Absorptive photonic crystals in 1D

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Abstract
Light propagation in one-dimensional (1D) periodic structures (photonic crystals), including absorption, is studied analytically. Oblique incidence on a slab with an arbitrary complex valued refractive index is treated. The transfer matrix method and Bragg conditions are modified accordingly. The propagation in a sinusoidal absorptive crystal is computed as an example.

Keywords: absorptive photonic crystal, oblique incidence, transfer matrix, Bragg conditions

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Periodic optical structures (including periodic arrangements of thin films) have been widely used in the design of bandpass filters and omnidirectional mirrors. Their bandgaps for incident light are due to Bragg reflection, and may cover several frequency (wavelength) ranges in the spectrum of incident light. The transmittance and reflectance are calculated with the aid of the transfer matrix method which is particularly suitable for a layered periodic slab [1–5].

We begin with a brief overview of known results for layered, periodic, and layered periodic structures. We consider a nonmagnetic ($\mu = 1$) absorptive slab with a complex valued refractive index, stratified in the z direction, i.e. $\tilde{n}(z) \equiv n(z) + i\kappa(z) = \tilde{n}(z + d)$, where $n(z)$ and $\kappa(z)$ stand for the real refractive index and extinction coefficient, respectively. They are related to a complex valued dielectric permittivity $\tilde{\varepsilon}$ of the slab as follows: $n^2 - \kappa^2 = \text{Re}(\tilde{\varepsilon})$ and $2n\kappa = \text{Im}(\tilde{\varepsilon})$. The slab is surrounded by semi-infinite incident and exit media with constant real refractive indices $\tilde{n}_i = n_i$ and $\tilde{n}_f = n_f$. We take the plane of incidence to be the (z, x) plane. Maxwell's equations for linear polarized monochromatic fields of frequency ω reduce to

$$\mathbf{\Psi}(\mathbf{r}, t) = \Psi(z)\, e^{i(k_x x - \omega t)}\, \hat{\mathbf{y}},$$

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}z^2} + \left[\tilde{k}_z^2(z) + \frac{1}{\tilde{n}(z)}\frac{\mathrm{d}^2\tilde{n}}{\mathrm{d}z^2} - \frac{2}{\tilde{n}^2(z)}\left(\frac{\mathrm{d}\tilde{n}}{\mathrm{d}z}\right)^2 \right]\Psi(z) = 0, \tag{1}$$

for s and p polarized light, respectively. The function $\Psi(z)$ stands for the electric field $E(z)$ in the case of s polarized light, and for the field $h(z) \equiv H(z)/\tilde{n}(z)$, where $H(z)$ is the

magnetic field, in the case of p polarized light. The equation for $H(z)$ itself involves a first-derivative term:

$$\frac{\mathrm{d}^2 H}{\mathrm{d}z^2} + \tilde{k}_z^2(z)H(z) - \frac{2}{\tilde{n}(z)}\frac{\mathrm{d}\tilde{n}(z)}{\mathrm{d}z}\frac{\mathrm{d}H}{\mathrm{d}z} = 0,$$

which is eliminated by the transformation to Ψ. But that assumes that $\tilde{n}(z)$ is analytic, i.e. has no discontinuities, within the range of integration. The x component of the complex valued wavevector $\tilde{\mathbf{k}}(z)$ inside a slab is a real constant given by

$$k_x = k_0 n_i \sin\theta_i \equiv k_0\beta, \tag{2}$$

where $k_0 = \omega/c = 2\pi/\lambda$ is the wavenumber in vacuum, and θ_i is the angle of incidence. The z component of the complex valued wavevector $\tilde{\mathbf{k}}(z)$ inside the slab is

$$\tilde{k}_z(z) = k_0\sqrt{[n(z) + i\kappa(z)]^2 - \beta^2} \equiv k_0\tilde{N}_z(z), \tag{3}$$

where the effective complex valued refractive index $\tilde{N}_z(z)$ is introduced. For a nonabsorptive slab, $\tilde{N}_z(z) = \sqrt{n^2(z) - \beta^2}$ is either real (conventional case) or pure imaginary (total internal reflection case). In the conventional case, $\sqrt{n^2(z) - \beta^2} = n(z)\cos\theta(z)$, where $\theta(z)$ is the angle between the real wavevector $\mathbf{k}(z)$ with the absolute value $k(z) = k_0 n(z)$, and the normal.

