Flow in Porous Media

Module 1.b
Fundamental Properties of Porous Media
Shahab Gerami
Wettability & Capillary Pressure
Porous Mediums As a Capillarity System

• In a capillarity system the surfaces separating the bulks phases play a significant role in determining the physical-chemical state of the system.

• Mechanical equilibrium is determined not only by hydrostatic pressure and gravitational forces, but also by forces associated with interfacial tensions. Most porous mediums are capillarity systems.
Multiphase System Considerations

The simultaneous existence of two or more fluids in a porous rock requires the definition of the following terms:

– **Wettability**
  - Wettability is defined as the tendency of one fluid to spread on or adhere to a solid surface in the presence of other immiscible fluids.

– **Capillary pressure**
  - When two immiscible fluids are in contact in the interstices of a porous medium, a discontinuity in pressure exists across the interface separating them. The difference in pressure $P_c$ is called capillary pressure.

– **Relative permeability**
  - The effective permeability of any reservoir fluid is a function of the reservoir fluid saturation and the wetting characteristics of the formation. The absolute permeability is a property of the porous medium and is a measure of the capacity of the medium to transmit fluids. When two or more fluids flow at the same time, relative permeability is used to describe the flow behavior of each phase.
Surface and Interfacial Tension

- Surface and interfacial tension of fluids result from **molecular properties** occurring at the surface or interface.

- Surface tension is the tendency of a liquid to expose a *minimum free surface*.

- **Surface tension** may be defined as the contractile tendency of a liquid surface exposed to gases.

- The **interfacial tension** is a similar tendency which exists when two **immiscible liquids** are in contact.
Factors Affecting Wettability

The wettability of a reservoir rock system depends on many factors:

- Reservoir rock material
- Pore geometry
- Geological mechanisms
- Composition and amount of oil and brine
- Pressure and temperature
- Changes in saturation, pressure and composition during production.

Wettability play an important role in the production of oil and gas as it not only determines initial fluid distributions, but also is a main factor in the flow processes in the reservoir rock.

The degree of wetting of solid by liquids is usually measured by the contact angle that a liquid-liquid interface makes with a solid.
Figure 1.25: Microscopic view of non-wetting (left side) and wetting fluid on mineral surfaces
Contact Angle

• The angle of contact of the interface with the solid surface, measured through the denser phase is called “contact angle.”
• Force balance at the point where the fluid-fluid interface meets the solid gives:

![Diagram showing contact angles and forces]

\[ A_t = \sigma_{so} - \sigma_{sw} = \sigma_{wo} \cos \theta_{wo} \]

It is generally not possible to measure \( \sigma_{os} \) and \( \sigma_{ws} \) directly, but the angle \( \theta \) and the interfacial tension \( \sigma_{ow} \) can be easily measured.
Silica Surface

Water

θ = 30°

θ = 83°

θ = 158°

Calcite Surface

Water

θ = 30°

θ = 48°

θ = 106°

Isooctane + 5.7% Isoquinoline

Isoquinoline

Naphthenic Acid

θ = 54°
FIGURE 13. Contact angle as a function of molecular weight.
**ST** and **IFT** vary significantly depending on the system. Typical IFT values are shown in the below Table. IFT depends strongly on temperature. It decreases as temperature increases. For water, the rate of IFT increase with temperature is approximately -0.2 dyne/cm °C.

<table>
<thead>
<tr>
<th>Wetting phase</th>
<th>Non-wetting phase</th>
<th>Conditions</th>
<th>θ</th>
<th>σ (dynes/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brine</td>
<td>Oil</td>
<td>Reservoir, <em>T,P</em></td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Brine</td>
<td>Oil</td>
<td>Laboratory, <em>T,P</em></td>
<td>30</td>
<td>48</td>
</tr>
<tr>
<td>Brine</td>
<td>Gas</td>
<td>Laboratory, <em>T,P</em></td>
<td>0</td>
<td>72</td>
</tr>
<tr>
<td>Brine</td>
<td>Gas</td>
<td>Reservoir, <em>T,P</em></td>
<td>0</td>
<td>(50)</td>
</tr>
<tr>
<td>Oil</td>
<td>Gas</td>
<td>Reservoir, <em>T,P</em></td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Gas</td>
<td>Mercury</td>
<td>Laboratory, <em>T,P</em></td>
<td>140</td>
<td>480</td>
</tr>
</tbody>
</table>
The following problems with contact angle measurement must be addressed when determining reservoir properties:

1. Surface roughness and history of which fluid first contacted the surface affects the measured value of contact angle.

2. Rock-fluid interaction (e.g. solubility, pH, type of ions in the aqueous phase, type of polar groups in crude oil, etc.) affects the value of contact angle.

3. Solids as a polished plate (quartz, calcite) may not be representative of solid surfaces in porous media.

4. Time to reach equilibrium whereby the contact angle is independent of time may vary from seconds to days or years.

Consequently the contact angle measured in the laboratory may not represent the natural wettability of the system.
Contact angle Hysteresis and Displacement

**Advancing Contact Angle** (Imbibition type of displacement), $\theta_A$ - is the contact angle when the interface is forced to move in the direction of the wetting phase displacing the non-wetting phase. (Usually measured through the wetting phase and solid surface).

