem's Paradox and Mathematical Practice

Mansooreh Kimiagari
MA in Philosophy
Tarbiat Modarres University (Tehran, Iran)
mnkimiagari@gmail.com

Introduction

It is claimed that one of the philosophical conclusions of Skolem's paradox and similar puzzles (if it has any) is that it shows first-order theories' deficiency and inadequacy for formalizing mathematical practice. The adequacy of first-order theories in formalizing mathematical practice has occupied eminent mathematicians and logicians including Skolem, Zermelo, Bernays, Godel and Shapiro (Shapiro 714).

In his paper Shapiro claimed that his argument "rules out any language whose logic is either complete or compact" and then he suggests that "nothing short of a language with second-order variables will do" (Shapiro 715). Since one of the purposes of logic is to codify correct inference, if Shapiro's conclusions were correct, the underlying logic of many branches of mathematics was (at least) second-order (Shapiro 716). One of Shapiro's corroborating premises is that Skolem's paradox and similar puzzles imply first-order theories' inadequacy.

First I will summarize Shapiro [1985]' main points. Then I will introduce new puzzles for some second-order theories which cannot be solved and I will explain why these puzzles can eventuate irrelevancy of Skolem's paradox as a premise to conclude first-order theories inadequacy for formalizing mathematical practice. Finally, I will show that Bay's solution to Skolem's Paradox cannot be generalized to this second-order puzzle.

Shapiro [1985] tried to assess the adequacy of first-order languages in formalizing actual mathematical practice. He concluded that no first-order language is sufficient for axiomatizing such branches as arithmetic, real and complex analysis and set theory (branches whose languages have "intended interpretations"). He argued that the semantics of first-order language is not adequate for the preformal semantics of mathematical practice.

In the first section of Shapiro [1985], he wrote about the importance of categoricity in understanding and communicating mathematics. He takes categoricity as one of the main purposes of an axiomatization for describing a particular structure, an intended interpretation of a branch of mathematics. The Lowenheim-Skolem theorems imply that no set of sentences in a first-order language can be a categorical description of an infinite structure. Following Myhill [1959], Shapiro believes that in order to communicate structures, categoricity is important and thus first-order axiomatizations are inadequate.

In section 2, Shapiro calls different forms of Skolem's paradox such as the forms about finitude, minimal closure and well-foundedness, inadequacies of first-order axiomatization of mathematics' branches. These concepts form an important part of general mathematical practice, but they cannot be formulated in first-order languages. These concepts are clear and unambiguous as for instance when a mathematician asserts something is finite, his listeners understand what he means. It fallows that a language used to formalize mathematical practice must be capable of expressing these properties. But if for each natural number n, there is a model of a first-order theory T in which the extension of ϕ (a formula with one free variable in a theory like T) has at least n members, then there is a model of T in which the extension of ϕ (or the domain of discourse) is infinite. The following second-order formula is satisfied all and only those models of T in which the extension of ϕ is finite:

$$\forall f\,[(\forall x\,(\phi(x) \rightarrow \phi\,(fx)) \wedge \forall y\,\forall z\,(fy = fz \rightarrow y = z)) \rightarrow \forall y\,(\phi(y) \rightarrow \exists x\,(fx = y))]$$

Similarly Boolos [1981] shows that first-order theories cannot express even simple cardinality comparisons as "the extension of ϕ is at least as large as the extension of ψ".

Another example is about minimal closure. Like finitude, this concept is clear and a straightforward compactness shows that this concept is not first-order describable. The fallowing formula characterize the minimal closure of the extension of ϕ under the function denoted by f:

$$\forall X\,\{\forall y\,[(\phi(y) \rightarrow Xy) \wedge (Xy \rightarrow Xfy)] \rightarrow Xx\}.$$

Finally about a well-founded relation, as we know well-foundedness is well-understood. Again a well-founded relation E cannot be characterized in a first-order language. The second-order formula which characterize the well-founded relation E is as follow:

$$\forall X\, [\exists x\, Xx \rightarrow \exists x\, (Xx \wedge \forall y\, (Xy \rightarrow \neg yEx))].$$

In addition to Skolem's paradoxes Shapiro uses other premises for showing first-order theories inadequacy. For instance he compares the second-order versions of arithmetic, Set theory and real analysis with their first-order analogous. After advocating first-order theories inadequacy, in section 3 Shapiro examines several alternatives including infinitary languages, ω-languages and free-variable versions of second-order language. He concludes that only the later substantially overcomes the deficiencies of first-order language.

