تمرینات مروری فصل ۵

۱)

$$\sum_{n=2}^{6}|1-3n| = |1-6|+|1-9|+|1-12|+|1-15|$$

$$+|1-18| = 5+8+11+14+17=55$$

۲)

$$\sum_{n=3}^{8}(n^2-2n) = (9-6)+(16-8)+(25-10)+$$
$$(36-12)+(49-14)+(64-16)=133$$

۳)

$$\sum_{n=0}^{3}\frac{1-n^2}{1+n^2} = 1+\frac{1-4}{1+4}+\frac{1-9}{1+9}+\frac{1-16}{1+16}=\frac{-109}{85}$$

۴)

$$\sum_{n=1}^{7}\tan\frac{2n-1}{4}\pi = \tan\frac{\pi}{4}+\tan\frac{3\pi}{4}+\tan\frac{5\pi}{4}+$$
$$\tan\frac{7\pi}{4}+\tan\frac{9\pi}{4}+\tan\frac{11\pi}{4}+\tan\frac{13\pi}{4}=1$$

۵)

با استفاده از مشتق‌گیری نشان دهید که بر بازه غیر شامل نقاط $x=\pm\frac{\pi}{2}, x=\pm\frac{3\pi}{2}$ داریم: $\tan^2 x = \sec^2 x + c$ پس ثابت c را بیابید

$$f(x)=\tan^2 x \qquad g(x)=\sec^2 x$$
$$f'(x)=2(1+\tan^2 x)\tan x$$
$$g'(x)=2\cdot\sec^2 x\cdot\tan x=$$
$$f'(x)=g'(x)\Rightarrow f(x)=g(x)+c$$
$$f(0)=g(0)+c\Rightarrow \tan^2(0)=\sec^2(0)+c$$
$$0=1+c \qquad c=-1$$

۶) $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$

$$p(1)=\sum_{i=1}^{1} i^3 = (\sum_{i=1}^{1} i)^2 = 1$$
$$p(k)=\sum_{i=1}^{k} i^3 = (\sum_{i=1}^{k} i)^2$$
$$p(k+1)=\sum_{i=1}^{k+1} i^3 = (\sum_{i=1}^{k+1} i)^2$$
$$\sum_{i=1}^{k} i^3 + (k+1)^3 = (\sum_{i=1}^{k} i)^2 + (k+1)^3 = (\sum_{i=1}^{k+1} i)^2$$

۷) $\lim_{n\to\infty}\sum_{i=1}^{n}(8\sqrt{i}/n^{\frac{3}{2}})$

$$\sum_{i=1}^{n}(\frac{8\sqrt{i}}{n^{\frac{3}{2}}}) = \sum_{i=1}^{n}\frac{1}{n}(\frac{8\sqrt{i}}{\sqrt{n}}) = \frac{1}{n}\times\sum_{i=1}^{n}8\sqrt{\frac{i}{n}}$$
$$\Delta x=\frac{b-a}{n}=\frac{1}{n}\Rightarrow b=1 \quad a=0$$
$$x_i=\frac{i}{n} \qquad f(x_i)=8\sqrt{\frac{i}{n}}\Rightarrow f(x)=8\sqrt{x}$$
$$\sum_{i=1}^{n}8\sqrt{\frac{i}{n^{\frac{3}{2}}}} = \int_0^1 8\sqrt{x}dx = \frac{16}{3}\sqrt[2]{x^3}\Big|_0^1 = \frac{16}{3}$$

۸) $\sum_{i=1}^{n} ai \le \sum_{i=1}^{n}|ai|$

$$p(1)=|a_1|\le|a_1|$$
$$p(k)=|a_1+a_2+\cdots+a_k|\le|a_1|+|a_2|+\cdots+$$
$$|a_k|+|a_{k+1}|$$
$$p(k+1):|a_1+a_2+\cdots+a_k+a_{k+1}|\le|a_1|+|a_2|$$
$$+\cdots+|a_k|+|a_{k+1}|$$
$$|a_1+a_2+\cdots+a_k+a_{k+1}|\le|a_1|+|a_2|+$$
$$\cdots+|a_{k+1}| \qquad (1)$$
$$|a_1+a_2+\cdots+a_k|+|a_{k+1}|\le|a_1|+\cdots+|a_{k+1}| \quad (2)$$

