تمرینات مروری فصل ۷

۱) $f(x)=3^{5x} \Rightarrow f'(x)=53^{5x}\ln 3$

۲)

$f(x)4^{3x^2} \to f'(x)=6x4^{3x^2}\times \ln 4=12x4^{3x^2}\ln 2$

۳) $f(x)=4^{Sin2x} \Rightarrow f'(x)=2Cos2x4^{Sin2x}\ln 4$

۴) $g(x)=2^{5x}3^{4x^2}$

$g'(x)=5\times 2^{5x}\ln 2\times 3^{4x^2}+2^{5^x}8x3^{4x^2}\ln 3=$

$2^{5x}\cdot 3^{4x^2}(5\ln 2+8\ln 3)$

۵) $h(x)=\dfrac{\log_{10}^{x}}{x}$

$$=\frac{\dfrac{x}{x\ln 10}}{x^2}=\frac{\dfrac{1}{\ln 10}-\log_{10}^{x}}{x^2}$$

۶) $f(x)=\sqrt{\log_a^x}$

$$y'=\frac{1}{2}\left(\frac{1}{x\ln a}\right)\left(\log_a^x\right)^{\frac{-1}{2}}=\frac{1}{2x\ln a\sqrt{\log_a^x}}$$

۷) $f(x)=\log_{10}\left[\log_{10}(x+1)\right]=\dfrac{1}{(x+1)\ln 10}$

$=\log_{10}(x+1)$

۸) $f(t)=Cos3^{t^2} \to$

$y'=-2t3^{t^2}\times \ln 3\times Sin3^{t^2}$

۹) $f(x)=x^{\sqrt{x}}\quad x>0\quad \ln y=\sqrt{x}\ln x$

$$\frac{y'}{y}=\frac{\ln x}{2\sqrt{x}}+\frac{\sqrt{x}}{x}\Rightarrow y'=x^{\sqrt{x}}\left(\frac{\ln x}{2\sqrt{x}}+\frac{\sqrt{x}}{x}\right)$$

۱۰) $g(z)=z^{Sinz}\qquad z>0$

$g'(z)=Sin\,z\cdot z^{Sin2-1}+Cos\,z\cdot z^{Sinz}\ln z=$

$$z^{Sinz}\left(\frac{Cos\,z}{z}+Cos\,z\ln z\right)$$

۱۱) $h(x)=(Cos\,x)^{Sinx}\qquad Cos\,x>0$

$\ln y=Sinx\ln(Cosx)\Rightarrow$

$\dfrac{y'}{y}=Cos\,x\ln(Cosx)\dfrac{Sin^2x}{Cosx}\Rightarrow$

$$y'=(Cosx)^{Sinx}\left(Cos\ln(Cosx)-\frac{Sin2x}{Cosx}\right)$$

۱۲) $f(x)=x^{e^x}\qquad x>0$

$f'(x)=e^x\times e^x\ln x+e^xx^{e^x-1}$

$\ln x+e^xx^{e^x-1}=x^{e^x}e^x\left(\ln x+\dfrac{1}{x}\right)$

۱۳) $\int 3^{2x}dx=\dfrac{1}{2\ln 3}3^{2x}+c$

۱۴) $\int a^te^tdt=\int (ae)^t\,dt=$

$$\frac{(ae)^t}{\ln(ae)}+c=\frac{(a\,e)^0}{\ln at}+c$$

۱۵) $\int x^2 10^{x^3}dx=\dfrac{1}{3\ln 10}10^{x^3}+c$

۱۶) $\int e^yr^{e^y}3^{e^y}dy$

۱۷) $\int_0^2 x^3e^{x^2}dx=2xe^{x^2}=du\to u=e^{x^2}\to$

$e^{x^2(x^2-1)}\Big]_0^2=3e^4+1$

۱۸) $\displaystyle\int_1^8\frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}+4}dx=\frac{3}{4}\ln\left(x^{\frac{4}{3}}+4\right)$

