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Second Edition

A First Course in Structural Equation Modeling

TENKO RAYKOV • GEORGE A. MARCOULIDES

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in
Structural Equation Modeling

Second Edition

**A First Course
in
Structural Equation Modeling**

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and

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Preface to the Second Edition

The idea when working on the second edition of this book was to provide a current text for an introductory structural equation modeling (SEM) course similar to the ones we teach for our departments at Michigan State University and California State University, Fullerton. Our goal is to present an updated, conceptual and nonmathematical introduction to the increasingly popular in the social and behavioral sciences SEM methodology. The readership we have in mind with this edition consists of advanced undergraduate students, graduate students, and researchers from any discipline, who have limited or no previous exposure to this analytic approach. Like before, in the past six years since the appearance of the first edition we could not locate a book that we thought would be appropriate for such an audience and course. Most of the available texts have what we see as significant limitations that may preclude their successful use in an introductory course. These books are either too technical for beginners, do not cover in sufficient breadth and detail the fundamentals of the methodology, or intermix fairly advanced issues with basic ones.

This edition maintains the previous goal of providing an alternative attempt to offer a first course in structural equation modeling at a coherent introductory level. Similarly to the first edition, there are no special prerequisites beyond a course in basic statistics that included coverage of regression analysis. We frequently draw a parallel between aspects of SEM and their apparent analogs in regression, and this prior knowledge is both helpful and important. In the main text, there are only a few mathematical formulas used, which are either conceptual or illustrative rather than computational in nature. In the appendixes to most of the chapters, we give the readers a glimpse into some formal aspects of topics discussed in the

pertinent chapter, which are directed at the mathematically more sophisticated among them. While desirable, the thorough understanding and mastery of these appendixes are not essential for accomplishing the main aims of the book.

The basic ideas and methods for conducting SEM as presented in this text are independent of particular software. We illustrate discussed model classes using the three apparently most widely circulated programs—EQS, LISREL, and *Mplus*. With these illustrations, we only aim at providing readers with information as to how to use these software, in terms of setting up command files and interpreting resulting output; we do not intend to imply any comparison between these programs or impart any judgment on relative strengths or limitations. To emphasize this, we discuss their input and output files in alphabetic order of software name, and in the later chapters use them in turn.

The goal of this text, however, is going well beyond discussion of command file generation and output interpretation for these SEM programs. Our primary aim is to provide the readers with an understanding of fundamental aspects of structural equation modeling, which we find to be of special relevance and believe will help them profitably utilize this methodology. Many of these aspects are discussed in Chapter 1, and thus a careful study of it before proceeding with the subsequent chapters and SEM applications is strongly recommended especially for newcomers to this field.

Due to the targeted audience of mostly first-time SEM users, many important advanced topics could not be covered in the book. Anyone interested in such topics could consult more advanced SEM texts published throughout the past 15 years or so (information about a score of them can be obtained from <http://www.erlbaum.com/>) and the above programs' manuals. We view our book as a stand-alone precursor to these advanced texts.

Our efforts to produce this book would not have been successful without the continued support and encouragement we have received from many scholars in the SEM area. We feel particularly indebted to Peter M. Bentler, Michael W. Browne, Karl G. Jöreskog, and Bengt O. Muthén for their path-breaking and far-reaching contributions to this field as well as helpful discussions and instruction throughout the years. In many regards they have profoundly influenced our understanding of SEM. We would also like to thank numerous colleagues and students who offered valuable comments and criticism on earlier drafts of various chapters as well as the first edition. For assistance and support, we are grateful to all at Lawrence Erlbaum Associates who were involved at various stages in the book production process. The second author also wishes to extend a very special thank you to the following people for their helpful hand in making the completion of this project a possibility: Dr. Keith E. Blackwell, Dr. Dechen Dolkar, Dr. Richard E. Loyd, and Leigh Maple along with the many other support staff at the UCLA

and St. Jude Medical Centers. Finally, and most importantly, we thank our families for their continued love despite the fact that we keep taking on new projects. The first author wishes to thank Albena and Anna; the second author wishes to thank Laura and Katerina.

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Fundamentals of Structural Equation Modeling

WHAT IS STRUCTURAL EQUATION MODELING?

Structural equation modeling (SEM) is a statistical methodology used by social, behavioral, and educational scientists as well as biologists, economists, marketing, and medical researchers. One reason for its pervasive use in many scientific fields is that SEM provides researchers with a comprehensive method for the quantification and testing of substantive theories. Other major characteristics of structural equation models are that they explicitly take into account measurement error that is ubiquitous in most disciplines, and typically contain latent variables.

Latent variables are theoretical or hypothetical constructs of major importance in many sciences, or alternatively can be viewed as variables that do not have observed realizations in a sample from a focused population. Hence, latent are such variables for which there are no available observations in a given study. Typically, there is no direct operational method for measuring a latent variable or a precise method for its evaluation. Nevertheless, manifestations of a latent construct can be observed by recording or measuring specific features of the behavior of studied subjects in a particular environment and/or situation. Measurement of behavior is usually carried out using pertinent instrumentation, for example tests, scales, self-reports, inventories, or questionnaires. Once studied constructs have been assessed, SEM can be used to quantify and test plausibility of hypothetical assertions about potential interrelationships among the constructs as well as their relationships to measures assessing them. Due to the mathematical complexities of estimating and testing these relationships and assertions,

computer software is a must in applications of SEM. To date, numerous programs are available for conducting SEM analyses. Software such as AMOS (Arbuckle & Wothke, 1999), EQS (Bentler, 2004), LISREL (Jöreskog & Sörbom, 1993a, 1993b, 1993c, 1999), *Mplus* (Muthén & Muthén, 2004), SAS PROC CALIS (SAS Institute, 1989), SEPATH (Statistica, 1998), and RAMONA (Browne & Mels, 2005) are likely to contribute in the coming years to yet a further increase in applications of this methodology. Although these programs have somewhat similar capabilities, LISREL and EQS seem to have historically dominated the field for a number of years (Marsh, Balla, & Hau, 1996); in addition, more recently *Mplus* has substantially gained in popularity among social, behavioral, and educational researchers. For this reason, and because it would be impossible to cover every program in reasonable detail in an introductory text, examples in this book are illustrated using only the LISREL, EQS, and *Mplus* software.

The term *structural equation modeling* is used throughout this text as a generic notion referring to various types of commonly encountered models. The following are some characteristics of structural equation models.

1. The models are usually conceived in terms of not directly measurable, and possibly not (very) well-defined, theoretical or hypothetical constructs. For example, anxiety, attitudes, goals, intelligence, motivation, personality, reading and writing abilities, aggression, and socioeconomic status can be considered representative of such constructs.

2. The models usually take into account potential errors of measurement in all observed variables, in particular in the independent (predictor, explanatory) variables. This is achieved by including an error term for each fallible measure, whether it is an explanatory or predicted variable. The variances of the error terms are, in general, parameters that are estimated when a model is fit to data. Tests of hypotheses about them can also be carried out when they represent substantively meaningful assertions about error variables or their relationships to other parameters.

3. The models are usually fit to matrices of interrelationship indices—that is, covariance or correlation matrices—between all pairs of observed variables, and sometimes also to variable means.¹

¹It can be shown that the fit function minimized with the maximum likelihood (ML) method used in a large part of current applications of SEM, is based on the likelihood function of the raw data (e.g., Bollen, 1989; see also section “Rules for Determining Model Parameters”). Hence, with multinormality, a structural equation model can be considered indirectly fitted to the raw data as well, similarly to models within the general linear modeling framework. Since this is an introductory book, however, we emphasize here the more direct process of fitting a model to the analyzed matrix of variable interrelationship indices, which can be viewed as the underlying idea of the most general asymptotically distribution-free method of model fitting and testing in SEM. The maximization of the likelihood function for the raw data is equivalent to the minimization of the fit function with the ML method, F_{ML} , which quantifies

This list of characteristics can be used to differentiate structural equation models from what we would like to refer to in this book as classical linear modeling approaches. These classical approaches encompass regression analysis, analysis of variance, analysis of covariance, and a large part of multivariate statistical methods (e.g., Johnson & Wichern, 2002; Marcoulides & Hershberger, 1997). In the classical approaches, typically models are fit to raw data and no error of measurement in the independent variables is assumed.

Despite these differences, an important feature that many of the classical approaches share with SEM is that they are based on linear models. Therefore, a frequent assumption made when using the SEM methodology is that the relationships among observed and/or latent variables are linear (although modeling nonlinear relationships is increasingly gaining popularity in SEM; see Schumacker & Marcoulides, 1998; Muthén & Muthén, 2004; Skrondal & Rabe-Hesketh, 2004). Another shared property between classical approaches and SEM is model comparison. For example, the well-known F test for comparing a less restricted model to a more restricted model is used in regression analysis when a researcher is interested in testing whether to drop from a considered model (prediction equation) one or more independent variables. As discussed later, the counterpart of this test in SEM is the difference in chi-square values test, or its asymptotic equivalents in the form of Lagrange multiplier or Wald tests (e.g., Bentler, 2004). More generally, the chi-square difference test is used in SEM to examine the plausibility of model parameter restrictions, for example equality of factor loadings, factor or error variances, or factor variances and covariances across groups.

Types of Structural Equation Models

The following types of commonly used structural equation models are considered in this book.

1. *Path analysis models.* Path analysis models are usually conceived of only in terms of observed variables. For this reason, some researchers do not consider them typical SEM models. We believe that path analysis models are worthy of discussion within the general SEM framework because, although they only focus on observed variables, they are an important part of the historical development of SEM and in particular use the same underlying idea of model fitting and testing as other SEM models. Figure 1 presents an example of a path analysis model examining the effects of several explanatory variables on the number of hours spent

the distance between that matrix and the one reproduced by the model (see section “Rules for Determining Model Parameters” and the Appendix to this chapter).

watching television (see section “Path Diagrams” for a complete list and discussion of the symbols that are commonly used to graphically represent structural equation models).

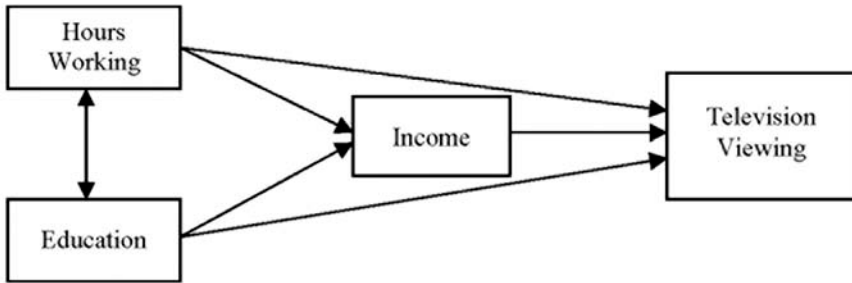


FIG. 1. Path analysis model examining the effects of some variables on television viewing. Hours Working = Average weekly working hours; Education = Number of completed school years; Income = Yearly gross income in dollars; Television Viewing = Average daily number of hours spent watching television.

2. *Confirmatory factor analysis models.* Confirmatory factor analysis models are frequently employed to examine patterns of interrelationships among several latent constructs. Each construct included in the model is usually measured by a set of observed indicators. Hence, in a confirmatory factor analysis model no specific directional relationships are assumed between the constructs, only that they are potentially correlated with one another. Figure 2 presents an example of a confirmatory factor analysis model with two interrelated self-concept constructs (Marcoulides & Hershberger, 1997).

3. *Structural regression models.* Structural regression models resemble confirmatory factor analysis models, except that they also postulate particular explanatory relationships among constructs (latent regressions) rather than these latent variables being only interrelated among themselves. The models can be used to test or disconfirm theories about explanatory relationships among various latent variables under investigation. Figure 3 presents an example of a structural regression model of variables assumed to influence returns of promotion for faculty in higher education (Heck & Johnsrud, 1994).

4. *Latent change models.* Latent change models, often also called latent growth curve models or latent curve analysis models (e.g., Bollen & Curran, 2006; Meredith & Tisak, 1990), represent a means of studying change over time. The models focus primarily on patterns of growth, decline, or both in longitudinal data (e.g., on such aspects of temporal change as initial status and rates of growth or decline), and enable re-

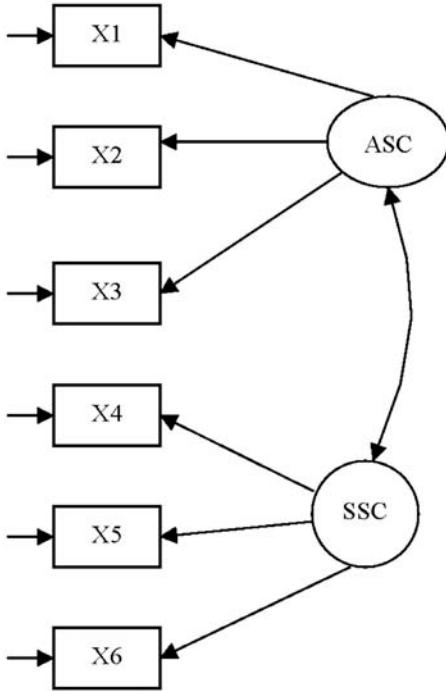


FIG. 2. Confirmatory factor analysis model with two self-concept constructs. ASC = Academic self-concept; SSC = Social self-concept.

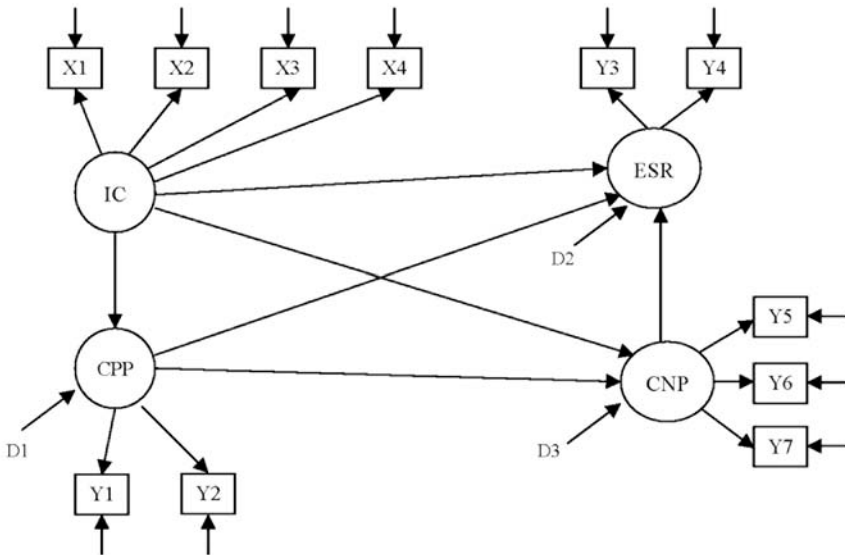


FIG. 3. Structural regression model of variables influencing return to promotion. IC = Individual characteristics; CPP = Characteristics of prior positions; ESR = Economic and social returns to promotion; CNP = Characteristics of new positions.

searchers to examine both intraindividual temporal development and interindividual similarities and differences in its patterns. The models can also be used to evaluate the relationships between patterns of change and other personal characteristics. Figure 4 presents the idea of a simple example of a two-factor growth model for two time points, although typical applications of these models occur in studies with more than two repeated assessments as discussed in more detail in Chapter 6.

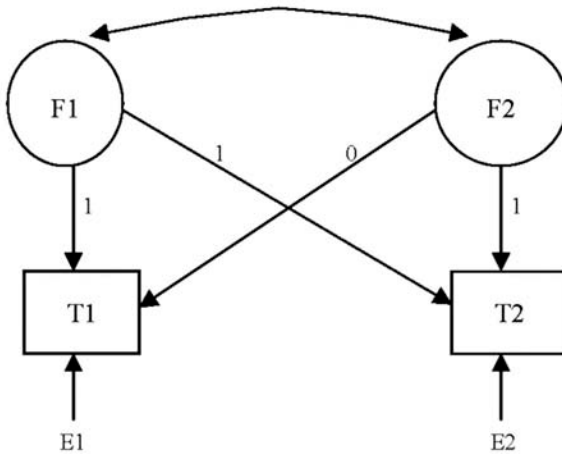


FIG. 4. A simple latent change model.

When and How Are Structural Equation Models Used?

Structural equation models can be utilized to represent knowledge or hypotheses about phenomena studied in substantive domains. The models are usually, and should best be, based on existing or proposed theories that describe and explain phenomena under investigation. With their unique feature of explicitly modeling measurement error, structural equation models provide an attractive means for examining such phenomena. Once a theory has been developed about a phenomenon of interest, the theory can be tested against empirical data using SEM. This process of testing is often called *confirmatory mode* of SEM applications.

A related utilization of structural models is *construct validation*. In these applications, researchers are interested mainly in evaluating the extent to which particular instruments actually measure a latent variable they are supposed to assess. This type of SEM use is most frequently employed when studying the psychometric properties of a given measurement device (e.g., Raykov, 2004).

Structural equation models are also used for theory development purposes. In *theory development*, repeated applications of SEM are carried

out, often on the same data set, in order to explore potential relationships between variables of interest. In contrast to the confirmatory mode of SEM applications, theory development assumes that no prior theory exists—or that one is available only in a rudimentary form—about a phenomenon under investigation. Since this utilization of SEM contributes both to the clarification and development of theories, it is commonly referred to as *exploratory mode* of SEM applications. Due to this development frequently occurring based on a single data set (single sample from a studied population), results from such exploratory applications of SEM need to be interpreted with great caution (e.g., MacCallum, 1986). Only when the findings are replicated across other samples from the same population, can they be considered more trustworthy. The reason for this concern stems mainly from the fact that results obtained by repeated SEM applications on a given sample may be capitalizing on chance factors having lead to obtaining the particular data set, which limits generalizability of results beyond that sample.

Why Are Structural Equation Models Used?

A main reason that structural equation models are widely employed in many scientific fields is that they provide a mechanism for explicitly taking into account measurement error in the observed variables (both dependent and independent) in a given model. In contrast, traditional regression analysis effectively ignores potential measurement error in the explanatory (predictor, independent) variables. As a consequence, regression results can be incorrect and possibly entail misleading substantive conclusions.

In addition to handling measurement error, SEM also enables researchers to readily develop, estimate, and test complex multivariable models, as well as to study both direct and indirect effects of variables involved in a given model. *Direct effects* are the effects that go directly from one variable to another variable. *Indirect effects* are the effects between two variables that are mediated by one or more intervening variables that are often referred to as a mediating variable(s) or mediator(s). The combination of direct and indirect effects makes up the *total effect* of an explanatory variable on a dependent variable. Hence, if an indirect effect does not receive proper attention, the relationship between two variables of concern may not be fully considered. Although regression analysis can also be used to estimate indirect effects—for example by regressing the mediating on the explanatory variable, then the effect variable on the mediator, and finally multiplying pertinent regression weights—this is strictly appropriate only when there are no measurement errors in the involved predictor variables. Such an assumption, however, is in general unrealistic in empirical research in the social and behavioral sciences. In addition, standard errors for relevant estimates are difficult to compute using this sequential ap-

plication of regression analysis, but are quite straightforwardly obtained in SEM applications for purposes of studying indirect effects.

What Are the Key Elements of Structural Equation Models?

The key elements of essentially all structural equation models are their parameters (often referred to as model parameters or unknown parameters). Model parameters reflect those aspects of a model that are typically unknown to the researcher, at least at the beginning of the analyses, yet are of potential interest to him or her. *Parameter* is a generic term referring to a characteristic of a population, such as mean or variance on a given variable, which is of relevance in a particular study. Although this characteristic is difficult to obtain, its inclusion into one's modeling considerations can be viewed as essential in order to facilitate understanding of the phenomenon under investigation. Appropriate sample statistics are used to estimate parameter(s). In SEM, the parameters are unknown aspects of a phenomenon under investigation, which are related to the distribution of the variables in an entertained model. The parameters are estimated, most frequently from the sample covariance matrix and possibly observed variable means, using specialized software.

The presence of parameters in structural equation models should not pose any difficulties to a newcomer to the SEM field. The well-known regression analysis models are also built upon multiple parameters. For example, the partial regression coefficients (or slope), intercept, and standard error of estimate are parameters in a multiple (or simple) regression model. Similarly, in a factorial analysis of variance the main effects and interaction(s) are model parameters. In general, parameters are essential elements of statistical models used in empirical research. The parameters reflect unknown aspects of a studied phenomenon and are estimated by fitting the model to sampled data using particular optimality criteria, numeric routines, and specific software. The topic of structural equation model parameters, along with a complete description of the rules that can be used to determine them, are discussed extensively in the following section "Parameter Estimation."

PATH DIAGRAMS

One of the easiest ways to communicate a structural equation model is to draw a diagram of it, referred to as *path diagram*, using special graphical notation. A path diagram is a form of graphical representation of a model under consideration. Such a diagram is equivalent to a set of equations defining a model (in addition to distributional and related assumptions), and is typically used as an alternative way of presenting a model pictorially. Path diagrams not only enhance the understanding of structural equation models and their communication among researchers with various backgrounds,

but also substantially contribute to the creation of correct command files to fit and test models with specialized programs. Figure 5 displays the most commonly used graphical notation for depicting SEM models, which is described in detail next.

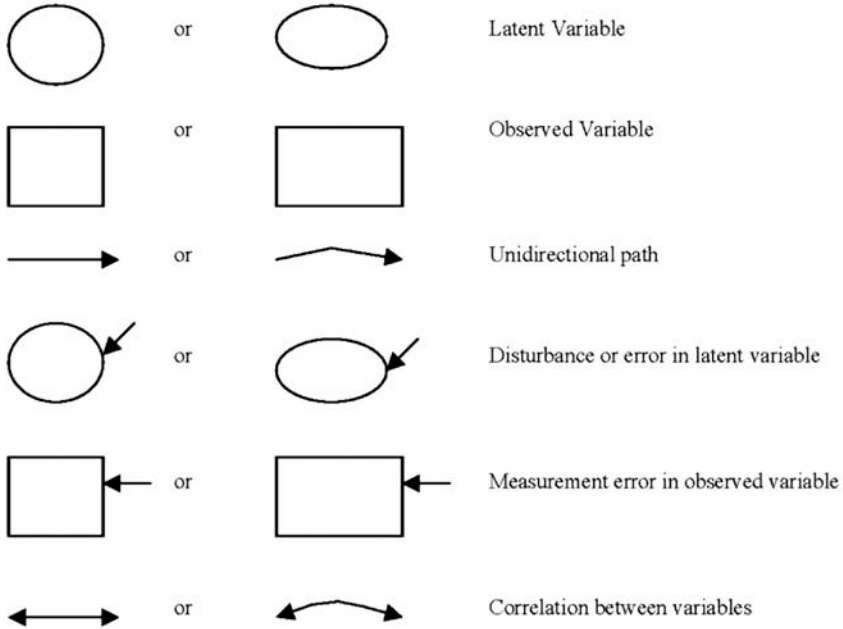


FIG. 5. Commonly used symbols for SEM models in path diagrams.

Latent and Observed Variables

One of the most important initial issues to resolve when using SEM is the distinction between observed variables and latent variables. *Observed variables* are the variables that are actually measured or recorded on a sample of subjects, such as manifested performance on a particular test or the answers to items or questions in an inventory or questionnaire. The term manifest variables is also often used for observed variables, to stress the fact that these are the variables that have actually been measured by the researcher in the process of data collection. In contrast, *latent variables* are typically hypothetically existing constructs of interest in a study. For example, intelligence, anxiety, locus of control, organizational culture, motivation, depression, social support, math ability, and socioeconomic status can all be considered latent variables. The main characteristic of latent vari-

ables is that they cannot be measured directly, because they are not directly observable. Hence, only proxies for them can be obtained using specifically developed measuring instruments, such as tests, inventories, self-reports, testlets, scales, questionnaires, or subscales. These proxies are the indicators of the latent constructs or, in simple terms, their measured aspects. For example, socioeconomic status may be considered to be measured in terms of income level, years of education, bank savings, type of occupation. Similarly, intelligence may be viewed as manifested (indicated) by subject performance on reasoning, figural relations, and culture-fair tests. Further, mathematical ability may be considered indicated by how well students do on algebra, geometry, and trigonometry tasks. Obviously, it is quite common for manifest variables to be fallible and unreliable indicators of the unobservable latent constructs of actual interest to a social or behavioral researcher. If a single observed variable is used as an indicator of a latent variable, it is most likely that the manifest variable will generally contain quite unreliable information about that construct. This information can be considered to be one-sided because it reflects only one aspect of the measured construct, the side captured by the observed variable used for its measurement. It is therefore generally recommended that researchers employ multiple indicators (preferably more than two) for each latent variable in order to obtain a much more complete and reliable picture of it than that provided by a single indicator. There are, however, instances in which a single observed variable may be a fairly good indicator of a latent variable, e.g., the total score on the Stanford-Binet Intelligence Test as a measure of the construct of intelligence.

The discussed meaning of latent variable could be referred to as a traditional, classical, or 'psychometric' conceptualization. This treatment of latent variable reflects a widespread understanding of unobservable constructs across the social and behavioral disciplines as reflecting proper subject characteristics that cannot be directly measured but (a) could be meaningfully assumed to exist separately from their measures without contradicting observed data, and (b) allow the development and testing of potentially far-reaching substantive theories that contribute significantly to knowledge accumulation in these sciences. This conceptualization of latent variable can be traced back perhaps to the pioneering work of the English psychologist Charles Spearman in the area of factor analysis around the turn of the 20th century (e.g., Spearman, 1904), and has enjoyed wide acceptance in the social and behavioral sciences over the past century. During the last 20 years or so, however, developments primarily in applied statistics have suggested the possibility of extending this traditional meaning of the concept of latent variable (e.g., Muthén, 2002; Skrondal & Rabe-Hesketh, 2004). According to what could be referred to as its modern conceptualization, any variable without observed realizations in a studied sample from a population of interest can be considered

a latent variable. In this way, as we will discuss in more detail in Chapter 6, patterns of intraindividual change (individual growth or decline trajectories) such as initial true status or true change across the period of a repeated measure study, can also be considered and in fact profitably used as latent variables. As another, perhaps more trivial example, the error term in a simple or multiple regression equation or in any statistical model containing a residual, can also be viewed as a latent variable. A common characteristic of these examples is that individual realizations (values) of the pertinent latent variables are conceptualized in a given study or modeling approach—e.g., the individual initial true status and overall change, or error score—which realizations however are not observed (see also Appendix to this chapter).²

This extended conceptualization of the notion of latent variable obviously includes as a special case the traditional, ‘psychometric’ understanding of latent constructs, which would be sufficient to use in most chapters of this introductory book. The benefit of adopting the more general, modern understanding of latent variable will be seen in the last chapter of the book. This benefit stems from the fact that the modern view provides the opportunity to capitalize on highly enriching developments in applied statistics and numerical analysis that have occurred over the past couple of decades, which allow one to consider the above modeling approaches, including SEM, as examples of a more general, latent variable modeling methodology (e.g., Muthén, 2002).

Squares and Rectangles, Circles and Ellipses

Observed and latent variables are represented in path diagrams by two distinct graphical symbols. Squares or rectangles are used for observed variables, and circles or ellipses are employed for latent variables. Observed variables are usually labeled sequentially (e.g., X_1 , X_2 , X_3), with the label centered in each square or rectangle. Latent variables can be abbreviated according to the construct they present (e.g., *SES* for socioeconomic status) or just labeled sequentially (e.g., F_1 , F_2 ; F standing for “factor”) with the name or label centered in each circle or ellipse.

Paths and Two-Way Arrows

Latent and observed variables are connected in a structural equation model in order to reflect a set of propositions about a studied phenomenon,

²Further, individual class or cluster membership in a latent class model or cluster analysis can be seen as a latent variable. Membership to a constituent in a finite mixture distribution may also be viewed as a latent variable. Similarly, the individual values of random effects in multi-level (hierarchical) or simpler variance component models can be considered scores on latent dimensions as well.

which a researcher is interested in examining (testing) using SEM. Typically, the interrelationships among the latent as well as the latent and observed variables are the main focus of study. These relationships are represented graphically in a path diagram by one-way and two-way arrows. The one-way arrows, also frequently called paths, signal that a variable at the end of the arrow is explained in the model by the variable at the beginning of the arrow. One-way arrows, or paths, are usually represented by straight lines, with arrowheads at the end of the lines. Such paths are often interpreted as symbolizing causal relationships—the variable at the end of the arrow is assumed according to the model to be the effect and the one at the beginning to be the cause. We believe that such inferences should not be made from path diagrams without a strong rationale for doing so. For instance, latent variables are oftentimes considered to be causes for their indicators; that is, the measured or recorded performance is viewed to be the effect of the presence of a corresponding latent variable. We generally abstain from making causal interpretations from structural equation models except possibly when the variable considered temporally precedes another one, in which case the former could be interpreted as the cause of the one occurring later (e.g., Babbie, 1992, chap. 1; Bollen, 1989, chap. 3). Bollen (1989) lists three conditions that should be used to establish a causal relation between variables—*isolation, association, and direction of causality*. While association may be easier to examine, it is quite difficult to ensure that a cause and effect have been isolated from all other influences. For this reason, most researchers consider SEM models and the causal relations within them only as approximations to reality that perhaps can never really be proved, but rather only disproved or disconfirmed.

In a path diagram, two-way arrows (sometimes referred to as two-way paths) are used to represent covariation between two variables, and signal that there is an association between the connected variables that is not assumed in the model to be directional. Usually two-way arrows are graphically represented as curved lines with an arrowhead at each end. A straight line with arrowheads at each end is also sometimes used to symbolize a correlation between variables. Lack of space may also force researchers to even represent a one-way arrow by a curved rather than a straight line, with an arrowhead attached to the appropriate end (e.g., Fig. 5). Therefore, when first looking at a path diagram of a structural equation model it is essential to determine which of the straight or curved lines have two arrowheads and which only one.

Dependent and Independent Variables

In order to properly conceptualize a proposed model, there is another distinction between variables that is of great importance—the differentiation

between dependent and independent variables. Dependent variables are those that receive at least one path (one-way arrow) from another variable in the model. Hence, when an entertained model is represented as a set of equations (with pertinent distributional and related assumptions), each dependent variable will appear in the left-hand side of an equation. Independent variables are variables that emanate paths (one-way arrows), but never receive a path; that is, no independent variable will appear in the left-hand side of an equation, in that system of model equations. Independent variables can be correlated among one another, i.e., connected in the path diagram by two-way arrows. We note that a dependent variable may act as an independent variable with respect to another variable, but this does not change its dependent-variable status. As long as there is at least one path (one-way arrow) ending at the variable, it is a dependent variable no matter how many other variables in the model are explained by it.

In the econometric literature, the terms exogenous variables and endogenous variables are also frequently used for independent and dependent variables, respectively. (These terms are derived from the Greek words *exo* and *endos*, for being correspondingly of external origin to the system of variables under consideration, and of internal origin to it.) Regardless of the terms one uses, an important implication of the distinction between dependent and independent variables is that there are no two-way arrows connecting any two dependent variables, or a dependent with an independent variable, in a model path diagram. For reasons that will become much clearer later, the variances and covariances (and correlations) between dependent variables, as well as covariances between dependent and independent variables, are explained in a structural equation model in terms of its unknown parameters.

An Example Path Diagram of a Structural Equation Model

To clarify further the discussion of path diagrams, consider the factor analysis model displayed in Fig. 6. This model represents assumed relationships among Parental dominance, Child intelligence, and Achievement motivation as well as their indicators.

As can be seen by examining Fig. 6, there are nine observed variables in the model. The observed variables represent nine scale scores that were obtained from a sample of 245 elementary school students. The variables are denoted by the labels V_1 through V_9 (using V for 'observed Variable'). The latent variables (or factors) are Parental dominance, Child intelligence, and Achievement motivation. As latent variables (factors), they are denoted F_1 , F_2 , and F_3 , respectively. The factors are each measured by three indicators, with each path in Fig. 6 symbolizing the factor loading of the observed variable on its pertinent latent variable.