$$(1),(2)\Rightarrow\left|\sum_{i=1}^{n}a_i\right|\le\sum_{i=1}^{n}|a_i|$$

۹) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}(\frac{i}{n})^2$

$$\sum_{i=1}^{n}\frac{1}{n^3}(i)^2=\frac{1}{n^3}\frac{(n+1)n(2n+1)}{6}$$
$$\lim_{n\to\infty}\frac{1}{6n^3}+\frac{1}{3}+\frac{n^2+2n}{6n^3}=\frac{1}{3}$$

۱۰) $S_n=\frac{1}{n}\left[\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\cdots+\frac{n-1}{n}\right]$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}(\frac{i}{n})^3=\int_0^1 x^3dx=\frac{1}{4}x^4\Big|_0^1=\frac{1}{4}$$

ب) $\lim\frac{1}{n^{17}}(1^{15}+\cdots n^{15})=$

$$\frac{1}{n}\left[\frac{1}{n^{16}}\left[1^{15}+2^{15}+\cdots+n^{15}\right]\right]$$

$$\frac{1}{n}\int_0^1 x^{15}+\frac{1}{n}\times\frac{1}{16}x^{16}\Big|_0^1=0\times\frac{1}{16}=0$$

ت) $\lim_{n\to\infty}\frac{1}{n^{15}}(1^{15}+2^{15}+\cdots+n^{15})$

$$\lim n(\frac{1}{n^{16}}(1^{15}+2^{15}+\cdots+n^{15}))$$

$$n\int_0^1 x^{15}=n\times\frac{1}{16}=\infty \quad n\to\infty$$

۱۴)

تابع $h(x)$ زوج و به ازای تمام x ها پیوسته باشد

الف) $f(x)=h(x)\cdot\sin x$

$$f(-x)=h(-x)\sin(-x)=-h(x)\sin x=-f(x)$$

ب) $\int_{-a}^{0}h(x)\sin xdx=-\int_0^a h(x)\sin xdx$

$u=-x\Rightarrow du=-dx$

$$-\int_0^a -h(-u)\sin(-u)du=-\int_0^a h(-u)\cdot\sin ud$$

پ) $\int_{-a}^{a}h(x)\sin xdx=0$

$$\int_{-a}^{0}h(x)\sin xdx=-\int_0^a h(x)\sin xdx$$

$$\Rightarrow\int_{-a}^{0}h(x)\sin xdx\neq\int_0^a h(x)\sin xdx=0$$

$$\Rightarrow\int_{-a}^{a}h(x)\sin xdx=0$$

ت)

$$\int_{-\frac{n}{4}}^{\frac{n}{4}}\sec x\sin x\Rightarrow f(-x)=\sec(-x)\cdot\sin(-x)=$$

$$-\sec\times\sin x=+f(-x)$$

۱۵) $\int_0^2(3x^2-1)dx=?$

$$Ax=\frac{200}{n}=\frac{2}{n}\quad x_0=0\quad x_1=\frac{2}{n}\Rightarrow x_i=\frac{2i}{n}$$

$$f(x_i)=3(\frac{2i}{n})^2-1\Rightarrow\int_0^2(3x^2-1)dx$$

$$=\frac{1}{n}(\sum_{i=1}^{n-1}\frac{i}{n})\Rightarrow\Delta x=\frac{b-a}{n}=\frac{1}{n}$$

$$x_i=\frac{i}{n}\quad f(x_i)=\frac{i}{n}\quad\begin{cases}a=0\\b=1\end{cases}$$

$$f\frac{i}{n}=\frac{i}{n}$$

$$f(x)=x\Rightarrow S_n=\int_0^1 xdx$$

$$S_n\frac{x^2}{2}\Big|_0^1=\frac{1}{2}$$

۱۱) $S_n=\frac{1^2}{u^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+\cdots+\frac{(n-1)^2}{n^3}$

$$S_n=\frac{1}{n}\left[\frac{1}{n^2}+\frac{2^2}{n^2}+\cdots+\frac{(n-1)^2}{n^2}\right]$$

