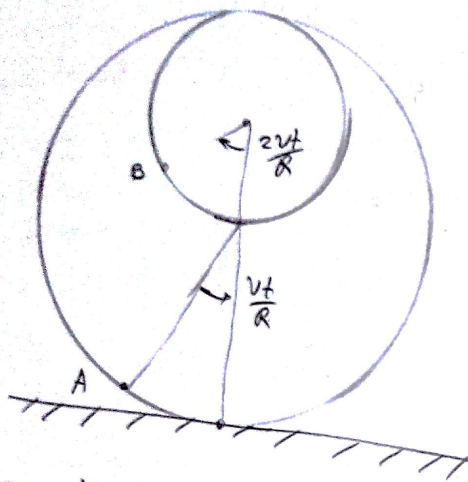


آزاد السال 90

مسئله الف



$$\vec{r}_A = \left[ vt - R \sin\left(\frac{vt}{R}\right) \right] \hat{x} + R \left[ 1 - \cos\left(\frac{vt}{R}\right) \right] \hat{y}$$

$$\begin{aligned} \vec{r}_B &= \left[ vt - R \sin\left(\frac{2vt}{R}\right) \right] \hat{x} + R \left[ \left(1 - \cos\left(\frac{2vt}{R}\right)\right) \frac{v}{2+1} \right] \hat{y} \\ &= \left[ vt - R \sin\left(\frac{2vt}{R}\right) \right] \hat{x} + R \left[ \frac{3}{2} - \frac{1}{2} \cos\left(\frac{2vt}{R}\right) \right] \hat{y} \end{aligned}$$

$$\theta = n\pi \rightarrow \tan \theta = 0 = \frac{y_B - y_A}{x_B - x_A} \rightarrow y_B = y_A$$

$$\frac{3}{2} - \frac{1}{2} \cos\left(\frac{2vt}{R}\right) = 1 - \cos\left(\frac{vt}{R}\right) \rightarrow 2 \sin^2\left(\frac{vt}{R}\right) = -2 \cos\left(\frac{vt}{R}\right)$$

$$\rightarrow 2 \left( 1 - \cos^2\left(\frac{vt}{R}\right) \right) + 2 \cos\left(\frac{vt}{R}\right) = 0 \rightarrow \cos^2\left(\frac{vt}{R}\right) - \cos\left(\frac{vt}{R}\right) - 1 = 0 \rightarrow \cos\left(\frac{vt}{R}\right) = \frac{1 \pm \sqrt{5}}{2} = \frac{1 - \sqrt{5}}{2}$$

$$\rightarrow \frac{vt}{R} = 2n\pi \pm \cos^{-1}\left(\frac{1 - \sqrt{5}}{2}\right) =$$

129°



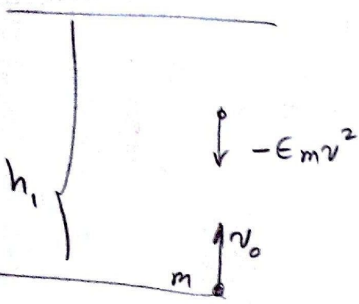
(c)



$$\downarrow -\epsilon/2 m v^2$$

$$v_0 t - \frac{gt^2}{2} = h_1 = \frac{3v_0^2}{8g}$$

(الف)



$$\rightarrow \frac{gt^2}{2} - v_0 t + \frac{3v_0^2}{8g} = 0 \rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 - \frac{3v_0^2}{4}}}{g}$$

$$t^{(0)} = \frac{v_0 - \frac{v_0}{2}}{g} = \frac{v_0}{2g}$$

$$m\ddot{x} = -mg - \epsilon m v^2 \rightarrow \ddot{x} = -g - \epsilon (v_0 - gt)^2 \rightarrow \dot{x} = -gt + v_0 - \epsilon$$

$$m\ddot{x} = -mg - \epsilon m v^2 \rightarrow \ddot{x}^{(1)} = -\epsilon v^2 dt \quad dv = -g dt \rightarrow \ddot{x}^{(1)} = \epsilon v \frac{dv}{g}$$

WAW

$$\dot{x}^{(1)} = \frac{\epsilon}{3g} (v^3 - v_0^3) \rightarrow x^{(1)} = \frac{\epsilon}{3g} \left( -\frac{v^3}{g} - v_0^3 t \right) = \frac{\epsilon}{3g} \left( -\frac{v^4}{4g} - v_0^3 t \right)$$

$$v_0 t^{(1)} - \frac{g}{2} \times \left( \frac{v_0}{2g} \right) t^{(1)} + \frac{\epsilon}{3g} \left( \frac{v_0^4}{4g} - \frac{v_0^3}{2g} \times \frac{v_0}{2g} \right) = 0 \quad v^2 - v_0^2 = -2g \times \frac{3v_0^2}{8g} + v^2 = \frac{v_0^2}{4}$$

$$m\ddot{x} = -mg - \epsilon m v^2 \rightarrow \ddot{x}^{(1)} = -\epsilon v^2 dt \quad dv = -g dt \rightarrow \dot{x}^{(1)} = \epsilon \frac{v^2}{g}$$

$$\dot{x}^{(1)} = \frac{\epsilon}{3g} (v^3 - v_0^3) \rightarrow x^{(1)} = \frac{\epsilon}{3g} \left( -\frac{v^3}{g} - v_0^3 t \right) = \frac{\epsilon}{3g} \left( -\frac{v^4}{4g} - v_0^3 t \right)$$

$$v_0 t^{(1)} - \frac{g}{2} \times \cancel{t^{(1)2}} + \frac{\epsilon}{3g} \left( \frac{v_0^4}{4g} - v_0^3 \times \frac{v_0}{2g} \right) = 0 \quad v^2 - v_0^2 = -2g \times \frac{3v^2}{8g} + v^2 = \frac{v_0^2}{4}$$

$$\rightarrow \frac{v_0 t^{(1)}}{2} = + \frac{\epsilon}{3g} \left( \frac{15 v_0^4}{64g} + \frac{32 v_0^4}{64 \cdot 2g} \right) = \frac{\epsilon v_0^4}{3g^2} \frac{17}{64} \rightarrow \boxed{t^{(1)} = \frac{\epsilon v_0^3}{g^2} \times \frac{17}{96}}$$

$$v^{(1)} = v_0 - \frac{v_0}{2} = \frac{v_0}{2} \quad v^{(1)} = -g t^{(1)} + \frac{\epsilon}{3g} \left( \frac{v_0^3}{8} - \frac{v_0^3}{8} \right) = -g \times \frac{\epsilon v_0^3}{g^2} \times \frac{17}{96} - \frac{7}{24} \frac{\epsilon v_0^3}{g}$$

$$= -\frac{\epsilon v_0^3}{g} \frac{-17 - 28}{96} = \boxed{\frac{\epsilon v_0^3}{g} \frac{45}{96} = v^{(1)}}$$

$$t_x^{(0)} = \frac{v_0}{2g} \quad \dot{x}^{(1)} = \frac{\epsilon}{2 \times 3g} \left( \frac{v_0^3}{8} - \frac{v_0^3}{8} \right) - g t^{(1)} = \frac{45}{96} \frac{\epsilon v_0^3}{g}$$