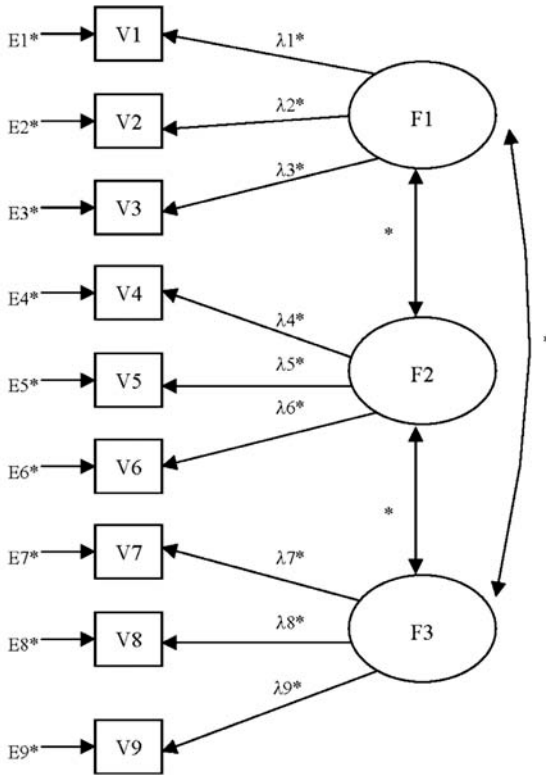


FIG. 6. Example factor analysis model. F_1 = Parental dominance; F_2 = Child intelligence; F_3 = Achievement motivation.

The two-way arrows in Fig. 6 designate the correlations between the latent variables (i.e., the factor correlations) in the model. There is also a residual term attached to each manifest variable. The residuals are denoted by E (for Error), followed by the index of the variable to which they are attached. Each residual represents the amount of variation in the manifest variable that is due to measurement error or remains unexplained by variation in the corresponding latent factor that variable loads on. The unexplained variance is the amount of indicator variance unshared with the other measures of the particular common factor. In this text, for the sake of convenience, we will frequently refer to residuals as errors or error terms.

As indicated previously, it is instrumental for an SEM application to determine the dependent and the independent variables of a model under consideration. As can be seen in Fig. 6, and using the definition of error, there are a total of 12 independent variables in this model—these are the

three latent variables and nine residual terms. Indeed, if one were to write out the 9 model definition equations (see below), none of these 12 variables will ever appear in the left-hand side of an equation. Note also that there are no one-way paths going into any independent variable, but there are paths leaving each one of them. In addition, there are three two-way arrows that connect the latent variables—they represent the three factor correlations. The dependent variables are the nine observed variables labeled V_1 through V_9 . Each of them receives two paths—(i) the path from the latent variable it loads on, which represents its factor loading; and (ii) the one from its residual term, which represents the error term effect.

First let us write down the *model definition equations*. These are the relationships between observed and unobserved variables that formally define the proposed model. Following Fig. 6, these equations are obtained by writing an equation for each observed variable in terms of how it is explained in the model, i.e., in terms of the latent variable(s) it loads on and corresponding residual term. The following system of nine equations is obtained in this way (one equation per dependent variable):

$$\begin{aligned}
 V_1 &= \lambda_1 F_1 + E_1, \\
 V_2 &= \lambda_2 F_1 + E_2, \\
 V_3 &= \lambda_3 F_1 + E_3, \\
 V_4 &= \lambda_4 F_2 + E_4, \\
 V_5 &= \lambda_5 F_2 + E_5, \\
 V_6 &= \lambda_6 F_2 + E_6, \\
 V_7 &= \lambda_7 F_3 + E_7, \\
 V_8 &= \lambda_8 F_3 + E_8, \\
 V_9 &= \lambda_9 F_3 + E_9,
 \end{aligned} \tag{1}$$

where λ_1 to λ_9 (Greek letter *lambda*) denote the nine factor loadings. In addition, we make the usual assumptions of uncorrelated residuals among themselves and with the three factors, while the factors are allowed to be interrelated, and that the nine observed variables are normally distributed, like the three factors and the nine residuals that possess zero means. We note the similarity of these distributional assumptions with those typically made in the multiple regression model (general linear model), specifically the normality of its error term, having zero mean and being uncorrelated with the predictors (e.g., Tabachnick & Fidell, 2001).

According to the factor analysis model under consideration, each of the nine Equations in (1) represents the corresponding observed variable as the sum of the product of that variable's factor loading with its pertinent factor, and a residual term. Note that on the left-hand side of each equation there is only one variable, the dependent variable, rather than a combination of variables, and also that no independent variable appears there.

Model Parameters and Asterisks

Another important feature of path diagrams, as used in this text, are the asterisks associated with one-way and two-way arrows and independent variables (e.g., Fig. 6). These asterisks are symbols of the unknown parameters and are very useful for understanding the parametric features of an entertained model as well as properly controlling its fitting and estimation process with most SEM programs. In our view, a satisfactory understanding of a given model can only then be accomplished when a researcher is able to locate the unknown model parameters. If this is done incorrectly or arbitrarily, there is a danger of ending up with a model that is unduly restrictive or has parameters that cannot be uniquely estimated. The latter problematic parameter estimation feature is characteristic of models that are unidentified—a notion discussed in greater detail in a later section—which are in general useless means of description and explanation of studied phenomena. The requirement of explicit understanding of all model parameters is quite unique to a SEM analysis but essential for meaningful utilization of pertinent software as well as subsequent model modification that is frequently needed in empirical research.

It is instructive to note that in difference to SEM, in regression analysis one does not really need to explicitly present the parameters of a fitted model, in particular when conducting this analysis with popular software. Indeed, suppose a researcher were interested in the following regression model aiming at predicting depression among college students:

$$\text{Depression} = a + b_1\text{Social-Support} + b_2\text{Intelligence} + b_3\text{Age} + \text{Error}, \quad (2)$$

where a is the intercept and b_1 , b_2 , and b_3 are the partial regression weights (slopes), with the usual assumption of normal and homoscedastic error with zero mean, which is uncorrelated with the predictors. When this model is to be fitted with a major statistical package (e.g., SAS or SPSS), the researcher is not required to specifically define a , b_1 , b_2 and b_3 , as well as the standard error of estimate, as the model parameters. This is due to the fact that unlike SEM, a regression analysis is routinely conducted in only one possible way with regard to the set of unknown parameters. Specifically, when a regression analysis is carried out, a researcher usually only needs to provide information about which measures are to be used as explanatory variables and which as the dependent variables; the utilized software automatically determines then the model parameters, typically one slope per predictor (partial regression weight) plus an intercept for the fitted regression equation and the standard error of estimate.

This automatic or default determination of model parameters does not generally work well in SEM applications and in our view should not be encouraged when the aim is a meaningful utilization of SEM. We find it particularly important in SEM to explicitly keep track of the unknown parameters in order to understand and correctly set up the model one is interested in fitting as well as subsequently appropriately modify it if needed. Therefore, we strongly recommend that researchers always first determine (locate) the parameters of a structural equation model they consider. Using default settings in SEM programs will not absolve a scientist from having to think carefully about this type of details for a particular model being examined. It is the researcher who must decide exactly how the model is defined, not the default features of a computer program used. For example, if a factor analytic model similar to the one presented in Fig. 6 is being considered in a study, one researcher may be interested in having all factor loadings as model parameters, whereas others may have reasons to view only a few of them as unknown. Furthermore, in a modeling session one is likely to be interested in several versions of an entertained model, which differ from one another only in the number and location of their parameters (see chapters 3 through 6 that also deal with such models). Hence, unlike routine applications of regression analysis, there is no single way of assuming unknown parameters without first considering a proposed structural equation model in the necessary detail that would allow one to determine its parameters. Since determination of unknown parameters is in our opinion particularly important in setting up structural equation models, we discuss it in detail next.

RULES FOR DETERMINING MODEL PARAMETERS

In order to correctly determine the parameters that can be uniquely estimated in a considered structural equation model, six rules can be used (cf. Bentler, 2004). The specific rationale behind them will be discussed in the next section of this chapter, which deals with parameter estimation. When the rules are applied in practice, for convenience no distinction needs to be made between the covariance and correlation of two independent variables (as they can be viewed equivalent for purposes of reflecting the degree of linear interrelationship between pairs of variables). For a given structural equation model, these rules are as follows.

Rule 1. All variances of independent variables are model parameters. For example, in the model depicted in Fig. 6 most of the variances of independent variables are symbolized by asterisks that are associated with each error term (residual). Error terms in a path diagram are generally attached to each dependent variable. For a latent dependent variable, an associated error term symbolizes the structural regression

disturbance that represents the variability in the latent variable unexplained by the variables it is regressed upon in the model. For example, the residual terms displayed in Fig. 3, D_1 to D_3 , encompass the part of the corresponding dependent variable variance that is not accounted for by the influence of variables explicitly present in the model and impacting that dependent variable. Similarly, for an observed dependent variable the residual represents that part of the variance of the former, which is not explained in terms of other variables that dependent variable is regressed upon in the model. We stress that all residual terms, whether attached to observed or latent variables, are (a) unobserved entities because they cannot be measured and (b) independent variables because they are not affected by any other variable in the model. Thus, by the present rule, the variances of all residuals are, in general, model parameters. However, we emphasize that this rule identifies as a parameter the variance of any independent variable, not only of residuals. Further, if there were a theory or hypothesis to be tested with a model, which indicated that some variances of independent variables (e.g., residual terms) were 0 or equal to a pre-specified number(s), then Rule 1 would not apply and the corresponding independent variable variance will be set equal to that number.

Rule 2. All covariances between independent variables are model parameters (unless there is a theory or hypothesis being tested with the model that states some of them as being equal to 0 or equal to a given constant(s)). In Fig. 6, the covariances between independent variables are the factor correlations symbolized by the two-way arrows connecting the three constructs. Note that this model does not hypothesize any correlation between observed variable residuals—there are no two-way arrows connecting any of the error terms—but other models may have one or more such correlations (e.g., see models in Chap. 5).

Rule 3. All factor loadings connecting the latent variables with their indicators are model parameters (unless there is a theory or hypothesis tested with the model that states some of them as equal to 0 or to a given constant(s)). In Fig. 6, these are the parameters denoted by the asterisks attached to the paths connecting each latent variable to its indicators.

Rule 4. All regression coefficients between observed or latent variables are model parameters (unless there is a theory or hypothesis tested with the model that states that some of them should be equal to 0 or to a given constant(s)). For example, in Fig. 3 the regression coefficients are represented by the paths going from some latent variables and ending at other latent variables. We note that Rule 3 can be considered a special case of Rule 4, after observing that a factor loading can be conceived of as a regression coefficient (slope) of the observed variable when regressed on the pertinent factor. However, performing this regression is typically impossi-

ble in practice because the factors are not observed variables to begin with and, hence, no individual measurements of them are available.

Rule 5. The variances of, and covariances between, dependent variables as well as the covariances between dependent and independent variables are never model parameters. This is due to the fact that these variances and covariances are themselves explained in terms of model parameters. As can be seen in Fig. 6, there are no two-way arrows connecting dependent variables in the model or connecting dependent and independent variables.

Rule 6. For each latent variable included in a model, the metric of its latent scale needs to be set. The reason is that, unlike an observed variable there is no natural metric underlying any latent variable. In fact, unless its metric is defined, the scale of the latent variable will remain indeterminate. Subsequently, this will lead to model-estimation problems and unidentified parameters and models (discussed later in this chapter). For any independent latent variable included in a given model, the metric can be fixed in one of two ways that are equivalent for this purpose. Either its variance is set equal to a constant, usually 1, or a path going out of the latent variable is set to a constant (typically 1). For dependent latent variables, this metric fixing is achieved by setting a path going out of the latent variable to equal a constant, typically 1. (Some SEM programs, e.g., LISREL and *Mplus*, offer the option of fixing the scales for both dependent and independent latent variable).

The reason that Rule 6 is needed stems from the fact that an application of Rule 1 on independent latent variables can produce a few redundant and not uniquely estimable model parameters. For example, the pair consisting of a path emanating from a given latent independent variable and this variable's variance, contains a redundant parameter. This means that one cannot distinguish between these two parameters given data on the observed variables; that is, based on all available observations one cannot come up with unique values for this path and latent variance, even if the entire population of interest were examined. As a result, SEM software is not able to estimate uniquely redundant parameters in a given model. Consequently, one of them will be associated with an arbitrarily determined estimate that is therefore useless. This is because both parameters reflect the same aspect of the model, although in a different form, and cannot be uniquely estimated from the sample data, i.e., are not identifiable. Hence, an infinite number of values can be associated with a redundant parameter, and all of these values will be equally consistent with the available data. Although the notion of identification is discussed in more detail later in the book, we note here that unidentified parameters can be made identified if one of

them is set equal to a constant, usually 1, or involved in a relationship with other parameters. This fixing to a constant is the essence of Rule 6.

A Summary of Model Parameters in Fig. 6

Using these six rules, one can easily summarize the parameters of the model depicted in Fig. 6. Following Rule 1, there are nine error term parameters, viz. the variances of E_1 to E_9 , as well as three factor variances (but they will be set to 1 shortly, to follow Rule 6). Based on Rule 2, there are three factor covariance parameters. According to Rule 3, the nine factor loadings are model parameters as well. Rule 4 cannot be applied in this model because no regression-type relationships are assumed between latent or between observed variables. Rule 5 states that the relationships between the observed variables, which are the dependent variables of the model, are not parameters because they are supposed to be explained in terms of the actual model parameters. Similarly, the relationships between dependent and independent variables are not model parameters.

Rule 6 now implies that in order to fix the metric of the three latent variables one can set their variances to unity or fix to 1 a path going out of each one of them. If a particularly good, that is, quite reliable, indicator of a latent variable is available, it may be better to fix the scale of that latent variable by setting to 1 the path leading from it to that indicator. Otherwise, it may be better to fix the scale of the latent variables by setting their variances to 1. We note that the paths leading from the nine error terms to their corresponding observed variables are not considered to be parameters, but instead are assumed to be equal to 1, which in fact complies with Rule 6 (fixing to 1 a loading on a latent variable, which an error term formally is, as mentioned above). For the latent variables in Fig. 6, one simply sets their variances equal to 1, because all their loadings on the pertinent observed variables are already assumed to be model parameters. This setting latent variances equal to 1 means that these variances are no more model parameters, and overrides the asterisks that would otherwise be attached to each latent variable circle in Fig. 6 to enhance pictorially the graphical representation of the model.

Therefore, applying all six rules, the model in Fig. 6 has altogether 21 parameters to be estimated—these are its nine error variances, nine factor loadings, and three factor covariances. We emphasize that testing any specific hypotheses in a model, e.g., whether all indicator loadings on the Child intelligence factor have the same value, places additional parameter restrictions and inevitably decreases the number of parameters to be estimated, as discussed further in the next section. For example, if one assumes that the three loadings on the Child intelligence factor in Fig. 6 are equal to one another, it follows that they can be represented by a single model parameter.

In that case, imposing this restriction decreases by two the number of unknown parameters to 19, because the three factor loadings involved in the constraint are not represented by three separate parameters anymore but only by a single one.

Free, Fixed, and Constrained Parameters

There are three types of model parameters that are important in conducting SEM analyses—free, fixed, and constrained. All parameters that are determined based on the above six rules are commonly referred to as *free parameters* (unless a researcher imposes additional constraints on some of them; see below), and must be estimated when fitting the model to data. For example, in Fig. 6 asterisks were used to denote the free model parameters in that factor analysis model. *Fixed parameters* have their value set equal to a given constant; such parameters are called fixed because they do not change value during the process of fitting the model, unlike the free parameters. For example, in Fig. 6 the covariances (correlations) among error terms of the observed variables V_1 to V_5 are fixed parameters since they are all set equal to 0; this is the reason why there are no two-way arrows connecting any pair of residuals in Fig. 6. Moreover, following Rule 6 one may decide to set a factor loading or alternatively a latent variance equal to 1. In this case, the loading or variance in question also becomes a fixed parameter. Alternatively, a researcher may decide to fix other parameters that were initially conceived of as free parameters, which might represent substantively interesting hypotheses to be tested with a given model. Conversely, a researcher may elect to free some initially fixed parameters, rendering them free parameters, after making sure of course that the model remains identified (see below).

The third type of parameters are called constrained parameters, also sometimes referred to as restricted or restrained parameters. *Constrained parameters* are those that are postulated to be equal to one another—but their value is not specified in advance as is that of fixed parameters—or involved in a more complex relationship among themselves. Constrained parameters are typically included in a model if their restriction is derived from existing theory or represents a substantively interesting hypothesis to be tested with the model. Hence, in a sense, constrained parameters can be viewed as having a status between that of free and of fixed parameters. This is because constrained parameters are not completely free, being set to follow some imposed restriction, yet their value can be anything as long as the restriction is preserved, rather than locked at a particular constant as is the case with a fixed parameter. It is for this reason that both free and constrained parameters are frequently referred to as model parameters. Oftentimes in the literature, all free parameters plus a representative(s) for the parameters involved in each

restriction in a considered model, are called independent model parameters. Therefore, whenever we refer to number of model parameters in the remainder, we will mean the number of independent model parameters (unless explicitly mentioned otherwise).

For example, imagine a situation in which a researcher hypothesized that the factor loadings of the Parental dominance construct associated with the measures V_1 , V_2 , and V_3 in Fig. 6 were all equal; such indicators are usually referred to in the psychometric literature as tau-equivalent measures (e.g., Jöreskog, 1971). This hypothesis amounts to the assumption that these three indicators measure the same latent variable in the same unit of measurement. Hence, by using constrained parameters, a researcher can test the plausibility of this hypothesis. If constrained parameters are included in a model, however, their restriction should be derived from existing theory or formulated as a substantively meaningful hypothesis to be tested. Further discussion concerning the process of testing parameter restrictions is provided in a later section of the book.

PARAMETER ESTIMATION

In any structural equation model, the unknown parameters are estimated in such a way that the model becomes capable of “emulating” the analyzed sample covariance or correlation matrix, and in some circumstances sample means (e.g., Chap. 6). In order to clarify this feature of the estimation process, let us look again at the path diagram in Fig. 6 and the associated model definition Equations 1 in the previous section. As indicated in earlier discussions the model represented by this path diagram, or system of equations, makes certain assumptions about the relationships between the involved variables. Hence, the model has specific implications for their variances and covariances. These implications can be worked out using a few simple relations that govern the variances and covariances of linear combinations of variables. For convenience, in this book these relations are referred to as the four laws of variances and covariances; they follow straightforwardly from the formal definition of variance and covariance (e.g., Hays, 1994).

The Four Laws for Variances and Covariances

Denote variance of a variable under consideration by ‘Var’ and covariance between two variables by ‘Cov.’ For a random variable X (e.g., an intelligence test score), the first law is stated as follows:

Law 1:

$$\text{Cov}(X,X) = \text{Var}(X).$$

Law 1 simply says that the covariance of a variable with itself is that variable's variance. This is an intuitively very clear result that is a direct consequence of the definition of variance and covariance. (This law can also be readily seen in action by looking at the formula for estimation of variance and observing that it results from the formula for estimating covariance when the two variables involved coincide; e.g., Hays, 1994.)

The second law allows one to find the covariance of two linear combinations of variables. Assume that $X, Y, Z,$ and U are four random variables—for example those denoting the scores on tests of depression, social support, intelligence, and a person's age (see Equation 2 in the section "Rules for Determining Model Parameters"). Suppose that $a, b, c,$ and d are four constants. Then the following relationship holds:

Law 2:

$$\text{Cov}(aX + bY, cZ + dU) = ac \text{Cov}(X,Z) + ad \text{Cov}(X,U) + bc \text{Cov}(Y,Z) + bd \text{Cov}(Y,U).$$

This law is quite similar to the rule of disclosing brackets used in elementary algebra. Indeed, to apply Law 2 all one needs to do is simply determine each resulting product of constants and attach the covariance of their pertinent variables. Note that the right-hand side of the equation of this law simplifies markedly if some of the variables are uncorrelated, that is, one or more of the involved covariances is equal to 0. Law 2 is extended readily to the case of covarying linear combinations of any number of initial variables, by including in its right-hand side all pairwise covariances pre-multiplied with products of pertinent weights.³

Using Laws 1 and 2, and the fact that $\text{Cov}(X,Y) = \text{Cov}(Y,X)$ (since the covariance does not depend on variable order), one obtains the next equation, which, due to its importance for the remainder of the book, is formulated as a separate law:

³Law 2 reveals the rationale behind the rules for determining the parameters for any model once the definition equations are written down (see section "Rules for Determining Model Parameters" and Appendix to this chapter). Specifically, Law 2 states that the covariance of any pair of observed measures is a function of (i) the covariances or variances of the variables involved and (ii) the weights by which these variables are multiplied and then summed up in the equations for these measures, as given in the model definition equations. The variables mentioned in (i) are the pertinent independent variables of the model (their analogs in Law 2 are $X, Y, Z,$ and U); the weights mentioned in (ii) are the respective factor loadings or regression coefficients in the model (their analogs in Law 2 are the constants $a, b, c,$ and d). Therefore, the parameters of any SEM model are (a) the variances and covariances of the independent variables, and (b) the factor loadings or regression coefficients (unless there is a theory or hypothesis tested within the model that states that some of them are equal to constants, in which case the parameters are the remaining of the quantities envisaged in (a) and (b)).

Law 3:

$$\begin{aligned}\text{Var}(aX + bY) &= \text{Cov}(aX + bY, aX + bY) \\ &= a^2 \text{Cov}(X, X) + b^2 \text{Cov}(Y, Y) + ab \text{Cov}(X, Y) + ab \text{Cov}(X, Y),\end{aligned}$$

or simply

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y).$$

A special case of Law 3 that is used often in this book involves uncorrelated variables X and Y (i.e., $\text{Cov}(X, Y) = 0$), and for this reason is formulated as another law:

Law 4: If X and Y are uncorrelated, then

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y).$$

We also stress that there are no restrictions in Laws 2, 3, and 4 on the values of the constants a , b , c , and d —in particular, they could take on the values 0 or 1, for example. In addition, we emphasize that these laws generalize straightforwardly to the case of linear combinations of more than two variables.

Model Implications and Reproduced Covariance Matrix

As mentioned earlier in this section, any considered model has certain implications for the variances and covariances (and means, if included in the analysis) of the involved observed variables. In order to see these implications, the four laws for variances and covariances can be used. For example, consider the first two manifest variables V_1 and V_2 presented in Equations 1 (see the section “Rules for Determining Model Parameters” and Fig. 6). Because both variables load on the same latent factor F_1 , we obtain the following equality directly from Law 2 (see also the first two of Equations (1)):

$$\begin{aligned}\text{Cov}(V_1, V_2) &= \text{Cov}(\lambda_1 F_1 + E_1, \lambda_2 F_1 + E_2) \\ &= \lambda_1 \lambda_2 \text{Cov}(F_1, F_1) + \lambda_1 \text{Cov}(F_1, E_2) + \lambda_2 \text{Cov}(E_1, F_1) + \text{Cov}(E_1, E_2) \\ &= \lambda_1 \lambda_2 \text{Cov}(F_1, F_1) \\ &= \lambda_1 \lambda_2 \text{Var}(F_1) \\ &= \lambda_1 \lambda_2.\end{aligned}\tag{3}$$

To obtain Equation 3, the following two facts regarding the model in Fig. 6 are also used. First, the covariance of the residuals E_1 and E_2 , and the covariance of each of them with the factor F_1 , are equal to 0 according to our earlier assumptions when defining the model (note that in Fig. 6 there are no two-headed arrows connecting the residuals or any of them with F_1); second, the variance of F_1 has been set equal to 1 according to Rule 6 (i.e., $\text{Var}(F_1) = 1$).

We stress that the elements of $\Sigma(\gamma)$ are all nonlinear functions of model parameters. In addition, each element of $\Sigma(\gamma)$ has as a counterpart a corresponding numerical element (entry) in the observed empirical covariance matrix that is obtained from the sample at hand for the nine observed variables under consideration here. Assuming that this observed sample covariance matrix, denoted by S , is as follows:

$$S = \begin{bmatrix} 1.01 & & & & & & & & & \\ 0.32 & 1.50 & & & & & & & & \\ 0.43 & 0.40 & 1.22 & & & & & & & \\ 0.38 & 0.25 & 0.33 & 1.13 & & & & & & \\ 0.30 & 0.20 & 0.30 & 0.70 & 1.06 & & & & & \\ 0.33 & 0.22 & 0.38 & 0.72 & 0.69 & 1.12 & & & & \\ 0.20 & 0.08 & 0.07 & 0.20 & 0.27 & 0.20 & 1.30 & & & \\ 0.33 & 0.19 & 0.22 & 0.09 & 0.22 & 0.12 & 0.69 & 1.07 & & \\ 0.52 & 0.27 & 0.36 & 0.33 & 0.37 & 0.29 & 0.50 & 0.62 & 1.16 & \end{bmatrix},$$

then the top element value of S (i.e., 1.01) corresponds to $\lambda_1^2 + \theta_1$ in the reproduced matrix $\Sigma(\gamma)$. Similarly, the counterpart of the element .72 in the sixth row and fourth column of S , is $\lambda_4\lambda_6$ in the $\Sigma(\gamma)$ matrix; conversely, the counterpart of the element in the 3rd row and 1st column in $\Sigma(\gamma)$, viz. $\lambda_1\lambda_3$, is .43 in S , and so on.

Now imagine setting the pairs of counterpart elements in S and $\Sigma(\gamma)$ equal to one another, from the top-left corner of S to its bottom-right corner; that is, for the model displayed in Fig. 6, set $1.01 = \lambda_1^2 + \theta_1$, then $0.32 = \lambda_1\lambda_2$, and so on until for the last elements, $1.16 = \lambda_9^2 + \theta_9$ is set. As a result of this equality setting, for the model in Fig. 6 a system of 45 equations (viz., the number of nonredundant elements in its matrix $\Sigma(\gamma)$ or S) is generated with as many unknowns as there are model parameters—that is, 21, as there are 21 asterisks in Fig. 6. Hence, one can conceive of the process of fitting a structural equation model as solving a system of possibly nonlinear equations. For each equation, its left-hand side is a subsequent numerical entry of the sample covariance matrix S , whereas its right-hand side is its counterpart element of the matrix $\Sigma(\gamma)$, i.e., the corresponding expression of model parameters at the same position in the model reproduced covariance matrix. Therefore, fitting a structural equation model is conceptually equivalent to solving this system of equations obtained according to the consequences of the model, whereby this solution is sought in an optimal way that is discussed in the next section.

The preceding discussion in this section also demonstrates that the model presented in Fig. 6 implies, as does any structural equation model, a specific structuring of the elements of the covariance matrix (and sometimes means; e.g., Chapter 6) that is reproduced by that model in terms of

particular expressions—in general, nonlinear functions—of unknown model parameters. Hence, if certain values for the parameters were entered into these expressions or functions, one would obtain a covariance matrix that has numbers as elements. In fact the process of fitting a model to data with SEM programs can be thought of as a repeated insertion of appropriate values for the parameters in the matrix $\Sigma(\gamma)$ until a certain optimality criterion, in terms of its proximity to the sample covariance matrix S , is satisfied (see below).

Every available SEM software has built into its memory the exact way in which these functions of model parameters in $\Sigma(\gamma)$ can be obtained (e.g., see Appendix to this chapter). For ease of computation, most programs make use of matrix algebra, with the software in effect determining each of the parametric expressions involved in these $p(p+1)/2$ equations. This occurs quite automatically once a researcher has communicated to the program the model with its parameters (and a few other related details discussed in the next chapter).

How Good Is a Proposed Model?

The previous section illustrated how a given structural equation model leads to a reproduced covariance matrix $\Sigma(\gamma)$ that is fit to the observed sample covariance matrix S through appropriate choice of values for the model parameters. Now it would seem that the next logical question is, “How can one measure or evaluate the extent to which the matrices S and $\Sigma(\gamma)$ differ?” This is a particularly important question in SEM because it permits one to assess the goodness of fit of the model. Indeed, if the difference between S and $\Sigma(\gamma)$ is small for a particular (optimal) set of values of the unknown parameters, then one can conclude that the model represents the observed data reasonably well. On the other hand, if this difference is large, one can conclude that the model is not consistent with the observed data. There are at least two reasons for such inconsistencies: (a) the proposed model may be deficient, in the sense that it is not capable of emulating well enough the analyzed matrix of variable interrelationships even with most favorable parameter values, or (b) the data may not be good, i.e., are deficient in some way, for example not measuring well the aspects of the studied phenomenon that are reflected in the model. Hence, in order to proceed with model fit assessment, a method is needed for evaluating the degree to which the reproduced matrix $\Sigma(\gamma)$ differs from the sample covariance matrix S .

In order to clarify this method, a new concept needs to be introduced, that of distance between matrices. Obviously, if the values to be compared were scalars, i.e., single numbers, a simple subtraction of one from the other (and possibly taking absolute value of the resulting difference) would suffice to evaluate the distance between them. However, this cannot be

done directly with the two matrices S and $\Sigma(\gamma)$. Subtracting the matrix S from the matrix $\Sigma(\gamma)$ does not result in a single number; rather, according to the rules of matrix subtraction (e.g., Johnson & Wichern, 2002) a matrix consisting of counterpart element differences is obtained.

There are some meaningful ways to evaluate the distance between two matrices, however, and the resulting distance measure ends up being a single number that is easier to interpret. Perhaps the simplest way to obtain this number involves taking the sum of the squares of the differences between the corresponding elements of the two matrices. Other more complicated ways involve the multiplication of these squares with some appropriately chosen weights and then taking their sum (discussed later). In either case, the single number obtained in the end represents a generalized measure of the distance between two matrices considered, in particular S and $\Sigma(\gamma)$. The larger this number, the more different these matrices are; the smaller the number, the more similar they are. Since in SEM this number typically results after comparing the elements of S with those of the model-implied covariance matrix $\Sigma(\gamma)$, this generalized distance is a function of the model parameters as well as the observed variances and covariances. Therefore, it is customary to refer to the relationship between the matrix distance, on the one hand, and the model parameters and S , on the other hand, as a *fit function*. Being defined as the distance between two matrices, the fit function value is always positive or 0. Whenever its value is 0, and only then, the two matrices involved are identical.

It turns out that depending on how the matrix distance is defined, several fit functions result. These fit functions, along with their corresponding methods of parameter estimation, are discussed next.

Methods of Parameter Estimation

There are four main estimation methods and types of fit functions in SEM: unweighted least squares, maximum likelihood, generalized least squares, and asymptotically distribution free (often called weighted least squares). The application of each estimation approach is based on the minimization of a corresponding fit function.

The unweighted least squares (ULS) method uses as a fit function, denoted F_{ULS} , the simple sum of squared differences between the corresponding elements of S and the model reproduced covariance matrix $\Sigma(\gamma)$. Accordingly, the estimates for the model parameters are those values for which F_{ULS} attains its smallest value. The ULS estimation approach can typically be used when the same or similar scales of measurement underlie the analyzed variables.

The other three estimation methods are based on the same sum of squares as the ULS approach, but after specific weights have been used to multiply

each of the squared element-wise differences between S and $\Sigma(\gamma)$, resulting in corresponding fit functions. These functions are designated F_{GLS} , F_{ML} and F_{ADF} for the generalized least squares (GLS), maximum likelihood (ML), and asymptotically distribution free (ADF) method, respectively (see Appendix to this chapter for their definition). The ML and GLS methods can be used when the observed data are multivariate normally distributed. This assumption is quite frequently made in multivariate analyses and can be examined using general-purpose statistical packages (e.g., SAS or SPSS); a number of consequences of it can also be addressed with SEM software. As discussed in more detail in alternative sources (e.g., Tabachnick & Fidell, 2001; Khattree & Naik, 1999), examining multinormality involves several steps. The simplest way to assess univariate normality, an implication of multivariate normality, is to consider skewness and kurtosis, and statistical tests are available for this purpose. Skewness is an index that reflects the lack of symmetry of a univariate distribution. Kurtosis has to do with the shape of the distribution in terms of its peakedness relative to a corresponding normal distribution. Under normality, the univariate skewness and kurtosis coefficients are 0; if they are markedly distinct from 0, the univariate and hence multivariate normality assumption is violated. (For statistical tests of their equality to 0, see Tabachnick & Fidell, 2001.) There is also a measure of multivariate kurtosis called Mardia's multivariate kurtosis coefficient, and its normalized estimate is of particular relevance in empirical research (e.g., Bentler, 2004). This coefficient measures the extent to which the multivariate distribution of all observed variables has tails that differ from the ones characteristic of the normal distribution, with the same component means, variances and covariances. If the distribution deviates only slightly from the normal, Mardia's coefficient will be close to 0; then its normalized estimate, which can be considered a standard normal variable under normality, will probably be nonsignificant.

