سئلہ 1

$$F = [l]^{\alpha_1}[A]^{\alpha_2}[Y]^{\alpha_3}[\Delta\theta]^{\alpha_4}[\alpha]^{\alpha_5} = L^{\alpha_1}L^{2\alpha_2}(ML^{-1}T^{-2})^{\alpha_3}\theta^{\alpha_4}\theta^{-\alpha_5}$$

الف)

$$\rightarrow \boxed{F = Y l^2 f\left(\alpha\Delta\theta, \frac{A}{l^2}\right)}$$

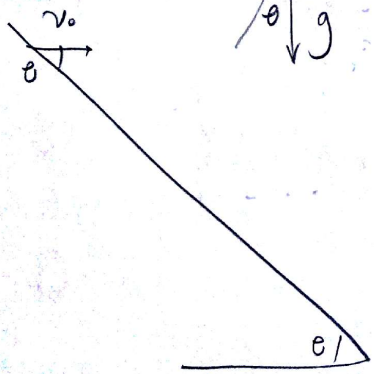
ب)

$$\boxed{F = Y l^2 \alpha\Delta\theta \; f\left(\frac{A}{l^2}\right)}$$

ج)

$$F' = 2F \rightarrow A' = 2A \;\text{یا}\; 1 \rightarrow F = Y l^2 \alpha\Delta\theta \frac{A}{l^2} = \boxed{Y\alpha\Delta\theta A = F}$$

سئلہ 2




الف)

$$S = v_0\cos\theta\, t + \frac{g\sin\theta}{2}t^2 \quad , t = \frac{2v_0\sin\theta}{g\cos\theta}$$

$$S_0 = v_0\cos\theta \times \frac{2v_0\sin\theta}{g\cos\theta} + \frac{g\sin\theta}{2} \times \frac{4v_0^2\sin^2\theta}{g^2\cos^2\theta}$$

$$S_0 = \frac{2v_0^2\sin\theta}{g}(1+\tan^2\theta) = \boxed{\frac{2v_0^2\sin\theta}{g\cos^2\theta} = S_0}$$

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e1 $S_0 = \frac{2v_0 \sin\theta}{g}(1+\tan^2\theta) = \boxed{\frac{2v_0^2 \sin\theta}{g\cos^2\theta} = S_0}$

ب) $v_1 = v_0\cos\theta + g\sin\theta \times \frac{2v_0\sin\theta}{g\cos\theta} = v_0\cos\theta + \frac{2v_0\sin^2\theta}{\cos\theta}$

$S_1 = \frac{2v_0\sin^2\theta}{\cos^2\theta} \times \frac{2v_0^2\sin\theta}{g\cos\theta} + S_0 = \boxed{\frac{2v_0^2\sin\theta}{g}\left(1+\tan^2\theta+2\tan^2\theta\right) = S_1}$

ج) $\frac{S_1}{S_0}$ تغییر می کند

$\boxed{S_n = \frac{2v_0^2\sin\theta}{g}\left(1+\tan^2\theta+2n\tan^2\theta\right)}$

د)

$\boxed{d_n = \frac{2v_0^2\sin\theta}{g}\left(\frac{1}{\cos^2\theta} + \cancel{2}\tan^2\theta\,\frac{n(n-1)}{\cancel{2}}\right)}$

ه)

$$\frac{S_n}{S_{n-1}} = \frac{\frac{1}{\cos^2\theta} + 2n\tan^2\theta}{\frac{1}{\cos^2\theta} + 2(n-1)\tan^2\theta} = \frac{1+\frac{1}{2n\tan^2\theta\cos^2\theta}}{1+\frac{\frac{1}{\cos^2\theta}-2\tan^2\theta}{2n\tan^2\theta}} = 1+\frac{2\tan^2\theta}{2n\tan^2\theta} = \boxed{1+\frac{1}{n}} = \frac{S_n}{S_{n-1}}$$

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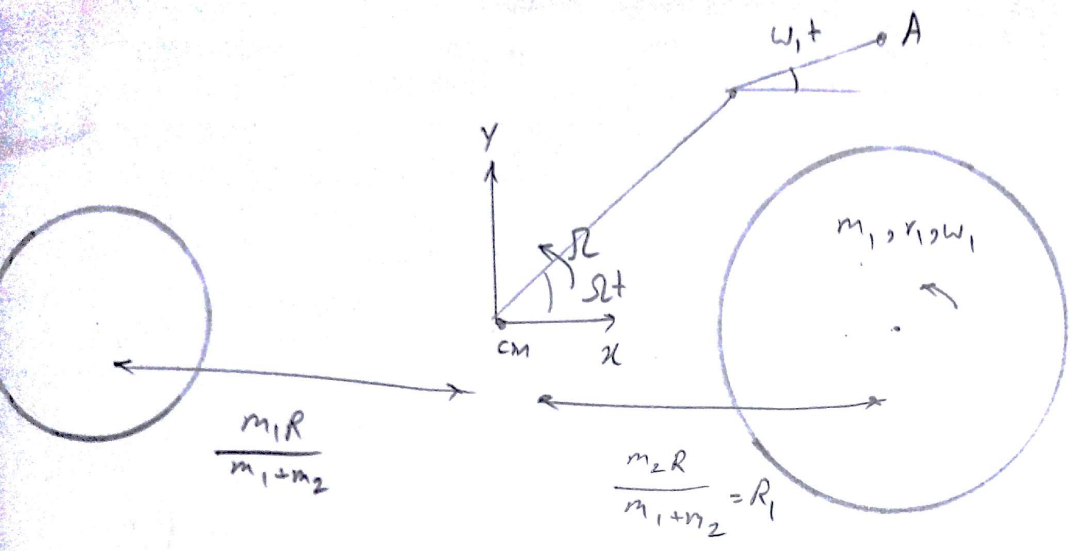


(الف)

$$\vec{r}_A = \left(R_1\cos(\Omega t) + r_1\cos(\omega_1 t)\right)\hat{x} + \left(R_1\sin(\Omega t) + r_1\sin(\omega_1 t)\right)\hat{y}$$

(ب)

$$\vec{v}_A = \left(-\Omega R_1\sin(\Omega t) - r_1\omega_1\sin(\omega_1 t)\right)\hat{x} + \left(R_1\Omega\cos(\Omega t) + r_1\omega_1\cos(\omega_1 t)\right)\hat{y}$$

(ج)

$$\vec{a}_A = \left(-\Omega^2 R\cos(\Omega t) - r_1\omega_1^2\cos(\omega_1 t)\right)\hat{x} + \left(-R_1\Omega^2\sin(\Omega t) - r_1\omega_1^2\sin(\omega_1 t)\right)\hat{y}$$