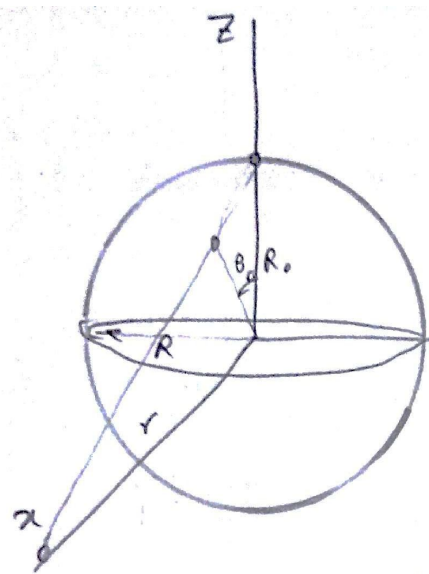
$$g t^{(1)} = \frac{-\epsilon v_0^3}{48g} - \frac{45}{96} \frac{\epsilon v_0^3}{g} = \frac{-47}{96} \frac{\epsilon v_0^3}{g} \rightarrow \boxed{t^{(1)} = \frac{-47}{96} \frac{\epsilon v_0^3}{g^2}}$$

$$x^{(1)} = v^{(1)} t^{(1)} + v^{(0)} t^{(1)} - g t^{(0)} t^{(1)} + \frac{\epsilon}{6g} \left( \frac{v_0^4}{4g \times 2^4} - v_0^3 \times \frac{v_0}{2g} \right) \quad (C)$$

$$= - \frac{\epsilon v_0^3}{g} \times \frac{45}{96} \times \frac{v_0}{2g} + \frac{v_0}{2} \times \frac{-47}{96} \frac{\epsilon v_0^3}{g^2} - g \times \frac{v_0}{2g} \times \frac{-47}{96} \frac{\epsilon v_0^3}{g^2} + \frac{\epsilon}{12g^2} v_0^4 \left( \frac{-31}{32} \right)$$

$$= \frac{\epsilon v_0^4}{g^2} \left( - \frac{45}{2 \times 96} - \frac{47}{96 \times 2} + \frac{47}{2 \times 96} - \frac{31}{32} \right) = \frac{-\epsilon v_0^4}{g^2} \frac{15+62}{64} = \boxed{\frac{-\epsilon v_0^4}{g^2} \frac{77}{64}} = x^{(1)}$$

سؤال 3



$$\vec{r}_A = R_0 \cos\left(\frac{\theta_0}{2}\right) \hat{x}$$

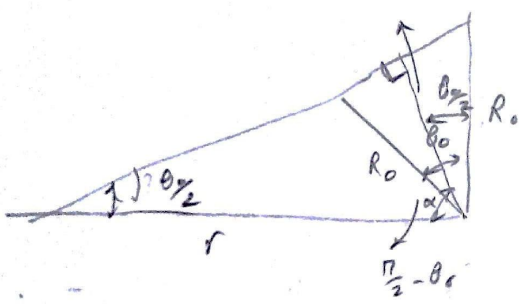
(الف)

$$r = (R_0 + vt) \cot\left(\frac{\theta_0 + \omega t}{2}\right)$$

(ب)

$$\omega T = 2\pi \rightarrow T = \frac{2\pi}{\omega}$$

(ج)



$$R_0 \cot\left(\frac{\theta_0}{2}\right) = \left(R_0 + \frac{2v\alpha}{\omega}\right) \cot\left(\frac{\theta_0}{2} + \frac{\alpha}{\omega}\right)$$

$$\frac{\pi}{\omega} = \frac{\alpha}{\omega} \rightarrow \tan\left(\frac{\theta_0}{2} + \alpha\right) R_0 \cot\left(\frac{\theta_0}{2}\right) = R_0 + \frac{2v\alpha}{\omega}$$

$$\tan\left(\frac{\theta_0}{2} + \alpha\right) R_0 \cot\left(\frac{\theta_0}{2}\right) = R_0 + \frac{2v\alpha}{\omega}$$

$$\frac{\tan\left(\frac{\theta_0}{2}\right) + \tan(\alpha)}{1 - \tan\left(\frac{\theta_0}{2}\right) \tan(\alpha)} \times R_0 \cot\left(\frac{\theta_0}{2}\right) = R_0 + \frac{2v\alpha}{\omega}$$

$$\left(\tan\left(\frac{\theta_0}{2}\right) + \tan(\alpha)\right) \times R_0 \cot\left(\frac{\theta_0}{2}\right) = \left(R_0 + \frac{2v\alpha}{\omega}\right) \left(1 - \tan\left(\frac{\theta_0}{2}\right) \tan(\alpha)\right)$$

$$\tan(\alpha) \left( R_0 \cot\left(\frac{\theta_0}{2}\right) + \left(R_0 + \frac{2v\alpha}{\omega}\right) \tan\left(\frac{\theta_0}{2}\right) \right)$$

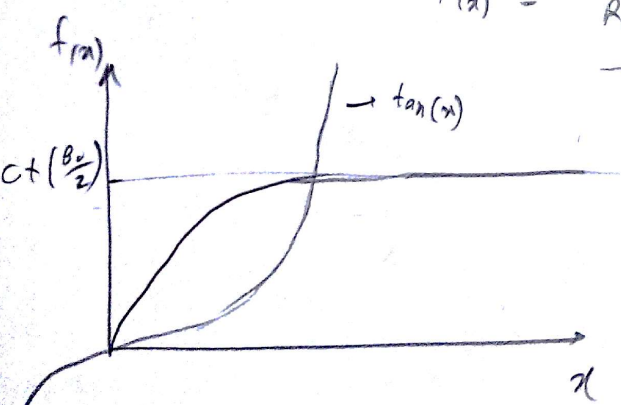
$$\frac{\tan(\frac{\theta_0}{2}) + \tan(x)}{1 - \tan(\frac{\theta_0}{2}) \tan(x)} \times R_0 \cot(\frac{\theta_0}{2}) = R_0 + \frac{2v_x}{\omega}$$

$$\left( \tan(\frac{\theta_0}{2}) + \tan(x) \right) \times R_0 \cot(\frac{\theta_0}{2}) = \left( R_0 + \frac{2v_x}{\omega} \right) \left( 1 - \tan(\frac{\theta_0}{2}) \tan(x) \right)$$

$$\tan(x) \left( R_0 \cot(\frac{\theta_0}{2}) + \left( R_0 + \frac{2v_x}{\omega} \right) \tan(\frac{\theta_0}{2}) \right) = \cancel{R_0} + \frac{2v_x}{\omega} - \cancel{R_0}$$

$$\rightarrow \tan(x) = \frac{\frac{2v_x}{\omega}}{\frac{R_0}{\sin(\frac{\theta_0}{2}) \cos(\frac{\theta_0}{2})} + \frac{2v_x}{\omega} \tan(\frac{\theta_0}{2})} = \frac{2v_x}{\omega R_0} \times \frac{\sin \frac{\theta_0}{2} \cos \frac{\theta_0}{2}}{1 + \sin^2 \frac{\theta_0}{2} \times \frac{2v_x}{R\omega}}$$