$$S_n=\sum_{i=1}^{n-1}\frac{1}{n}(\frac{i^2}{n^2})\Rightarrow\Delta x=\frac{b-a}{n}=\frac{1}{n}\quad b=1\quad a=0$$

$$f(x_i)=(\frac{i}{n})^2\Rightarrow f(\frac{i}{n})=(\frac{i}{n})^2$$
$$f(x)=x^2$$

$$S_n=\int_0^1 x^2dx=\frac{1}{3}x^3\Big]_0^1=\frac{1}{3}-0=\frac{1}{3}$$

۱۲) $\lim_{n\to\infty}\frac{1^5+2^5+\cdots+n^5}{n^6}=\lim_{n\to\infty}\sum_{i=1}^{\infty}\frac{i^5}{n^6}$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}(\frac{i}{n})^5$$

$$\Delta=\frac{1}{n}\quad b=1\quad a=0$$

$$f(x_i)=(\frac{i}{n})^5\Rightarrow f(x)=x^5$$

$$\int_0^1 x^5dx=\frac{x^6}{6}\Big|_0^1=\frac{1}{6}$$

۱۳) الف) $\lim_{n\to\infty}\frac{1}{n^4}\left[1^3+2^3+\cdots+n^3\right]=$

$$\sum_{i=1}^{n}\frac{1}{n^4}\left[i^3\right]=\sum_{i=1}^{n}\frac{1}{n}\frac{i^3}{n^3}$$

$$x_i=\frac{i}{n}\quad f(x_i)=f(\frac{i}{n})=(\frac{i}{n})^3\Rightarrow f(x)=x^3$$

$$= \lim_{n\to+\infty}\sum_{i=1}^{n}\left(3\left(\frac{2i}{n}\right)^2-1\right)\times\frac{2}{n}=\lim_{x\to+\infty}\frac{2}{n}\sum_{i=1}^{n}\left(\frac{2^2}{n^2}-1\right)$$

$$\lim_{n\to+\infty}\frac{2}{n}\cdot\left(\frac{12}{n^2}\times\frac{n(n+1)(2n+1)}{\underset{n\to\infty}{6}}-n\right)=6$$

۱۶) $\int_{-2}^{1}(x^3+2x)dx=?$

$$\Delta x=\frac{۱+۲}{n}=\frac{۳}{n}\qquad x_۰=-۲\qquad x_۱=-۲+\frac{۳}{n}\cdots x_i=-۲+\frac{۳i}{n}$$

$$f(x_i)=(-2+\frac{3i}{n})^3+2(-2+\frac{3i}{n})$$

$$\lim_{n\to+\infty}\sum_{i=1}^{n}(-2+\frac{3i}{n})^3+2(-2+\frac{2i}{n})\times\frac{3}{n}$$

$$=\frac{3}{n}\sum_{i=1}^{\infty}(\frac{27i^3}{n^3}-\frac{54i^2}{n^3}+\frac{i}{n}-12)=$$

$$l\ im_{n\to+\infty}\frac{3}{n}\left[\frac{27}{n^3}(\frac{n(n+1)}{2})^2-\frac{54}{n^2}\times\frac{n(n+1)(2n+1)}{6}+\frac{36}{n}\times\frac{n(n+1)}{2}-12n\right]=15/75$$

۱۷) $\int_{-۲}^{۲}(t^۳+۳t)dt$

$$f(x)=x^۳+۳x\qquad f(-x)=-(t^۳+۳t^۲)=-f(x)$$

بنابراین تابع f فرد است و سطح زیر نمودار صفر

۱۸) $\int_{۱}^{۵}\frac{dx}{\sqrt{۳x-۱}}=\frac{۱}{۳}\int_{۱}^{۵}۳(۳x-۱)^{-\frac{۱}{۲}}dx=^{۵}_{۱}$

$$\frac{۱}{۳}\times\frac{(۳x-۱)^{\frac{۱}{۲}}}{\frac{۱}{۲}}\Bigg|=\frac{۲}{۳}\sqrt{۲}(\sqrt{۷}-۱)$$

