Problem 1 : Electrical conductivity in two dimensions - Answer Sheet *(10 points)*

Part A. Four-point-probe (4PP) measurements (1.2 points)

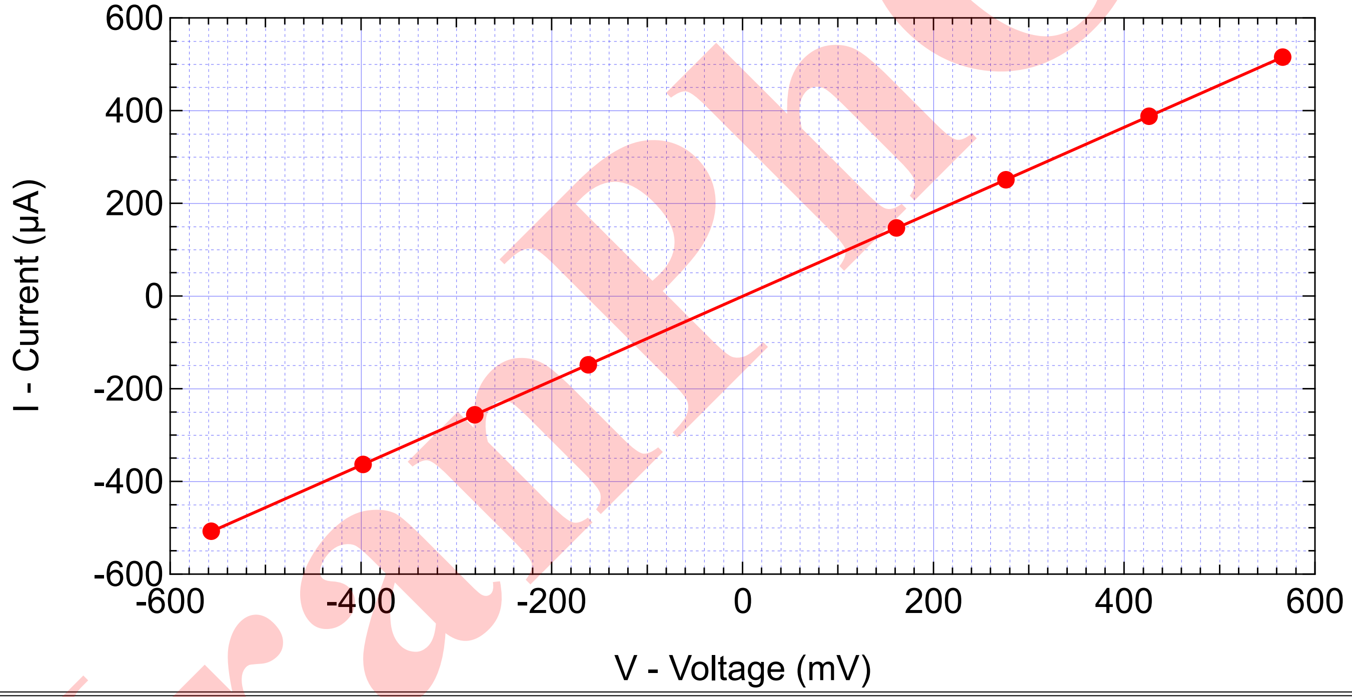
A1 (0.6 pts)

$s = 2$ cm

I (μA)	V (mV)	I (μA)	V (mV)
251	276	-148	-162
516	566	-256	-281
388	426	-363	-398
147	161	-507	-557

Plot your data in the graph B1

Graph B1: I vs V



A2 (0.2 pts)

$R = 1.08$ kΩ

A3 (0.4 pts)

$\Delta R = \pm 1\ \Omega$

Part B. Sheet resistivity (0.3 points)

B1 (0.3 pts)

$\rho_{\square} \equiv \rho_{\infty} = 4.89\ \mathrm{k\Omega}$

Part C. Measurements for different sample dimensions (3.2 points)

C1 (3 pts) and **C2 (0.2 pts)**

$s = 20$ mm

$\rho_\infty = 4.89$ kΩ

w/s	I (μA)	V (mV)	R(w/s) (kΩ)	$R_{average}$ (kΩ)		$\hat{R}$
0.3	92	1477	16.1	15.9		14.7
0.3	74	1184	16			
0.3	57	914	16			
0.3	41	651	15.9			
0.3	23	358	15.6			
0.5	154	1306	8.5	8.5		7.8
0.5	127	1079	8.5			
0.5	97	824	8.5			
0.5	67	567	8.5			
0.5	38	321	8.4			
1	233	1071	4.6	4.6		4.3
1	174	799	4.6			
1	135	621	4.6			
1	101	465	4.6			
1	59	271	4.6			
2.5	389	749	1.9	1.9		1.8
2.5	319	635	2			
2.5	237	457	1.9			
2.5	151	291	1.9			
2.5	74	143	1.9			
5	467	648	1.4	1.4		1.3
5	419	577	1.4			
5	363	499	1.4			
5	289	398	1.4			
5	185	254	1.4			

Part D. Geometrical correction factor (1.9 points)

D1 (1.0 pts)

Plot your data on the appropriate graph paper: linear (Graph E1a), semi-logarithmic (D1b) **or** double-logarithmic (D1c) on the following pages.