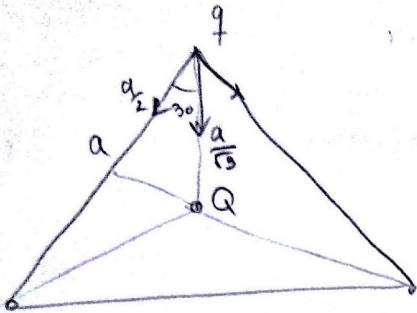
$$f(x) = \frac{v_x}{R_0 \omega} \frac{\sin \theta_0}{1 + \sin^2 \frac{\theta_0}{2} \frac{2v_x}{R\omega}}$$



$$\frac{v \sin \theta_0}{R_0 \omega} > 1$$

(c)

(c)



$$0 = \frac{kq^2}{a^2} \times \sqrt{3} + \frac{3kqQ}{a^2}$$

4 dx

$$4\sqrt{3} + 3Q = 0 \rightarrow \frac{q}{Q} = -\frac{3}{\sqrt{3}} = -\sqrt{3} = \frac{q}{Q} \quad (a)$$

b

$$-\frac{ma}{\sqrt{3}} \omega^2 = \frac{kq^2}{a^2} \times \sqrt{3} + \frac{3kqQ}{a^2} = \frac{kq}{a^2} (\sqrt{3}q + 3Q) = -\frac{ma\omega^2}{\sqrt{3}}$$

$$\rightarrow \omega^2 = \frac{-kq \times \sqrt{3}}{ma^3} (\sqrt{3}q + 3Q) = \frac{-kq}{ma^3} (3q + 3\sqrt{3}Q) = \frac{-3kq}{ma^3} (q + \sqrt{3}Q) = \omega^2$$

(c)

$$r_i(t) = \frac{q}{\sqrt{3}} + p(t)$$

$$m(\ddot{p} - p\omega^2) = \frac{kq}{ma^3} \times -2\sqrt{3}a(\sqrt{3}q + 3Q) + \frac{50a}{\sqrt{3}} + \delta a \left( \frac{2kq}{ma^3} (\sqrt{3}q + 3Q) - \frac{\omega^2}{\sqrt{3}} \right) = 0$$

$$\frac{\frac{2kq}{ma^3} (\sqrt{3}q + 3Q) - \frac{\omega^2}{\sqrt{3}}}{1} = \frac{\frac{2\sqrt{3}kq}{ma^3} (q + \sqrt{3}Q) - \frac{\omega^2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \left( -\frac{2\sqrt{3}}{\sqrt{3}} \times \sqrt{3} - 1 \right) \omega^2 = -3\omega^2 = \omega^2$$



$$-\frac{ma}{\sqrt{3}} \omega^2 = \frac{kq^2}{a^2} \times \sqrt{3} + \frac{3kqQ}{a^2} = \frac{kq}{a^2} (\sqrt{3}q + 3Q) = \frac{-ma\omega^2}{\sqrt{3}}$$

b

$$\rightarrow \omega^2 = \frac{-kq \times \sqrt{3}}{ma^3} (\sqrt{3}q + 3Q) = \frac{-kq}{ma^3} (3q + 3\sqrt{3}Q) = \boxed{\frac{-3kq}{ma^3} (q + \sqrt{3}Q) = \omega^2}$$

$$r_i(t) = \frac{q}{\sqrt{3}} + p(t)$$

c

$$m(\ddot{p} - p\omega^2) = \frac{kq}{ma^3} \times -2\sqrt{3}a(\sqrt{3}q + 3Q) + \frac{50a}{\sqrt{3}} + \sqrt{3}a \left( \frac{2kq}{ma^3} (\sqrt{3}q + 3Q) - \frac{\omega^2}{\sqrt{3}} \right) = 0$$

$$p + p \left( \frac{\frac{2kq}{ma^3} (\sqrt{3}q + 3Q) - \frac{\omega^2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \right) = \frac{\frac{2\sqrt{3}kq}{ma^3} (q + \sqrt{3}Q) - \frac{\omega^2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \left( -\frac{2\sqrt{3}}{\sqrt{3}} \times \sqrt{3} - 1 \right) = (-2 + 1)\omega^2 = \boxed{-3\omega^2 = \omega^2}$$

$$\vec{F} = \frac{q}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^3} \left( \frac{3\vec{p}_1 \cdot (\vec{r}_2 - \vec{r}_1) (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^2} - \vec{p}_1 \right)$$

مسألة 5  
الف

$$\vec{F} = \vec{F}(q, \vec{r} + \frac{\vec{p}_2}{q}) + \vec{F}(-q, \vec{r})$$

ب

$$|\vec{r} + \frac{\vec{p}_2}{q}| = \left( (\vec{r} + \frac{\vec{p}_2}{q})^2 \right)^{1/2} = \left( r^2 + 2\vec{r} \cdot \frac{\vec{p}_2}{q} \right)^{1/2} = r \left( 1 + \frac{\vec{r} \cdot \vec{p}_2}{qr^2} \right)$$

$$\vec{F} = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r^3} \left( 1 - \frac{3\vec{r} \cdot \vec{p}_2}{qr^2} \right) \left[ \frac{3\vec{p}_1 \cdot (\vec{r} + \frac{\vec{p}_2}{q}) (\vec{r} + \frac{\vec{p}_2}{q})}{r^2} \left( 1 - \frac{2\vec{r} \cdot \vec{p}_2}{qr^2} \right) - \vec{p}_1 \right] \right]$$

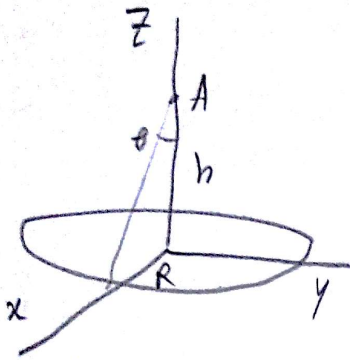
$$= \frac{q}{4\pi\epsilon_0 r^3} \left( 1 - \frac{3\vec{r} \cdot \vec{p}_2}{qr^2} \right) \left[ \frac{1}{r^2} \left( \frac{3\vec{p}_1 \cdot \vec{p}_2}{q} \vec{r} + 3\vec{p}_1 \cdot \vec{r} \frac{\vec{p}_2}{q} + 3\vec{p}_1 \cdot \vec{r} \vec{r} \right) \left( 1 - \frac{2\vec{r} \cdot \vec{p}_2}{qr^2} \right) - \vec{p}_1 \right]$$

