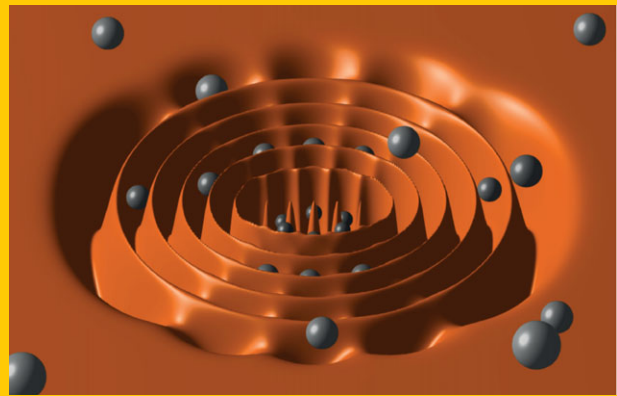


**Abstract** Optical tweezers, a simple and robust implementation of optical micromanipulation technologies, have become a standard tool in biological, medical and physics research laboratories. Recently, with the utilization of holographic beam shaping techniques, more sophisticated trapping configurations have been realized to overcome current challenges in applications. Holographically generated higher-order light modes, for example, can induce highly structured and ordered three-dimensional optical potential landscapes with promising applications in optically guided assembly, transfer of orbital angular momentum, or acceleration of particles along defined trajectories. The non-diffracting property of particular light modes enables the optical manipulation in multiple planes or the creation of axially extended particle structures. Alongside with these concepts which rely on direct interaction of the light field with particles, two promising adjacent approaches tackle fundamental limitations by utilizing non-optical forces which are, however, induced by optical light fields. Optoelectronic tweezers take advantage of dielectrophoretic forces for adaptive and flexible, massively parallel trapping. Photophoretic trapping makes use of thermal



forces and by this means is perfectly suited for trapping absorbing particles. Hence the possibility to tailor light fields holographically, combined with the complementary dielectrophoretic and photophoretic trapping provides a holistic approach to the majority of optical micromanipulation scenarios.

## Advanced optical trapping by complex beam shaping

Mike Woerdemann\*, Christina Alpmann, Michael Esseling, and Cornelia Denz

### 1. Introduction to optical trapping

It was as early as 1619 that Kepler suggested that light radiation can exert forces on particles of matter, explaining the observation that the tail of comets in general points away from the sun [1]. With the electromagnetic theory by Maxwell, a natural description of light waves was achieved which does not only associate energy with the wave but also linear momentum [2]. Subsequent observations with elaborate experimental apparatus proved the existence of radiation pressure qualitatively [1, 3] and quantitatively [4]. Ehrenhaft, finally, showed theoretically and experimentally that, in addition to the direct transfer of momentum from a light wave to matter, secondary effects can become important [5]. While the majority of classical optical trapping concepts rely on direct light forces, important developments in the utilization of secondary effects as dielectrophoresis or photophoresis have been demonstrated recently.

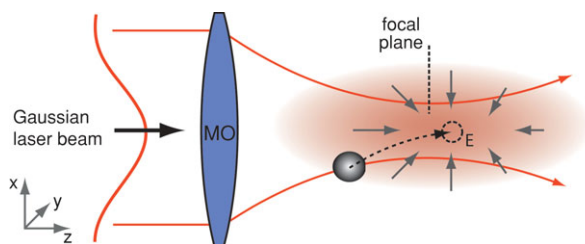
The actual field of optical trapping was initiated about 40 years ago by Ashkin with his seminal paper on “acceleration and trapping of particles by radiation pressure” [6]. In this early paper, the author proposed and demonstrated different basic concepts of optical trapping which are still important for recent applications. Besides the optical guiding of microscopic particles with a laser beam, he developed the basic concept of counter-propagating optical trapping,

where the opposed radiation pressure of two beams leads to the stable three-dimensional confinement of particles [6]. Shortly afterwards, other stable optical traps were demonstrated, including the optical levitation trap where gravitational forces counteract the radiation pressure [7]. Early applications of optical trapping include the investigation of resonant modes in optically levitated microspheres, a modern and extremely precise version of the Millikan experiment, and the guiding and focusing of atomic beams, just to mention a few [8].

A milestone in optical trapping was the discovery that even a single, tightly focused laser beam can create a stable optical trap for microscopic, transparent particles. This originally termed “single beam gradient force trap” [9] nowadays is widely known as *optical tweezers* (cf. Fig. 1). The emergence of optical tweezers, and the fact that they can be integrated into almost any optical microscope, has leveraged optical trapping so that it is now available as a routine tool in many laboratories of various disciplines. Optical tweezers have found a huge range of applications in biophotonics and biomedicine [10], including the manipulation of biological cells [11], force measurements in general [12, 13] and the investigation of molecular motor properties in particular [14, 15]. Other important applications of optical tweezers [16–22] can be found in colloidal sciences [23], microfluidics [24],

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**Figure 1** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Basic principle of optical tweezers. A laser beam is tightly focused through a microscope objective (MO) so that the axial gradient forces on a transparent particle due to the intensity gradient surpass the scattering forces along the optical axis. In transverse direction, gradient forces point towards the beam axis, resulting in a stable three-dimensional equilibrium position (E) for the particle. From [22]. Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission.

microscopic particle alignment [25–27], or particle sorting [28, 29]. Moreover, optical tweezers can be used to address fundamental physical questions concerning light-matter interaction [30–32], hydrodynamic forces [33, 34], or Brownian motion at very short time scales [35].

In the present paper, in Section 2, a concise overview of established micromanipulation concepts utilizing different beam patterning approaches that already go far beyond basic optical tweezers is provided. The state-of-the-art of complexly patterned light fields and their applications in advanced optical trapping will be reviewed in Section 3, where also the important concept of optical potential energy landscapes is introduced. While a majority of applications relies on direct transfer of light forces to matter, the second emphasis within this paper will be on applications where the indirect exertion of forces via dielectrophoresis and photophoresis is employed (Section 4).

## 2. Beyond standard optical tweezers

The basic concept of optical tweezers is versatile and flexible. However, with the emergence of more advanced application scenarios came a strong need for more sophisticated optical trapping concepts. These application driven needs are met by a vast number of innovations, ranging from optimizations of the mechanical and optical layout of the actual optical tweezers to ingenious automation schemes, including time-shared optical tweezers [36] as an important example. Of particular importance and versatility, however, are the various approaches that involve structuring of the light field that is utilized for optical trapping [21]. Enabled by the outstanding flexibility of holographic beam shaping, suitably shaped light fields can be tailored to specific applications [37], ranging from multiple optical traps over continuous potential landscapes to optical traps that can transfer optical angular momentum.

In particular the concept of optical angular momentum, which is closely related to vortices in the light field and different polarization states, has developed into an advanced

field on its own which is beyond the scope of this paper but excellent recent reviews are available elsewhere [27, 38]. Except for a very few examples, also the whole mature field of biological applications will be neglected in this paper because it has been intensively reviewed from various perspectives, for example in [10, 12, 13].

In this section, the focus will be on holographic and non-holographic beam patterning methods commonly employed in optical micromanipulation, using the examples of holographic optical tweezers and phase-contrast methods. Furthermore, with counter-propagating geometries, a complementary approach for the creation of complex, three-dimensional light patterns and its applications in optical micromanipulation are reviewed.

