Hybrid Control for Longitudinal Speed and Traction of Vehicles

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Abstract **– This paper presents a hybrid system approach to longitudinal speed and traction control of a vehicle with wheel slip constraints. The vehicle system is modeled as a hybrid system where the system is divided into local subsystems and each subsystem is selected to control the vehicle, in terms of control modes and operating region. Controllers are designed to track a desired speed value while maintaining the safety constraint that the absolute value of the slip between wheels and ground must be less than a given limit value to prevent the wheels from skidding or spinning. Simulation results are provided to show the feasibility of the proposed control system.**

I. INTRODUCTION

With the increase of the number of vehicles, driving safety has been one major concern in the community, automotive industry, and control engineering. There have been considerable efforts to improve driving safety, along with efforts to improve the performance of vehicles (see [6], [11] and references therein). Among the situations that threaten driving safety, skidding is a major dangerous situation as drivers may encounter, for example, ice patches on the road in wintertime or early springtime and these patches can cause μ-split [1].

Vehicle traction control has received considerable attention since it can provide the driving safety in skidding situations as well as improve vehicle motion in longitudinal and lateral directions [8]. In vehicle traction control, the wheel slip between the tire and ground is an important variable and considered as a source of generating the tractive friction force to accelerate or decelerate the vehicle. This slip is considered as a key to anti-skidding. Many results on identifying the relationship between the wheel slip and the tractive friction force, however, indicate that its nonlinearity and uncertainties make vehicle traction control a challenging problem.

Tai and Tomizuka [13] proposed a vehicle speed control method using traction and brake control. The method is designed based on backstepping which helps construct a candidate Lyapunov function to guarantee stability. Kachroo *et al.* [7] proposed a fuzzy supervisory control method for a vehicle speed and traction control system. They establish a vehicle model for traction control, which combines the wheel and vehicle dynamics, and use wheel slip characteristics to distinguish the vehicle state (whether the state is in accelerating or in braking) for applying a particular control algorithm. De Koker *et al.* [5] proposed a fuzzy control algorithm for traction control and showed that if the wheel slip is kept around an optimal value, the vehicle will be free for skidding.

All the methods described above use nonlinear methods that guarantee stability, but the complexity in the algorithms requires much computing power and even makes it questionable to implement the algorithm since vehicles in market have embedded computer systems and many of these systems do not have enough computing power. Therefore, this problem calls for a simple and efficient method for vehicle traction control.

In this paper, a hybrid system approach to longitudinal speed and traction control of a vehicle is presented. This work was motivated by the fact that hybrid system frameworks have been reported to provide a means of simplifying complex systems [2], [10]. For example, one can obtain a simplified (linearized) system for a nonlinear system and achieve a performance comparable to that of nonlinear control, using hybrid system frameworks.

Recently, hybrid system frameworks have been comprehensively investigated [2], [4], [10], [12]. With the help of wide spread use of computers for decision and control, hybrid systems can be seen in various control areas and have become an important paradigm in control engineering. Switched systems are a class of hybrid systems and have attracted much interest in control community since many systems in control applications can be seen as switched systems. As mentioned above, the main reason that switched systems have an increased attention by control community is that switched systems provide a means to avoid dealing directly with a complex system. For example, one can work with a set of simpler systems (e.g., linear systems) instead of working with a complex system (e.g., nonlinear system) and select a particular system by switching in terms of operating conditions [2].

The paper is organized as follows. Section II describes a vehicle longitudinal model for traction control. In section III, the obtained model is recast as a switched system model which consists of local subsystems corresponding to the operating modes. Then the controller for this switched model is proposed, which incorporates a decision-making logic to select a particular subsystem in terms of switching conditions. Simulation results are presented in section IV and conclusions are drawn in section V.