(د)

$$\rho = \frac{\left(\Omega^2 R_1^2 + r_1^2\omega_1^2 + 2R_1 r_1\omega_1\Omega\left(\cos(\Omega t)\cos(\omega_1 t) + \sin(\Omega t)\sin(\omega_1 t)\right)\right)^{3/2}}{\left(\quad\right)}$$

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$$\rho = \frac{\left(\Omega^2 R_1^2 + r_1^2 \omega_1^2 + 2 R_1 r_1 \omega_1 \Omega \left(\cos(\Omega t)\cos(\omega_1 t) + \sin(\Omega t)\sin(\omega_1 t)\right)\right)^{3/2}}{\left(\Omega^2 R_1 \cos(\Omega t) + r_1 \omega_1^2 \cos(\omega_1 t)\right)\left(R_1 \Omega \cos(\Omega t) + r_1 \omega_1 \cos(\omega_1 t)\right) + \left(\Omega^2 R_1 \sin(\Omega t) + r_1 \omega_1^2 \sin(\omega_1 t)\right)\left(\Omega R_1 \sin(\Omega t) + r_1 \omega_1 \sin(\omega_1 t)\right)}$$

(2)

$$R^2\Omega^3\cos^2(\Omega t) + r_1^2\omega_1^3\cos^2(\omega t) + \Omega^2 R r_1 \omega_1 \cos(\omega t)\cos(\Omega t) + \omega^2 R r_1 \Omega \cos(\omega t)\cos(\Omega t) + \cdots$$

($\cos^2 = 1$; $\cos(\omega t)\cos(\Omega t) \to \cos((\omega-\Omega)t)$; $\cos(\omega t)\cos(\Omega t) \to \cos(\omega - \Omega) +$)

$$\rho = \frac{\left(\Omega^2 R_1^2 + \omega_1^2 r_1^2 + 2 R_1 r_1 \omega_1 \Omega \cos\left((\omega - \Omega)t\right)\right)^{3/2}}{R_1^2\Omega^3 + r_1^2\omega_1^3 + R_1 r_1 \omega_1 \Omega \cos\left((\omega_1 - \Omega)t\right)(\omega_1 + \Omega)}$$

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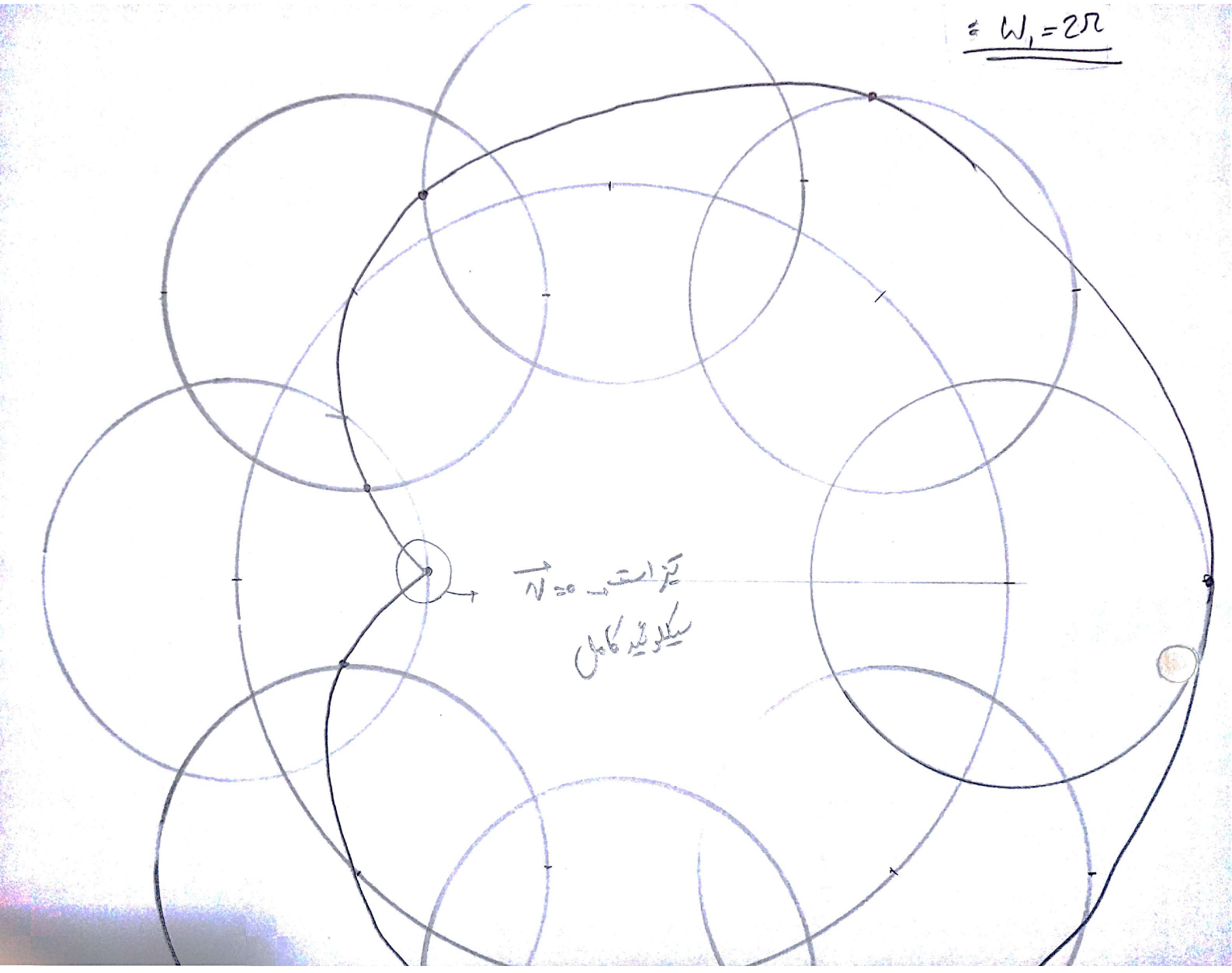
$\omega_1 = 2\pi$

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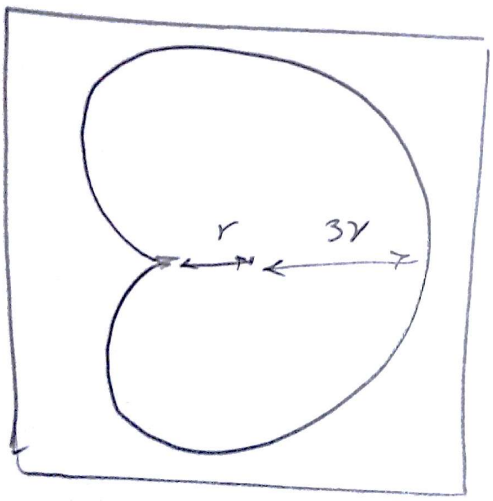