### 2.1. Holographic optical tweezers

The most obvious further development of single optical tweezers is a system that enables the simultaneous use of multiple optical traps, ideally steerable independently from each other. Besides the time-shared scheme [36], holographic beam shaping is an elegant approach to this extension [22, 39, 40]. The basic idea of *holographic optical tweezers* (HOT) is that a computer-generated hologram (CGH) can be utilized to generate a light field that focuses to a number of individual spots at defined three-dimensional positions within the sample volume [41]. Each spot corresponds to an optical trap and the number and position of the trapping sites can be changed by a suitable change of the CGH [39, 40, 42].

Hologram design for HOT can be challenging and highly sophisticated. The basic idea, however, is simple. It is well known from diffraction gratings that they diffract a laser beam into higher diffraction orders. Different orders correspond to different solid angles and – if the diffraction grating is positioned in a Fourier plane with respect to the focal plane of a lens – to particular transverse positions of the focal spots. The task to address is thus to design a complex diffraction grating, i. e. a CGH, which has a defined number of diffraction orders, which correspond to the same number of individual optical tweezers. Each diffraction order has a defined diffraction angle, which corresponds to the transverse position of the tweezer, and defined divergence properties, which correspond to the axial position of the tweezer. An intuitive and practical algorithm for the design of CGH for HOT is the superposition of diffractive blazed gratings and Fresnel lenses so as to create a single optical tweezer at a defined three-dimensional position [43]. For multiple traps, the phase argument of the complex sum of a corresponding number of thus prepared diffraction gratings is taken as the CGH [44]. This intuitive algorithm outputs CGHs which can generate arbitrary numbers of optical traps at different positions in the vicinity of the focal plane. The simplicity, however, comes at the cost of low control of the hologram quality. Depending on the symmetry of the trapping geometry, homogeneity of the traps can be poor and also additional undesired traps, known as ghost traps, can appear at symmetry dictated positions [45, 46].

There are many modifications to the basic lenses and gratings algorithm, including phase randomization, spatial disordering [45], binarization [46], and random mask encoding [47]. Also, direct search algorithms have been exploited successfully [48]. The most flexible way to design CGHs, however, relies on iterative Fourier transformation algorithms (iFTAs) [49,50]. These kind of algorithms propagate the light field numerically (using Fourier transformations) between the focal plane where the optical tweezers are supposed to appear and the hologram plane. In each plane the individual constraints, such as the trapping light pattern, the shape of the illumination beam, possible pixelation, discretisation of the phase levels, but also desired homogeneity of the traps or suppression of ghost traps can be accounted for [51].

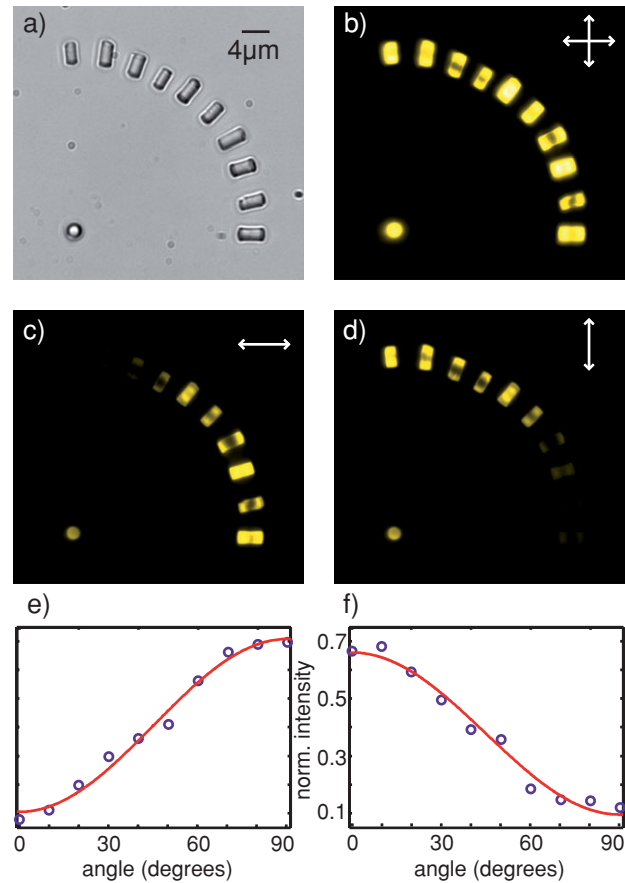
While an optimal hologram can create almost any arbitrary number of high quality but static optical traps, dynamic applications require the CGH to be updated with any change of the desired trapping geometry. For this purpose, usually computer-addressable phase-only spatial light modulators (SLMs) are employed [52]. Frequently used phase-only SLMs consist of a two-dimensional pixel matrix of individually addressed liquid crystal cells. Each pixel can retard the optical phase between 0 and  $2\pi$ , according to the calculated CGH, and thus the wave front can be tailored dynamically with the SLM's refresh rate [53]. Using an SLM, it is even possible to correct for aberrations in the optical path by suitable pre-shaping of the wave front [54–56].

The emergence of HOT was early perceived as “a revolution in optical manipulation” [17] and has enabled a number of exciting applications, including the arrangement and orientation of non-spherical objects [27], operating of and sensing in lab-on-chip devices [24], the assembly of complex structures from microscopic building blocks [58], and the experimental modeling of infection scenarios at the single cell level [59], just to name a few.

Figure 2 shows an example where elongated microscopic container-particles are employed as building blocks to construct complex, functional structures with HOT [57]. As these particles are not spherical but rather cylindrical, a scheme of multiple holographically generated optical tweezers can be used to position, rotate and orient the particles at will in three dimensions [26,60]. The container-particles are loaded with dye molecules that align strictly parallel within the nano-cavities of the particles. Using a microfluidic environment that defined the optical chemical conditions, the particles are attached to a glass substrate in the desired structure. The resulting structures can have exciting functionalities that are only enabled by the strict alignment of the molecules within the container-particles combined with the highly defined relative orientations of the container-particles themselves.

## 2.2. Phase-contrast approaches

An alternative approach for the patterning of light fields, complementary to holographic beam shaping, is the structuring in an image plane of the optical trapping plane rather



**Figure 2** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) A sophisticated assembly of microscopic container particles loaded with fluorescent dye molecules. The building blocks were aligned and assembled with HOT within an optical tweezers assembly line (a). The luminescent response of the structure is sensitive to the polarisation state of the excitation light. While with unpolarized light all particles show fluorescence (b), with linearly polarized light only particles with particular angles respond (c,d). The intensity response can be quantified (e,f), suggesting applications as microscopic polarisation sensor. From [57]. Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission.

than a Fourier plane. This approach has the clear advantage that no calculation of any CGH is necessary at all [61]. The desired trapping geometry is displayed by the phase-only SLM – which is placed in a conjugate plane of the optical trapping plane for this purpose – and a subsequent phase-contrast filter transfers the phase pattern into a corresponding intensity pattern of optical traps. The original scheme for the phase-to-intensity transfer was termed *generalized phase contrast* and is a quantitative extension of the well-known Zernike phase contrast [62]. In terms of dynamic capabilities, the phase-contrast approach is conceptually superior to HOT [63] even though the real-time computation of high quality CGHs is usually not an issue anymore with modern computer hardware [64]. Moreover, many possible drawbacks of holographic beam shaping approaches are avoided completely, including

zeroth order losses, ghost traps, and inhomogeneities [65]. Axial re-positioning of individual optical traps, however, is not straight-forward with this approach which is why it is usually operated in counter-propagating geometry (see below) in order to achieve full three-dimensional control [66]. Furthermore, the overall efficiency can be low compared to HOT if only a small number of traps are to be generated [65]. Based on the phase-contrast approach, highly sophisticated, multi-functional micromanipulation workstations have been developed and employed, for example, for the optical control and investigation of tailored two-photon polymerization microstructures [67].