II. LONGITUDINAL VEHICLE MODEL

In this section, a longitudinal vehicle model presented by Kachroo *et al.* [7] is described. In the model, tire longitudinal

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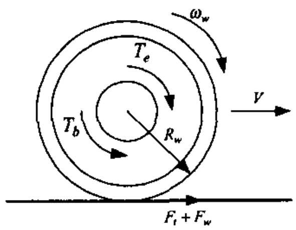

Fig. 1. Wheel diagram [7].

force-slip characteristics and wheel dynamics are included. Here, the wheel speed and the vehicle speed are used as the state variables and the wheel torque is used as the input variable. By using Newton's law of motion, wheel dynamics and longitudinal vehicle dynamics are obtained.

Consider the wheel shown in Fig. 1 [7]. In this figure, the parameters are defined: R_w is the radius of the wheel, V is the longitudinal vehicle speed, ω_w is the wheel angular speed, T_e the shaft torque from the engine, T_b the brake torque, F_t the tractive friction force, and F_w the wheel viscous friction. The term *tractive friction force* is regarded as the force to make the vehicle accelerated or decelerated while the tires are interacting with ground.

When the wheel is rotating to accelerate the vehicle, it experiences slip against the ground, i.e., $\omega_w > \omega_v$, where ω_v is the vehicle angular speed defined as $\omega_v = V / R_w$ [7]. This wheel slip generates the tractive friction force to accelerate the vehicle towards right in Fig. 1. In the deceleration case, the wheel also experiences slip against the ground, i.e., $\omega_w < \omega_v$. This wheel slip also generates the tractive friction force to decelerate the vehicle towards left in Fig. 1.

The phenomenon that wheel slip generates tractive friction force has been comprehensively studied in automotive and control engineering communities [3], [6]–[9], [11], [13], [14]. It is well known that the tractive force is a function of wheel slip and different road conditions. Several models to describe the phenomenon have been proposed. Along with the development of Anti-lock Brake Systems (ABS), a class of wheel slip models have been developed, which are often referred to as *static* models [14]. Static models assume that tire is rigid (non-deformable) and braking time delay due to the deformation of the tire is neglected. In these models, only steady-state response is considered. Recently, Canudas de Wit *et al.* [3] and Wallén [14] proposed *dynamic* models in which tire deformation is considered and therefore braking time delay and transient response are mainly focused.

Dynamic models are reported to be well suited for traction control, but dynamic models in general require complex dynamics and high computing power. In this paper, a static model is used since the static model is relatively simple and accepted to satisfy a certain performance specification.

As mentioned above, wheel slip is crucial for modeling the wheel dynamics. The adhesion coefficient, μ, is a function of the wheel slip, λ. Fig. 2 [13] shows a typical (static) characteristic curve between the adhesion coefficient and the wheel slip. Wheel slip, the difference between the vehicle speed and wheel speed, is defined as

$$\lambda = \frac{\omega_w - \omega_v}{\max(\omega_w, \omega_v)}. \tag{1}$$

Here, the difference is normalized by the maximum value between the vehicle speed and the wheel speed.

The tire tractive force is obtained as

$$F_t = \mu(\lambda) N_v, \tag{2}$$

where the normal reaction force, N_v, depends on parameters of vehicle dynamics such as the mass of the vehicle, the location of the center of gravity, and the steering and suspension dynamics [7]. Note that the adhesion coefficient is dependent on various road conditions as well as wheel slip, and therefore it has different characteristics in different road conditions.

From the variables defined in Fig. 1 and by applying Newton's law, the wheel angular dynamic equation is obtained as [7]

$$\dot{\omega}_w = \frac{1}{J_w}(T_e - T_b - R_w F_t - R_w F_w), \tag{3}$$

where J_w is the moment of inertia of the wheel.

The vehicle considered is depicted in Fig. 3 [7]. Since only longitudinal motion is considered, it is assumed that the vehicle model is a bicycle model for force balance. It is also

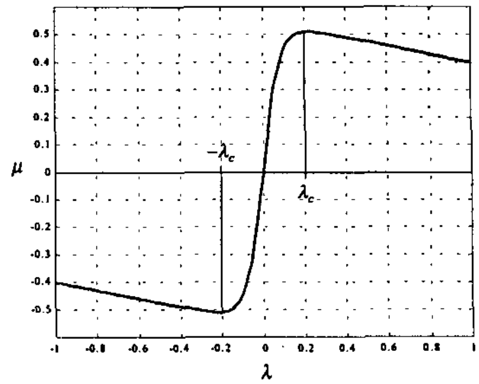

Fig. 2. A typical $\mu - \lambda$ characteristic curve [13].