$\theta_A = \text{advancing contact angle (ie. Wetting phase is advancing)}$

**Receding Contact Angle** (Drainage type of displacement) $\theta_R$ - is the contact angle when the wetting phase is displaced by the non-wetting phase.

$\theta_R = \text{receding contact angle (i.e. wetting phase receding)}$
When variable pore geometries are involved, the effect of the pore geometry is reflected on the apparent value of the contact angle.

- The contact angle of an **advancing interface** is always greater than or equal to the value of the **receding** contact angle for the same fluid interface. This value has important implications in the recovery of residual oil.
Figure 2.7: Surfactants present in the oil can change the surface to be oil-wet.
In their natural state, rocks may be either water-wet or oil-wet according to the atoms exposed in the grain or pore surface.

Naturally water-wet rocks include quartz, calcite, and dolomite, while naturally oil-wet rocks include coal, sulfur, and some silicates.

In the case of rocks, which tend to be water-wet, they become thus when the oxygen atoms exposed at their surfaces attract hydrophilic hydrogen from the water molecules.

If polar impurities such as resins or asphaltenes can reach the surface, they will substitute lipophilic radicals, rendering the surfaces oil-wet.

Note that while only basic impurities will attach to quartz, both basic and acidic impurities can attach to calcite. This might explain the large percentage of carbonate reservoirs found by several researchers to be oil-wet (Treibel et al, 1972). In their study of 155 carbonate reservoirs worldwide, Chillinger et al found that 65% are moderately to strongly oil-wet (contact angles 100-160).

Oil wettability can be artificially induced by treating the surface of the pores with substances that bond to the surfaces and render them oil-wet, such as chlorosilane compounds.
Contact angle Hysteresis and Displacement

Intrinsic or Equilibrium Contact Angle, $\theta_E$ - is the contact angle when system attained equilibrium position over time.

\[ \theta_E = \text{equilibrium or static contact angle} \]

\[ P_c = \frac{2\sigma \cos \theta_E}{r} \]

Equilibrium is implied when a capillary tube is drawn.
FIGURE 17. Influence of aging on laboratory-measured contact angle.\(^{48}\)
B. Adhesion Tension, $A_T$

Adhesion tension, which is a function of the interfacial tension, determines which fluid preferentially wets the solid. In the case of water-oil-solid, the adhesion tension $A_T$ is defined as

$$A_T = \sigma_{WS} - \sigma_{OS} = \sigma_{OW} \cos \theta$$

- A positive adhesion tension $A_T$ indicates that water preferentially wets the solid surface (water wet).
- An $A_T$ of zero indicates that both phases have an equal affinity for the surface (neutral system).
- A negative $A_T$ indicates the oil wets the solid surface (oil wet).
- The magnitude of the adhesion tension determines the ability of the wetting phase to adhere to the solid and to spread over the surface of the solid.
Water Wet & Oil-Wet Comparison

**WATER-WET**
- $\theta < 90^\circ$
- SOLID (ROCK)
- WATER
- FREE WATER
- BOUND WATER
- GRAIN

**OIL-WET**
- $\theta > 90^\circ$
- SOLID (ROCK)
- OIL
- FREE WATER
- BOUND WATER
- GRAIN
- OIL RIM
Water Wet & Oil-Wet Comparison

\[ A_T = \sigma_{WS} - \sigma_{OS} = \sigma_{OW} \cos \theta \]

**if the solid is "water-wet"

\[ \sigma_{WS} \geq \sigma_{OS} \]

\[ A_T = + \]

\[ \cos \theta = + \]

\[ 0^\circ \leq \theta \leq 90^\circ \]

if \( \theta = 0^\circ \) - strongly water-wet

**if the solid is "oil-wet"

\[ \sigma_{OS} \geq \sigma_{WS} \]

\[ A_T = - \]

\[ \cos \theta = - \]

\[ 90^\circ \leq \theta \leq 180^\circ \]

if \( \theta = 180^\circ \) - strongly oil-wet
Wetting Phase Fluid

- **Wetting phase** fluid preferentially wets the solid rock surface.

- Attractive forces between rock and fluid draw the wetting phase into small pores.

- Wetting phase fluid often has low mobile.

- Attractive forces limit reduction in wetting phase saturation to an irreducible value (irreducible wetting phase saturation).

- Many hydrocarbon reservoirs are either totally or partially water-wet.
Nonwetting Phase Fluid

- **Nonwetting phase** does not preferentially wet the solid rock surface

- Repulsive forces between rock and fluid cause nonwetting phase to occupy largest pores.