### 2.3. Counter-propagating light fields

In contrast to optical tweezers, counter-propagating (CP) optical traps – where two laser beams propagating in opposite directions overlap – are slightly more complex to implement but they also allow for trapping geometries that are not accessible with single beam approaches [21]. First, in CP optical traps the scattering force in direction of beam propagation is canceled out. This enables to employ focusing lenses with lower numerical aperture and thus higher working distances. And, second, the counter-propagating beams can be mutually coherent and thus enable the induction of highly complex intensity patterns [68].

Although it was a counter-propagating geometry that provided the very first all-optical trap [6], the basic concept still is of topical interest and has been refined and extended, in particular in the last decade. One example for a sophisticated implementation of CP traps is the *optical stretcher* [69] which is implemented with two opposing optical fibers. The light beams exiting both fibers overlap and form a CP optical trap. In this trap, biological cells are not only confined, but they are also stretched by radiation forces. The degree of stretching depends on the laser power and the compliance of the cell, which makes it possible to differentiate between different cell types in a high-throughput system.

Owing to the relatively low numerical apertures required for CP optical traps, there is a high potential for larger working distances, a larger field of view, and lower light exposure of the sample. These properties can be highly advantageous, in particular for biological samples, which can undergo photo-damage in intense light fields [70]. Biological applications also might require a high penetration depth or large field of view if, for example, observed organisms are larger than a few micrometers or highly motile, respectively [71]. Moreover, larger working distances are mandatory for a side view of the trapped particles which is advantageous for many applications [72, 73].

In terms of complex beam patterning, the possibly most important advantage of the CP optical trapping geometry is that the available wave vectors which can be utilized to pattern the light field interferometrically include the whole sphere of solid angles [74]. This is in contrast to the case of any single beam optical trap where only a hemisphere

of wave vectors can be used. Moreover, the two CP beams can be mutually coherent, incoherent, or partly coherent, opening up a whole range of opportunities for the complex three-dimensional patterning of the light field and exciting applications. Examples include optical trapping and sorting in standing wave traps [29, 75], or the realization of an optical conveyor belt [76] that transports particles along the optical axis.

A potential challenge of CP optical trapping geometries can be their relatively high requirement for accurate mutual alignment and mechanical stability of the system – compared to single beam approaches [66]. This challenge has been addressed by automated alignment schemes [77] and different concepts for the generation of the “back-propagating” wave from the (single-beam) incident wave. One ingenious realization is the “holographic twin trap” where two co-propagating beams are generated by holographic beam shaping in such a way that they focus at different axial positions. Back-reflections from the rear surface of the sample chamber – which often is specially coated for this purpose – can then be employed to overlap the two foci in CP geometry [71, 78]. A complementary approach which does not require specially prepared samples is the utilization of optical phase-conjugation in order to create CP beams almost automatically at the cost, however, of less control on the relative position of the two foci [79]. This approach can be used with complexly patterned input light fields [80] and by this means enables the creation of highly interesting three-dimensional intensity patterns for optical trapping [81].

There are many other concepts for the patterning of light fields which we cannot even touch within the framework of this concise review, including evanescent field traps [30, 82, 83], plasmon-based optical trapping [84], Talbot traps [85] using self-imaging [86], or specially manufactured lenses [87]. In the following, the emphasis will be on the generation of complex light patterns, mainly with advanced holographic methods which probably is the most versatile approach and can be combined with many established concepts.

## 3. Tailored optical energy potential landscapes

Beside classical HOT, the field of optical potential landscapes evokes an upcoming interest on topical research in optical micromanipulation. The idea of molding energy potential landscapes with two-dimensional or even three-dimensional extent in space gives an alternative approach for organized arrangements or the manipulation of complex-shaped particles.

The transition from multiple holographic point traps to optical potential landscapes is a smooth one, but underlying techniques differ essentially. While classical HOT already allow for the handling of multiple particles into two- and three-dimensional crystal-like structures [46, 58], the creation of differently shaped traps offers even more



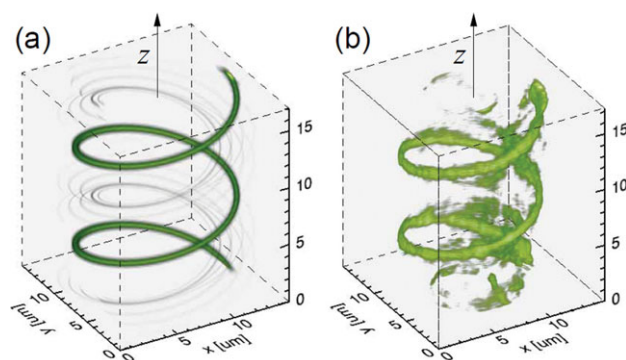
possibilities, as they can be tailored to individual applications or, for example, reveal complex dynamic properties [88]. As counterpart to point-traps, one-dimensionally extended optical traps – so called line tweezers – provide a highly asymmetric optical potential landscape where trapped objects are free to move in one transverse direction while they are confined in the other [89].

The technique of shape-phase holography has been proposed to reach independent control over both the intensity and phase distribution of a line trap [90]. This advantage in comparison with earlier used interferometric [91] or cylindrical lens techniques, avoids astigmatism and allows for three-dimensional trapping in a single-beam approach. Later on, efficient methods for the creation of tunable optical line traps by control of gradient and scattering forces have been demonstrated [92,93]. Line tweezers find many applications in optical micromanipulation connected with the investigation of light-particle or particle-particle interactions. Especially the orientation of elongated particles or nanorods [94] and the phenomenon of optical binding [30,95] are of interest.

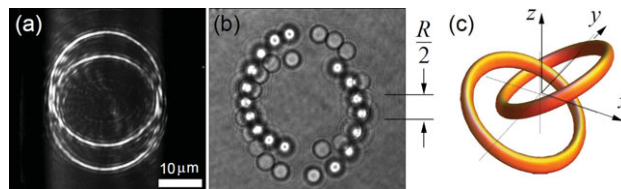
Different configurations and methods employing complex-shaped beams have been investigated, most importantly the utilization of interference to create new optical potentials [85,96]. The interference of an annularly shaped laser beam with a reference beam [97], for example, or a holographic micro-projection method [98] allows for the controlled rotation of optically trapped objects in a spiral intensity pattern. This idea has been developed even further to create three-dimensional optical twister structures which are phase-engineered relative to one another [99]. In this configuration, tunable helically stacked multi-layered micro-rotors have been realized. Highly interesting potential landscapes also can be realized by the superposition of incoherent beams [88].

