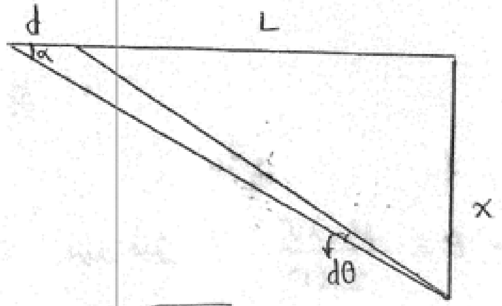
دورهٔ شانزدهم

۱)

الف)



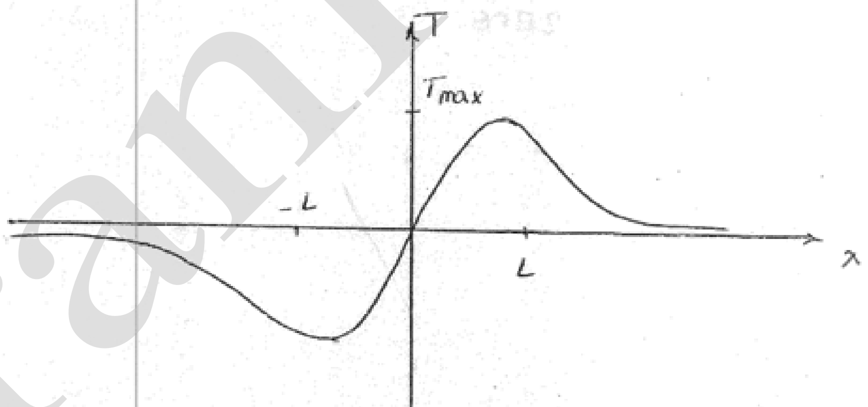
تقریب sin ها: $\sqrt{L^2+x^2}\; d\theta = (d)\sin\alpha$

$$\sin\alpha = \frac{x}{\sqrt{x^2+(L+d)^2}} \Rightarrow$$

چون $\frac{d}{L} \ll 1$ پس $\frac{d}{L}$ را تا مرتبه‌ی اول تقریب داریم

$$d\theta\,\sqrt{L^2+x^2} = \frac{x\,d}{\sqrt{x^2+L^2}} \Rightarrow d\theta = \frac{x\,d}{x^2+L^2}$$

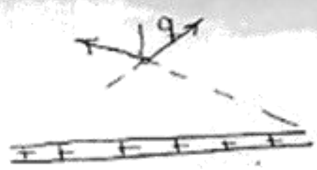
$$d\theta = \omega T \Rightarrow T = \frac{x\,d}{\omega(x^2+L^2)}$$



$$\frac{dT}{dx} = 0 \rightarrow \frac{dT}{dx} = \frac{d}{\omega}\left(\frac{L^2+x^2-2x^2}{(L^2+x^2)^2}\right) = \frac{d}{\omega}\left(\frac{L^2-x^2}{L^2+x^2}\right) = 0 \Rightarrow x = \pm L$$

ب) $T_{max} = \frac{dL}{2\omega L^2} = \frac{d}{2\omega L} = \frac{1}{30 \times 2 \times 6} = \frac{1}{360} \simeq 0.003\,s$

2) الف)



در جهت $+y$

ب) $\vec{F}_E = \frac{q\lambda}{2\pi\epsilon_0 r}$

ج) $I = \lambda v$

د) $2\pi r B = \mu_0 I \rightarrow B = \frac{\mu_0 \lambda v}{2\pi r}$ بسمت بیرون صفحه

ه) $\vec{F}_B = q\,\vec{v}\times\vec{B} = -\frac{q\, v\, \mu_0 \lambda v}{2\pi r}\hat{y} = -\frac{q\mu_0\lambda v^2}{2\pi r}\hat{y}$

و) $\vec{F} = \vec{F}_E + \vec{F}_B = \frac{q}{2\pi r}\left(\frac{\lambda}{\epsilon_0} - \mu_0\lambda v^2\right)\hat{y}$

$$= \frac{q\lambda}{2\pi r}\left(\frac{1}{\epsilon_0} - \mu_0 v^2\right)\hat{y}$$

ز) $\frac{F}{F_E} = \frac{\frac{q\lambda}{2\pi r\epsilon_0}\left(1-\mu_0\epsilon_0 v^2\right)\hat{y}}{\frac{q\lambda}{2\pi r\epsilon_0}} = 1-\mu_0\epsilon_0 v^2$

3)