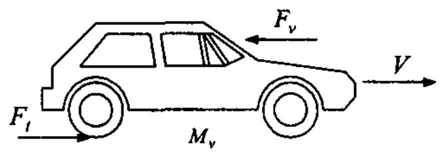

Fig. 3. Vehicle diagram [7].

assumed that all wheels are used for both acceleration and braking, and the road elevation is zero. Therefore, the tractive force, F_t, is assumed to be an average sum of the forces generated by all wheels.

The dynamic equation of the vehicle motion is

$$\dot{V} = \frac{1}{M_v}(F_t - F_v), \tag{4}$$

where M_v is the mass of the vehicle and F_v is the wind drag force which is a function of vehicle speed. The wind drag force is usually modeled as $F_v = kV^2$, where k is a constant dependent on weather conditions [7].

If the state variables are defined as

$$x_1 = \omega_v = \frac{V}{R_w} \tag{5}$$

$$x_2 = \omega_w, \tag{6}$$

then vehicle dynamic equations are obtained as [7]

$$\dot{x}_1 = -f_1(x_1) + \alpha\,\mu(\lambda) \tag{7}$$

$$\dot{x}_2 = -f_2(x_2) - \beta\,\mu(\lambda) + \gamma u, \tag{8}$$

where the input is $u = T_e - T_b$, the wheel slip is $\lambda = (x_2 - x_1)/\max(x_1, x_2)$, and nonlinear functions and constants are $f_1(x_1) = k R_w x_1^2 / M_v$ and $f_2(x_2) = F_w(x_2)/J_w$, and $\alpha = N_v/(M_v R_w)$, $\beta = R_w N_v / J_w$, and $\gamma = 1/J_w$, respectively.

III. HYBRID CONTROLLER

A. Hybrid Modeling and Problem Statement

From the curve in Fig. 2 and the definition of wheel slip in (1), it is obvious that $-1 \le \lambda \le 1$ and that λ is positive when the vehicle is in acceleration or negative in braking. When $-\lambda_c \le \lambda \le \lambda_c$, the curve can be approximated by a linear function, $\mu = c\lambda$, where c is a positive constant (see Fig. 4).

Local models are defined in terms of the modes: acceleration mode and braking mode, each of which is also divided into normal mode and emergency mode. These local models are switched in-between by a decision-making logic.

The acceleration mode is defined when $x_2 > x_1 > 0$. Thus, in the acceleration mode, the wheel slip is greater than zero, which generates tractive friction force to accelerate the vehicle in the forward direction. On the contrary, the braking mode is defined when $x_1 > x_2 > 0$, where the wheel slip is less than zero making the force as a braking force.

In the normal modes, the vehicle is in normal situation where the absolute value of the wheel slip does not exceed a given limit (i.e., $|\lambda_c - \alpha|, \alpha > 0$). The emergency modes indicate that the absolute value of the wheel slip exceeds the limit and unless a particular action for such an emergency is made, the vehicle would be in either skidding or spinning that

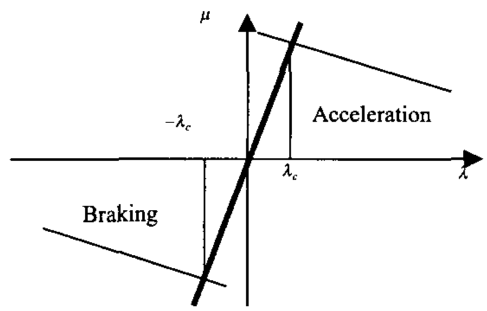

Fig. 4. Approximation of slip characteristic curve.

may lead the vehicle to dangerous situations. Here, it is assumed that the situation that the wheel slip is greater than λ_c when the vehicle is in acceleration or less than $-\lambda_c$ in braking does not occur if these emergency modes are active. This assumption is valid by the fact that if vehicle wheels are released to be passive (by some control actions), then the wheel slip is getting smaller to be negligible. Thus, the model in this paper considers the cases only when $-\lambda_c \le \lambda \le \lambda_c$.