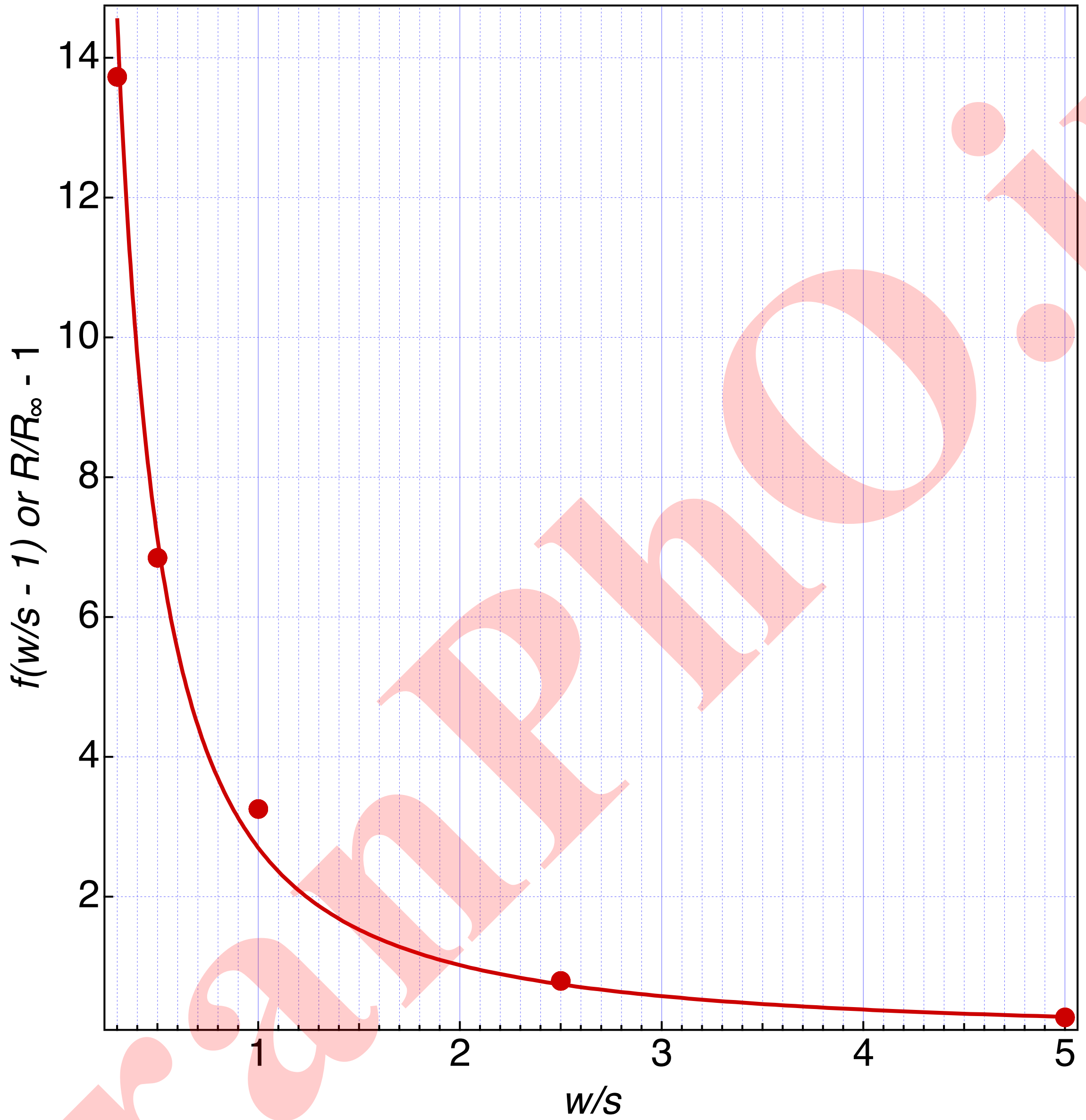
D2 (0.9 pts)