Although it may happen that multivariate normality holds when all observed variables are individually normally distributed, it is desirable to also examine bivariate normality that is generally not a consequence of univariate normality. In fact, if the observations are from a multivariate normal distribution, each bivariate distribution should also be normal, like each univariate distribution. (We stress that the converse does not hold—univariate and/or bivariate normality does not imply multivariate normality.) A graphical method for examining bivariate normality involves looking at the scatter plots between all pairs of analyzed variables to ensure that they have (at least approximately) cigar-shaped forms (e.g., Tabachnick & Fidell, 2001). A formal method for judging bivariate normality is based on a plot of the chi-square percentiles and the mean distance measure of individual observations (e.g., Johnson & Wichern, 2002; Khattree & Naik, 1999; Marcoulides & Hershberger, 1997). If the distribution is normal, the plot of appropriately chosen chi-square percentiles and the individual score distance to mean

should resemble a straight line. (For further details on assessing multivariate normality, see also Marcoulides & Hershberger, 1997, pp. 48–52.)

In recent years, research has shown that the ML method can also be employed with minor deviations from normality (e.g., Bollen, 1989; Jöreskog & Sörbom, 1993b; see also Raykov & Widaman, 1995), especially when one is primarily interested in parameter estimates. In this text for a first course in SEM that is chiefly concerned with the fundamentals of SEM, we consider only the ML method for parameter estimation and model testing purposes with continuous latent and manifest variables, and refer to its robustness features in cases with slight deviations from observed variable normality (e.g., Bentler, 2004). In more general terms, the ML method aims at finding estimates for the model parameters that maximize the likelihood of observing the available data if one were to collect data from the same population again. (For a nontechnical introduction to ML, in particular in the context of missing data analysis via SEM, see, e.g., Raykov, 2005). This maximization is achieved by selecting, using a numerical search procedure across the space of all possible parameter values, numerical values for all model parameters in such a way that they minimize the corresponding fit function, F_{ML} .

With more serious deviations from normality, the asymptotically distribution free (or weighted least squares) method can be used as long as the analyzed sample is fairly large. Sample size plays an important role in almost every statistical technique applied in empirical research. Although there is universal agreement among researchers that the larger the sample relative to the population the more stable the parameter estimates, there is no agreement as to what constitutes large, due to the exceeding complexity of this matter. This topic has received a considerable amount of attention in the literature, but no easily applicable and clear-cut general rules of thumb have been proposed. To give only an idea of the issue involved, a cautious and simplified attempt at a rule of thumb might suggest that sample size would desirably be more than 10 times the number of free model parameters (cf. Bentler, 1995; Hu, Bentler, & Kano, 1992). Nevertheless, it is important to emphasize that no rule of thumb can be applied indiscriminately to all situations. This is because the appropriate size of a sample depends on many factors, including the psychometric properties of the variables, the strength of the relationships among the variables considered and size of the model, and the distributional characteristics of the variables (as well as, in general, the amount of missing data). When all these above mentioned issues are considered, samples of varying magnitude may be needed to obtain reasonable parameter estimates. If the observed variable distribution is not quite normal and does not demonstrate piling of cases at one end of the range of values for manifest variables, researchers are encouraged to use the Satorra-Bentler robust ML method of parameter estimation (e.g., Bentler, 2004; Muthén & Muthén, 2004; du Toit & du Toit, 2001). This promising approach is based on corrected statistics

obtainable with the ML method, and for this reason is frequently referred to as robust ML method. It is available in all three programs used later in this book, EQS, LISREL, and *Mplus* (see, e.g., Raykov, 2004, for evaluation of model differences with this method) and provides overall model fit test statistics and parameter standard errors (see below) that are all robust to mild deviations from normality.

Another alternative to dealing with nonnormality is to make the data appear more normal by introducing some normalizing transformation on the raw data. Once the data have been transformed and closeness to normality achieved, normal theory analysis can be carried out. In general, transformations are simply reexpressions of the data in different units of measurement. Numerous transformations have been proposed in the literature. The most popular are (a) power transformations, such as squaring each data point, taking the square root of it, or reciprocal transformations; and (b) logarithmic transformations. Last but not least, one may also want to consider alternative measures of the constructs involved in a proposed model, if such are readily available.

When data stem from designs with only a few possible response categories, the asymptotically distribution free method can be used with polychoric or polyserial correlations, or a somewhat less restrictive categorical data analysis approach can be utilized that is available within a general, latent variable modeling framework (e.g., Muthén, 2002; Muthén & Muthén, 2004). For example, suppose a questionnaire included the item, "How satisfied are you with your recent car purchase?", with response categories labeled, "Very satisfied", "Somewhat satisfied", and "Not satisfied". A considerable amount of research has shown that ignoring the categorical attributes of data obtained from items like these can lead to biased SEM results obtained with standard methods, such as that based on minimization of the ordinary ML fit function. For this reason, it has been suggested that use of the polychoric-correlation coefficient (for assessing the degree of association between ordinal variables) and the polyserial-correlation coefficient (for assessing the degree of association between an ordinal variable and a continuous variable) can be made, or alternatively the above mentioned latent variable modeling approach to categorical data analysis may be utilized. Some research has also demonstrated that when there are five or more response categories, and the distribution of data could be viewed as resembling normal, the problems from disregarding the categorical nature of responses are likely to be relatively minimal (e.g., Rigdon, 1998), especially if one uses the Satorra-Bentler robust ML approach. Hence, once again, examining the data distribution becomes essential.

From a statistical perspective, all four mentioned parameter estimation methods lead to consistent estimates. *Consistency* is a desirable feature insuring that with increasing sample size the estimates converge to the un-

known, true population parameter values of interest. Hence, consistency can be considered a minimal requirement for parameter estimates obtained with a given estimation method in order for the latter to be recommendable. With large samples, the estimates obtained using the ML and GLS approaches under multinormality, or the ADF method, possess the additional important property of being normally distributed around their population counterparts. Moreover, with large samples these three methods yield under their assumptions efficient estimates, which are associated with the smallest possible variances across the set of consistent estimates using the same data information and therefore allow one to evaluate most precisely the model parameters.

Iterative Estimation of Model Parameters

The final question now is, “Using any of the above estimation methods, how does one actually evaluate the parameters of a given model in order to render the empirical covariance matrix S and the model reproduced covariance matrix $\Sigma(\gamma)$ as close as possible?” In order to answer this question, one must resort to special numerical routines. Their goal is to minimize the fit function corresponding to the chosen method of estimation. These numerical routines proceed in a consecutive, or iterative, manner by selecting values for model parameters according to the following principle. At each step, the method specific distance—that is, fit function value—between S and $\Sigma(\gamma)$ with the new parameter values, should be smaller than this distance with the parameter values available at the preceding step. This principle is followed until no further improvement in the fit function can be achieved. At that moment, there is no additional decrease possible in the generalized distance between the empirical covariance matrix S and the model reproduced covariance matrix $\Sigma(\gamma)$, as defined by the used estimation method (e.g., Appendix to this chapter).

This iterative process starts with initial estimates of the parameters, i.e., start values for all parameters. Quite often, these values can be automatically calculated by the SEM software used, although researchers can provide their own initial values if they so choose with some complicated models. The iteration process terminates (i.e., converges) if at some step the fit function does not change by more than a very small amount (typically .000001 or a very close number, yet this value may be changed by the researcher if they have a strong reason). That is, the iterative process of parameter estimation converges at that step where practically no further improvement is possible in the distance between sample covariance matrix S and model reproduced covariance matrix $\Sigma(\gamma)$. The numerical values for the parameters obtained at that final iteration step represent the required estimates of the model parameters. We emphasize that in order for a set of

parameter estimates to be meaningful, it is necessary that the iterative process converges, i.e., terminates and thus yields a final solution. If convergence does not occur, a warning sign is issued by the software utilized, which is easily spotted in the output, and the parameter estimates at the last iteration step are in general meaningless, except perhaps being useful for tracking down the problem of lack of convergence.

All converged solutions also provide a measure of the sampling variability for each obtained parameter estimate, called *standard error*. The magnitude of the standard error indicates how stable the pertinent parameter estimate is if repeated sampling, at the same size as the analyzed sample, were carried out from the studied population. With plausible models (see following section on fit indices), the standard errors are used to compute t values that provide information about statistical significance of the associated unknown parameter. The t values are computed by the software as the ratio of parameter estimate to its standard error. If for a free parameter, for example, its t value is greater than $+2$ or less than -2 , the parameter is referred to as significant at the used significance level (typically .05) and can be considered distinct from 0 in the population. Conversely, if its t value lies between $+2$ and -2 , the parameter is nonsignificant and may be considered 0 in the population. Furthermore, based on the large-sample normality feature of parameter estimates, adding 1.96 times the standard error to and subtracting 1.96 times the standard error from the parameter estimate yields a confidence interval (at the 95% confidence level) for that parameter. This confidence interval represents a range of plausible values for the unknown parameter in the population, and can be conventionally used to test hypotheses about a prespecified value of that parameter there. In particular, if this interval covers the preselected value, the hypothesis that the parameter equals it in the population can be retained (at significance level .05); otherwise, this hypothesis can be rejected. Moreover, the width of the interval permits one to assess the precision with which the parameter has been estimated. Wider confidence intervals are associated with lower precision (and larger standard errors), and narrower confidence intervals go together with higher precision of estimation (and smaller standard errors). In fact, these features of the standard errors as measures of sampling variability of parameter estimates make them quite useful—along with a number of model goodness-of-fit indices discussed in the next section—for purposes of assessing goodness of fit of a considered model (see below).

We note that a reason for the numerical procedure of fit function minimization not to converge could be that the proposed model may simply be misspecified. Misspecified models are inadequate for the analyzed data, that is, they contain unnecessary parameters or omit important ones (and/or such variables); in terms of path diagrams, misspecified models contain wrong paths or two-way arrows, and/or omit important one-way paths or covariances. Another reason for lack of convergence is lack of

model identification. A model is not identified, frequently also referred to as unidentified, if it possesses one or more unidentified parameters. These are parameters that cannot be uniquely estimated—unlike identified parameters—even if one gathers data on the analyzed variables from the whole population and calculates their population covariance matrix to which the model is subsequently fitted. It is therefore of utmost importance to deal only with identifiable parameters, and thus make sure one is working with an identified model. Due to its special relevance for SEM analyses, the issue of parameter and model identification is discussed next.

PARAMETER AND MODEL IDENTIFICATION

As indicated earlier in this chapter, a model parameter is unidentified if there is not enough empirical information to allow its unique estimation. In that case, any estimate of an unidentified parameter computed by a SEM program is arbitrary and should not be relied on. A model containing at least one unidentified parameter cannot generally be relied on either, even though some parts of it may represent a useful approximation to the studied phenomenon. Since an unidentified model is generally useless in empirical research (although it may be useful in some theoretical discussions), one must ensure the positive identification status of a model, which can be achieved by following some general guidelines.

What Does It Mean to Have an Unidentified Parameter?

In simple terms, having an unidentified parameter implies that it is impossible to compute a defensible estimate of it. For example, suppose one considers the equation $a + b = 10$, and is faced with the task of finding unique values for the two unknown constants a and b . One solution obviously is $a = 5, b = 5$, yet another is also $a = 1, b = 9$. Evidently, there is no way in which one can determine unique values for a and b satisfying the above equation, because one is given a single equation with two unknowns. For this reason, there is an infinite number of solutions to the equation since there are more unknown values (*viz.* two parameters) than known values (one equation, *i.e.*, one known number—namely 10—related to the two parameters). Hence, the ‘model’ represented by this equation is underidentified, or unidentified, and any pair of estimates that could be obtained for a and b —like the above two pairs of values for them—are just as legitimate as any other pair satisfying the equation and hence there is no reason to prefer any of its infinitely many solutions. As will become clear later, the most desirable condition to encounter in SEM is to have more equations than are needed to obtain unique solutions for the parameters; this condition is called overidentification.

A similar identification problem occurs when the only information available is that the product of two unknown parameters λ and ϕ , say, is equal to 55. Knowing only that $\lambda\phi = 55$ does not provide sufficient information to come up with unique estimates of either λ or ϕ . Obviously, one could choose a value of λ in a completely arbitrary manner (except, of course, 0 for this example) and then take $\phi = 55/\lambda$ to provide one solution. Since there are an infinite number of solutions for λ and ϕ , neither of these two unknowns is identified until some further information is provided, for example a given value for one of them.

The last product example is in fact quite relevant in the context of our earlier consideration of Rule 6 for determining model parameters. Indeed, if neither the variance ϕ of a latent variable nor a path λ going out of it are fixed to a constant, then a situation similar to the one just discussed is created in any routinely used structural equation model, with the end result that the variance of the latent variable and the factor loading for a given indicator of it become entangled in their product and hence unidentified.

Although the two numerical examples in this section were deliberately simple, they nonetheless illustrate the nature of similar problems that can occur in the context of structural equation models. Recall from earlier sections that SEM can be thought of as an approach to solving, in an optimal way, a system of equations—those relating the elements of the sample covariance matrix S with their counterparts in the model reproduced covariance matrix $\Sigma(\gamma)$. It is possible then, depending on the model, that for some of its parameters the system may have infinitely many solutions. Clearly, such a situation is not desirable given the fact that SEM models attempt to estimate what the parameter values would be in the population. Only identified models, and in particular estimates of identified parameters, can provide this type of information; models and parameters that are unidentified cannot supply such information. A straightforward way to determine if a model is unidentified is presented next.

A Necessary Condition for Model Identification

The above introduced parallel between SEM and solving a system of equations is also useful for understanding a simple and necessary condition for model identification. Specifically, if the system of equations relating the elements of the sample covariance matrix S with their counterparts in the model-implied covariance matrix $\Sigma(\gamma)$ contains more parameters than equations, then the model will be unidentified. This is because that system has more unknowns than could possibly be uniquely solved for, and hence for at least some of them there are infinitely many solutions. Although this condition is easy to check, it should be noted that it is not a suf-

ficient condition. That is, having less unknown parameters than equations in that system does not guarantee that a model is identified. However, if a model is identified then this condition must hold, i.e., there must be as many or fewer parameters in that system of equations than nonredundant elements of the empirical covariance matrix S .

To check the necessary condition for model identification, one simply counts the number of (independent) parameters in the model and subtracts it from the number of nonredundant elements in the sample covariance matrix S , i.e., from $p(p+1)/2$ where p is the number of observed variables to which the model is fitted. The resulting difference,

$$p(p + 1)/2 - (\text{Number of model parameters}), \quad (6)$$

is referred to as *degrees of freedom* of the considered model, and is usually denoted *df*. If this difference is positive or zero, the model degrees of freedom are nonnegative. As indicated above, having nonnegative degrees of freedom represents a necessary condition for model identification.

If the difference in Equation 6 is 0, however, the degrees of freedom are 0 and the model is called saturated. *Saturated models* have as many parameters as there are nonredundant elements in the sample covariance matrix. In such cases, there is no way that one can test or disconfirm the model. This is because a saturated model will always fit the data perfectly, since it has just as many parameters as there are nonredundant elements in the empirical covariance matrix S . The number of parameters then equals that of nonredundant equations in the above mentioned system obtained when one sets equal the elements of S to their counterpart entries in the model reproduced covariance matrix $\Sigma(\gamma)$, which system therefore has a unique solution.

This lack of testability for saturated models is not unique to the SEM context, and in fact is analogous to the lack of testability of any other statistical hypothesis when pertinent degrees of freedom equal zero. For example, in an analysis of variance design with at least two factors and a single observation per cell, the hypothesis of factorial interaction is not testable because its associated degrees of freedom equal 0. In this situation, the interaction term for the underlying analysis of variance model (decomposition) is confounded with the error term and cannot be disentangled from it. For this reason, the hypothesis of interaction cannot be tested, as a result of lack of empirical information bearing upon the interaction term. The same sort of lack of empirical information renders a saturated structural equation model untestable as well.

Conversely, if the difference in Equation 6 is negative, the model degrees of freedom are negative. In such cases, the model is unidentified since it violates the necessary condition for identification. Then the system of equa-

tions relating the elements of S to their counterpart entries of the implied covariance matrix $\Sigma(\gamma)$ contains more unknown than equations. Such systems of equations, as is well known from elementary algebra, do not possess unique solutions, and hence the corresponding structural equation model is not associated with a unique set of values for its parameters that could be obtained from the sample data, no matter how large the sample size is. This is the typical deficiency of unidentified structural equation models, which renders them in general useless in practice.

From this discussion it follows that since one of the primary reasons for conducting a SEM analysis is to test the fit of a proposed model, the latter must have positive degrees of freedom. That is, there must be more nonredundant elements in the data covariance matrix than unknown model parameters. This insures that there are over-identifying restrictions placed on the model parameters, which are obtained when the elements of S are set equal to the corresponding entries of $\Sigma(\gamma)$. In such cases, it becomes of interest to evaluate to what extent these restrictions are consistent with the data. This evaluation is the task of the process of model fit evaluation.

Having said that, we stress that as mentioned earlier the condition of non-negative degrees of freedom is only a necessary but not a sufficient condition for model identification. As a matter of fact, there are many situations in which the degrees of freedom for a model are positive and yet some of its parameters are unidentified. Hence, passing the check for non-negative degrees of freedom, which sometimes is also referred to as the t -rule for model identification, does not guarantee identification and that a model could be a useful means of description and explanation of a studied phenomenon. Model identification is in general a rather complex issue that requires careful consideration and handling, and is further discussed next.

How to Deal with Unidentified Parameters in Empirical Research?

If a model under consideration is carefully conceptualized, the likelihood of unidentified parameters will usually be minimized. In particular, using Rules 1 to 6 will most likely ensure that the proposed model is identified. However, if a model is found to be unidentified, a first step toward identification is to see if all its parameters have been correctly determined or whether all the latent variables have their scales fixed. In many instances, a SEM program will signal an identification problem with an error message and even correctly point to the unidentified parameter, but in some cases the software may point to the wrong parameter or even may miss an unidentified parameter and model. Hence, the best strategy is for the researcher to examine the issue of model identification and locate the unidentified parameter(s) in an unidentified model, rather than rely com-

pletely on a SEM program. This could be accomplished on a case-to-case basis, by trying to solve the above system of equations obtained when elements of the empirical covariance matrix S are set equal to their counterpart entries in the model implied covariance matrix $\Sigma(\gamma)$. If for at least one model parameter there are infinitely many solutions possible from the system, that parameter is not identified and the situation needs to be rectified (see below). If none of the parameters is unidentified, all of them are identified and the model is identified as well.

Once located, as a first step one should see if the unidentified parameter is a latent variance or factor loading, since omitting to fix the scale of a latent variable will lead to lack of identification of at least one of these two parameters pertaining to that variable. When none of the unidentified parameters results because of such an omission, a possible way of dealing with these parameters is to impose appropriate, substantively plausible constraints on them or functions of them that potentially involve also other parameters. This way of attempting model identification may not always work, in part because of lack of such constraints. In those cases, either a completely new model may have to be contemplated—one that is perhaps simpler and does not contain unidentified parameters—or a new study and data-collection process may have to be designed.

MODEL-TESTING AND -FIT EVALUATION

The SEM methodology offers researchers a method for quantification and testing of theories. Substantive theories are often representable as models that describe and explain phenomena under investigation. As discussed previously, an essential requirement for all such models is that they be identified. Another requirement, one of no lesser importance, is that researchers consider for further study only those models that are meaningful from a substantive viewpoint and present plausible means of data description and explanation.

SEM provides a number of inferential and descriptive indices that reflect the extent to which a model can be considered an acceptable means of data representation. Using them together with substantive considerations allows one to make a decision whether a given model should reasonably be rejected as a means of data explanation or could be tentatively relied on (to some extent). The topic of structural equation *model fit evaluation*, i.e., the process of assessing the extent to which a model fits an analyzed data set, is very complex and in some aspects not necessarily uncontroversial. Due to the introductory nature of this book, this section develops a relatively brief discussion of the underlying issues, which can be considered a minimalist scheme for carrying out model evaluation. For further elaboration on these issues, we refer the reader to Bentler (2004), Bollen (1989), Byrne (1998), Jöreskog and Sörbom (1993a, 1993b), Muthén & Muthén (2004).

Substantive Considerations

A major aspect of fit evaluation involves the substantive interpretations of results obtained with a proposed model. All models considered in research should first be conceptualized according to the latest knowledge about the phenomenon under consideration. This knowledge is usually obtained via extensive study of the pertinent literature. Fitted models should try to embody in appropriate ways the findings available from previous studies. If a researcher wishes to critically examine aspects of available theories, alternative models with various new restrictions or relationships between involved variables can also be tested. However, the initial conceptualization of any proposed model can only come after an informed study of the phenomenon under consideration that includes also a careful study of past research and accumulated knowledge in the respective subject-matter domain.

Regardless of the specifics of a model in this regard, the advantages of the SEM methodology can only be used with variables that have been validly and reliably assessed. Even the most intricate and sophisticated models are of no use if the variables included in the model are poorly assessed. A model cannot do more than what is contained in the data themselves. If the data are poor, in the sense of reflecting substantial unreliability in assessing aspects of a studied phenomenon, the results will be poor, regardless of the particulars of used models.

Providing an extensive discussion of the various ways of ensuring satisfactory measurement properties of variables included in structural equation models is beyond the scope of this introductory book. These issues are usually addressed at length in books dealing specifically with psychometrics and measurement theory (e.g., Allen & Yen, 1979; Crocker & Algina, 1986; Suen, 1990). The present text instead assumes that the researcher has sufficient knowledge of how to organize a reliable and valid assessment process for the variables included in a given model.

Model Evaluation and the True Model

Before particular indexes of model fit are discussed, a word of warning is in order. Even if all fit indexes point to an acceptable model, one cannot claim in empirical research to have found the true model that has generated the analyzed data. (The cases in which data are simulated according to a known model are excluded from this consideration.) This fact is related to another specificity of SEM that is different from classical modeling approaches. Whereas classical methodology is typically interested in rejecting null hypotheses because the substantive conjecture is usually reflected in the alternative rather than null hypotheses (e.g., alternative hypotheses of difference or change), SEM is pragmatically concerned with finding a model that does not contradict the data. That is, in an empirical SEM session, one is typically

interested in retaining a proposed model whose validity is the essence of a pertinent null hypothesis. In other words, statistically speaking, when using SEM one is usually 'interested' in not rejecting the null hypothesis.

However, recall from introductory statistics that not rejecting a null hypothesis does not mean that it is true. Similarly, because model testing in SEM involves testing the null hypothesis that the model is capable of perfectly reproducing with certain values of its unknown parameters the population matrix of observed variable interrelationship indices, not rejecting a fitted model does not imply that it is the true model. In fact, it may well be that the model is not correctly specified (i.e., wrong), yet due to sampling error it appears plausible. Similarly, just because a model fits a data set well does not mean that it is the only model that fits the data well or nearly as well. There can be a plethora of other models that fit the data equally well, better, or only marginally worse. In fact, there can be a number (possibly very many; e.g., Raykov & Marcoulides, 2001; Raykov & Penev, 1999) of equivalent models that fit the data just as well as a model under consideration. Unfortunately, at present there is no statistical means for discriminating among these equivalent models—especially when the issue is choosing one (or more) of them for further consideration or interpretation. Which one of these models is better and which one is to be ruled out, can only be decided on the basis of a sound body of substantive knowledge about the studied phenomenon. This is partly the reason why substantive considerations are so important in model-fit evaluation. In addition, one can also evaluate the validity of a proposed model by conducting replication studies. The value of a given model is greatly enhanced if it can be replicated in new samples from the same studied population.

Parameter Estimate Signs, Magnitude, and Standard Errors

It is worth reiterating at this point that one cannot in general meaningfully interpret a model solution provided by SEM software if the underlying numerical minimization routine has not converged, that is has not ended after a finite number of iterations. If this routine does not terminate, one cannot trust the program output for purposes of solution interpretation (although the solution may provide information that is useful for tracking down the reasons for lack of convergence).

For a model to be considered further for fit evaluation, the parameter estimates in the final solution of the minimization procedure should have the right sign and magnitude as predicted or expected by available theory and past research. In addition, the standard errors associated with each of the parameter estimates should not be excessively large. If a standard error of a parameter estimate is very large, especially when compared to other parameter estimate standard errors, the model does not provide reliable information with regard to that parameter and should be interpreted with great

caution; moreover, the reasons for this finding should be clarified before further work with the model is undertaken.

Goodness-of-Fit Indices

The Chi-Square Value Evaluation of model fit is typically carried out on the basis of an inferential goodness-of-fit index as well as a number of other descriptive or alternative indices. This inferential index is the so-called chi-square value. The index represents a test statistic of the goodness of fit of the model, and is used when testing the null hypothesis that the model fits the corresponding population covariance matrix perfectly. This test statistic is defined as

$$T = (N - 1) F_{\min}, \quad (7)$$

where N is the sample size and F_{\min} denotes the minimal value of the fit function for the parameter estimation method used (e.g., ML, GLS, ADF).

The name chi-square value derives from the fact that with large samples the distribution of T approaches a chi-square distribution if the model is correct and fitted to the covariance matrix S . This large-sample behavior of T follows from what is referred to as likelihood ratio theory in statistics (e.g., Johnson & Wichern, 2002). The test statistic in (7) can be obtained in the context of comparing a proposed model with a saturated model, which as discussed earlier fits perfectly the data, using the so-called likelihood ratio. After multiplication with -2 , the distribution of the logarithm of this ratio approaches with increasing sample size a chi-square distribution under the null hypothesis (e.g., Bollen, 1989), a result that for our purposes is equivalent to a chi-square distribution of the right-hand side of Equation (7).

The degrees of freedom of this limiting chi-square distribution are equal to those of the model. As mentioned previously, they are determined by using the formula $df = (p(p+1)/2) - q$, where p is the number of observed variables involved in the model and q is the number of model parameters (see Equation 6 in section "Parameters and Model Identification").

When a considered model is fit to data using SEM software, the program will judge the obtained chi-square value T in relation to the model's degrees of freedom, and output its associated p -value. This p -value can be examined and compared with a preset significance level (often .05) in order to test the null hypothesis that the model is capable of exactly reproducing the population matrix of observed variable relationship indices. Hence, following the statistical null hypothesis testing tradition, one may consider rejection of the model when this p -value is smaller than a preset significance value (e.g., .05), and alternatively retention of the model if this p -value is higher than that significance level.

Although this way of looking at statistical inference in SEM may appear to be the reverse of the one used within the framework of traditional hypothesis testing, as exemplified with the framework of the general linear model, it turns out that at least from a philosophy-of-science perspective the two are compatible. Indeed, following Popperian logic (Popper, 1962), one's interest lies in rejecting models rather than confirming them. This is because there is in general no scientific way of proving the validity of a given model. That is, in empirical research no structural equation model can be proved to be the true model (see discussion in previous section).

In this context, it is also important to note that in general there is a preference for dealing with models that have a large number of degrees of freedom. This is because an intuitive meaning of the notion of degree of freedom is as a dimension along which a model can be disconfirmed. Hence, the more degrees of freedom a model has, the more dimensions there are along which one can reject the model, and hence the higher the likelihood of disconfirming it when it is tested against data. This is a desirable feature of the testing process because, according to Popperian logic, empirical science can only disconfirm and not confirm models. Therefore, if one has two models that are plausible descriptions of a studied phenomenon, the one with more degrees of freedom is a stronger candidate for consideration as a means of data description and explanation. The reason is that the model with more degrees of freedom has withstood a greater chance of being rejected; if the model was not rejected then, the results obtained with it may be viewed as more trustworthy. This reasoning is essentially the conceptual basis of the parsimony principle widely discussed in the SEM literature (e.g., Raykov & Marcoulides, 1999, and references therein). Hence, Popperian logic, which maintains that a goal of empirical science is to formulate theories that are falsifiable, is facilitated by an application of the parsimony principle. If a more parsimonious model is found to be acceptable, then one may also place more trust in it because it has withstood a higher chance of rejection than a less parsimonious model. However, researchers are cautioned that rigid and routine applications of the parsimony principle can lead to conclusions favoring an incorrect model and implications that are incompatible with those of the correct model. (For a further discussion, see Raykov & Marcoulides, 1999.)

The chi-square value T has received a lengthy discussion in this section for two reasons. First, historically and traditionally, it has been the index that has attracted a great deal of attention over the past 40 years or so and especially in the 1970s through 1990s. In fact, most of the fit indexes devised over the past several decades in the SEM literature are functions of the chi-square value. Second, the chi-square value has the important feature of being an inferential fit index. That is to say, by using it one is in a position to make a generalization about the fit of the model in a studied population.

This is due to the fact that the large-sample distribution of T is known—namely central chi-square, when a correct model is fitted to the covariance matrix—and a p -value can be attached to each particular sample's value of T . This feature is not shared by most of the other goodness of fit indexes.

However, it may not be always wise to strictly follow this statistical evaluation of plausibility of a model using the chi-square value T , due to the fact that with very large samples T cannot be really relied on. The reason is readily seen from its definition in Equation 7. Since the value of T is obtained by multiplying $N - 1$ (sample size less 1) by the attained minimum of the fit function, increasing the sample size typically leads to an increase in T as well. Yet the model's degrees of freedom remain the same because the model has not changed, and hence so does the reference chi-square distribution against which T is judged for significance. Consequently, with very large samples there is a spurious tendency to obtain large values of T , which tend to be associated with small p -values. Therefore, if one were to use only the chi-square's p -value as an index of model fit, there will be an artificial tendency with very large samples to reject models even if they were only marginally inconsistent with the analyzed data.

Alternatively, there is another spurious tendency with small samples for the test statistic T to remain small, which is also explained by looking at the above Equation (7) and noting that the multiplier $N - 1$ is then small. Hence, with small samples there is a tendency for the chi-square fit index to be associated with large p -values, suggesting a considered model as a plausible data-description means. Thus, the chi-square index and its p -value alone cannot be fully trusted in general as means for model evaluation. Other fit indices must also be examined in order to obtain a better picture of model fit.

Descriptive Fit Indices The above limitations of the chi-square value indicate the importance of the availability of other fit indices to aid the process of model evaluation. A number of descriptive-fit indices have been proposed mostly in the 1970s and 80s that provide a family of fit measures useful in the process of assessing model fit.

The first developed descriptive-fit index is the goodness-of-fit index (GFI). It can be loosely considered a measure of the proportion of variance and covariance that a given model is able to explain. The GFI may be viewed as an analog in the SEM field of the widely used R^2 index in regression analysis. If the number of parameters is also taken into account in computing the GFI, the resulting index is called adjusted goodness-of-fit index (AGFI). Its underlying logic is similar to that of the adjusted R^2 index also used in regression analysis. The GFI and AGFI indexes range between 0 and 1, and are usually fairly close to 1 for well-fitting models. Unfortunately, as with many other descriptive indices, there are no strict norms for the GFI and AGFI below which a model cannot be considered a plausible description of the

analyzed data and above which one could rest assured that the model approximates the data reasonably well. As a rough guide, it may be suggested that models with a GFI and AGFI in the mid-.90s or above may represent a reasonably good approximation of the data (Hu & Bentler, 1999).

There are two other descriptive indices that are also very useful for model-fit evaluation purposes. These are the normed fit index (NFI) and the non-normed fit index (NNFI) (Bentler & Bonnet, 1980). The NFI and NNFI are based on the idea of comparing the proposed model to a model in which no interrelationships at all are assumed among any of the variables. The latter model is referred to as the independence model or the null model, and in some sense may be seen as the least attractive, or “worst”, model that could be considered as a means of explanation and description of one’s data. The name independence or null model derives from the fact that this model assumes the variables only have variances but that there are no relationships at all among them, that is, all their covariances are zero. Thus, the null model represents the extreme case of no relationships among the studied variables, and interest lies in comparing a proposed model to the corresponding null model. When the chi-square value of the null model is compared to that of a model under consideration, one gets an idea of how much better the model of concern fits the data relative to how bad a means of data description and explanation that model could possibly be. This is the basic idea that underlies the NFI and NNFI descriptive-fit indices.

The NFI is computed by relating the difference of the chi-square value for a proposed model to the chi-square value for the independence or null model. The NNFI is a variant of the NFI that takes into account also the degrees of freedom of the proposed model. This is done in order to account for model complexity, as reflected in the degrees of freedom. The reason this inclusion is meaningful, is that for a given data set more complex models have more parameters and hence fewer degrees of freedom, whereas less complex models have less parameters and thus more degrees of freedom. Therefore, one can consider degrees of freedom as an indicator of complexity of a model (given a set of observed variables to which it is fitted).

Similar to the GFI and AGFI, models with NFI and NNFI close to 1 are considered to be more plausible means of describing the data than models for which these indices are further away from 1. Unfortunately, once again, there are no strict norms above which one can consider the indices as supporting model plausibility and below which one can safely reject the model. As a rough guide, models with NNFI and NFI in the mid-.90s or higher are viewed likely to represent reasonably good approximations to analyzed data (Hu & Bentler, 1999).