The field $\Psi(z)$ can be expressed as

$$\Psi(z) = \begin{cases} A_i\, e^{ik_{iz}z} + B_i\, e^{-ik_{iz}z}, & z < 0, \\ AF(z) + BG(z), & 0 < z < l, \\ A_f\, e^{ik_{fz}(z-l)} + B_f\, e^{-ik_{fz}(z-l)}, & z > l, \end{cases} \tag{4}$$

where $k_{iz,fz} = k_0 n_{i,f} \cos\theta_{i,f} = k_0\sqrt{n_{i,f}^2 - \beta^2}$, and $F(z)$, $G(z)$ are two arbitrary linearly independent solutions of equation (1) in the region $0 < z < l$. The coefficients in equation (4) on the two external edges of a slab are connected by the overall transfer matrix T:

$$\begin{bmatrix} A_i \\ B_i \end{bmatrix} = T \begin{bmatrix} A_f \\ B_f \end{bmatrix}, \qquad T = L_{0^-}^{-1} W_{0^+ l^-} L_{l^+}, \qquad (5)$$

where the matrix $W_{0^+ l^-}$ links $E(z)$ and its derivative $E'(z)$ on the internal edges of the slab, in the case of s waves, as

$$\begin{bmatrix} E(0^+) \\ E'(0^+) \end{bmatrix} = W_{0^+ l^-}^{\mathrm{s}} \begin{bmatrix} E(l^-) \\ E'(l^-) \end{bmatrix}, \qquad (6)$$

or $H(z) = \tilde{n}(z) h(z)$ and $H'(z)/\tilde{n}^2(z) = [h(z)\tilde{n}(z)]'/\tilde{n}^2(z)$ in the case of p waves as

$$\begin{bmatrix} h(0^+)\tilde{n}(0^+) \\ \frac{[h(0^+)\tilde{n}(0^+)]'}{\tilde{n}^2(0^+)} \end{bmatrix} = W_{0^+ l^-}^{\mathrm{p}} \begin{bmatrix} h(l^-)\tilde{n}(l^-) \\ \frac{[h(l^-)\tilde{n}(l^-)]'}{\tilde{n}^2(l^-)} \end{bmatrix}. \qquad (7)$$

The slab edge-matrices $L_{0^-}^{-1}$ and L_{l^+} take the form

$$[L_{0^-}^{-1}]^{\mathrm{s}} = 1/2 \begin{bmatrix} 1 & -\mathrm{i}/k_{iz} \\ 1 & \mathrm{i}/k_{iz} \end{bmatrix},$$

$$[L_{l^+}]^{\mathrm{s}} = \begin{bmatrix} 1 & 1 \\ \mathrm{i}k_{fz} & -\mathrm{i}k_{fz} \end{bmatrix},$$

$$[L_{0^-}^{-1}]^{\mathrm{p}} = 1/2 \begin{bmatrix} 1/n_i & -\mathrm{i}n_i/k_{iz} \\ 1/n_i & \mathrm{i}n_i/k_{iz} \end{bmatrix}, \qquad (8)$$

$$[L_{l^+}]^{\mathrm{p}} = \begin{bmatrix} n_f & n_f \\ \mathrm{i}k_{fz}/n_f & -\mathrm{i}k_{fz}/n_f \end{bmatrix}.$$

In order to find the amplitude reflection r and transmission t coefficients, one should put $A_i = 1$ and $B_f = 0$ in equation (5). Then, those coefficients are obtained by setting $r = B_i$ and $t = A_f$. This leads to

$$r^{\mathrm{s}} = \frac{W_{11}^{\mathrm{s}} - \frac{k_{fz}}{k_{iz}} W_{22}^{\mathrm{s}} + \mathrm{i}[k_{fz} W_{12}^{\mathrm{s}} + \frac{1}{k_{iz}} W_{21}^{\mathrm{s}}]}{W_{11}^{\mathrm{s}} + \frac{k_{fz}}{k_{iz}} W_{22}^{\mathrm{s}} + \mathrm{i}[k_{fz} W_{12}^{\mathrm{s}} - \frac{1}{k_{iz}} W_{21}^{\mathrm{s}}]},$$

$$t^{\mathrm{s}} = \frac{2}{W_{11}^{\mathrm{s}} + \frac{k_{fz}}{k_{iz}} W_{22}^{\mathrm{s}} + \mathrm{i}[k_{fz} W_{12}^{\mathrm{s}} - \frac{1}{k_{iz}} W_{21}^{\mathrm{s}}]},$$

