Chapter 9

9.1-1 This is clearly a non-stationary process. For example, amplitudes of all sample functions are zero at same instants (one is shown with a dotted line). Hence, the statistics clearly depend on t.

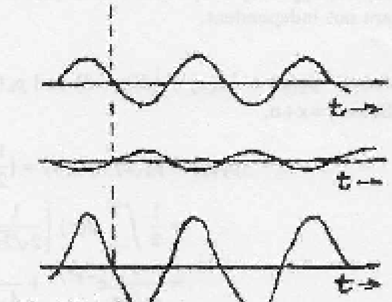


Fig. S9.1-1

9.1-2 Ensemble statistics varies with t. This can be seen by finding

$$\overline{x(t)} = \overline{A\cos(\omega t+\theta)} = A\int_0^{100}\cos(\omega t+\theta)p(\omega)d\omega$$

$$= \frac{A}{100t}\int_0^{100}\cos(\omega t+\theta)d\omega$$. This is a function of t.

Hence, the process is non-stationary.

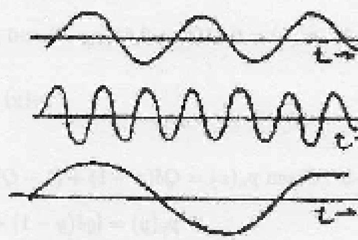


Fig. S9.1-2

9.1-3 This is clearly a non-stationary process since its statistics depend on t. For example, at $t = 0$, the amplitudes of all sample functions is b. This is not the case at other values of t.

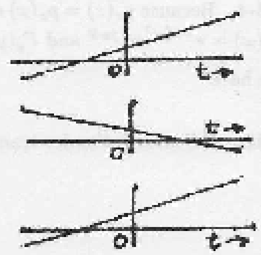


Fig. S9.1-3

9.1-4 $x(t) = a\cos(\omega t+\theta)$

$$\overline{x(t)} = \overline{a\cos(\omega t+\theta)} = \bar{a}\cos(\omega t+\theta) = \cos(\omega t+\theta)\int_{-A}^{A} a p_a(a)da$$

$$= \left[\cos(\omega t+\theta)/2A\right]\int_{-A}^{A} a\,da = 0$$

$$R_x(t_1,t_2) = \overline{a^2\cos(\omega t_1+\theta)\cos(\omega t_2+\theta)} = \cos(\omega t_1+\theta)\cos(\omega t_2+\theta)\overline{a^2}$$

$$= \cos(\omega t_1+\theta)\cos(\omega t_2+\theta)\int_{-A}^{A}\frac{a^2}{2A}da$$

$$= \frac{A^2}{3}\cos(\omega t_1+\theta)\cos(\omega t_2+\theta)$$

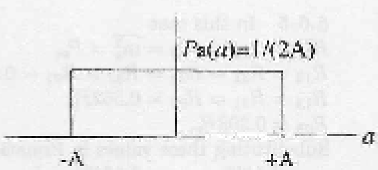


Fig. S9.1-4

9-1.5

$$\overline{x(t)} = \overline{a\cos(\omega t + \theta)} = \int_0^{100} \cos(\omega t + \theta)\, p(\omega)\, d\omega$$

$$= \frac{a}{100}\int_0^{100} \cos(\omega t + \theta)\, d\omega = \frac{a}{100t}\sin(\omega t + \theta)\Big|_0^{100}$$

$$= \frac{a}{100t}\left[\sin(100t + \theta) - \sin\theta\right]$$

Using this result, we obtain

$$R_x(t_1, t_2) = a^2\,\overline{\cos(\omega t_1 + \theta)\cos(\omega t_2 + \theta)} = \frac{a^2}{2}\,\overline{\cos\left[\omega(t_1 + t_2) + 2\theta\right]} + \overline{\cos\omega(t_1 - t_2)}$$

$$= \frac{a^2}{200(t_1 + t_2)}\left[\sin\left[100(t_1 + t_2) + 2\theta\right] - \sin 2\theta\right] + \frac{a^2}{200(t_1 - t_2)}\left[\sin 100(t_1 - t_2)\right]$$