In addition to the GFI, AGFI, NNFI, and NFI, there are more than a dozen other descriptive-fit indices that have been proposed in the SEM literature

over the past 30 years or so. Despite this plethora of descriptive-fit indices, most of them are directly related to the chi-square value T and represent reexpressions of it or its relationships to other models' chi-square values and related quantities. The interested reader may refer to more advanced SEM books that provide mathematical definitions of each of these indexes (e.g., Bollen, 1989) as well as the program manuals for EQS, LISREL, and *Mplus* (Bentler, 2004; Jöreskog & Sörbom, 1993a; Muthén & Muthén, 2004).

Alternative Fit Indices A family of alternative fit indices are based on an altogether different conceptual approach to the process of hypothesis testing in SEM, which can be referred to as an alternative approach to model assessment. These indices have been developed over the past 25 years and largely originate from an insightful paper by Steiger and Lind (1980). The basis for alternative-fit indices is the noncentrality parameter (NCP), denoted δ . The NCP basically reflects the extent to which a model does not fit the data. For example, if a model is correct and the sample large, the test statistic T presented in Equation 7 follows a (central) chi-square distribution, but if the model is not quite correct, i.e., is misspecified to a small degree, then T follows a noncentral chi-square distribution. As an approximation, a noncentral chi-square distribution can roughly be thought of as resulting when the central chi-square distribution is shifted to the right by δ units (and its variance correspondingly enlarged). In this way, the NCP can be viewed as an index reflecting the degree to which a model under consideration fails to fit the data. Thus, the larger the NCP, the worse the model; and the smaller the NCP, the better the model. It can be shown that with not-too-misspecified models, normality, and large samples, δ approximately equals $(N - 1)F_{ML,0}$, where $F_{ML,0}$ is the value of the maximum likelihood fit function when the model is fit to the population covariance matrix. The NCP is estimated in a given sample by $\hat{\delta} = T - d$ if $T \geq d$, and by 0 if $T < d$, where for simplicity d denotes the model degrees of freedom.

Within the alternative approach to model testing, the conventional null hypothesis that a proposed model perfectly fits the population covariance matrix is relaxed. This is explained by the observation that in practice every model is wrong even before it is fitted to data. Indeed, the reason why a model is used when studying a phenomenon of interest is that the model should represent a useful simplification and approximation of reality rather than be a precise replica of it. That is, by its very nature, a model cannot be correct because then it would have to be an exact copy of reality and therefore useless. Hence, in the alternative approach to model testing the conventional null hypothesis of perfect model fit that has been traditionally tested in SEM by examining the chi-square index and its p -value, is really of

no interest. Instead, one is primarily concerned with evaluating the extent to which the model fails to fit the data. Consequently, for the reasonableness of a model as a means of data description and explanation, one should impose weaker requirements for degree of fit.

This is the logic of model testing that is followed by the so-called root mean square error of approximation (RMSEA) index that has recently become quite a popular index of model fit. In a given sample, the RMSEA is evaluated as

$$\pi = \sqrt{(T-d)/(dn)} \quad (8)$$

when $T \geq d$, or as 0 if $T < d$, where $n = N - 1$ is sample size less 1. The RMSEA, similar to other fit indices, also takes into account model complexity, as reflected in the degrees of freedom. It has been suggested that a value of the RMSEA of less than .05 is indicative of the model being a reasonable approximation to the analyzed data (Browne & Cudeck, 1993). Some research has found that the RMSEA is among the fit indices least affected by sample size; this feature sets the RMSEA apart from many other fit indices that are sample-dependent or have characteristics of their distribution, such as the mean, depending on sample size (Marsh et al., 1996; Bollen, 1989).

The RMSEA is not the only index that can be obtained as a direct function of the noncentrality parameter. The comparative-fit index (CFI) also follows the logic of comparing a proposed model with the null model assuming no relationships between the observed measures (Bentler, 1990). The CFI is defined as the ratio of improvement in noncentrality when moving from the null to a considered model, to the noncentrality of the null model. Typically, the null model has considerably higher noncentrality than a proposed model because the former could be expected to fit the data poorly. Hence, values of CFI close to 1 are considered likely to be indicative of a reasonably well-fitting model. Again, there are no norms about how high the CFI should be in order to safely retain or reject a given model. CFI's in the mid-.90s or above are usually associated with models that are plausible approximations of the data.

The expected cross-validation index (ECVI) was also introduced as a function of the noncentrality parameter (Browne & Cudeck, 1993). The ECVI represents a measure of the degree to which one would expect a given model to replicate in another sample from the same population. In a set of several proposed models for the same studied phenomenon, a model is preferred if it minimizes the value of ECVI relative to the other models. The ECVI was developed partly as a reaction to the fact that because the RMSEA is only weakly related to sample size, it cannot account for the fact that with small samples it would be unwise to fit a very complex

model (i.e., one with many parameters). The ECVI accounts for this possibility, and when the maximum likelihood method of estimation is used it will be identical up to a multiplicative constant to the Akaike information criterion (AIC). The AIC is a special type of fit index that takes into account both the measure of fit and model complexity (Akaike, 1987), and resembles the so-called Bayesian information criterion (BIC). The two indices, AIC and BIC, are widely used in applied statistics for purposes of model comparison. Generally, models with lower values of ECVI, AIC, and BIC are more likely to be better means of data description than models with higher such indexes. The ECVI, AIC, and BIC have become quite popular in SEM and latent variable modeling applications, particularly for the purpose of examining competing models, i.e., when a researcher is considering several models and wishes to select from them the one with best fit. According to these indices, models with smaller values on them are preferred to models with higher values.

Another important feature of this alternative approach to model assessment involves the routine use of confidence intervals, and specifically for the noncentrality parameter and RMSEA. Recall from basic statistics that a confidence interval provides a range of plausible values for the population parameter being estimated, at a given confidence level. The width of the interval is also indicative of the precision of estimation of the parameter using the data at hand. Of special interest to the alternative approach of model testing is the left endpoint of the 90% confidence interval of the RMSEA index for an entertained model. In particular, if this endpoint is considerably smaller than .05 and the interval not too wide (e.g., the right endpoint not higher than .08), it can be argued that the model is a plausible means of describing the analyzed data. Hence, if the RMSEA is smaller than .05 or the left endpoint of its confidence interval markedly smaller than .05, with this interval being not excessively wide, the pertinent model could be considered a reasonable approximation of the analyzed data.

In conclusion of this section on model testing and fit evaluation, we would like to emphasize that no decision on goodness of fit should be based on a single index, no matter how favorable for the model that index may appear. As indicated earlier, every index represents a certain aspect of the fit of a proposed model, and in this sense is a source of limited information as to how good the model is or how well it can be expected to perform in the future (e.g., on another sample from the same population). Therefore, a decision to reject or retain a model should always be based on multiple goodness-of-fit indices (and if possible on the results of replication studies). In addition, as indicated in the next section, important insights regarding model fit can be sometimes obtained by also conducting an analysis of residuals.

Analysis of Residuals

All fit indices discussed in the previous section should be viewed as overall measures of model fit. In other words, they are summary measures of fit and none of them provides information about the fit of individual parts of the model. As a consequence, it is possible for a given model to be seriously misspecified in some parts (i.e., incorrect with regard to some of the variables and their relationships) but very well fitting in others, so that an evaluation of the previously discussed fit criteria suggests that the model may be judged plausible.

For example, consider a model that is substantially off the mark with respect to an important relationship between two particular observed variables (e.g., the model omits this relationship). In such a case, the difference between the sample covariance and the covariance reproduced by the model at the final solution—called residual for that pair of variables—may be substantial. This result would suggest that the model cannot be considered a plausible means of data description. However, at the same time the model may do an excellent job of explaining all of the remaining covariances and variances in the sample covariance matrix S , and overall result in a nonsignificant chi-square value and favorable descriptive as well as alternative fit indices. Such an apparent paradox may emerge because the chi-square value T , or any of the other fit indices discussed above, is a measure of overall fit. Hence, all that is provided by overall measures of model fit is a summary picture of how well a model fits the entire analyzed matrix, but no information is contained in them about how well the model reproduces the individual elements of that matrix.

To counteract this possibility, the so-called covariance residuals—often also referred to as model residuals—can be examined. There are as many generic residuals of this kind as there are nonredundant elements of the sample covariance matrix of the variables to which a model is fitted. They result from an element-wise comparison of each sample variance and covariance to the value of its counterpart element in the implied covariance matrix obtained with the parameter estimates when the model is fitted to data. In fact, there are two types of model residuals that can be examined in most SEM models and are provided by the used software. The unstandardized residuals index the amount of unexplained variable covariance in terms of the original metric of the raw data. However, if this metric is quite different across measured variables, it is impossible to examine meaningfully these residuals and determine which are large and which are small. A standardization of the residuals to a common metric, as reflected in the standardized residuals, makes this comparison much easier.

A standardized residual above 2 generally indicates that the model considerably underexplains a particular relationship between two variables.

Conversely, a standardized residual below -2 generally indicates that the model markedly overexplains the relationship between the two variables. Using this residual information, a researcher may decide to either add or remove some substantively meaningful paths or covariance parameters, which could contribute to a smaller residual associated with the involved two variables and hence a better-fitting model with regard to their relationship (see Appendix to this chapter).

Overall, good-fitting models will typically exhibit a stem-and-leaf plot of standardized residuals that closely resembles a symmetric distribution. In addition, examining the so-called Q plot of the standardized residuals is a useful means of checking the plausibility of a proposed model. The Q plot graphs the standardized residuals against their expectations if the model were a good means of data description. With well-fitting models, a line drawn through the marks of the residuals on that plot will be close to the dotted, equidistant line provided on it by the software (Jöreskog & Sörbom, 1993c). Marked departures from a straight line indicate serious model misspecifications or possibly violations of the normality assumption (e.g., nonlinear trends in the relationships between some observed variables).

An important current limitation in SEM applications is the lack of evaluation of estimated individual-case residuals. Individual-case residuals are routinely used in applications of regression analysis because they help researchers with model evaluation and modification. In regression analysis, residuals are defined as the differences between individual raw data and their model-based predictions. Unfortunately, SEM developers have only recently begun to investigate more formally ways in which individual-case residuals can be defined within this framework (e.g., Bollen & Arminger, 1991; Raykov & Penev, 2001). The development of means for defining individual-case residuals is also hampered by the fact that most structural equation models are based on latent variables, which cannot be directly observed or precisely measured. Therefore, very important pieces of information that are needed in order to arrive at individual-case residuals similar to those used in regression analysis are typically missing.

Modification Indices

A researcher usually conducts a SEM analysis by fitting a proposed model to available data. If the model does not fit, one may accept this fact and leave it at that (which is not really commonly recommended), or alternatively may consider answering the question, "How could the model be altered in order to improve its fit?" In the SEM literature, the modification of a specified model with the aim of improving fit has been termed a specification search (Long, 1983; MacCallum, 1986). Accordingly, a specification search is conducted with the intent to detect and correct specification error in a pro-

posed model, that is, its deviation from the true model characterizing a studied population and relationships among analyzed variables.

Although in theory researchers should fully specify and deductively hypothesize a model prior to data collection and model testing, in practice this often may not be possible, either because a theory is poorly formulated or because it is altogether nonexistent. As a consequence, specification searches have become nearly a common practice in many SEM applications. In fact, most currently available SEM programs provide researchers with options to conduct specification searches to improve model fit, and some new search procedures (e.g., using genetic algorithms, ant colony optimization, and Tabu search) have also been developed to automate this process (see Marcoulides, Drezner, & Schumacker, 1998; Marcoulides & Drezner, 2001, 2002; Scheines, Spirtes, Glymour, Meek & Richardson, 1998).

Specification searches are clearly helpful for improving a model that is not fundamentally misspecified but is incorrect only to the extent that it has some missing paths or some of its parameters are involved in unnecessarily restrictive constraints. With such models, it can be hypothesized that their unsatisfactory fit stems from overly strong restriction(s) on its parameters that are either fixed to 0 or set equal to other parameter(s), or included in a more complex relationship. The application of any means of model improvement is only appropriate when the model modification suggested is theoretically sound and does not contradict the results of previous research in a particular substantive domain. Alternatively, the results of any specification search that do not agree with past research should be subjected to further analysis based on new data before any real validity can be claimed.

The indexes that can be used as diagnostic statistics about which parameters could be changed in a model are called modification indices (a term used in the LISREL and *Mplus* programs) or Lagrange multiplier test statistics (a term used in the EQS program). The value of a *modification index* (term is used generically) indicates approximately how much a proposed model's chi-square would decrease if a particular parameter were freely estimated or freed from a constraint it was involved in the immediately preceding modeling session. There is also another modification index, called the Wald index, which takes an alternative approach to the problem. The value of the *Wald index* indicates how much a proposed model's chi-square would increase if a particular parameter were fixed to 0 (i.e., if the parameter were dropped from a model under consideration).

The modification indexes address the question of how to improve an initially specified model that does not fit satisfactorily the data. Although no strict rules-of-thumb exist concerning how large these indexes must be to warrant a meaningful model modification, based on purely statistical considerations one might simply consider making changes to parameters asso-

ciated with the highest modification indices (see below for a guideline regarding their magnitude). If there are several parameters with high modification indices, one may consider freeing them one at a time, beginning with the largest, because like in the general linear modeling framework a single change in a structural equation model can affect other parts of the solution (Jöreskog & Sörbom, 1990; Marcoulides et al., 1998). When LISREL or *Mplus* are used, modification indices larger than 5 generally merit close consideration. Similarly, when EQS is used, parameters associated with significant Lagrange-multiplier or Wald-index statistics also deserve close consideration.

It must be emphasized, however, that any model modification must first be justified on theoretical grounds and be consistent with already available theories or results from previous research in the substantive domain under consideration, and only second must be in agreement with statistical optimality criteria such as those mentioned. Blind use of modification indices can turn out to be a road to models that lead researchers astray from their original substantive goals. It is therefore imperative to consider changing only those parameters that have a clear substantive interpretation. Additional statistics, in the form of the estimated change for each parameter, can also be taken into account before one reaches a final decision regarding model modification.

In conclusion, we emphasize that results obtained from any model-improvement specification search may be unique to the particular data set, and that capitalization on chance can occur during the search (e.g., MacCallum, 1986). Consequently, once a specification search is conducted a researcher is entering a more exploratory phase of analysis. This has also purely statistical implications in terms of not keeping the overall significance level at the initially prescribed nominal value (the preset significance level, usually .05). Hence, the possibility exists of arriving at such statistically significant results regarding aspects of the model due only to chance fluctuations. Thus, the likelihood of falsely declaring at least one of the conducted statistical tests of the model or any of its parameters to be significant, is increased rather than being the same as that of any single test. For this reason, any models that result from specification searches must be cross-validated before real validity can be claimed for any of its findings.

APPENDIX TO CHAPTER 1

The structural equation models considered in this book are special cases of the so-called *general Linear Structural RELationships (LISREL) model*. To define it formally, denote by Y the vector of p observed variables in a considered study ($p > 1$), by η that of q latent factors assumed in it ($q > 0$), and by ε the vector of p pertinent residuals (error terms). Designating by Λ the $p \times q$ matrix of factor loadings, by v the $p \times 1$ vector of observed variable mean intercepts, by α the $q \times 1$ vector of latent variable intercepts, and by ζ the $q \times 1$ vector of corresponding latent disturbance terms, the general LISREL model is represented by the following pair of equations:

$$Y = v + \Lambda\eta + \varepsilon \text{ and} \quad (\text{A1.1})$$

$$\eta = \alpha + B\eta + \zeta, \quad (\text{A1.2})$$

where in addition the matrix $I - B$ is assumed invertible. In this introductory text, for simplicity the additional assumption of normality of the variables in Y , η , ε , and ζ is made, which are continuous, as well as that of uncorrelatedness of ε and ζ , ε and η , and ζ and η . Components of each of the vectors ε , ζ , and η may be correlated among themselves, however, assuming overall model identification. Further, the model classes considered in this book also assume $v = \alpha = 0$, which does not pose any special restriction of generality in many social and behavioral studies when the units and origins of measurement are not meaningful (or the analyzed variables may be considered mean-centered without loss of relevant information for a particular investigation.) The validity of Equations (A1.1) and (A1.2) is assumed for each individual in a sample, but for simplicity of notation we suppress the subject subindex in this Appendix. We stress that the LISREL model defined by Equations (A1.1) and (A1.2) also includes the case where manifest variables influence observed and/or latent variables, which is seen by noting that observed predictors can be formally represented by error-free latent variables (i.e., η 's with corresponding ε 's being 0 and unitary pertinent elements of Λ) in the right-hand sides of each of these two equations.

Under these assumptions, via simple algebra Equations (A1.1) and (A1.2) entail that the implied, observed variable covariance matrix Σ has the following form:

$$\Sigma = \Lambda(I-B)^{-1}\Psi(I-B')^{-1}\Lambda' + \Theta, \quad (\text{A1.3})$$

where priming denotes transposition, $\Psi = \text{Cov}(\xi)$ is the latent disturbance terms covariance matrix, and $\Theta = \text{Cov}(\varepsilon)$ is that of the residuals.

A simple inspection of the right-hand side of Equation (A1.3) demonstrates that the parameters of the general LISREL model are among: (a) the

factor loadings (elements of Λ); (b) the latent regression coefficients (elements of B); and (c) the variances and covariances of the latent disturbance terms and residuals (the elements of Ψ and Θ) that collectively represent the independent variables of the model. (In case a considered model has no latent dependent variable, as in confirmatory factor analysis models, set $\eta = \zeta$ and $B = 0$ in (A1.2) and use the immediately preceding sentence in this paragraph to obtain its model parameters.) This observation presents the rationale behind the rules for determining model parameters discussed in this chapter. Also, denoting by γ the vector of all model parameters (i.e., all variances of and covariances between independent variables as well as all regression coefficients and factor loadings), Equation (A1.3) states that $S = \Sigma(\gamma)$. That is, an implication of any considered structural equation model is the structuring of all elements of the pertinent population covariance matrix in terms of fewer in number, more fundamental parameters, viz. those in γ . This is the essence of the model parameterization that is invoked as a consequence of adopting a particular structural equation model.

The fit function for the model fitting and estimation approach used throughout this book, that of the maximum likelihood method, is defined as

$$F_{ML} = \text{“distance”}(S, \Sigma(\gamma)) = -\ln|S \Sigma(\gamma)^{-1}| + \text{tr}(S \Sigma(\gamma)^{-1}) - p, \quad (\text{A1.4})$$

where $|\cdot|$ denotes matrix determinant and $\text{tr}(\cdot)$ trace (sum of main diagonal elements). Using special numerical optimization algorithms, this fit function is minimized across the parameter space, i.e., the set of all admissible values of all model parameters (in particular, typically positive variances). When the model is correct and fitted to the covariance matrix for a large sample, $T = n F_{ML, min}$ follows a central chi-square distribution with degrees of freedom being those of the fitted model, where $F_{ML, min}$ is the minimum of (A1.4), $n = N - 1$ and N is sample size. This fit function is derivable in the context of the likelihood ratio test theory, when comparing a given model to a saturated model fitted to the same set of observed variables (e.g., Bollen, 1989). The minimizer of (A1.4), $\hat{\gamma}$, consists of the point estimates of all model parameters; their standard errors are obtained as the corresponding elements on the main diagonal of the inverted observed information matrix.

The unweighted least squares (ULS) method is based on minimizing, across the set of all possible values for γ , the fit function

$$F_{ULS} = .5 \text{tr}[(S - \Sigma(\gamma))^2], \quad (\text{A1.5})$$

while the generalized least squares (GLS) approach minimizes the fit function

$$F_{GLS} = .5 \text{tr}[(I - S^{-1}\Sigma(\gamma))^2]. \quad (\text{A1.6})$$

The asymptotically distribution free (ADF) method (also called weighted least squares method) minimizes the fit function

$$F_{ADF} = (s - \sigma(\gamma))'W^{-1}(s - \sigma(\gamma)) , \quad (\text{A1.7})$$

where s denotes the strung-out vector of nonredundant elements of S , $\sigma(\gamma)$ the similar vector of their counterparts in $\Sigma(\gamma)$, and W is a weight matrix that represents a consistent estimate of the large sample covariance matrix of the elements of S (considered as random variables themselves). As shown in the literature (e.g., Bollen, 1989), with large samples the ULS, ML, and GLS estimation methods can be considered special cases of the ADF method, obtained with appropriate choices of the matrix W . Corrections of the chi-square value and parameter standard errors, which are functions of the matrix W , yield from the ML test statistic and standard errors robust ML test statistics and robust standard errors, respectively (e.g., Bentler, 2004).

The matrix of covariance residuals, $S - \Sigma(\hat{\gamma})$, which is also often referred to as matrix of model residuals, contains information about local goodness of fit of the model. These residuals, i.e., the elements of $S - \Sigma(\hat{\gamma})$, are expressed in the original metrics of manifest variables that may be quite dissimilar across variables, and thus are in general hard to interpret. Their standardized counterparts, called standardized residuals, are expressed in a uniform metric across all variables and can therefore be used to locate pairs of variables whose interrelationship indices are markedly misfit. Like the individual case residuals in regression analysis, in SEM the model residuals are not unrelated to one another. However, those of them whose standardized versions are in absolute value higher than 2 generally indicate parts of the model that are considerably inconsistent with the data. A positive residual means underprediction by the model of the covariance for the two variables involved, and may be made smaller by introducing a parameter additionally contributing to their interrelationship index (as reflected in its counterpart element of $\Sigma(\hat{\gamma})$). Conversely, a negative residual means overprediction by the model of that covariance, and may be rendered smaller in magnitude (i.e., absolute value) by deleting a parameter contributing to this interrelationship index (as reflected in its counterpart in $\Sigma(\hat{\gamma})$). An examination of model residuals, in particular standardized residuals, is therefore recommendable as an essential step in the process of model fit evaluation.

Getting to Know the EQS, LISREL, and *Mplus* Programs

In Chapter 1, the basic concepts of the structural equation modeling methodology were discussed. In this chapter, essential elements of the notation and syntax used in the EQS, LISREL, and *Mplus* programs are introduced. The chapter begins by presenting an easy-to-follow flowchart depicting the general principles behind constructing command files for SEM software. This material is followed by a discussion of the EQS, LISREL, and *Mplus* programming languages. The discussion begins with EQS since the preceding chapter has already familiarized the reader with a number of important concepts and notation substantially facilitating an introduction to this software. The LISREL program, with its somewhat different structure, is dealt with second, followed then by *Mplus*. For more detailed and extensive information going beyond the confines of this text, a study of the latest versions of pertinent software manuals is recommended (Bentler, 2004; duToit & duToit, 2001; Jöreskog & Sörbom, 1999; Muthén & Muthén, 2004).

STRUCTURE OF INPUT FILES FOR SEM SOFTWARE

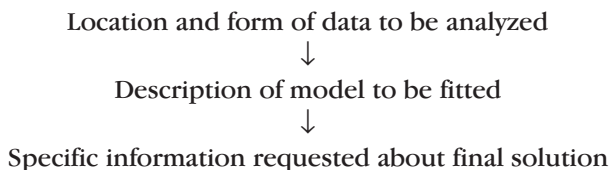
Each SEM program may be considered a separate language with specific syntax and rules that must be precisely followed. The syntax is used to communicate to the program all needed details about observed data and models of concern to the researcher. These details are provided to the software as commands. The results of the program acting on them are presented in the associated output.

Obviously, an essential piece of information that must be given to the software is what a model under consideration looks like. Conveying information about the model begins with an appropriate and software specific reference to the observed and unobserved variables, as well as the way they relate to one another. As discussed in the previous chapter, each program has in its memory the formal way in which the model reproduced matrix $\Sigma(\gamma)$ can be obtained, once the software is informed about the model. As illustrated in Chapter 1, especially for newcomers to SEM, it is highly recommendable that one first draws the path diagram of the model and determines its parameters using Rules 1 to 6, and then proceeds with the actual model fitting process.

In addition, one must communicate to the program a few details about the data to which the model will be fit. To this end, information concerning the location of the data, the name of its file, and possibly the data format (unless free) must be provided. For example, with respect to data format, it must be specified whether the data is in raw form, in the form of a covariance matrix (and means, if needed; see Chap. 6), or in the form of a correlation matrix. While the pertinent rules to accomplish this are software specific, generally it is important to indicate the number of variables (or their names) in the data set or the location of the analyzed variables within the file, and to provide information about sample size if other than raw data is used. SEM programs echo in the first part of their output the command file (specifically, the number of variables and observations as well as all submitted commands), and also display the analyzed data in the form of either a covariance or correlation matrix or alternatively a reference to raw data file is made (depending on how the data have been supplied to the software). This feature allows the researcher to quickly check the top part of the output in order to ensure that the program has indeed correctly read the data to be analyzed.

SEM software is built upon output defaults that routinely provide most of the relevant information for many empirical research settings, but on occasion one must specifically communicate to the program which other type of analysis is desired, e.g., particulars about the model estimation process, number of iterations to be conducted, particular measures of model fit, or particular analysis results.

In simple terms, the flowchart given next represents in general the backbone of a command file for a model to be fitted with a SEM program.



The next three sections in the present chapter follow this flow chart for setting up command files in EQS, LISREL, and *Mplus*. Although this is done first using only the information likely to be most frequently necessary to provide to the software, it will prove sufficient for fitting most of the models encountered in this text. The discussion is extended when more complicated models are dealt with in later chapters (such as multi-sample or mean structure models in Chap. 6).

INTRODUCTION TO THE EQS NOTATION AND SYNTAX

In Chapter 1, we have in fact laid the grounds for introducing the specific elements needed to set up command files using the EQS syntax and notation. An input file in EQS is made up of various commands. The beginning of each command is signaled by a forward slash (/) and its end by a semicolon (;). A command is typically followed by several keywords, or subcommands, and can go on for several lines. To begin the introduction to EQS, a list of commands that arguably are most often used in practice when conducting SEM analyses are presented next (for further details see Bentler, 2004). Each command is illustrated using the factor analysis model originally displayed in Fig. 6 in Chapter 1. For ease of discussion, the same figure is displayed again in Fig. 7. In addition, and to keep all command files visually separate from the regular text, all command lines are capitalized throughout the book.

Title Command

One of the first things needed to create an EQS input file is a title command. This command simply describes in plain English the type of model examined (e.g., a confirmatory factor analysis model or a path analytic model) and perhaps some of its specifics (e.g., a study of college students or middle managers). The title command is initiated by the keyword /TITLE. On the line immediately following, and for as many lines as needed, an explanatory title is provided. For example, suppose that the model of concern is the factor analysis model displayed in Fig. 7. The title command could then be listed as

```
/TITLE  
EQS INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE  
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE  
INDICATORS;
```

We emphasize that although a title command can be kept to a single or a couple of descriptive lines, like in this example, the more details are pro-

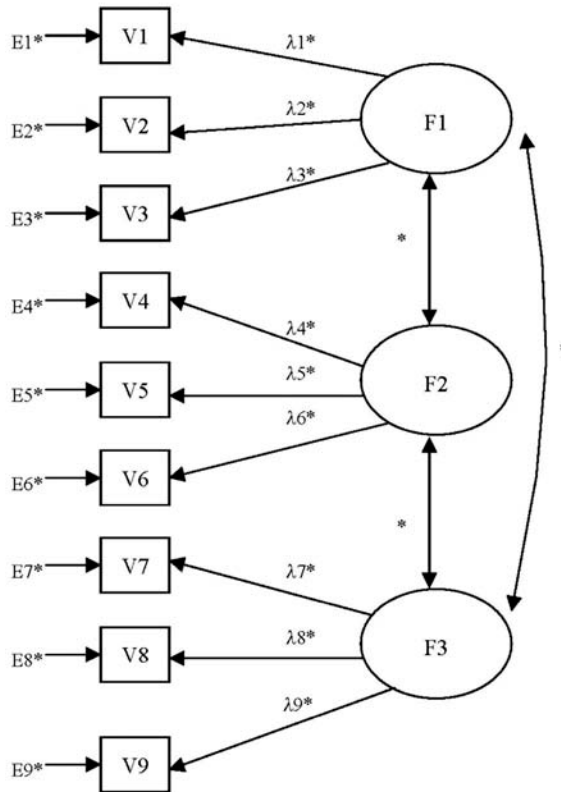


FIG. 7. Example factor analysis model. F1 = Parental dominance; F2 = Child intelligence; F3 = Achievement motivation.

vided in it the better one will recall the purpose of the modeling, especially when revisiting the command file later.

Data Command

The data command lists details about the data to be analyzed. This is initiated by using the keyword `/SPECIFICATIONS`. On the next line, the exact number of variables in the data set are provided using the keyword (sub-command) `VARIABLES=`, followed by that number. Then information about the number of observations in the data set is given using the keyword `CASES=`, followed by sample size. The data to be used in the study may be placed directly in the input file or in a separate data file. If the data are available as a covariance matrix, the keyword `MATRIX=COV` can be used (although it is not strictly needed as it is the default option). If the data are in

raw form (one column per variable, in as many lines as sample size), then the keyword `MATRIX=RAW` is used and the name of the file enclosed in apostrophes (including its location, i.e., path to it) are provided with the keyword `DATA_FILE=` (e.g., `DATA_FILE='C:\DATA\DATAFILE'`). If the data are in the form of a covariance matrix and to be placed directly in the input file (e.g., the covariance matrix S in Chap. 1), the command `/MATRIX` is used later in the input file followed by the matrix itself. (Although the covariance matrix typically would appear at the very end of the input file for convenience reasons—see final EQS input file below—for continuity reasons we mention its inclusion into the command file at this point.) If a matrix of variable interrelationships other than the covariance matrix is to be analyzed, this information is provided using the keyword `ANALYSIS=`, followed by `MOMENTS` if the mean structure is to be analyzed (i.e., variable means along with covariance matrix, as in Chap. 6), or by `CORRELATION` if the correlation matrix is to be analyzed.

The default method of estimation in EQS is maximum likelihood (ML). If a method other than ML is to be used for estimation purposes, it is stated after the keyword `METHOD=`, which is followed by its abbreviation in the program language (e.g., `GLS` or `LS` for unweighted least squares, and `ROBUST` for the robust ML method; e.g., Bentler, 2004). Although EQS provides the option of selecting from among several estimation procedures, as indicated in Chapter 1, only the use of the ML method will be exemplified in this introductory text. Utilizing the factor analysis model in Fig. 7 as an illustration, the data command line can then be listed as (in case the model is fitted to data from 245 subjects; we note that the last 3 subcommands of the command `/SPECIFICATIONS` are defaults and do not need to be explicitly stated in the input file):

```

/SPECIFICATIONS
  VARIABLES=9;
  CASES=245;
  METHOD=ML;
  MATRIX=COV;
  ANALYSIS=COV;
/MATRIX
1.01
.32   1.50
.43   .40   1.22
.38   .25   .33   1.13
.30   .20   .30   .7   1.06
.33   .22   .38   .72   .69   1.12
.20   .08   .07   .20   .27   .20   1.30
.33   .19   .22   .09   .22   .12   .69   1.07
.52   .27   .36   .33   .37   .29   .50   .62   1.16

```

It should be noted that because each of the subcommands ends with a semicolon, they can all be stated also in a single line; the only command in which no semicolon is needed to mark its end is `/MATRIX` (in addition to the data lines).

Model-Definition Commands

The next commands needed in the EQS input file, following the flowchart presented earlier, deal with model description. To accomplish this aim, one can take a close look at the path diagram of the model to be fitted and provide to the software information about its parameters. Setting up the model definition commands involves: (a) writing out the equations relating each dependent variable to its explanatory variables; (b) determining the status of all variances of independent variables (whether free or fixed, and at what value in the latter case); and (c) determining the status of the covariances for all independent variables. These activities are achieved by using the commands `/EQUATIONS`, `/VARIANCES`, and `/COVARIANCES`, respectively.

Each free or constrained parameter in the model is denoted in EQS by an asterisk. Following closely the path diagram in Fig. 7 results in 21 asterisks appearing in the model-definition equations and commands, which number as discussed in Chap. 1 equals that of free model parameters. The command `/EQUATIONS` initiates a listing of the model equations, which in the current example of Fig. 7 is as follows

```

/EQUATIONS
V1 = *F1 + E1;
V2 = *F1 + E2;
V3 = *F1 + E3;
V4 = *F2 + E4;
V5 = *F2 + E5;
V6 = *F2 + E6;
V7 = *F3 + E7;
V8 = *F3 + E8;
V9 = *F3 + E9;

```

We stress that model parameters (i.e., the nine λ 's in Equations 1 in Chap. 1) have been explicitly represented by asterisks in the listing of model equations. In addition, if needed, one can also assign a special start value to any model parameter by writing that value immediately before the asterisk in the corresponding equation (e.g., `V9 = .9*F3 + E9` assigns a start value of .9 to the loading of V9 upon the third latent variable). This does not change the status of the parameter in question (e.g., from free to fixed), but only signals to the software that this value will be the one this parameter will receive at the initial step of the numerical iteration process. Alternatively, fixed factor loadings are represented by their value placed immediately be-

fore the latent variable they belong to (i.e., they are not followed by an asterisk, unlike free parameters).