- Nonwetting phase fluid is often the most mobile fluid, especially at large nonwetting phase saturations

- Natural gas is never the wetting phase in hydrocarbon reservoirs
Wettability Classification

• **Strongly oil- or water-wetting**

• **Neutral wettability** – no preferential wettability to either water or oil in the pores

• **Fractional wettability** – reservoir that has local areas that are strongly oil-wet, whereas most of the reservoir is strongly water-wet - Occurs where reservoir rock have variable mineral composition and surface chemistry

• **Mixed wettability** – smaller pores area water-wet are filled with water, whereas larger pores are oil-wet and filled with oil
  - Residual oil saturation is low
  - Occurs where oil with polar organic compounds invades a water-wet rock saturated with brine
Wettability Classification

In larger grains/pores, the water film ruptures, and oil and rock in direct contact, locally altering wettability to preferentially oil-wet.

In smaller grains/pores, the water film covers the grain surfaces completely, thus maintaining those grains water-wet.
F. Wettability-consolidated sand

1. Pendular-ring distribution - wetting phase is not continuous, occupies the small interstices - non-wetting phase is in contact with some of the solid.

2. Funicular distribution - wetting phase is continuous, completely covering the surface of solid.

Idealized representation of distribution of wetting and nonwetting fluid phase about intergrain contacts of spheres. (a) Pendular-ring distributions; (b) funicular distribution.
Wettability Classification

Generally

- Silicate minerals have acidic surfaces
  - Repel acidic fluids such as major polar organic compounds present in some crude oils
  - Attract basic compounds
  - Neutral to water-wet surfaces

- Carbonate minerals have basic surfaces
  - Attract acidic compounds of crude oils
  - Neutral to oil-wet surfaces
Wettability Classification

When regarding oil reservoirs it is necessary to consider the specific rock and fluid properties in order to determine whether the reservoir rock is water or oil wet. Rocks which are neither water nor oil wet are called intermediate or neutral wet. The data published by Treiber et al. (1972) show that most of the carbonate reservoirs are oil wet, while the sandstone reservoirs can be equally water or oil wet.

<table>
<thead>
<tr>
<th>Wettability</th>
<th>Triebert et al.</th>
<th>Chillingarian &amp; Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Silicates, Percent</td>
<td>Carbonates, Percent</td>
</tr>
<tr>
<td>Water-wet</td>
<td>43</td>
<td>8</td>
</tr>
<tr>
<td>Intermediate-wet</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Oil-wet</td>
<td>50</td>
<td>88</td>
</tr>
</tbody>
</table>

n = 30 silicate and 25 carbonates

CONTACT ANGLE: Triber et al.
- Water-wet = 0 – 75 degrees
- Intermediate-wet = 75 – 105 degrees
- Oil-wet = 105 – 180 degrees

CONTACT ANGLE: Chillingarian & Yen
- Water-wet = 0 – 80 degrees
- Intermediate-wet = 80 – 100 degrees
- Oil-wet = 100 – 180 degrees
Wettability and Pore Size Distribution

The wettability of reservoir rocks to the fluids is important in that the distribution of the fluids in the porous media is a function of wettability. Because of the attractive forces, the wetting phase tends to occupy the smaller pores of the rock and the nonwetting phase occupies the more open channels.

Distribution
The shape, spread, and location of a data set. Knowing the distribution of a data set can tell you a great deal about the data itself, and can be critical in selecting appropriate analyses and interpreting their results. You can assess a distribution through graphs, descriptive statistics, or more formally with a distribution identification tool.

The frequency distribution defines the relationship between the values of a variable and the probability of “those values”.
If a pore size frequency curve is available, distribution of the wetting and non-wetting phases will be as shown in figure 4.35 where the wetting phase fills small pores and the non-wetting phase fills the large pores. Thus, saturation in the irreducible wetting phase will depend on the average pore size. Saturation in irreducible water will be greater in small pores than in large pores (as shown in figure 4.36 where $d_1 < d_2$).

4.35 – Pore size frequency curve and fluid saturation
If all three phases, water, oil and gas, coexist simultaneously in the reservoir, their distribution will follow the same pattern. According to the degree of wettability, water will fill the smallest pores and gas the largest pores, leaving the intermediate pores for oil (figure 4.37).

4.37 – Three-phase saturation distribution.
Properties Characterizing the Wettability

<table>
<thead>
<tr>
<th>Parameter</th>
<th>water wet</th>
<th>oil wet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{wir}$</td>
<td>&gt;0.2</td>
<td>&lt;0.15</td>
</tr>
<tr>
<td>$S_w$ at $k_{rw}=k_{ro}$</td>
<td>&gt;0.5</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>$k_{rw}$ at $1-S_{or}$</td>
<td>&lt;0.3</td>
<td>&gt;0.5</td>
</tr>
</tbody>
</table>
The following figure illustrates the fluid distribution during a waterflood of a preferentially water-wet rock. Oil has been isolated and trapped as globules by the invasion of water. At flood-out only trapped isolated oil exists and it is likely to be in the more conductive portions of the pore channels.