The following theorem implies that there is a concept which cannot be characterized by second-order language. Let M be a model of the second-order language, define a cardinal λ to be second-order describable if there is a sentence ϕ of the second-order language (with no non-logical terminology) such that $M \vDash \phi$ iff the cardinality of M is λ. For $n \geq 3$, define λ to be nth-order describable if there is a sentence ϕ of L_nK, with no non-logical terminology, such that $M \vDash \phi$ iff the cardinality of M is λ. One might think that the set of second-order describable cardinals is exactly the set of mth-order describable cardinals, for any $m > 1$. Alas, the following is stated, but not proved, by Montague [1965].

Theorem. Let $n > 3$ and let A be the smallest cardinal that is not nth-order describable. Then A is $(n\text{-} 1)$th-order describable.

It is obvious that the nth-order describable cardinal concept, like uncountablity, finitude, well-orderness, etc. cannot be characterized by (n-1)th-order language (Shapiro 141). Thus, this could amount to another form of the Skolem's paradox.

We might appeal the notions "Lowenheim number" and "Hanf number" to formulate this in another form. Although the Lowenheim-Skolem theorems do not hold in the standard semantics of second-order languages (Shapiro, Foundations without foundationalism 86), the fallowing notions are analogues to this results. Let K be a set of non-logical terminology and let LK be a language which contains L1K= (the first-order language) and has a semantics with the same class of models as that of L1K=, Definition. The Lowenheim number for LK is the smallest cardinal κ such that for every formula ϕ of LK, if ϕ is satisfiable at all, then ϕ has a model whose domain has cardinality at most κ.

Definition. The Hanf number for LK is the smallest cardinal κ such that for every formula ϕ of LK, if ϕ has a model whose domain has cardinality at least κ, then there is no upper bound on the size of the models of K, i.e. if ϕ has a model of cardinality κ or greater, then for each cardinal δ, ϕ has a model whose domain has cardinality at least δ.

The Hanf number and Lowenheim number of the first-order L1K= (and L1K) are $\aleph_0$, however it gets more complicated in the case of L2K for it involves large cardinals. The following theorem proves Hanf and Lowenheim number's existence for LK:

Theorem. If the collection of formulas of LK is a set (i.e. not a proper class), then LK has a Lowenheim number and a Hanf number.

You can find the proof in (Shapiro, Foundations without foundationalism 148).

For a cardinal larger than L2K's Hanf number we can find a model whose cardinality is L2K's Lowemheim number, and this model satisfies "there exists a set whose cardinality is larger than L2k's Lowenheim number." Again, this is a variation of Skolem's paradox. This could be solved by going to a higher-order theory, but its analogous can be formulated in L3K and again it will be solved in L4K. this implies that Shapiro goes wrong in using variations of Skolem's paradox to prove first-order language inadequacy, as there are infinite such paradoxes in second-order and and higher order theories.

Shapiro may respond that these new paradoxes are not as philosophically valuable as the classical Skolem's paradox, for the large cardinals are not as much involved as concepts like uncountablity. But mathematical practice is not something constant. It is entirely possible that one day, large cardinals are more involved in mathematical practice. Thus variations of Skolem's paradox do not imply that second-order theories are more suitable for modelling mathematical practice (even if they really are).

Bays [2000] provides a solution for Skolem's Paradox and analogous puzzles which involves an equivocation between model-theoretic and plain English interpretations of ∃x "x is uncountable". I argue that Bays' solution cannot be generalized to these second-order paradoxes:

Let $\Omega_E(x)$ and $\Omega_M(x)$ be as in Bays [2000] the pain English and model theoretic interpretations, respectively.