The command `/VARIANCES` is used to inform the software about the status of the independent variable variances (recall Rule 1 in Chap. 1). According to Fig. 7, there are nine residual variances and three factor variances, which represent all variances of independent variables in this model. The factor variances, however, will be fixed to 1 following Rule 6 in order to insure that the latent variable metrics are set. Hence, we add the following lines to the input file:

```
/VARIANCES
F1 TO F3 = 1; E1 TO E9 = *;
```

Note that one can use the TO convention in order to save tedious writing of all independent variables in the model, which becomes a particularly handy feature with large models having many error and/or latent variable variances.

Finally, information about independent variable covariances—that is, the three factor covariances in Fig. 7—must be communicated to the program. This is accomplished with the command `/COVARIANCES`:

```
/COVARIANCES
F2,F1=*; F3,F1=*; F3,F2=*;
```

Using the TO convention, the last line can be shortened to `F1 TO F3 = *; .`

Once the commands dealing with model definition have been completed, it is important to ensure that Rules 5 and 6 have not been contradicted in the input file. Thus, for the model in Fig. 7, a final check should make sure that each of the three factor variances is indeed fixed at 1 (Rule 6) and in particular that no variance or covariance of dependent variables as well as no covariance of a dependent and an independent variable have been declared model parameters (Rule 5). Lastly, in the example of Fig. 7, counting the number of asterisks one finds 21 model parameters declared in the input file—just as many as there are asterisks in the path diagram. Obviously, if the two numbers differ, some model parameters have either been left out or incorrectly declared as such (for the case of no parameter constraints).

In conclusion of this section, we note that the example considered does not include any requests for particular information from the final solution (see last part of earlier flow chart). This is because no additional output information beyond that provided by the default settings of EQS was needed. Later in this book we will include such requests, however, which either relate to the execution of the iteration process or ask the program to list information that it otherwise does not routinely provide.

Complete EQS Command File for the Model in Fig. 7

Based on the above discussion, the following complete EQS command file emerges for the confirmatory factor analysis model of concern in this section. The use of the command /END signals the end of the EQS input file for this model.

```

/TITLE
EQS INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
INDICATORS;
/SPECIFICATIONS
VARIABLES=9; CASES=245; METHOD=ML; MATRIX=COV;
ANALYSIS=COV;
/EQUATIONS
V1 = *F1 + E1;
V2 = *F1 + E2;
V3 = *F1 + E3;
V4 = *F2 + E4;
V5 = *F2 + E5;
V6 = *F2 + E6;
V7 = *F3 + E7;
V8 = *F3 + E8;
V9 = *F3 + E9;
/VARIANCES
F1 TO F3 = 1; E1 TO E9 = *;
/COVARIANCES
F1,F2=*; F1,F3=*; F2,F3=*;
/MATRIX
1.01
.32    1.50
.43    .40    1.22
.38    .25    .33    1.13
.30    .20    .30    .7    1.06
.33    .22    .38    .72    .69    1.12
.20    .08    .07    .20    .27    .20    1.30
.33    .19    .22    .09    .22    .12    .69    1.07
.52    .27    .36    .33    .37    .29    .50    .62    1.16
/END;

```

A Useful Abbreviation

A particularly helpful feature of EQS (as well as other software) is that each command and keyword/subcommand can be abbreviated to its first three

letters. This often saves a considerable amount of time for the researcher when setting up command files. In this way, the input file of the immediately preceding subsection can be shortened as follows.

```

/TIT
EQS INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
INDICATORS;
/SPE
VAR=9; CAS=245; MET=ML; MAT=COV; ANA=COV;
/EQU
V1 = *F1 + E1;
V2 = *F1 + E2;
V3 = *F1 + E3;
V4 = *F2 + E4;
V5 = *F2 + E5;
V6 = *F2 + E6;
V7 = *F3 + E7;
V8 = *F3 + E8;
V9 = *F3 + E9;
/VAR
F1 TO F3 = 1; E1 TO E9 = *;
/COV
F1,F2=*; F1,F3=*; F2,F3=*;
/MAT
1.01
.32    1.50
.43    .40    1.22
.38    .25    .33    1.13
.30    .20    .30    .7    1.06
.33    .22    .38    .72    .69    1.12
.20    .08    .07    .20    .27    .20    1.30
.33    .19    .22    .09    .22    .12    .69    1.07
.52    .27    .36    .33    .37    .29    .50    .62    1.16
/END;

```

We note that the last three subcommands of the specifications command can be omitted as they are default options.

Imposing Parameter Restrictions

An issue that frequently arises in empirical research is testing substantively meaningful hypotheses. For example, suppose that when dealing with the model in Fig. 7 one were interested in examining the plausibility of the hy-

pothesis that the first three observed variables—the indicators of the Parental domination factor—have equal factor loadings (i.e., represent a triplet of tau-equivalent tests). This assumption is tantamount to the three measures assessing the same construct, Parental dominance, in the same units of measurement. In order to introduce this constraint in the model under consideration, a new command handling parameter restrictions needs to be included in the input file. This command is `/CONSTRAINTS` and contains the following specification of the imposed parameter equalities:

```
/CONSTRAINTS
(V1,F1)=(V2,F1)=(V3,F1);
```

We note that within parentheses first comes the dependent and then independent variable to which the parameter pertains, in case it is a regression coefficient; this order is immaterial for variance and covariance parameters. For consistency reasons, we suggest that all constraints imposed in a model be included immediately after the `/COVARIANCE` command, which usually ends the model-definition part of an EQS input file.

INTRODUCTION TO THE LISREL NOTATION AND SYNTAX

This section deals with the notation and syntax used in the general LISREL model (also referred to as submodel 3B in the LISREL manual; Jöreskog & Sörbom, 1993b). In order to keep the discussion simple, the same factor analysis model dealt with in the previous section is considered here as well. Although the particular notation and syntax of the LISREL command lines are quite different from those of EQS, the underlying elements needed to set up input files are very similar.

The general LISREL model assumes that a set of observed variables (denoted Y) is used to measure a set of latent variables (denoted as η —the lowercase Greek letter *eta*; for a more formal discussion of the model, see Appendix to Chap. 1). The relationships between observed and latent variables are represented by a factor analysis model, also referred to as measurement model, whose residual terms are denoted by ε (the lowercase Greek letter *epsilon*).¹ Alternatively, the explanatory relationships among

¹In LISREL, the observed variables can actually be denoted either by Y or X , the latent variables correspondingly either by η or ξ (lowercase Greek letters *eta* and *ksi*), and the residual terms either by ε or δ (lowercase Greek letters *epsilon* or *delta*), respectively. For ease of presentation, however, only the Y , η and ε notation is used in the present chapter—specifically, this is the notation within submodel 3B in the LISREL manual (Jöreskog & Sörbom, 1993). As one becomes more familiar with the LISREL program, selecting which notation to use for representing a measurement model may become a matter of taste. When a structural equation model includes both a measurement part and a structural part, X may be used for the indicators of the independent latent variables and Y for these of the dependent constructs.

latent variables constitute what is referred to as structural model. To avoid any confusion resulting from this tautology, however, throughout this book these two models are instead correspondingly called *measurement part* and *structural part* of a given structural equation model.

Measurement Part Notation

Consider again the factor analysis example displayed in Fig. 7. The measurement part of this model can be written using the following notation (note the slight deviation in notation from Equations 1 in Chap. 1; see also Appendix to Chap. 1 for formal description):

$$\begin{aligned}
 Y_1 &= \lambda_{11} \eta_1 + \varepsilon_1, \\
 Y_2 &= \lambda_{21} \eta_1 + \varepsilon_2, \\
 Y_3 &= \lambda_{31} \eta_1 + \varepsilon_3, \\
 Y_4 &= \lambda_{42} \eta_2 + \varepsilon_4, \\
 Y_5 &= \lambda_{52} \eta_2 + \varepsilon_5, \\
 Y_6 &= \lambda_{62} \eta_2 + \varepsilon_6, \\
 Y_7 &= \lambda_{73} \eta_3 + \varepsilon_7, \\
 Y_8 &= \lambda_{83} \eta_3 + \varepsilon_8, \\
 Y_9 &= \lambda_{93} \eta_3 + \varepsilon_9.
 \end{aligned} \tag{9}$$

To facilitate the following discussion, the model in Fig. 7 is reproduced again in Fig. 8 using this new notation; we stress that this is the same model, with the only difference being variable notation.

Note that Equations 9 are formally obtained from Equations 1 in Chap. 1 after several simple modifications. First, change the symbols of the observed variables from V_1 through V_9 to Y_1 through Y_9 , respectively; then change the symbols of the factors from F_1 through F_3 to η_1 through η_3 , respectively. Next, change the symbols of the residual terms from E_1 through E_9 to ε_1 through ε_9 , respectively; and finally, add a second subscript to the factor loadings, which is identical to the factor on which the manifest variable loads and further discussed next.

The last step represents a rather helpful notation in developing LISREL command files. Specifically, each factor loading or regression coefficient in a given model is subscripted with two indices. The first equals the index of the dependent variable, and the second that of the independent variable in the pertinent equation. For example, in the fifth of Equations 9, $Y_5 = \lambda_{52} \eta_2 + \varepsilon_5$, the factor loading λ has two subscripts; the first is that of the dependent variable (Y_5), and the second is the one of the latent variable (η_2) that with regard to Y_5 plays the role of an independent variable. Independent variable variances and covariances have as subscripts the indices of the variables they are related to. Therefore, every variance has as subscripts twice the index of the variable it belongs to, whereas a covariance has as sub-

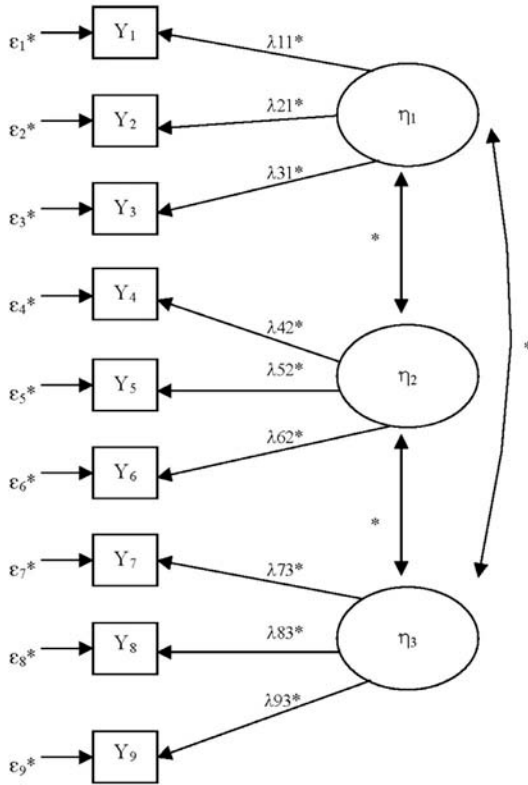


FIG. 8. Example factor analysis model using LISREL notation. η_1 = Parental dominance; η_2 = Child intelligence; η_3 = Achievement motivation.

scripts the indices of the two variables it relates with one another, with the order of these subscripts being immaterial (since the covariance is symmetric with respect to the two variables involved).

Structural Part Notation

In the model displayed in Fig. 8, there is no structural part because no explanatory relationships are assumed among the constructs. As an alternative example, however, if one assumed that the latent variable η_2 was regressed upon η_1 and that η_3 was regressed upon η_2 , then the structural part for this model would be

$$\begin{aligned}\eta_2 &= \beta_{21} \eta_1 + \zeta_2, \\ \eta_3 &= \beta_{32} \eta_2 + \zeta_3.\end{aligned}\tag{10}$$

In Equations 10, the structural slopes of the two regressions are denoted by β (the Greek letter *beta*), whereas the corresponding residual terms are symbolized by ζ (the Greek letter *zeta*). Note again the double indexing of the β 's in which, as mentioned earlier, the first index is that of the dependent variable and the second is the one of the pertinent independent variable in the corresponding equation. The indices of the ζ 's are identical to those of the latent dependent variables to which they belong as residual terms. (This type of structural equation models with latent variable regressions are extensively discussed in Chap. 4.)

Two-Letter Abbreviations

A highly useful feature of the LISREL notation and syntax is the possibility to use abbreviations consisting of the first two letters of keywords or parameter names. For example, within the general LISREL model, a factor loading can be referred to using the notation LY (derived from *Lambda for a Y variable*) followed by its indices as discussed in the preceding subsection. A structural regression coefficient is referred to using BE (for *BEta*), followed by its indices. Hence, using the above indexing principle, the loading of the fifth manifest variable on the second factor (see Equations 9) is denoted LY(5,2). Similarly, the coefficient of the third factor when regressed upon the second factor (see Equations 10) is denoted BE(3,2). (Although the brackets and delimiting comma are not really required, they make the presentation here easier to follow and for this reason we adopt them in this section.)

Variances of, and covariances between, independent latent variables are denoted by PS (the Greek letter *psi*, ψ), followed by the indices of the constructs involved. For example, the covariance between the first two factors in Fig. 8 is denoted by PS(2,1), whereas the variance of the third factor is PS(3,3). Finally, variances and covariances for residual terms are denoted by TE (for the Greek letters *Tbeta* and *Epsilon*). For instance, the variance of the seventh residual term ε_7 is symbolized by TE(7,7), whereas the covariance (assuming such a parameter is contained in a model of interest) between the first and fourth error terms would be denoted TE(4,1).

Hence, in the general LISREL model parameters are among (a) the factor loadings, denoted as LY's; (b) the structural regression coefficients, symbolized as BE's; (c) the latent variable variances and covariances, designated as PS's; and (d) the residual variances and covariances denoted as TE's. Each parameter is thereby subscripted by an appropriate pair of indices.

Matrices of Parameters—A Helpful Notational Tool

The representation of the indices of model parameters as numbers delineated by a comma and placed within brackets, strongly resembles that of

matrix elements. Indeed, in the LISREL notation, parameters can be thought of as being conveniently collected into matrices. For example, the loading of the fourth observed variable upon the second factor, λ_{42} , denoted in the LISREL notation as LY(4,2), can be thought of as the second element (from left to right) in the fourth row (from top to bottom) of a matrix designated LY. Similarly, the structural regression slope of η_3 on η_1 , β_{31} , denoted in the LISREL notation as BE(3,1), is the first element in the third row of the matrix BE. The covariance between the third and fifth factors, ψ_{53} , is the third element in the fifth row of the matrix PS, viz. PS(5,3); whereas the covariance between the error terms associated with the first and second observed variables, θ_{21} , is the first element in the second row of the matrix TE, namely TE(2,1) or simply TE 2 1. These four matrices may be rectangular or square, symmetric or nonsymmetric (also referred to as full matrices), have only 0 elements, or be diagonal.

That is, in the LISREL notation of relevance in this text all factor loadings are considered elements of a matrix denoted LY, which is most often a rectangular (rather than square) matrix because in empirical research there are frequently more observed variables than factors. (This general statement does not exclude cases where LY may be square, as it would be in some special models.) The structural regression coefficients (the regression coefficients when predicting a latent variable in terms of other constructs) are elements of a matrix called BE that is square because it represents the explanatory relationships among one and the same set of latent variables. The factor variances and covariances are entries of a symmetric square matrix PS (since PS is a covariance matrix), whereas the error variances and covariances are the elements of a symmetric square matrix TE. In empirical research, unless one has some theoretical justification for considering the presence of covariances among errors, the TE matrix is usually assumed to be diagonal, that is, containing on its diagonal the error variances of all observed variables.

Hence, in order to use the general LISREL model, within the LISREL notation and syntax one must consider the following four matrices: LY, BE, PS, and TE. Describing the model parameters residing in them constitutes a major part of constructing the LISREL input file.

Setting up a LISREL Command File

Based on the preceding discussion, the process of constructing the input for a LISREL model can now be examined in more detail. As outlined in the flowchart presented in section “Structure of Command Files for SEM Programs”, there are three main parts to an input file—data description, model description, and user-specified output.

It is recommendable that every LISREL command file begins with a title. For example, using the factor analysis model displayed in Fig. 8, the title could be

```
LISREL COMMAND FILE FOR A FACTOR ANALYSIS MODEL OF THREE
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
INDICATORS
```

We note that unlike EQS, the LISREL title does not end with a semicolon. In case the title continues for more than a single line, each line following the first may not begin with the letters DA (because the program will interpret that line as a data definition line; see next).

Next, the data to be analyzed are described in what is referred to as a data definition line. This line includes information about the number of variables in the data file, sample size, and type of data to be analyzed, e.g., covariance or correlation matrix. (When models are fit to data from more than one group, the number of groups must also be provided in this line; see Chap. 6.) Therefore, using just the first two letters of any keyword, for the factor analysis model in Fig. 8 the LISREL command file continues as follows:

```
DA NI=9 NO=245
```

where DA is the abbreviation for “Data definition line”, NI stands for “Number of Input variables”, and NO for “Number of Observations”. We do not need to explicitly state that we wish to fit the model to the covariance matrix, since this analysis is the default option in LISREL.

Immediately after this line comes the command CM, for “Covariance Matrix”, followed by the actual covariance matrix to be analyzed:

```
CM
1.01
.32  1.50
.43  .40  1.22
.38  .25  .33  1.13
.30  .20  .30  .7  1.06
.33  .22  .38  .72  .69  1.12
.20  .08  .07  .20  .27  .20  1.30
.33  .19  .22  .09  .22  .12  .69  1.07
.52  .27  .36  .33  .37  .29  .50  .62  1.16
```

If the data are available in raw format in a separate file, one can refer to it here by just stating RA=, followed by the name of the file (e.g., RA=C:\DATA\DATAFILE). Alternatively, if one has already computed the sample covariance

matrix and saved it in a separate file, one can also use `CM=`, followed by the name of that file. This completes the first part of the LISREL input file, which pertains to a description of the data to be analyzed.

The description of a model under consideration comes next. This is accomplished in what is referred to as the model definition line and subsequent specifications. This line contains information about the number of observed variables (Y 's) and the number of latent variables (η 's) in the model. For example, because there are nine observed and three latent variables in Fig. 8, the beginning of that model definition line would read

```
MO NY=9 NE=3
```

where MO stands for "MOdel definition line", NY for "Number of Y variables", and NE for "Number of Eta variables", i.e., number of latent variables. However, this is only the initial part of the model-description information. As was done when creating the EQS command file, one must communicate to LISREL details about the model parameters. In order to accomplish this, one must define the status of the four matrices in the general LISREL model discussed in this book (i.e., the matrices LY, BE, PS and TE), and if needed provide additional information about their elements that are model parameters. We note that to describe the confirmatory factor analysis model in Fig. 8, only the matrices LY, PS, and TE are necessary because there are no explanatory relationships assumed among the latent variables, and thus the matrix BE is equal to 0 (which is a default option in LISREL).

Based on our experience with the LISREL syntax, the following status definition of these matrices resembles a great deal of cases encountered in practice and permits a full description of models with relatively minimal additional effort. Accordingly, LY is initially defined as a rectangular (full) matrix with fixed elements, and subsequently the model parameters residing in it are explicitly declared. This definition is accomplished by stating `LY=FU,FI`, where FU stands for "Full" and FI for "Fixed". (Although this is a default option in LISREL, it is mentioned explicitly here in order to emphasize defining the free factor loadings in a subsequent line(s)). Since as mentioned earlier there are no explanatory relationships among latent variables, the matrix BE is equal to 0 (i.e., consists only of 0 elements), and given that this is also a default option in LISREL the matrix BE is not mentioned in the complete model definition line that looks now as follows for the model in Fig. 8:

```
MO NY=9 NE=3 LY=FU,FI PS=SY,FR TE=DI,FR
```

As previously indicated, the factor loading matrix LY is defined as full and fixed, keeping in mind that some of its elements will be freed next; these el-

ements are the factor loadings in the model, which according to Rule 3 are model parameters. The matrix PS is defined as symmetric and consisting of free parameters, as invoked by PS=SY,FR (SY standing for “Symmetric” and FR for “Free”, i.e., consisting of free parameters). These parameters are the variances and covariances of the latent variables, which by Rules 1 and 2 are model parameters since in this model the latent variables are independent variables. (Some of these parameters will subsequently be fixed at 1, following Rule 6, in order to set the latent variable metrics.) The error covariance matrix TE contains as diagonal elements the remaining model parameters, the error variances, and is defined as TE=DI,FR. (This definition of TE is also a default option in LISREL, but it is included here to emphasize its effect of declaring the error variances to be the remaining model parameters.)

With respect to factor loading parameters, the model definition line has only prepared the grounds for their definition. Their complete definition includes a line that specifically declares the pertinent factor loadings to be free parameters:

```
FR LY(1, 1) LY(2, 1) LY(3, 1) LY(4, 2) LY(5, 2)
FR LY(6, 2) LY(7, 3) LY(8, 3) LY(9, 3)
```

or more simply

```
FR LY 1 1 LY 2 1 LY 3 1 LY 4 2 LY 5 2 LY 6 2 LY 7 3 LY 8 3 LY 9 3
```

This definition line starts with the keyword FR that frees the following factor loading parameters in the model under consideration. Throughout the command file, it is essential to use the previously mentioned double-indexing principle correctly—first state the index of the dependent variable (observed variable in case of a factor loading) and then the index of the independent variable (latent variable in this case). Because there are nine observed variables and three latent variables, there are potentially 27 factor loadings to deal with. However, based on Fig. 8 most of them are 0 because not all variables load on every factor. Rather, every observed variable loads only on its corresponding factor. Having declared the factor loading matrix LY as full and fixed in the model definition line above, one can now quickly free the relatively limited number of factor-loading model parameters with the next line.

Up until this point, the factor loadings, independent variable variances and covariances, and error variances have been communicated to LISREL as model parameters. That is, all rules outlined in Chap. 1 have been applied except Rule 6. According to Rule 6, however, the metric of each latent variable in the model must be set. As with EQS, the easiest option here is to fix their variances to a value of 1. This metric setting can be achieved by first de-

declaring the factor variances to be fixed parameters (because they were defined in the model definition line as free), and then assigning the value of 1 to each one of them. These two steps are accomplished with the following two input lines:

```
FI PS(1, 1) PS(2, 2) PS(3, 3)
VA 1 PS(1, 1) PS(2, 2) PS(3, 3)
```

where FI stands for “Fix” and VA for “Value”. That is, we first fix the three factor variances and then assign on the next line the value of 1 to each of them. This finalizes the second part of the LISREL input file and completes the definition of all features of the model under consideration.

The final section of the input file is minimal and refers to the kind of output information requested from the software. There is a simple way of asking for all possible output—although with more complex models such a request can often lead to an enormous amount of output—and for now this option is used. Hence,

```
OU ALL
```

is the final line of the discussed LISREL command file, where OU stands for “output” and ALL requests all available output from the software. (With more complicated models where the entire output is likely to be voluminous, we only use OU in the last command line, unless having specific further requests.)

The Complete LISREL Input File

The following complete LISREL command file can now be used to fit the model in Fig. 8:

```
LISREL INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
INDICATORS
DA NI=9 NO=245
CM
1.01
.32    1.50
.43    .40    1.22
.38    .25    .33    1.13
.30    .20    .30    .7    1.06
.33    .22    .38    .72    .69    1.12
.20    .08    .07    .20    .27    .20    1.30
```

```
.33   .19   .22   .09   .22   .12   .69   1.07
.52   .27   .36   .33   .37   .29   .50   .62   1.16
MO NY=9 NE=3 LY=FU,FI PS=SY,FR TE=DI,FR
FR LY(1, 1) LY(2, 1) LY(3, 1) LY(4, 2) LY(5, 2)
FR LY(6, 2) LY(7, 3) LY(8, 3) LY(9, 3)
FI PS(1, 1) PS(2, 2) PS(3, 3)
VA 1 PS(1, 1) PS(2, 2) PS(3, 3)
OU ALL
```

INTRODUCTION TO THE *Mplus* NOTATION AND SYNTAX

Like EQS and LISREL, the *Mplus* programming language is based on commands that are each associated with several subcommands, or options. The number of basic commands is only 10, and they can be used in any order. Different models and analyses may happen to use different subsets of these commands, but all will require two of the 10 commands. They specify the location of the data and assign names to variables in the data file so that reference to the appropriate variables can be readily made when building the *Mplus* command file for a given model. Each command, except that stating the title, must end with a semicolon.

Although the TITLE command is not required, as with the other software described in this book, it is recommended that a title command be used to describe the essence of an analysis to be carried out with *Mplus*; the title can be of any length. The command is simply invoked by the word TITLE, followed by colon and some appropriate description. For the confirmatory factor analysis model example we have been using throughout this chapter (see, e.g., Fig. 7), one could use the following title:

```
TITLE: MPLUS INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE
INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
INDICATORS
```

The first required command in all models fit with *Mplus*, is that providing the location of the data to be analyzed and name of data file, and is similarly invoked by DATA . This is accomplished with the keyword FILE IS. In case of raw data, only the name of the file (with its path) needs to be provided. If the data is in a covariance matrix form, as will mostly be the case in this text, after giving the name of the file where that matrix is stored one needs to add the subcommand (option) TYPE = , followed by COVARIANCE, as well as indicate sample size with the subcommand NOBSERVATIONS = , followed by sample size. Thus, for the example model in Fig. 7, given that the data from 245 subjects is available in a covariance matrix form, the data command looks now as follows:

```
DATA:          FILE IS 'COVARIANCE-MATRIX-FILE-NAME'
              TYPE = COVARIANCE;
              NOBSERVATIONS = 245;
```

where 'covariance-matrix-file-name' provides the name of the file of the covariance matrix to be analyzed, possibly preceded by its path (directory location).

The second required command assigns names to the variables in the data file and is invoked by `VARIABLES`. In this command, after the keywords `NAMES ARE`, the researcher needs to give names to all variables in the data file, whereby a dash can be used to shorten long lists of names. For example, if the data format is free (e.g., at least one blank space separates variables within each line), the number of variables equals that of columns in the data file, and the simplest variable name assignment with this command could be in the following form:

```
VARIABLES: NAMES ARE V1-V#;
```

where # symbolizes the number of columns in the data file. For the above confirmatory factor analysis example, the simplest form of this command would be:

```
VARIABLES: NAMES ARE V1-V9;
```

Usually, detailed descriptive names will be more helpful to the analyst, but we keep here the simplest reference to variable names as used in Fig. 7.

Frequently it may be desirable to create within a single modeling session new variables from existing ones in a given data file, or transform some initial variables. This is accomplished using the command `DEFINE`, followed by the equations defining new variables or transforming already existing ones. Furthermore, SEM analyses vary in a number of aspects (e.g., covariance structure versus mean structure analyses; see Chap. 6) so the command `ANALYSIS`, followed by a selection from its options, states the particulars of an analysis to be carried out with the model and data under consideration. The default option for this command is covariance structure analysis, the one most frequently used in this text.

To define a model that is going to be estimated, one uses the `MODEL` command. For the models of concern in this introductory text, *Mplus* utilizes mostly the special keywords 'BY', 'ON', and 'WITH' for this purpose. A useful notation here is that latent variables can oftentimes be denoted by `F#`, with # standing for their consecutive numbering in a list of constructs for a considered model. The keyword 'BY' is followed by a listing of the indicators of a latent variable under consideration (for "measured by"), thus

automatically signaling to the software the pertinent factor loadings and error term variances as parameters. The keyword “ON” indicates which variable(s) are regressed upon which explanatory measures, whereby the names of the former are mentioned before this keyword while the names of the latter after it. Furthermore, the keyword ‘WITH’ indicates in the models of this book covariance (i.e., communicates to the software particular independent variable covariances). Residual variances are by default model parameters as are factor variances and covariances. The default metric setting for latent variables is internal—that is, the researcher does not need to explicitly carry it out—and is accomplished by *Mplus* through fixing to 1 of the loading of the first listed observed variable declared to load on a given latent variable. To illustrate, for the example in Fig. 7 the model definition can be accomplished with the following three lines:

```
MODEL:      F1 BY V1-V3;
           F2 BY V4-V6;
           F3 BY V7-V9;
```

The remaining of the 10 *Mplus* commands deal with special output requests, saving particular analysis results as data files, producing graphical plots, and generation of simulated data. In particular, the command OUTPUT asks that additional output results be provided, which are not included by default. In general, it can be recommended that when fitting a given model one requests model residuals, which is accomplished by OUTPUT: RESIDUAL;.² The command SAVEDATA invokes saving various results from the current analysis as well as auxiliary data. Requesting graphical displays of analyzed data as well as results from an analysis is accomplished with the command PLOT . Last but not least, the command MONTECARLO is utilized when one is interested in carrying out a simulation study with the software (for further details on the topic see Muthén & Muthén, 2002). Use of the four commands briefly discussed in this paragraph goes beyond the confines of this introductory text.

For the confirmatory factor analysis example in Fig. 7, the entire *Mplus* command file now looks as follows:

```
TITLE: MPLUS INPUT FILE FOR A FACTOR ANALYSIS MODEL OF THREE
      INTERRELATED CONSTRUCTS EACH MEASURED BY THREE
      INDICATORS
```

²In order to save space and not repeatedly request essentially identical information about model residuals when in addition to EQS (where they are provided by default) also LISREL and/or *Mplus* are used on a particular example, in the rest of this text we will not routinely require residuals to be included in the output of the latter two programs (but generally recommend a researcher requests them when only one or both of the latter two software is used).

```
DATA:          FILE IS 'COVARIANCE-MATRIX-FILE-NAME'  
              TYPE = COVARIANCE;  
              NOOBSERVATIONS = 245;  
VARIABLES:    NAMES ARE V1-V9;  
MODEL:        F1 BY V1-V3;  
              F2 BY V4-V6;  
              F3 BY V7-V9;  
OUTPUT:       RESIDUAL;
```

This listing of *Mplus* commands completes the present introductory chapter on notation and syntax underlying three of the most popular SEM programs, EQS, LISREL, and *Mplus*. Beginning with the next chapter, we will be concerned with several widely used types of structural equation models that are frequently employed in empirical research in the social and behavioral sciences.

Path Analysis

WHAT IS PATH ANALYSIS?

Path analysis is an approach to modeling explanatory relationships between observed variables. The explanatory variables are assumed to have no measurement error (or to contain error that is only negligible). The dependent variables may contain error of measurement that is subsumed in the residual terms of the model equations, that is, the part left unexplained by the explanatory variables. A special characteristic of path analysis models is that they do not contain latent variables.¹

Path analysis has a relatively long history. The term was first used in the early 1900s by the English biometrician Sewell Wright (Wright, 1920, 1921). Wright's approach was developed within a framework that is conceptually analogous to the one underlying structural equation modeling, as discussed in Chap. 1. The basic idea of path analysis is similar to solving a system of equations obtained when setting the elements of the sample covariance matrix S equal to their counterpart elements of the model reproduced covariance matrix $\Sigma(\gamma)$, where γ denotes the model parameter vector. Wright first demonstrated the application of path analysis to biology by developing models for predicting the birth weight of guinea pigs, examining the relative importance of hereditary influence and environment, and studying human intelligence (Wolfe, 1999). Wright (1921, 1934) provided

¹Throughout this chapter (as well as in chapters 4 and 5), whenever referring to 'latent variable' we imply its traditional psychometric definition, i.e., as a latent construct that is not observable directly and represents a subject's ability, attitude, trait, or latent dimension of substantive interest per se; hence path analysis models can, and typically will, contain residual (error) terms associated with their dependent variables (see Chap. 1).

a useful summary of the approach, with estimate calculations and a demonstration of the basic theorem of path analysis by which variable correlations in models can be reproduced by connecting chains of paths. More specifically, Wright's path analysis approach included the following steps. First, write out the model equations relating measured variables. Second, work out the correlations among them in terms of the unknown model parameters. Finally, try to solve in terms of the parameters the resulting system of equations in which correlations are replaced by sample correlations.

Using the SEM framework, one can easily fit models constructed within the path analysis tradition. This is because path analysis models can be viewed as special cases of structural equation models. Indeed, one can consider any path analysis model as resulting from a corresponding structural equation model that assumes (i) explanatory relationships between its latent variables, (ii) the independent variables to be associated with no error of measurement, and (iii) all latent variables to be measured by single indicators with (iv) unitary loadings on them (see Appendix to this chapter). Hence, to fit a path analysis model one can use a SEM program like EQS, LISREL, or *Mplus*. Although this SEM application capitalizes on the original idea of fitting models to matrices of interrelationship indices, as developed by Wright (1934), the actual model-fitting procedure is slightly modified. In particular, even though the independent variables are still treated as measured without error, the SEM approach considers all model equations simultaneously.