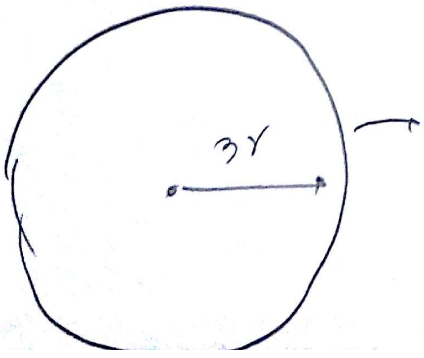


$\therefore W_1 = R$

$\dot{\rho} = 0 \rightarrow \frac{3}{2}\left(\Omega^2 R_1^2 + \omega_1^2 r_1^2 + 2Rr\omega\Omega\cos((\omega-\Omega)t)\right)^{\frac{1}{2}}\left(R_1^2\Omega^3 + r_1^2\omega^3 + R_1 r\omega\Omega(\omega+\Omega)\cos((\omega-\Omega)t)\right)$ (و

$\times -2Rr\omega\Omega(\omega-\Omega)\sin(\omega-\Omega)t = -Rr\omega\Omega(\omega+\Omega)\sin(\omega_1-\Omega)t \times$

$\beta\left(\beta R_1^2\Omega^3 + \beta r_1^2\omega^3 + \beta R_1 r\omega\Omega(\omega+\Omega)\cos(\omega-\Omega)t\right) = (\omega+\Omega)\left(\Omega^2 R_1^2 + \omega^2 r_1^2 + 2Rr\omega\Omega\cos((\omega-\Omega)t)\right)$

$= \Omega^2\omega R^2 + \Omega^3 R^2 + \omega^3 r^2 + \omega^2\Omega r^2$

$\therefore 2r^2\omega\Omega(\omega+\Omega)\cos(\omega-\Omega)t = \Omega^2\omega\cdot 4r^2 + \omega^2\Omega r^2 - 2\times(2r)^2\Omega^3 - 2r^2\omega^3$

$$\cos(\omega-\Omega)t = \frac{4\Omega^2\omega + \omega^2\Omega - 8\Omega^3 - 2\omega^3}{2\omega\Omega(\omega+\Omega)}$$

ل $\sin(\omega-\Omega)t = 0 \rightarrow (\omega-\Omega)t = n\pi$

$R = 4r \qquad R_1 = \frac{R}{2} = 2r$

$\left(\Omega^2 \quad r^2 \quad \Omega^2 \quad r^2 \quad \omega^2\right)^{3/2}$

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$$\rho_{max} = \frac{\left(\Omega^2 \times 4r_1^2 + \omega_1^2 r_1^2 + 4 r_1^2 \omega_1 \Omega\right)^{3/2}}{4 r_1^2 \Omega^3 + r_1^2 \omega_1^3 + 2 r_1^2 \omega_1 \Omega}$$

$$\boxed{\rho_{max} = r_1 \frac{\left(4\Omega^2 + \omega_1^2 + 4\omega_1\Omega\right)^{3/2}}{4\Omega^3 + \omega_1^3 + 2\omega_1\Omega} \quad \because \theta = 0}$$

$$\rho_{min} = \frac{\left(4\Omega^2 + \omega_1^2 + \frac{2\left(4\Omega^2\omega + \omega^2\Omega - 8\Omega^3 - 2\omega^3\right)}{\omega + \Omega}\right)^{3/2}}{4\Omega^3 + \omega_1^3 + 4\Omega^2\omega + \omega^2\Omega - 8\Omega^3 - 2\omega^3} r_1$$

$$= \frac{\left(4\Omega^2\omega + 4\Omega^3 + \omega_1^3 + \Omega\omega_1^2 + 8\Omega^2\omega + 2\omega^2\Omega - 16\Omega^3 - 4\omega^3\right)^{3/2}}{(\omega+\Omega)^{3/2}\left(-4\Omega^3 - \omega^3 + 4\Omega^2\omega + \omega^2\Omega\right)} r_1$$

$$\boxed{= \frac{\left(12\Omega^2\omega - 12\Omega^3 - 3\omega^3 + 3\Omega\omega^2\right)^{3/2}}{(\omega+\Omega)^{3/2}\left(-4\Omega^3 - \omega^3 + 4\Omega^2\omega + \omega^2\Omega\right)} r_1 = \rho_{min} \quad \cos\theta = \text{Ans}}$$

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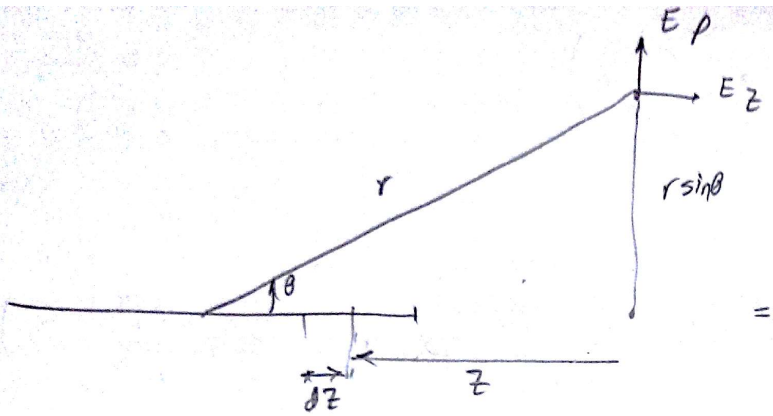
مسئلہ 4

(الف)




$$E_\rho = \frac{Q\,dx\,r\sin\theta}{8\pi\varepsilon_0 a(r^2\sin^2\theta + x^2)^{3/2}}$$

$$= \frac{Q}{8\pi\varepsilon_0 a r\sin\theta} \int_{r\cos\theta - a}^{r\cos\theta + a} \frac{dx}{\left(1 + \frac{x^2}{r^2\sin^2\theta}\right)^{3/2}}$$

$$E_\rho = \frac{Q}{8\pi\varepsilon_0 a r\sin\theta}\left(\frac{r\cos\theta + a}{(r^2 + a^2 + 2ar\cos\theta)^{1/2}} - \frac{r\cos\theta - a}{(r^2 + a^2 - 2ar\cos\theta)^{1/2}}\right)$$

$$E_z = \frac{Q}{8\pi\varepsilon_0 a(r^2\sin^2\theta + x^2)^{3/2}} = \frac{Q}{8\pi\varepsilon_0 a r\sin\theta} \int \frac{\frac{x}{r\sin\theta}\frac{dx}{r\sin\theta}}{\left(1 + \frac{x^2}{r^2\sin^2\theta}\right)^{3/2}}$$

$$= \frac{Q}{8\pi\varepsilon_0 a}\left(\frac{1}{((r\cos\theta - a)^2 + r^2\sin^2\theta)^{1/2}} - \frac{1}{((r\cos\theta + a)^2 + r^2\sin^2\theta)^{1/2}}\right)$$

$$E_z = \frac{Q}{8\pi\varepsilon_0 a}\left(\frac{1}{(r^2 + a^2 - 2ar\cos\theta)^{1/2}} - \frac{1}{(r^2 + a^2 + 2ar\cos\theta)^{1/2}}\right)$$