$$= \frac{q}{4\pi\epsilon_0 r^3} \left[ \left( \frac{3\vec{p}_1 \cdot \vec{p}_2}{q} \vec{r} + 3\vec{p}_1 \cdot \vec{r} \frac{\vec{p}_2}{q} \right) \frac{1}{r^2} - 6\vec{p}_1 \cdot \vec{r} \vec{r} \frac{\vec{r} \cdot \vec{p}_2}{qr^3} - \frac{3\vec{r} \cdot \vec{p}_2}{r^2} \left( \frac{3\vec{p}_1 \cdot \vec{r} \vec{r}}{r^2} - \vec{p}_1 \right) \right]$$

$$= \frac{q}{4\pi\epsilon_0 r^3} \left( 1 - \frac{3\vec{r} \cdot \vec{P}_2}{qr^2} \right) \left[ \frac{1}{r^2} \left( \frac{3\vec{P}_1 \cdot \vec{P}_2}{q} \vec{r} + 3\vec{P}_1 \cdot \vec{r} \frac{\vec{P}_2}{q} + 3\vec{P}_1 \cdot \vec{r} \vec{r} \right) \left( 1 - \frac{2\vec{r} \cdot \vec{P}_2}{qr^2} \right) - \vec{P}_1 \right]$$

$$= \frac{q}{4\pi\epsilon_0 r^3} \left[ \left( \frac{3\vec{P}_1 \cdot \vec{P}_2}{q} \vec{r} + 3\vec{P}_1 \cdot \vec{r} \frac{\vec{P}_2}{q} \right) \frac{1}{r^2} - 6\vec{P}_1 \cdot \vec{r} \vec{r} \frac{\vec{r} \cdot \vec{P}_2}{qr^3} - \frac{3\vec{r} \cdot \vec{P}_2}{r^2} \left( \frac{3\vec{P}_1 \cdot \vec{r} \vec{r}}{r^2} - \vec{P}_1 \right) \right]$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0 r^3} \left[ \frac{3\vec{P}_1 \cdot \vec{P}_2}{r^2} \vec{r} + \frac{3\vec{P}_1 \cdot \vec{r} \vec{P}_2}{r^2} + \frac{3\vec{P}_2 \cdot \vec{r}}{r^2} \vec{P}_1 - 15 \frac{\vec{P}_2 \cdot \vec{r} \vec{P}_1 \cdot \vec{r}}{r^4} \vec{r} \right]$$



$$E = \frac{\sigma}{2\epsilon_0} (1 - \cos\theta)$$

6 د

(الف)

$$dz \times \frac{\phi(z) - \phi(z+dz)}{dz} = \frac{-d\phi}{dz} \stackrel{dE}{=} E dz$$

$$\phi = \frac{\sigma dz (1 - \cos\theta)}{2\epsilon_0} = \boxed{\frac{D}{2\epsilon_0} \left(1 - \frac{h}{\sqrt{R^2+h^2}}\right) = \phi}$$

$$F = -\frac{d\phi}{dh} q = \frac{Dq}{2\epsilon_0}$$

$$\frac{\sqrt{R^2+h^2} - \frac{h^2}{\sqrt{R^2+h^2}}}{R^2+h^2} = \boxed{\frac{Dq}{2\epsilon_0} \frac{R^2}{(R^2+h^2)^{3/2}} = F}$$

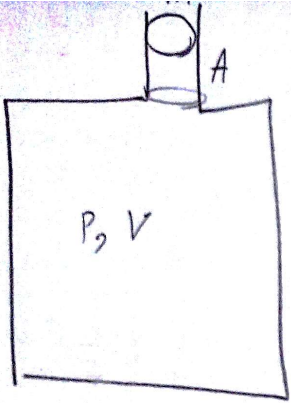
(ب)

$$\frac{dF}{dh} = \frac{DqR^2}{2\epsilon_0} \times -\frac{3}{2} \frac{2h}{(R^2+h^2)^{5/2}} = \frac{-3hDqR^2}{2\epsilon_0(R^2+h^2)^{5/2}}$$

(ج)

$$\omega^2 = \frac{2.3hDqR^2}{m\epsilon_0(R^2+h^2)^{5/2}}$$

آیزون 2 ج ل 90



$$T = m^{\alpha_1} v^{\alpha_2} p^{\alpha_3} A^{\alpha_4}$$

$$= M^{\alpha_1} (L^2 T)^{\alpha_2} \alpha^{2\alpha_3} \left(\frac{M L}{T^2 L^2}\right)^{\alpha_3}$$

$$= M^{\frac{1}{2}} (L^2 T)^{\frac{1}{2}} \alpha^{-2} \left(\frac{M L}{T^2 L^2}\right)^{-\frac{1}{2}}$$

مسئله 1

(الف)

$$\rightarrow T \propto \sqrt{\frac{m v}{A^2 p}}$$

$$\frac{h}{T} = E \rightarrow [h] = \frac{M L^2}{T^2}$$

$$F \propto h^{\alpha_1} c^{\alpha_2} d^{\alpha_3}$$

$$\frac{M L}{T^2} = \left(\frac{M L^2}{T}\right)^{\alpha_1} \left(\frac{L}{T}\right)^{\alpha_2} L^{\alpha_3 - 2}$$

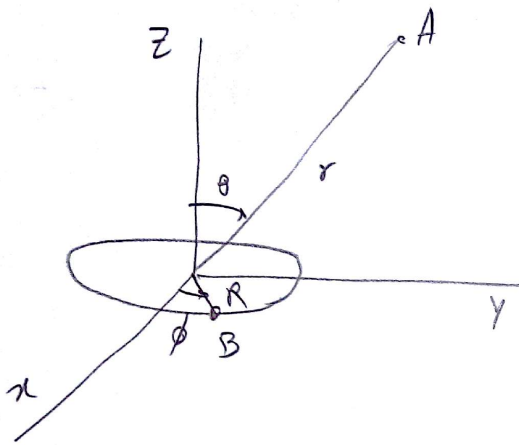
(ب)

$$\rightarrow F \propto \frac{h c}{d^2}$$



$\vec{r} = r \hat{r}$  ...