$a = 2.7$

$b = -1.4$

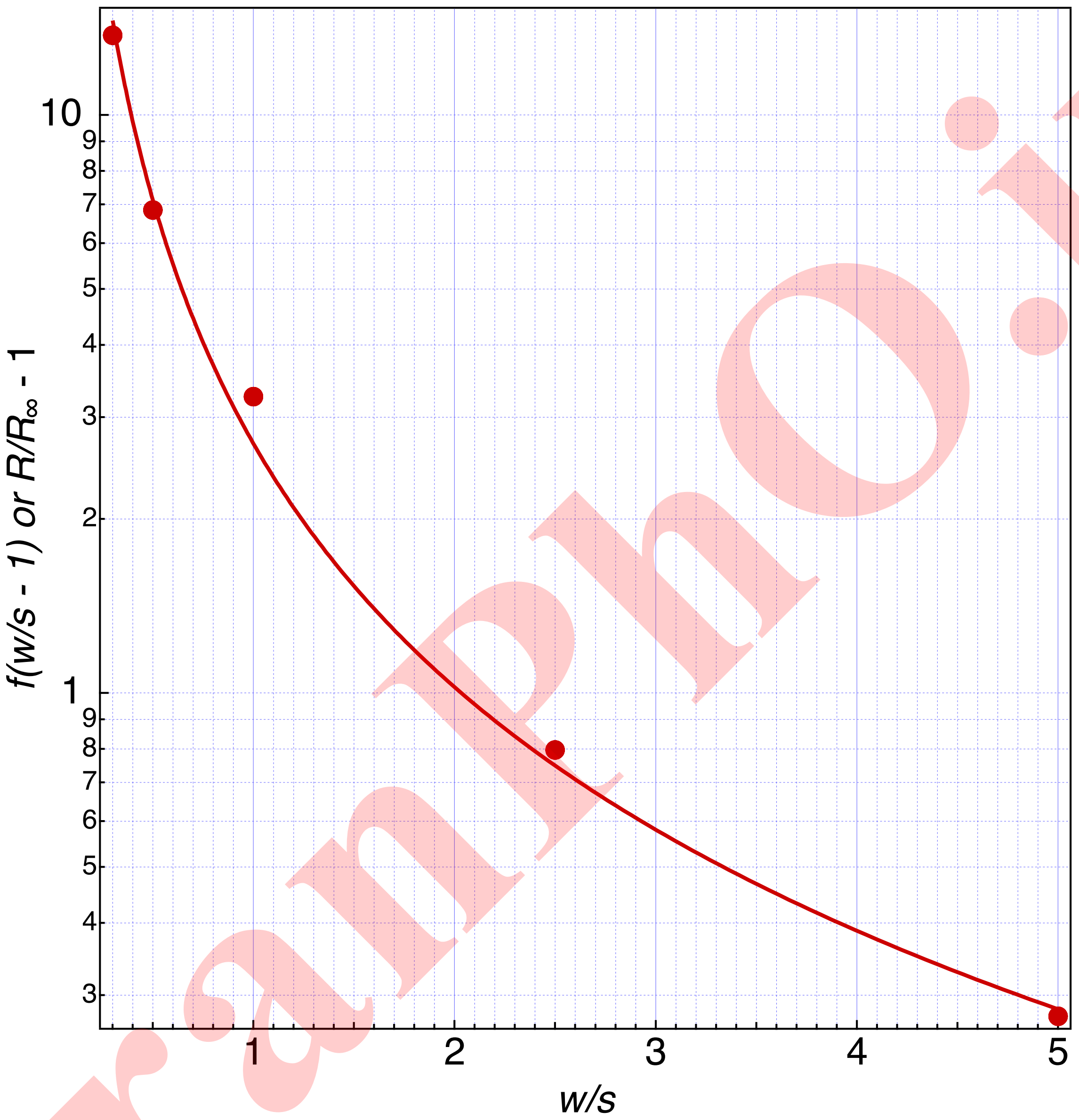
Graph D1a: linear scale: I vs V

Wrong. The usage of linear scale does not allow for deduction of the parameters.