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$$8\pi\varepsilon_0 a \left( \frac{}{((r\cos\theta - a)^2 + r^2\sin^2\theta)^{1/2}} - \frac{}{((r\cos\theta + a)^2 + r^2\sin^2\theta)^{1/2}} \right)$$

$$\boxed{E_z = \frac{Q}{8\pi\varepsilon_0 a}\left( \frac{1}{(r^2 + a^2 - 2ar\cos\theta)^{1/2}} - \frac{1}{(r^2 + a^2 + 2ar\cos\theta)^{1/2}} \right)}$$

$$E_\rho = \frac{Q}{8\pi\varepsilon_0 a r \sin\theta}\left( \frac{r\cos\theta + a}{r}\left(1 + \frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-1/2} - f(-a) \right) \quad (f(a))$$

(ب)

$$f(a) = \left(\cos\theta + \frac{a}{r}\right)\left(1 - \frac{a}{r}\cos\theta - \frac{a^2}{2r^2} + \frac{3}{2}\left(\frac{4a^2\cos^2\theta}{r^2} + \frac{4a^3\cos\theta}{r^3}\right) - \frac{5}{2} \times \frac{8a^3}{r^3}\cos^3\theta\right)$$

$$\frac{f(a) + f(-a)}{2} = -\frac{a}{r}\cos^2\theta + \frac{3}{2}\frac{a^3\cos^2\theta}{r^3} - \frac{5}{2}\frac{a^3}{r^3}\cos^4\theta + \frac{a}{r} - \frac{a^3}{2r^3} + \frac{3}{2}\frac{a^3\cos^2\theta}{r^3}$$

$$E_\rho = \frac{Q}{4\pi\varepsilon_0 r^2 \sin\theta}\left( \sin^2\theta + \frac{a^2}{r^2}\left(3\cos^2\theta - \frac{5}{2}\cos^4\theta - \frac{1}{2}\right)\right)$$

$$E_\rho = \frac{Q}{4\pi\varepsilon_0 r^2}\left( \sin\theta + \frac{a^2}{r^2}\left(\frac{3\cos^2\theta}{2\sin\theta} - \frac{5\cos^4\theta}{2\sin\theta} - \frac{1}{2\sin\theta}\right)\right)$$

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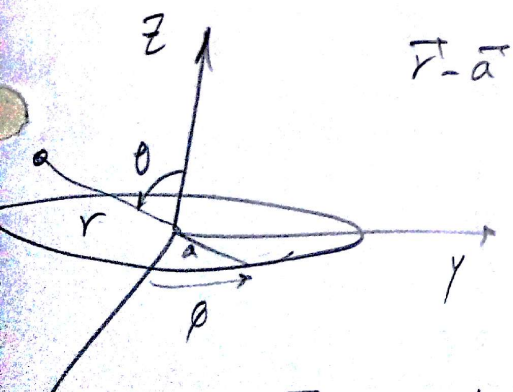
$$E_z = \frac{Q}{8\pi\varepsilon_0 r}\left( \underbrace{\left(1 - \frac{2a}{r}\cos\theta + \frac{a^2}{r^2}\right)^{-1/2}}_{G(a)} - G(-a) \right)$$

$$\frac{G(a) - G(-a)}{2} = \frac{a}{r}\cos\theta + \frac{3}{8} \times \frac{4a^2}{r^2}\cos^2\theta - \frac{3}{8} \times \frac{4a}{r}\cos\theta \times \frac{a^2}{r^2} + \frac{5}{16} \times 8\frac{a^3}{r^3}\cos^3\theta$$

$$\therefore E_z = \frac{Q}{4\pi\varepsilon_0 r^2}\left(\cos\theta + \frac{a^2}{2r^2}(-3\cos\theta + 5\cos^3\theta)\right)$$

$$\vec{a} = a(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}) \qquad \vec{r} = r(\sin\theta\,\hat{x} + \cos\theta\,\hat{z})$$




$$\vec{r} - \vec{a} = (r\sin\theta - a\cos\varphi)\hat{x} - a\sin\varphi\,\hat{y} + r\cos\theta\,\hat{z}$$

$$\vec{E} = \frac{Q\,d\varphi}{2\pi \times 4\pi\varepsilon_0 r^3}\left(\frac{(r\sin\theta - a\cos\varphi)\hat{x} + r\cos\theta\,\hat{z} - a\sin\varphi\,\hat{y}}{\left(1 - \frac{2a}{r}\sin\theta\cos\varphi + \frac{a^2}{r^2}\right)^{3/2}}\right)$$

$$\vec{E} = \frac{Q\,d\varphi}{\quad}\left( \quad \right)$$

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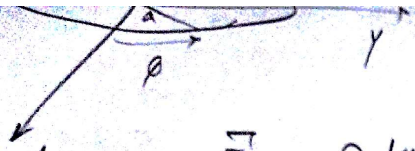


$$E = \frac{q}{2\pi \times 4\pi\varepsilon_0 r^3}\left(\frac{(r\sin\theta - a\cos\varphi)\hat{x} + r\cos\theta\hat{z} - a\sin\varphi\hat{y}}{\left(1 - \frac{2a}{r}\sin\theta\cos\varphi + \frac{a^2}{r^2}\right)^{3/2}}\right)$$

$$\vec{E} = \frac{Q\,d\varphi}{8\pi^2\varepsilon_0 r^2}\left(\left(\sin\theta - \frac{a}{r}\cos\varphi\right)\hat{x} + \cos\theta\hat{z}\right)\left(1 + \frac{3a}{r}\sin\theta\cos\varphi - \frac{3}{2}\frac{a^2}{r^2} + \frac{15}{8}\times\frac{4a^2}{r^2}\sin^2\theta\times\cos^2\varphi\right)$$