1. According to Bays' formulation of the Skolem's Paradox $\Omega_E(x)$ and $\Omega_M(x)$ are first-order sentences which their semantical difference leads to the Bays' solution.
2. It is obvious that for each puzzle, $\Omega_E(x)$ is at most a sentence in a language with a finite order.
3. If $\Psi_E(x)$ and $\Psi_M(x)$ are plain English and model theoretic interpretations of the statement "there is a set which has a cardinality larger than L2K's Lowenheim number" respectively, an analogous puzzle can be formulated.
4. The puzzle in premise 3 would be solved, if we assume our language third-order.
5. For each n, we can formulate a new puzzle using the statement "there is a set which has a cardinality larger than LnK's Lowenheim number".
6. In order to solve the puzzles generated by premise 5 using Bays' solution, we must assume $\Psi_E(x)$ is n+1th-order, for each n.
7. Thus, there is no plain English language with a finite order in which we can talk in a precise sense about all Lowenheim numbers.

So, Bays' solution cannot undermine these new puzzles completely since it leads to this controversial consequence that there is no particular metalanguage in which we can talk about those Lowenheim numbers.

How ordinary people think of meaning: Towards finding essential properties of meaningful structures in natural languages

Erme Pinar
National Embassy of Education,
Diyarbakir, Turkey
pinaremre000@gmail.com

Amirhossein Kiani
Amirkabir University of Technology
mathelogics1939@gmail.com

The study of meaning and truth has been of philosophers' concern since ancient times. There is an old tradition of considering some *laws of thought* which are fundamental axiomatic rules upon which rational discourse itself is often considered to be based. Generally, they are taken as laws that underlie everyone's thinking, thoughts, expressions, discussions, etc. A modern tradition that originated in the early contributions of Frege [1] and Russell [2] in the philosophy of language opened a new window towards a systematic study of meaning and truth, in the light of discussions about *sense and reference* of linguistic expressions. Depending on how meaning and truth are defined, some theories of semantics consider meaning for expressions without reference, and some would reject such a view. As another approaches to the problem of meaning, we have in one side Chomsky, who in 1957 proposed his idea at [3] about existence of grammatically correct sentences that are semantically nonsensical, by the well-known example "Colorless green ideas sleep furiously." As an example of a *category mistake* (coined so after Ryle's [4]), it was used to show inadequacy of the then-popular probabilistic models of grammar, and the need for more structured models. On the other side, we have Grice who concerned about meaning in context. In [5] he proposed his so called cooperative principle, which is intended as a description of how people normally behave in a conversation. He introduced four maxims as constitutes of the principle, commitment to which is presupposed by both parties in a conversation, and infringing any of which would result conversational implicatures, which roughly are interpretations made by one of the parties, to make the used statement seem meaningful. One of the maxims is *relevance.*

In this study, we will consider the above theories concerning different aspects of meaning, in a broad sense. Our primary goal is to investigate how the (native) ordinary speakers of the languages Persian, English and Turkish - tend to think about meaning and truth in sentences. More specifically, we wanted to know the answer to the following questions: given a grammatically correct sentence with which lacks a specific property - as will be explicitly stated as we go further - how do ordinary[1] people react to its meaning and truth? Do they regard it as meaningful or true merely because it is grammatically well-formed? If the answer is no, what factors would affect their conception of meaning?

Inspired by Chomsky's [1], we hypothesized that more than having a grammatically well-formed structure, there are fixed inevitable properties in many natural languages that participants tend to avoid, in order to call an expression "meaningful". We examined four such hypothetical properties for the three target languages, in the study: *contradictions*, *category mistake*, *lack of relevance* between sentence's parts, and *failure of reference* for the constitution parts in sentences.

