

پاسخ ۱.۴:

$$i_L = -\frac{v_L}{r} \Rightarrow v_L = -ri_L, i_L = i_C + ri_L \Rightarrow i_C = ri_L$$

$$-v_L + v_L + v_C = 0 \Rightarrow ri_L + \frac{1}{f} \frac{di_L}{dt} + \frac{1}{f} \int i_C dt' = 0 \Rightarrow r \frac{di_L}{dt} + \frac{1}{f} \frac{d^2 i_L}{dt^2} + \frac{1}{f} i_C = 0$$

$$\frac{d^2 i_L}{dt^2} + \lambda \frac{di_L}{dt} + \gamma i_L = 0, s^2 + \lambda s + \gamma = 0 \Rightarrow s_1 = -1, s_2 = -\gamma$$

$$i_L(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} = k_1 e^{-t} + k_2 e^{-\gamma t}$$

$$i_L(0) = k_1 + k_2 = 1$$

$$v_L = v_L - v_C \Rightarrow v_L = -ri_L(0) - v_C(0) \Rightarrow \frac{1}{f} \frac{di_L(0)}{dt} = -1 \times 1 - (-2) \times f = 0$$

$$\frac{di_L(0)}{dt} = -k_1 - \gamma k_2 = 0, i_L(t) = \frac{\gamma}{f} e^{-t} - \frac{1}{f} e^{-\gamma t}$$

پاسخ ۲.۴:

$$i_L(\cdot^-) = i_L(\cdot^+) = i_C(\cdot^-) = 0, v_C(\cdot^-) = v_C(\cdot^+) = \Delta$$

$$-\Delta + i_L + v_L + v_C = 0 \Rightarrow -\Delta + i_C - ru(t) + r \frac{di_L}{dt} + v_C = 0$$

$$-\Delta + i_C - ru(t) + r \frac{di_L}{dt} - f \delta(t) + r \int i_C t' = 0 \quad [I]$$

$$\frac{di_C}{dt} - r \delta(t) + r \frac{d^2 i_C}{dt^2} - f \delta'(t) + ri_C = 0 \Rightarrow \frac{d^2 i_C}{dt^2} + \frac{1}{r} \frac{di_C}{dt} + i_C = \delta(t) + r \delta'(t) \quad [II]$$

$$s^2 + \frac{1}{r} s + 1 = 0 \Rightarrow s_{1,2} = -\frac{1}{f} \pm j \frac{\sqrt{1-\Delta}}{f}$$

$$i_C(t) = e^{-\frac{1}{f}t} \left(A \cos \frac{\sqrt{1-\Delta}}{f} t + B \sin \frac{\sqrt{1-\Delta}}{f} t \right) + C$$

پاسخ پله واحد: $C = \frac{1}{f}$

با جایگذاری معادله بالا در معادله II داریم: $C = \frac{1}{f}$

$$i_C(0) = A + \frac{1}{f} = 0 \Rightarrow A = -\frac{1}{f}$$

$$-\Delta + i_C(0) - r \times 0 + r \frac{di_C(0)}{dt} - f \times 0 + v_C(0) = 0 \Rightarrow \frac{di_C(0)}{dt} = 0 \Rightarrow B = -\frac{1}{2\sqrt{1-\Delta}}$$

$$\frac{di_C}{dt} = e^{-\frac{1}{4}t} \left[\frac{2}{\sqrt{15}} \sin \frac{\sqrt{15}}{4} t \right] \quad \text{پاسخ ضربه:}$$

$$\frac{d^2 i_C}{dt^2} = e^{-\frac{1}{4}t} \left[\frac{1}{2} \cos \frac{\sqrt{15}}{4} t - \frac{1}{2\sqrt{15}} \sin \frac{\sqrt{15}}{4} t \right] \quad \text{پاسخ مشتق ضربه:}$$

$$2 \frac{d^2 i_C}{dt^2} + \frac{di_C}{dt} = e^{-\frac{1}{4}t} \left[2 \cos \frac{\sqrt{15}}{4} t + \frac{2}{\sqrt{15}} \sin \frac{\sqrt{15}}{4} t \right] \quad \text{پاسخ کل:}$$

پاسخ ۳.۴:

$$t = 0^- \Rightarrow v_o(0^-) = v_C(0^-) = v_C(0^+) = 0, i_{L_1}(0^-) = i_{L_1}(0^+) = \frac{10}{1} = 10 \text{ A}$$

$$t = 0^+ \Rightarrow -v_C(0^+) + v_{L_1}(0^+) + 15i_{L_1}(0^+) + v_{L_2}(0^+) = 0 \Rightarrow \frac{di_{L_1}}{dt}(0^+) = 0$$

$$t > 0 \Rightarrow -v_C + v_{L_1} + 15i_{L_1} + v_{L_2} = 0 \Rightarrow -10 + 10(i_C + i_{L_1}) + v_{L_1} + 15i_{L_1} + v_{L_2} = 0$$

$$-10 + 10 \cdot \frac{d}{dt} \left(2 \frac{di_{L_1}}{dt} + 15i_{L_1} + 2 \frac{di_{L_1}}{dt} \right) + 10i_{L_1} + 3 \frac{di_{L_1}}{dt} + 15i_{L_1} + 2 \frac{di_{L_1}}{dt} = 0$$

$$\frac{d^2 i_{L_1}}{dt^2} + 3.1 \frac{di_{L_1}}{dt} + 0.5i_{L_1} = 0.2, s^2 + 3.1s + 0.5 = 0 \Rightarrow s_1 = -0.17, s_2 = -2.93$$

$$i_{L_1}(t) = Ae^{-0.17t} + Be^{-2.93t} + C, 0.5C = 0.2 \Rightarrow C = 0.4, A = -0.57, B = 0.17$$

$$v_o(t) = 15i_{L_1}(t) + 2 \frac{di_{L_1}}{dt}(t) = -8.36e^{-0.17t} + 1.55e^{-2.93t} + 6$$