۱۹) $\int_{۰}^{\frac{\pi}{۶}}\frac{\sin t dt}{\cos^۲ t}$

$$=-\int_{۰}^{\frac{\pi}{۶}}\frac{-\sin t}{\cos^۲ t}dt=\frac{۱}{\cos t}+\Bigg|_{۰}^{\frac{\pi}{۶}}=\frac{۲}{\sqrt{۳}}-۱$$

۲۰) $\int_{۰}^{۲}x\sqrt{۳x+۴}dx$

$$u=۳x+۴\Rightarrow x=\frac{u-۴}{۳}\ ,\ dx=\frac{du}{۳}$$

$$=\int_4^{10}(\frac{u-4}{3})\sqrt{u}\times\frac{du}{3}=\frac{1}{9}\int_4^{10}(u^{\frac{3}{2}}-4u^{\frac{1}{2}})du$$

$$=\frac{1}{9}(\frac{2}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}})\Bigg|_4^{10}=\frac{1}{9}(\frac{2}{5}\times10^{\frac{5}{2}}-\frac{8}{3}\times10^{\frac{3}{2}})$$

$$=-\frac{۲}{۵}\times۴^{\frac{۵}{۲}}+\frac{۸}{۳}\times۴^{\frac{۳}{۲}}$$

۲۱) $\int_{-۱}^{۷}\frac{x^۲dx}{\sqrt{x+۲}}\quad \sqrt{x+۲}=u\Rightarrow x=u^۲-۲\Rightarrow$

$$x^۲=(u^۲-۲)^۲\qquad dx=۲udu$$

$$=\int_1^3\frac{(u^2-2)^2\,2u}{u}du=2\int_1^3(u^4+4-4u^2)du$$

$$=2(\frac{1}{5}u^5-\frac{4}{3}u^3+4u)\Bigg|_1^3=32$$

۲۲) $\int_{-۳}^{۳}|x-۲|^۳dx$

$$=\int_{-۳}^{۲}(۲-x)^۳dx+\int_{۲}^{+۳}(x-۲)^۳dx=-\frac{(۲-x)^۴}{۴}\Bigg|_{-۳}^{۲}$$

$$+\frac{(x-۲)^۴}{۴}\Bigg|_{۱}^{۲}=۱۵۶$$

۲۳) $\int_{۰}^{\frac{\pi}{۲}}\sin۵x\cos۳xdx$

$$=\frac{۱}{۲}\int_{۰}^{\frac{\pi}{۲}}(\sin۲x+\sin۸x)dx=\frac{۱}{۲}(\int_{۰}^{\frac{\pi}{۲}}\sin۲xdx+$$

$$\int_{۰}^{\frac{\pi}{۲}}\sin۸xdx)=-\frac{۱}{۴}\cos۲x\Bigg|_{۰}^{\frac{\pi}{۲}}-\frac{۱}{۱۲}\cos۸x\Bigg|_{۰}^{\frac{\pi}{۲}}=\frac{۱}{۲}$$

۲۴) $\frac{۱}{۶}<\int_{۰}^{۲}\frac{۱}{۱۰+x}dx<\frac{۱}{۵}$ تحقیق کنید :

$$\int_0^2\frac{1}{10+x}dx=1n|10+x|\Big|_0^2=1n12-1n10=0/0177$$

$$\frac{1}{6}=0/166<0/0177<0/20$$

۲۵) مقدار میانگین توابع را بیابید

$f(x)=\frac{۱}{x^۲}$ بر $[-۳,۱]$

$$\int_{-۳}^{-۱}\frac{۱}{x^۲}dx=-\frac{۱}{x}\Big|_{-۳}^{-۱}=۱-\frac{۱}{۳}=\frac{۲}{۳}$$

$$\frac{\frac{۲}{۳}}{۲}=\frac{۲}{٦}=\frac{۱}{۳}$$

۲۶) $f(x)=\cos x[۰\,,\,۲\pi)$

$$\int_۰^{۲\pi}\cos x dx=sinx\Big|_۰^{۲\pi}=۰$$

$$\frac{۰}{۲\pi}=۰$$

۲۷) $f(x)=\sec^۲(x)\ \left[-\frac{\pi}{۳},\frac{\pi}{۳}\right]$

$$\int_{-\frac{\pi}{٤}}^{\frac{\pi}{٤}}\sec^۲(x)^{dx}=\int_{-\frac{\pi}{٤}}^{\frac{\pi}{٤}}(۱+\tan^۲x)dx=\tan x\Big|_{-\frac{\pi}{٤}}^{\frac{\pi}{٤}}$$