(1) EARLY IN DRIVE  (2) MIDWAY IN DRIVE  (3) FLOOD-OUT
The following figure illustrates similar history for an oil-wet rock, initially saturated with oil and flooded with water. As the non-wetting phase (water) enters the rock it first forms tortoise but continuous flow channels through the largest pores. As water injection continues, successively smaller pores are invaded and join to form other continuous channels. When sufficient flow channels form to permit almost unrestricted water flow, oil flow practically ceases, resulting in lower displacement efficiently compared to the water-wet system.
Imbibition

• **Imbibition** is a fluid flow process in which the saturation of the wetting phase increases and the nonwetting phase saturation decreases. (e.g., waterflood of an oil reservoir that is water-wet).

• Mobility of wetting phase increases as wetting phase saturation increases
  
  – mobility is the fraction of total flow capacity for a particular phase
Imbibition

**Water-wet Reservoir**

- Water will occupy the smallest pores
- Water will wet the circumference of most larger pores
- In pores having high oil saturation, oil rests on a water film

**Imbibition** - If a *water-wet rock* saturated with oil is placed in water, it will *imbibe water* into the smallest pores, *displacing oil*

**Oil-wet Reservoir**

- Oil will occupy the smallest pores
- Oil will wet the circumference of most larger pores
- In pores having high water saturation, water rests on a water film

**Imbibition** - If an *oil-wet rock* saturated with *water* is placed in oil, it will *imbibe oil* into the smallest pores, *displacing water*

  e.g., Oil-wet reservoir – accumulation of oil in trap
Drainage

• Fluid flow process in which the saturation of the nonwetting phase increases

• Mobility of nonwetting fluid phase increases as nonwetting phase saturation increases
  – e.g., waterflood of an oil reservoir that is oil-wet
  – Gas injection in an oil- or water-wet reservoir
  – Pressure maintenance or gas cycling by gas injection in a retrograde condensate reservoir
  – Water-wet reservoir – accumulation of oil or gas in trap
Measurement of Wettability

Different methods have been used for the categorization of the wettability. A detailed discussion of all methods has been published by Anderson. Two groups of methods are distinguished:

1. **Quantitative methods:**
   - determination of the contact angle
   - Amott
   - Amott-Harvey
   - USBM wettability indices (centrifuge method)

2. **Qualitative methods:**
   - imbibition rate
   - microscopic examination
   - capillary pressure curves
   - relative permeability/saturation relationship
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   - imbibition rate
   - microscopic examination
   - capillary pressure curves
   - relative permeability/saturation relationship
Wetting Phase
This phase spreads over the solid surface and preferentially wets the solid. The contact angle approaches zero.

Nonwetting Phase
This phase has little or no affinity for the solid. The contact angle exceeds 90 degrees.

The rise or depression of liquids in fine bore tubes is a result of surface tension and wetting preference and is called capillarity.
Capillary Pressure

- When two immiscible fluids are in contact in the interstices of a porous medium, a discontinuity in pressure exists across the interface separating them. The difference in pressure $P_c$ is called capillary pressure, which is pressure in the non-wetting phase minus the pressure in the wetting phase.
- Using the example of an oil drop in a water environment:

The curvature of the interface suggests that the oil phase pressure $P_o$ is greater than the water phase pressure $P_w$. The capillary pressure $P_c$ is defined as the difference between the two phase pressures.
Importance of Capillary Pressure

- Determine fluid distribution in reservoir (initial conditions)
- Determine recoverable oil for water flooding applications
- Pore Size Distribution Index, $\lambda$
- Reservoir Flow
- Input data for reservoir simulation models
Air-water system
Capillary Rise in a Thin Tube

Considering the force balance in the vertical direction:

\[ 2\pi r \sigma \cos \theta = \pi r^2 (p_2 - p_1) \]

\[ P_c = p_2 - p_1 = \frac{2\sigma \cos \theta}{r} \]

And since \( p_2 = p_{\text{atm}} \) and \( p_1 = p_{\text{atm}} - \rho gh \), then:

\[ P_c = p_2 - p_1 = \frac{2\sigma \cos \theta}{r} = \rho gh \]
Consider the following figure. It represents the various forces that act on an infinitesimal segment of a fluid-fluid interface. The shown infinitesimal element of a curved interface is between a \( w \)-fluid (water or wetting), which occupies the convex side of the interface, and an \( a \)-fluid (air or non-wetting), which occupies the concave side of the latter. Assuming the interfacial tension between these two fluids to be constant, a balance of force components normal to this element requires that at equilibrium

\[
p'_{c} = p_{a} - p_{w} = \gamma_{aw} \left( \frac{1}{r^1} + \frac{1}{r^m} \right) = 2\gamma_{aw} \frac{1}{r^*}.
\]
Capillary Pressure as a Function of Fluid Saturation and the Rock Wettability

The derivation of capillary pressure equations thus far has been based on a single uniform capillary tube. Porous geologic materials, however, are composed of interconnected pores of various sizes. In addition, the wettability of the pore surfaces varies from point to point within the rock due to the variation in the mixture of minerals in contact with the fluids. This leads to variation of the capillary pressure as a function of fluid saturation and an overall mean description of the rock wettability.