$$(9)$$

$$r^{\mathrm{p}} = \frac{\frac{n_f}{n_i} W_{11}^{\mathrm{p}} - \frac{n_i k_{fz}}{n_f k_{iz}} W_{22}^{\mathrm{p}} + \mathrm{i}\left[\frac{k_{fz}}{n_i n_f} W_{12}^{\mathrm{p}} + \frac{n_i n_f}{k_{iz}} W_{21}^{\mathrm{p}}\right]}{\frac{n_f}{n_i} W_{11}^{\mathrm{p}} + \frac{n_i k_{fz}}{n_f k_{iz}} W_{22}^{\mathrm{p}} + \mathrm{i}\left[\frac{k_{fz}}{n_i n_f} W_{12}^{\mathrm{p}} - \frac{n_i n_f}{k_{iz}} W_{21}^{\mathrm{p}}\right]},$$

$$t^{\mathrm{p}} = \frac{2}{\frac{n_f}{n_i} W_{11}^{\mathrm{p}} + \frac{n_i k_{fz}}{n_f k_{iz}} W_{22}^{\mathrm{p}} + \mathrm{i}\left[\frac{k_{fz}}{n_i n_f} W_{12}^{\mathrm{p}} - \frac{n_i n_f}{k_{iz}} W_{21}^{\mathrm{p}}\right]}.$$

Finally, the absorptance A can be obtained from

$$A = 1 - |r|^2 - \frac{k_{fz}}{k_{iz}} |t|^2. \qquad (10)$$

We emphasize here that equations (1)–(10) are valid for any absorptive slab (layered or not, periodic or not).

In the case of a layered slab, each layer with complex refractive index $\tilde{n}_j$ and width d_j can be represented by a transfer matrix

$$W_j^{\mathrm{s}} = \begin{bmatrix} \cos(\tilde{k}_{jz} d_j) & -\sin(\tilde{k}_{jz} d_j)/\tilde{k}_{jz} \\ \tilde{k}_{jz}\sin(\tilde{k}_{jz} d_j) & \cos(\tilde{k}_{jz} d_j) \end{bmatrix},$$

$$W_j^{\mathrm{p}} = \begin{bmatrix} \cos(\tilde{k}_{jz} d_j) & -\tilde{n}_j^2\sin(\tilde{k}_{jz} d_j)/\tilde{k}_{jz} \\ \tilde{k}_{jz}\sin(\tilde{k}_{jz} d_j)/\tilde{n}_j^2 & \cos(\tilde{k}_{jz} d_j) \end{bmatrix},$$

$$(11)$$

where $\tilde{k}_{jz} = k_0\sqrt{\tilde{n}_j^2 - \beta^2} = k_0\sqrt{(n_j - \mathrm{i}\kappa_j)^2 - \beta^2} = k_0\tilde{N}_j$. The total matrix $W_{0^+ l^-}$ can be calculated as the product of these layer matrices. Equations (5), (8) and (11) summarize how to construct the transfer matrix for a layered absorptive slab.

The first goal of this paper is to construct the transfer matrix, in particular, the matrix $W_{0^+ l^-}$ for an arbitrary (not necessarily layered) absorptive slab.

Insight into the reflection/transmission properties of a layered slab obviously cannot be gained by simple numerical multiplications of the corresponding matrices. For a layered periodic slab a key tool is the well-known theorem for the Mth power of a unimodular 2×2 matrix which states that

$$W_{0^+ l^-} = \frac{1}{\sin\phi}[W_d \sin M\phi - \hat{1}\sin(M-1)\phi], \qquad (12)$$

where W_d is the transfer matrix for one cell, M is the number of cells, $\hat{1}$ is the unit matrix, and ϕ is the Bloch phase given by

$$2\cos\phi = (W_d)_{11} + (W_d)_{22}. \qquad (13)$$

Each layer matrix given by equation (11) is unimodular (det $W_j = 1$). Therefore W_d, which is the product of all layer matrices constituting the system, is itself unimodular, i.e. equation (12) applies to any absorptive layered periodic slab.

The next goal of this paper is to validate equations (12) and (13) for an arbitrary periodic (not necessarily layered) absorptive slab.