$$=۱-(-۱)=۲\qquad \frac{۲}{\frac{\pi}{۲}}=\frac{٤}{\pi}$$

۲۸) $f(x)=(۱-x)^۳\quad[-۲\,,\,۲]$

$$\int_{-۲}^{۲}(۱-x)^۳dx=-\frac{۱}{٤}(۱-x)^٤\Big|_{-۲}^{۲}=-\frac{۱}{٤}+\frac{۸۱}{٤}=۲۰$$

$$\frac{۲۰}{٤}=٥$$

۲۹) $\sqrt[3]{(x+1)^2}\qquad[-1,7]$

$$\int_{-۱}^{۷}\sqrt[۳]{(x+۱)^۲}dx=\frac{۳}{٥}\sqrt[۳]{(x+۱)^٥}\Big|_{-۱}^{۷}=\frac{۳}{٥}\sqrt[۳]{۸^٥}$$

$$=۳۲\times\frac{۳}{٥}$$

۳۰) $f(x)=۲\cos x-۳\sin x\left[۰,\frac{\pi}{۲}\right]$

$$\int_۰^{\frac{\pi}{۲}}۲\cos x-۳\sin x=۲\sin x+۳\cos x\Big|_۰^{\frac{\pi}{۲}}$$

$$=۲-۳=-۱\qquad\frac{-۱}{\frac{\pi}{۲}}=-\frac{۲}{\pi}$$

۳۱) $f(x)=(-\cos x+x)\qquad[-\pi,\pi]$

$$\int_{-\pi}^{\pi}x-\cos x dx=\frac{۱}{۲}x^۲-\sin x\Big]_{-\pi}^{\pi}$$

$$\frac{\pi^۲}{۲}-\frac{\pi^۲}{۲}=۰\qquad\frac{۰}{\pi}=۰$$

۳۲) الف) $\int_{-۲}^{-۱}\frac{dx}{x-۳}\ge\int_{۲}^{-۱}\frac{dx}{x}$

$$\forall x\in[-۲,-۱]\Rightarrow\frac{۱}{x-۳}>\frac{۱}{x}\Rightarrow\int_{-۲}^{-۱}\frac{dx}{x-۳}\ge\int_{-۲}^{-۱}\frac{dx}{x}$$

ب) $\int_۱^۲\frac{dx}{x}\ge\int_۱^۲\frac{dx}{x-۳}$

$$\forall x\in[۱,۲]\Rightarrow\frac{۱}{x}>\frac{۱}{x-۳}\Rightarrow\int_۱^۲\frac{dx}{x}\ge\int_۱^۲\frac{dx}{x-۳}$$

پ) $\int_٤^٥\frac{dx}{x-۳}\ge\int_٤^٥\frac{dx}{x}$

$$\forall x\in[٤,٥]\Rightarrow\frac{۱}{x-۳}>\frac{۱}{x}\Rightarrow\int_٤^٥\frac{dx}{x-۳}\ge\int_٤^٥\frac{dx}{x}$$