The value of capillary pressure is dependent on:

- the saturation of each phase,
- the continuity of each phase, and
- the shape and size of the pores and pore throats.
Capillary Pressure Determination by Drainage

• The upper and lower cells are separated by a diaphragm, which is impermeable to the non-wetting fluid. The sample (core), which is placed into the upper chamber, and the diaphragm are both saturated with the wetting fluid. The non-wetting fluid surrounding the core, e.g. air, is then set under constant pressure and is thus pressed into the pore-space of the core. The wetting fluid, displaced by the non-wetting fluid, flows through the diaphragm into a graduated pipette to be measured. This procedure is then repeated several times at higher pressures.

• After completing the described drainage and measurement procedure, it is possible to establish the capillary pressure curve for imbibition by reducing the pressure stepwise inside the upper chamber.
1. Saturate both the core sample and the diaphragm with the fluid to be displaced.
2. Place the core in the apparatus as shown.
3. Apply a level of pressure, wait for the core to reach static equilibrium.  
   The capillary pressure = height of liquid column + applied pressure

\[
\text{Saturation} = \frac{\text{Pore volume} - \text{Volume produced}}{\text{Pore volume}}
\]

4. Increase the pressure and repeat step (3)
5. Plot capillary pressure versus saturation
Porous Plate Capillary Pressure Apparatus
If the pore surfaces are preferentially wet by water, a finite pressure (the threshold pressure, $P_{et}$) will be required before any of the water is displaced from the core.
• There will be a threshold pressure for each pore radius which has to be overcome by the applied pressure differential in order to move wetting phase fluid from that pore.
• The relationship between applied pressure differential (equivalent to capillary pressure) and saturation thus gives a characterization of pore size distribution.
• The laboratory test results may look like those shown in Fig. 6.5. The first applied pressure differential does not cause any desaturation of wetting phase and is interpreted as meaning that the threshold capillary pressure of the largest pore sizes has not been reached. Between 0.5 psi and 3.0 psi some desaturation is achieved and the minimum threshold pressure ($P_{ct}$) lies in this region. For pressures greater than the minimum threshold pressure, a decreasing pore size is invaded by non-wetting phase fluid until an irreducible wetting phase saturation $S_{wirr}$ is reached and no further increase in differential pressure causes further desaturation.

The crosshatched region in Fig. 6.5 which lies between $P_{ct}$ and $P_C$ ($S_{wirr}$) is known as the transition zone region. In higher permeability reservoir rocks (500 mD) the value of $P_C$, may be indistinguishable from zero applied pressure.
\[ S_w = f(P_c) \]

\[ P_c = \frac{2 \sigma \cos \theta}{r} \]

Fluid Property

Wettability

Rock-Fluid Property

Rock Property (K & \phi)
Drainage and Imbibition
Capillary Pressure Curves

DRAINAGE
• Fluid flow process in which the saturation of the nonwetting phase increases

IMBIBITION
• Fluid flow process in which the saturation of the wetting phase increases

Saturation History - Hysteresis
- Capillary pressure depends on both direction of change, and previous saturation history
- Blue arrow indicates probable path from drainage curve to imbibition curve at $S_{wt}=0.4$
- At $S_m$, nonwetting phase cannot flow, resulting in residual nonwetting phase saturation (imbibition)
- At $S_{wi}$, wetting phase cannot flow, resulting in irreducible wetting phase saturation (drainage)
For porous media modeled as a bundle of tubes with varying diameters, a given capillary pressure exhibits a higher fluid saturation on the drainage curve than on the imbibition curve.
If the core is saturated completely with the wetting phase (e.g. water) at the beginning of the measurement, then a certain pressure must be applied to enable the non-wetting phase to intrude the pore space. This pressure is the threshold pressure which depends on the largest pore diameter.
The Free water level (FWL) in a reservoir is the level at which the oil-water capillary pressure vanishes. It is the oil-water interface that would exist at equilibrium in an observation borehole, free of capillary effects.

When the pressure of a static fluid is plotted against depth, each fluid will have a particular slope depending on the density of the fluid. The slope or static fluid pressure gradient in psi/ft for any fluid can be calculated by multiplying the density in g/cc by 0.433.
Capillary Transition Zone

Capillary pressure difference in a reservoir

$H = \frac{P_c}{\Delta \rho g}$

Capillary transition zone in a reservoir
Capillary Transition Zone

In a reservoir, a similar process causes the wetting phase (commonly water) to rise up into the non-wetting phase (typically oil, then gas) above the apparent “water-oil contact” (WOC) which clearly represents a transition zone rather than a sharp interface. This is often referred to as the *capillary fringe* or the capillary zone. The height depends on the capillary pressure, which in turn depends on the saturation. So the saturation becomes a function of height, the same in shape as the capillary pressure curve.

\[
\begin{align*}
\text{In the oil: } p_o + \rho_o g H &= p \\
\text{In the water: } p_w + \rho_w g H &= p \\
\end{align*}
\]

So \( P_c = p_o - p_w = \Delta \rho g H \), or \( H = \frac{P_c (S_w)}{\Delta \rho g} \)

Capillary pressure difference in a reservoir
To convert capillary-pressure-saturation data to height saturation, it is only necessary to rearrange the terms in Eq. (3-12) so as to solve for the height instead of the capillary pressure so that

\[ h = \frac{P_c \times 144}{\rho_w - \rho_o} \]

where \( h \) = height above free water surface, ft
\( \rho_w \) = density of water at reservoir conditions, lb/cu ft
\( \rho_o \) = density of oil at reservoir conditions, lb/cu ft
\( P_c \) = capillary pressure at some particular saturation for reservoir conditions, which means it must first be converted from laboratory data.