Assume that wind drag force and wheel viscous friction are negligible. Then, the dynamic equations in (7) and (8) are rewritten in terms of the acceleration and braking modes as follows.

For the acceleration mode ($x_2 > x_1$):

$$\begin{aligned} \dot{x}_1 &= a_1(1 - x_1/x_2) \\ \dot{x}_2 &= -a_2(1 - x_1/x_2) + a_3 u. \end{aligned} \tag{9}$$

For the braking mode ($x_1 > x_2$):

$$\begin{aligned} \dot{x}_1 &= a_1(x_2/x_1 - 1) \\ \dot{x}_2 &= -a_2(x_2/x_1 - 1) + a_3 u. \end{aligned} \tag{10}$$

Here, a_i, $i = 1,2,3$, are positive constants determined by the vehicle parameters.

Problem Statement: For the vehicle system represented by (9) and (10), find a control law (with a decision making logic) to track a given speed reference input while maintaining the safety constraint ($|\lambda| < \lambda_c$) to prevent the wheels from skidding ($\lambda < -\lambda_c$) or spinning ($\lambda > \lambda_c$).

B. Controller Design

With the obtained model in (9) and (10), a control law is designed, which lenders the vehicle system to comply with the control objective described in the problem stated above. The proposed control law consists of a decision-making logic to determine a particular local system by switching among the local systems in terms of the system mode, and local controllers to regulate the system (continuous) dynamics.

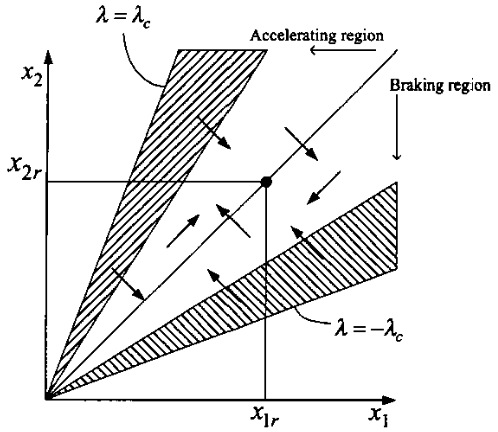

Fig. 5. Partitioned regions in the state-space.

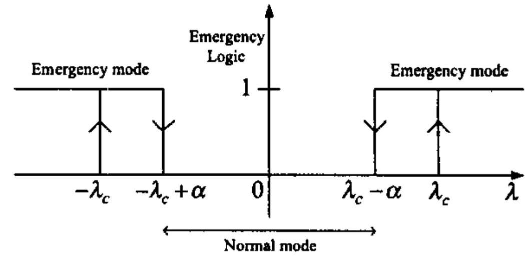

Fig. 6. Hysteresis in the emergency logic.

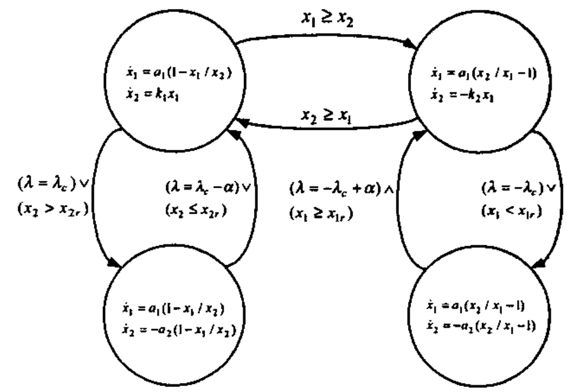

Fig. 7. Automaton diagram of the proposed control system.

Thus, the controller yields a switched system.

