بسم الله الرحمن الرحیم

Reducibility

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What we are going to discuss?

- Undecidable problems from language theory
  - Reductions via computation histories
- Mapping reducibility
  - Computable functions
  - Formal definition of mapping reducibility
- Post correspondence problem, or PCP

Reduction

A way of converting one problem to another problem in such a way that a solution to the second problem can be used to solve the first problem.

$$HALT_{TM} = \{\langle M, w\rangle | M \text{ is a TM and halts on input } w\}$$

Undecidable

Theorem 5.1

$$E_{TM} = \{\langle M \rangle | M \text{ is a TM and } L(M) = \emptyset\}$$

Undecidable

Theorem 5.2

$$\text{Regular}_{TM} = \{\langle M \rangle | M \text{ is a TM and } L(M) \text{ is regular}\}$$

Undecidable

Theorem 5.3

Exercise 5.28
(Rice's theorem)

$S =$ "On input $\langle M, w \rangle$, where M is a TM and w is a string:

1. Construct the following TM M_2.
   $M_2 =$ "On input x:
   1. If x has the form 0^n1^n, *accept*.
   2. If x does not have this form, run M on input w and *accept* if M accepts w."
2. Run R on input $\langle M_2 \rangle$.
3. If R accepts, *accept*; if R rejects, *reject*."

$$EQ_{TM} = \{\langle M_1, M_2\rangle | M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$$

Undecidable

Theorem 5.4

S = "On input $\langle M\rangle$, where M is a TM:

1. Run R on input $\langle M, M_1\rangle$, where M_1 is a TM that rejects all inputs.
2. If R accepts, *accept*; if R rejects, *reject*."