۳۳) $\int_۱^٤\sqrt{٥+٤x-x^۲}\qquad f(x)=\sqrt{٥+٤x-x^۲}$

$$f'(x)=\frac{٤-۲x}{۲\sqrt{٥+٤x-x^۲}}=۰\Rightarrow x=۲\ \text{ماکزیمم}$$

$\max=f(۲)=۳$

$\min=f(٤)=\sqrt{٥}\qquad ۳\sqrt{٥}<\int_۱^٤\sqrt{٥-٤x-x^۲}<۳\times۳$

$\Rightarrow\left[۳\sqrt{٥},۹\right]$

۳۴) $\int_{-\frac{\pi}{۲}}^{\frac{\pi}{۲}}\sqrt{\cos t}\,dt$

$\max=f(۰)=۱\qquad\min=f(\frac{\pi}{۲}$ یا $-\frac{\pi}{۲})=۰$

$$۰\le\int_{-\frac{\pi}{۲}}^{\frac{\pi}{۲}}\sqrt{\cos t}\,dt\le\pi$$

$$\sqrt[2]{(3t^2-4)^3}\Rightarrow3t^2-4\ge0\Rightarrow t^2\ge\frac{4}{3}$$

$\Rightarrow t\ge\frac{۲}{\sqrt{۳}}$ یا $t\le\frac{-۲}{\sqrt{۳}}\Rightarrow$

$$D_x=(-\infty,-\frac{۲}{\sqrt{۳}})\cup\left[\frac{۲}{\sqrt{۳}},٤\right]$$

$$D_x=-\sqrt[۲]{(۳t^۲-٤)^۳}$$

۳۶)

چون $g(x)$ تغییر علامت نمی‌دهد پس $\int_a^b g(x)dx \neq ۰$

$\int_a^b f(x)g(x)\cdot dx = f(c')\cdot g(c')\cdot (b-a)$

$\int_a^b g(x)dx - g(c'')(b-a)$

پس

$f(c')\cdot g(c')(b-a) = f(c)g(c'')(b-a) \Rightarrow$

$f(c')g(c') = f(c)g(c'')$

$\{c, c', c''\} \in [a,b]$

۳۷) $y = ۹ - x^۲$

$S = \int_۰^۳ (۹-x^۲)dx = ۹x - \frac{۱}{۳}x^۳ \Big]_۰^۳ = ۱۸$

$S_n = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}(\frac{27i}{n} - \frac{9i^2}{n^2})$

۳۸) $y = ۲\sqrt{x-۱}$

$S_n = \int_۵^{۱۷} ۲\sqrt{x-۱}\cdot dx - ۲\int_۵^{۱۷}\sqrt{x-۱}dx =$

$2(\frac{2}{3}\sqrt{(x-1)^3})\Big|_5^{17} = \frac{4}{3}(64-8) = \frac{4}{3}\times 56$

$S_n = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{12}{n}(2\sqrt{12\frac{i}{n}} - 1)$

۳۹) $y = x\sqrt{x+۵}$

$S = \int_{-۱}^{۱} x\sqrt{x+۵}dx \qquad x+۵ = u \Rightarrow du = dx$

$x = u - ۵ = \int_۴^۹ (u-۵)\sqrt{u}\,du =$

$\int_۴^۹ (u^{\frac{۳}{۲}} - ۵\sqrt{u})du = \frac{۲}{۵}\sqrt{u^۵} - \frac{۱۰}{۳}{}^{۲}\sqrt{u^۳}$

$\Big|_4^9 = (\frac{2}{5}\sqrt{9^5} - \frac{10}{3}\sqrt{9^3}) - (\frac{2}{5}\sqrt{4^5} - \frac{10}{3}\sqrt{4^3})$

$= ۲۳۴ \times \frac{۲}{۵} - ۳۲ \times \frac{۲}{۵} + ۸ \times \frac{۱۰}{۳} - ۲۷ \times \frac{۱۰}{۳} = \frac{۳۱۶}{۱۵}$

$S_n = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}(\frac{5i}{n}\sqrt{\frac{5i}{n}+5})$

۴۰) $\int_۱^{\frac{\sqrt{۸}}{۷}} (-\frac{۱۵}{۸} + \frac{۷}{۸}x^۲ + \frac{۱}{x^۲})dx =$

$\left[\frac{۱۵}{۸}x + \frac{۷}{۲۴}x - \frac{۱}{x}\right]_۱^{\frac{\sqrt{۸}}{۷}} \cong ۱/۸۵$

۴۱) $f(x) = \begin{cases} x & ۰ \le x \le a \\ a\frac{۱-x}{۱-a} & a \le x \le ۱ \end{cases}$

$\int_۰^۱ f(x)dx = \int_۰^a f(x)dx + \int_a^۱ f(x)dx =$

$\int_۰^a xdx + \int_a^۱ a\frac{۱-x}{۱-a}dx = \frac{x^۲}{a}\Big|_۰^a + \frac{a}{۱-a}(x - \frac{x^۲}{۲})\Big|_a^۱$