$$\vec{E} = \frac{Q}{8\pi^2\varepsilon_0 r^2}\left(\left(2\pi\sin\theta - \frac{3}{2}\frac{a^2}{r^2}\sin\theta\times 2\pi + \frac{15}{2}\frac{a^2}{r^2}\sin^3\theta\times\pi - \frac{3a^2}{r^2}\sin\theta\times\pi\right)\hat{x}\right.$$
$$\left. + \left(\cos\theta\times 2\pi - \frac{3}{2}\times 2\pi\frac{a^2}{r^2}\cos\theta + \frac{15}{2}\frac{a^2}{r^2}\sin^2\theta\cos\theta\times\pi\right)\hat{z}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}\left(\left(\sin\theta - \frac{3}{2}\frac{a^2}{r^2}\sin\theta + \frac{15}{4}\frac{a^2}{r^2}\sin^3\theta - \frac{3a^2}{r^2}\sin\theta\right)\hat{\rho} + \left(\cos\theta - \frac{3}{2}\frac{a^2}{r^2}\cos\theta + \frac{15}{4}\frac{a^2}{r^2}\sin^2\theta\cos\theta\right)\hat{z}\right)$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\times\frac{a^2}{r^2}\left[\left(\frac{3\cos^2\theta}{\sin\theta} - \frac{5}{2}\frac{\cos^4\theta}{\sin\theta} - \frac{1}{2\sin\theta} - \frac{15}{4}\sin^3\theta + 3\sin\theta\right)\hat{\rho}\right.$$
$$\left. + \left(-\frac{3\cos\theta}{2} + \frac{5}{2}\cos^3\theta + \frac{3}{2}\cos\theta - \frac{15}{4}\sin^2\theta\cos\theta\right)\hat{z}\right]$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{a^2}{r^4}\left[\left(\frac{5}{2\sin\theta} - \frac{5}{2}\frac{\cos^4\theta}{\sin\theta} - \frac{15}{4}\sin^3\theta\right)\hat{\rho} + \left(\frac{5}{2}\cos^3\theta - \frac{15}{4}\sin^2\theta\cos\theta\right)\hat{z}\right]$$

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آزمون 2 سال 92

سئله 1) الف)

$$[H] = [K]^{\alpha_1}[c]^{\alpha_2}[v]^{\alpha_3}[R]^{\alpha_4}[\Delta T]^{\alpha_5}$$

$$= (HL^{-1}T^{-1}\theta^{-1})^{\alpha_1}(HL^{-3}\theta^{-1})^{\alpha_2}(LT^{-1})^{\alpha_3}L^{\alpha_4}\theta^{\alpha_5}$$

$$= HT^{-1}$$

$$\begin{cases} \alpha_1+\alpha_2=1 \\ -\alpha_1-3\alpha_2+\alpha_3+\alpha_4=0 \\ -\alpha_1-\alpha_3=-1 \\ -\alpha_1-\alpha_2+\alpha_5=0 \end{cases}$$

$\alpha_2 = 1-\alpha_1$

$-\alpha_1-3+3\alpha_1+1-\alpha_1+\alpha_4=0 \rightarrow \alpha_4 = 2-\alpha_1$

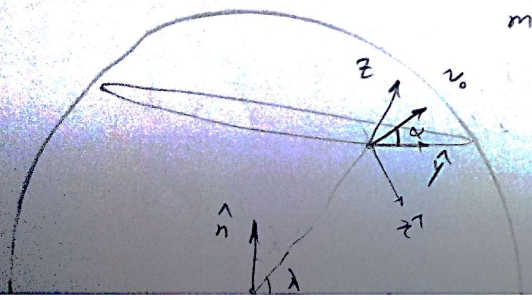
$\alpha_3 = 1-\alpha_1$

$-\alpha_1-1+\alpha_1+\alpha_5=0 \rightarrow \alpha_5 = 1$

$$H = K^{\alpha_1}(cv)^{1-\alpha_1}R^{2-\alpha_1}\Delta T = \left(\frac{K}{RCv}\right)^{\alpha_1} CvR^2\Delta T = H \rightarrow \boxed{H = CvR^2\Delta T f\left(\frac{K}{RCv}\right)}$$

$$\frac{h_1}{h_2} = \frac{2\times 3^2}{3\times 2^2} = \boxed{\frac{3}{2} = \frac{h_1}{h_2}}$$

ب)




سئله 2)

$$m\ddot{\vec{r}} = -mg\hat{k} - 2m\omega(-\hat{i}\cos\lambda + \hat{k}\sin\lambda) \times (v_0\cos\alpha\,\hat{j} + (v_0\sin\alpha - gt)\hat{k})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\cos\lambda & 0 & \sin\lambda \end{vmatrix}$$

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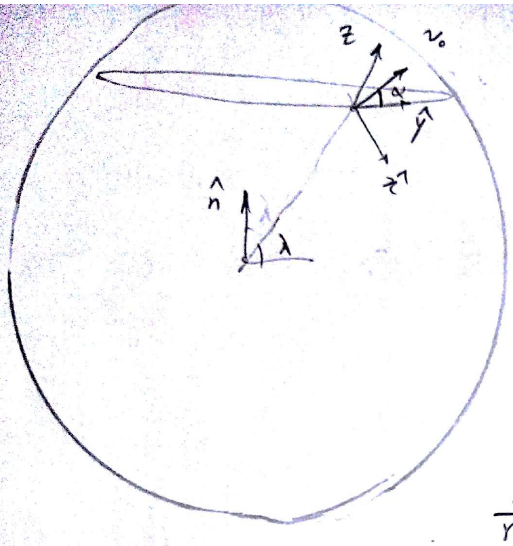


$-2m\omega(-\cos\lambda\,\hat{j}+\hat{k}\sin\lambda)\times(v_0\cos\alpha\,\hat{j}+(v_0\sin\alpha-gt)\hat{k})$ (2 MI