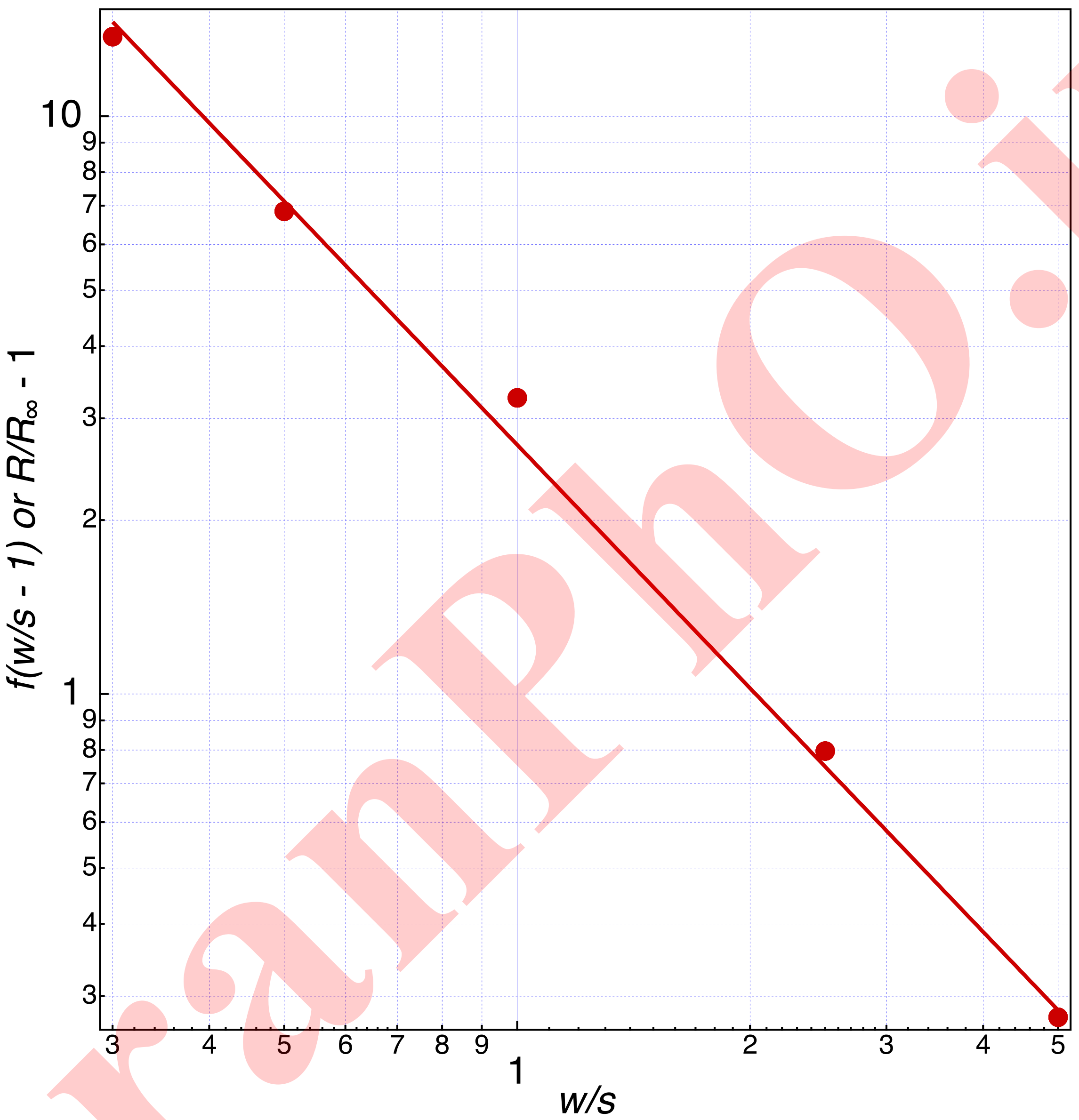
Graph D1b: semi-log scale: I vs V

Wrong. The usage of semi-log scale does not allow for deduction of the parameters.



Graph D1c: double-log scale: I vs V

Correct. The parameters can be deduced by fitting a line.



Part E. van der Pauw-method (3.4 points)