$\displaystyle\int_1^8=\frac{3}{4}\ln\left(8^{\frac{4}{3}}+4\right)-\frac{3}{4}\ln(5)=\frac{3}{4}\ln 20-\frac{3}{4}\ln 5=$

$$\frac{3}{4}\ln 4=\frac{3}{2}\ln 2$$

۱۹) $\int_0^{\ln 2} \frac{e^{2x}}{e^x-5}dx \Rightarrow e^x-5=t\Rightarrow$

$e^x dx = dt \int \frac{(t+5)dt}{t} = \int \left(1+\frac{5}{t}\right)dt$

$= t+5\ln t = \left[(e^x-5)+5\ln\left|e^x-5\right|\right]_0^{\ln 2} =$

$(-3+5\ln 3)+4-5\ln 4 = 1+5\left(\ln\frac{3}{4}\right)$

۲۰) $\log_5^e = \frac{\ln e}{\ln 5} = \frac{1}{\ln 5} = \frac{1}{1/6} = 0/625$

۲۱) $\log 997 = \frac{\ln 997}{\ln 10} = \frac{6/904}{2/302} = 2/999$

۲۲) $a^x \div a^y = a^{x-y} \xrightarrow{\text{طرف اول}} e^{\ln a^x} \div e^{\ln_a y} =$

$e^{x\ln a}\div e^{y\ln a} = e^{x\ln a - y\ln a} = e^{\ln a(x-y)} =$

$e^{\ln a^{x-y}} = a^{xy}$ طرف دوم

۲۳) $(ab)^x = a^x b^x$

$(ab)^x = e^{x\ln(ab)} = e^{x\ln a + x\ln b} =$

$e^{x\ln a}\cdot e^{x\ln b} = e^{\ln a^x}\cdot e^{\ln b^x} = a^x\cdot b^x$

۲۴) $\log_a^1 = 0 \qquad \log_a^1 = \frac{\ln 1}{\ln a} = \frac{0}{\ln a} = 0$

۲۵) $\log_a x^y = y\log_a^x = \frac{\ln x y}{\ln a} =$

$\frac{y\ln x}{\ln a} = y\log_a^x$

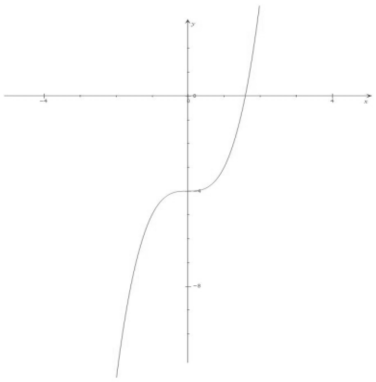
۲۶) $f(x)=x^3-4$

$y'=3x^2 \to$ صعودی $\to 1-1 \to$ معکوس‌پذیری

الف) $y=x^3-4\to x^3=y+4\to x=\sqrt[3]{y+4}$

$f(x)=\sqrt[3]{x+4} \quad D_{f-1}=R \quad R_{f-1}=R$

ب) $y'=3x^2\to y''6x\to x=0$ نقطه عطف



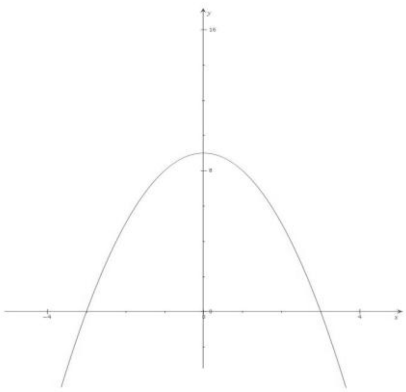
۲۷) $f(x)=9-x^2$

$f(x_1)=f(x_2)\Rightarrow 9-x_1^2=9-x_2^2\Rightarrow$

$x_1^2=x_2^2\Rightarrow x_1=t\,x_2$

تابع ۱-۱ نیست پس معکوس‌پذیر نیست دو خط افقی نمودار را در دو نقطه قطع می‌کند.