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\cos\lambda & 0 & \sin\lambda \\ 0 & v_0\cos\alpha & v_0\sin\alpha - gt \end{vmatrix} = -v_0\cos\alpha\sin\lambda\,\hat{i} + \cos\lambda(v_0\sin\alpha - gt)\,\hat{j} - v_0\cos\alpha\cos\lambda\,\hat{k}$$

$$\ddot{\vec{r}} = -g\hat{k} - 2\omega(-v_0\cos\alpha\sin\lambda\,\hat{i} + \cos\lambda(v_0\sin\alpha - gt)\,\hat{j} - v_0\cos\alpha\cos\lambda\,\hat{k})$$

$$\dot{\vec{r}} = (-gt + v_0\sin\alpha)\hat{k} + v_0\cos\alpha\,\hat{j} - 2\omega(-v_0\cos\alpha\sin\lambda\,t\,\hat{i} + \hat{j}\cos\lambda(v_0\sin\alpha\,t - \frac{gt^2}{2}) - \hat{k}\,v_0\cos\alpha\cos\lambda\,t)$$

$$\vec{r} = (-\frac{gt^2}{2} + v_0\sin\alpha\,t)\hat{k} + v_0\cos\alpha\,t\,\hat{j} - 2\omega(-v_0\cos\alpha\sin\lambda\frac{t^2}{2}\hat{i} + \hat{j}\cos\lambda(v_0\sin\alpha\frac{t^2}{2} - \frac{gt^3}{6}) - \hat{k}\,v_0\cos\alpha\cos\lambda\frac{t^2}{2})$$

$$\rightarrow -gt^{(0)}t^{(1)} + v_0\sin\alpha\,t^{(1)} + 2\omega v_0\cos\alpha\cos\lambda\frac{t^{(1)2}}{2} = 0 \qquad t^{(0)} = \frac{2v_0\sin\alpha}{g}$$

$$-t^{(1)}v_0\sin\alpha + \omega v_0\cos\alpha\cos\lambda\frac{4v_0^2\sin^2\alpha}{g^2} = 0 \rightarrow t^{(1)} = \frac{4\omega v_0^2\sin\alpha\cos\alpha\cos\lambda}{g^2}$$

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$$R = v_0 \cos\alpha \left( \frac{2v_0 \sin\alpha}{g} + \frac{4\omega v_0^2 \sin\alpha \cos\alpha \cos\lambda}{g^2} \right) - 2\omega \cos\lambda \left( \frac{v_0 \sin\alpha}{2} \times \frac{4v_0^2 \sin^2\alpha}{g^2} - \frac{g}{6} \times \frac{8v_0^3 \sin^3\alpha}{g^3} \right)$$

$$= \frac{2v_0^2 \sin\alpha\cos\alpha}{g} + \frac{4\omega v_0^3 \sin\alpha \cos^2\alpha \cos\lambda}{g^2} - 2\omega\cos\lambda \times \frac{2}{3} \frac{v_0^3 \sin^3\alpha}{g^2}$$

$$= \boxed{\frac{2v_0^2 \sin\alpha\cos\alpha}{g} + \frac{4\omega v_0^3}{g^2} \sin\alpha\cos\lambda \left( \cos^2\alpha - \frac{\sin^2\alpha}{3} \right) = R}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g} + \frac{4\omega v_0^3}{g^2} \cos\lambda \left( \sin\alpha \cos^2\alpha - \frac{\sin^3\alpha}{3} \right)$$

(ب)

$$0 = \frac{2v_0^2 \cos 2\alpha}{g} + \frac{4\omega v_0^3}{g^2} \cos\lambda \left( \cos^3\alpha - \sin^2\alpha \times 2\cos\alpha - \sin^2\alpha\cos\alpha \right)$$

$$\rightarrow \alpha = \frac{\pi}{4} + \delta\alpha$$

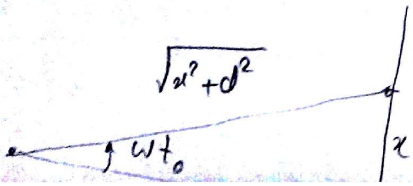
$$-\sin(2\delta\alpha) = \frac{2v_0\omega}{g}\cos\lambda \times \left( \frac{3}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \right) = \frac{2v_0\omega\cos\lambda}{\sqrt{2}g} = -2\delta\alpha$$

$$\rightarrow \boxed{\delta\alpha = \frac{-v_0\omega\cos\lambda}{\sqrt{2}g}}$$

$$x = d\tan(\omega t) \rightarrow \dot{x} = \frac{\omega d}{\cos^2(\omega t)} = \omega d \left( 1 + \frac{x^2}{d^2} \right)$$

سؤال 3

(الف)




$$\rightarrow \boxed{\dot{x} = \omega d \left( 1 + \frac{x^2}{d^2} \right)}$$

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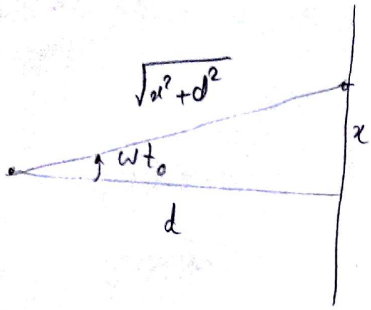
$x = d\tan(\omega t) \rightarrow \dot{x} = \frac{\omega d}{\cos^2(\omega t)} = \omega d\left(1+\frac{x^2}{d^2}\right)$ سئلہ 3

(الف)




$$\dot{x} = \omega d\left(1+\frac{x^2}{d^2}\right)$$

(ب)

$$t' = t_0 + \frac{d}{c\cos(\omega t_0)}$$

$$x_0 = d\tan(\omega t_0)$$

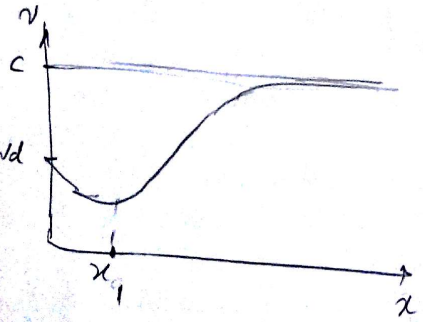
(ج)

$x(t') = d\tan(\omega t_0) \rightarrow \dot{x} = \frac{dx}{dt'} = \frac{dx}{dt} \times \frac{dt}{dt'}$

$t' = t_0 + \frac{d}{c\cos(\omega t_0)}$ $\quad \frac{dt'}{dt} = 1 + \frac{\omega d\sin(\omega t)}{c\cos^2(\omega t)} \rightarrow \dot{x} = \frac{\omega d}{\cos^2(\omega t)} \frac{1}{1+\frac{\omega d\sin(\omega t)}{c\cos^2(\omega t)}}$

$$\dot{x} = \frac{\omega d}{\cos^2(\omega t) + \frac{\omega d}{c}\sin(\omega t)}$$

$$\dot{x} = \frac{\omega d\left(1+\frac{x^2}{d^2}\right)}{1+\frac{\omega d}{c} \times \frac{\frac{x}{\sqrt{x^2+d^2}}}{\frac{d^2}{x^2+d^2}}} = \frac{\omega d\left(1+\frac{x^2}{d^2}\right)}{1+\frac{\omega x\sqrt{x^2+d^2}}{dc}} = \dot{x}$$