$$\rightarrow \left[ F \propto \frac{hc}{d^2} \right]$$



$$\vec{r}_B = R (\cos\phi \hat{x} + \sin\phi \hat{y}) \rightarrow \vec{r}_A = r (\sin\theta \hat{y} + \cos\theta \hat{z}) \quad (\text{ان})$$

$$|\vec{r}_A - \vec{r}_B| = \left| R \cos\phi \hat{x} + (R \sin\phi - r \sin\theta) \hat{y} - r \cos\theta \hat{z} \right|$$

$$= \left[ R^2 + r^2 - 2Rr \sin\theta \sin\phi \right]^{1/2}$$

$$\phi^{(0)} = \frac{Q}{4\pi\epsilon_0 r}$$

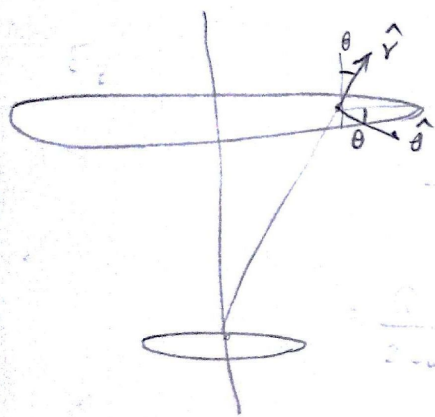
$$\phi = \int \frac{Q d\phi}{2\pi \times 4\pi\epsilon_0 (R^2 + r^2 - 2Rr \sin\theta \sin\phi)^{1/2}} = \frac{Q}{8\pi^2\epsilon_0} \int \frac{d\phi}{r \left( 1 - \frac{2R}{r} \sin\theta \sin\phi + \frac{R^2}{r^2} \right)^{1/2}}$$

$$= \frac{Q}{8\pi^2\epsilon_0 r} \int_0^{2\pi} d\phi \left( 1 + \frac{R}{r} \sin\theta \sin\phi - \frac{R^2}{2r^2} + \frac{3}{8} \times \frac{AR^2}{r^2} \sin^2\theta \sin^2\phi \right)$$

$$= \frac{Q}{4\pi^2\epsilon_0 r} \left( \cancel{2\pi} \left( 1 - \frac{R^2}{2r^2} \right) + \frac{3}{4} \frac{R^2}{r^2} \sin^2\theta \times \pi \right) = \frac{Q}{4\pi\epsilon_0 r} \left( 1 + \frac{R^2}{r^2} \left( \frac{3}{4} \sin^2\theta - \frac{1}{2} \right) \right) = \phi$$

$$\vec{E} = -\frac{Q}{4\pi\epsilon} \left[ -\frac{\hat{r}}{r^2} + \frac{R^2}{r^4} \times \frac{3}{2} \sin\theta \cos\theta \hat{\theta} - \frac{3R^2}{r^4} \hat{r} \left( \frac{3}{4} \sin^2\theta - \frac{1}{2} \right) \right]$$

$$\vec{E} = \frac{Q}{4\pi\epsilon \cdot r^2} \left[ \hat{r} \left( 1 + \frac{3R^2}{r^2} \left( \frac{3}{4} \sin^2\theta - \frac{1}{2} \right) \right) - \frac{3}{2} \frac{R^2}{r^2} \sin\theta \cos\theta \hat{\theta} \right]$$



$$E_z = \frac{Q}{4\pi\epsilon \cdot r^2} \left[ \cos\theta \left( 1 + \frac{3R^2}{r^2} \left( \frac{3}{4} \sin^2\theta - \frac{1}{2} \right) \right) + \frac{3}{2} \frac{R^2}{r^2} \sin^2\theta \cos\theta \right] \quad (\text{C})$$

$$\Phi = \frac{Q}{2\epsilon} \left[ \frac{1 - \cos^4\theta}{5} + \frac{R^2}{H^2} \left( -\frac{8}{21} - \frac{1}{28} \cos^7\theta + \frac{5}{12} \cos^9\theta \right) \right]$$

$$\Phi = \frac{Q}{2\epsilon_0} \left[ \frac{1 - \cos^4 \theta}{5} + \frac{R^2}{H^2} \left( -\frac{9}{21} - \frac{9}{28} \cos^7 \theta + \frac{5}{12} \cos^9 \theta \right) \right]$$

$$\Phi = \frac{Q \cos^2 \theta}{4\pi \epsilon_0 H^2} \left[ \cos \theta \left( 1 + \frac{3R^2 \cos^2 \theta}{H^2} \left( \frac{3}{4} \sin^2 \theta - \frac{1}{2} \right) \right) + \frac{3}{2} \frac{R^2}{H^2} \sin^2 \theta \cos^3 \theta \right] 2\pi H^2 \frac{\sin \theta d\theta}{\cos^3 \theta}$$

$$= \frac{Q}{2\epsilon_0} \left[ \sin \theta \left( 1 + \frac{3R^2 \cos^2 \theta}{H^2} \left( \frac{3}{4} \sin^2 \theta - \frac{1}{2} \right) \right) + \frac{3}{2} \frac{R^2}{H^2} \sin^3 \theta \cos^2 \theta \right] d\theta$$

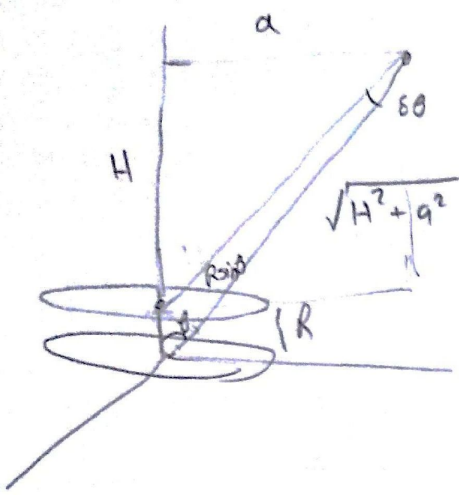
$$= \frac{Q}{2\epsilon_0} \left[ \sin \theta + \frac{R^2}{H^2} \left( \frac{15}{4} \sin \theta (1 - \cos^2 \theta) - \frac{9}{4} \cos^2 \theta \sin^3 \theta - \frac{3}{2} \cos^2 \theta \sin \theta + \frac{3}{2} \sin^3 \theta \cos^2 \theta \right) \right] d\theta$$

$$= \frac{Q}{2\epsilon_0} \left[ \sin \theta + \frac{R^2}{H^2} \left( \frac{9}{4} \cos^2 \theta \sin \theta - \frac{15}{4} \cos^4 \theta \sin \theta \right) \right]$$

$$= \frac{Q}{2\epsilon_0} \left[ 1 - \cos \theta + \frac{R^2}{H^2} \left( \frac{9}{4} \cos^3 \theta + \frac{3}{5 \times 4} \cos^5 \theta \right) \right] = \frac{Q}{2\epsilon_0} \left[ 1 - \cos \theta - \frac{3R^2}{4H^2} \cos^3 \theta \sin^2 \theta \right] = \Phi$$

$$\Phi = \frac{Q}{2\epsilon_0} \left[ 1 - \cos \theta - \frac{3R^2}{4H^2} \cos^3 \theta \sin^2 \theta \right]$$