ب)



۲۸) $f(x)=\frac{3x-4}{x} \qquad y'=\frac{4}{x^2}>0$ صعودی

$1-1\to$ معکوس‌پذیر

الف) $y=\frac{3x-4}{x}\to y(x)=3x-4$

$x(y-3)=-4 \qquad x=\frac{-4}{y-3}=\frac{4}{3-y}\Rightarrow$

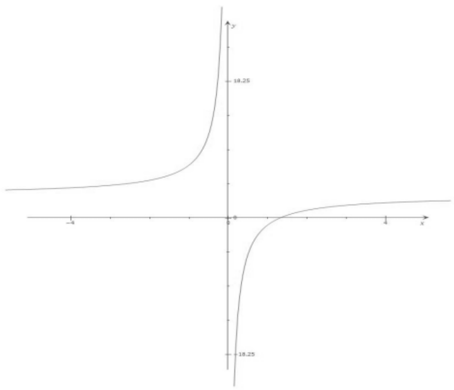
$f^{-1}(x)=\frac{4}{3-x} \quad D_{f^{-1}}=R-\{3\} \quad D_f\ -1=R-\{0\}$

ب) $y=\frac{3x-4}{x}$

$y'=\frac{4}{x^2}$ همواره صعودی

$x\to 0 \quad y\to\infty$ مجانب قائم

$x\to\pm\infty \quad y\to 3$ مجانب افقی



۲۵) $f(x)=\frac{x^2-4}{x}$

$\left.\begin{matrix}(2\ ,\ 0)\in f(x)\\(-2\ ,\ 0)\in f(x)\end{matrix}\right\}\Rightarrow$ یک به یک نیست پس معکوس ندارد.

۳۰) $\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\log_a e \overset{\text{اثبات}}{\Longrightarrow}$

طرف اول $\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\lim_{x\to 0}\frac{\ln(1+x)}{x\ln a}=\frac{0}{0}$

$H:\lim_{x\to 0}\frac{\frac{1}{1+x}}{\ln a}=\frac{1}{\ln a}=\frac{\ln e}{\ln a}=\log_a^e$

۳۱) $\lim_{x\to 1}\frac{x^b-1}{x-1}=b$

با استفاده از قاعده هوپیتال داریم

$\lim_{x\to 1}\frac{x^b-1}{x-1}=\lim_{x\to 1}\frac{bx^{b-1}}{1}=b$

۳۲) $\lim_{x\to 0}\frac{e^{ax}-1}{x}=a \overset{\text{فرض می‌کنیم}}{\rightarrow} f(x)=e^{ax}\Rightarrow$

$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\frac{e^{ax-1}}{x} \qquad (1)$

از طرفی

دیگر $f'(x)=a\,e^{ax}\Rightarrow f'(0)=a \qquad (2)$

$\overset{1,2}{\rightarrow}\lim_{x\to 0}\frac{e^{ax}-1}{x}=a$

۳۳) $D_x^n(\ln x)=(-1)^{n-1}\frac{(n-1)}{x^n}$

$f(x)=\ln x \qquad f^{(1)}(x)=\frac{1}{x}=(-1)^{1-1}\frac{0!}{x^1}$

$f^{(2)}(x)=\frac{-1}{x^2}=(-1)^{2-1}\frac{1!}{x^2}$

$f^{(3)}(x)=\frac{+1}{x^3}=(-1)^{3-1}\frac{2!}{x^3}$

$f^{(n)}(x)=(-1)^{n-1}\frac{(n-1)!}{x^n}$

با استفاده از استقراء

۳۴) $y=e^{Sinx} \qquad -\pi\le x\le 2\pi$

$y'=Cosx\cdot e^{Sinx} \qquad y'=0 \Rightarrow Cosx=0$

$\Rightarrow x=k\pi+\frac{\pi}{2}$

$\Rightarrow x=-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}$

محل جدول

۴۰) $y=e^{-x} \quad , \quad x=t>0$

$A(t)=\int_0^t e^{-x}dx=-e^{-x}\Big]_0^t=-\left(\frac{1}{e^t}-1\right)=1-\frac{1}{e^t}$

$V(t)=\int_0^t\pi\left(e^{-x}\right)^2dx=\int_0^t\pi e^{-2x}dx=\left(-\frac{\pi}{2}e^{-2x}\right]_0^t$