The Bloch phase ϕ is the key parameter of the Floquet–Bloch theory of periodic structures [6–9]. According to this theory a fundamental system within a periodic slab can be chosen in terms of two Bloch waves:

$$\Psi(z) = C P(z)\,\mathrm{e}^{\mathrm{i}\phi z/d} + D Q(z)\,\mathrm{e}^{-\mathrm{i}\phi z/d}, \qquad (14)$$

where $P(z) = P(z+d)$ and $Q(z) = Q(z+d)$, i.e. they are periodic functions, and C and D are arbitrary constants. For nonabsorptive periodic structures the Bloch phase is complex only in the bandgaps: $\phi = q\pi + \mathrm{i}\phi''$, $q = 1, 2, \ldots$ (the integer q numbers the bandgaps) and the forward Bloch wave $P(z)\exp(\mathrm{i}\phi z/d)$ is exponentially damped. This damping is maximal at what are called *Bragg resonances*. The bandgaps can be found from equation (13) by requiring

$$|\cos\phi(k_0)| > 1, \qquad \text{or} \qquad |\mathrm{e}^{\mathrm{i}\phi(k_0)} + \mathrm{e}^{-\mathrm{i}\phi(k_0)}| > 2. \quad (15)$$

The Bragg resonances (which are located close to the centres of the bandgaps) can be approximated by the same expressions for either s or p waves. Conventionally, one has

$$[k_{\mathrm{av}}]_z \cong q\frac{\pi}{d}, \qquad q = 1, 2, \ldots,$$

$$[k_{\mathrm{av}}]_z = k_{\mathrm{av}}\cos\theta_{\mathrm{av}} = k_0 n_{\mathrm{av}}\cos\theta_{\mathrm{av}},$$

$$(16)$$

where

$$n_{av} = \frac{1}{d}\int_0^d n(z)\,dz, \qquad n_{av}\cos\theta_{av} = \sqrt{n_{av}^2 - \beta^2}, \quad (17)$$

and to derive the last expression we took into account that $n_{av}\sin\theta_{av} = n_i\sin\theta_i \equiv \beta$. One can see that if $n_{av}^2 < \beta^2$, the conventional approximation for Bragg resonances is not defined.

In [10–12] we suggested using a somewhat better approximation for estimating Bragg resonances in the sense that the values obtained are often closer to the actual points of maximal damping. For the case $n^2(z) > \beta^2$ for all z within a periodic stack, we had

$$[k_z]_{av} \cong q\frac{\pi}{d}, \qquad q = 1, 2, \ldots,$$

$$[k_z]_{av} = k_0[n(z)\cos\theta(z)]_{av} = k_0[N_z]_{av}, \tag{18}$$

where $[N_z]_{av} = \frac{1}{d}\int_0^d \sqrt{n^2(z) - \beta^2}\,dz$. This can be easily extended to cases at some (or even at all) points within a stack $n^2(z) < \beta^2$ if we assign

$$[N_z]_{av} = \frac{1}{d}\int_0^d \text{Re}[\sqrt{n^2(z) - \beta^2}]\,dz. \tag{19}$$

In the case of normal incidence $\theta_i = 0$, and both equations (16) and (17) and equations (18) and (19) lead to the same condition which is $k_0 n_{av} = q\pi/d$.

Our final task is to modify equation (15) and equations (18) and (19) to include absorptive periodic slabs.

2. The transfer matrix for an arbitrary absorptive potential

The basic idea is to separate the complex valued potential determined by $\tilde{n}(z)$ into segments, within which a fundamental system of solutions $F(z)$ and $G(z)$ is known, and use appropriate boundary conditions. Suppose that

$$\Psi(z) = A_1 F_1(z) + B_1 G_1(z), \qquad z < a^-,$$
$$= A_2 F_2(z) + B_2 G_2(z), \qquad z > a^+. \tag{20}$$

Then the coefficients are related by

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = I_{a^-} I_{a^+} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}, \tag{21}$$

where the so-called half-interface matrices I_{a^-}, I_{a^+} are

$$I_{a^-}^s = \begin{bmatrix} \frac{G_1'(a^-)}{w_1} & \frac{G_1(a^-)}{-w_1} \\ \frac{F_1'(a^-)}{-w_1} & \frac{F_1(a^-)}{w_1} \end{bmatrix}, \qquad I_{a^+}^s = \begin{bmatrix} F_2(a^+) & G_2(a^+) \\ F_2'(a^+) & G_2'(a^+) \end{bmatrix},$$

$$I_{a^-}^p = \begin{bmatrix} \frac{[G_1(a^-)\tilde{n}(a^-)]'}{\tilde{n}^2(a^-)w_1} & \frac{G_1(a^-)\tilde{n}(a^-)}{-w_1} \\ \frac{[F_1(a^-)\tilde{n}(a^-)]'}{-\tilde{n}^2(a^-)w_1} & \frac{F_1(a^-)\tilde{n}(a^-)}{w_1} \end{bmatrix},$$