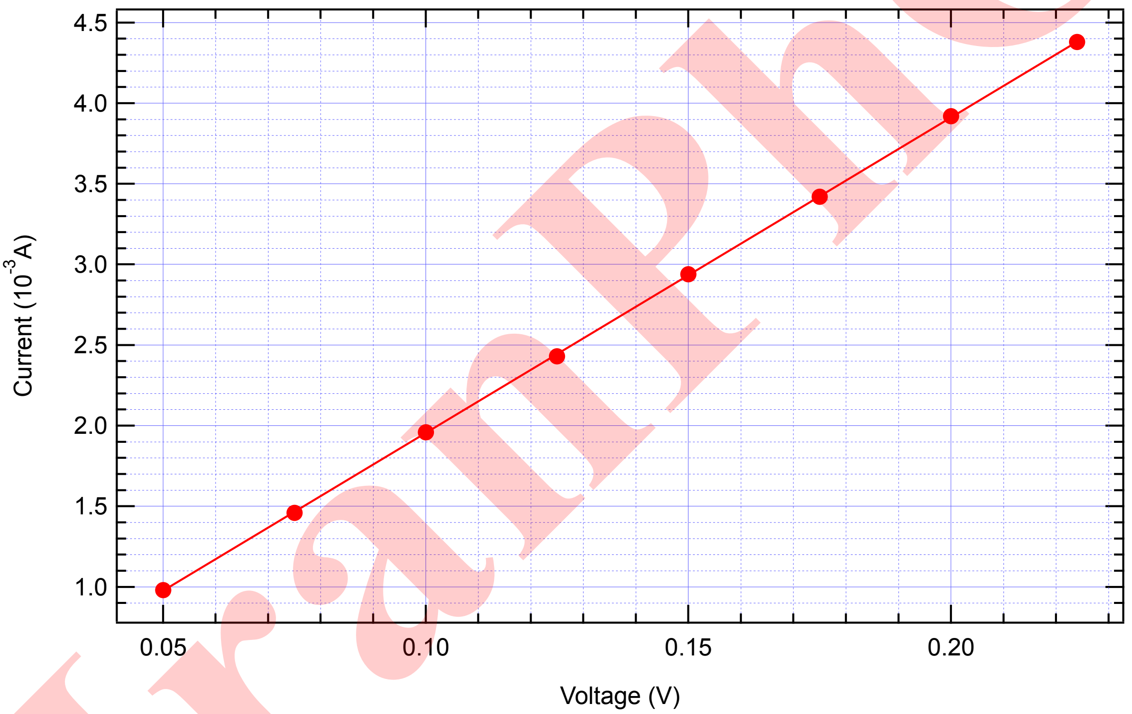
Note the number of your wafer here: 99 (between 1 - 450)

E1 (0.4 pts)

I (mA)	V (mV)	I (mA)	V (mV)
0.98	50	2.94	150
1.46	75	3.42	175
1.96	100	3.92	200
2.43	125	4.38	224

E2 (0.4 pts)

Graph F2: I vs V



$R_{4PP} = 51.1\Omega$

E3 (0.2 pts)

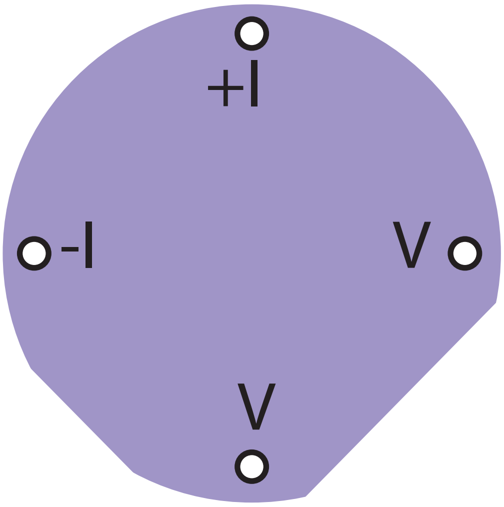
$w = 10$ cm $\rightarrow$ $w/s = 5$ $f(w/s) = 1.284$

E4 (0.1 pts)

$\rho_{\square} = 180\,\Omega$

E5 (0.6 pts)

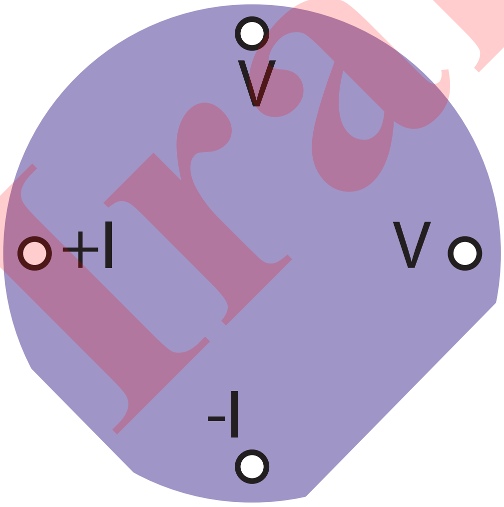
Sketch (orientation of the current):



V mV	I mA
140	3.71
120	3.18
100	2.66
80	2.12
60	1.58
40	1.06
20	0.53
-20	-0.54
-40	-1.06
-60	-1.61
-80	-2.13
-100	-2.68
-120	-3.2
-136	-3.62

E6 (0.6 pts)