$=-\frac{\pi}{2}\left(e^{-2t}-1\right)=\frac{\pi}{2}\left(1-e^{-2t}\right)$

$\lim_{t\to+\infty}A(t)=\lim_{t\to+\infty}\frac{e^t-1}{e^t}=\lim_{t\to+\infty}\frac{e^t}{e^t}=1$

$$\lim_{t\to+\infty} V(t)/A(t)=\lim_{t\to+\infty}\frac{\frac{\pi}{2}\left(1-\frac{1}{e^{2t}}\right)}{1-\frac{1}{e^{2t}}}=\frac{\pi}{2}$$

$$\lim_{t\to+\infty}\left(\frac{x+a}{x-a}\right)^x=\lim_{t\to+\infty}\left(\frac{x-a+2a}{x-a}\right)^x=\lim_{t\to+\infty}\left(1+\frac{2x}{x-a}\right)$$

$$=\lim_{t\to+\infty}\left(1+\frac{2a}{x-a}\right)^{\left(\frac{x-a}{2a}\right)\frac{2ax}{(x-a)}}$$

$$=\lim_{t\to+\infty} e^{\frac{2ax}{x-a}}=e^{2a}=e\Rightarrow a=\frac{1}{2}$$

۳۹) $y=e^{Sinx}\quad -\pi\le x\le 2\pi$

$y'=Cos\,x\cdot e^{Sinx}\qquad y'=0\Rightarrow Cos\,x=0$

$$\Rightarrow x=k\pi+\frac{\pi}{2}$$

$$\Rightarrow x=-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}$$

محل جدول

۴۰) $y=e^{-x}\quad ,\quad x=t>0$

$$A(t)=\int_0^t e^{-x}dx=-e^{-x}\Big]_0^t=-\left(\frac{1}{e^t}-1\right)=1-\frac{1}{e^t}$$

$$V(t)=\int_0^t\pi\left(e^{-x}\right)^2dx=\int_0^t\pi e^{-2x}dx=\left(-\frac{\pi}{2}e^{-2x}\right]_0^t$$

$$=-\frac{\pi}{2}\left(e^{-2t}-1\right)=\frac{\pi}{2}\left(1-e^{-2t}\right)$$

$$\lim_{t\to+\infty}A(t)=\lim_{t\to+\infty}\frac{e^t-1}{e^t}=\lim_{t\to+\infty}\frac{e^t}{e^t}=1$$

$$\lim_{t\to+\infty}V(t)/A(t)=\lim_{t\to+\infty}\frac{\frac{\pi}{2}\left(1-\frac{1}{e^{2t}}\right)}{1-\frac{1}{e^{20}}}=\frac{\pi}{2}$$

$$\lim_{t\to0^+}V(t)/A(t)=\lim_{t\to0^+}\frac{\frac{\pi}{2}\left(1-\frac{1}{e^{2t}}\right)}{1-\frac{1}{e^t}}=\frac{\frac{\pi}{2}\left(2e^{-2t}\right)}{e^{-t}}$$

$$=\lim_{t\to0^+}\frac{\pi}{e^{-t}}=\pi$$

۴۱) $y=x^{yx}\qquad x>0$ (الف

$$\ln y=\frac{1}{x}\ln x\qquad \frac{y'}{y}=\frac{-1}{x^2}\ln x+\frac{1}{x^2}=$$

$$=\frac{1}{x^2}(1-\ln x)\to y'=x^{yx}\left(\frac{1-\ln x}{x^2}\right)$$

$y'=0\to 1-\ln x=0\quad x=e\quad y=1$

$$\to\begin{matrix}0\le x<ye & y'>0\\ x>e & y'<0\end{matrix}$$

محل جدول

ب) $y=x^{\frac{1}{x^2}}\quad x>0\quad \ln y=\frac{\ln x}{x^2}$

$$\Rightarrow\frac{y'}{y}=\frac{x-2\ln x}{x^4}=\frac{1-2\ln x}{x^3}$$

$$y'=x^{\frac{1}{x^2}}\left(\frac{1-2\ln x}{x^3}\right)\quad 1-2\ln x=0\quad x=\sqrt{e}$$

$$\begin{matrix}0<x<\sqrt{e} & y'>0\\ x>\sqrt{e} & y'<0\end{matrix}$$

محل جدول

پ)

$$y=x^{\frac{1}{x^n}}\qquad \ln y=\frac{\ln x}{x^n}\quad \frac{y'}{y}=\frac{\frac{1}{x}\cdot x^n-nx^{n-1}\ln x}{x^{2n}}$$