In general, the aim is to go from two- to three-dimensional optical potential landscapes which consist of continuous potentials opposed to discrete potentials of classic HOT. In particular, defined propagation properties of the light field, in the sense of known or even controllable propagation, is of highest interest for three-dimensional particle manipulation. For example, three-dimensional ring like structures, which have sufficiently strong axial intensity gradients to trap objects in three dimensions, have been implemented with the before mentioned technique of shape-phase holography [101]. Furthermore, a three-dimensional intensity distribution where the intensity peak spirals around the optical axis, and whose wavefronts carry an independent helical pitch, has been introduced as optical solenoid beam [100]. Figure 3 shows the theoretical and experimentally realized intensity distribution of such a beam. Solenoid beams can trap microscopic objects in three dimensions while their phase-gradients can drive the particle around the spiral. Resulting forces can be used to transport trapped objects not only down the optical axis but also up [100].

If additionally an inclination of the ring like structures is applied, even more sophisticated optical potentials become possible [102]. For example, a pair of interlocking bright



**Figure 3** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) (a) Calculated three-dimensional intensity distribution of a solenoid beam propagating in the  $z$  direction. (b) Volumetric rendering of the measured intensity in an experimental realization. From [100].



**Figure 4** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) (a) Intensity in the focal plane of the microscope of two tilted ring traps projected simultaneously with opposite inclination, (b) Colloidal silica spheres trapped in three dimensions within the focused Hopf link. (c) Schematic representation of the three-dimensional intensity distribution responsible for the image in (a). From [102].

rings in the form of a Hopf link has been created. Colloidal silica spheres are organized in these interpenetrating rings which still act as three-dimensional optical traps, as shown in Fig. 4. Remarkably, the trapped spheres may pass freely past each other along the knotted rings [102].

Beside an application-oriented construction of complex-shaped three-dimensional optical potentials as described before, it is desirable to find a possibility to classify the optical potentials themselves in some way. One option is to categorize them by their propagation properties as this is the characteristic property which differs from classical HOT. As mathematical solutions of the (paraxial) Helmholtz equation offer known propagation properties and by their higher-order light modes build different classes of transverse modes, their classification can be adapted for three-dimensional optical potential landscapes. We will concentrate on higher-order light modes in the following section.

### 3.1. Higher order light modes

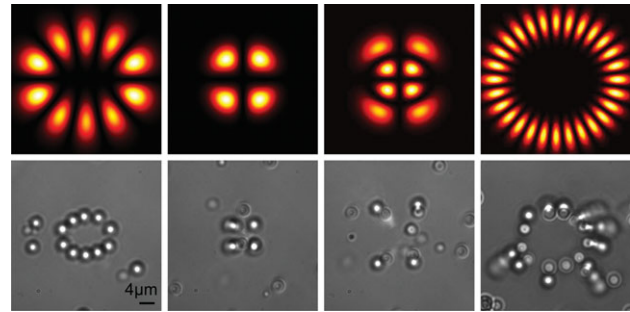
The fundamental Gaussian laser beam, as basis for most laser experiments, is the zeroth-order beam of the class of self-similar beams, also generally denoted as Gaussian beams. Although the fundamental Gaussian beam is the

most common representative, the whole beam class consists of an immense number of higher-order modes featuring highly interesting transverse beam profiles which can be employed to form extended optical potential landscapes. At the same time, they provide defined propagation behavior, resulting in extended and continuous longitudinal potential landscapes, making them attractive for three-dimensional applications. Mathematically, these beams can be described as solutions of the paraxial Helmholtz equation, which is solvable in different geometries. This is also the reason why self-similar beams are solutions of the resonator problem and hence often available directly from a laser source [103, 104].

The family of Hermite-Gaussian (HG) beams is given by solutions of the paraxial Helmholtz equation in transverse Cartesian coordinates. In the field of optical micromanipulation, these modes have been used, for example, to align non-spherical particles [105] and are promising candidates for the creation of highly ordered, extended potential landscapes [106].

Laguerre-Gaussian (LG) beams, contrary to HG beams, are solutions in transverse polar coordinates, leading to radially symmetric transverse beam profiles. The reason, why especially helical LG beams are of highest interest in optical micromanipulation, is not only the improvement of the axial trapping efficiency compared to a fundamental Gaussian beam, if an LG laser mode is used as trapping beam [107, 108]. Even more significant for the application in optical micromanipulation is the ability of helical LG beams – so called vortex beams – to transfer orbital angular momentum from light to matter [31]. From this feature many applications have emerged, including three-dimensional particle transport [109] or microfluidic applications where artificial micro-machines are driven [110]. The fundamentals of optical angular momentum and the whole range of state-of-the-art applications in optical micromanipulation have been reviewed recently [27, 38].

Beside single LG modes, also superpositions of two or more modes have been employed. The three-dimensional optical potential landscapes with radial symmetry, generated by the interference of two co-propagating LG beams, have been applied to optical micromanipulation [111]. Coherent counter-propagating LG beams have been proposed for optical trapping [112]. With the optical “cogwheel”-tweezers, which are collinear superpositions of two helical LG beams of equal but opposite helical index, size selective trapping has been shown [113]. The light intensity of the cogwheel is periodically modulated around the circumference of a sphere with a precisely adjustable diameter. Even three-dimensional movements can be induced by a single-beam trap given as a collinear superposition of LG modes [109], where microparticles follow different trajectories of the formed optical funnels. Solutions of the paraxial Helmholtz equation are not restricted to Cartesian and polar coordinates but also can be derived in elliptical symmetry. These solutions build the third family of self-similar beams and are known as Ince-Gaussian (IG) beams [115]. They have the ellipticity as an additional parameter and therefore join all favorable properties of HG and LG beams

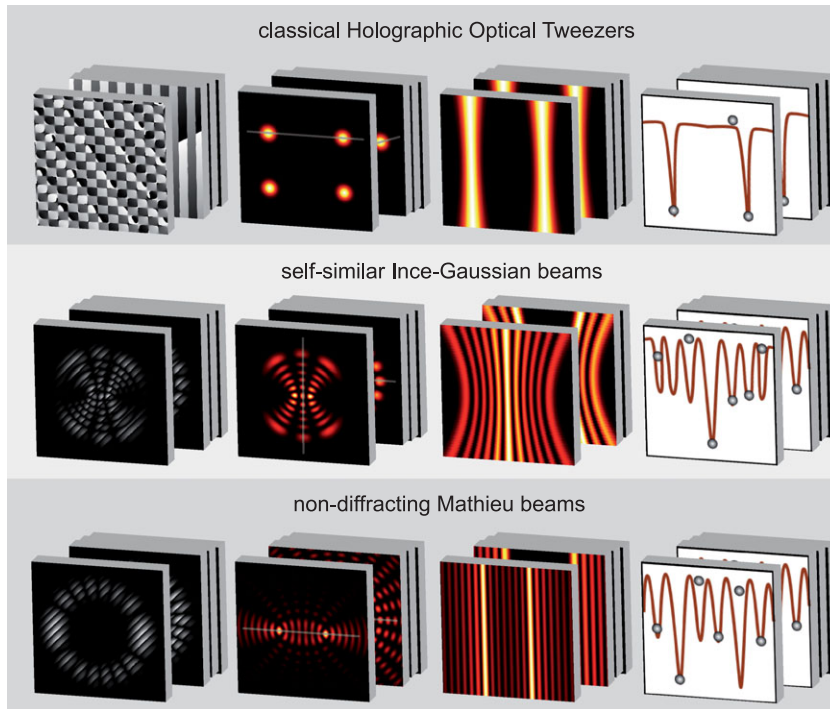


**Figure 5** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Silica spheres optically assembled into microstructures (bottom) and corresponding IG intensity pattern (top). Fourth column shows a three-dimensional optical particle structure including the phenomenon of optical binding. Reprinted with permission from [114]. Copyright 2011, American Institute of Physics.

to a more general class. For zero and unity ellipticity, IG beams become equivalent to LG and HG modes, respectively [22]. Their potential to optical micromanipulation has been demonstrated recently as depicted in Fig. 5 [114]. Their even higher amount of transverse modes combined with the well-known propagation properties of Gaussian beams, makes them attractive for future applications. The subset of helical IG beams offers non-circular but elliptical intensity distributions combined with multiple phase singularities aligned on one axis, which might predestine them for investigations of spatial behavior of phase singularities. In addition to the intensity pattern and the phase front features, also the polarization of the light field can be tailored [116], enhancing the possible complexity of the optical potential landscapes even more. Figure 6 gives an overview of typical optical potential landscapes, ranging from classical HOT with discrete, point-like potential wells (top row) to continuous landscapes induced by self-similar IG beams (middle row). In the bottom row, axially expanded potentials of nondiffracting beams are indicated. The potential of this highly interesting class of light fields will be discussed in the following section.