$x \rightarrow \infty \rightarrow \dot{x} = c\left(\frac{d\dot{x}}{dx}\right)_{x=0} = \frac{\omega d}{1+\frac{\omega x}{dc}} = \omega d\left(1-\frac{\omega x}{dc}\right)$

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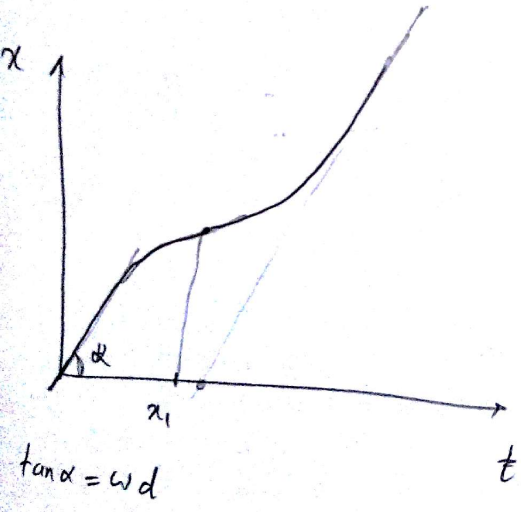
$$\frac{dx}{dx} = 0 \rightarrow \frac{d\dot{x}}{dt_0} \times \frac{dt_0}{dx} = 0 \qquad x = d\tan(\omega t_0) \rightarrow \frac{dx}{dt_0} = \frac{\omega d}{\cos^2(\omega t_0)}$$

$$\frac{dt_0}{dx} = 0 \rightarrow \frac{\cos^2(\omega t)}{\omega d} = 0 \rightarrow \omega t = \frac{\pi}{2} \quad \text{זמן}$$

$$\rightarrow \frac{d}{dt}\left(\cos^2(\omega t) + \frac{\omega d}{c}\sin(\omega t)\right) = 0 \rightarrow -2\cos(\omega t_0)\sin(\omega t_0) + \frac{\omega d}{c}\cos(\omega t_0) = 0$$

$$\rightarrow \sin(\omega t_0) = \frac{\omega d}{2c} \rightarrow x = d\,\frac{\frac{\omega d}{2c}}{\sqrt{1 - \frac{\omega^2 d^2}{4c^2}}} = \boxed{\frac{\omega d^2}{\sqrt{4c^2 - \omega^2 d^2}} = x_1}$$




$\tan\alpha = \omega d$

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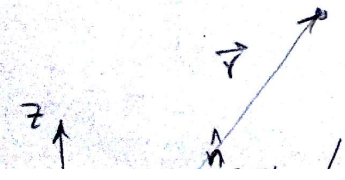


$\hat{n} = \cos\alpha\,\hat{z} - \sin\alpha\,\hat{\rho}$

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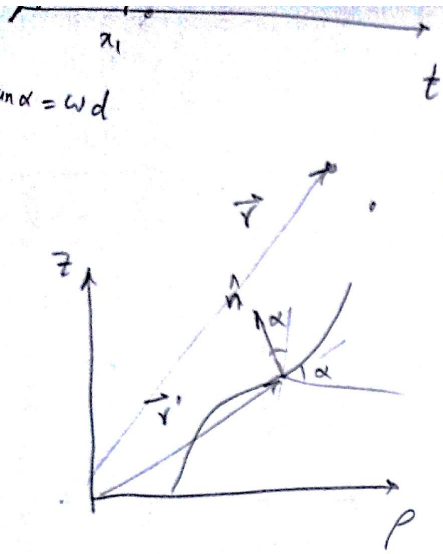



$\tan\alpha = \omega d$




$\hat{n} = \cos\alpha\,\hat{z} - \sin\alpha\,\hat{\rho}$

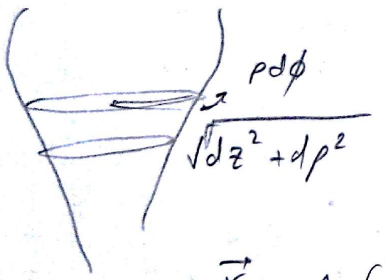
$$\hat{n} = \frac{\hat{z}}{\sqrt{1+f'^2}} - \frac{f'\hat{\rho}}{\sqrt{1+f'^2}}$$

مسئله ۴

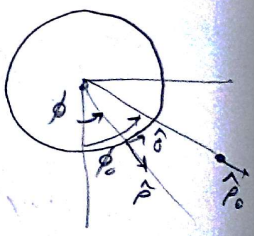
(الف)

(ب)

$$\phi = \int \frac{D\left(\frac{\hat{z}}{\sqrt{1+f'^2}} - \frac{f'\hat{\rho}}{\sqrt{1+f'^2}}\right)\cdot(\vec{r}-\vec{r}')}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}'|^3} \times \rho\,d\phi \times d\rho\,\sqrt{1+f'(\rho)^2}$$




$$\phi = \frac{D}{4\pi\varepsilon_0} \int \frac{(\hat{z}-f'\hat{\rho})\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,\rho\,d\rho\,d\phi$$




$\vec{r} = \rho_0\,\hat{\rho}_0 + z_0\,\hat{z}$ ، $\vec{r}' = \rho\,\hat{\rho} + f\,\hat{z}$

$$\vec{r} = \rho_0\left(\cos(\phi_0-\phi)\,\hat{\rho} + \sin(\phi_0-\phi)\,\hat{\theta}\right) + z_0\,\hat{z}$$

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$$V = \frac{D}{4\pi\varepsilon_0} \int \frac{(\hat{z} - f'\hat{\rho}) \cdot \left[(-\rho + \rho_0 \cos(\phi_0 - \phi))\hat{\rho} + (z_0 - f)\hat{z}\right]}{\left|(\rho_0 \cos(\phi_0 - \phi) - \rho)\hat{\rho} + \rho_0 \sin(\phi_0 - \phi)\hat{\theta} + (z_0 - f)\hat{z}\right|^3} \rho\, d\rho\, d\phi$$

$$\boxed{V = \frac{D}{4\pi\varepsilon_0} \int \frac{\left[z_0 - f - f'(-\rho + \rho_0 \cos(\phi_0 - \phi))\right] \rho\, d\rho\, d\phi}{\left(\rho_0^2 + \rho^2 - 2\rho_0\rho\cos(\phi_0 - \phi) + (z_0 - f)^2\right)^{3/2}}}$$

$$\begin{cases} f = -R\cos\theta \\ \rho = R\sin\theta \end{cases} \qquad f' = R\sin\theta \times \frac{d\theta}{d\rho} = \frac{R\sin\theta}{R\cos\theta} = \tan\theta$$