$x>0$

$$\frac{y'}{y}=\frac{x^{n-1}-nx^{n-1}\ln x}{x^{2n}}=\frac{x^{n-1}(1-n\ln x)}{x^{2n}}$$

$$y' = x^{\frac{1}{x^n}} \cdot \frac{1}{x^{n+1}} \cdot (1 - n\ln x)$$

$1 - n\ln x = 0 \quad x = \sqrt[n]{e}$

$0 < x < \sqrt[n]{e} \qquad y' > 0$

$x < \sqrt[n]{e} \qquad y' < 0$

محل جدول

۴۲) $\lim\limits_{x\to+\infty} x^{\frac{1}{x^n}} = 1 \to \lim\limits_{x\to+\infty} \frac{1}{x^n}\ln x =$

$$\lim_{x\to+\infty}\frac{\ln x}{x^n} \overset{H}{=} \lim_{x\to+\infty}\frac{\frac{1}{x}}{nx^{n-1}} =$$

$$\lim_{x\to+\infty}\frac{1}{nx^n} = 0 \Rightarrow \lim_{x\to+\infty} x^{\frac{1}{x^n}}$$

۴۳) $x^{(x)^x} = \left(x^x\right)^x$ از دو طرف $\ln$ می‌گیریم

$x^x\ln(x) = x\ln\left(x^x\right) \Rightarrow x^{x-1}\ln(x) =$

$\ln\left(x^x\right) \to x^{x-1}\ln(x) = \ln(x\ln(x)) \Rightarrow x = 1$

۴۴) $\lim\limits_{x\to\infty}\frac{\ln x}{x^{\frac{1}{n}}} \overset{H}{=} \lim\limits_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{n}x^{\frac{1}{n}-1}} =$

$\lim\limits_{x\to\infty}\frac{n}{x^{\frac{1}{n}}} = 0 \Rightarrow$ $\ln x$ کمتر از $x^{\frac{1}{n}}$ رشد می‌کند

۴۵) $ni \approx \sqrt{2\pi n}^{\left(n+\frac{1}{2}\right)} e^{-n} \qquad e^{-n} \approx 10^{-mn}$

$10^{-mn} - e^{-n} \Rightarrow (-mn)\ln 10 = -n$

$\Rightarrow (m\ln 10 = 1) \Rightarrow m = \frac{1}{\ln 10}$

۴۶) $\lim\limits_{x\to+\infty}(\ln x)x^n = \lim\limits_{x\to\infty}\frac{\frac{1}{x}}{hx^{h-1}}$

$\lim\limits_{x\to\infty}\frac{1}{hx^h} = 0 \qquad \lim\limits_{x\to\infty}\frac{x^n}{e^x} \overset{H}{=}$

$\lim\limits_{x\to\infty}\frac{nx^{n-1}}{e^x} \overset{H}{=} \lim\limits_{x\to\infty}\frac{n(n-1)x^{n-2}}{e^n}$

$\lim\limits_{x\to\infty}\frac{n^1}{e^x} = 0$

۴۷)

$$\lim_{x\to\infty}\frac{e^{\frac{1}{n}} + e^{\frac{2}{n}} + \cdots + e^{\frac{n-1}{n}} + e^{\frac{n}{n}}}{n} = \lim_{x\to\infty}\sum_{i=1}^{n}\frac{1}{n}e^{\frac{i}{n}} =$$

$$\int_0^1 e^x dx = e^x\Big]_0^1 = e - 1$$

۴۸) $f(x) = \ln(x-1) \qquad f^{(n)}(x) = ?$

$f^{(1)}(x) = \frac{1}{x-1}$

$f^{(2)}(x) = \frac{-1}{(x-1)^2}$

$f^{(3)}(x) = \frac{-2}{(x-1)^3}$

$\vdots$

$f^{(4)}(x) = \frac{-6}{(x-1)^4} \Rightarrow f^{(n)}(x) = \frac{(n-1)!}{(x-1)^n}(-1)^{n+1}$