$$I_{a^+}^p = \begin{bmatrix} F_2(a^+)\tilde{n}(a^+) & G_2(a^+)\tilde{n}(a^+) \\ \frac{[F_2(a^+)\tilde{n}(a^+)]'}{\tilde{n}^2(a^+)} & \frac{[G_2(a^+)\tilde{n}(a^+)]'}{\tilde{n}^2(a^+)} \end{bmatrix}, \tag{22}$$

for s and p waves, respectively. The quantity $w_1 \equiv F_1(a^-)G_1'(a^-) - G_1(a^-)F_1'(a^-)$ is the Wronskian of a second-order linear differential equation with no first-derivative term

(see equation (1)); hence it is constant and can be calculated at any point $z \leqslant a^-$. Now we are in a position to express all transfer matrices in terms of the half-interface matrices introduced above.

Firstly, we choose $a = 0$ and assume a constant $\tilde{n}(z) = n_i$ for $z < 0$. We obtain that $L_{0^-}^{-1} = I_{0^-}$, provided that we take the fundamental system in the region $z < 0$ to be $e^{ik_{iz}z}$ and $e^{-ik_{iz}z}$.

Secondly, we choose $a = l$ and assume a constant $\tilde{n}(z) = n_f$ for $z > 0$. Then $L_{l^+} = I_{l^+}$, provided we take the fundamental system in the region $z > 0$ to be $e^{ik_{fz}(z-l)}$ and $e^{-ik_{fz}(z-l)}$.

Thirdly, the transfer matrix $W_{0^+l^-}$ can be written as the product

$$W_{0^+l^-} = I_{0^+} I_{l^-}. \tag{23}$$

The important property of this matrix is as follows. It does not matter which particular fundamental system of equation (1) in the region $0 < z < l$ is chosen; we always end up with the same matrix. For example, if $\tilde{n}(z) = \tilde{n}_j$ for $0 < z < l$ (a homogeneous layer), we can take $F(z) = e^{i\tilde{k}_{jz}z}$ and $G(z) = e^{-i\tilde{k}_{jz}z}$ (or $F(z) = \cos(\tilde{k}_{jz}z)$ and $G(z) = \sin(\tilde{k}_{jz}z)$) to find that $W_{0^+l^-}$ is given by equation (11). To prove this property in general, one should recognize that equations (6) and (7) express a one to one correspondence between single-valued physical functions (EM fields and their derivatives). One can now see that

$$\det W_{0^+l^-} = \det I_{0^+} I_{l^-} = 1, \quad \text{and} \quad \det T = \frac{k_{fz}}{k_{0z}}. \tag{24}$$

To find the $W_{0^+l^-}$ matrix in a specific situation, we need to specify a fundamental system within the slab, either in analytical or in numerical form. The freedom to choose such a system allows us to impose on $F(z)$ and $G(z)$ the boundary conditions

$$F(0^+) = 1, \quad F'(0^+) = 0, \quad G(0^+) = 0, \quad G'(0^+) = 1. \tag{25}$$

Integrating from 0 to l, we arrive at the corresponding values $F(l^-)$, $F'(l^-)$ and $G(l^-)$, $G'(l^-)$. This integration can be done numerically for almost any absorptive potential $\tilde{n}(z)$. Therefore, the above choice of the fundamental system (see equation (25)) is especially useful when explicit solutions are not available in analytical form. For that choice of $F(z)$ and $G(z)$, the elements of the $W_{0^+l^-}^s$ matrix become

$$W_{0^+l^-}^s = \begin{bmatrix} G'(l) & -G(l) \\ -F'(l) & F(l) \end{bmatrix}, \tag{26}$$

and the elements of the $W_{0^+l^-}^p$ matrix become

$$(W_{0^+l^-}^p)_{11} = \frac{n(0^+)G'(l^-)}{n(l^-)} + \frac{n(0^+)n'(l^-)G(l^-)}{n^2(l^-)},$$

$$(W_{0^+l^-}^p)_{21} = \frac{n'(0^+)G'(l^-)}{n^2(0^+)n(l^-)} + \frac{n'(0^+)n'(l^-)G(l^-)}{n^2(0^+)n^2(l^-)}$$
$$\qquad - \frac{F'(l^-)}{n(0^+)n(l^-)} - \frac{n'(l^-)F(l^-)}{n(0^+)n^2(l^-)},$$

$$(W_{0^+l^-}^p)_{12} = -n(0^+)n(l^-)G(l^-),$$

$$(W_{0^+l^-}^p)_{22} = -\frac{n'(0^+)n(l^-)G(l^-)}{n^2(0^+)} + \frac{n(l^-)F(l^-)}{n(0^+)}. \tag{27}$$