If the state-space is partitioned into sub-regions in terms of the conditions by which a particular system mode is selected (switched in), one may obtain partitions as shown in Fig. 5, where $x_{ir}, i = 1, 2$, are the reference states. In Fig. 5, the shaded regions indicate emergency modes where the vehicle is in state right before skidding or spinning. If the trajectories of the states intersect, for example, the line $\lambda = -\lambda_c$, it means that the vehicle is subject to skidding.

The proposed control law is simple and efficient to avoid the skidding or spinning while regulating the vehicle speed. The fundamental concept of this law lies in the fact that vehicles do not experience skidding or spinning when the wheels of the vehicle are released (i.e., passive or not actuated). With the help of hybrid system frameworks, one can obtain such a law to prevent the skidding or spinning while maintaining the vehicle speed to a reference speed, by partitioning the working regions into normal regions and emergency regions as described in the previous subsection.

One idea of controlling the system described in (9) and (10) is to make the trajectories of the system evolve toward the reference states. This idea is illustrated in Fig. 5 as the arrows indicate the directions to which the rates of the states correspond. The control law is derived from this idea.

Consider the system described by (9) and (10) and assume that the system is observable and the states and the rates of the states are available from the sensors. If the control input u is determined by the following criterion:

$$u = \begin{cases} k_1 x_1 + \dfrac{a_2}{a_3}(1 - x_1/x_2), & \text{if acceleration normal} \\ 0, & \text{if acceleration emergency or } x_2 > x_{2r} \\ -k_2 x_1 + \dfrac{a_2}{a_3}(x_1/x_2 - 1), & \text{if braking normal} \\ 0, & \text{if braking emergency or } x_1 < x_{1r}, \end{cases} \tag{11}$$

where $k_i, i = 1, 2$, are positive constants, then the system trajectories are driven toward the direction indicated by the arrows shown in Fig. 5.

The feasibility of this law is proved by the fact that when the control input (11) is applied to the system in terms of the mode switching conditions, the rates of the states are as follows:

if the vehicle is in the acceleration normal mode,

$$\begin{aligned} \dot{x}_1 &= a_1(1 - x_1/x_2) > 0 \\ \dot{x}_2 &= k_1 x_1 > 0, \end{aligned} \tag{12}$$

if the vehicle is in the acceleration emergency mode or $x_2 > x_{2r}$,

$$\begin{aligned} \dot{x}_1 &= a_1(1 - x_1/x_2) > 0 \\ \dot{x}_2 &= -a_2(1 - x_1/x_2) < 0, \end{aligned} \tag{13}$$

if the vehicle is in the braking normal mode,

$$\begin{aligned} \dot{x}_1 &= a_1(x_2/x_1 - 1) < 0 \\ \dot{x}_2 &= -k_2 x_1 < 0, \end{aligned} \tag{14}$$

and if the vehicle is in the braking emergency mode or

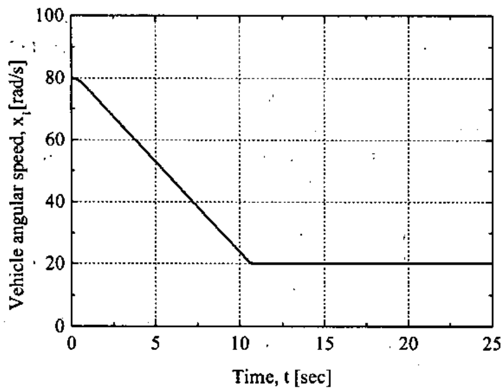

Fig. 8. Vehicle angular speed trajectory.

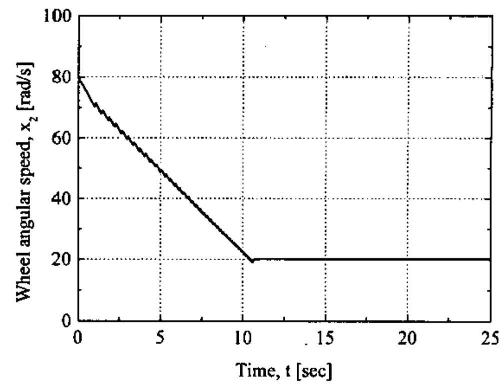

Fig. 9. Wheel angular speed trajectory.