الف) $F = +NK(\Delta x - \Delta z)$

ب) $\Delta x = \frac{L}{N} \rightarrow N = \frac{L}{\Delta x}$

$F = +NK(\Delta x - \Delta z) = + \quad LK + \frac{LK\Delta z}{\Delta x}$

$\lim_{n \to \infty} F = -KL\left(\frac{dz}{dx} - 1\right)$

ج) $(L - x)\omega = -KL\left(\frac{dz}{dx} - 1\right)$

$\frac{dz}{dx} = -\frac{(L-x)\omega}{KL} + 1$

$\rightarrow \int dz = \int \left(-\frac{L-x}{KL}\omega + 1\right) dx$

$z = \left(-\frac{\omega}{K} + 1\right)x + \frac{\omega x^2}{2KL}$

د) $x = L \rightarrow z(L) = \left(-\frac{\omega}{K} + 1\right)L + \frac{\omega L^2}{2KL} = L\left(1 - \frac{\omega}{2K}\right)$

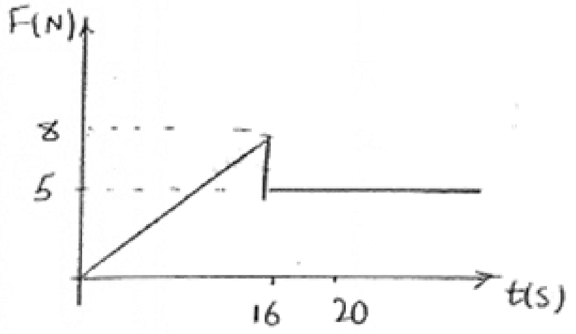
$\Delta = L - z(L) = \frac{\omega}{2K}L$

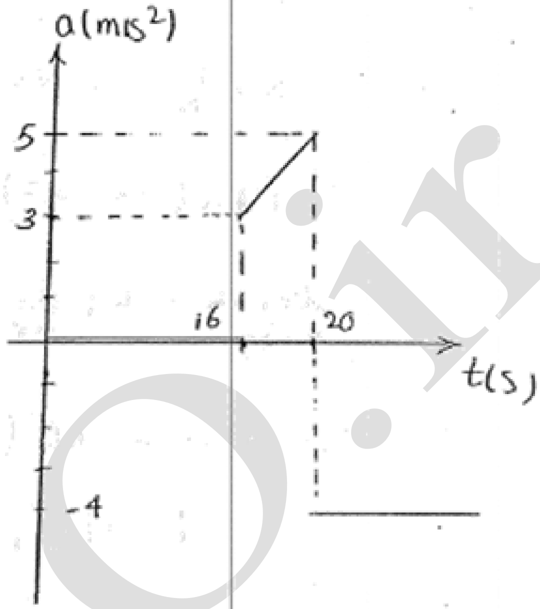
ه) $\omega L = K\Delta L \rightarrow \Delta L = \frac{\omega L}{K}$

4)

الف) $f_s = 0.8 \times 1g = 8N$

$8N = \alpha t \Rightarrow t = 16s$





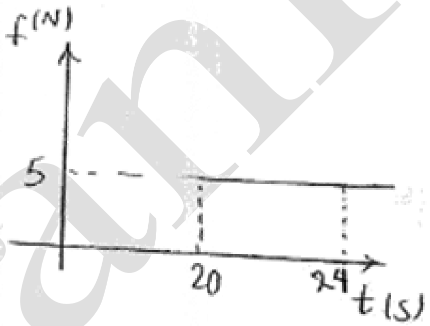
شتاب از ثانیه 16 تا 20 ، $a = \alpha t - 5$

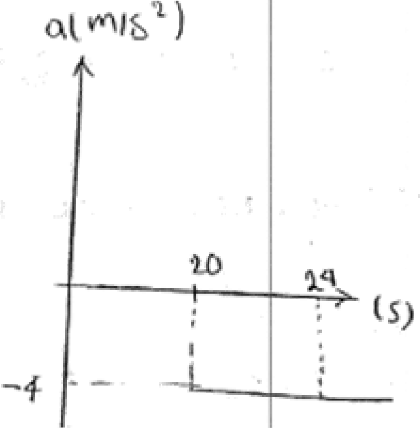
شتاب بعد از ثانیه 20 ، $a = 1 - 5 = -4\ m/s^2$

ب) $V = \frac{(5+3)\times 4}{2} = 16\ m/s$

ج) $16 = 4(t-20) \Rightarrow t = 24s$

د)





5)

الف) برای هر همسایگی به هر مولکول $-\frac{U}{2}$ انرژی می‌رسد

$$\Rightarrow E = -\frac{NU}{2}$$

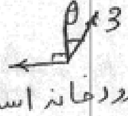
ب) $TS = -aSn\left(-\frac{NU}{2}\right) \Rightarrow \tau = \frac{anNU}{2}$

ج) $Q = -m \times \frac{1}{f} \times n \times \left(-\frac{NU}{2}\right) = \frac{mnNU}{2f}$