EXAMPLE PATH ANALYSIS MODEL

To demonstrate a path analysis model, consider the following example study (cf. Finn, 1974; see also Jöreskog & Sörbom, 1993b, sec. 4.1.4). The study examined the effects of several variables on university freshmen's academic performance. Five educational measures were collected from a sample of $N = 150$ university freshmen (for which the normality assumption was plausible). The following observed variables were used in the study:

1. Grade point average obtained in required courses (abbreviated below to AV-REQRD = V_1).
2. Grade point average obtained in elective courses (AV-ELECT = V_2).
3. High school general knowledge score (SAT = V_3).
4. Intelligence score obtained in the last year of high school (IQ = V_4).
5. Educational motivation score obtained the last year of high school (ED-MOTIV = V_5).

The path analysis model of interest in this section is presented in Fig. 9 where EQS notation is used to denote the observed variables by V_1 to V_5 , and the residual (error) terms associated with the two dependent grade point average variables (AV-REQRD and AV-ELECT) by E_1 and E_2 .

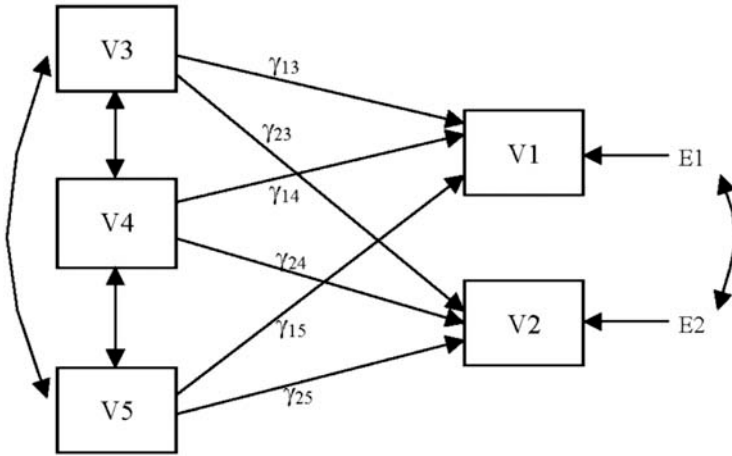


FIG. 9. Example path analysis model.

In Fig. 9, there are three two-headed arrows connecting all independent variables among themselves and representing the interrelationships among high school general knowledge score (SAT = V3), intelligence score (IQ = V4), and motivation score (ED-MOTIV = V5). No measurement errors are present in any of the independent variables. The dependent variables, however, are associated with residual terms that may contain measurement error along with prediction error (which would be confounded in these terms). The curved two-headed arrow connecting the residuals E_1 and E_2 symbolizes their possible interrelation that has also been included in the model. (We note in passing that allowing correlation of residual terms differs somewhat from the original path analysis method.)

The purpose of this path analysis study is to examine the predictive power of high school knowledge, intelligence, and motivation on university freshmen's academic performance as evaluated by grade point averages in required and elective courses. In terms of path analysis, one's interest lies in regressing simultaneously the two dependent variables (V_1 and V_2) on the three independent variables (V_3 , V_4 , and V_5). Note that regressing the two dependent variables on those predictors is not the same as typically done in a routine multiple regression analysis, in which a single dependent variable is considered, since here the model represents a multivariate multiple regression. Hence, in terms of equations, the following relationships are simultaneously postulated:

$$\begin{aligned} V_1 &= \gamma_{13}V_3 + \gamma_{14}V_4 + \gamma_{15}V_5 + E_1, \text{ and} \\ V_2 &= \gamma_{23}V_3 + \gamma_{24}V_4 + \gamma_{25}V_5 + E_2, \end{aligned} \tag{11}$$

where γ_{13} to γ_{25} are the six parameters of main interest—partial regression coefficients, also called path coefficients. These coefficients reflect the predictive power, in the particular metric used, of SAT, intelligence and motivation on the two dependent variables that assess generally interrelated aspects of freshmen performance. Furthermore, in Equations 11 the variables E_1 and E_2 represent residuals of model equations, which as indicated earlier may contain measurement error in addition to all influences on the pertinent dependent variables over and above those captured by a linear combination of their presumed predictors.

To determine the parameters of the model in Fig. 9, we follow the six rules outlined in Chap. 1. Since the model does not contain latent variables, Rule 6 is not applicable. Using the remaining rules, its parameters are (a) the six regression coefficients (i.e., all γ 's in Equations 11), which represent the paths in Fig. 9 that connect each of the dependent variables V_1 and V_2 with their predictors V_3 , V_4 , and V_5 ; and (b) the variances and covariances of the independent variables—i.e., the variances and covariances of V_3 , V_4 , and V_5 —as well as the variances and covariance of the residuals E_1 and E_2 .

Hence, the model under consideration has altogether 15 parameters—six path coefficients, six variances and covariances of independent variables, as well as two variances and a covariance of residual terms. Observe that there are no model implications for the variances and covariances of the predictors. This is because none of them is a dependent variable. Hence, the covariance matrix of the three predictors SAT, IQ, and ED-MOTIV is not restricted, other than being positive definite of course, in the sense that the model does not have any consequences with regard to its elements. Therefore, the estimates of the pertinent six parameters (three predictor variances and three covariances) will be based entirely on the corresponding values in the sample covariance matrix S .

We now move on to presenting the command files for this model with the three SEM programs used in this text, and discuss the associated output in a later section.

EQS, LISREL, AND *Mplus* INPUT FILES

EQS Command File

The EQS input file is constructed following the guidelines outlined in Chap. 2. Accordingly, the file begins with a title command:

```
/TITLE
PATH ANALYSIS MODEL;
```

Next, the number of variables in the model, sample size, and method of estimation are stated in the specification command. Since in this example the variable normality assumption was plausible as mentioned earlier, we employ the maximum likelihood (ML) method that is the default option in all three programs used in this text and hence need not be mentioned explicitly:

```
/SPECIFICATIONS
VARIABLES=5; CASES=150;
```

To facilitate interpretation of the output, labels are given to all variables. Using the command `/LABELS`, the following names are assigned to each variable in the model. (We mention in passing that variable labels should not be longer than eight symbols in any of the programs utilized in this book, and their provision in the input file is not an essential software requirement.)

```
/LABELS
V1=AV-REQRD; V2=AV-ELECT; V3=SAT; V4=IQ; V5=ED-MOTIV;
```

Next come the two model definition equations (cf. Equations 11):

```
/EQUATIONS
V1 = *V3 + *V4 + *V5 + E1;
V2 = *V3 + *V4 + *V5 + E2;
```

followed by the remaining model parameters in the variance and covariance commands:

```
/VARIANCES
V3 TO V5 = *; E1 TO E2 = *;
/COVARIANCES
V3 TO V5 = *; E1 TO E2 = *;
```

Finally, the data are provided along with the end-of-input file command:

```
/MATRIX
.594
.483 .754
3.993 3.626 47.457
.426 1.757 4.100 10.267
.500 .722 6.394 .525 2.675
/END;
```

Using the three-letter abbreviations, the complete EQS input file now looks as follows:

```

/TIT
PATH ANALYSIS MODEL;
/SPE
VAR=5; CAS=150;
/LAB
V1=AV-REQRD; V2=AV-ELECT; V3=SAT; V4=IQ; V5=ED-MOTIV;
/EQU
V1 = *V3 + *V4 + *V5 + E1;
V2 = *V3 + *V4 + *V5 + E2;
/VAR
V3 TO V5 = *; E1 TO E2 = *;
/COV
V3 TO V5 = *; E1 TO E2 = *;
/MAT
.594
.483 .754
3.993 3.626 47.457
.426 1.757 4.100 10.267
.500 .722 6.394 .525 2.675
/END;

```

LISREL Command File

The LISREL input file is most conveniently presented with a slight extension of the general LISREL model notation discussed in Chap. 2, which is not really essential but simplifies a great deal the application of this software for purposes of fitting any path analysis model (cf. Appendix to this chapter and Note 1 to Chap. 1). The discussion in this subsection is substantially facilitated by the particular symbols used for path coefficients (partial-regression coefficients) in Equations 11. According to this notation extension, observed predictor variables are denoted X whereas dependent variables remain symbolized by Y . That is, V_1 and V_2 now become Y_1 and Y_2 in this notation, and V_3 , V_4 , and V_5 become X_1 , X_2 , and X_3 , respectively. The covariance matrix of the predictor variables is referred to as Φ (the Greek letter *phi*), denoted PH in the LISREL syntax, and the same guidelines discussed in Chap. 2 apply when referring to its elements. The six regression coefficients in Equations 11 are collected in a matrix Γ (the Greek letter *gamma*), denoted GA in the syntax. The columns of Γ correspond to the predictors, X_1 to X_3 , and its rows are associated with the dependent variables, Y_1 and Y_2 . Each entry of the matrix Γ represents a coefficient for the regression of a dependent (Y) on an independent (X) variable. Thus, the elements of the matrix Γ in this example are GA(1, 1), GA(1, 2), GA(1, 3), GA(2, 1), GA(2, 2), and GA(2, 3).

The following LISREL command file is constructed adhering to the guidelines outlined in Chap. 2, whereby this input file also includes some variable labels using the keyword `Labels`; we discuss each line of this command file immediately after presenting it.

```

PATH ANALYSIS MODEL
DA NI=5 NO=150
CM
.594
.483 .754
3.993 3.626 47.457
.426 1.757 4.100 10.267
.500 .722 6.394 .525 2.675
LA
AV-REQRD AV-ELECT SAT IQ ED-MOTIV
MO NY=2 NX=3 GA=FU,FR PH=SY,FR PS=SY,FR
OU

```

Using the same title, the data-definition line declares that the model will be fit to data on five variables collected from 150 subjects. The sample covariance matrix signaled by `CM` is provided next, along with the variable labels that were also used earlier in this chapter. In the model definition line, the notation `NX` stands for “Number of X variables”. The matrix `GA` is, accordingly, declared to be full of free model parameters—these are the six γ 's in Equations 11. The (symmetric) covariance matrix of the predictors, `PH`, is defined as containing free model parameters (as was done when creating the EQS input file). The elements of the `PH` matrix correspond to the variances and covariances of the predictors `SAT`, `IQ`, and `ED-MOTIV`. The covariance matrix of the residual terms of the dependent variables Y_1 and Y_2 , that is, the matrix `PS`, is also defined as symmetric and containing free model parameters, viz. the residual variances and covariance. Finally, to save space and redundant information presentation relative to the EQS output discussed first in the next section, we request from LISREL the default output provided routinely (see Note 2 to Chap. 2).

***Mplus* Command File**

In order to create the *Mplus* command file for the model in Fig. 9, according to the pertinent discussion in Chap. 2 we begin with the `TITLE` command. Then the `DATA` command needs to provide information about the analyzed data. Subsequently, the `VARIABLE` command assigns names to the variables in the data set analyzed. Last but not least, the `MODEL` command describes the model to be fitted. As mentioned in Chap. 2, when using the `DATA` command it is necessary that we also indicate the type of the data if they are not in raw form, as in the case in this example since we are dealing with a covari-

ance matrix only; hence, we also need to provide the sample size. With this in mind, the following *Mplus* command file results.

```
TITLE:          PATH ANALYSIS MODEL
DATA:          FILE IS EX3.1.COV;
              TYPE = COVARIANCE;
              NOBSERVATIONS=150;
VARIABLE:     NAMES ARE AV_REQRD AV_ELECT SAT IQ
              ED_MOTIV;
MODEL:        AV_REQRD AV_ELECT ON SAT IQ ED_MOTIV;
```

Note that we specify in the DATA command the name of the file containing the covariance matrix since we are in possession only of the covariance matrix for the analyzed variables, and then give the sample size. (As indicated in the preceding chapter, we would not need to provide sample size if the data were in raw form.) At least as importantly, we stress the particularly useful feature of *Mplus* that it needs only a listing of the dependent variables to appear before the “ON” keyword, and similarly requires merely a listing of the putative predictors after that keyword. In particular, we do not need to write out the model equations or indicate matrices containing the model parameters. A similarly convenient feature of this software, which is readily capitalized in this path-analysis context, is also the implemented default options by which the model parameters are accounted for internally without the researcher having to explicitly point to them (see also Note 2 to Chap. 2).

MODELING RESULTS

In this section, we discuss in turn the outputs produced by EQS, LISREL, and *Mplus* when the corresponding command files discussed earlier in this chapter are submitted to them. At appropriate places, we insert comments that aim to clarify parts of the immediately preceding portion of output presented unless it is self-explanatory, and occasionally annotate the output. We introduce at this point the convention of displaying output results in a different and proportionate font, so that they stand out from the main text in the book. In the remainder of this section, for the sake of saving space, we dispense with repeatedly presenting the command file title as found at the beginning of each consecutive page of software output, as well as recurring statements regarding estimation method after their first appearance.

EQS Results

The EQS output begins with echoing back the input file submitted to the program:

```

PROGRAM CONTROL INFORMATION
1  /TIT
2  PATH ANALYSIS MODEL;
3  /SPE
4  VAR=5; CAS=150;
5  /LAB
6  V1=AV-REQRD; V2=AV-ELECT; V3=SAT; V4=IQ; V5=ED-MOTIV;
7  /EQU
8  V1 = *V3 + *V4 + *V5 + E1;
9  V2 = *V3 + *V4 + *V5 + E2;
10 /VAR
11 V3 TO V5 = *; E1 TO E2 = *;
12 /COV
13 V3 TO V5 = *; E1 TO E2 = *;
14 /MAT
15 .594
16 .483 .754
17 3.993 3.626 47.457
18 .426 1.757 4.100 10.267
19 .500 .722 6.394 .525 2.675
20 /END;
    20 RECORDS OF INPUT MODEL FILE WERE READ

```

In this part of the output any mistakes made while creating the input can be easily spotted. It is quite important, therefore, that this section of the output always be carefully examined before looking at other output parts.

COVARIANCE MATRIX TO BE ANALYZED: 5 VARIABLES (SELECTED FROM 5 VARIABLES)
BASED ON 150 CASES.

		AV-REQRD	AV-ELECT	SAT	IQ	ED-MOTIV
		V1	V2	V3	V4	V5
AV-REQRD	V1	.594				
AV-ELECT	V2	.483	.754			
SAT	V3	3.993	3.626	47.457		
IQ	V4	.426	1.757	4.100	10.267	
ED-MOTIV	V5	.500	.722	6.394	.525	2.675

BENTLER-WEEKS STRUCTURAL REPRESENTATION:
NUMBER OF DEPENDENT VARIABLES = 2
DEPENDENT V'S : 1 2

NUMBER OF INDEPENDENT VARIABLES = 5
INDEPENDENT V'S : 3 4 5
INDEPENDENT E'S : 1 2

NUMBER OF FREE PARAMETERS = 15
NUMBER OF FIXED NONZERO PARAMETERS = 2

*** WARNING MESSAGES ABOVE, IF ANY, REFER TO THE MODEL PROVIDED.
CALCULATIONS FOR INDEPENDENCE MODEL NOW BEGIN.
*** WARNING MESSAGES ABOVE, IF ANY, REFER TO INDEPENDENCE MODEL.
CALCULATIONS FOR USER'S MODEL NOW BEGIN.

3RD STAGE OF COMPUTATION REQUIRED 2026 WORDS OF MEMORY.
PROGRAM ALLOCATED 2000000 WORDS

DETERMINANT OF INPUT MATRIX IS .26607D+02

In this second portion of the output, EQS provides details about the data (number of independent and dependent variables, and analyzed covariance matrix) as well as the internal organization of the memory to accomplish the underlying computational routine. The software then confirms the number of dependent and independent variables as well as model parameters. The bottom of this second output page also contains a message that can be particularly important for detecting numerical difficulties. The message refers to the DETERMINANT OF THE INPUT MATRIX, which is, in simple terms, a number reflecting a “generalized variance” of the analyzed covariance matrix. If this determinant is 0, important matrix calculations cannot be conducted (in matrix-algebra terminology, the matrix is singular). In cases where that determinant is very close to 0, matrix computations can be unreliable and obtained numerical solutions may be quite unstable. The presence of a determinant rather close to 0 is a clue that there is a problem of a nearly perfect linear dependency (i.e., multicollinearity) among observed variables involved in the fitted model. For example, in a regression analysis the presence of multicollinearity implies that one is using redundant information in the model, which can easily lead to unstable regression coefficient estimates. A simple solution may be to drop an offending variable that is (close to) linearly related to other analyzed variables and respecify correspondingly the model. Hence, examining the determinant of the input matrix in this output section provides important information about the accuracy of conducted analyses.

PARAMETER ESTIMATES APPEAR IN ORDER,
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.

This is also a very important message as it indicates that the program has not encountered problems stemming from lack of model identification. Otherwise, this is the place where one would see a warning message entitled CONDITION CODE that would indicate which parameters are possibly unidentified. For this empirical example, the NO SPECIAL PROBLEMS message is a reassurance that the model is technically sound and identified.

RESIDUAL COVARIANCE MATRIX (S-SIGMA) :

		AV-REQRD V1	AV-ELECT V2	SAT V3	IQ V4	ED-MOTIV V5
AV-REQRD	V1	.000				
AV-ELECT	V2	.000	.000			
SAT	V3	.000	.000	.000		
IQ	V4	.000	.000	.000	.000	
ED-MOTIV	V5	.000	.000	.000	.000	.000

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = .0000
AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = .0000

STANDARDIZED RESIDUAL MATRIX:

		AV-REQRD	AV-ELECT	SAT	IQ	ED-MOTIV
		V1	V2	V3	V4	V5
AV-REQRD	V1	.000				
AV-ELECT	V2	.000	.000			
SAT	V3	.000	.000	.000		
IQ	V4	.000	.000	.000	.000	
ED-MOTIV	V5	.000	.000	.000	.000	.000

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0000
 AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0000

The RESIDUAL COVARIANCE MATRIX contains the resulting variance and covariance residuals. As discussed in Chap. 1 (see also Appendix to Chap. 1), these are the differences between the counterpart elements of the empirical covariance matrix S , given in the last input section /MATRIX, and the one reproduced by the model at the final solution, $\Sigma(\hat{\gamma})$, where $\hat{\gamma}$ is the vector comprising the values of the model parameters at the last iteration step of the numerical fit function minimization routine. That is, the elements of the residual covariance matrix equal the corresponding differences between the two matrices, $S - \Sigma(\hat{\gamma})$. In this sense, the residual covariance matrix is a complex measure of model fit for the variances and covariances, as opposed to a single number provided by most fit indices. The unstandardized residuals, presented first in this output section, evaluate the model-to-data discrepancy in the original metrics of variable assessment, and for this reason cannot in general be readily evaluated. Instead, their standardized versions, called standardized residuals, represent the model-to-data inconsistency in a uniform metric across all variables and are therefore easier to interpret. In particular, any large standardized residual—i.e., larger in absolute value than 2—is indicative of possibly serious deficiencies of the model with regard to the variable variance or covariance pertaining to the residual. For a given model, the unstandardized and standardized residuals are often referred to as model residuals or covariance residuals. We note that like in regression analysis model residuals are not unrelated to one another, with this degree of interrelationship becoming less pronounced in models based on a larger number of observed variables.

As seen from the last presented output section, in this example there are no non-zero residuals. The reason is that the fitted model is saturated and hence exhibits perfect fit to the data. Here we see another aspect of best possible fit, namely that the model exactly reproduces the analyzed covariance matrix—all residuals are 0. This finding results from the fact that we are dealing with a model that has as many parameters as there are nonredundant elements of the covariance matrix (see Chap. 1). Indeed, recall that the model has 15 parameters, and that with five observed variables there are $p(p + 1)/2 = 5(5 + 1)/2 = 15$ nonredundant elements in the analyzed covariance matrix—therefore the model is saturated. Since saturated models will always

fit the data perfectly, there is no way one can test or disconfirm them (see Chap. 1 for a more detailed discussion of saturated models).

MAXIMUM LIKELIHOOD SOLUTION (NORMAL DISTRIBUTION THEORY)

LARGEST STANDARDIZED RESIDUALS:

NO.	PARAMETER	ESTIMATE	NO.	PARAMETER	ESTIMATE
1	V1, V1	.000	11	V4, V4	.000
2	V3, V1	.000	12	V4, V3	.000
3	V4, V2	.000	13	V2, V1	.000
4	V3, V2	.000	14	V3, V3	.000
5	V5, V1	.000	15	V5, V5	.000
6	V4, V1	.000			
7	V2, V2	.000			
8	V5, V4	.000			
9	V5, V3	.000			
10	V5, V2	.000			

DISTRIBUTION OF STANDARDIZED RESIDUALS

					RANGE	FREQ	PERCENT				
!				!							
20-				-							
!				!							
!				!							
!				!							
!				!							
15-				-							
!				!	1	-0.5	- 0 .00%				
!				!	2	-0.4	- -0.5 0 .00%				
!				!	3	-0.3	- -0.4 0 .00%				
!		*		!	4	-0.2	- -0.3 0 .00%				
10-		*		-	5	-0.1	- -0.2 0 .00%				
!		*		!	6	0.0	- -0.1 4 26.67%				
!		*		!	7	0.1	- 0.0 11 73.33%				
!		*		!	8	0.2	- 0.1 0 .00%				
!		*		!	9	0.3	- 0.2 0 .00%				
5-		*		-	A	0.4	- 0.3 0 .00%				
!		* *		!	B	0.5	- 0.4 0 .00%				
!		* *		!	C	++	- 0.5 0 .00%				
!		* *		!							
!		* *		!							
						TOTAL	15 100.00%				
1	2	3	4	5	6	7	8	9	A	B	C

EACH "*" REPRESENTS 1 RESIDUALS

This section of the output provides only rearranged information about the fit of the model as judged by the model residuals, immediately after a statement of the employed parameter estimation method (in this case ML, as indicated earlier in this chapter). In the case of a less-than-perfect fit, a fair amount of information about model fit can be obtained from this section. For instance, the upper part of this output section would then provide a convenient summary of where to find the largest standardized residuals.

The lower part of the section contains information about the distribution of the standardized residuals. In this example, due to numerical estimation involved in fitting the model, the obtained residuals are not precisely equal to 0 to all decimal places used by the software. This is the reason there is a spike in the center of the residual distribution, and why a few residuals happen to fall off it. With well-fitting models, one should expect all residuals, in particular standardized residuals, to be small and concentrated in the central part of the distribution of asterisks symbolizing the latter, and the distribution to be in general symmetric.

GOODNESS OF FIT SUMMARY FOR METHOD = ML

INDEPENDENCE MODEL CHI-SQUARE = 460.156 ON 10 DEGREES OF FREEDOM

INDEPENDENCE AIC = 440.15570 INDEPENDENCE CAIC = 400.04935
 MODEL AIC = .00000 MODEL CAIC = .00000

CHI-SQUARE = .000 BASED ON 0 DEGREES OF FREEDOM
 NONPOSITIVE DEGREES OF FREEDOM. PROBABILITY COMPUTATIONS ARE UNDEFINED.

FIT INDICES

BENTLER-BONETT NORMED FIT INDEX = 1.000

NON-NORMED FIT INDEX WILL NOT BE COMPUTED BECAUSE A DEGREES OF FREEDOM IS ZERO.

RELIABILITY COEFFICIENTS

CRONBACH'S ALPHA = .527
 COEFFICIENT ALPHA FOR AN OPTIMAL SHORT SCALE = .835
 BASED ON THE FOLLOWING 2 VARIABLES
 AV-REQRD AV-ELECT
 GREATEST LOWER BOUND RELIABILITY = .832
 GLB RELIABILITY FOR AN OPTIMAL SHORT SCALE = .835
 BASED ON THE FOLLOWING 2 VARIABLES
 AV-REQRD AV-ELECT
 BENTLER'S DIMENSION-FREE LOWER BOUND RELIABILITY = .682
 SHAPIRO'S LOWER BOUND RELIABILITY FOR A WEIGHTED COMPOSITE = .901
 WEIGHTS THAT ACHIEVE SHAPIRO'S LOWER BOUND:
 AV-REQRD AV-ELECT SAT IQ ED-MOTIV
 .338 .395 .692 .431 .255

In this output section, several goodness-of-fit indices are initially presented. The first feature to note concerns the 0 degrees of freedom for the model. As discussed previously, this is a consequence of the fact that the model has as many parameters as there are nonredundant elements in the observed covariance matrix. The chi-square value is also 0 and indicates, again, perfect fit. In contrast, the chi-square value of the so-called independence model is quite large. This result is also expected because that model assumes no relationships between the variables, and hence represents in general a poor means of describing analyzed data from variables that are in-

terrelated as are the ones in the present example. In fact, large chi-square values for the independence model are quite frequent in practice—particularly when the variables of interest exhibit marked interrelationships. The Akaike information criterion (AIC) and its consistent-estimator version (CAIC) are reported as 0 for the fitted model, and much smaller than those of the independence model. AIC and CAIC indices are generally smaller for better fitting models, as can also be seen demonstrated in this case when one compares the model of interest with the independence model. Finally, because a saturated model was fit in this example, which by necessity has both a 0 chi-square value and 0 degrees of freedom, the Bentler-Bonett nonnormed fit index cannot be computed. The reason is that, by definition, this index involves division by the degrees of freedom of the fitted model, which equal 0 here. By way of contrast, the Bentler-Bonett normed fit index is 1, which is its maximum possible value associated with a perfect fit. The reliability measures routinely reported in this part of the output are not of concern in this empirical example, since we are not dealing with scale development issues or latent variables.

ITERATIVE SUMMARY

	ITERATION	PARAMETER	
		ABS CHANGE	ALPHA FUNCTION
1	14.688357	1.00000	5.14968
2	14.360452	1.00000	0.00000
3	0.000000	1.00000	0.00000

The iterative summary provides an account of the numerical routine performed by EQS to minimize the maximum likelihood fit function (matrix distance) used by the ML method. In fact, it took the program three iterations to find the final solution. This was achieved after the fit function quickly diminished to 0 (see last row). Since this is a saturated model with perfect fit, the absolute minimum of 0 is achieved by the fit function, which is generally not the case with nonsaturated models, as demonstrated in later examples.

MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

AV-REQRD=V1	=	.086*V3	+	.008*V4	-	.021*V5	+	1.000 E1
		.007		.013		.031		
		11.642@		.616		-.676		
AV-ELECT=V2	=	.046*V3	+	.146*V4	+	.131*V5	+	1.000 E2
		.007		.013		.030		
		6.501@		11.560@		4.434@		

This is the final solution, presented in nearly the same form as the model equations submitted to EQS with the command file. However, in this out-

put the asterisks are preceded by the obtained parameter estimates. (We emphasize that asterisks used throughout the EQS input and output denote estimated parameters, and do not have any relation to significance; for the latter the symbol '@' is used in this program.) Immediately beneath each parameter estimate is its standard error. The standard errors are measures of estimate stability, i.e., precision of estimation. Dividing each parameter estimate by its standard error yields what is referred to as a *t* test value and provided beneath the standard error. As mentioned in Chap. 1, if the *t* value is outside the interval (-2; +2) one can suggest that the pertinent parameter is likely to be non-zero in the population; otherwise, it could be treated as 0 in the population. The *t* value, therefore, represents a simple test statistic of the null hypotheses that the corresponding model parameter equals 0 in the population.

As can be seen in the last presented output section, the *t* values indicate that all path coefficients are significant except the impacts of IQ (V4) and ED-MOTIV (V5) on AV-REQRD (V1), which are associated with nonsignificant *t* values—viz. ones that fall inside the interval (-2; +2). Hence, IQ and ED-MOTIV seem to be unimportant predictors of AV-REQRD. In other words, there appears to be only weak evidence that IQ and ED-MOTIV might matter for student performance in required courses (AV-REQRD) once the impact of SAT is accounted for. (This example is reconsidered later in the chapter and more qualified statements are made then.)

VARIANCES OF INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

		V		F	
		-----		-----	
V3	- SAT		47.457*I		I
			5.498 I		I
			8.631@I		I
			I		I
V4	- IQ		10.267*I		I
			1.190 I		I
			8.631@I		I
			I		I
V5	- ED-MOTIV		2.675*I		I
			.310 I		I
			8.631@I		I

The entries in this table are the estimated variances of independent manifest variables—SAT, IQ, and ED-MOTIV—along with their standard errors and *t* values. (From this and next output sections, we observe that none of the variance estimates in the fitted model is negative; in the remainder of this text, none of the fitted models will be associated with a negative variance estimate and while checking for it we will not report this observation for each of them; see next.)

VARIANCES OF INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	-----		-----	
E1 - AV-REQRD		.257*I		I
		.030 I		I
		8.631@I		I
		I		I
E2 - AV-ELECT		.236*I		I
		.027 I		I
		8.631@I		I

These are the variances of the residual terms along with their standard errors and *t* values that are significant for both residuals. We stress that these estimates are nonnegative, as they should be. A negative variance estimate, regardless of model fit, is an indication of an inadmissible solution and that the results in the entire output may not be trustworthy. One therefore should get into the habit of examining all variance estimates in a model, regardless of its type, to make sure that none of them is negative before proceeding with result interpretation.

COVARIANCES AMONG INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	V		F	
	-----		-----	
V4 - IQ		4.100*I		I
V3 - SAT		1.839 I		I
		2.229@I		I
		I		I
V5 - ED-MOTIV		6.394*I		I
V3 - SAT		1.061 I		I
		6.025@I		I
		I		I
V5 - ED-MOTIV		.525*I		I
V4 - IQ		.431 I		I
		1.217 I		I

These statistics are the covariances among the predictor variables along with their standard errors and *t* values.

COVARIANCES AMONG INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	-----		-----	
E2 - AV-ELECT		.171*I		I
E1 - AV-REQRD		.025 I		I
		6.971@I		I

This is the covariance between the two residuals that exhibits a rather high *t*-value and is significant. This suggests that the unexplained parts in the two dependent variables—graduate point average in required and elective subjects—in terms of the used three predictors, are markedly interrelated. We return to this finding later in this section.

STANDARDIZED SOLUTION:

AV-REQRD=V1	=	.771*V3	+	.034*V4	-	.044*V5	+	.657 E1	R-SQUARED	.568
AV-ELECT=V2	=	.366*V3	+	.539*V4	+	.247*V5	+	.559 E2		.688

The STANDARDIZED SOLUTION results from standardizing all variables in the model. This solution uses a metric that is uniform across all measures and, hence, makes possible some assessment of the relative importance of the predictors. A related way to use the information in the standardized solution involves squaring the coefficients associated with the error terms in it. The resulting values reveal the percentage of unexplained variance in the dependent variables. These squared values are analogs to the complements to 1 of the R^2 indices corresponding to regression models for each equation, when all model equations are simultaneously estimated. As can be seen from the last column in this standardized solution section, called “R-squared”, some 43% (= 0.657² in percentage) of individual differences in AV-REQRD could not be predicted by SAT, IQ, and ED-MOTIV; similarly, some 31% of individual differences in AV-ELECT were not explained by SAT, IQ, and ED-MOTIV.

CORRELATIONS AMONG INDEPENDENT VARIABLES

	V		F	
	-----		-----	
V4 - IQ		.186*I		I
V3 - SAT		I		I
		I		I
		I		I
V5 - ED-MOTIV		.567*I		I
V3 - SAT		I		I
		I		I
		I		I
V5 - ED-MOTIV		.100*I		I
V4 - IQ		I		I

CORRELATIONS AMONG INDEPENDENT VARIABLES

	E		D	
	-----		-----	
E2 - AV-ELECT		.696*I		I
E1 - AV-REQRD		I		I

E N D O F M E T H O D

Finally, at the end of the EQS output, the correlations of all independent model variables are presented. These values are just reexpressions in a correlation metric of earlier parts of the output dealing with estimated independent variances and covariances. The magnitude of the correlation between the two equations' residuals suggests that the degree of relationship between the unexplained portions of individual differences in AV-ELECT and AV-REQRD is quite sizeable. One may suspect that this could be a consequence of common omitted variables. This possibility can be addressed by a subsequent and more comprehensive study that includes as predictors other appropriate variables in addition to the SAT, IQ, and ED-MOTIV used in this example.