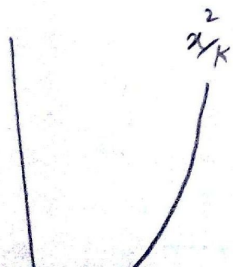
$$\delta\theta = \frac{R \sin\theta}{\sqrt{H^2 + a^2}}$$

$$\cos(\theta + \delta\theta) = \cos\theta - \frac{R \sin^2\theta}{\sqrt{H^2 + a^2}}$$

$$\Phi = \frac{Q}{2\epsilon_0} \left[ 2(1 - \cos\theta) + \frac{R \sin^2\theta}{\sqrt{H^2 + a^2}} \right] = \Phi_0$$

$$\Phi_0 = \frac{Q}{2\epsilon_0} \left[ 2 \left( 1 - \frac{H}{\sqrt{H^2 + a^2}} \right) + \frac{R a^2}{(H^2 + a^2)^{3/2}} \right]$$

$$\Phi_0 = \frac{Q}{2\epsilon_0} \left[ 2 \left( 1 - \frac{z}{\sqrt{z^2 + \rho^2}} \right) + \frac{R \rho^2}{(z^2 + \rho^2)^{3/2}} \right]$$



$$\frac{h^2}{2} + gh = \frac{v^2}{2} \rightarrow v^2 = h^2 + 2gh$$

$$vA = \frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = \pi x^2 h = \pi k h^2 = vA$$

مسئله 3  
(الف)

$$vA = \frac{dV}{dt} = \frac{dV}{dh} \times \dot{h} = \pi x^2 \dot{h} = \pi k h \dot{h} = vA \quad (\text{الف})$$

$$\left(\frac{\pi k h}{A}\right)^2 h^2 = \dot{h}^2 = h^2 + 2gh \rightarrow \dot{h}^2 \left(\left(\frac{\pi k h}{A}\right)^2 - 1\right) = 2gh \rightarrow \dot{h} = \left(\frac{2gh}{\left(\frac{\pi k h}{A}\right)^2 - 1}\right)^{\frac{1}{2}}$$

$$\epsilon = \frac{A}{\pi k} \rightarrow \dot{h} = \left(\frac{2gh}{\left(\frac{\pi \epsilon}{h}\right)^2 - 1}\right)^{\frac{1}{2}} = \left(\frac{2gh}{\left(\frac{\pi \epsilon}{h}\right)^2 \left(1 - \frac{\epsilon^2}{h^2}\right)}\right)^{\frac{1}{2}} = \frac{\epsilon \sqrt{2gh}}{\pi} \left(1 + \frac{\epsilon^2}{2h^2}\right) = \frac{dh}{dt} \quad (\text{ب})$$

$$\rightarrow \frac{t}{\pi} = \int \frac{dh}{\epsilon \sqrt{2gh} \left(1 + \frac{\epsilon^2}{2h^2}\right)} = \frac{dh}{\epsilon \sqrt{2gh}} \left(1 - \frac{A^2}{2\pi^2 h^2 k^2}\right) = \sqrt{\frac{h}{2g}} \times \frac{k}{A} dh \left(1 - \frac{A^2}{2\pi^2 k^2 h^2}\right)$$

$$= \frac{k}{A \sqrt{2g}} \left( \frac{2}{3} (h^{3/2} - h_0^{3/2}) + \frac{A^2}{\pi^2 k^2} \left( \frac{1}{\sqrt{h}} - \frac{1}{\sqrt{h_0}} \right) \right) \quad v = \pi x^2 dy = \pi x^2 \times \frac{2x dx}{k}$$

$$v \propto x^4 = h^2 \rightarrow \frac{v}{2} \rightarrow \frac{h_0}{\sqrt{2}} \rightarrow t = \frac{\pi k}{A \sqrt{2g}} \left( \frac{2}{3} h_0^{3/2} \left( \frac{1}{(\sqrt{2})^{3/2}} - 1 \right) + \frac{A^2}{\pi^2 k^2 \sqrt{h_0}} (\sqrt{2} - 1) \right)$$

$$t = \frac{\pi k}{A \sqrt{2g}} \left( \frac{h_0^{3/2}}{3} \left( 2^{1/4} - 2 \right) + \frac{A^2}{\pi^2 k^2 \sqrt{h_0}} \left( 2^{1/4} - 1 \right) \right)$$

4.18

$$v \frac{t_1+t_2}{2} = l \rightarrow v = \frac{2l}{t_1+t_2}$$

$$P = \frac{v_2 - \frac{2l}{t_1+t_2}}{v_2 - v_1}$$

$$v_1 t_1 = l, v_2 t_2 = l$$

الف  $\frac{1}{2}$  ب

$$\frac{2l}{t_1+t_2} = \frac{2l}{\frac{l}{v_1} + \frac{l}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

$$P = \frac{v_2 - \frac{2v_1 v_2}{v_1 + v_2}}{v_2 - v_1} = \frac{v_2^2 - 2v_1 v_2}{v_2^2 - v_1^2} = \frac{v_2(v_2 - v_1)}{(v_2 - v_1)(v_2 + v_1)} = \frac{v_2}{v_2 + v_1}$$

$$\frac{v_2}{v_2 + v_1} = P$$

$$\int_{v_1}^{v_2} \alpha e^{-\lambda v} dv = 1 = -\frac{\alpha}{\lambda} e^{-\lambda v} \Big|_{v_1}^{v_2} = -\frac{\alpha}{\lambda} (e^{-\lambda v_2} - e^{-\lambda v_1}) = 1$$

$$\alpha = \frac{\lambda}{e^{-\lambda v_1} - e^{-\lambda v_2}}$$

$$\alpha \int_{\frac{v_1}{2}}^{\frac{v_1+v_2}{2}} e^{-\lambda v} dv = -\frac{\alpha}{\lambda} \left( e^{-\lambda \frac{v_1+v_2}{2}} - e^{-\lambda \frac{v_1}{2}} \right)$$

$$= \frac{e^{-\lambda \frac{v_1+v_2}{2}} - e^{-\lambda \frac{v_1}{2}}}{e^{-\lambda v_1} - e^{-\lambda v_2}} = P$$

2v<sub>1</sub>v<sub>2</sub>

$v_1 + v_2$

$$P = \frac{e^{-\lambda v_2} - e^{-\lambda \frac{2v_1 v_2}{v_1 + v_2}}}{e^{-\lambda v_2} - e^{-\lambda v_1}}$$

(c)

$$P = \frac{1 - \lambda v_2 + \frac{\lambda^2 v_2^2}{2} - \left( 1 - \lambda \frac{2v_1 v_2}{v_1 + v_2} + \frac{\lambda^2 \frac{4v_1^2 v_2^2}{(v_1 + v_2)^2}}{2} \right)}{1 - \lambda v_2 + \frac{\lambda^2 v_2^2}{2} - \left( 1 - \lambda v_1 + \frac{\lambda^2 v_1^2}{2} \right)}$$

(c)

$$= \frac{\frac{v_2(v_1 - v_2)}{v_1 + v_2} + \frac{\lambda}{2} \frac{v_2^2 (v_1 + v_2)^2 - 4v_1^2 v_2^2}{(v_1 + v_2)^2}}{v_1 - v_2 + \frac{\lambda}{2} (v_2^2 - v_1^2)} = \frac{\frac{v_2(v_1 - v_2)}{v_1 + v_2} + \frac{\lambda}{2} v_2^2 \frac{(v_1 + v_2)^2 - 4v_1^2}{(v_1 + v_2)^2}}{(v_1 - v_2)}$$

$$= \frac{v_2}{v_1 + v_2} + \frac{\lambda}{2} v_2^2 \frac{(v_1 + v_2)^2 - 4v_1^2}{(v_1 + v_2)^2 (v_1 - v_2)} + \frac{\lambda}{2} v_2 = P$$