$= \frac{a^۲}{۲} + \frac{a}{۱-a}(\frac{۱}{۲} - a + \frac{a^۲}{۲}) = \frac{a}{۲}$

۴۲) $y' = x^۵ - x^۳ \qquad y(۰) = ۴$

$\int (x^۵ - x)^۳ dx = \frac{x^۶}{۶} - \frac{x^۴}{۴} + c$

$y(۰) = ۴ \qquad ۰ + ۰ + c = ۴ \Rightarrow c = ۴$

$y = \frac{x^۶}{۶} - \frac{x^۴}{۴} + ۴$

۴۳) $y' = \cos^۲ x - \sin x \qquad y(۰) = ۷$

$\int \cos^۲ x - \sin x = \frac{۱}{۲}x + \frac{۱}{۴}\sin^۲ x + \cos x + c$

$۰ + ۰ + ۱ + c = ۷ \Rightarrow c = ۶$

$y = \frac{۱}{۲}x + \frac{۱}{۴}\sin cx + \cos x + ۶$

۴۴) $y'' = \sqrt{x} \qquad y(۱) = ۲ \qquad y'(۱) = ۱$

$\int \sqrt{x}\,dx = \frac{۲}{۳}\sqrt{x^۳} + c'$

$y'(1) = 1 \Rightarrow \frac{2}{3} + c' = 1 \Rightarrow c = \frac{1}{3}$

$\int (\frac{۲}{۳}\sqrt{x^۳} + \frac{۱}{۳})dx = \frac{۲}{۳} \times \frac{\sqrt{x^۵}}{\frac{۵}{۲}} + \frac{۱}{۳}x + c$

$y(1) = 2 \Rightarrow \frac{2}{3} \times \frac{2}{5} + \frac{1}{3} + c = 2 \qquad c = \frac{21}{15}$

$y = \frac{۲}{۳} \times \frac{۲}{۵}\sqrt{x^۵} + \frac{۱}{۳}x + \frac{۲۱}{۱۵}$

۴۵) $y'' = \sqrt{x^{\frac{۱}{۲}}} \qquad y(-۱) = -۲\ ,\ y'(۱) = ۰$

$54) Dx\int_{-x}^{x}\frac{dt}{3+t^2}=\frac{1}{3+x^2}+\frac{1}{3+x^2}=\frac{2}{3+x^2}$

$55) Dx\int_{2x}^{3x}\frac{\upsilon-1}{\upsilon+1}du=3\times\frac{3x-1}{3x+1}-2\times\frac{2x-1}{2x+1}$

$56) Dx\int_{\sqrt{x}}^{x^3}\sqrt{t}\,Sin\,t\,dt=$

$3x^2\sqrt{x^3}Sin\,x^3-\frac{1}{2\sqrt{x}}\sqrt{\sqrt{x}}Sin\sqrt{x}$

$57) \frac{1}{4}\int_0^4 8x-x^2=$

$\frac{1}{4}\left(\frac{4x^8}{32}-\frac{1}{3}x^3\right]_0^4=-\frac{4}{3}$

$58) \frac{1}{1}\int_4^5 3x\sqrt{x^2-16}$

$\upsilon=x^2-16 \qquad d\upsilon=2x\,dx$

$\frac{3}{2}\int_4^5\upsilon\,d\upsilon-\frac{2}{3}\times\frac{3}{2}\frac{\upsilon^{\frac{3}{2}}}{3}\Bigg]_4^5=$

$\frac{3}{2}\sqrt[2]{(x^2-16)^3}\Bigg]_4^5=\sqrt{729}$

$59) A=\int_0^1\left(\sqrt[3]{x}-x^3\right)dx=\frac{3}{4}x^{\frac{4}{3}}-\frac{1}{4}x^4\Bigg]_0^1$