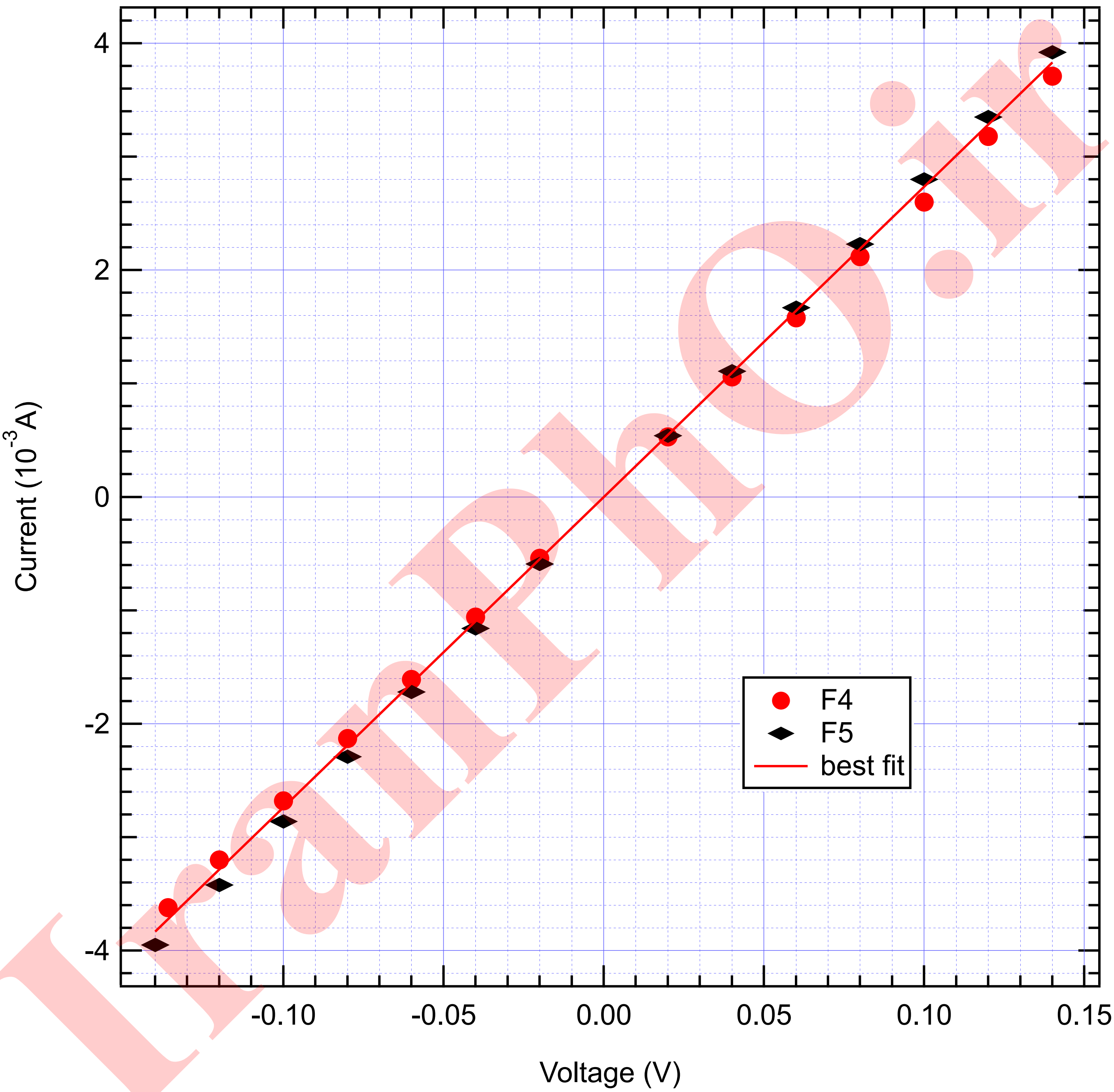
Sketch (orientation of the current):



I	V
140	3.92
120	3.35
100	2.8
80	2.23
60	1.67
40	1.11
20	0.54
-20	-0.59
-40	-1.16
-60	-1.72
-80	-2.29
-100	-2.86
-120	-3.42
-140	-3.95

E7 (0.5 pts)

Graph F6: I vs V



$\langle R \rangle = 36.5\ \Omega$

E8 (0.4 pts) Calculation:

$2 \cdot e^{-\pi \cdot \langle R \rangle / \rho_\square} = 1 \qquad e^{-\pi \cdot \langle R \rangle / \rho_\square} = 1/2$

$-\dfrac{\pi \cdot \langle R \rangle}{\rho_\square} = \ln(1/2) \qquad \dfrac{\pi \cdot \langle R \rangle}{\rho_\square} = \ln(2)$

$\rho_\square = \dfrac{\pi \cdot \langle R \rangle}{\ln(2)}$

$\rho_\square = 165\,\Omega$

E9 (0.1 pts)

$\frac{\Delta \rho_\square}{\rho_\square} = \quad 0.091 \quad = \quad 9.1 \quad \%$

E10 (0.1 pts)

