The Variational Iteration Method for the Newell-Whitehead-Segel Equation

By:

Mohsen Soori

University:

Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran

Date:

25 February 2016

Composition of the Jury:

Prof. **S. Salman Nourazar**

Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic)

Dr. **Akbar Nazar-Golshan**

Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic)

Abstract: The Variational Iteration Method (VIM) is used to solve the Newell-Whitehead-Segel non-linear differential equations. Three case study problems of Newell-Whitehead-Segel are solved by using the VIM and the exact solutions are obtained. The capability and great potential of the VIM in solving the non-linear differential equations is also proved by applying the method to the Newell-Whitehead-Segel equation.

Keywords: Newell-Whitehead-Segel equation, Variational Iteration Method, Nonlinear differential equations
2010 Mathematics Subject Classification: 35D35

---

1. Introduction

In order to model behavior and effects of many phenomena in different fields of science and engineering by mathematical concepts, nonlinear differential equations are introduced. Semi-analytical method as the Variational Iteration Method (VIM) is considered as a powerful and flexible algorithm to solve and obtain exact solution of nonlinear differential equations.
The idea of the VIM was first pioneered by He [1]. Then, the VIM is applied by He [2,3] in order to solve autonomous ordinary differential equation as well as delay differential equation. Also, new development and applications of the VIM to nonlinear wave equation, nonlinear fractional differential equations, nonlinear oscillations and nonlinear problems arising in various engineering applications is presented by He and Xh [4]. Variational iteration method is strong and efficient method which can be widely used to handle linear and nonlinear models. The VIM has no specific requirements for nonlinear operators. The method gives the solution in the form of rapidly convergent successive approximations that may give the exact solution if such a solution exists.
Nourazar et al. [5] obtained the exact solution of the Newell-Whitehead-Segel equation by using the homotopy perturbation method. Also, Pue-on [6] presented application of the laplace adomian decomposition method for solving the Newell-Whitehead-Segel Equation. Analytic solution for Newell–Whitehead–Segel equation by differential transform method is presented by Aasaraai [7]. Application of the the homotopy perturbation method to the exact solution of nonlinear differential equations is presented by Nourazar et al. [8,9].
Soori et al. [10] , [11] presented application of the Variational Iteration Method and the Homotopy Perturbation Method to the fisher type equation.

The Newell-Whitehead-Segel equation describes the appearance of the stripe pattern in two dimensional systems. The equation has wide range of applications in mechanical and chemical engineering, ecology, biology, Rayleigh-Benard convection, faraday instability, nonlinear optics, chemical reactions and bio-engineering.

The Newell-Whitehead-Segel equation is written as:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + au - bu^q\ , \tag{1.1}$$

Where a, b and k are real numbers with $k > 0$, and q is a positive integer.

In the present research work, the Variational Iteration Method (VIM) is applied to obtain the closed form solution of the non-linear Newell-Whitehead-Segel equation. The main aim of the study is to present trend of rapid convergence of the sequences constructed by the VIM toward the exact solution of the equation. So, three case study problems of non-linear Newell-Whitehead-Segel equations are solved by using the VIM and the exact solution is obtained.

The ideas of variational iteration method is presented in the section 2. Application of the variational iteration method to the exact solution of Newell-Whitehead-Segel equation is presented in the section 4.

2. The idea of variational iteration method

The idea of the variational iteration method is based on constructing a correction functional by a general Lagrange multiplier. The multiplier is chosen in such a way that its correction solution is improved with respect to the initial approximation or to the trial function. To illustrate the basic idea of the variational iteration method, consider the following nonlinear equation:

$$Lu(t) + Nu(t) = g(t), \tag{2.1}$$

Where L is a linear operator, N is a nonlinear operator, and g (t) is a known analytic function. According to the variational iteration method, we can construct the following correction functional:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\big(Lu_n(\xi) + N\tilde{u}_n(\xi) - g(t)\big)\ d\xi, \tag{2.2}$$

Where λ is a general Lagrange multiplier which can be identified optimally via variational theory and $\tilde{u}_n$ is considered as a restricted variation which means $\delta\tilde{u}_n = 0$.