For each target language, an online questionnaire with 8 items was given to participants. Questionnaires were made in the form of documents in the platform of Google-Docs and were spread among participants via online mailing lists and social media apps such as Viber and Telegram, and in some cases, printed and spread by hand. Each questionnaire was designed in two pages, where the first page asked some personal background of the participants such as their academic degree and their major of study. The second page consisted of some instructions about answering the items followed by a caution to the reader, after which the items were began. (See the Appendix.)

In each language's questionnaire, and except for the 8th item[2], each of the items 1-7 was a grammatically well-formed sentence in that language which lacked one of the essential elements mentioned earlier: items 1-3 were self-contradictory, while each of the items 4-6 violated relevance between sentence's components, and item 7 consisted a word without any known/defined reference. For each item, there were considered 5 choices [i.e.

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[1] During this paper, this adjective specifies people who has not who do not have professional trainings in the academic fields related to philosophy or linguistics.

[2] The 8th item asked the participant to "clarify" a given sentence, due to their own understanding. The purpose of this item is beyond the extent of this report, and is a subject for another investigation. Therefore, in this report, we are merely concerned with the first 7 items.

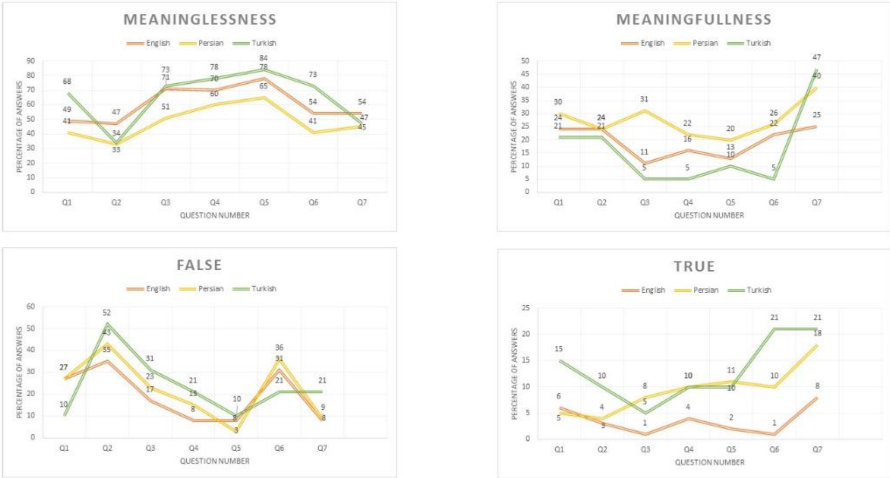
meaningful, not meaningful, True, False and Other (with a place to write the answer other than the first four choices)] that however answering to at least one choice was necessary, each participant was allowed to fill out as many as the choices he/she wants, which was considered as a platform for expression of pluralistic thoughts. Besides a place for argument (optional) was sat up in order to let the participants explain their answer.

For each target language, there was a separate questionnaire, including sentences with special properties lack of which were hypothesized to have effects on their meaningfulness. The Turkish version was a translation of the English version, by the second author, which is a native of Turkish, while the English and the Persian versions were created by the first author, who is a native speaker of Persian, with professional background in English. In general, the following number of participants afforded to complete the questionnaires in each language: 87 for English, 86 for Persian and 19 for Turkish. The Persian and English forms were spread online, using social media apps and online mailing lists, while the Turkish forms were spread online and in some cases, they were printed one paper and spread by hands among the locals in Turkey.

As for the analysis of answers for each item, participants were divided into groups of speakers of the three target languages. The general results, as they were expected, show that in each of the languages, most of the population tend to call the sentences meaningless, when the sentences don't have the essential components that were considered. Another considerable result which was not anticipated before doing the experiment, shows that there might be a *meaning spectrum* in each language: as the items change or get complicated, the percentage of people changes in a considerable way towards the answer choices. The notable fact is that this behavior is universal to all of the target languages, in almost every item. Therefore, it seems that each anticipated component of meaningfulness, bears a meaning spectrum, which ranges probably depending on the internal complexity of the sentence with its special property.[٣] The results are observable in the following comparative graphical representation of the data for 4 important answer types.