**Fig. 3-21.** Comparison of water distribution as determined by capillary-pressure and electric-log data. (From Owen)
The threshold capillary pressure found in reservoir rocks is proportional to the height above the free water level (FWL) datum, where a region of 100% water saturation will be found.

The FWL is thus a property of the reservoir system, while an oil-water contact observed in a particular well in the reservoir will depend on the threshold pressure of the rock type present in the vicinity of the well and there may then be a zone of 100% water saturation from some height above the FWL.

The relationship between height above free water level and capillary pressure is derived from consideration of the gravity-capillary pressure force equilibrium.

The Oil-water contact (OWC) is the level at which the hydrocarbon saturation starts to increase from some minimum saturation. In a water-wet rock, that minimum saturation is essentially zero.
\[ D_{FWL} = D_{OWC} + H_t \]

The FWL depth is usually determined by noting an observed OWC in a well and conducting a drainage laboratory capillary pressure test on a rock sample from the interval to find the threshold capillary pressure, then the depth of the FWL (= D_{FWL}).

The residual oil saturation (S_{or}) is the oil saturation level above which the oil starts to be moveable.
Fluid Distribution in Reservoirs

Capillary pressure difference between gas and oil phases in core ‘B’
\[ P_{c,go} = h_2 g (\rho_o - \rho_g) \]

Capillary pressure difference between oil and water phases in core ‘A’
\[ P_{c,ow} = h_1 g (\rho_w - \rho_o) \]
Characteristic Mercury Injection Capillary Pressure Curve Shapes
Brooks-Corey Capillary Pressure Model

Drainage Capillary Pressure Curve

\[ P_c = p_e \left( \frac{S - S_{wr}}{1 - S_{wr}} \right)^{-1/\lambda} \]

Imbibition Capillary Pressure Curve

\[ P_c = p_e \left( S_{eff}^{-1/\lambda} - 1 \right) \]

\[ S_{eff} = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{nwr}} \]

\( \lambda \) is a parameter that depends on pore size distribution:

- \( \lambda = 2 \) for a wide range of pore sizes.
- \( \lambda = 4 \) for a medium range of pore sizes.
- \( \lambda = \infty \) for a single uniform pore size.
The capillary pressure at a given saturation is a measure of the smallest pore being entered by the nonwetting phase at that point, suggesting the curvature of the capillary pressure curve is a function of the pore size distribution. The level of the curve is determined by the mean pore size.
$S_w^*$ Power Law Model

- $S_w^*$ rescales x-axis

Capillary Pressure vs. Wetting Phase Saturation

$P_c$, psia

$S_w^*$, fraction

$S_{wi} = 0.20$

$P_d = 3.0$

$S_w^* = 0$

$S_w^* = 1$
$S_w^*$ Power Law Model

- Power Law Equations plot as Log-Log straight line

Capillary Pressure Data Plotted vs. $S_w^*$ (for $S_{wi}=0.20$)

![Graph showing Capillary Pressure Data Plotted vs. $S_w^*$](image)

- slope = $-1/\lambda = -1/2.0$
- $P_d = 3.0$
Relationship of lambda versus pore size range distribution, pore distribution, and capillary pressure curve

<table>
<thead>
<tr>
<th>PORE SIZE RANGE DISTRIBUTION</th>
<th>FREQUENCY DISTRIBUTION</th>
<th>CAPILLARY PRESSURE CURVE</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VERY WIDE</td>
<td>f</td>
<td>( P_c )</td>
<td>0.5</td>
</tr>
<tr>
<td>WIDE</td>
<td>f</td>
<td>( P_c )</td>
<td>2</td>
</tr>
<tr>
<td>MEDIUM RANGE</td>
<td>f</td>
<td>( P_c )</td>
<td>4</td>
</tr>
<tr>
<td>UNIFORM</td>
<td>f</td>
<td>( P_c )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
\[ P_{cD} = (1 - S_{wD})^{-1/\lambda} \]

\[ P_{cD} = \frac{P_c}{P_d} \]

\[ S_{wD} = 1 - S^*_w = \frac{1 - S_w}{1 - S_{wi}} \]
Effect of Permeability on Capillary Pressure in a Given Rock Type
Correlation of Capillary Pressure with Permeability in a Given Rock Type