(ب)

$$\rightarrow V = \frac{2\pi D}{4\pi\varepsilon_0} \int \frac{R\cos\theta - \tan\theta \times -R\sin\theta}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}} R^2 \sin\theta\cos\theta\, d\theta$$

$$= \frac{D}{2\varepsilon_0} \int \left(\cos^2\theta + \frac{\sin^2\theta}{\cos\theta}\right) \sin\theta\sec\theta\, d\theta = \frac{D}{2\varepsilon_0} \int \sin\theta\, d\theta = \frac{D}{2\varepsilon_0}(1 - \cos\theta) = \boxed{\frac{D}{2\varepsilon_0} = V}$$

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$$= \frac{D}{2\varepsilon_0}\int\left(\cos^2\theta + \frac{\sin^2\theta}{\cos\theta}\right)\sin\theta\csc\theta\,d\theta = \frac{D}{2\varepsilon_0}\int\sin\theta\,d\theta = \frac{D}{2\varepsilon_0}(1-\cos\theta) = \boxed{\frac{D}{2\varepsilon_0} = V}$$

$$V = \frac{D\times 2\pi}{4\pi\varepsilon_0}\int\frac{(z_0+\alpha-\beta\rho-\beta\alpha-\rho)\rho\,d\rho}{\left(\rho^2+(z_0+\alpha-\beta\rho)^2\right)^{\frac{3}{2}}} = \frac{D(z_0+\alpha)}{2\varepsilon_0}\int\frac{\rho\,d\rho}{\underbrace{\left(\rho^2+(z_0+\alpha-\beta\rho)^2\right)^{3/2}}_{F}}$$ (ans)

$$F = \rho^2+(z_0+\alpha-\beta\rho)^2 = \rho^2(1+\beta^2)+(z_0+\alpha)^2-2\beta\rho(z_0+\alpha) = (1+\beta^2)\left(\rho^2-\frac{2\beta\rho(z_0+\alpha)}{1+\beta^2}\right)+(z_0+\alpha)^2$$

$$= (1+\beta^2)\left(\rho-\frac{\beta(z_0+\alpha)}{1+\beta^2}\right)^2 - \cancel{(1+\beta^2)}\frac{\beta^2(z_0+\alpha)^2}{\cancel{(1+\beta^2)^2}} + (z_0+\alpha)^2$$

$$= (1+\beta^2)\left(\rho-\frac{\beta(z_0+\alpha)}{1+\beta^2}\right)^2 + \frac{(z_0+\alpha)^2}{1+\beta^2} = A(\rho-B)^2+C$$

$$\int\frac{\rho\,d\rho}{(A(\rho-B)^2+C)^{3/2}} = \frac{1}{C^{3/2}\times\frac{A}{C}}\int\frac{\frac{A}{C}\rho\,d\rho}{\left(\frac{A}{C}(\rho-B)^2+1\right)^{3/2}} = \frac{1}{A\sqrt{C}}\int_{-\infty}^{\infty}\frac{\left(\sqrt{\frac{A}{C}}(\rho-B)+B\sqrt{\frac{A}{C}}\right)\sqrt{\frac{A}{C}}\,d\rho}{\left(\frac{A}{C}(\rho-B)^2+1\right)^{3/2}}$$

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$$\rightarrow \frac{1}{A\sqrt{C}}\left(-\frac{1}{\left(\frac{A}{C}(\rho-B)^2+1\right)^{1/2}}+B\sqrt{\frac{A}{C}}\cdot\frac{\sqrt{\frac{A}{C}}(\rho-B)}{\left(\frac{A}{C}(\rho-B)^2+1\right)^{1/2}}\right)\Bigg|_0^{\infty}$$

$$=\frac{1}{A\sqrt{C}}\left(\frac{1}{\left(\frac{A}{C}B^2+1\right)^{1/2}}+\frac{B^2\frac{A}{C}}{\left(\frac{A}{C}B^2+1\right)^{1/2}}+B\sqrt{\frac{A}{C}}\right)$$

$$=\frac{1}{A\sqrt{C}}\left(\sqrt{1+\frac{A}{C}B^2}+B\sqrt{\frac{A}{C}}\right)$$

$$\frac{A}{C}B^2=\frac{(1+\beta^2)\,\beta^2(z_0+\alpha)^2}{(1+\beta^2)^2(z_0+\alpha)^2}=\beta^2$$

$$\frac{1}{A\sqrt{C}}=\frac{1\times\sqrt{1+\beta^2}}{(1+\beta^2)(z_0+\alpha)}=\frac{1}{\sqrt{1+\beta^2}(z_0+\alpha)}$$

$$B\sqrt{\frac{A}{C}}=\frac{\beta(z_0+\alpha)}{1+\beta^2}\times\frac{(1+\beta^2)^{1/2}(1+\beta^2)^{1/2}}{(z_0+\alpha)}$$

$$=\beta$$

$$\rightarrow=\frac{1}{\sqrt{1+\beta^2}(z_0+\alpha)}\left(\sqrt{1+\beta^2}+\beta\right)$$

$$\boxed{V=\frac{D}{2\varepsilon_0}\times\frac{\sqrt{1+\beta^2}+\beta}{\sqrt{1+\beta^2}}}$$

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$$= \frac{1}{A\sqrt{C}} \left( \frac{}{\left(\frac{A}{C}B^2+1\right)^{1/2}} + \frac{}{\left(\frac{A}{C}B^2+1\right)^{1/2}} + B\sqrt{\frac{}{C}} \right)$$

$$= \frac{1}{A\sqrt{C}} \left( \sqrt{1+\frac{A}{C}B^2} + B\sqrt{\frac{A}{C}} \right)$$

$$\frac{A}{C}B^2 = \frac{(1+\beta^2)\,\beta^2(z_0+\alpha)^2}{(1+\beta^2)^2\,(z_0+\alpha)^2} = \beta^2$$

$$\frac{1}{A\sqrt{C}} = \frac{1}{(1+\beta^2)(z_0+\alpha)}\sqrt{1+\beta^2} = \frac{1}{\sqrt{1+\beta^2}\,(z_0+\alpha)}$$

$$\rightarrow B\sqrt{\frac{A}{C}} = \frac{\beta(z_0+\alpha)}{1+\beta^2} \times \frac{(1+\beta^2)^{1/2}(1+\beta^2)^{1/2}}{(z_0+\alpha)}$$

$$= \beta$$

$$\rightarrow = \frac{1}{\sqrt{1+\beta^2}\,(z_0+\alpha)} \left( \sqrt{1+\beta^2} + \beta \right)$$

$$V = \frac{D}{2\varepsilon_0} \times \frac{\sqrt{1+\beta^2}+\beta}{\sqrt{1+\beta^2}}$$

(ج)

$$V = 0$$

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