### 3.2. Nondiffracting beams

Every electromagnetic wave field underlies diffraction during propagation, which results in a transverse spreading of the field distribution. However, there are beam classes which maintain their profile and appear as propagation invariant beams. These beams became famous as nondiffracting or diffraction-free beams [117]. Their transverse intensity profile is defined by interference of plane waves whose wave vectors have the same projection to the optical axis [118], which means that both their structure and their spatial extent is maintained during propagation [119]. Mathematically, nondiffracting (ND) beams can be described as particular solutions of the Helmholtz equation. If we consider general cylindrical coordinates  $\vec{r} = (u_1, u_2, z)$ , a ND beam separates in a transverse part ( $\varphi_t$ ) which is independent



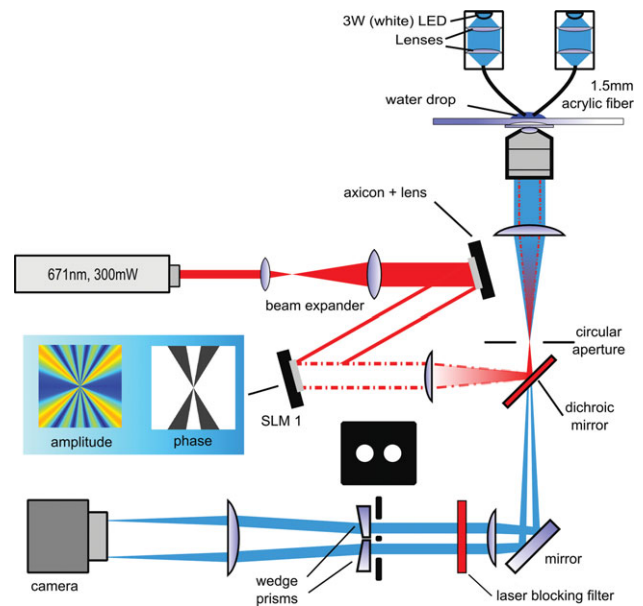
**Figure 6** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Overview of key properties of holographically shaped light fields. From left to right, typical holograms, transverse intensity distributions, axial intensity distributions (obtained along indicated transverse line), and transverse optical potential landscapes are shown. From [37]. Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission.

of the propagation direction  $z$ , and a propagation function ( $\varphi_p$ ):

$$\varphi_{ND} = \varphi_t(u_1, u_2) \cdot \varphi_p(z) = \varphi_t(u_1, u_2) \cdot e^{ik_z z}, \quad (1)$$

with the axial wave vector  $k_z$ . Nondiffracting beams have been studied in different configurations. In the context of optical micromanipulation, first experiments were realized with Bessel beams [120, 121], which provide radial-symmetric trapping potentials and allow for simultaneous micromanipulation in multiple planes [122]. As solutions of the Helmholtz equation in polar coordinates, they possess a whole class of higher-order modes, which are typically indicated by one integer. Additionally, their higher-order modes carry optical orbital angular momentum [123, 124], which can be transferred to trapped particles, similar to LG beams. Bessel beams are already wide spread because they can easily be generated with conical lenses, so called axicons. A detailed review of applications of Bessel beams in optical micromanipulation can be found in Ref. [125].

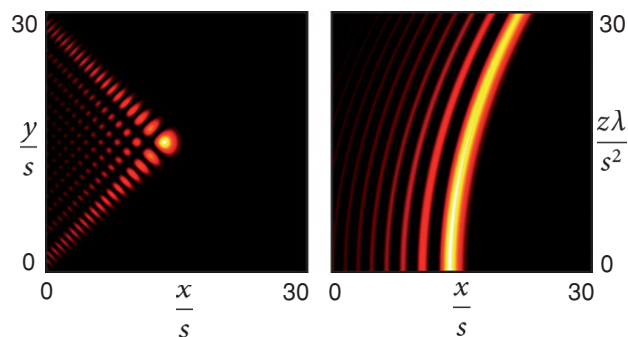
A second family of nondiffracting beams are the so-called Mathieu beams which are solutions of the Helmholtz equation in elliptical symmetry. They feature a high diversity of intensity and phase distributions, similar to Ince-Gaussian beams. Their propagation properties, however, are exactly complementary. Compared to Bessel beams, their generation is more complicated, as there are no simple refractive devices commercially available. One option is the holographic generation using an SLM. If Mathieu beams are to be integrated in an existing setup of classical HOT this might result in low diffraction efficiencies. The main reason is that nondiffracting beams form a ring of spatial frequencies in the Fourier plane that overlaps only poorly with the active area of the SLM. To overcome



**Figure 7** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Generation of optical light molds: Light field tailoring setup for propagation invariant beams in stereoscopic tweezers. From [126].

this challenge, the idea of pre-shaping the hologram illumination has been proposed, as indicated in the red beam path of the trapping laser beam in Fig. 7 [126], where two SLMs have been used. Figure 6 shows a phase hologram, the transverse beam structure and propagation properties of a higher order Mathieu beam from left to right, respectively. The ability to use the axially expanded optical potential of nondiffracting beams to stack particles in the

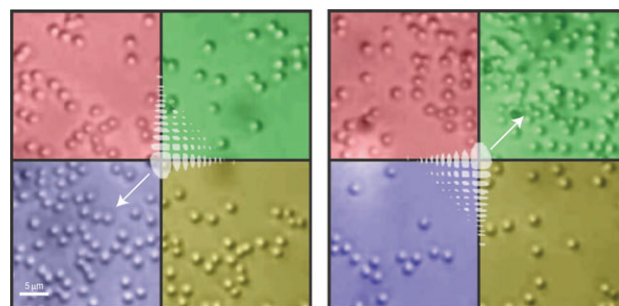




**Figure 8** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Numerical simulation of the transverse intensity distribution (left) and the intensity distribution in propagation direction (right) of a two-dimensional Airy beam. The wavelength is denoted with  $\lambda$  and  $s$  is a scaling parameter.

direction of propagation is supported by their self-healing property [127]. A nondiffracting beam which is distorted by an obstacle within the beam path, reconstructs its original transverse shape after a distinct propagation length. This feature makes nondiffracting beams highly suited for three-dimensional applications. Moreover, Mathieu beams allow to trap and orient non-spherical particles by utilizing single or multiple substructures of their variety of transverse modes. Both properties can be combined elegantly to create expanded three-dimensional particle structures [126]. To visualize those structures, a stereoscopic microscope [128] can be implemented in the optical trapping setup, as depicted in Fig. 7. This technique is one possibility to directly image three-dimensional particle structures and may be useful for any nondiffracting beam implementation. Furthermore, Mathieu beams also offer superpositions of even and odd modes, resulting in helical Mathieu beams which provide continuous phase variations, as already described for helical IG beams. Thus, these beams have also been used to investigate transfer of orbital angular momentum [123] to particles in optical micromanipulation [129].