The second method of evaluating capillary-pressure data is to analyze a number of representative samples and treat the data statistically to derive correlations which, together with the porosity and permeability distribution data, can be used to compute the connate-water saturations for a field. A first approximation for the correlation of capillary-pressure data is to plot water saturation against the logarithm of permeability for constant values of capillary pressure. An approximately linear relationship usually results such as those shown in Fig. 3-20. A straight line can be fitted to the data for each value of capillary pressure, and average capillary-pressure curves computed from permeability distribution data for the field. The resulting straight-line equation takes the general form of

$$S_w = a \log k + C$$  \hspace{1cm} (3-17)

There are indications, however, that water saturation at constant capillary pressure is not only a function of permeability but also some function of porosity. In Fig. 3-24, the results of fitting an equation

$$S_w = a_0 + a_1 \phi + a_2 \log k + C$$  \hspace{1cm} (3-18)

to the field data for 5-psi capillary pressure is shown. The upper portion of Fig. 3-24 shows the three-dimensional aspect of such a correlation. The lower part shows lines of constant porosity (light dashed lines) fitted to the data. The heavy black dashed line is the straight line [Eq. (3-17)] fitted to the data wherein the effect of porosity is omitted.

In Eq. (3-17) and (3-18), $S_w$ is the water saturation, $\phi$ the porosity, $k$ the permeability, and $a_1$, $a_2$, and $C$ are constants which must be determined from the sample data. The method of least squares can be used to determine the constants of the best fitting lines as described by (3-17) and (3-18). The effect of ignoring the porosity is to predict lower water saturations for low-permeability materials. Equation (3-18) can be modified to a polynomial form so that

$$S_w = a_0 + a_1 \phi + a_2 \phi^2 + a_3 \log k + a_4 (\log k)^2 + C$$  \hspace{1cm} (3-19)
Averaging Capillary Pressure Curves

Consider a reservoir cross-section from which four core samples are taken at different depth as shown below. Each core will generate its own complete capillary pressure curve in the laboratory which can be converted to a reservoir capillary pressure curve. Thus four different laboratory capillary pressure curves are obtained as shown below. The question then arises: **How do we get a single Pc or height versus $S_w$ curve to represent the reservoir?**

The *answer* is to use the Leverett J-function
Correlation of Capillary Ressure Data from a Given Rock Type (The Leverett J-function)

• From our definition of $P_C = 2(\sigma \cos \theta)/r$ where $r$ is a mean radius, we may note that the grouping $(r^* P_C)/(\sigma \cos \theta)$ will be dimensionless. Since permeability has the dimension $L^2$ (the unit of area), then we could substitute $(k^{0.5})$ for $r$ and maintain the dimensionless nature of the group.

• Leverett in fact defined a dimensionless capillary pressure group in this way, with the exception that $(k/\Phi)^{0.5}$ was preferred. Since capillary pressure is a function of saturation, then the dimensionless capillary pressure term ($J$) is also a function of saturation.

\[
J_{(sw)} = \frac{P_C(sw)}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}}
\]

This relationship (Figure) will apply as a correlating group for all measurements of capillary pressure using different fluid systems, so long as the porous rocks have similar pore geometries.
The capillary pressure at a given saturation is a measure of the smallest pores entered by the continuous phase.

- The shape of the capillary pressure curve is a function of the pore size distribution.
- The magnitude of the capillary pressure depends on the mean pore size.
- The J-function combines these dependencies to remove the mean pore size and therefore provide a single defining function for similar rocks.

The J-function has the effect of normalizing all curves to approach a single curve and is based on the assumption that the porous medium can be modelled as a bundle of non-connecting capillaries. Obviously the more capillary bundle assumption deviates from reality, the less effective the J-function correlation becomes.
This correlation (J-function) is not unique, but seems to work better when the rocks are classed as to rock types, eg; limestone, dolomite, etc.
Leverett J-function for Conversion of $P_c$ Data

J-function usually not accurate correlation for different lithologies

\[
J(S_w) = \left[ \frac{CP_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} \right]_{\text{Lab}} = \left[ \frac{CP_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} \right]_{\text{Reservoir}}
\]

\[
P_{cR} = \frac{\cos \theta_R \sigma_R}{\cos \theta_L \sigma_L} \cdot P_{cL}
\]

where

- $P_{cR}$ is the capillary pressure under reservoir conditions,
- $P_{cL}$ is the capillary pressure measured under laboratory conditions,
- $\sigma_R$ is the interfacial tension under reservoir conditions,
- $\sigma_L$ is the interfacial tension measured under laboratory conditions,
- $\theta_R$ is the contact angle measured under reservoir conditions,
- $\theta_L$ is the contact angle measured under laboratory conditions.
We consider an assumed sharp boundary (AB) between two porous medium domains, I and II (say, fine sand and a coarse one), with water and air occupying the void space.