Resistivity of the Cr thin film $\rho = 1.32 \cdot 10^{-6}\,\Omega \cdot \mathrm{m}$

Problem 2 : Solution – Jumping Beads - a model for phase transitions and instabilities *(10 points)*

Part A. Critical driving amplitude (3.3 points)

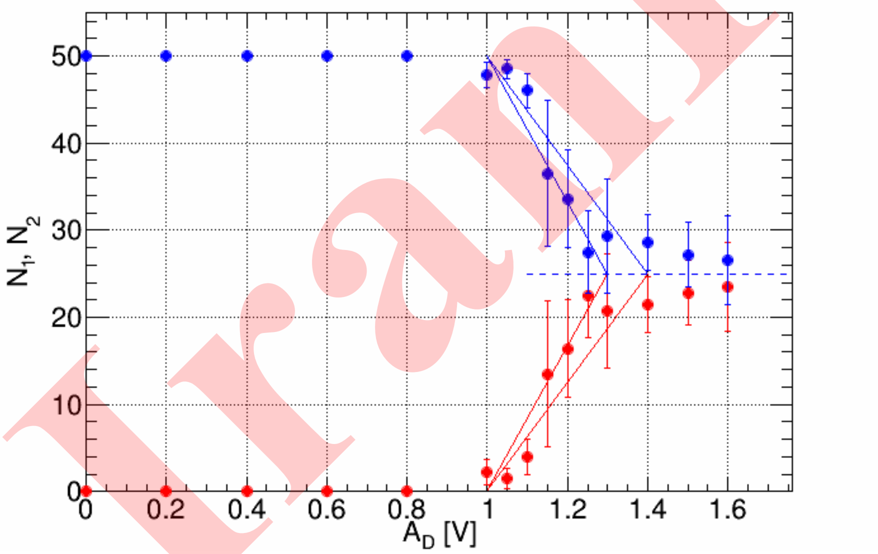
A1 (1.2 pts)

Total number of seeds: $N_0 = 50$.
Number of readings: $n = 6$.

A_D, [V]	N_1						$\bar{N}_1 = \frac{1}{n}\Sigma_{i=1}^{n} N_1^i$	$\bar{N}_2 = N_0 - \bar{N}_1$	$\sigma = \sqrt{\frac{\Sigma_{i=1}^{n}(N_i - \bar{N})^2}{n-1}}$	$\mathrm{SE} = \frac{\sigma}{\sqrt{n}}$
1.00	1	5	2	1	2	2	**2.2**	**47.8**	**1.5**	**0.6**
1.05	1	0	2	3	1	2	**1.5**	**48.5**	**1.1**	**0.5**
1.10	4	4	1	7	3	5	**4.0**	**46.0**	**2.0**	**0.8**
1.15	26	5	18	7	18	7	**13.5**	**36.5**	**8.4**	**3.4**
1.20	13	16	27	12	17	13	**16.4**	**33.7**	**5.6**	**2.3**
1.25	26	28	22	22	14	23	**22.5**	**27.5**	**4.8**	**2.0**
1.30	27	24	8	22	22	21	**20.7**	**29.3**	**6.6**	**2.7**
1.40	22	18	17	23	23	25	**21.4**	**28.7**	**3.2**	**1.3**
1.50	19	27	27	24	19	21	**22.8**	**27.2**	**3.7**	**1.5**
1.60	27	15	23	23	23	30	**23.5**	**26.5**	**5.1**	**2.1**

Plot the data in the graph A2.

A2 (1.1 pts)



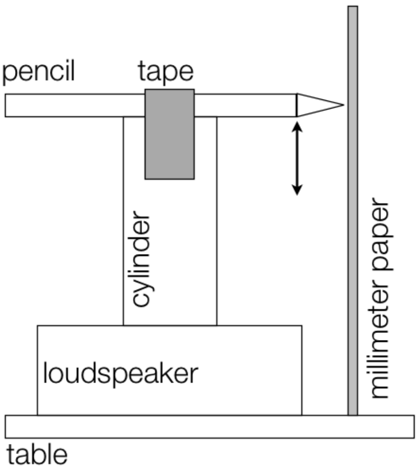
Error bars represent either standard deviation (σ) or standard error (SE).

