The Coil Misalignment Model of Inductively Coupled Wireless Power Transfer System: Mutual Inductance Analysis and Transfer Efficiency Optimization

X. L. Huang¹, H. Qiang¹, ², and L. L. Tan¹

¹School of Electrical Engineering, Southeast University, Nanjing 210096, China
²School of Information Science and Engineering, Changzhou University, Changzhou 213164, China

Abstract—A novel means of optimizing transfer efficiency of Inductively Coupled wireless power transfer (WPT) system is presented for the first time. In the applications envisaged, such as electric vehicles and biomedical implants, generally the receiving coil is laterally and angularly misaligned from the transmitting coil. The numerical solution of mutual inductance between the two coils is derived and simulation result shows there is a partial optimal solution in the ranges of lateral and angular misalignments. The bigger is mutual inductance, the higher transfer efficiency is. The novel means presented in this study allows the coil to be removable and rotatable for achieving bigger mutual inductance and optimizing transfer efficiency. Finally the experimental results show the means is efficient and greatly improves the transfer efficiency. If the transfer distance is 15 cm and there is no lateral misalignment, the transfer efficiency can be improved about from 38.6% to 60.5% by turning the transmitting coil to make the tilt angle varied from 0 to \(\pi/4\). This introduced technique can be widely applied to WPT system to optimize the transfer efficiency.

1. INTRODUCTION

As the development of mobile devices such as electric vehicles and portable devices [1–3], the requirement of wireless charging made WPT technology become increasing important. Inductive coupling and resonant coupling are two major means, and in inductive or magnetic coupled systems, power is transferred from a primary transmitter (Tx) coil to a secondary receiver (Rx) coil with the aid of an alternating magnetic field as shown in Fig. 1. In the applications envisaged, such as electric vehicle and biomedical implants [4], the misalignment between the two coils including lateral displacement and angular tilt generally took place. And the impact of coil misalignment on the transfer efficiency has received little attention by researchers.

The aim of this work is to optimize the transfer efficiency of inductive coupled WPT system. A novel means is presented based on the mutual inductance analysis of coil misalignment. In experiments, the primary Tx coil is designed to be removable and rotatable for obtaining the partial optimal solution of mutual inductance so that optimizing the transfer efficiency. The experimental results show that if the transfer distance is 15 cm and there is no lateral misalignment, the transfer efficiency can be improved about from 38.6% to 60.5% by turning the transmitting coil to make the tilt angle varied from 0 to \(\pi/4\). And if the lateral misalignment occurs, changing the tilt angle can also change the transfer efficiency and the optimal efficiency is according to the partial maximum mutual inductance.

Figure 1: Structure of the wireless power transfer system.

Figure 2: Model of the wireless power transfer system.
2. INDUCTIVE COUPLED WPT SYSTEM MODEL

In the inductive coupled WPT operation represented by the equivalent circuit of Fig. 2 [5, 6], \( d \) is the coil separation distance, \( M \) is the mutual induction of two coils, \( U_{in} \) is the induced potential source; \( R_P, R_S, C_P \) and \( C_S \) are the parasitic parameters in HF and \( L_P, L_S \) are the self-inductances of the two coils (the subscripts “\( P \)” and “\( S \)” stand for the primary and secondary respectively); \( R_L \) stands for the load resistance. The \( Rx \) coil is situated within the near-field of the \( Tx \) coil and the interaction between two coils is considered as magnetoquasistatic [7].

Suppose the system operating angular frequency equals to \( \omega \), the KVL equations of primary and secondary circuits are easily deduced from Fig. 2.

\[
\begin{bmatrix}
    R_P + jX_P & j\omega M \\
    j\omega M & R_S + R_L + jX_S
\end{bmatrix}
\begin{bmatrix}
    I_P \\
    I_S
\end{bmatrix} =
\begin{bmatrix}
    U_{in} \\
    0
\end{bmatrix}, \quad \begin{cases}
    X_P = \omega L_P - 1/(\omega C_P) \\
    X_S = \omega L_S - 1/(\omega C_S)
\end{cases}
\]

The physical meaning of the power transfer efficiency studied in this paper is defined as the ratio of the output power \( P_{out} \) in the load \( R_L \) to the input power \( P_{in} \) generated by the \( Tx \) coil, denoted \( \eta \).

\[
P_{in} = \frac{u_{in}^2 Z_S}{Z_P Z_S + (\omega M)^2}, \quad P_{out} = \frac{u_{in}^2 (\omega M)^2 R_L}{Z_P Z_S + (\omega M)^2}, \quad \eta = \frac{P_{out}}{P_{in}} = \frac{(\omega M)^2 R_L}{Z_S Z_P Z_S + (\omega M)^2} \times 100\%
\]

When resonance occurs in \( Tx \) and \( Rx \) circuits, the maximum power is delivered to the resistive component of the load and the losses of the reactive components are canceled [8, 9]. Then \( \eta \) can be rewrite as

\[
\eta = \frac{\omega M^2 R_L}{(R_S + R_L)[R_P(R_S + R_L) + (\omega M)^2]} \times 100\%.
\]

From (3), the impact of mutual inductance on transfer efficiency is obvious and big mutual inductance implies high efficiency. The mutual inductance \( M \) can be derived by solving the double integral in Neumann’s formula.

\[
M = \frac{N_{Tx} N_{Rx} \mu_0}{4\pi} \int_{l_{Tx}} \int_{l_{Rx}} \frac{dl_{Tx} \cdot dl_{Rx}}{R}
\]

where \( N_{Tx}, N_{Rx}, l_{Tx}, l_{Rx}, dl_{Tx} \) and \( dl_{Rx} \) define the coil turns, the length of each turn and infinitesimal of \( l \) of the resonant \( Tx \) and \( Rx \) coils, respectively. \( R \) is the distance between \( dl_{Tx} \) and \( dl_{Rx} \), and \( \mu_0 \) is the magnetic permeability of free-space.