LISREL Results

The LISREL command file described previously produces the following results rounded off to two decimals, which is the default option in LISREL (unlike EQS and *Mplus*, in which the default is three digits after the decimal point). As in the preceding section, when presenting the LISREL output portions, the font is changed, comments are inserted at appropriate places (after each output section is discussed), and the logo of the program as well as recurring first title line are dispensed with in order to save space.

The following lines were read from file PA.LSR:

```

PATH ANALYSIS MODEL
DA NI=5 NO=150
CM
.594
.483          .754
3.993         3.626         47.457
.426         1.757         4.1         10.267
.500         .722         6.394         .525         2.675
LA
AV-REQRD AV-ELECT SAT IQ ED-MOTIV
MO NY=2 NX=3 GA=FU,FR PH=SY,FR PS=SY,FR
OU

```

As in EQS, the LISREL output first echoes the input file. This is very useful for checking whether the model actually fitted is indeed the one intended to be analyzed.

```

NUMBER OF INPUT VARIABLES          5
NUMBER OF Y - VARIABLES            2
NUMBER OF X - VARIABLES            3
NUMBER OF ETA - VARIABLES          2
NUMBER OF KSI - VARIABLES          3
NUMBER OF OBSERVATIONS             150

```

Next, a summary of the variables in the model is given, in terms of observed, unobserved variables, and sample size. This section is also quite useful for checking whether the number of variables in the model have been correctly specified. Although in this example the number of ETA and KSI variables is irrelevant since the model does not contain latent variables, the former will prove to be quite important later in this book when dealing with the general LISREL model.

Covariance Matrix

	<u>AV-REQRD</u>	<u>AV-ELECT</u>	<u>SAT</u>	<u>IQ</u>	<u>ED-MOTIV</u>
AV-REQRD	0.59				
AV-ELECT	0.48	0.75			
SAT	3.99	3.63	47.46		
IQ	0.43	1.76	4.10	10.27	
ED-MOTIV	0.50	0.72	6.39	0.53	2.67

The covariance matrix contained in the LISREL input file is also echoed in the output, and should be examined for potential errors.

Parameter Specifications

GAMMA			
	<u>SAT</u>	<u>IQ</u>	<u>ED-MOTIV</u>
AV-REQRD	1	2	3
AV-ELECT	4	5	6
PHI			
	<u>SAT</u>	<u>IQ</u>	<u>ED-MOTIV</u>
SAT	7		
IQ	8	9	
ED-MOTIV	10	11	12
PSI			
	<u>AV-REQRD</u>	<u>AV-ELECT</u>	
AV-REQRD	13		
AV-ELECT	14	15	

This is a rather important section that identifies the model parameters as declared in the input file, and then numbers them consecutively. Each free model parameter is assigned a separate number. Although not of relevance in the present example, we note a general rule that all parameters that are constrained to be equal to one another are given the same number, where-

as parameters that are fixed are not consecutively numbered—they are instead assigned a 0 (see discussion in next section). According to the last presented output section, LISREL interpreted that the model had altogether 15 parameters (see Fig. 9). These include the six regression (path) coefficients in the GAMMA matrix that relate each of the predictors to the dependent variables (with predictor variables listed as columns and dependent variables as rows of the matrix); the PHI matrix, containing all six variances and covariances of the predictors; and the PSI matrix, containing the variances and covariance of the residual terms of the dependent variables (because both the PHI and PSI matrices are symmetric, only the elements along the diagonal and beneath it are nonredundant, and hence only they are assigned consecutive numbers).

LISREL Estimates (Maximum Likelihood)

GAMMA					
	SAT	IQ	ED-MOTIV		
	—	—	—		
AV-REQRD	0.09 (0.01)	0.01 (0.01)	-0.02 (0.03)		
AV-ELECT	11.52 0.05 (0.01)	0.61 0.15 (0.01)	-0.67 0.13 (0.03)		
	6.44	11.44	4.39		
Covariance Matrix of Y and X					
	AV-REQRD	AV-ELECT	SAT	IQ	ED-MOTIV
	—	—	—	—	—
AV-REQRD	0.59				
AV-ELECT	0.48	0.75			
SAT	3.99	3.63	47.46		
IQ	0.43	1.76	4.10	10.27	
ED-MOTIV	0.50	0.72	6.39	0.53	2.67
PHI					
	SAT	IQ	ED-MOTIV		
	—	—	—		
SAT	47.46 (5.55)				
	8.54				
IQ	4.10 (1.86)	10.27 (1.20)			
	2.21	8.54			
ED-MOTIV	6.39 (1.07)	0.53 (0.44)	2.67 (0.31)		
	5.96	1.20	8.54		
PSI					
	AV-REQRD	AV-ELECT			
	—	—			
AV-REQRD	0.26 (0.03)				
	8.54				

AV-ELECT	0.17	0.24
	(0.02)	(0.03)
	6.90	8.54

Squared Multiple Correlations for Structural Equations

AV-REQRD	AV-ELECT
-----	-----
0.57	0.69

This is the final solution provided by LISREL. In a way similar to EQS, LISREL presents the parameter estimates along with their standard errors and t values in a column format. Note that the matrix presented in this section under the title COVARIANCE MATRIX OF Y AND X is actually the model reproduced covariance matrix $\Sigma(\hat{\gamma})$ at the solution point. As can be seen, in this case $\Sigma(\hat{\gamma})$ is identical to the sample covariance matrix S because the model is saturated and hence fits or reproduces the latter perfectly. The section ends by providing the squared multiple correlations for the two structural equations of the model, one per dependent variable. The squared multiple correlations provide information about the percentage of explained variance in the dependent variables. As mentioned earlier, they are analogs to the R^2 indices corresponding to regression models for each equation, accounting for the fact that both of them are fitted simultaneously (and are the same up to rounding-off error as those obtained using EQS).

Goodness of Fit Statistics

Degrees of Freedom = 0
 Minimum Fit Function Chi-Square = 0.0 (P = 1.00)
 Normal Theory Weighted Least Squares Chi-Square = 0.00 (P = 1.00)

The Model is Saturated, the Fit is Perfect !

The goodness-of-fit statistics section of the output is yet another indication of perfect fit of this saturated model. We stress that with most models examined in empirical research, which are not saturated, a researcher should inspect the goodness-of-fit statistics provided in this section of the output before interpreting parameter estimates. In this way it is ensured that a researcher's interpretation of estimates is carried out only for models that are reasonable approximations of the analyzed data. For models that are rejected as means of data representation, parameter estimates should not be generally interpreted because they can yield substantively misleading results.

Mplus Results

Like EQS and LISREL, *Mplus* commences its output by echoing back the command file submitted to it.

```

INPUT INSTRUCTIONS

TITLE:      PATH ANALYSIS MODEL
DATA:       FILE IS EX3.1.COV;
            TYPE = COVARIANCE;
            NOBSERVATIONS=150;
VARIABLE:   NAMES ARE AV_REQRD AV_ELECT SAT IQ ED_MOTIV;
MODEL:      AV_REQRD AV_ELECT ON SAT IQ ED_MOTIV;

INPUT READING TERMINATED NORMALLY

```

The last line of this section is important to also look for, since it is reassuring to know that there is no miscommunication between the analyst and the software as far as the submitted command file is concerned.

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	150
Number of dependent variables	2
Number of independent variables	3
Number of continuous latent variables	0

Observed dependent variables

Continuous		
AV_REQRD	AV_ELECT	

Observed independent variables

SAT	IQ	ED_MOTIV
-----	----	----------

In this summary output section, the software lists the details obtained from the command file that pertain to the particular analysis intended to be carried out.

Estimator	ML
Information matrix	EXPECTED
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)
EX3.1.COV

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

This portion provides information about the invoked estimation procedure, the upper limit of iteration steps within which *Mplus* will be monitoring the underlying numerical minimization process for convergence, and applicable criterion for the latter. After restating the covariance matrix data

file name and its free format, a very important statement comes that confirms convergence has been achieved. Typically in empirical research, this statement needs to be present in an output in order for the analyst to trust the output results and move on with interpreting following sections of the output.

TESTS OF MODEL FIT

Chi-Square Test of Model Fit

Value	0.0000
Degrees of Freedom	0
P-Value	0.0000

Chi-Square Test of Model Fit for the Baseline Model

Value	399.669
Degrees of Freedom	7
P-Value	0.0000

CFI/TLI

CFI	1.000
TLI	1.000

Loglikelihood

H0 Value	-1307.784
H1 Value	-1307.784

Information Criteria

Number of Free Parameters	9
Akaike (AIC)	2633.567
Bayesian (BIC)	2660.663
Sample-Size Adjusted BIC	2632.180
(n* = (n + 2) / 24)	

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.000
90 Percent C.I.	0.000 0.000
Probability RMSEA <= .05	0.000

SRMR (Standardized Root Mean Square Residual)

Value	0.000
-------	-------

The last presented output part contains information about the fit of the model. Again, since this is a saturated model, its fit is perfect. The maximum values of the log-likelihood for the fitted model and saturated model (in this case identical to the former) are also presented; they are closely related to the fit function values and will typically differ in empirical research that commonly deals with non-saturated models. The information criteria values

listed become especially important when one is considering several competing models for a given data set, as later in the book. Specifically, better fitting models typically are associated with smaller values on these criteria.

MODEL RESULTS	Estimates	S.E.	Est./S.E.
AV_REQRD ON			
SAT	0.086	0.007	11.681
IQ	0.008	0.013	0.618
ED_MOTIV	-0.021	0.031	-0.678
AV_ELECT ON			
SAT	0.046	0.007	6.523
IQ	0.146	0.013	11.599
ED_MOTIV	0.131	0.029	4.449

The path coefficient estimates are essentially identical, within round-off or numerical error, to those obtained with EQS and LISREL.

AV_ELECT WITH			
AV_REQRD	0.170	0.024	6.995
Residual Variances			
AV_REQRD	0.256	0.029	8.660
AV_ELECT	0.235	0.027	8.660

Similarly, essentially identical results are also found for the estimates of the variances and covariance of the residual terms associated with the dependent variables in this model. We note in conclusion that *t*-values are presented in the final column “Est./S.E.,” which equal the parameter estimates divided by standard error.

TESTING MODEL RESTRICTIONS IN SEM

The model examined in the previous section was a very special one, a saturated model. As a result, the model fit the data perfectly—something that one cannot expect to see with most models considered in empirical research. Being saturated, it had 0 degrees of freedom and could not really be tested. The reason was that models with 0 degrees of freedom can never be disproved when tested against observed data, regardless of their implications or how that data look. As a consequence, saturated models are generally not very interesting in substantive research. However, in addition to standard errors for its parameters, a saturated model provides a very useful baseline against which other models with positive degrees of freedom can be tested. This is the reason why a saturated model was examined in the first empirical example in this book.

Issues of Fit with Nested Models

The model presented in the previous section and displayed in Fig. 9 also facilitates the introduction of a very important topic in structural equation model testing—that of nested models. A model M is said to be *nested* in another model M' , if M can be obtained from M' by constraining one or more of the parameters in M' to be either (a) equal to 0 or some other constant(s), or (b) following a particular relationship (e.g., a linear combination of them to equal 0 or another constant). For example, a model obtained from a given one by setting two or more of its parameters to equal one another, is nested in the given model. More generally, there is potentially no limit to the number of nested models in a considered sequence of models, or to the number/values of parameters that are fixed or constrained in one of them in order to obtain the more restricted model(s).

Another interesting aspect about a nested model is that it will always have a greater number of degrees of freedom than the model in which it is nested—recall that the nested model M is obtained from M' by constraining one or more parameters to 0 or another constant(s), or to follow a given relationship. As it turns out, the difference in fit of the two models, relative to their degrees of freedom, can be used as a test of the hypothesis that the restriction imposed on one or more parameters in M is plausible in a studied population. This test is generally referred to as the chi-square difference test and utilizes a chi-square distribution with degrees of freedom equal to the difference between the degrees of freedom of the two models compared.

In empirical research, researchers typically consider only those nested models that impose substantively interesting restrictions on parameters. By comparing the fit of nested models and taking into account the difference in the number of their parameters, researchers examine the plausibility of imposed restrictions. In general, the more restricted model (being a constrained version of another model) will typically be associated with a higher chi-square value because it imposes more restrictions on the model parameters. These restrictions, in effect, make the model less able to emulate the analyzed covariance matrix, and hence the resulting difference in the chi-square values will be positive (in case of identified models, as assumed throughout the rest of this book). Similarly, the difference in degrees of freedom obtained when the degrees of freedom of the more general model are subtracted from those of the more restricted one, is also positive because the more restricted model has fewer parameters and hence more degrees of freedom.

It can be shown that with large samples the difference in chi-square values of nested models follows approximately a chi-square distribution with

degrees of freedom equal to the difference in their degrees of freedom, under the assumption that the imposed restriction is valid in the studied population. For simplicity, in the remainder of this text the difference in chi-square values is denoted by ΔT , and the difference between the degrees of freedom by Δdf . If the increment in the chi-square value, ΔT , is significant when assessed against the chi-square distribution with Δdf degrees of freedom, it is an indication that the imposed restrictions in the more constrained model resulted in a significant decrement in model fit and hence the constraints can be considered violated in the population. Conversely, if this difference ΔT is nonsignificant, one can retain the hypothesis of validity of the imposed constraints. We stress that model comparison can also proceed in the other direction—models can have parameters sequentially freed, and the resulting successive model versions can be compared to assess whether freeing a particular parameter led to a significant improvement in model fit. Hence, in general terms the chi-square difference test can be considered analogous to the test of change in R^2 when adding predictors to (or removing them from) a set of explanatory variables in conventional regression analysis, or equivalently to the F test for considering dropping predictors in a regression model.

Testing Restrictive Hypotheses in the Path-Analysis Example

Returning to the specific model presented in Fig. 9, the hypothesis can be examined that intelligence (IQ) and motivation (ED-MOTIV) have the same impact—as reflected in their metrics—on grade point average obtained in elective subjects (AV-ELECT). To accomplish this, the equality restriction on the paths leaving IQ and ED-MOTIV and ending at AV-ELECT is introduced. This restriction can be included in the EQS command file by adding a /CONSTRAINT section, in LISREL by adding an input line that imposes the particular parameters' identity (equality), and in *Mplus* by adding a line assigning the same number to the parameters in question (details given below). The necessary changes in the EQS command file are as follows (note the extended title, indicating the additional restriction feature of this model):

```

/TIT
PATH ANALYSIS MODEL WITH IMPOSED CONSTRAINTS;
/SPE
VAR=5; CAS=150;
/LAB
V1=AV-REQRD; V2=AV-ELECT; V3=SAT; V4=IQ; V5=ED-MOTIV;
/EQU
V1 = *V3 + *V4 + *V5 + E1;

```

```

V2 = *V3 + *V4 + *V5 + E2;
/VAR
V3 TO V5 = *; E1 TO E2 = *;
/COV
V3 TO V5 = *; E1 TO E2 = *;
/CON
(V2,V4) = (V2,V5); ! THIS IS THE INTRODUCED RESTRICTION;
/MAT
.594
.483 .754
3.993 3.626 47.457
.426 1.757 4.100 10.267
.500 .722 6.394 .525 2.675
/END;

```

As indicated by the added comment in the constraint command (after the exclamation mark in the actual constraint line, which signals beginning of a comment in all three programs used in this text), the restriction under consideration is imposed by requiring the identity of the path coefficients of the predictors in question. Note the syntax used in this constraint command—within brackets symbolizing parameter, first comes the dependent variable and then the independent one for the two measures pertaining to that parameter.

In the corresponding LISREL command file the parameter restriction is introduced by adding an EQuality statement that sets the two involved parameters equal to one another as follows:

```

PATH ANALYSIS MODEL WITH IMPOSED CONSTRAINTS
DA NI=5 NO=150
CM
.594
.483 .754
3.993 3.626 47.457
.426 1.757 4.100 10.267
.500 .722 6.394 .525 2.675
LA
AV-REQRD AV-ELECT SAT IQ ED-MOTIV
MO NY=2 NX=3 GA=FU,FR PH=SY,FR PS=SY,FR
EQ GA(2, 2) GA(2, 3) ! THIS IS THE INTRODUCED RESTRICTION
OU

```

The EQuality command line, followed by a listing of the parameters of concern, sets the two path coefficients equal in the restricted model.

In the pertinent *Mplus* command file, one needs to indicate the identity of the two path coefficients in question. This is accomplished with the last line of the following command file, which states that these two listed coefficients are set equal by assigning to them the same number (in this case it can be 1 as there are no other parameters already constrained for equality).

```
TITLE:      PATH ANALYSIS WITH IMPOSED CONSTRAINTS
DATA:      FILE IS EX3.1.COV;
           TYPE = COVARIANCE;
           NOBSERVATIONS=150;
VARIABLE:  NAMES ARE AV_REQRD AV_ELECT SAT IQ ED_MOTIV;
MODEL:    AV_REQRD ON SAT IQ ED_MOTIV;
           AV_ELECT ON SAT
           IQ ED_MOTIV (1);
```

Note that the earlier *Mplus* command line for the second dependent variable equation is 'broken' here into two lines, whereby the first one does not end with a semicolon while the second one contains the names of the predictors whose path coefficients are set equal to one another.

Because the results produced with the three programs are identical, only the pertinent EQS output sections are discussed next (interested readers may verify this by using the corresponding LISREL and/or *Mplus* input files as well); the first two pages of the EQS output, which are identical to those of the model without the constraint in question, are not presented.

```
PARAMETER ESTIMATES APPEAR IN ORDER,
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.
ALL EQUALITY CONSTRAINTS WERE CORRECTLY IMPOSED
```

As indicated before, this message is important because it indicates that the program has not encountered any problems stemming from lack of model identification and, in particular, that the model constraints were correctly introduced.

```
RESIDUAL COVARIANCE MATRIX (S-SIGMA) :

           AV-REQRD    AV-ELECT    SAT    IQ    ED-MOTIV
           V1         V2         V3    V4     V5
AV-REQRD V1    .000
AV-ELECT V2    .000    .000
SAT V3        .000    .000    .000
IQ V4         .017    .023    .000    .000
ED-MOTIV V5   -.017   -.023    .000    .000    .000

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = .0054
AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = .0080
```

STANDARDIZED RESIDUAL MATRIX:

	AV-REQRD V1	AV-ELECT V2	SAT V3	IQ V4	ED-MOTIV V5
AV-REQRD V1	.001				
AV-ELECT V2	.000	.000			
SAT V3	.000	.000	.000		
IQ V4	.007	.008	.000	.000	
ED-MOTIV V5	-.013	-.016	.000	.000	.000

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0031
 AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0045

LARGEST STANDARDIZED RESIDUALS:

NO.	PARAMETER	ESTIMATE	NO.	PARAMETER	ESTIMATE
1	V5, V2	-.016	11	V4, V4	.000
2	V5, V1	-.013	12	V2, V2	.000
3	V4, V2	.008	13	V5, V3	.000
4	V4, V1	.007	14	V5, V4	.000
5	V1, V1	.001	15	V5, V5	.000
6	V2, V1	.000			
7	V3, V1	.000			
8	V3, V3	.000			
9	V3, V2	.000			
10	V4, V3	.000			

DISTRIBUTION OF STANDARDIZED RESIDUALS

					RANGE	FREQ	PERCENT
20-	!		!				
	!		!				
	!		!				
	!		!				
	!		!				
15-	!		!	1	-0.5 -	0	.00%
	!	*	!	2	-0.4 -	-0.5 0	.00%
	!	*	!	3	-0.3 -	-0.4 0	.00%
	!	*	!	4	-0.2 -	-0.3 0	.00%
10-	!	*	!	5	-0.1 -	-0.2 0	.00%
	!	*	!	6	0.0 -	-0.1 2	13.33%
	!	*	!	7	0.1 -	0.0 13	86.67%
	!	*	!	8	0.2 -	0.1 0	.00%
	!	*	!	9	0.3 -	0.2 0	.00%
5-	!	*	!	A	0.4 -	0.3 0	.00%
	!	*	!	B	0.5 -	0.4 0	.00%
	!	*	!	C	++ -	0.5 0	.00%
	!	* *	!	TOTAL			15 100.00%
	!	* *	!				

1 2 3 4 5 6 7 8 9 A B C EACH "*" REPRESENTS 1 RESIDUALS

As expected, the constrained model does not perfectly reproduce the observed covariance matrix, unlike the saturated model discussed earlier in

this chapter. This is because most models fit in empirical research are not perfect representations of the analyzed data and therefore do not completely reproduce the sample matrices of variable interrelationships. In the case under consideration, the largest standardized residuals are, however, all fairly small. This result suggests that the model reproduces all elements of the sample covariance matrix fairly well.

GOODNESS OF FIT SUMMARY FOR METHOD = ML

INDEPENDENCE MODEL CHI-SQUARE = 460.156 ON 10 DEGREES OF FREEDOM

INDEPENDENCE AIC = 440.15570 INDEPENDENCE CAIC = 400.04935
 MODEL AIC = -1.77899 MODEL CAIC = -5.78963

CHI-SQUARE = .221 BASED ON 1 DEGREES OF FREEDOM
 PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .63827

THE NORMAL THEORY RLS CHI-SQUARE FOR THIS ML SOLUTION IS .221.

FIT INDICES

BENTLER-BONETT NORMED FIT INDEX = 1.000
 BENTLER-BONETT NON-NORMED FIT INDEX = 1.017
 COMPARATIVE FIT INDEX (CFI) = 1.000

RELIABILITY COEFFICIENTS

CRONBACH'S ALPHA = .527
 COEFFICIENT ALPHA FOR AN OPTIMAL SHORT SCALE = .835
 BASED ON THE FOLLOWING 2 VARIABLES
 AV-REQRD AV-ELECT
 GREATEST LOWER BOUND RELIABILITY = .832
 GLB RELIABILITY FOR AN OPTIMAL SHORT SCALE = .835
 BASED ON THE FOLLOWING 2 VARIABLES
 AV-REQRD AV-ELECT
 BENTLER'S DIMENSION-FREE LOWER BOUND RELIABILITY = .682
 SHAPIRO'S LOWER BOUND RELIABILITY FOR A WEIGHTED COMPOSITE = .901
 WEIGHTS THAT ACHIEVE SHAPIRO'S LOWER BOUND:
 AV-REQRD AV-ELECT SAT IQ ED-MOTIV
 .338 .395 .692 .431 .255

ITERATIVE SUMMARY

ITERATION	PARAMETER	ABS CHANGE	ALPHA	FUNCTION
1		14.687760	1.00000	5.15066
2		14.361550	1.00000	.00170
3		.000366	1.00000	.00148

As mentioned before, a saturated model provides a highly useful benchmark against which the fit of any nonsaturated model can be tested. Consider the fact that the model examined here is nested in the saturated model in Fig. 9. The current model differs only in one aspect from that sat-

urated model—namely in the imposed restriction of equality on the two path coefficients that pertain to the substantive query—and therefore can be examined by using the chi-square difference test. Subtracting the values of the two chi-squares, i.e., $0.221 - 0 = 0.221$, and subtracting the degrees of freedom of the two model, i.e., $1 - 0 = 1$, yields a nonsignificant chi-square value difference when judged against the chi-square distribution with 1 degree of freedom. This is because the cut-off value for the chi-square distribution with 1 degree of freedom is 3.84 at a significance level .05 (and obviously higher for any smaller level), as can be found in any table of chi-square critical values. Therefore, the conclusion is that there is no evidence in the data to warrant rejection of the introduced restriction. That is, the hypothesis of equality of the impacts of IQ and ED-MOTIV on AV-ELECT can be retained.

MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

AV-REQRD = V1	=	.085*V3	+	.006*V4	-	.012*V5	+ 1.000	E1
		.007		.013		.024		
		12.109@		.508		-.488		
AV-ELECT = V2	=	.045*V3	+	.144*V4	+	.144*V5	+1.000	E2
		.006		.012		.012		
		7.057@		12.337@		12.337@		

As requested in the input file, the solution equalizes the impact of IQ (V4) and ED-MOTIV (V5) on AV-ELECT (V2). Indeed, both pertinent path coefficients are estimated at .144, have the same standard errors and *t*-values, and are significant. Note that all other path coefficients are significant except the ones relating IQ (V4) and ED-MOTIV (V5) to AV-REQRD (V1). This issue is considered further at the end of the discussion of the entire output.

VARIANCES OF INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	V	F
	---	---
V3 - SAT	47.457*I	I
	5.498 I	I
	8.631@I	I
	I	I
V4 - IQ	10.267*I	I
	1.190 I	I
	8.631@I	I
	I	I
V5 - ED-MOTIV	2.675*I	I
	.310 I	I
	8.631@I	I

VARIANCES OF INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	---		---	
E1 - AV-REQRD		.257*I		I
		.030 I		I
		8.631@I		I
		I		I
E2 - AV-ELECT		.236*I		I
		.027 I		I
		8.631@I		I

COVARIANCES AMONG INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	V		F	
	---		---	
V4 - IQ		4.100*I		I
V3 - SAT		1.839 I		
		2.229@I		I
		I		I
V5 - ED-MOTIV		6.394*I		I
V3 - SAT		1.061 I		I
		6.025@I		I
		I		I
V5 - ED-MOTIV		.525*I		I
V4 - IQ		.431 I		I
		1.217 I		I

COVARIANCES AMONG INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	---		---	
E2 - AV-ELECT		.171*I		I
E1 - AV-REQRD		.025 I		I
		6.974@I		I

As can also be seen from the saturated model in the preceding section, the covariance between IQ and ED-MOTIV is not significant. This suggests that there is weak evidence in the analyzed data for a discernible (linear) relationship between these two high school variables.

STANDARDIZED SOLUTION:

R-SQUARED

AV-REQRD=V1	=	.761*V3	+	.027*V4	-	.025*V5	+	.658	E1	.567
AV-ELECT=V2	=	.354*V3	+	.530*V4	+	.271*V5	+	.559	E2	.687

We emphasize that the standardized solution has not upheld the introduced restriction. Indeed, the standardized path coefficients of V4 and V5 on V2 are not equal, unlike in the immediately preceding unstandardized solution. This violation of constraints in the standardized solution is a common result with restricted models, partly because in practice constraints are typically imposed on parameters that do not have identical units of measurement to begin with. Therefore, standardization of the variable metrics, which underlies the standardized solution, destroys the restrictions since it affects differentially the original units of assessment in various variables. Nonetheless, the restriction imposed in the model of this section has an effect and leads to a standardized solution distinct from the one for the unrestricted model presented earlier, as shown in the last presented output portion. The reason for this is that the standardization is carried out on the solution obtained with the constrained model, by subsequently modifying its parameter estimates appropriately.

CORRELATIONS AMONG INDEPENDENT VARIABLES

V4 - IQ	.186*
V3 - SAT	
V5 - ED-MOTIV	.567*
V3 - SAT	
V5 - ED-MOTIV	.100*
V4 - IQ	

CORRELATIONS AMONG INDEPENDENT VARIABLES

E2 - AV-ELECT	.696*
E1 - AV-REQRD	

 E N D O F M E T H O D

As with the model presented in Fig. 9, the last output portion shows that there is a substantial degree of interrelationship between the error terms. That is, those parts of AV-REQRD and AV-ELECT that are unexplained (linearly) by the common set of predictors used are markedly interrelated. This finding again provides some hint concerning the possibility of common omitted variables, as mentioned at the end of the discussion of the unconstrained model.

MODEL MODIFICATIONS

The restricted model examined in the preceding section indicated that the path coefficients relating the predictors IQ and ED-MOTIV to the dependent variable AV-REQRD were nonsignificant. An interesting question to consider is whether one or both of these paths can be excluded in a modified model while maintaining a good fit to the analyzed data. As indicated in

Chap. 1, modification of a given model with the aim of fit improvement has been termed a specification search (Long, 1983; MacCallum, 1986). A specification search is typically conducted with the intent to detect and correct specification errors between a proposed model and the unknown, true model characterizing the population and variables in a study. Although some new search procedures (e.g., ant colony, genetic algorithms, Tabu search) have been developed to automate the process of model modification using computer algorithms, in this introductory book only model modifications are discussed that can be carried out with available SEM programs (for more extensive discussions, see Marcoulides et al., 1998; Marcoulides & Drezner, 2001, 2003).

Consider a modification of the model in which the path coefficients relating IQ and ED-MOTIV to AV-REQRD were found to be nonsignificant. Since it is well known that a single change in a model can affect other parts of the solution, only one change at a time is to be made in the model. By proceeding step by step—i.e., by setting equal to 0 one parameter at a time—the possibility of missing a tenable restrictive model or misspecifying a model, which might occur if all the nonsignificant parameters were fixed to 0 at once, is excluded. In a sense, this strategy is analogous to backward selection in regression analysis, particularly the rule not to drop from the regression equation more than a single predictor at a time (although automated model-selection search procedures have also been developed for regression analyses; see Drezner, Marcoulides, & Salhi, 1999; Marcoulides & Drezner, 1999).

With user-specified (nonautomated) model specification searches there are no rules concerning which nonsignificant parameters to fix first to 0. One strategy might be to consider setting to 0 the parameter with the smallest nonsignificant t value taken at its absolute value, which in a sense appears 'most nonsignificant'. In the present empirical example, this is the path coefficient of ED-MOTIV on AV-REQRD. Fixing this path coefficient to 0 yields a nested model because the resulting model is obtained from that in Fig. 9 after introducing this particular parameter restriction in addition to the equality of intelligence and motivation effect upon performance on elective subjects, which was examined in the preceding subsection and found plausible. This fixing at 0 is achieved by simply not mentioning the constrained parameter as free when defining the model or, alternatively, by fixing it after the model definition line. Provided here is only the LISREL input file for this model with the restricted parameter between ED-MOTIV and AV-REQRD, GA (1, 3). (The earlier EQS and *Mplus* command files are only minimally modified to introduce this restraint, following exactly the same notational guidelines used in the constrained model of the preceding section.)

```

PATH ANALYSIS MODEL FOR RESTRICTED MODEL WITH THE PATH OF ED-MOTIV TO
AV-REQRD FIXED AT ZERO
  DA NI=5 NO=150
  CM
  .594
  .483          .754
3.993          3.626          47.457
  .426          1.757          4.100          10.267
  .500          .722          6.394          .525          2.675
  LA
  AV-REQRD AV-ELECT SAT IQ ED-MOTIV
  MO NY=2 NX=3 GA=FU,FR PH=SY,FR PS=SY,FR
  EQ GA(2, 2) GA(2, 3)
  FI GA(1, 3) ! THIS IS THE ADDED LINE IN ORDER TO FIX THE PARAMETER  $\gamma_{13}$ 
  OU

```

From here onward, to save space, we will only report those parts of the output, or fit indices if pertinent, that are relevant for answering the substantive query that has led to a fitted model under consideration.

The model for the LISREL command file just presented yields a chi-square value $T = 0.46$ for 2 degrees of freedom. Hence, the difference in chi-square values between this model and the immediately preceding one with only the constraint of two equal path coefficients, is $\Delta T = 0.24$ for $\Delta df = 1$. Because this difference is nonsignificant when compared to the cutoff of 3.84 for the chi-square distribution with 1 degree of freedom, one can conclude that there is no evidence in the data concerning any influence of ED-MOTIV on AV-REQRD, after accounting for the impact of SAT and IQ on that dependent variable. (Note that the last mentioned two predictors are still present in the equation for the dependent variable in this modified model.)

Since the imposed restriction was found to be plausible, it is retained in the model and the next nonsignificant path, that is, the one between IQ and AV-REQRD, is now considered for fixing at 0. For this model version, we present the *Mplus* command files and results. Introducing this model constraint is readily carried out in the *Mplus* syntax by setting the parameter in question equal to 0, which is achieved with the following input:

```

TITLE:          THIS IS AN EXAMPLE OF PATH ANALYSIS WITH
                IMPOSED CONSTRAINTS: TWO PATHS ARE SET
                EQUAL AND TWO OTHERS ARE FIXED AT 0
DATA:           FILE IS EX3.1.COV;
                TYPE = COVARIANCE;
                NOBSERVATIONS=150;
VARIABLE:       NAMES ARE AV_REQRD AV_ELECT SAT IQ ED_MOTIV;
MODEL:          AV_REQRD ON SAT IQ ED_MOTIV;
                AV_ELECT ON SAT

```

```

      IQ ED_MOTIV (1);
      AV_REQRD ON IQ@0;
      AV_REQRD ON ED_MOTIV@0;
OUTPUT:  RESIDUAL;

```

Fixing a path coefficient to 0, or any other constant, is accomplished by stating the path followed by the symbol “@” and the value to which it is to be set equal to; alternatively, in this case of fixing paths to 0, the same effect is accomplished by simply not mentioning the predictors in question when writing in *Mplus* code the equation for the pertinent dependent variable. The final command, OUTPUT, requests the matrix of model residuals. This command file yields a tenable model version: $T = 0.88$ for $df = 3$ that is non-significant and a pertinent RMSEA index being 0 with a 90%-confidence interval (0, .080). The increase in the chi-square goodness of fit is $\Delta T = 0.42$ for $\Delta df = 1$ and is similarly nonsignificant. One can therefore conclude that there is no evidence in the data for an impact of IQ on AV-REQRD, once that of SAT on the latter variable is accounted for.