$u_0(t)$ is an initial approximation with possible unknowns. We first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. With λ determined, then several approximations$u_n(t), n \geq 0$ follow immediately. Consequently, the exact solution may be obtained as:

$$u(t) = \lim_{n\to\infty} u_n(t), \tag{2.3}$$

The correction functional of the Eq. (2.1) gives several approximations. Therefore, the exact solution can be obtained as the limit of resulting successive approximations.

3. The Newell-Whitehead-Segel equation

To illustrate the capability and reliability of the method, three cases of nonlinear diffusion equations are presented.

Case I: In Eq. (1.1) for $a = 2, b = 3, k = 1$ and $q = 2$ the Newell-Whitehead-Segel equation is written as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2u - 3u^2, \tag{3.1}$$

Subject to a constant initial condition:

$$u(x,0) = \lambda, \tag{3.2}$$

The correction functional for Eq. (3.1) is in the following form:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - 2u_n(x,\xi) + 3u_n^2(x,\xi)\right) d\xi, \tag{3.3}$$

Where u_n is restricted variation $\partial u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary we have:

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - 2u_n(x,\xi) + 3u_n^2(x,\xi)\right) d\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\delta \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi}\right) d\xi, \tag{3.4}$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1 + \lambda(\xi)\big) - \delta \int_0^t \lambda'(\xi) u_n(x,\xi)\, d\xi,$$

By using the following stationary conditions:

$$\delta u_n :\ 1 + \lambda(\xi) = 0, \tag{3.5}$$

$$\delta u_n :\ \lambda'(\xi) = 0, \tag{3.6}$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$, therefore the following iteration formula becomes as:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - 2u_n(x,\xi) + 3u_n^2(x,\xi)\right) d\xi, \tag{3.7}$$

We can select $u_0(x,y) = \lambda$, from the given condition.

Using this selection into the Eq. (3.7) the following successive approximation can be obtained as:

$$u_0(x,t) = \lambda,$$

$$u_1(x,t) = \lambda - \int_0^t \left(\frac{\partial u_0(x,\xi)}{\partial \xi} - \frac{\partial^2 u_0(x,\xi)}{\partial x^2} - 2u_0(x,\xi) + 3u_0^2(x,\xi)\right) d\xi = \lambda + \lambda(2-3\lambda)t\,,$$

$$\begin{aligned} u_2(x,t) &= \lambda + \lambda(2-3\lambda)t - \int_0^t \left(\frac{\partial u_1(x,\xi)}{\partial \xi} - \frac{\partial^2 u_1(x,\xi)}{\partial x^2} - 2u_1(x,\xi) + 3u_1^2(x,\xi)\right) d\xi \\ &= \lambda + \lambda(2-3\lambda)t + \big(\lambda(2-3\lambda) - 3\lambda^2(2-3\lambda)\big)t^2 \\ &- \lambda^2(2-3\lambda)^2 t^3, \end{aligned} \tag{3.8}$$

$$\begin{aligned} u_3(x,t) = {} & \lambda + \lambda(2-3\lambda)t + \big(\lambda(2-3\lambda) - 3\lambda^2(2-3\lambda)\big)t^2 - \lambda^2(2-3\lambda)^2 t^3 \\ & - \int_0^t \left(\frac{\partial u_2(x,\xi)}{\partial \xi} - \frac{\partial^2 u_2(x,\xi)}{\partial x^2} - 2u_2(x,\xi) + 3u_2^2(x,\xi)\right) d\xi \\ & = \lambda + \lambda(2-3\lambda)t + \lambda(3\lambda-1)(3\lambda-2)t^2 - \lambda(3\lambda-2)(27\lambda^2 - 18\lambda + 2)\frac{t^3}{3} \\ & + 2\lambda^2(3\lambda-1)(3\lambda-2)^2 t^4 - 3\lambda^2(3\lambda-2)^2(15\lambda^2 - 10\lambda + 1)\frac{t^5}{5} + \lambda^3(3\lambda-1)(3\lambda-2)^3 t^6 \\ & - 3\lambda^4(3\lambda-2)^4\frac{t^7}{7}, \end{aligned}$$