Beside the before mentioned nondiffracting Bessel and Mathieu beams, there exist many others, including parabolic [130] as well as kaleidoscopic [131] or discrete [132] nondiffracting beams, which might also be interesting for further applications in optical micromanipulation. In recent years, Airy beams attracted interest in different fields of optics and photonics [133–136]. In contrast to the hitherto discussed *straight* nondiffracting beams, they belong to the class of *accelerating* nondiffracting beams and propagate on curved trajectories [119]. Figure 8 shows the transverse (left) and longitudinal (right) intensity distribution of a two-dimensional Airy beam, where the curved trajectory can be well observed. In optical micromanipulation, they have been proposed as “optical snow blower” due to their ability of two-dimensional particle transport effected by the transverse acceleration of their intensity profile [137]. Figure 9 shows the corresponding experiments where in the particles are accelerated from the lower left to the upper right corner and vice versa. As an alternative, the creation of



**Figure 9** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) The micrometer-scale “optical snow blower” at work. Microparticles are levitated and transported along the intensity profile of an accelerating Airy beam, from the lower left to the upper right corner (left) and vice versa (right). Adapted by permission from Macmillan Publishers Ltd: Nature Photonics [137], copyright 2008.

a curved beam trajectory also can be achieved by variations of the phase term applied to a zero-order Bessel beam and the resulting light field can be used for multidimensional particle transport [138].

## 4. Optical trapping by indirect light-particle interaction

The above described techniques all represent methods of *direct* optical trapping in the sense that matter directly interacts with the high intensity light field. Such a behavior has enabled a multitude of flexible trapping scenarios, however, there may be situations where such an interaction is prohibited by the microparticle properties or where the use of other light-induced mechanisms may be beneficial. Since the following techniques rely on secondary effects modulated through light, they will be referred to as *indirect interactions*.

### 4.1. Optoelectronic Tweezers

Possibly the most significant challenge in typical applications of optical tweezers is the fact that a strongly focused laser beam of sufficient intensity is necessary to hold particles. In cases where little laser power is available or a large field of view must be visible at once, such prerequisites may prohibit the use of (holographic) optical tweezers. As a very efficient alternative approach to optically mediated trapping, the effect of light-induced dielectrophoresis (DEP) has been employed in the development of optoelectronic tweezers (OET) [139]. A number of intriguing schemes for the all-optical addressing of microbeads had been developed before [140, 141], but none of them reached the versatility of dielectrophoretic forces, which appear instantly and are able to manipulate uncharged matter. Dielectrophoresis, a phenomenon that has been known since the late 1950s, in general describes the interaction of suspended matter with an external electric field [142].



Such a field induces multipole moments inside a particle, which can interact with the inducing field. A net force is exerted only if the external field is spatially inhomogeneous, since then the (linear) multipole experiences different magnitudes of forces on both ends. In general, the calculation of the forces that originate can be arbitrarily complex and must accurately account not only for the particle shape, but also for the inner structure, e.g. in the case of cells, which in good approximation represent multi-layered spheres [143]. Readers interested in these calculations are referred to the literature [142, 144]. However, in order to get an insight into the most interesting properties of the DEP force, it is sufficient to restrict the calculation to spherical particles and truncate the induced moments after the second term, i.e. the dipole moment, which leads to an effective force of

$$F_{\text{DEP}} = 2\pi r^3 \varepsilon_m K(\omega) \nabla E^2, \quad (2)$$

where  $r$  is the radius of the sphere,  $\varepsilon_m$  the static permittivity of the surrounding medium,  $E$  the electric field and  $K(\omega)$  the frequency-dependent *Clausius-Mossotti factor*:

$$K(\omega) = \frac{\varepsilon_p^* - \varepsilon_m^*}{\varepsilon_p^* + 2\varepsilon_m^*}. \quad (3)$$

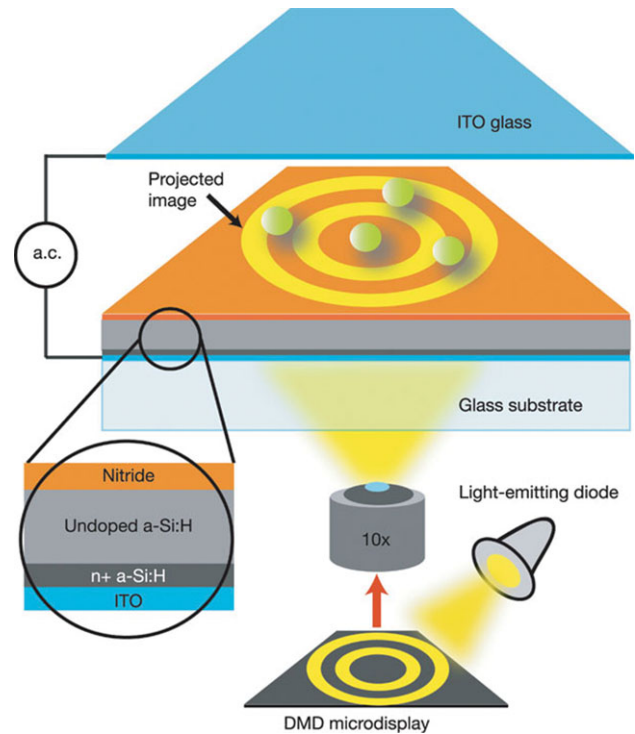
In this equation,  $\varepsilon^*$  ist the complex permittivity of particle (p) or surrounding medium (m):

$$\varepsilon_i^* = \varepsilon_0 \varepsilon_r - i \frac{\sigma_i}{\omega}. \quad (4)$$

These equations describe some of the basic properties very well:

1. Dielectrophoresis only occurs in the presence of gradients in the electric field.
2. The force scales with the volume of particles, i.e. larger particles experience a larger force.
3. It may have positive or negative values, depending on the sign of the Clausius-Mossotti factor, which is determined by the electric properties of particle and surrounding medium, hence particles are attracted to/repelled from regions of high field intensity.

A number of realizations for a dielectrophoretic system exists which make use of static electrode arrays. For further information, the reader is redirected to [145] and references therein. The revolutionary transfer of this effect to the field of optical manipulation was accomplished by the integration of a photoconductive layer into a sandwich structure of two electrodes for the external voltage supply (cf. Fig. 10 and [139]). In order to simultaneously obtain a high optical and electrical transmission, these electrodes typically consist of glass coated with indium tin oxide (ITO) to which an external voltage of varying magnitude and frequency is applied. An amplitude modulating device (for example a digital mirror device (DMD) is illuminated by a laser source and its image is demagnified onto the photoconductive layer. In the case of no illumination, the applied voltage drops homogeneously over the whole area, mostly across



**Figure 10** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) General concept of OET: A photoconductive layer is sandwiched between two ITO-coated glass electrodes, the voltage drop across the photoactive layer can be efficiently modulated using patterned illumination. Reprinted by permission from Macmillan Publishers Ltd: Nature [139], copyright 2005.

