In the Name of GOD

**Problem 1-1** Two solid cylindrical rods $AB$ and $BC$ are welded together at $B$ and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod $AB$ and 150 MPa in rod $BC$, determine the smallest allowable values of $d_1$ and $d_2$.

**Answer:** $d_1 = 22.6$ mm  $d_2 = 15.96$ mm

**Problem 1-2** Two solid cylindrical rods $AB$ and $BC$ are welded together at $B$ and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (a) rod $AB$, (b) rod $BC$.

**Answer:** (a) 35.7 MPa  (b) 42.4 MPa

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**SOLUTION 1-1**

(a) **Rod $AB$**

\[ P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N} \]

\[ \sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\pi d_1^2} = \frac{4P}{\pi d_1^2} \]

\[ d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi(175 \times 10^6)}} = 22.6 \times 10^{-3} \text{ m} \quad d_1 = 22.6 \text{ mm} \]

(b) **Rod $BC$**

\[ P = 30 \text{ kN} = 30 \times 10^3 \text{ N} \]

\[ \sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\pi d_2^2} = \frac{4P}{\pi d_2^2} \]

\[ d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi(150 \times 10^6)}} = 15.96 \times 10^{-3} \text{ m} \quad d_2 = 15.96 \text{ mm} \]

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**SOLUTION 1-2**

(a) **Rod $AB$**

\[ P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N} \]

\[ A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2 \]

\[ \sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa} \quad \sigma_{AB} = 35.7 \text{ MPa} \]

(b) **Rod $BC$**

\[ P = 30 \text{ kN} = 30 \times 10^3 \text{ N} \]

\[ A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2 \]

\[ \sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa} \quad \sigma_{BC} = 42.4 \text{ MPa} \]
Problem 1-3 Three forces, each of magnitude $P = 4\, \text{kN}$, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod $BE$ for which the normal stress in that portion is 1100 MPa.

**Answer:** $285 \, \text{mm}^2$

**SOLUTION**

Draw free body diagrams of $AC$ and $CD$.

Free Body $CD$:

$\Sigma M_D = 0: \quad 0.150P - 0.250C = 0$

$C = 0.6P$

Free Body $AC$:

$\Sigma M_A = 0: \quad 0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$

$F_{BE} = \frac{1.07}{0.150} \cdot P = 7.133 \cdot P = (7.133)(4 \, \text{kN}) = 28.533 \, \text{kN}$

Required area of $BE$:

$\sigma_{BE} = \frac{F_{BE}}{A_{BE}}$

$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533 \times 10^3}{100 \times 10^6} = 285.33 \times 10^{-6} \, \text{m}^2 \quad A_{BE} = 285 \, \text{mm}^2$

Problem 1-4 A couple $M$ of magnitude 1500 N.m is applied to the crank of an engine. For the position shown, determine (a) the force $P$ required to hold the engine system in equilibrium, (b) the average normal stress in the connecting rod $BC$, which has a 450 mm$^2$ uniform cross section.

**Answer:** (a) $17.86 \, \text{kN}$ (b) $-41.4 \, \text{MPa}$
Problem 1-5 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length \( d \) of the cuts if the joint is to withstand an axial load of magnitude \( P = 7.6 \, \text{kN} \). Answer: 60.2 mm
Problem 1-6 When the force $P$ reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.  Answer: 5.93 MPa

**SOLUTION**

Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$. Let $t = 22 \text{ mm}$.

Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \quad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2 = 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \quad d = 60.2 \text{ mm}$$

Problem 1-7 The load $P$ applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter $d$ of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.  Answer: 63.3 mm

**SOLUTION**

Area being sheared: $A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

Force: $P = 8 \times 10^3 \text{ N}$

Shearing stress: $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6 \text{ Pa} \quad \tau = 5.93 \text{ MPa}$
Problem 1-8 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (a) the largest load $P$ that can be safely applied, (b) the corresponding tensile stress in the splice.

**Answer:** (a) 13.95 kN  (b) 620 kPa
**Problem 1-9** Link $BC$ is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480 MPa ultimate strength in tension. What is the safety factor used if the structure shown was designed to support a 16 kN load $P$?

**Answer:** 3.60

**SOLUTION**

Use bar $ACD$ as a free body and note that member $BC$ is a two-force member.

$$\Sigma M_A = 0:$$

$$(480)F_{BC} - (600)P = 0$$

$$F_{BC} = \frac{600}{480}P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{ N}$$

Ultimate load for member $BC$: $F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{ N}$$

Factor of safety:

$$\text{F.S.} = \frac{F_U}{F_{BC}} = \frac{72 \times 10^3}{20 \times 10^3} = \text{F.S.} = 3.60$$

**Problem 1-10** Link $AB$ is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area of $AB$ for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at $A$ and $B$. **Answer:** 168.1 mm$^2$
Problem 1-11 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load P that can be applied at A. Note that link BD is not reinforced around the pin holes.

Answer: 1.683 kN
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Problem 1-12 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.

Answer: 3.72 kN
Problem 1-13 A centric load P is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (a) the magnitude of P, (b) the orientation of the surface on which the maximum shearing stress occurs, (c) the normal stress exerted on that surface, (d) the maximum value of the normal stress in the block.

**Answer:**
(a) 706 kN  (b) $\theta = 45^\circ$  (c) 18.00 MPa  (d) 36.0 MPa (compression)
SOLUTION

\[ A_0 = (140 \text{ mm})(140 \text{ mm}) = 19.6 \times 10^3 \text{ mm}^2 = 19.6 \times 10^{-3} \text{ m}^2 \]

\[ \tau_{\text{max}} = 18 \text{ MPa} = 18 \times 10^6 \text{ Pa} \quad \theta = 45^\circ \text{ for plane of } \tau_{\text{max}} \]

(a) Magnitude of \( P \). \[ \tau_{\text{max}} = \frac{|P|}{2A_0} \text{ so } P = 2A_0 \tau_{\text{max}} \]

\[ P = (2)(19.6 \times 10^{-3})(18 \times 10^6) = 705.6 \times 10^3 \text{ N} \quad P = 706 \text{ kN} \]

(b) Orientation. \[ \sin 2\theta \text{ is maximum when } 2\theta = 90^\circ \quad \theta = 45^\circ \]

(c) Normal stress at \( \theta = 45^\circ \).

\[ \sigma = \frac{P \cos^2 \theta}{A_0} = \frac{(705.8 \times 10^3) \cos^2 45^\circ}{19.6 \times 10^{-3}} = 18.00 \times 10^6 \text{ Pa} \quad \sigma = 18.00 \text{ MPa} \]

(d) Maximum normal stress: \( \sigma_{\text{max}} = \frac{P}{A_0} \)

\[ \sigma_{\text{max}} = \frac{705.8 \times 10^3}{19.6 \times 10^{-3}} = 36.0 \times 10^6 \text{ Pa} \quad \sigma_{\text{max}} = 36.0 \text{ MPa} \text{ (compression)} \]