As a result, the series of the exact solution of the Eq. (3.1) can be constructed as:

$$u_n(x,t) = \lambda + \lambda(2-3\lambda)t + \lambda(3\lambda-1)(3\lambda-2)t^2 - \lambda(3\lambda-2)(27\lambda^2-18\lambda+2)\frac{t^3}{3}$$
$$+ 2\lambda^2(3\lambda-1)(3\lambda-2)^2 t^4 - 3\lambda^2(3\lambda-2)^2(15\lambda^2-10\lambda+1)\frac{t^5}{5} + \lambda^3(3\lambda-1)(3\lambda-2)^3 t^6$$
$$- 3\lambda^4(3\lambda-2)^4\frac{t^7}{7} + \cdots \tag{3.9}$$

Using the identity:

$$u\,(x,t) = \lim_{n\to\infty} u_n(x,t), \tag{3.10}$$

We can write Eq. (3.9) in the closed form as:

$$u\,(x,t) = \frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3}+\lambda-\lambda e^{2t}}\,, \tag{3.11}$$

This is the exact solution of the problem, Eq. (3.1).

Case II: In Eq. (1.1) for $a = 1, b = 1, k = 1$ and $q = 2$ the Newell-Whitehead-Segel equation is written as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^{\,2}, \tag{3.12}$$

Subject to initial condition:

$$u(x,0) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2}\,, \tag{3.13}$$

The correction functional for Eq. (3.12) is in the following form:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^2(x,\xi)\right)\,d\xi, \tag{3.14}$$

Where u_n is restricted variation $\partial u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary we have:

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^2(x,\xi)\right)d\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\delta\int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi}\right)d\xi, \tag{3.15}$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1+\lambda(\xi)\big) - \delta\int_0^t \acute{\lambda}(\xi)u_n(x,\xi)\,d\xi,$$

By using the following stationary conditions:

$$\delta u_n:\ 1+\lambda(\xi) = 0, \tag{3.16}$$

$$\delta u_n:\ \acute{\lambda}(\xi) = 0, \tag{3.17}$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$, therefore the following iteration formula becomes as:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^2(x,\xi) \right) d\xi, \tag{3.18}$$

We can select $u_0(x,y) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2}$, from the given condition.

Using this selection into the Eq. (3.18) the following successive approximation can be obtained as:

$$u_0(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2},$$

$$u_1(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} - \int_0^t \left(\frac{\partial u_0(x,\xi)}{\partial \xi} - \frac{\partial^2 u_0(x,\xi)}{\partial x^2} - u_0(x,\xi) + u_0^2(x,\xi) \right) d\xi$$

$$= \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} + \left(\frac{5}{3} \frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} \right) t,$$

$$u_2(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} + \left(\frac{5}{3} \frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} \right) t - \int_0^t \left(\frac{\partial u_1(x,\xi)}{\partial \xi} - \frac{\partial^2 u_1(x,\xi)}{\partial x^2} - u_1(x,\xi) + u_1^2(x,\xi) \right) d\xi$$

$$= \frac{1}{5} \frac{5e^{\frac{x}{\sqrt{6}}} + \left(5e^{\frac{x}{\sqrt{6}}}\right)^2 - 1}{e^{\frac{x}{\sqrt{6}}}\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} + \left(\frac{5e^{\frac{x}{\sqrt{6}}}}{3\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} \right) t + \left(\frac{25}{36} \frac{e^{\frac{x}{\sqrt{6}}}\left(2e^{\frac{x}{\sqrt{6}}} - 1\right)}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^4} \right) t^2 - \left(\frac{25\left(e^{\frac{x}{\sqrt{6}}}\right)^2}{27\left(1+e^{\frac{x}{\sqrt{6}}}\right)^6} \right) t^3, \tag{3.19}$$