Besides, results show that there is a direct relation between "meaningful" and "true" answer choices; however, when it turns to the pair "false" and "meaningless" answers, the relation becomes inverse: the more

[٣] The authors believe, however, a comprehensive judgment for this issue is not possible at this stage and requires more investigations.



votes go to represent an item meaningless, the less votes target it as false, which by itself can be an interesting subject of investigation for further studies.

Figure 1- Graphical charts comparing the distribution of answer choices over the target languages. Four notable answer choices are presented.

Some factors may have affected the results, including the intended hypothesis that meaning can exist out of any specific interpretations (i.e. the given caution). Besides, since we had a wide access to English speaking people via mailing lists with contacts from across the world, there is a chance that some of the participants have not been native speakers of English, where in that case, they will not be appropriate candidates for the experiment[4]. Finally, since we had less access to Turkish speaking people, the number of their participants is observably less than the other two language groups. It might have been effective on the results of compared to the other two languages.

To conclude, in this study we examined reactions of ordinary people from three languages regarding the meaning and truth of well-formed sentences with specific intended structures, avoiding of which were hypothesized to affect the results. The final results show supportive evidence

[4] This, however, doesn't hold in for Persian and Turkish participants, as they have been chosen by the knowledge that they are natives of the languages.

regarding this: sentences which have any of the four intended properties [i.e. contradictions, category mistake, internal irrelevance, reference failure] will be regarded as meaningless/false rather than meaningful/true by a higher percentage of people. Evidence also show that there is a chance that the nature of meaning inherits a fuzziness: there is a seemingly universal pattern in the responses towards meaningfulness of the items when ranging from item 1 to 7. The pattern is shared by all the target languages. Our guess is that this pattern depends on the internal complexity of the items with regard to the intended properties they have. However, we believe that this problem can be considered as subject of a more comprehensive study for future investigations. Also, there is a direct relation between "true" and "meaningful" answers, while it is the converse, when it turns to the pair of "false" and "not meaningful" answers. Finally, it seems that Gricean maxims - such as relevance - may play role beyond a cooperative principle in a conversational context: they may range over wider contexts as well, such as written language, as is observable in the results of this experiment. We believe, the obtained evidence should be considered in any theory of semantics that is aimed to model natural language semantics.

APPENDIX

Here we represent the items appeared in the questionnaire:

1- The (totally) bald man behind the camera braids his long hair 3 times a day.

- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]

Not necessary

2- There was a lot of paintings on Berlin wall, including triangles of four sides.

- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]
Not necessary

3- The bicycle that was talking by an iPhone in its hand is a really good football player.
- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]
Not necessary

4- Once I liked hamburgers and so, Steve's pet is male.
- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]
Not necessary

5- While I was drinking soda, America is a vast country; although my grandma makes such soups that you have no idea how hard Tyler punched my face.
- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]
Not necessary

6- Last day I and Amir were walking through a humid jungle and while we were talking about the plants there, a shark suddenly attacked our boat and caused Amir to die in that hot desert.
- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]

Not necessary

7- George touched the jfgm&&# in a tricky way.
Supposing that jfgm&&# has no meaning.

- ☐ Meaningful
- ☐ Not meaningful
- ☐ True
- ☐ False
- ☐ Other [Type here]

Argument1 [Type here]
Not necessary

The part "instructions" at top of the second page stated the following:

- It is possible to filled out more than 1 answer choice.
- Answers are better to be in accordance with the participant's own knowledge/intuition.
- The participants can optionally argue about their choices.

The caution after the guidelines stated the following: "See each text AS WHAT IT IS. They should NOT be seen as METAPHORS, IRONIES, etc. Do NOT INTERPRET the texts. Just consider each one AS A WHOLE and FREE OF ANY INTERPRETATIONS."

An objection to branching model for time

Maysam Zandi
MA in Philosophy
Tarbiat Modarres University (Tehran, Iran)
maysam.zandi@yahoo.com

Introduction

While Aristotle believes events, pertinent to the future, happen contingently, Diodorus claims at the present time it is determined that the sea battle happens tomorrow or not. He presents an argument which is called "Master Argument." Based on the result of it, these three premises are not consistent.