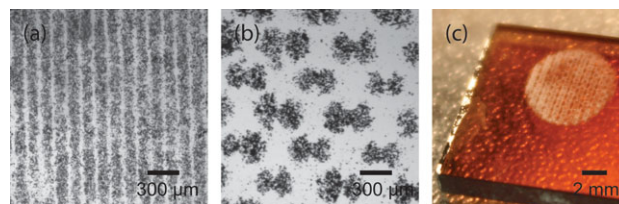
the silicon layer since, in comparison to the sandwiched liquid, it has a much higher resistance. In the case that this layer is illuminated, however, the conductivity increases by several orders of magnitude. In that case, most of the voltage drop near this *virtual electrode* occurs across the solvent, with the highest field density near the illuminated area. This inhomogeneous electric field leads to the occurrence of dielectrophoretic trapping forces. Since these are induced by laser illumination but do not directly originate from a light-matter interaction, this can be classified as indirect trapping. Using this scheme, the developers of the OET demonstrated the generation of 15.000 trapping sites on a 1.3 mm<sup>2</sup> area, each capable of attracting a 4.5 μm polystyrene bead [139]. It has been verified that the traps stiffness per mW is almost 500 times higher for OET than it is for standard optical tweezers, hence requiring only a fraction of the laser power necessary for optical tweezers [146]. OET not only enable the direct control of trapping patterns by light, but also inherit all of the features from classical DEP. This includes, for example, the frequency dependence of the Clausius-Mossotti factor, which means that for cells of different constitution one can obtain different signs of the DEP force by adjusting the external field frequency. This feature enables a very efficient differentiation, not only between live and dead human B cells [139], but also between motile and non-motile sperm cells [143].

Since the Clausius-Mossotti factor for non-spherical particles, such as rods, can be much larger than in the case of spheres, it could be shown that even nanometric wires can be arranged [147, 148]. There has also been a thorough investigation of the different physical mechanisms present in OET. For example, it should be mentioned that particle transport in an OET device is supported by light-actuated AC electroosmosis (LACE) where the electroosmotic flow along the boundary layer between liquid and photoconductor is manipulated by the illumination pattern [149]. The induced tangential flow vortices originating from LACE can be used for particle transport over much longer distances than DEP, whose magnitude decays rapidly away from the virtual electrode [150].

The first demonstration of light-induced dielectrophoresis has inspired a lot of contributions that extend the basic idea of OET and further contribute to their versatility. The main prerequisite for the construction of efficient OET is that the conductivity of the photoactive surface in the illuminated state exceeds that of the sandwiched liquid medium. For the case of a cell culture medium, such a high conductivity cannot be reached by amorphous silicon-based photoconductors, thus the use of the previously described devices is prohibited. To address this problem, the concept of OET was expanded towards photoaddressable transistors with a 500 times higher photoconductivity [151], sacrificing the featureless electrodes but winning the ability to observe and manipulate cells in their natural habitat for long times. In the struggle for an improvement of the photoconductive surface, several other materials and compositions have also been investigated such as crystalline single layer [152] or polymer coatings [153], which hold the promise of a further simplification of the production process.

Another challenge in the construction of OET is that the occurrence of forces is bound to the photoactive surface [150]. In the case of positive DEP forces, the vertical force component attracts particles towards the surface, where they may adhere due to non-specific interactions. The use of negative DEP can only partially prevent such behaviour, since it presses particles against the top ground layer where similar problems might occur. The construction of 3D OET [155], which features two equal photoconductive surfaces as the top and bottom electrodes, leads a way out of this dilemma. It enables the vertical focusing of beads and cells by means of negative DEP from both sides. Utilizing a grayscale illumination pattern rather than a binary one, the trapping strength of OET can be optically tuned while the externally applied voltage remains unchanged [156]. Taking advantage of the fact that the wettability of a surface can be changed by laser illumination [157], the two principles of OET and *optoelectrowetting* have been combined to be able to move single droplets and manipulate beads inside the liquid using the same device at different frequency ranges [158].

The vision of an easy integration of OET into small devices has motivated concepts which can possibly economize some of the equipment that is necessary for the construction of OET. Instead of using an expanded beam to illuminate a pixelated device for the modulation of the beam ampli-

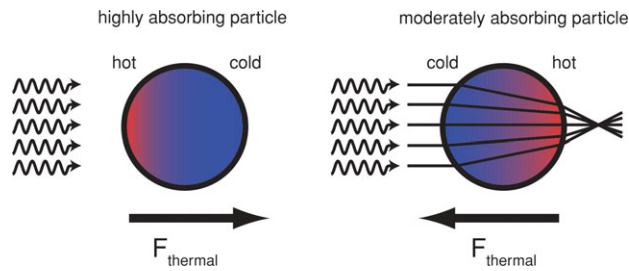


**Figure 11** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Dielectrophoretic trapping on a lithium niobate crystal: (a)-(b) microscopic images of regular patterns created by trapping graphite flakes on LN OET, (c) large area trapping of Zeolite L nanocontainers on a LN crystal; see also [154]

tude, the implementation of a pixelated LED source in the first place enabled the independent control of 64 trapping spots using a very small CMOS chip [159]. It is straightforward to envision that such a technique may in the future benefit from a higher integration level in semiconductor fabrication, which could dramatically increase the number of trapping sites available. The external voltage supply can be saved if photorefractive lithium niobate (LN) samples are used as the photoconductive material [160]. Although limited to the occurrence of quasi-DC electric fields, several interesting one- and two-dimensional applications of aligning matter [154] and liquid polymers [161] could be demonstrated. LN furthermore possesses the advantage that the magnitude of the internal fields is optically accessible by the photorefractive effect and persists for long times even if the illumination is switched off [162], which allows to structure large areas in advance (see also Fig. 11). These various implementations have made OET a very powerful alternative to optical tweezing by gradient forces, especially when an application requires simultaneous control over a very large number of objects in two dimensions.

#### 4.2. Photophoretic trapping

Although OET provide an efficient tool to confine any dielectric particle near a surface, applications that require a true three-dimensional manipulation cannot benefit from this technique. In addition, when particles under investigation possess a non-negligible absorption, the applicability of direct optical tweezing is very challenging [163]. Despite the fact that scattering forces in the axial direction will always drive particles out of a gradient trap, a very efficient way to construct optical traps for such particles is the use of the secondary effect of photophoresis [164]. Photophoresis describes the light-induced thermophoretic forces on matter surrounded by a (gaseous) medium. Thermophoresis can be explained by a transfer of momentum from the surrounding molecules to the micro-object [cf. Fig. 12]. In the case of a uniform temperature distribution around a particle, no net force is exerted since statistically all surrounding molecules impinge with the same momentum on the surface. If the particle, however, is located in a thermal gradient, the molecules on the hotter side move

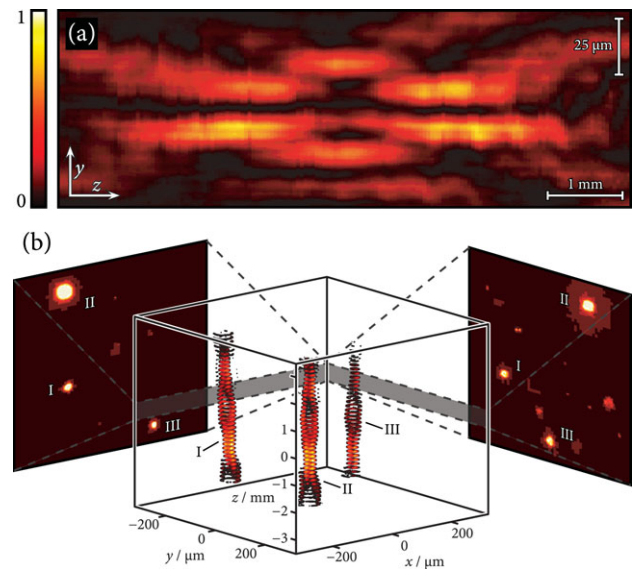


**Figure 12** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Different forms of photophoresis: Positive photophoresis for highly absorbing particles (left) drives particles away from regions of high light intensity. In the case of moderately absorbing particles (right), the photon density is higher at the back side of the particle and negative photophoresis attracts particles towards high intensity regions. Redrawn after [167].

with a larger velocity and hence transfer more momentum on the particle surface driving it towards the cooler region [165, 166].