As a result, the series of the exact solution of the Eq. (3.12) can be constructed as:

$$u_n(x,t) = \frac{1}{5} \frac{5e^{\frac{x}{\sqrt{6}}} + \left(5e^{\frac{x}{\sqrt{6}}}\right)^2 - 1}{e^{\frac{x}{\sqrt{6}}}\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} + \left(\frac{5e^{\frac{x}{\sqrt{6}}}}{3\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} \right) t + \left(\frac{25}{36} \frac{e^{\frac{x}{\sqrt{6}}}\left(2e^{\frac{x}{\sqrt{6}}} - 1\right)}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^4} \right) t^2 - \left(\frac{25\left(e^{\frac{x}{\sqrt{6}}}\right)^2}{27\left(1+e^{\frac{x}{\sqrt{6}}}\right)^6} \right) t^3 + \cdots \tag{3.20}$$

Using the identity:

$$u(x,t) = \lim_{n\to\infty} u_n(x,t), \tag{3.21}$$

We can write Eq. (3.20) in the closed form as:

$$v(x,t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}} - \frac{5}{6}t}\right)^2} \ , \qquad (3.22)$$

This is the exact solution of the problem, Eq. (3.12).

Case III: In Eq. (1.1) for $a = 1, b = 1, k = 1$ and $q = 4$ the Newell-Whitehead-Segel equation becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^4, \qquad (3.23)$$

Subject to initial condition:

$$u(x,0) = \left(\frac{1}{1 + e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}}, \qquad (3.24)$$

The correction functional for Eq. (3.23) is in the following form:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^4(x,\xi)\right) d\xi, \qquad (3.25)$$

Where u_n is restricted variation $\partial u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary we have:

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^4(x,\xi)\right) d\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\delta \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi}\right) d\xi, \qquad (3.26)$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1 + \lambda(\xi)\big) - \delta \int_0^t \acute{\lambda}(\xi) u_n(x,\xi)\, d\xi,$$

By using the following stationary conditions:

$$\delta u_n : \ 1 + \lambda(\xi) = 0, \qquad (3.27)$$

$$\delta u_n : \ \acute{\lambda}(\xi) = 0, \qquad (3.28)$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$, therefore the following iteration formula becomes as:

$$u_{n+1}(x,t) = \ u_n(x,t) - \int_0^t \left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial x^2} - u_n(x,\xi) + u_n^4(x,\xi)\right) d\xi, \qquad (3.29)$$

We can select $u_0(x,y) = \left(\frac{1}{1+e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}}$, from the given condition.

Using this selection into the Eq. (3.29) the following successive approximation can be obtained as:

$$u_0(x,t)=\left(\frac{1}{1+e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}},$$

$$u_1(x,t)=\left(\frac{1}{1+e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}}-\int_0^t\left(\frac{\partial u_0(x,\xi)}{\partial\xi}-\frac{\partial^2 u_0(x,\xi)}{\partial x^2}-u_0(x,\xi)+u_0^4(x,\xi)\right)d\xi$$

$$=\left(\frac{1}{1+e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}}+\left(\frac{7e^{\frac{3x}{\sqrt{10}}}}{5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t,$$

$$u_2(x,t)=\left(\frac{1}{1+e^{\frac{3x}{\sqrt{10}}}}\right)^{\frac{2}{3}}+\left(\frac{7e^{\frac{3x}{\sqrt{10}}}}{5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t-\int_0^t\left(\frac{\partial u_1(x,\xi)}{\partial\xi}-\frac{\partial^2 u_1(x,\xi)}{\partial x^2}-u_1(x,\xi)+u_1^4(x,\xi)\right)d\xi$$

$$=\frac{7e^{\frac{3x}{\sqrt{10}}}+7\left(e^{\frac{3x}{\sqrt{10}}}\right)^2-1}{7e^{\frac{3x}{\sqrt{10}}}\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}+\left(\frac{7e^{\frac{3x}{\sqrt{10}}}}{5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t+\left(\frac{49}{100}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)\left(2e^{\frac{3x}{\sqrt{10}}}-3\right)}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}}\right)t^2$$

$$-\left(\frac{98}{25}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^2}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{14}{3}}}\right)t^3-\left(\frac{343}{125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^3}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{17}{3}}}\right)t^4$$

$$-\left(\frac{2401}{3125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^4}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{20}{3}}}\right)t^5, \tag{3.30}$$