پاسخ ۴.۴:

$$v_C = v_{L_2}(\beta + 1)i_b = i_L + i_C, -v_s + 25i_b + v_C = 0$$

$$\frac{25}{\beta + 1} \times 10 \int v_C dt' + \frac{25}{\beta + 1} \frac{dv_C}{dt} + v_C = v_s$$

$$\frac{d^2 v_C}{dt^2} + \frac{\beta + 1}{25} \frac{dv_C}{dt} + 10 \cdot v_C = \frac{\beta + 1}{25} \frac{dv_s}{dt}, s^2 + \frac{\beta + 1}{25} s + 10 = 0 \Rightarrow s_1 = -9.39, s_2 = -10.65$$

$$2\alpha = \frac{\beta + 1}{25} = 0 \Rightarrow \beta = -1 \quad \text{الف)}$$

$$\alpha < \omega_o \Rightarrow \frac{\beta + 1}{50} < 10 \Rightarrow \beta < 499 \quad \text{ب)}$$

(ج)

$$v_C(t) = Ae^{-9.39t} + Be^{-10.65t}, v_C(\cdot^-) = v_C(\cdot^+) = \cdot, \frac{dv_C(\cdot^+)}{dt} = \frac{50.1}{25}$$

$$\Rightarrow A = -15.9, B = 15.9, v_C(t) = 15.9(-e^{-9.39t} + e^{-10.65t})$$

پاسخ ۵.۴:

$$\begin{cases} 2i_1 + 2(i_1 - i_2) + v_L = \cdot \\ 2(i_2 - i_1) + v_C + 3(i_2 - i_2) = \cdot, v_L = L \frac{di_L}{dt} = \frac{di_1}{dt} - \frac{di_2}{dt}, v_C = \frac{1}{C} \int i_C t' + I_o = \frac{1}{C} \int i_2 t' + I_o, v = i_2 \\ -v_L + 3(i_2 - i_1) + i_2 = \cdot \end{cases}$$

$$\begin{cases} 4i_1 - 2i_2 + \frac{di_1}{dt} - \frac{di_2}{dt} = \cdot \\ -2 \frac{di_1}{dt} + 5 \frac{di_2}{dt} - 3 \frac{di_2}{dt} + 2i_2 = \cdot \Rightarrow 15 \frac{d^2 i_2}{dt^2} + 44 \frac{di_2}{dt} + 32 = \cdot, s^2 + \frac{44}{15}s + \frac{32}{15} = \cdot \Rightarrow s_1 = -1.33, s_2 = -1.6 \\ -3i_2 + 4i_2 - \frac{di_1}{dt} + \frac{di_2}{dt} = \cdot \end{cases}$$

پاسخ ۶.۴:

$$t = \cdot^- \Rightarrow i_L(\cdot^-) = i_L(\cdot^+) = \cdot, v_C(\cdot^-) = v_C(\cdot^+) = \cdot$$

$$t = \cdot^+ \Rightarrow v_1(\cdot^+) = R_1 I_s, v_2(\cdot^+) = \cdot$$

$$t > \cdot \Rightarrow v_1 = R_1(I_s - i_L), v_2 = R_2(i_L - i_C), v_3 = v_C$$

$$\frac{dv_1}{dt} = R_1 \frac{dI_s}{dt} - R_1 \frac{di_L}{dt} \Rightarrow \frac{dv_1(\cdot^+)}{dt} = -\frac{R_1}{L} I_s$$

$$\frac{dv_2(\cdot^+)}{dt} = \frac{dv_C(\cdot^+)}{dt} = \frac{i_C(\cdot^+)}{C} = \cdot$$

$$\frac{d^2 v_2(\cdot^+)}{dt^2} = \frac{1}{C} \left(\frac{di_L(\cdot^+)}{dt} - \frac{di_2(\cdot^+)}{dt} \right) = \frac{R_1}{LC} I_s$$

پاسخ ۷.۴:

$$v_C = v_L + v_{R_1} = -v_{R_2} = -R_2 i_{R_2}, i_{R_2} = i_L + i_C$$

$$v_C = -\frac{di_C}{dt} + \frac{di_{R_2}}{dt} + R_2 i_{R_2} - R_1 i_C \Rightarrow \frac{dv_C}{dt} + \left(R_1 + \frac{1}{R_2}\right) \frac{dv_C}{dt} + \left(1 + \frac{R_1}{R_2}\right) v_C = 0$$

$$\alpha = R_1 + \frac{1}{R_2} = 0 \Rightarrow R_1 = \frac{1}{R_2}$$