In the case of photophoresis, this thermal gradient is induced by (laser) light. When an absorbing particle interacts with a light beam of sufficient intensity, its surface is heated up and energy is transferred to the surrounding (gas) molecules. The resulting forces can be distinguished between negative and positive photophoresis. Negative photophoresis occurs in the case of moderately absorbing particles, where light is focused at the back side of particles, driving them towards the light source [168]. However, for strongly absorbing particles, most of the light is absorbed at the front side which pushes particles away from regions of high light intensity. This positive photophoresis has recently been utilized for the three-dimensional confinement of strongly absorbing particles [169, 170]. Since the occurring photophoretic forces are orders of magnitude larger than gradient forces, tightly focused beams, as necessary for direct optical tweezers, are not required [169]. A direct result of this advantage is that also lenses with very long focal lengths may be used for the generation of a light field, enabling the photophoretic manipulation of particles over meter-scale distances, for example using CP vortex beams [171, 172]. In this geometry, two hollow beams – overlapping in the trapping volume – balance the axial transfer of momentum on the microparticle. An arbitrary one-dimensional positioning along this light tube is possible by adjusting the relative powers of the two beams [173].

Since the use of CP beams requires sophisticated adjustment processes, the creation of a photophoretic trap by a single beam is highly desirable. This method has the further advantage that it requires less optical components and saves physical space. Generally speaking, the problem can be related back to the construction of a closed hollow light field. For example, such closed regions with a minimum of the optical intensity occur naturally in a speckle field that originates from the illumination of a scattering target [174]. It has been demonstrated that such a speckle field can act as a photophoretic sieve, since the size of particles that can be trapped is limited by the characteristic size of the



**Figure 13** (online color at: [www.lpr-journal.org](http://www.lpr-journal.org)) Holographic optical bottle beams: (a) measured intensity distribution of a single optical bottle, (b) visualization of the independent trapping of absorbing matter in three individually controllable holographic bottle beams. Reprinted with permission from [170]. Copyright 2012, American Institute of Physics.

speckle minima [175]. In contrast to the arbitrary positions of minima in a speckle field, the realization of a regular pattern of trapping sites is possible by superimposing multiple diffraction orders of a grating [176].

In order to obtain a higher degree of control over the position of individual particles, beams with a single defined intensity minimum are the laser field of choice. Due to their form, with closed light walls surrounding a trapping volume, these light fields have been depicted as optical bottle beams (cf. [177] and Fig. 13). Such an intensity distribution can be produced by a multitude of methods. One of the easiest is to use aberrations of a lens, that inevitably lead to the formation of intensity minima, in which the confined particles may be robustly manipulated by moving the trapping lens [178]. To increase the flexibility even further, holographic beam shaping can be utilized to form optical bottle beams, either by a moiré technique [179] or by a convolution between trapping geometry and discrete trapping sites [170]. The latter approach has the advantage that it exploits the output from the well-known lenses and gratings algorithm. This allows a very fast calculation and an independent manipulation of several confined microparticles (cf. Fig. 13).

Apart from these demonstration of optical trapping in optical bottle beams, a hollow light field may be produced by a variety of methods, among them laser-written diffractive optical elements (DOE) [180], circular Airy beams [181] or (higher order) Bessel beams [182, 183]. Another interesting aspect stimulating research in this area is that many results accomplished in the field of photophoresis may be transferred to the field of atom trapping [184, 185],



where atoms are confined in blue-detuned laser fields of similar hollow shape [186, 187].

## 5. Summary

The complex patterning of light fields provides entirely new perspectives in advanced optical micromanipulation. While single optical tweezers can already hold and manipulate a microscopic particle, even the relatively simple beam shaping involved in classical HOT can increase the versatility of optical tweezers dramatically. Holographic beam shaping is not limited to the creation of point-like optical traps used in HOT. Using specially designed computer-generated holograms, light fields with exciting properties can be tailored. These properties include complex three-dimensional intensity distributions which can serve as sophisticated optical potential landscapes, special features of the wave front such as optical vortices, and propagation properties which can be tuned from strongly diverging to non-diffracting. Particles can be manipulated with light directly or by employing indirect interactions as dielectrophoresis or photophoresis. Optoelectronic tweezers elegantly combine the versatility of patterned light fields with the stronger forces achievable with dielectrophoretic trapping and enable the use of much lower light intensities or, alternatively, the manipulation over much larger areas, compared to optical tweezers. Photophoretic traps can manipulate absorbing particles, even in air and over macroscopic distances with microscopic precision. This is enabled by advanced beam shaping using higher order light modes with tailored propagation properties. Although the level of sophistication already is very high in the field of optical micromanipulation with specially patterned light fields, there are various white spots and even more fantastic opportunities for the future. The potential of higher order light modes with defined, complex propagation properties has only been exploited on the surface so far and many more exciting concepts and applications will certainly arise soon. In particular biological applications – a field where standard optical tweezers have revolutionized the tool set of researchers – have almost been neglected so far. Interesting developments will be seen with the utilization of indirect light forces. These new concepts address fundamental limitations of (holographic) optical tweezers and thus are perfectly complementary to them. Moreover, they can profit dramatically from the holographic patterning of the light field and from all the advances made in that rapidly developing field.

**Acknowledgements.** The authors would like to thank Patrick Rose for helpful discussions about Airy beams and for providing the numerical data shown in Fig. 8.

**Received:** 13 July 2012, **Revised:** 6 September 2012,

**Accepted:** 24 September 2012

**Published online:** 27 December 2012

**Key words:** Optical micromanipulation, optical tweezers, holographic beam shaping, structured light fields, non-diffracting beams, dielectrophoretic trapping, optoelectronic tweezers, photophoretic trapping, optical bottle beams.



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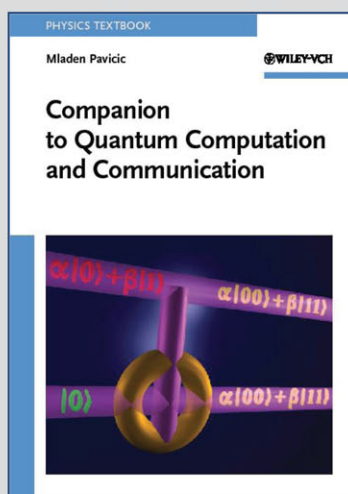
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