A3 (1.0 pts)

$A_{D,\mathrm{crit}} = (1.25 \pm 0.05)$ V

Part B. Calibration (3.2 points)

B1 (0.5 pts)

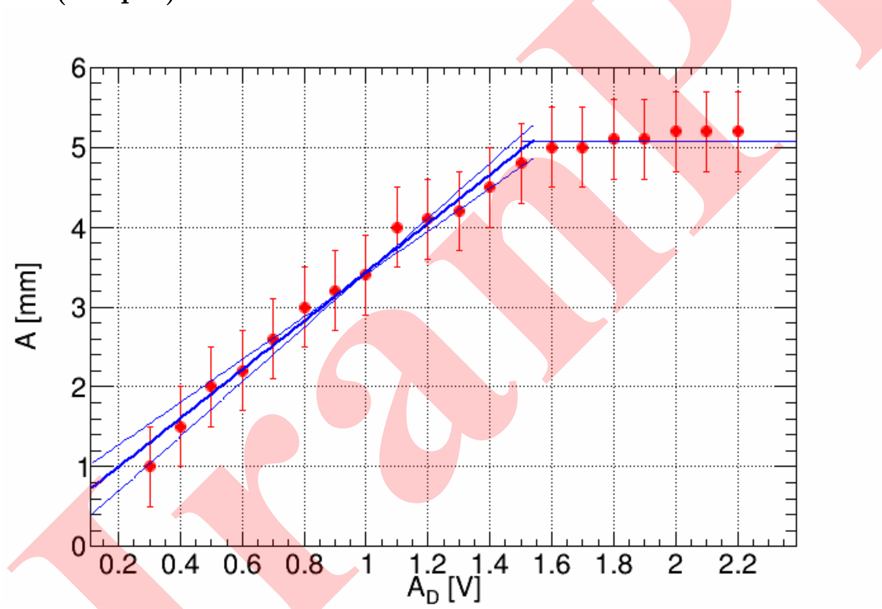


B2 (0.8 pts)

A_D [V]	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1
A [mm]	1.0	1.5	2.0	2.2	2.6	3.0	3.2	3.4	4.0	4.1	4.2	4.5	4.8	5.0	5.0	5.1	5.1	5.2	5.2

Instrumental error ± 0.5 mm.

B3 (1.0 pts)



B4 (0.8 pts)

$A = k_0 + k_1 \times A_D$,

where:

$k_0 = 0.2$ [mm], $k_1 = 3.1$ [mm/V]

B5 (0.1 pts)

$A_{\text{crit}} = (4.4 \pm 0.1)$ mm

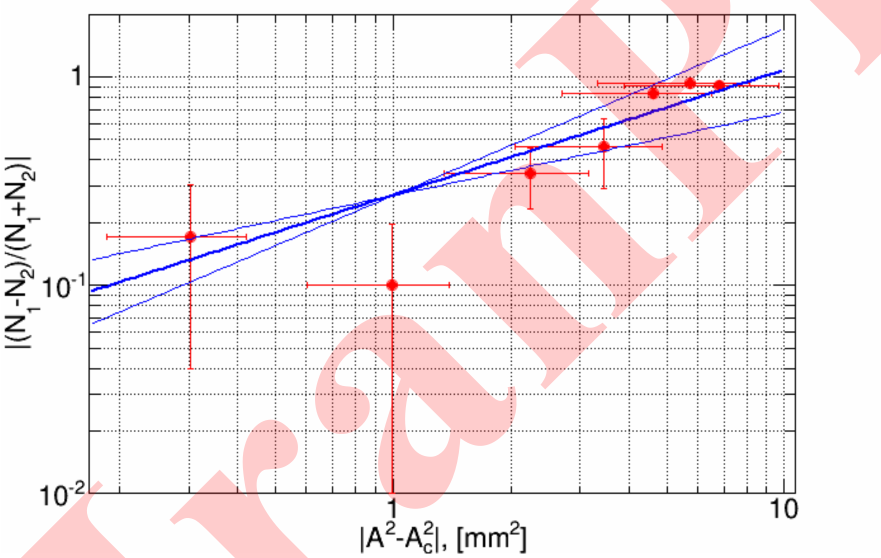
Part C. Critical exponent (3.5 points)

C1 (1.1 pts)

A_D, [V]	A, [mm]	$\left\|\frac{N_1-N_2}{N_1+N_2}\right\|$	$\|A^2 - A_0^2\|$
1.00	3.5	0.91	6.8
1.05	3.6	0.94	5.7
1.10	3.8	0.84	4.6
1.15	3.9	0.46	3.5
1.20	4.1	0.35	2.2
1.25	4.2	0.10	1.0
1.30	4.4	0.17	0.3
1.40	4.7	0.15	
1.50	5.0	0.09	
1.60	5.3	0.06	

Plot the data in the graph C2.

C2 (1.0 pts)



C3 (1.4 pts)

$y = a \cdot x^b$, where $x = |A^2 - A_0^2|$, $y = |\frac{N_1-N_2}{N_1+N_2}|$.

Critical exponent $b = 0.6 \pm 0.2$.