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If the slab can be separated into, let us say, two regions within which the fundamental systems are known, then

$$W_{0^+l^-} = I_{0^+} I_{a^-} I_{a^+} I_{l^-} = W_{0^+a^-} W_{a^+l^-},\qquad(28)$$

and all three W-matrices are unimodular.

Finally, for an arbitrary absorptive periodic slab,

$$W_{(qd)^+(qd+d)^-} = I_{(qd)^+} I_{(qd+d)^-} = I_{0^+} I_{d^-} = W_{0^+d^-}$$
$$\equiv W_d, \qquad q = 0, 1, \ldots, M-1. \quad(29)$$

To prove this last result, one needs to choose the fundamental system in terms of Bloch waves $P(z)\exp(i\alpha z)$ and $Q(z)\exp(-i\alpha z)$. Since $\det W_d = \det I_{(qd)^+} I_{(qd+d)^-} = 1$, we can finally state that equations (12) and (13) are valid for an arbitrary absorptive periodic slab.

3. The Bragg condition for an absorptive periodic potential

As we have just mentioned, equation (13) remains valid for absorptive periodic structures. However, a noticeable difference is that the Bloch phase ϕ becomes complex ($\phi = \alpha d = \phi' + i\phi''$) everywhere (not only in the bandgaps, as it was for a real valued refractive index) and, as a result, $\cos\phi$ becomes complex valued itself.

In the light of the above, equation (15) and equations (18) and (19) are modified as follows. The bandgaps can be found by setting

$$|\operatorname{Re}[\cos\phi(k_0)]| > 1,\qquad(30)$$

and the Bragg resonances are approximated by

$$k_0[N_z]_{\text{av}} \cong q\frac{\pi}{d},$$
$$[N_z]_{\text{av}} = \frac{1}{d}\int_0^d \operatorname{Re}[\sqrt{[n(z) + i\kappa(z)]^2 - \beta^2}]\,dz.\qquad(31)$$

In [13] we showed that

$$N_z(z) \equiv \operatorname{Re}[\tilde{N}_z(z)] \equiv \operatorname{Re}[\sqrt{[n(z) + i\kappa(z)]^2 - \beta^2}]$$
$$= [\tfrac{1}{2}[n^2(z) - \kappa^2(z) - \beta^2]$$
$$+ \tfrac{1}{2}\sqrt{[n^2(z) - \kappa^2(z) - \beta^2]^2 + 4n^2(z)\kappa^2(z)}]^{1/2},\qquad(32)$$

which is in agreement with the results of [14, 15]. We should emphasize again that the Bragg resonances are approximated by the same expression, equations (31), for s and p waves. In the case of normal propagation there is no difference between those waves and $N_z(z) = n(z)$ and also $[N_z]_{\text{av}} = n_{\text{av}}$.

4. The sinusoidal absorptive periodic potential

To illustrate the formalism of the above two sections, we calculate the Bloch phase, reflectance, transmittance, and absorptance of a periodic structure with a sinusoidal absorptive refractive index profile of the form