$x_1 < x_{1r}$,

$$\begin{aligned} \dot{x}_1 &= a_1(x_2 / x_1 - 1) < 0 \\ \dot{x}_2 &= -a_2(x_2 / x_1 - 1) > 0. \end{aligned} \tag{15}$$

C. Controller Improvement

Whenever the system trajectories come into the emergency regions, by the control law defined in (11), the system is governed by the inequalities in (13) or (15). However, in practice, since the emergency regions should be defined to ensure that the vehicle is in a safe state even though it is in the emergency modes, the entrance lines to the emergency modes in Fig. 5 are defined as $\lambda = \lambda_c - \alpha$ and $\lambda = -\lambda_c + \alpha$ for the acceleration and braking modes, respectively. This may lead to conservativeness; that is, the vehicle cannot use a peak tractive force that could be a maximal braking force without skidding or a maximal acceleration force without spinning.

To minimize the conservativeness, hysteresis is applied to the determination of the emergency modes as shown in Fig. 6. Thus, the system mode is determined as a normal mode by the decision-making logic until the trajectories reach the line $\lambda = \lambda_c$ or $\lambda = -\lambda_c$, and when the system mode is once determined as an emergency mode, the mode is retained until the trajectories return to the entrance line $\lambda = \lambda_c - \alpha$ or $\lambda = -\lambda_c + \alpha$. Note that α should be chosen to not cause chattering. If α is too small, the system experiences chattering; if α is too large, the system does not have a good performance.

In Fig. 7, the automaton diagram of the proposed control system is illustrated.

IV. SIMULATIONS

Simulations are carried out to verify the feasibility of the proposed control system. The parameters are $a_1 = 82.9958$, $a_2 = 198.1598$, and $a_3 = 0.0497$, which are taken from [7].

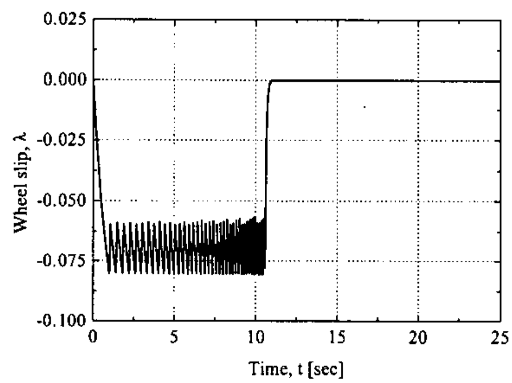

Fig. 10. Wheel slip trajectory.

The slip limit is given as $\lambda_c = 0.08$ and $\alpha = 0.02$. The simulations are carried out with the assumption that the vehicle is driven on an asphalt road, where the initial speed of the vehicle is 80.00 [rad/s], which corresponds to 89.28 [km/h] when $R = 0.31$ [m], and the final speed is 20 [rad/s], which corresponds to 22.32 [km/h] when $R = 0.31$ [m].

Fig. 8 and Fig. 9 show the state trajectories, and Fig. 10 shows the wheel slip experienced through the simulation. In Fig. 8 and Fig. 9, the system trajectories are seen to comply with the control objective. In Fig. 10, it is shown that the slip is maintained to satisfy the safety constraint.

V. CONCLUSION

This paper proposed a hybrid control method for longitudinal speed and traction control of a vehicle with wheel slip constraints. A longitudinal vehicle model with wheel slip is considered, which is crucial for the traction control. In terms of the modes defined by given conditions, the vehicle model is reformulated as a switched system that consists of local subsystems. Controllers for particular local subsystems are designed in the context of hybrid system frameworks.

The proposed control method is simple and efficient as it provides an intuitively simple algorithm and a good performance to comply with the control objective. The feasibility of the proposed method is verified by numerical simulations.

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