۴۹) $AP = 1/2m$

$OA = 40 \ cm$

$\frac{d\theta}{dt} = 360 \times 2\pi = 720\pi \ \ rad/\min$

۵۰) $T = 2 \times \sqrt{\frac{l}{g}}$

$$\frac{dT}{dt} = \frac{d}{dl}\left(2\times\sqrt{\frac{l}{g}}\right) = 2\pi \times \frac{1}{2} \times \frac{1}{g} \times \sqrt{\frac{g}{l}} = \frac{\pi}{\sqrt{gl}}$$

$$\frac{T}{2l} = 2\pi\sqrt{\frac{l}{g}} \times \frac{1}{2l} = \frac{\pi}{\sqrt{gl}} \Rightarrow \frac{dT}{\sqrt{T}} = \frac{dl}{2l}$$

$$\frac{dT}{T} = \frac{dl}{2l} \Rightarrow \frac{dl}{l} = \frac{2dT}{T} = 2 \times \frac{15}{3600} = \frac{1}{120}$$

$l = 120\,dl$

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = \frac{4\pi^2 l}{g}$$

$$\Rightarrow g=\frac{4\pi^2 l}{T^2}\Rightarrow \frac{dg}{dT}=-8\pi^2 lT^{-3}$$

$$\Rightarrow dg=\frac{-8\pi^2 l}{T^3}dT$$

۵۱) $a_n=n2^{-n}\Rightarrow a_n=\frac{n}{2^n}$

$$\lim_{x\to\infty} a_n=\lim_{x\to\infty}\frac{n}{2^n}=\lim_{x\to\infty}\frac{1}{n2^{n-1}}=0$$

۵۲) $a_n=n^{-\frac{1}{n}}$

$$\ln(a_n)=\frac{-1}{n}\ln n$$

$$\lim_{n\to\infty}\ln an=\lim_{n\to\infty}\frac{\ln n}{-n}=\lim_{n\to\infty}\frac{\frac{1}{n}}{-1}=0$$

$$\ln_{n\to\infty} an=1$$

۵۳)

$$a_n=\left(\frac{n}{n+1}\right)^n\Rightarrow \lim_{n\to\infty} an=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)^n$$

$$\overset{n+1=t}{\Rightarrow}\lim\left(1-\frac{1}{t}\right)^t\cdot\left(1-\frac{1}{t}\right)^{-1}=\frac{1}{e}$$

۵۴) $a_n=\frac{n^2}{2^n}$

چون رشد 2^n از n^2 بیشتر است داریم:

$$\lim_{n\to\infty}\frac{n^2}{2^n}=0$$

۵۵) $a_n=\ln\frac{1}{n}\Rightarrow \lim_{n\to\infty} an=\lim_{n\to\infty}\ln\frac{1}{x}=-\infty$

۵۶) $a_n=\frac{1}{n}\ln\frac{1}{n}\to \lim_{n\to\infty} a_n=\lim_{x\to\infty}\frac{\ln\frac{1}{x}}{x}=$

$$H=\lim_{x\to\infty}-\frac{1}{x}=0$$

۵۷) $f(x)=\sqrt{x^2+1}\ ,\ g(x)=\sqrt{x^2-4x}$

$$\lim\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{x}{x-2}=$$

$\lim_{x\to\infty}\frac{x}{x}=1$ مثل هم رشد می‌کنند.