$=\frac{1}{2}$

$60) \int_{-\frac{1}{2}}^{1}\left(3y+2-2y^2\right)dy=$

$\frac{3}{2}y^2+2y-\frac{2}{3}y^3\Bigg]_{-\frac{1}{2}}^{1}=\frac{81}{24}$

$61) \int_{-2}^{1}4-y^2-y^2-2y=\int_{-2}^{1}-2y^2-2y+4$

$=-\frac{2}{3}y^3-y^2+4y\Bigg]_{-2}^{1}=9$

$62) \int_{-1}^{1}Cos\,n\pi x\,Cos\,m\pi x\,dx=$

$\begin{cases}۰ & m\neq n\\ ۱ & m=n\end{cases}$

$\int ۷x^{\frac{۱}{۳}}dx=\frac{۲۱}{٤}\sqrt[۳]{x^٤}+c'$

$\frac{۲۱}{٤}+c'=۰\Rightarrow c'=-\frac{۲۱}{٤}$

$\int(\frac{21}{4}\sqrt[3]{x^4}-\frac{21}{4})dx=\frac{9}{4}\sqrt[3]{x^7}-\frac{21}{4}x+c$

$\frac{-۹}{٤}+\frac{۲۱}{٤}+c=-۲ \qquad ۳+c=-۲$

$c=-5 \qquad y=\frac{9}{4}\sqrt[3]{x^2}-\frac{21}{4}x-5$

۴۶) $v=\sqrt{۲gx}$ از $[۰,S]$

$\bar{v}=\int_0^4\sqrt{2g(x)}dx\times\frac{1}{S}+\sqrt{2g}\times\frac{S^{\frac{3}{2}}}{\frac{3}{2}}\times\frac{1}{S}=$

$\frac{2}{3}\sqrt{2gS}=\frac{2}{3}v$

۴۷) $m=2700(Ib)\quad a=$ (یکنواخـــــــت)

$t=105(s)$

$v=75(mph)\qquad v_0=0\qquad p=?$

$x=\frac{v_.+v}{۲}\times t\Rightarrow x=\frac{۰+۷۵}{۲}\times ۱۵\Rightarrow x=۵۶۲/۵$

$F=ma\Rightarrow F=2700a$

$w=Fx\Rightarrow w=1/5\times10^6a$

$p=\frac{w}{t}\Rightarrow p=\frac{1/5\times10^6a}{t}$

$50) \left(50t-\frac{14\times12}{\pi}Cos\frac{\pi t}{12}\right]_9^{21}$

$=\frac{1}{12}\left[345+\frac{168\sqrt{2}}{\pi}\right]$

$51) \Rightarrow\frac{12}{8}\int^8\frac{1}{\sqrt{x}}dx=$

$\left(3\sqrt{x}\right)_0^8=6\sqrt{2}$

$52) Dx\int_0^x\sqrt{4+t^6}=\sqrt{4+x^6}$

$53) Dx\int_x^3\sqrt{Sin\,t}\,dt=-\sqrt{Sin\,x}$

$$63)\int_{-1}^{1} Sin\, n\pi\, x\, Sin\, m\pi\, x\, dx$$

$$= \int_{1}^{0} \begin{matrix} m \neq n \\ m = n \end{matrix}$$

$$64)\ f(x) = x^2\sqrt{x-3} \quad [a,b] = [7,12]$$

$$\int_{7}^{12} x^2\sqrt{x-3}\, dx \quad \sqrt{x-3} = u\,,\, x-3 = u^2$$

$$\Rightarrow x = u^2 + 3 \qquad d\,x = d\,u$$

$$\int_{7}^{12} (u^2+3)^2 \times u\, du \int_{7}^{12} (u^4 + 6u^2 + 9) u\, du$$

$$\int_{7}^{12} (u^5 + 6u^3 + 9u) d\,u =$$

$$\frac{u^6}{6} + \frac{6u^4}{4} + \frac{9u^2}{2}\Big|_{7}^{12}$$

$$\Rightarrow \frac{(x-3)^3}{6} + \frac{3(x-3)^2}{2} + \frac{9(x-3)}{2}\Big|_{7}^{12}$$

$$65)\int_{1}^{3} \frac{Sin\, 2\, x}{x} d\, x$$

$$= 2\int_{1}^{3} \frac{Sin\, x\, Cos\, x}{x} d\, x = 2\int_{3}^{1} f(x)$$

$$Cos\, x\, d\, x \Rightarrow f\ x\, d\, x = d\, u$$

$$Cos\, x$$