$\left( 1 - \frac{\lambda}{2} (v_1 + v_2) \right)$   
 $\left( 1 + \frac{\lambda}{2} (v_1 + v_2) \right)$

$$P = \frac{v_2}{v_1 + v_2} + \frac{1}{2} v_2 \frac{v_2 (v_1 + v_2)^2 - 4v_1^2 v_2 + (v_1 + v_2)^2 (v_1 - v_2)}{(v_1 + v_2)^2 (v_1 - v_2)}$$

$$= \frac{v_2}{v_1 + v_2} + \frac{1}{2} v_2 v_1 \frac{(v_1 - v_2)^2}{(v_1 + v_2)^2 (v_1 - v_2)} = \boxed{\frac{v_2}{v_1 + v_2} + \frac{1}{2} v_1 v_2 \frac{v_1 - v_2}{(v_1 + v_2)^2} = P}$$

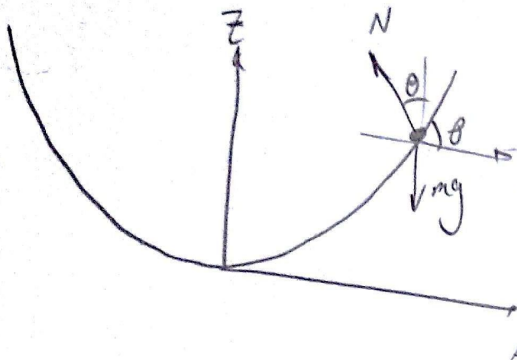
$$\int t P(t) dt = \int t P(v) dv = \int \frac{l}{v} \frac{dv}{v_2 - v_1} = \boxed{\frac{l}{v_2 - v_1} \ln\left(\frac{v_2}{v_1}\right) = \bar{t}} \quad (ع)$$

$$\bar{t} = \int t P(t) dt = \int \frac{l}{v} \frac{\lambda e^{-\lambda v} dv}{e^{-\lambda v_1} - e^{-\lambda v_2}} = \frac{\lambda l}{e^{-\lambda v_1} - e^{-\lambda v_2}} \int (1 - \lambda v) \frac{dv}{v}$$

$$= \frac{\lambda l}{\lambda - \lambda v_1 + \frac{\lambda^2 v_1^2}{2} - (\lambda - \lambda v_2 + \frac{\lambda^2 v_2^2}{2})} (\ln\left(\frac{v_2}{v_1}\right) - \lambda(v_2 - v_1))$$

$$= \frac{l (\ln\left(\frac{v_2}{v_1}\right) - \lambda(v_2 - v_1))}{v_2 - v_1 + \frac{\lambda}{2} (v_1^2 - v_2^2)} = \boxed{\frac{l}{v_2 - v_1} \left( \ln\left(\frac{v_2}{v_1}\right) - \lambda(v_2 - v_1) + \frac{\lambda}{2} (v_1 + v_2) \ln\left(\frac{v_2}{v_1}\right) \right) = \bar{t}}$$

$$\lambda \frac{\frac{v_1 + v_2}{2} \ln\left(\frac{v_2}{v_1}\right) - (v_2 - v_1)}{\ln\left(\frac{v_2}{v_1}\right)} = R \rightarrow \text{ظای بی} \quad (ع)$$



$$N \cos \theta = mg$$

$$N \sin \theta = m \rho \dot{\phi}^2 = mg \tan \theta = \rho \dot{\phi}^2$$

$$\dot{\phi}^2 = \frac{g}{R_0} \times 4K \rho^3 R_0^3 = 4KgR_0^2 = \dot{\phi}^2$$

$$\dot{\phi} = \sqrt{Kg \times 2R_0}$$

$$R_0^2 \times 2R_0 \sqrt{Kg} = \rho^2 \dot{\phi} \rightarrow \dot{\phi} = \frac{2R_0^3 \sqrt{Kg}}{\rho^2}$$

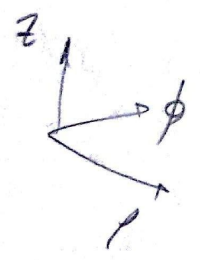
5 د

(الف)

(ب)

$$\vec{v} = \dot{\rho} \hat{\rho} + \rho \dot{\phi} \hat{\phi} + \dot{z} \hat{z} \quad \vec{B} = B \hat{\phi} \rightarrow \vec{F} = qB (\dot{\rho} \hat{z} - \dot{z} \hat{\rho})$$

(ج)



$$\begin{cases} m(\ddot{\rho} - \rho \dot{\phi}^2) = -N \sin \theta - qB \dot{z} \\ m\ddot{z} = N \cos \theta - mg + qB \dot{\rho} \end{cases}$$

$$m \left( \ddot{\rho} - \rho \frac{4R_0^6 Kg}{\rho^4} \right) = -N \times 4K \rho^3$$

$$mz = N \cos \theta - mg + qB\dot{\rho}$$

$$m \left( \ddot{\rho} - \rho \times \frac{4R \cdot kg}{\rho^3} \right) = - \frac{N \times 4K\rho^3}{\sqrt{1+16K^2\rho^6}} - \cancel{qB\dot{\rho}} + 4K\rho^3 \dot{\rho}$$

$$z = K\rho^4 \rightarrow \dot{z} = 4K\rho^3\dot{\rho} \rightarrow \ddot{z} = 4K(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho})$$

$$4Km(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho}) = \frac{N}{\sqrt{1+16K^2\rho^6}} - mg + qB\dot{\rho}$$

$$m \left( \ddot{\rho} - \frac{4R \cdot kg}{\rho^3} \right) = -4K\rho^3 \times (4Km(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho}) + mg - qB\dot{\rho}) - \cancel{4K\rho^3\dot{\rho}qB}$$

$$\ddot{\rho} (m + 16K^2m\rho^6) = -16K^2m\rho^3 \times 3\rho^2\dot{\rho}^2 - 4K\rho^3mg + \frac{4mR \cdot kg}{\rho^3}$$

$$= -48K^2m\rho^5\dot{\rho}^2 - 4K\rho^3mg + \frac{4mR \cdot kg}{\rho^3}$$

$$\ddot{\rho} (1 + 16K^2\rho^6) = -48K^2\rho^5\dot{\rho}^2 - 4K\rho^3g + \frac{4R \cdot kg}{\rho^3}$$



$$\delta \ddot{\phi} (1 + 16k^2 R_0^6) = \left( -12k \frac{R_0^2}{R_0^2} g - \frac{12 R_0^2 k g}{R_0^4} \right) \delta \rho$$