1. Every proposition that is true about the past is necessary.
2. An impossible proposition cannot follow from a possible one.
3. There is a proposition that is possible which neither is nor will be true.

Diodorus accepts first two premises and rejects the third one. In fact, the definition of the possibility is the negation of the third premise. The possible proposition, from his point of view, is the proposition which is true or will be true in the future. Medieval philosophers were also concerned about these premises. That the propositions about the future are necessary is very close to arguments for determinism. They present arguments in favor of determinism with the help of Diodorus's premises. Contemporary logicians and philosophers have been trying to formalize these arguments with the help of Formal Logic. In many logical systems which are built to reconstruct the arguments, only a special case of first premise is considered. I will present an argument similar to the medieval arguments. In what follows, I will show that some logical systems which are successful in refuting the special case of the first premise cannot refute the general version of the first premise.

New Argument for Determinism

2-1 Formalization of the necessity of the past

The first premise of Diodorus is regarded frequently like this:

Every true proposition about the past is necessary

In formal figure we could consider it (see Prior [5]) like this: $Pp \supset \Box Pp$
As I think it is the special version, we could also consider it generally like this:

Every true proposition at a time will be necessary afterwards.

Formally in first order logic we can present this premise as Rescher has done so (Rescher [6], p. 191:

$$\forall \mathbf{t}\ \forall \mathbf{t'}\ \{[\mathbf{T_t\ (p)\ \&\ t<t'}] \supset \mathbf{N_{t'}\ (p)}\}$$

Here $\mathbf{T_t\ (p)}$ means that p happens at time **t** and $\mathbf{N_{t'}\ (p)}$ means that **p** is necessary at time t'. It must be mentioned that in Rescher's formalization a proposition could be tensed. On the other hand, from a historical point of view a proposition could be necessary at a time while it is not so at another time. Equivalently, based on Prior formalization for this premise, we have these three formulas:

1. $P(x)p \supset \Box\, P(x)p$
2. $F(m+n)P(n)p \supset\ F(m+n)\Box P(n)p$
3. $p \supset F(x)\Box\, P(x)p$

The first formula is considered in all systems. Many solutions which reject the first premise are actually rejecting a special case of this formula. In fact, they think if the proposition, which is in front of a past operator, presents an event about the future, then it can be not-necessary. One of the best criteria is given by Plantinga [3]. He differentiates between hard facts which are about something that has happened in the past and soft facts which are about something that will happen in the future. According to him, if the event of a proposition is pertinent to the past, then the proposition is a hard fact. But if the event of a proposition is pertinent to the future, then the proposition is a soft fact. The hard facts must be necessary yet the soft fact is not necessarily necessary. With this approach the second formula should also be rejected, unless the proposition **p** is replaced by a formula about an event related to the past before m time units earlier. The third formula is acceptable, although it depends on our view to the present time. The present time is considered often as the last thing which belongs to the past. In all tensed theories about time, present is all or one part of the actual world. Its events are actual and necessary.

2-2 Argument for Determinism

In this section I present a new argument for determinism very similar to the argument based on the version of Ohrstrom for medieval argument ([2]). The main assumption which is needed for this new argument is the first premise of Diodorus in general version.

1. Either ***e*** is going to take place tomorrow or ***non-e*** is going to take place tomorrow. (Assumption)
2. If a proposition is true at a time, then it is necessary afterwards. (Assumption)
3. If ***e*** is going to take place tomorrow, then it is true that two days later it will be the case that ***e*** would take place yesterday. (Assumption)
4. If ***e*** is going to take place tomorrow, then it is now the case that two days later, it is necessary that ***e*** took place yesterday. (Follows from 2 and 3)
5. If it is now the case that two days later it is necessary that ***e*** took place yesterday, then it is now necessary that two days later e took place yesterday. (Assumption)
6. If it is now necessary that two days later ***e*** took place yesterday, it is now necessary that ***e*** would take place tomorrow. (Assumption)
7. If ***e*** would take place tomorrow, it is now necessary that ***e*** would take place tomorrow. (Follows from 4, 5 and 6)
8. If ***non-e*** is going to take place tomorrow, then ***non-e*** is necessarily going to take place tomorrow. (Follows by the same kind of reasoning as 6)
9. Either ***e*** is necessarily going to take place tomorrow or ***non-e*** is necessarily going to take place tomorrow. (Follows from 1, 6 and 7)
10. Therefore, what is going to happen tomorrow is going to happen with necessity. (Follows from 8)