د) $\begin{cases} 7\times10^{-2} = \frac{anNU}{2} \\ 2000\times10^{3} = \frac{nNU}{2f} \end{cases} \Rightarrow 7\times10^{-2} = 2000\times10^{3}\times10^{3}\times a$

$a = 3.5\times10^{-11}\ m$

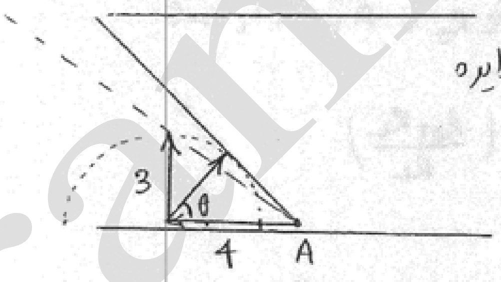
6)



$3\sin\theta = 2 \Rightarrow \theta = \arcsin\frac{2}{3}$

الف) چون که کوتاه‌ترین مسیر، مسیر مستقیم است که عمود بر رودخانه است.

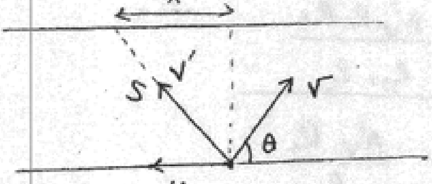
ب)



راه اول:

کوتاه‌ترین مسیر به ازای مماسی است که بر نیم دایره به شعاع 3 برخورد می‌کند

$\theta = \arccos\frac{3}{4}$



راه دوم:

$S^2 = x^2 + d^2$

$\Rightarrow \frac{dS}{d\theta}\times 2S = 0 \Rightarrow 2x\frac{dx}{d\theta} = 0$

$\Rightarrow \frac{dx}{d\theta} = 0$

$v' = \sqrt{v^2+u^2-2vu\cos\theta}$

$\left.\begin{array}{l} u - v\cos\theta = \frac{x}{t} \Rightarrow (u - v\cos\theta)t = x \\ (v\sin\theta)t = d \Rightarrow t = \frac{d}{v\sin\theta} \end{array}\right\} \Rightarrow x = \frac{(u - v\cos\theta)d}{v\sin\theta}$

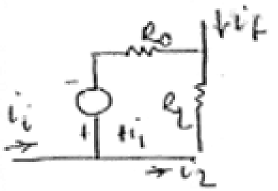
$\Rightarrow \frac{dx}{d\theta} = 0 \Rightarrow v^2 d\sin^2\theta - vud\cos\theta + v^2 d\cos^2\theta = 0 \Rightarrow v = u\cos\theta$

7) الف) $A_f = \frac{x_o}{x_s}$ $A = \frac{x_o}{x_i}$ $x_i = x_s - x_f$

$$x_i = \frac{x_o}{A} \to \frac{x_o}{A} = x_s - \beta x_o \to x_s = x_o\left(\beta + \frac{1}{A}\right)$$

$$\to A_f = \frac{A}{1+\beta A}$$

ب)



$x_f = \beta x_o$ $V_i = i_i R_i$

$i_f = \beta V_o$

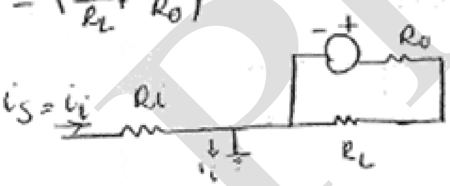
$A'_v V_i + R_o i_1 = V_o$

$$i_f = -\left(\frac{V_o}{R_L} + i_1\right) = \beta V_0$$

$$\Rightarrow -\left(\frac{V_o}{R_L} + \frac{V_o - A'_v V_i}{R_o}\right) = \beta V_o \Rightarrow \beta = -\left(\frac{1}{R_L} + \frac{1}{R_o}\right) + \frac{A'_v}{R_o} \cdot \frac{V_i}{V_o} \ll 1$$

$$\Rightarrow \beta = -\left(\frac{1}{R_L} + \frac{1}{R_o}\right)$$

ج) در



$$\begin{cases} A_f = \frac{A}{1+\beta A} \\ \beta = -\left(\frac{1}{R_L} + \frac{1}{R_o}\right) \end{cases}$$

$A = \frac{V_o}{i_i}$ $-V_o - \frac{V_o}{R_L} R_o + A'_v R_i i_i = 0$

$$A'_v R_i i_i = V_o\left(\frac{R_o + R_L}{R_L}\right)$$

$$A = \frac{A'_v R_i R_L}{R_o + R_L}$$

$$A' = \frac{A}{1+\beta A} = \frac{\frac{A'_v R_i R_L}{R_o + R_L}}{1 - \frac{A'_v R_i}{R_o}}$$