۵۸) $f(x)=e^x\quad ,\quad g(x)=x^n\quad ,\quad x\to+\infty$

$\lim_{n\to\infty}\frac{e^x}{x^n}=\lim_{n\to\infty}\frac{e^x}{nx^{n-1}}$ با استفاده از قاعده n هوپیتال

$$\lim_{x\to\infty}\frac{e^x}{n!}=\infty$$

۵۹) $f(x)=e^{\frac{x}{2}}\quad ,\quad g(x)=Cosx$

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{e^{\frac{x}{2}}}{Cosx}=\infty$$

f سریعتر رشد می‌کند

۶۰) $f(x)=\ln(\ln x)\ ,\ g(x)=\ln x$

$$\lim_{x\to\infty}\frac{\ln(\ln x)}{\ln x}=\lim_{x\to\infty}\frac{\frac{\frac{1}{x}}{\ln x}}{\frac{1}{x}}=\lim_{x\to\infty}\frac{1}{\ln x}=0$$

۶۱) $f(x)=10^{-x}\quad ,\quad g(x)=e^x$

$$\lim\frac{f(x)}{g(x)}\lim\frac{10^{-x}}{e^x}=\lim\frac{1}{10^{e^x}}=0$$

e^x آهنگ رشد سریعتر

۶۲) $y=(\ln x)^x\Rightarrow \ln y=x(\ln\ln x)\Rightarrow$

$$\frac{y'}{y}=\ln\ln x+\frac{\frac{1}{x}}{\ln x}\cdot x\Rightarrow$$

$$y'=\left(\ln\ln x+\frac{1}{\ln x}\right)(\ln x)^x$$

۶۳) $y=x^{\frac{1}{\ln x}}\qquad \ln y=\frac{1}{\ln x}\ln x\Rightarrow$

$$\ln y=1\qquad y'=\phi$$

۶۴) $y = x^{\frac{1}{x}} \qquad \ln y = \dfrac{1}{x}\ln x$

$$\frac{y'}{y} = \frac{-1}{x^2}\ln x + \frac{1}{x^2} \Rightarrow$$

$$y' = \left(\frac{1-\ln x}{x^2}\right)x^{\frac{1}{x}}$$

۶۵) $y = \left(x^x\right)^x \Rightarrow \ln y = x^2 \ln x \Rightarrow$

$$\frac{y'}{y} = 2x\ln x + x \Rightarrow y' = z(2\ln x + 1)x^{x^2}$$

۶۶) $f(x) = e^x \ln x \qquad f'(x) = e^x \ln x + \dfrac{e^x}{x} =$

$$e^x\left(\ln x + \frac{1}{x}\right)$$

۶۷) $\ln xy + x + y = 2$

$$D_x y = -\frac{f'_x}{f'_y} = -\frac{\frac{y}{xy}+1}{\frac{x}{xy}+1} = -\frac{\frac{1}{x}+1}{\frac{1}{y}+1}$$

۶۸) $\ln\dfrac{y}{x} + x\,y = 1$

$$\frac{y'}{x} = -\frac{f'_x}{f'_y} = -\frac{\frac{-\frac{y}{x^2}}{\frac{y}{x}}+y}{\frac{\frac{1}{y}}{\frac{y}{x}}+x} = -\frac{\frac{-1}{x}+y}{\frac{1}{y}+x}$$

$$= \frac{\frac{1}{x}-y}{\frac{1}{y}+x}$$

۶۹) $x = \ln(x+y+1) \qquad x - \ln(x+y+1) = 0$

$$y' = -\frac{1-\frac{1}{x+y+1}}{-\frac{1}{x+y+1}} = 1-x+y-1 = -x-y$$

۷۰) $\ln(x+y) - \ln(x-y) = 4$

$$y' = -\frac{\frac{1}{x+y}-\frac{1}{x-y}}{\frac{1}{x+y}+\frac{1}{x-y}} = -\frac{x-y-x-y}{x-y+x+y} =$$

$$= \frac{2y}{2x} = \frac{y}{x}$$

۷۱) $x + \ln x^2 y + 3y^2 = 2x^2 - 1$

$$y' = -\frac{1+\frac{2xy}{x^2y}-4x}{\frac{x^2}{x^2y}+6y} = \frac{-1-\frac{2}{x}+4x}{\frac{1}{y}+6y} = \frac{\left(4x^2-x-2\right)y}{x\left(1+6y^2\right)}$$

۷۲) $x\ln y + y\ln x = xy$

$$x\ln y + y\ln x - xy = \phi \rightarrow$$