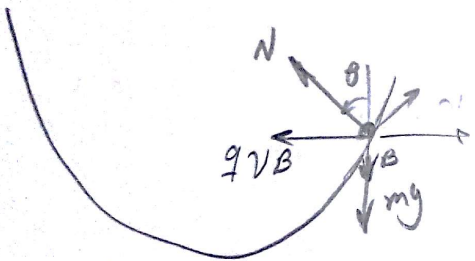
$$= -24k R_0^2 g$$

(C)

$$\omega^2 = \frac{24k R_0^2 g}{1 + 16k^2 R_0^6}$$

$$m R_1 \dot{\phi}^2 = N \sin \theta + q R_1 \dot{\phi} B$$

(C)



$$N \cos \theta = mg$$

$$m R_1 \dot{\phi}^2 = mg \times 4k \frac{R_1^3}{R_1^3} + q R_1 \dot{\phi} B$$

$$m R_1 \dot{\phi}^2 - 4kmg R_1^3 - q R_1 \dot{\phi} B = 0$$

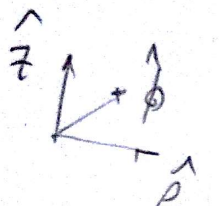
$$\dot{\phi} = \frac{q R_1 B \pm \sqrt{(q R_1 B)^2 + 16k m^2 g R_1^4}}{2m R_1}$$

$$= \frac{q B}{2m} + \frac{\sqrt{(q B)^2 + 16k m^2 g R_1^2}}{2m} = \dot{\phi}$$

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$$= \left[ \frac{qB}{2m} + \frac{\sqrt{(10) + 16 \text{ km}^2 g R_1^2}}{2m} \right] = \dot{\phi} \quad 2m R_1$$

$$\vec{v} = \dot{\rho} \hat{\rho} + \rho \dot{\phi} \hat{\phi} + \dot{z} \hat{z} \quad \vec{B} = -B_0 \hat{z} \rightarrow \vec{F} = -qB_0 (-\dot{\rho} \hat{\phi} + \rho \dot{\phi} \hat{\rho})$$



$$m(\rho \ddot{\phi} + 2\dot{\rho} \dot{\phi}) = qB_0 \dot{\rho} \rightarrow \frac{m}{\rho} \frac{d}{dt}(\rho^2 \dot{\phi}) = qB_0 \dot{\rho}$$

$$\frac{qB_0}{m} \rho d\rho = \rho^2 \dot{\phi} - R_1^2 \dot{\phi}_0$$

$$\dot{\phi}_c = \frac{qB}{2m} + \left( \frac{1}{4} \left( \frac{qB}{m} \right)^2 + 4 \times \frac{qR_1^2}{16} \times \frac{3}{64} \times \left( \frac{qB}{m} \right)^2 \right)^{1/2}$$

$$= \frac{qB}{2m} + \left( \frac{7}{16} \times \left( \frac{qB}{m} \right)^2 \right)^{1/2} = \frac{qB}{2m} \left( 1 + \frac{\sqrt{7}}{2} \right) = \dot{\phi}_c$$

$$\frac{qB_0}{2m} (\rho^2 - R_1^2) = \rho^2 \dot{\phi} - \frac{qB}{2m} R_1^2 \left( 1 + \frac{\sqrt{7}}{2} \right)$$

$$\dot{\phi} = \frac{qB}{2m} \left( \rho^2 + \frac{\sqrt{7}}{2} R_1^2 \right) = \left[ \frac{qB}{2m} \left( 1 + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} \right) \right] = \dot{\phi}$$

$$m(\ddot{\rho} - \rho\dot{\phi}^2) = -N \sin \theta - f B_c \rho \dot{\phi} \quad (E)$$

$$m\ddot{z} = N \cos \theta - mg, \quad z = K\rho^4 \rightarrow \dot{z} = 4K\rho^3\dot{\rho} \quad \ddot{z} = 4K(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho})$$

$$m\left(\ddot{\rho} - \rho \times \left(\frac{qB}{2m}\right)^2 \left(1 + \frac{7}{4} \frac{R_1^4}{\rho^4} + \sqrt{7} \frac{R_1^2}{\rho^2}\right)\right) = \frac{-N \times 4K\rho^3}{\sqrt{1 + (4K\rho^3)^2}} - f B_c \rho \dot{\phi}$$

$$\frac{Na}{\sqrt{1 + (4K\rho^3)^2}} = m \left( 4K(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho}) + mg \right)$$

$$m\left(\ddot{\rho} - \rho \times \frac{q^2 B^2}{4m^2} \left(1 + \frac{7}{4} \frac{R_1^4}{\rho^4} + \sqrt{7} \frac{R_1^2}{\rho^2}\right)\right) = -4K\rho^3 \left( 4Km(3\rho^2\dot{\rho}^2 + \rho^3\ddot{\rho}) + mg \right) - \frac{q^2 B^2}{2m} \left( 1 + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} \right)$$

$$\ddot{\rho} \left( m + 16K^2 m \rho^6 \right) = -\frac{4^8}{16} K^2 m \sqrt{7} \rho^5 \dot{\rho}^2 - 4Kmg\rho^3 - \frac{q^2 B^2}{2m} \left( 1 + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} \right) + \rho \frac{q^2 B^2}{2m} \left( 1 + \frac{7}{4} \frac{R_1^4}{\rho^4} + \sqrt{7} \frac{R_1^2}{\rho^2} \right)$$

$$\ddot{\rho} (m + 16k^2 m \rho^6) = -\frac{48}{16} k^2 m \sqrt{\rho} \dot{\rho}^2 - 4kmg\rho^3 - \frac{g^2 B^2}{2m} \left(1 + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2}\right) + \rho \frac{g^2 B^2}{2m} \left(1 + \frac{7}{4} \frac{R_1^4}{\rho^4} + \sqrt{7} \frac{R_1^2}{\rho^2}\right)$$

$$m \delta \ddot{\rho} (1 + 16k^2 R_0^6) = -12kmg R_0^2 \delta \rho - \frac{g^2 B^2}{2m} \left(\delta \rho + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} \delta \rho\right) + \frac{g^2 B^2}{2m} \left(\delta \rho + \frac{7}{4} \frac{R_1^4}{R_1^4} \times -3\delta \rho + \frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} \delta \rho\right) \quad (2)$$

$$m \delta \ddot{\rho} (1 + 16k^2 R_0^6) + 12kmg R_0^2 \delta \rho - \frac{g^2 B^2}{2m} \left(-\frac{21}{4} - \frac{\sqrt{7}}{2}\right) \delta \rho = 0$$

$$\omega^2 = \frac{\frac{g^2 B^2}{2m} \left(\frac{21}{4} + \frac{\sqrt{7}}{2}\right) + 12kmg R_0^2}{1 + 16k^2 R_0^6}$$

$$m \ddot{\rho} (1 + 16k^2 \rho^6) = -48 k^2 m \rho^5 \dot{\rho}^2 - 4kmg\rho^3 + \frac{\rho g^2 B^2}{2m} \left(\frac{\sqrt{7}}{2} \frac{R_1^2}{\rho^2} + \frac{7}{4} \frac{R_1^4}{\rho^4}\right)$$

جواب آخذ  
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