$$n(z) = n_{\text{av}} + n_a \sin\left(2\pi\frac{z}{d}\right),$$
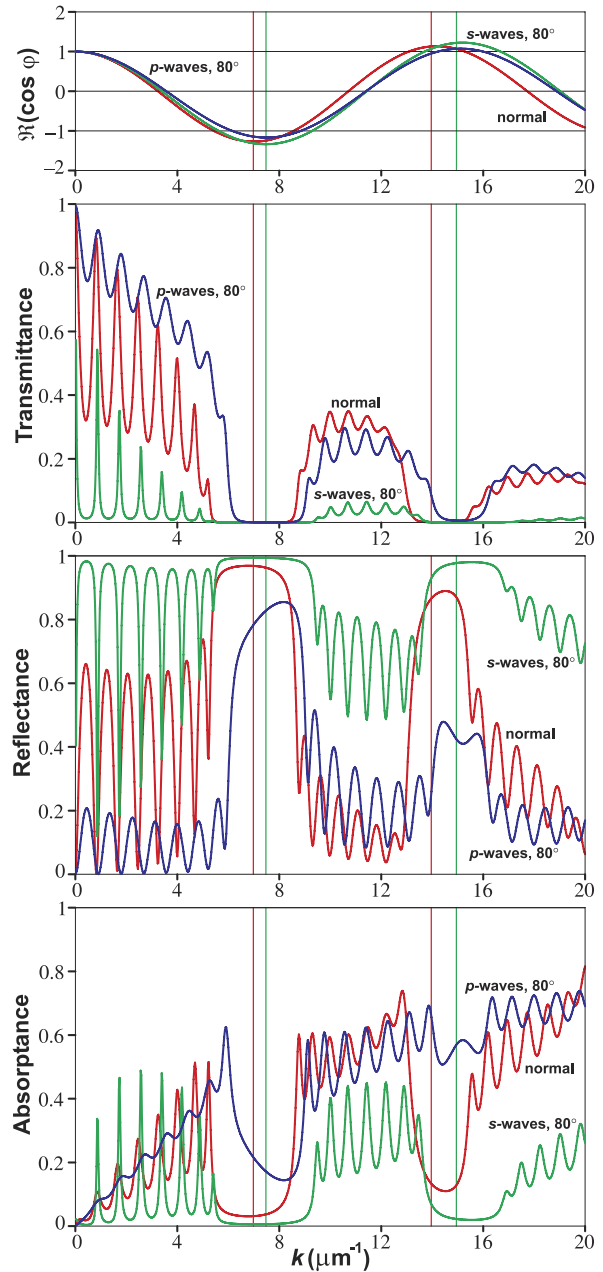$$\kappa(z) = \kappa_{\text{av}} + \kappa_a \sin\left(2\pi\frac{z}{d}\right).\qquad(33)$$

Figure 1. Bloch phase, transmittance, reflectance, and absorptance versus wavenumber $k = \omega/c$ for a sinusoidal absorptive periodic structure in the case of normal incidence (red curves), s polarized light at 80° angle of incidence (green curves), and p polarized light at 80° angle of incidence (blue curves). Two pairs of vertical lines indicate the positions of the first two Bragg resonances: the right green line in each pair corresponds to the resonances for s and p polarized light incident at 80°; the left red line corresponds to the resonances for normal incidence. The parameters of the structure are as described in the text. In the case of normal incidence, there is no difference between s and p waves.

The rugate filters are described by such a profile of the refractive index. An overview of rugates in the case of zero absorption, i.e. for $\kappa(z) \equiv 0$, was given in [16].

As a specific example, we take a rugate filter of $M = 8$ periods with overall thickness $l = 1.2\ \mu\text{m}$ ($d = 150$ nm) suspended in air, i.e. $n_i = n_f = 1$. The refractive indices and extinctions of the rugate are $n_{\text{av}} = 3.0$, $n_a = 1.5$, and
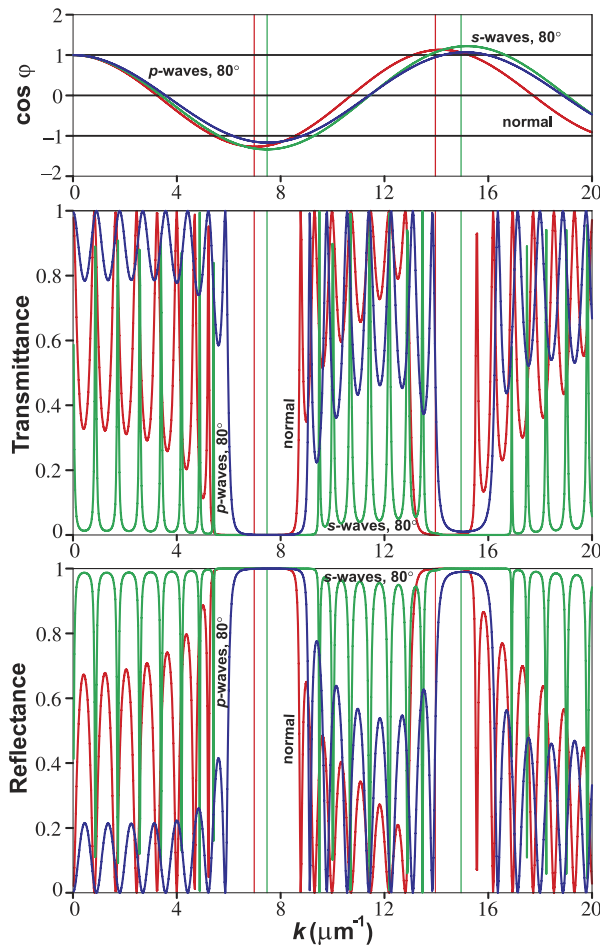
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Figure 2. Bloch phase, transmittance, and reflectance versus wavenumber $k = \omega/c$ for the structure as in figure 1 but with zero absorption.