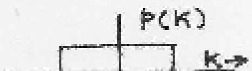
9-1.6 $\overline{x(t)} = \overline{at + b} = \bar{a}t + b$. But $\bar{a} = 0$ Hence, $\overline{x(t)} = b$

Also, $\bar{a} = 0$, $\overline{a^2} = \int_{-2}^{2} a^2 p(a)\, da = \frac{1}{4}\frac{a^3}{3}\Big|_{-2}^{2} = \frac{4}{3}$

$$R_x(t_1, t_2) = \overline{(at_1 + b)(at_2 + b)} = a^2 t_1 t_2 + a(t_1 b + t_2 b) + b^2$$

$$= \overline{a^2}\, t_1 t_2 + \bar{a} b(t_1 + t_2) + b^2 = \frac{4}{3} t_1 t_2 + b^2$$

9-1.7 (b) $\overline{x(t)} = \overline{K} = 0$



(c)

$$R_x(t_1, t_2) = \overline{KK} = \overline{K^2} = \int_{-1}^{1} K^2 p(K)\, dK = \frac{1}{2}\int_{-1}^{1} K^2\, dK = \frac{1}{3}$$

(d) The process is W.S.S. Since $\overline{x(t)} = 0$ and $R_x(t_1, t_2) = \frac{1}{3}$

(e) The process is not ergodic since the time mean of each sample function is different from that of the other and it is not equal to the ensemble mean $(\bar{x} = 0)$

(f) $\overline{x^2} = R_x(0) = \frac{1}{3}$

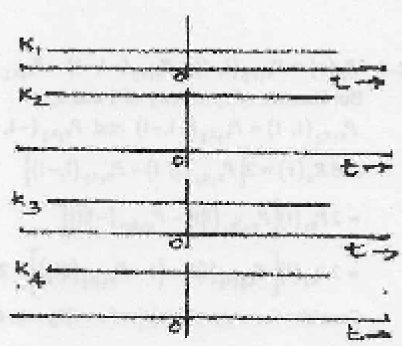


Fig. S9-1.7

9-1.8

$x(t) = a\cos(\omega_c t + \theta)$

$\bar{a} = 0 \qquad \overline{a^2} = \frac{1}{3}$

(b) $\overline{x(t)} = \overline{a\cos(\omega_c t + \theta)} = \bar{a}\,\overline{\cos(\omega_c t + \theta)} = 0$

(c) $R_x(t_1, t_2) = \overline{a^2}\,\overline{\cos(\omega_c t_1 + \theta)\cos(\omega_c t_2 + \theta)}$

$$= \frac{1}{3}\left\{\overline{\cos\omega_c(t_1 - t_2)} + \overline{\cos\left[\omega_c(t_1 + t_2) + 2\theta\right]}\right\}$$

$$= \frac{1}{3}\cos\omega_c(t_1 - t_2) + \frac{1}{2\pi}\int_0^{2\pi} \cos\left[\omega_c(t_1 - t_2) + 2\theta\right] d\theta$$

$$= \frac{1}{3}\cos\omega_c(t_1 - t_2)$$

(d) The process is W.S.S.

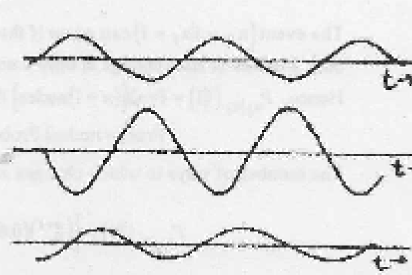


Fig. S9-1.8

(e) The process is not ergodic. Time means of each sample function is different and is not equal to the ensemble mean.

(f) $\overline{x^2} = R_x(0) = \frac{1}{3}$

9-2-1 (a), (d), and (e) are valid PSDs. Others are not valid PSDs. PSD is always a real, non-negative and even function of ω. Processes in (b), (c), (f), and (g) violate these conditions.