For accepting this argument, we must have these assumptions:

A1) $F(x)p \vee F(x)\neg p$

A2) $F(x)P(x)p \equiv p$
A3) $F(x+y)P(y)p \supset F(x+y)\Box P(y)p$
A4) $F(x)\Box p \supset \Box F(x)p$

Considering the above assumptions, we could formalize this argument so:

1.	$F(x)p \vee F(x)\neg p$	(A1)
2.	$F(x)F(x)P(x)p \supset F(x)F(x) \Box P(x)p$	(A3)
3.	$F(x)p \supset F(x)F(x)P(x)p$	(A2,substitution)
4.	$F(x)p \supset F(x)F(x) \Box P(x)p$	(2,3)
5.	$F(x)F(x) \Box P(x)p \supset \Box F(x)F(x)P(x)p$	(A4)
6.	$\Box F(x)F(x)P(x)p \supset \Box F(x)p$	(A2,substitution)
7.	$F(x)p \supset \Box F(x)p$	(4,5,6)
8.	$F(x)\neg p \supset \Box F(x) \neg p$	(similar to 1-7)
9.	$\Box F(x)p \vee \Box F(x) \neg p$	(1,7,8)

In what follows, I will show that Ockham and Thin Red Line systems do not reject the above argument. As a result, there would be two possible solutions. First, we could follow alternative systems like Nishimora's System which Ohrstrom called it Leibnizian system (See Nishimora [1]). Second, we could accept the first premise and follow the solutions based on rejecting other premises like the Principle of Future Excluded Middle.

Appraisal of Ockham and Thin Red Line systems

3-1 Ockham system

Based on the Ohstrom's formalization, in this system there is a function called TRUE. It assigns to every proposition in every time the value 0 or 1. The truth function called Ock is defined in the following:

(a)	$Ock(t, c, p) = 1$	Iff	$TRUE(p,t) = 1$
(b)	$Ock(t,c,p \wedge q) = 1$	Iff	both $Ock(t,c,p) = 1$ and $Ock(t,c,q) = 1$
(c)	$Ock(t,c, \neg p) = 1$	Iff	not $Ock(t,c,p) = 1$
(d)	$Ock(t,c,F(x)p) = 1$	Iff	$Ock(t',c,p) = 1$ for some $t' \in c$ with $dur(t,t',x)$
(e)	$Ock(t,c,P(x)p) = 1$	Iff	$Ock(t',c,p) = 1$ for some $t' \in c$ with $dur(t',t,x)$

(f) Ock(t,c,◊p) = 1 Iff Ock(t,c′,p) = 1 for some c′ ∈ C(t) v

Time in the future is branching, while in the past it is linear. Every **C**, which is a maximal ordered set of time points, is a history. In definition of possibility, **C(t)** is defined as all time points which are equivalent to the time point in which the truth is considered. Two equivalent histories are similar before the time of consideration. For every two histories, their intersection is not null. This semantics system has a tree structure and the necessity in it is meant to happen in all histories equivalent to the history under consideration at the same time.

Now consider the main premise of my argument, namely **F(x+y)P(y)p ⊃ F(x+y)□P(y)p**. Assume that we have **F(x+y)P(y)p**. This means the antecedent is true in the history **C1** and at the present time. This means at the x+y time units in the future, **p** is true at **y** time units before. This time point is in the history **C1**. If we do not have this premise, then we must have the negation of its consequence. Therefore, we must have **¬F(x+y)□P(y)p**.