$\kappa_{av} = 0.03$, $\kappa_a = 0.02$. The corresponding plots are shown in figure 1. For comparison, in figure 2 we show the plots for the Bloch phase, transmittance and reflectance for the same rugate in the case of zero absorption, i.e. when $\kappa_{av} = \kappa_a = 0$. One can see that a relatively small absorption (as in figure 1) leaves the formal positions and widths of the bandgaps almost unchanged (see the plots for the Bloch phases). However, it drastically changes the transmittance in allowed bands, and the reflectance in both allowed bands and bandgaps. The calculations were done according to equations (9) and (10). The elements of the W_d matrix were found with the aid of equations (25)–(27) where we replaced l with d; the elements of the W_{0+l-} matrix were then obtained from equation (12). Since the central point of this paper is to clarify how general methods available for nondissipative periodic structures can be extended to dissipative ones, we have neglected the dispersion of n and κ in the calculations reported here.

5. Conclusions

We have shown that the problem of oblique light propagation through one-dimensional periodic dissipative photonic crystals can be solved using well-established methods available for nondissipative crystals.

Our main results are as follows. Once the refractive indices and extinctions are defined, the W transfer matrix for one period can be constructed either in analytical or numerical form. The W transfer matrix for an M-period system is then given by the well-known relation for nondissipative crystals, equation (12). The validity of this relation for dissipative systems relies on the fact that the determinant of the W transfer matrix for one period remains 1. As in the case of nondissipative crystals the Bloch phase is given by equation (13). The bandgaps are defined, however, by equation (30) rather than by equation (15) and the Bragg resonances are approximated by equation (31) rather than equations (18) and (19).

To use the above theory for most absorptive periodic stacks one also needs to take into account the dispersion of the complex valued refractive indices; see [17–19]. However, the formalism remains valid without modification if the proper frequency-dependent values of n and κ are used from the beginning. The only exception is equation (31) where the average refractive index must be defined appropriately.

References

[1] Abeles F 1950 *Ann. Phys. Fr.* **5** 596
[2] Yeh P, Yariv A and Hong C-S 1977 *J. Opt. Soc. Am.* **67** 423
[3] Sprung D W L, Wu H and Martorell J 1993 *Am. J. Phys.* **61** 1118
[4] Lekner J 1994 *J. Opt. Soc. Am.* A **11** 2892
[5] Bendickson J M, Dowling J P and Scalora M 1996 *Phys. Rev. E* **53** 4107
[6] Brillouin L 1946 *Wave Propagation in Periodic Structures* (New York: McGraw-Hill)
[7] Levin M L 1948 *Zh. Tekh. Fiz.* **18** 1399
[8] Morozov G V, Maev R G and Drake G W F 1999 *Phys. Rev. E* **60** 4860
[9] Nusinsky I and Hardy A A 2006 *Phys. Rev. B* **73** 125104
[10] Morozov G V, Sprung D W L and Martorell J 2004 *Phys. Rev. E* **69** 016612
[11] Morozov G V, Sprung D W L and Martorell J 2004 *Phys. Rev. E* **70** 016606
[12] Martorell J, Sprung D W L and Morozov G V 2006 *J. Opt. A: Pure Appl. Opt.* **8** 630
[13] Morozov G V and Placido F 2010 *Opt. Quantum Electron.* at press doi:10.1007/s11082-010-9424-9
[14] Chang P C Y, Walker J C and Hopcraft K I 2005 *J. Quant. Spectrosc. Radiat. Transfer* **96** 327
[15] Shen J, Yu H and Lu J 2010 *Chin. Opt. Lett.* **8** 111
[16] Bovard B G 1993 *Appl. Opt.* **32** 5427
[17] Yu J, Shen Y, Liu X, Fu R, Zi J and Zhu Z 2004 *J. Phys.: Condens. Matter* **16** L51
[18] Wang D D 2007 *J. Phys.: Condens. Matter* **19** 248001
[19] Chen S, Wang Y, Yao D and Song Z 2009 *Opt. Appl.* **39** 473