It is evident that the mutual inductance \( M \) of (4) depends on the shapes and the orientations of the two coils. In this paper, the impact of coil orientation on the mutual inductance is studied in Section 3 that follows.

3. MISALIGNMENT AND MUTUAL INDUCTANCE ANALYSIS

Clearly, coil orientation is a key parameter in the design of the inductively coupled systems. In practical application, coil misalignment is normal. And there are three following forms of misalignment:

1) Lateral misalignment: In this case the pair of \( Tx \) and \( Rx \) coils are situated in parallel planes, which are separated by a vertical distance \( d \) and their centers are displaced by a lateral distance \( l \), as shown in Fig. 3(a).

2) Angular misalignment: In this case the plane of the \( Tx \) coil is tilted to form an angle \( \theta \) and the axis of one coil passes through the center of the other coil, as shown in Fig. 3(b).

3) Incorporated misalignment: This case incorporates both lateral displacement and angular tilt of the coils, as shown in Fig. 3(c).

A general misalignment case is the third case presented above. Suppose that the projection of \( O_2 \) in the \( Tx \) plane is \( O_2' \), using the vector \( O_1O_2' \) to indicate the direction of the \( x \) axis we can establish the Cartesian coordinate system, shown in Fig. 3(c). Then for mutual inductance analysis and efficiency optimization, we assume the angle \( \theta \) is formed around the \( x \) axis. In paper [10], Soma had demonstrated that there is no strong interaction between the lateral misalignment effect
and the angular misalignment effect. And paper [11] had studied the two displacement configurations independently for the maximum permissible angular and lateral displacements for different applications.

Any theoretical investigation of the mutual inductance in the misalignment cases is extremely complex due to the heavy work required to solve the double integral as (4) and the analytical solution is almost impossible to be derived. Hence, in this study, Matlab is used to obtain the numerical solution of the mutual inductance in varied misalignment cases with the separate distance of 15 cm as shown in Fig. 4.

In Fig. 4, the positive and negative of \( l \) and \( \theta \) just stand for the position in the special coordinate system and the direction of rotation (anticlockwise or clockwise), respectively. It is evident that in lateral misalignment, the mutual inductance is become smaller with the increase of the lateral distance \( l \). And in incorporated misalignment, the mutual inductance is varied with the lateral distance and tilt angle, and there are partial optimal solutions in their ranges shown in Fig. 4(b). It is also worth noting that with a specific separate distance, the peaks of the mutual inductance in Fig. 4(b) do not occur in perfect alignment. This implies that in inductively coupled WPT system, there are some constraints to guarantee the maximum power transfer occurs in perfect alignment.

4. EXPERIMENTAL RESULTS

In order to verify the above theory results and the proposed means of optimizing the transfer efficiency, an inductively coupled WPT system shown in Fig. 5 is set up according to the parameters listed in Table 1.

In Fig. 5, we adjust the power supply AG1017L to output the high-frequency signal of 0.58 MHz and read the RF output power as the system input power \( P_{in} \). Using the oscilloscope DSO5014A to detect the voltage signal of the load and reading the \( U_{RMS} \), we can calculate the output power \( P_{out} \) in the load by \( P_{out} = \frac{U_{RMS}^2}{R_L} \).
Setting the separate distance of 15 cm, the measured transfer efficiencies are shown in Fig. 6 with the removable and rotatable Tx coil and the fixed Rx coil.

It can be seen in Fig. 6 that in perfect alignment the system can only achieve about 38.6% efficiency, whereas turning the Tx coil to make a tilt angle of $\pi/4$ can improve the efficiency up to 60.5%. If the separate distance is constant, the smaller is the lateral distance, the higher transfer efficiency is. And if the lateral distance is constant, the efficiency is varied with the tilt angle and there is a partial optimal solution. It is also evident in Fig. 6 that the tilt angle corresponding to the partial solution becomes smaller with increasing lateral distance.

### 5. CONCLUSION

Coil misalignment is an inhere problem of inductively coupled links and its impact on transfer efficiency is complex. In this paper, we use Neumann’s formula to solve the numerical solutions of the mutual inductance in varied coil misalignment cases. Simulation results show that only meeting certain constraints, the mutual inductance and the transfer efficiency can achieve maximum and optimal in perfect alignment. If not, there are partial optimal solutions in coil misalignment. Then for some practical applications, such as electric vehicles, a novel method proposed in this study allows the coil to be removable and rotatable for achieving bigger mutual inductance and optimizing transfer efficiency. Finally, the experimental results verify the theoretical results and show the proposed method is feasible. With the distance of 15 cm, if there is no lateral distance, the transfer efficiency can reach up to about 60.5% by rotating the Tx coil to make a tilt angle $\pi/4$ from 38.6% in perfect alignment.

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