This means at a point **x+y** time units later in the **C1** we must have **◊P(y) ¬p**. This means at least at a time point concurrent with the considered time (**x+y** time unit later) whose history is equivalent to **C1**, we must have **P(y) ¬p**. But this proposition links all the points to the same point in which **p** holds. Therefore, my premise is not rejected.

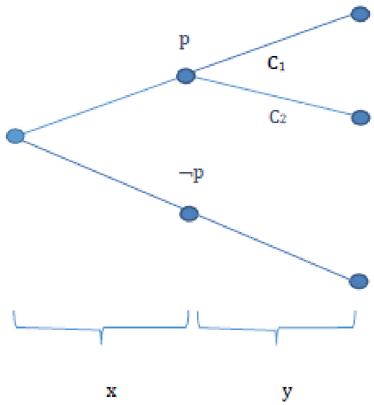


Figure 1

3-2 Thin Red Line system

The Thin Red Line system in its first version is similar to Ockham System. In the Thin Red Line, one history is considered as actual world. The semantics in this system is similar to Ockham system. The only difference is in the definition of F(x)p:

$$Trl(t,c,Fp) = 1 \quad \text{Iff} \quad Trl(t',TRL,p) = 1 \text{ for some } t' \in c \text{ with } t < t'$$

The TRL is the truth function in this system. I must mention the TRL is the actual world or history.

Again consider the main premise namely: **F(x+y)P(y)p ⊃ F(x+y) □P(y)p**. Assume in Thin Red Line system we have **F(x+y)P(y)p**. This means the antecedent is held in a history **C1** which must be the actual world namely TRL. This means in the actual world at **x+y** time units later we would have in **y** time units earlier p is held. This point is on the actual world. If we would not have this premise, we must have its negation of its consequence. Then, we have **¬F(x+y)□P(y)p**. This means in a point **x+y** time units later in the TRL history we should have **◊P(y) ¬p**. This also means in at least one point whose history is equivalent to TRL history at that time, we have **P(y) ¬p**. But such proposition converges all points to a same point. Therefore, my premise could not be rejected.

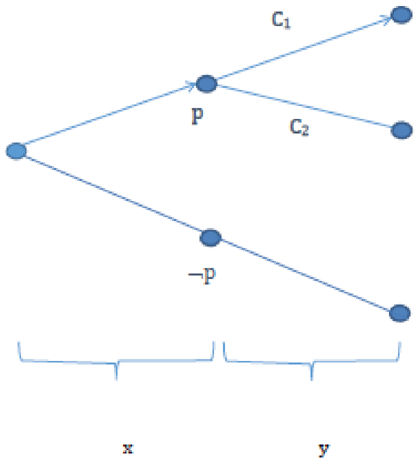


Figure 2

There are also other versions of Thin Red Line. Belnap and Green proposed that we have for every time point (actual or nonfactual) a thin red line.

Therefore, we have a function which defines for every time point a thin red line and has these conditions:

(TRL1) $t \in TRL(t)$
(TRL2) $(t1 < t2 \wedge t2 \in TRL(t1)) \supset TRL(t1) = TRL(t2)$

There are 2 approaches for exact definition of semantics in this system. The first approach suggests similar definitions to Ockhum system unless in the definition of F(x)q:

$T(t,F(x)q) = 1$ Iff there is some $t' \in TRL(t)$ with $t<t'$ and $T(t',q) = 1$

This approach accepts also my argument. In fact, in this system, for all points relative to the past, only one history exists.

Conclusion

Solving future contingency with considering Diodorus premises could be done in different ways. Some of the solutions are based on rejecting the first premise of Diodorus. We could consider the first premise like Rescher. On the other hand, systems which reject the first premise of Diodorus can be divided into two groups. The first group rejects the first premise in all conditions. The second group only rejects the premise in a special case. The first conclusion is that there is a difference between Ockham and Thin Red Line on one hand and Nishimoura, on the other hand. The second conclusion is that only Nishimoura could reject the first premise in all cases.