As a result, the series of the exact solution of the Eq. (3.23) can be constructed as:

$$u_n(x,t)=\frac{7e^{\frac{3x}{\sqrt{10}}}+7\left(e^{\frac{3x}{\sqrt{10}}}\right)^2-1}{7e^{\frac{3x}{\sqrt{10}}}\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}+\left(\frac{7e^{\frac{3x}{\sqrt{10}}}}{5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t+\left(\frac{49}{100}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)\left(2e^{\frac{3x}{\sqrt{10}}}-3\right)}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}}\right)t^2$$

$$-\left(\frac{98}{25}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^2}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{14}{3}}}\right)t^3-\left(\frac{343}{125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^3}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{17}{3}}}\right)t^4-\left(\frac{2401}{3125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^4}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{20}{3}}}\right)t^5$$

$$+\cdots \tag{3.31}$$

Using the identity:

$$u\,(x,t) = \lim_{n\to\infty} u_n(x,t), \tag{3.32}$$

We can write Eq. (3.31) in the closed form as:

$$v(x,t) = \left(\frac{1}{2}tanh\left(-\frac{3}{2\sqrt{10}}\left(x-\frac{7}{\sqrt{10}}t\right)\right)+\frac{1}{2}\right)^{\frac{2}{3}}, \tag{3.33}$$

This is the exact solution of the problem, Eq. (3.23).

4. Conclusion

In the present research work, the exact solution of the Newell-Whitehead-Segel nonlinear diffusion equation is obtained using the VIM. The validity and effectiveness of the VIM is shown by solving three non-homogenous non-linear differential equations. Convergence of sequences obtained by using the VIM towards the exact solutions of the Eq. (3.1), Eq. (3.12) and Eq. (3.23) indicates validity and great potential of the method as a powerful algorithm to solve the Newell-Whitehead-Segel equation. Furthermore, it can be concluded that the VIM is a very efficient and powerful technique with acceptable accuracy in order to construct the exact solution of linear and nonlinear differential equations.

References

[1] J.H. He, Variational iteration method - a kind of non-linear analytical technique: Some examples. Int. J. NonLinear Mech. 34 (1999), 699-708.
[2] J.H. He, Variational iteration method for autonomous ordinary differential systems. Applied Math. and Comput. 114 (2000), 115–123.
[3] J.H. He, Variational iteration method for delay differential equations. Commun. Nonlinear Sci. and Numer. Simul. 2 (1997), 235-236.
[4] J.H. He and X.h.Wu, Variational iteration method: new development and applications. Comput. and Math. Appl. 54 (2007), 881–894.
[5] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On The Exact Solution of Newell-Whitehead-Segel Equation Using the Homotopy Perturbation Method. Aust. J. Basic and Appl. Sci. 5 (2011), 1400-1411.
[6] P. Pue-on, Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation. Appl. Math. Sci. 7 (2013), 6593-6600.
[7] A. Aasaraai, Analytic solution for Newell–Whitehead–Segel equation by differential transform method. Middle-East J. Sci. Res. 10 (2011), 270-273.
[8] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On the Exact Solution of Burgers-Huxley Equation Using the Homotopy Perturbation Method. J. Appl. Math. and Phy. 3 (2015), 285-294.
[9] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On the Homotopy Perturbation Method for the Exact Solution of Fitzhugh–Nagumo Equation. Int. J. Math. Comput. 27 (2015), 32-43.
[10] M. Soori, S.S. Nourazar, and A. Nazari-Golshan, Application of the Variational Iteration Method and the Homotopy Perturbation Method to the Fisher Type Equation, Int. J. Math. Comput. 27 (2015), 1-9.
[11] M. Soori, The Homotopy Perturbation Method and the Variational Iteration Method to Nonlinear Differential Equations (2011).