9-2-2 (a) Let $x(t) = x_1$ and $x(t+\tau) = x_2$. Then,

$$\overline{(x_1 \pm x_2)^2} = \overline{x_1^2} + \overline{x_2^2} \pm 2\overline{x_1 x_2} \geq 0, \quad \overline{x_1^2} + \overline{x_2^2} \geq \pm 2\overline{x_1 x_2}$$

But, $\overline{x_1 x_2} = R_x(\tau)$ and $\overline{x_1^2} = \overline{x_2^2} = R_x(0)$. Hence, $R_x(0) \geq |R_x(\tau)|$

(b) $R_x(\tau) = \overline{x(t)x(t+\tau)}$, $\quad \lim_{\tau\to\infty} R_x(\tau) = \lim_{\tau\to\infty} \overline{x(t)x(t+\tau)}$

As $\tau \to \infty$, $x(t)$ and $x(t+\tau)$ become independent, so $\lim_{\tau\to\infty} R_x(\tau) = \overline{x(t)x(t+\tau)} = (\overline{x})(\overline{x}) = \overline{x}^2$

9-2-3 $R_x(\tau) = 0$ for $\tau = \pm\frac{n}{2B}$ and its Fourier transform $S_x(\omega)$ is bandlimited to B Hz. Hence, $R_x(\tau)$ is a waveform bandlimited to B Hz and according to Eq. 6.10b

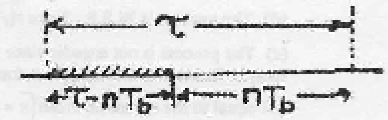
$$R_x(\tau) = \sum_{n=-\infty}^{\infty} R_x\left(\frac{n}{2B}\right) \text{sinc}\,(2\pi B\tau - n).$$ Since $R_x\left(\frac{n}{2B}\right) = 0$ for all n except $n = 0$,

$R_x(\tau) = R_x(0)\,\text{sinc}\,(2\pi B\tau)$ and $S_x(\omega) = \frac{R_x(0)}{2B}\,\text{rect}\left(\frac{\omega}{4\pi B}\right)$. Hence, $x(t)$ is a white process bandlimited to B Hz.

9-2-4 $R_x(\tau) = P_{x_1x_2}(1,1) + P_{x_1x_2}(-1,-1) - P_{x_1x_2}(-1,1) - P_{x_1x_2}(1,-1)$

But because of symmetry of 1 and 0,

$P_{x_1x_2}(1,1) = P_{x_1x_2}(-1,-1)$ and $P_{x_1x_2}(-1,1) = P_{x_1x_2}(1,-1)$



and $R_x(\tau) = 2\left[P_{x_1x_2}(1,1) - P_{x_1x_2}(1,-1)\right]$

$$= 2P_{x_1}(1)\left[P_{x_2|x_1}(1|1) - P_{x_2|x_1}(-1|1)\right]$$

$$= 2P_{x_1}(1)\left[P_{x_2|x_1}(1|1) - \left(1 - P_{x_2|x_1}(1|1)\right)\right] = 2P_{x_2|x_1}(1|1) - 1$$

Consider the case $nT_b < |\tau| < (n+1)T_b$. In this case, there are at least n nodes and a possibility of $(n+1)$ nodes

$$\text{Prob}\left[(n+1)\text{nodes}\right] = \frac{\tau - nT_b}{T_b} = \frac{\tau}{T_b} - n$$

$$\text{Prob}(n\text{ nodes}) = 1 - \text{Prob}\left[(n+1)\text{nodes}\right] = (n+1) - \frac{\tau}{T_b}$$

The event $(x_2 = 1|x_1 = 1)$ can occur if there are N nodes and no state change at any node or state change at only 2 nodes or state change at only 4 nodes, etc.

Hence, $P_{x_2|x_1}(1|1) = \text{Prob}\left[(n+1)\text{nodes}\right]\,\text{Prob(state change at even number of nodes)} +$

Prob(n nodes) Prob(State changes at eveen number of nodes)

The number of ways in which changes at K nodes out of N nodes occur is $\binom{N}{K}$. Hence,

$$P_{x_2|x_1}(1|1) = \left[\binom{n+1}{0}(0.6)^0(0.4)^{n+1} + \binom{n+1}{2}(0.6)^2(0.4)^{n-1} + \ldots\right]\left(\frac{|\tau|}{T_b} - n\right) +$$