**QUESTION:**
What is the situation with respect to saturation along the boundary AB? (beyond the obvious condition of continuity of water and air fluxes)

**ANSWER:**
A. Continuity in water saturation, $S_w$.
B. A JUMP in water saturation.
BOTH WATER PRESSURE AND AIR PRESSURE MUST BE CONTINUOUS along the microscopic boundary AB. This means that the capillary pressure (= difference between air pressure and water pressure) undergoes no jump as the boundary is crossed. From the continuity in microscopic capillary pressures, we may conclude that there exists also continuity in the macroscopic capillary pressure. We may write:

\[ P_{wl} = P_{wl}', \quad p_{al} = p_{al}', \quad p_{cl} = p_{cl}' \]

The continuity in \( \rho_c \) means a jump from \( S_{w1} \) to \( S_{w2} \) as the boundary is crossed.

The condition of equality of pressures at the interface between different porous media, remains valid also when the fluids are moving.
Clean rocks, sandy aquifers and surface soils with a low organic content are usually water-wet.

Reservoir rocks and surface soils with high organic content are often mixed-wet rather than completely water-wet, as typically some of the pores are water-wet and others are oil-wet.

Mixed-wet is not the same as a neutral-wet medium in which the contact angle is zero everywhere. The reason for mixed-wetness is that pores in contact with crude oil become oil-wet, whereas the small throats and crevices remain water-wet since crude oil never reaches them.
Figure 2.40: (a) Water wet, (b) oil wet, (c) mixed wet 1, (d) mixed wet 2.

We refer to *imbibition* as the increase in the wetting phase saturation. *Spontaneous* imbibition occurs when the capillary pressure is positive, *forced* imbibition occurs when the capillary pressure is negative. The only way to have forced imbibition of water (for example) is to have a connected network of oil-wet pores so that the water can displace the oil.
We have seen that a porous material can be defined as water-wet, oil-wet or mixed-wet. The degree to which a reservoir is one or another of these can be determined by considering the capillary pressure curve, or by characterizing it in terms of wettability indices. There are a number of different indices in common usage.

Figure 2.41: Capillary pressure diagram used to characterize wettability.
Amott Indices

Referring to the below-figure, we can define the Amott indices as:

\[ I_o = \frac{\Delta S_{os}}{1 - S_{wi} - S_{or}} \]
\[ I_w = \frac{\Delta S_{ws}}{1 - S_{wi} - S_{or}} \]

1. Completely water-wet material: \( I_o = 0 \) & \( I_w = 1 \)
2. Completely Oil-wet material: \( I_o = 1 \) & \( I_w = 0 \)
Amott-Harvey Index

\[ I_{AH} = I_w - I_o = \frac{\Delta S_{ws} - \Delta S_{os}}{1 - S_{wi} - S_{or}} \]

1. Completely water-wet material: \( I_{AH} = 1 \)
2. Completely Oil-wet material: \( I_{AH} = -1 \)
USBM Wettability Index

This index is based on the ratio of the two areas representing forced imbibition in the below-figure:

\[ N_w = \log\left(\frac{A_1}{A_2}\right) \]

The range is from \(+\infty\) for a completely water-wet material to \(-\infty\) for a completely oil-wet material. Typical values are in the range \(-1.5\) to \(+1.0\). In general this index is not used very much.
Summary

1: first drainage
2: spontaneous imbibition
3: forced imbibition
4: spontaneous drainage
5: forced drainage
A1: surface below the curve of the second drainage (5)
A2: surface above the curve of forced imbibition (3)

**Amott index**

$$I_{ow} = I_w - I_o = \frac{a}{a+b} - \frac{c}{c+d}$$

**USBM index**

$$I_{USBM} = \log\left(\frac{A_1}{A_2}\right)$$
Leverett J scaling does not work for mixed-wet rocks, because it is defined for water-wet materials.

<table>
<thead>
<tr>
<th>Water-wet</th>
<th>Oil-wet</th>
<th>Mixed-wet 1</th>
<th>Mixed wet 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta &lt; 30^\circ$</td>
<td>$\theta &gt; 150^\circ$</td>
<td>Patches of oil-wet and water-wet</td>
<td>$30^\circ &lt; \theta &lt; 150^\circ$</td>
</tr>
<tr>
<td>$P_c = p_o - p_w &gt; 0$</td>
<td>$P_c = p_o - p_w &lt; 0$</td>
<td>$S_{wi} &lt; S_{or}$</td>
<td>$S_{or}$ a bit less than when $\theta &lt; 30^\circ$ and $S_{wi}$ a bit more.</td>
</tr>
<tr>
<td>$S_{wi} &lt; S_{or}$</td>
<td>$S_{wi} &gt; S_{or}$</td>
<td>Because water remains connected but oil does not.</td>
<td>Because oil remains connected but water does not.</td>
</tr>
<tr>
<td>Water in crevices and small pores.</td>
<td>Oil in crevices and small pores.</td>
<td>Water remains connected.</td>
<td>Oil remains connected.</td>
</tr>
<tr>
<td>$I_w = 1$</td>
<td>$I_w = 0$</td>
<td>$I_w &gt; 0$</td>
<td>$I_w = 0$</td>
</tr>
<tr>
<td>$I_o = 0$</td>
<td>$I_o = 1$</td>
<td>$I_o &gt; 0$</td>
<td>$I_o = 0$</td>
</tr>
</tbody>
</table>