Following these modifications, the model does not have any more nonsignificant parameters and represents the most restrictive, yet tenable model examined in this chapter. The results of the last part of this specification search are presented here (with some redundant sections eliminated):

MODEL RESULTS					
	Estimates	S.E.	Est./S.E.		
AV_REQRD ON					
SAT	0.084	0.006	13.975		
IQ	0.000	0.000	0.000		
ED_MOTIV	0.000	0.000	0.000		
AV_ELECT ON					
SAT	0.045	0.006	7.472		
IQ	0.141	0.008	16.949		
ED_MOTIV	0.141	0.008	16.949		
AV_ELECT WITH					
AV_REQRD	0.171	0.024	6.999		
Residual Variances					
AV_REQRD	0.256	0.030	8.660		
AV_ELECT	0.234	0.027	8.660		
Residuals for Covariances/Correlations/Residual Correlations					
	AV_REQRD	AV_ELECT	SAT	IQ	ED_MOTIV
AV_REQRD	0.000				
AV_ELECT	0.006	0.008			
SAT	0.000	0.000	0.000		
IQ	0.080	0.047	0.000	0.000	
ED_MOTIV	-0.038	-0.019	0.000	0.000	0.000

Although this final model does not perfectly reproduce the analyzed covariance matrix, as seen from the very last output section (see Appendix to this chapter), the fit criteria mentioned above clearly indicate that it is a plausible model. Similar to the preceding two models considered in this section, the last one fitted shows that in order for a path-analysis model to be a plausible representation of analyzed data it is not necessary that it perfectly or nearly so reproduces the data (covariance matrix).

In conclusion of this chapter, we emphasize that despite the good fit criteria obtained with the last fitted model, results obtained from any model specification search may be unique to the particular data set analyzed and that chance elements may affect the search. In fact, once a specification search is begun, a researcher actually enters a more exploratory phase of analysis. Hence, the possibility exists that at least some results with intermediate as well as final model versions are due only to chance fluctuations at work in the analyzed data. For this reason, any models that result from specification searches should always be cross-validated, i.e., replicated on other data sets from the studied population, before any real validity for results obtained with them can be claimed.

APPENDIX TO CHAPTER 3

The general path analysis model can be written as follows, using the general LISREL model notation introduced in Chap. 1 and its Appendix:

$$Y = BY + \zeta, \quad (\text{A3.1})$$

where Y is the $p \times 1$ vector of observed variables ($p > 1$), B is the $p \times p$ regression coefficient (path coefficient) matrix, ζ is the $p \times 1$ vector of error terms ($p > 1$); and the matrix $I_p - B$ is assumed to be invertible, with I_p denoting the $p \times p$ identity matrix. The error terms are assumed normal and with zero mean, with those pertaining to dependent variables being uncorrelated with their predictors (see below). (In terms of the LISREL notation used in this chapter to refer to path coefficients, $\Gamma = B$ also holds.)

Equation (A3.1) can be thought of as relating several dependent variables (a subset of Y) simultaneously to its putative predictors (the remaining part of Y). Equation (A3.1) results from that of the general LISREL model, (A1.1), by assuming error-free measurement of p corresponding latent variables, i.e.,

$$Y = I_p \eta + 0, \quad (\text{A3.2})$$

where η is a $p \times 1$ vector of "unobservable constructs" (dummy latent variables, correspondingly equal to the observed variables). With this assumption, (A3.1) becomes identical to Equation (A1.2) in the Appendix to Chap. 1, and with $\Lambda = I_p$ Equation (A3.2) is the same as (A1.1).

Hence, all modeling developments presented in Chap. 1 and its Appendix apply to the general path-analysis model (and all its special cases of relevance in a given empirical setting).

The particular model used for demonstration purposes in this chapter is a special case of the general path-analysis model, and thus of the general LISREL model. Indeed, denoting by Y_1 through Y_5 the five observed variables of interest in that empirical setting, using (A3.2) one first assigns a dummy latent variable to each observed variable, i.e., sets

$$Y_i = \eta_i, \quad i = 1, 2, \dots, 5. \quad (\text{A3.3})$$

The path analysis model equations are then a special case of (A1.2) and look as follows:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \gamma_{13} & \gamma_{14} & \gamma_{15} \\ 0 & 0 & \gamma_{23} & \gamma_{24} & \gamma_{25} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix}, \quad (\text{A3.4})$$

where the residual covariance matrix is assumed block-diagonal, with the vector $(\zeta_1, \zeta_2)'$ being unrelated to $(\zeta_1, \zeta_2, \zeta_3)'$ (" denoting transposition).

Confirmatory Factor Analysis

WHAT IS FACTOR ANALYSIS?

Factor analysis is a modeling approach that was first developed by psychologists as a method to study unobservable, hypothetically existing variables, such as intelligence, motivation, ability, attitude, and opinion. Latent variables typically represent not directly measurable dimensions that are of substantive interest to social and behavioral scientists, and a widely accepted interpretation of a latent variable is that an individual's standing on this unobserved dimension can be indicated by various proxies of the dimension, which are generally referred to as indicators. These are directly measurable manifestations of the underlying latent dimension, such as scores on particular tests of intelligence that indicate one's intellectual ability (see Chap. 1).

Like path analysis, factor analysis has a relatively long history. The original idea dates back to the early 1900s, and it is generally acknowledged that the English psychologist Charles Spearman first applied early forms of this approach to study the structure of human abilities. Spearman (1904) proposed that an individual's ability scores were manifestations of a general ability (called general intelligence, or just *g*) and other specific abilities, such as verbal or numerical abilities. The general and specific factors combined to produce the ability performance. This idea was labeled the *two-factor theory of human abilities*. However, as more researchers became interested in this approach (e.g., Thurstone, 1935), the theory was extended to accommodate more factors and the corresponding analytic method was referred to as factor analysis.

In general terms, *factor analysis* is a modeling approach for studying hypothetical constructs by using a variety of observable proxies or indicators of them that can be directly measured. The analysis is considered exploratory, also referred to as exploratory factor analysis (EFA), when the concern is with determining how many factors, or latent constructs, are needed to explain well the relationships among a given set of observed measures. Alternatively, the analysis is confirmatory, formally referred to as confirmatory factor analysis (CFA), when a preexisting structure of the relationships among the measures is being quantified and tested. Thus, unlike EFA, CFA is not concerned with discovering a factor structure, but with confirming and examining the details of an assumed factor structure. In order to confirm a specific factor structure, one must have some initial idea about its composition. In this respect, CFA is considered to be a general modeling approach that is designed to test hypotheses about a factor structure, when the factor number and interpretation in terms of indicators are given in advance. Hence, in CFA (a) the theory comes first, (b) the model is then derived from it, and finally (c) the model is tested for consistency with the observed data. For the latter purpose, structural equation modeling can be used. Thereby, as discussed at length in Chap. 1, the unknown model parameters are estimated so that, in general, the model reproduced matrix $\Sigma(\gamma)$ comes as close as possible to the sample matrix S (i.e., the model is 'given' the best chance to emulate S). If the proposed model emulates S to a sufficient extent, as measured by the goodness-of-fit indices, it can be treated as a plausible description of the phenomenon under investigation and the theory from which the model has been derived is supported. Otherwise, the model is rejected and the theory—as embodied in the model—is disconfirmed. We stress that this testing rationale is valid for all applications of the SEM methodology, not only those within the framework of confirmatory factor analysis, with its origins being traditionally rooted partly in the factor analytic approach.

This discussion of confirmatory factor analysis (CFA) suggests an important limitation concerning its use. The starting point of CFA is a very demanding one, requiring that the complete details of a proposed model be specified before it is fitted to the data. Unfortunately, in many substantive areas this may be too strong a requirement since theories are often poorly developed or even nonexistent. Due to these potential limitations, Jöreskog & Sörbom (1993a) distinguished aptly between three situations concerning model fitting and testing: (a) a strictly confirmatory situation in which a single formulated model is either accepted or rejected; (b) an alternative-models or competing-models situation in which several models are formulated and preferably one of them is selected; and (c) a model-generating situation in which an initial model is specified and, in case of unsatisfactory fit to the data, is modified and repeatedly tested until acceptable fit is obtained.

The strictly confirmatory situation is rare in practice because most researchers are unwilling to reject a proposed model without at least suggesting some alternative model. The alternative- or competing-model situation is not very common because researchers usually prefer not to specify, or cannot specify, particular alternative models beforehand. Model generation seems to be the most common situation encountered in empirical research (Jöreskog & Sörbom, 1993a; Marcoulides, 1989). As a consequence, many applications of CFA actually bear some characteristic features of both explanatory and confirmatory approaches. In fact, it is not very frequent that researchers are dealing with purely exploratory or purely confirmatory analyses. For this reason, results of any repeated modeling conducted on the same data set should be treated with a great deal of caution and be considered tentative until a replication study can provide further information on the performance of these models.

AN EXAMPLE CONFIRMATORY FACTOR ANALYSIS MODEL

To demonstrate a confirmatory factor analysis model, consider the following example model with three latent variables: Ability, Achievement Motivation, and Aspiration. The model proposes that three variables are indicators of Ability, three variables are proxies of Motivation, and two variables are indicative of Aspiration. Here the primary interest lies in estimating the relationships among Ability, Motivation, and Aspiration. For the purposes of this chapter, assume data are available from a sample of $N = 250$ second-year college students for which the normality assumption is plausible. The following observed variables are used in this study:

1. A general ability score (ABILITY1).
2. Grade point average obtained in last year of high school (ABILITY2).
3. Grade point average obtained in first year of college (ABILITY3).
4. Achievement motivation score 1 (MOTIVN1).
5. Achievement motivation score 2 (MOTIVN2).
6. Achievement motivation score 3 (MOTIVN3).
7. A general educational aspiration score (ASPIRN1).
8. A general vocational aspiration score (ASPIRN2).

The example confirmatory factor analysis model is presented in Fig. 10 and the observed covariance matrix in Table 1. The model is initially depicted in EQS notation using V_1 to V_8 for the observed variables, E_1 to E_8 for the error terms associated with the observed variables, and F_1 to F_3 for the latent variables.

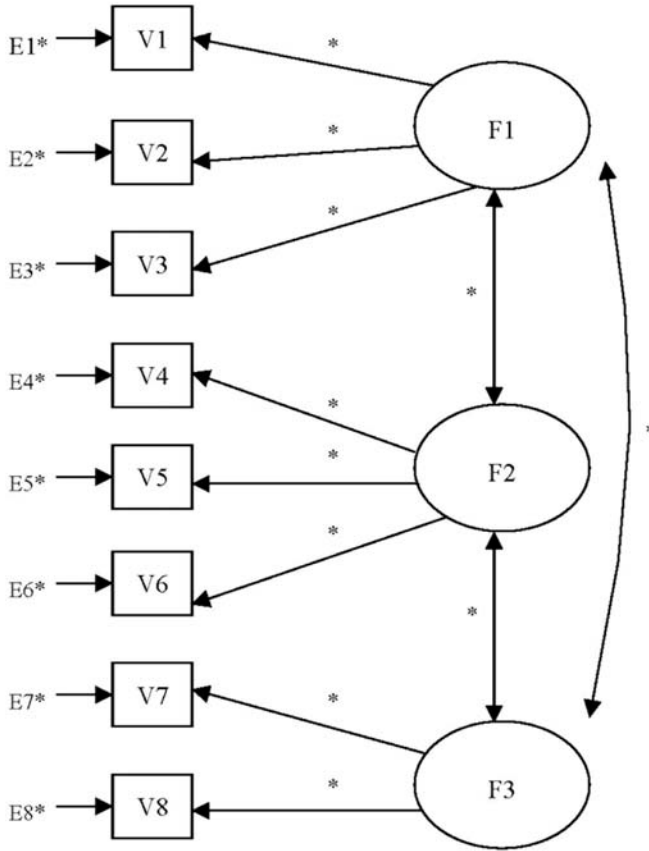


FIG. 10. Example confirmatory factor analysis model using EQS notation.

To determine the parameters of the model in Fig. 10, which are designated by asterisks there, we follow the six rules outlined in Chap. 1. According to Rule 1, all eight error-term variances are model parameters, and according to Rule 3 the eight factor loadings are also model parameters. In addition, the three construct variances are tentatively designated model parameters (but see the use of Rule 6 later in this paragraph). Following Rule 2, the three covariances between latent variables are also model parameters. Rule 4 is not applicable to this model with regard to latent variables because no explanatory relationships are assumed among any of them. For Rule 5, observe that there are no two-way arrows connecting dependent variables, or a dependent and independent variable in the model in Fig. 10. Finally, Rule 6 requires that the scale of each latent variable be fixed. Be-

TABLE 1
Covariance Matrix for Confirmatory Factor Analysis Example of Ability, Motivation, and Aspiration

<i>Variable</i>	<i>AB1</i>	<i>AB2</i>	<i>AB3</i>	<i>MOT1</i>	<i>MOT2</i>	<i>MOT3</i>	<i>ASP1</i>	<i>ASP2</i>
AB1	.45							
AB2	.32	.56						
AB3	.27	.32	.45					
MOT1	.17	.20	.19	.55				
MOT2	.20	.21	.18	.30	.66			
MOT3	.19	.25	.20	.30	.36	.61		
ASP1	.08	.12	.09	.23	.27	.22	.58	
ASP2	.11	.10	.07	.21	.25	.27	.39	.62

Note. AB denotes ability; MOT, motivation; ASP, aspiration. Sample size = 250.

cause this study's primary interest is in estimating the correlations between Ability, Motivation, and Aspiration (which are identical to their covariances if the variances of the latent variables are set equal to 1), the variances of the latent variables are fixed to unity. This decision makes the construct variances fixed parameters rather than free model parameters. Hence, the model in Fig. 10 has altogether 19 parameters (8 factor loadings + 3 factor covariances + 8 error variances = 19), which are symbolized by asterisks.

EQS, LISREL, and *Mplus* COMMAND FILES

EQS Input File

The EQS input file is constructed following the guidelines outlined in Chap. 2. Accordingly, the file begins with a title command line followed by a specification line providing the number of variables in the model and sample size.

```

/TITLE
EXAMPLE CONFIRMATORY FACTOR ANALYSIS;
/SPECIFICATIONS
CASES=250; VARIABLES=8;

```

To facilitate interpretation of the output, labels are provided for all variables included in the model using the command line `/LABELS`.

```
/LABELS
V1=ABILITY1; V2=ABILITY2; V3=ABILITY3; V4=MOTIVN1;
V5=MOTIVN2; V6=MOTIVN3; V7=ASPIRN1; V8=ASPIRN2;
F1=ABILITY; F2=MOTIVATN; F3=ASPIRATN;
```

Next the model definition equations are stated followed by the remaining model parameters in the variance and covariance commands. The `/LMTEST` command requests the modification indices discussed in Chapter 1.

```
/EQUATIONS
V1=*F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V4=*F2+E4;
V5=*F2+E5;
V6=*F2+E6;
V7=*F3+E7;
V8=*F3+E8;
/VARIANCES
F1 TO F3=1; E1 TO E8=*;
/COVARIANCES
F1 TO F3=*;
/LMTEST;
```

Finally, the data are provided along with the end of input file command.

```
/MATRIX
.45
.32 .56
.27 .32 .45
.17 .20 .19 .55
.20 .21 .18 .30 .66
.19 .25 .20 .30 .36 .61
.08 .12 .09 .23 .27 .22 .58
.11 .10 .07 .21 .25 .27 .39 .62
/END;
```

The complete EQS command file, using the appropriate abbreviations, looks now as follows:

```

/TIT
CONFIRMATORY FACTOR ANALYSIS MODEL;
/SPE
CAS=250; VAR=8;
/LAB
V1=ABILITY1; V2=ABILITY2; V3=ABILITY3; V4=MOTIVN1;
V5=MOTIVN2; V6=MOTIVN3; V7=ASPIRN1; V8=ASPIRN2;
F1=ABILITY; F2=MOTIVATN; F3=ASPIRATN;
/EQU
V1=*F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V4=*F2+E4;
V5=*F2+E5;
V6=*F2+E6;
V7=*F3+E7;
V8=*F3+E8;
/VAR
F1 TO F3=1; E1 TO E8=*;
/COV
F1 TO F3=*;
/LMTEST;
/MAT
.45
.32   .56
.27   .32   .45
.17   .20   .19   .55
.20   .21   .18   .30   .66
.19   .25   .20   .30   .36   .61
.08   .12   .09   .23   .27   .22   .58
.11   .10   .07   .21   .25   .27   .39   .62
/END;

```

LISREL Command File

After stating a title and data details, the LISREL input file describes as follows the CFA model under consideration that is based on 8 observed and 3 latent variables, and with model parameters being appropriate elements of corresponding matrices.

```

CONFIRMATORY FACTOR ANALYSIS MODEL
DA NI=8 NO=250
CM
.45
.32   .56
.27   .32   .45
.17   .20   .19   .55
.20   .21   .18   .30   .66
.19   .25   .20   .30   .36   .61
.08   .12   .09   .23   .27   .22   .58
.11   .10   .07   .21   .25   .27   .39   .62
LA
ABILITY1 ABILITY2 ABILITY3 MOTIVN1 MOTIVN2 MOTIVN3 C
ASPIRN1 ASPIRN2
MO NY=8 NE=3 PS=SY,FR TE=DI,FR LY=FU,FI
LE
ABILITY MOTIVATN ASPIRATN
FR LY(1, 1) LY(2, 1) LY(3, 1)
FR LY(4, 2) LY(5, 2) LY(6, 2)
FR LY(7, 3) LY(8, 3)
FI PS(1, 1) PS(2, 2) PS(3, 3)
VA 1 PS(1, 1) PS(2, 2) PS(3, 3)
OU MI

```

Specifically, after the same title the data definition line declares that the model will be fit to data on eight variables collected from 250 subjects. The sample covariance matrix CM is given next, along with the variable labels (note the use of C, for Continue, to wrap over to the second label line). The latent variables are also assigned labels by using the notation LE (for Labels for Etas, following the notation of the general LISREL model in which the Greek letter *eta* represents a latent variable). Next, in the model command line the three matrices PS, TE, and LY are defined. The latent covariance matrix PS is initially declared to be symmetric and free (i.e., all its elements are free parameters), which defines all factor variances and covariances as model parameters. Subsequently, for reasons discussed in the previous section, the variances in the matrix PS are fixed to a value of 1. The error covariance matrix TE is defined as diagonal (i.e., no error covariances are introduced) and therefore only has as model parameters the error variances along its main diagonal. Defining then the matrix of factor loadings LY as a fixed and full (rectangular) matrix relating the eight manifest variables to the three latent variables, permits those loadings that relate the corresponding indicators to their factors to be declared freed in the next lines. Last but not least, to illustrate use and interpretation of modification indices, which we may wish to examine if model fit comes out as unsatisfactory, we include on the OUput line the request for them with the keyword MI.

***Mplus* Command File**

Here we override some default options available in *Mplus*, given our interest in estimating factor correlations as model parameters, as mentioned earlier in this chapter. This is accomplished by fixing the factor variances at 1, while the default arrangement in this software is to alternatively fix at 1 the loading of the first listed indicator for each latent variable. The following command file will accomplish our aim.

```
TITLE: CONFIRMATORY FACTOR ANALYSIS MODEL
DATA: FILE IS EX4.COV;
      TYPE=COVARIANCE;
      NOOBSERVATIONS=250;
VARIABLE:NAMES ARE ABILITY1 ABILITY2 ABILITY3 MOTIVN1 MOTIVN2
      MOTIVN3 ASPIRN1 ASPIRN2;
MODEL: F1 BY ABILITY1*1 ABILITY2 ABILITY3;
      F2 BY MOTIVN1*1 MOTIVN2 MOTIVN3;
      F3 BY ASPIRN1*1 ASPIRN2;
      F1-F3@1;
OUTPUT: MODINDICES(5);
```

After giving a title to this modeling session, the data location is provided. Since we only have access to the covariance matrix of the eight analyzed variables, the type of data is declared and the sample size stated. Next we give names to the variables in the study, and in the model definition part declare each of the three constructs as being measured by its pertinent indicators. The default options built into *Mplus* consider the factor covariances as well as error term variances as model parameters, which is what we need. To override the other default arrangement regarding fixing a factor loading instead of latent variance as we would like, we list all latent variable indicators and after the first of them add an asterisk that signals a start value being stated next for that factor loading. In this way, we free all factor loadings per latent variable. With the last line of the MODEL command, we fix the latent variances at 1, which is not a default arrangement and therefore needs to be explicitly done. The OUTPUT command requests the printing of modification indices in excess of 5, which we may wish to examine if fit of this model turns out not to be satisfactory.

MODELING RESULTS

EQS Results

The EQS input described in the previous section produces the following results. In presenting the output sections with this and the other two pro-

grams, comments are inserted at appropriate places; similarly, the pages echoing the input and the recurring page titles as well as estimation method line, after its first occurrence, are omitted in order to save space.

PARAMETER ESTIMATES APPEAR IN ORDER,
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.

This message indicates that the program has not encountered problems stemming from lack of model identification or other numerical difficulties, and it is reassuring that the model is technically sound.

RESIDUAL COVARIANCE MATRIX (S-SIGMA):

	ABILITY1 V1	ABILITY2 V2	ABILITY3 V3	MOTIVN1 V4	MOTIVN2 V5
ABILITY1 V1	.000				
ABILITY2 V2	.001	.000			
ABILITY3 V3	-.001	-.001	.000		
MOTIVN1 V4	.001	.000	.020	.000	
MOTIVN2 V5	.004	-.022	-.017	-.004	.000
MOTIVN3 V6	-.007	.017	.002	-.004	.007
ASPIRN1 V7	-.008	.016	.001	.016	.022
ASPIRN2 V8	.019	-.008	-.022	-.011	-.007
	MOTIVN3 V6	ASPIRN1 V7	ASPIRN2 V8		
MOTIVN3 V6	.000				
ASPIRN1 V7	-.029	.000			
ASPIRN2 V8	.013	.000	.000		

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = .0077
 AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = .0099

STANDARDIZED RESIDUAL MATRIX:

	ABILITY1 V1	ABILITY2 V2	ABILITY3 V3	MOTIVN1 V4	MOTIVN2 V5
ABILITY1 V1	.000				
ABILITY2 V2	.002	.000			
ABILITY3 V3	-.001	-.001	.000		
MOTIVN1 V4	.002	.000	.041	.000	
MOTIVN2 V5	.007	-.037	-.031	-.006	.000
MOTIVN3 V6	-.012	.029	.005	-.008	.011
ASPIRN1 V7	-.016	.027	.003	.028	.035
ASPIRN2 V8	.036	-.013	-.041	-.019	-.010
	MOTIVN3 V6	ASPIRN1 V7	ASPIRN2 V8		
MOTIVN3 V6	.000				
ASPIRN1 V7	-.049	.000			
ASPIRN2 V8	.021	.000	.000		

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0137
 AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0176

MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

ABILITY1=V1	=	.519*F1	+	1.000	E1
		.039			
		13.372@			
ABILITY2=V2	=	.615*F1	+	1.000	E2
		.043			
		14.465@			
ABILITY3=V3	=	.521*F1	+	1.000	E3
		.039			
		13.461@			
MOTIVN1 =V4	=	.511*F2	+	1.000	E4
		.045			
		11.338@			
MOTIVN2 =V5	=	.594*F2	+	1.000	E5
		.049			
		12.208@			
MOTIVN3 =V6	=	.595*F2	+	1.000	E6
		.046			
		12.882@			
ASPIRN1 =V7	=	.614*F3	+	1.000	E7
		.049			
		12.551@			
ASPIRN2 =V8	=	.635*F3	+	1.000	E8
		.051			
		12.547@			

This is the final model solution presented in nearly the same form as the model equations submitted to EQS with the input file (recall that in EQS asterisks denote the estimated parameters rather than significance). Immediately beneath each parameter estimate its standard error appears, and below it the pertinent *t* value is listed. These measurement equations suggest that some of the latent variable indicators load to a very similar degree on their factors. (This issue is revisited in a later section of the chapter when some restricted hypotheses are tested.)

VARIANCES OF INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

V		F	
---		---	
	I	F1 -ABILITY	1.000 I
	I		I
	I		I
	I		I
	I	F2-MOTIVATN	1.000 I
	I		I
	I		I
	I		I
	I	F3-ASPIRATN	1.000 I
	I		I
	I		I
	I		I

VARIANCES OF INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E	D
E1 -ABILITY1	.181*I .023 I 7.936@I	- I I I
E2 -ABILITY2	.182*I .027 I 6.673@I	I I I I
E3 -ABILITY3	.178*I .023 I 7.846@I	I I I I
E4 -MOTIVN1	.288*I .032 I 8.965@I	I I I I
E5 -MOTIVN2	.308*I .037 I 8.362@I	I I I I
E6 -MOTIVN3	.256*I .033 I 7.760@I	I I I I
E7 -ASPIRN1	.203*I .040 I 5.109@I	I I I I
E8 -ASPIRN2	.217*I .042 I 5.118@I	I I I I

This output section begins with a restatement of the unitary factor variances, followed by the error term variances that are all significant. This finding indicates that for each of the indicators some nonnegligible portion of its variance is due to unaccounted by the model sources of variability, including measurement error. This type of result is typical in social and behavioral research that as widely appreciated is plagued by considerable and nearly ubiquitous error of measurement. We note in particular that none of these variances is estimated at a negative value (and similarly none of the variances that follow below in this model output). As indicated before, a negative variance estimate would render the entire output not trustworthy since it would represent a clear sign of an inadmissible solution that cannot be relied on.

COVARIANCES AMONG INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

V		F
—		—
I	F2 -MOTIVAT	.636*I
I	F1 -ABILITY	.055 I
I		11.678@I
I		I
I	F3-ASPIRATN	.276*I
I	F1 -ABILITY	.073 I
I		3.764@I
I		I
I	F3-ASPIRATN	.681*I
I	F2-MOTIVATN	.054 I
I		12.579@I

These are the latent variable correlations along with their standard errors and *t* values. Due to our fixing all latent variances to 1 in the model construction phase, these reported covariances in fact equal the factor correlations. Hence, the provided standard errors and *t*-values actually pertain to the factor correlations and allow one to readily test hypotheses about these correlations as well as construct confidence intervals for any of them. For example, if one were interested in interval estimation of the correlation between Aspiration and Motivation, adding and subtracting twice the indicated standard error to the estimate of this correlation, .681, renders an approximate (large-sample) 95%-confidence interval for it as (.573, .789). Hence, if one was concerned with testing a hypothesis of this correlation being equal to any prespecified number that happens to lie within this interval, that hypothesis could not be rejected at the significance level of $\alpha = .05$; if that number was not covered by the interval, the pertinent hypothesis would be rejected. (We stress that we are discussing here testing of hypotheses that are formulated before looking at the data and in particular the SEM analysis output.)

STANDARDIZED SOLUTION:		R-SQUARED
ABILITY1 = V1 =	.773*F1 + .634 E1	.598
ABILITY2 = V2 =	.822*F1 + .570 E2	.675
ABILITY3 = V3 =	.777*F1 + .629 E3	.604
MOTIVN1 = V4 =	.690*F2 + .724 E4	.476
MOTIVN2 = V5 =	.731*F2 + .683 E5	.534
MOTIVN3 = V6 =	.762*F2 + .648 E6	.581
ASPIRN1 = V7 =	.807*F3 + .591 E7	.651
ASPIRN2 = V8 =	.806*F3 + .592 E8	.650

As discussed in Chap. 3, this STANDARDIZED SOLUTION results from standardizing all variables in the model. Since the standardized solution uses a metric that is uniform across all measures, it is possible to address the issue

of relative importance of manifest variables in assessing the underlying constructs by comparing their loadings.

CORRELATIONS AMONG INDEPENDENT VARIABLES

V		F
I	F2-MOTIVATN	.636*I
I	F1 -ABILITY	I
I		I
I	F3-ASPIRATN	.276*I
I	F1 -ABILITY	I
I		I
I	F3-ASPIRATN	.681*I
I	F2-MOTIVATN	I

These are the same latent correlations that we discussed earlier in this section. There are medium-sized relationships between Ability and Motivation (estimated at .64), as well as between Motivation and Aspiration (estimated at .68). In contrast, the correlation between Ability and Aspiration appears to be much weaker (estimated at .28).

LAGRANGE MULTIPLIER TEST (FOR ADDING PARAMETERS)

ORDERED UNIVARIATE TEST STATISTICS:

NO	CODE	PARAMETER	CHI-SQUARE	PROB.	HANCOCK 17 DF PROB.	PARAMETER CHANGE	STANDARDIZED CHANGE
1	2 12	V5, F1	.958	.328	1.000	-.069	-.085
2	2 12	V6, F3	.515	.473	1.000	-.055	-.071
3	2 12	V5, F3	.387	.534	1.000	.049	.061
4	2 12	V3, F3	.324	.569	1.000	-.022	-.033
5	2 12	V6, F1	.246	.620	1.000	.034	.044
6	2 12	V4, F1	.244	.621	1.000	.032	.043
7	2 12	V8, F2	.110	.740	1.000	-.057	-.072
8	2 12	V7, F2	.110	.740	1.000	.055	.072
9	2 12	V8, F1	.110	.740	1.000	-.018	-.022
10	2 12	V7, F1	.110	.740	1.000	.017	.022
11	2 12	V1, F3	.088	.767	1.000	.011	.017
12	2 12	V2, F3	.067	.796	1.000	.011	.015
13	2 12	V4, F3	.016	.900	1.000	.009	.012
14	2 12	V3, F2	.011	.916	1.000	-.006	-.008
15	2 12	V2, F2	.003	.955	1.00	.003	.005
16	2 12	V1, F2	.002	.962	1.000	.003	.004
17	2 0	F2, F2	.000	1.000	1.000	.000	.000
18	2 0	F3, F3	.000	1.000	1.000	.000	.000
19	2 0	F1, F1	.000	1.000	1.000	.000	.000

***** NONE OF THE UNIVARIATE LAGRANGE MULTIPLIERS IS SIGNIFICANT,
 ***** THE MULTIVARIATE TEST PROCEDURE WILL NOT BE EXECUTED.

As discussed previously in Chapter 1, modification indices (in this case so-called Lagrange Multipliers) found to be larger than 5 could merit closer inspection. The above results indicate no large modification indices among those presented indicating that no changes should be made to the proposed model (see next section).

LISREL Results

As before, for brevity we dispense with the echoed input file, analyzed covariance matrix, and the recurring first title line.

Parameter Specifications

```

LAMBDA-Y
      ABILITY      MOTIVATN      ASPIRATN
ABILITY1          1              0              0
ABILITY2          2              0              0
ABILITY3          3              0              0
MOTIVN1           0              4              0
MOTIVN2           0              5              0
MOTIVN3           0              6              0
ASPIRN1           0              0              7
ASPIRN2           0              0              8

      PSI
      ABILITY      MOTIVATN      ASPIRATN
-----
ABILITY           0
MOTIVATN          9              0
ASPIRATN         10              11              0

THETA-EPS
ABILITY1  ABILITY2  ABILITY3  MOTIVN1  MOTIVN2  MOTIVN3
-----
      12      13      14      15      16      17

THETA-EPS
      ASPIRN1  ASPIRN2
-----
      18      19
    
```

We observe from this section that the command file we submitted has correctly communicated to the software the number and exact location of the 19 model parameters—the eight factor loadings, three factor covariances, and eight error variances.

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y	ABILITY	MOTIVATN	ASPIRATN
	- - - -	- - - - -	- - - - -
ABILITY1	0.52 (0.04) 13.37	- -	- -
ABILITY2	0.61 (0.04) 14.46	- -	- -
ABILITY3	0.52 (0.04) 13.46	- -	- -
MOTIVN1	- -	0.51 (0.05) 11.34	- -
MOTIVN2	- -	0.59 (0.05) 12.21	- -
MOTIVN3	- -	0.60 (0.05) 12.89	- -
ASPIRN1	- -	- -	0.61 (0.05) 12.55
ASPIRN2	- -	- -	0.63 (0.05) 12.55

This part of the output presents the factor loading estimates in the LY matrix along with their standard errors and *t* values, in a column format. The estimates of the loadings for each indicator appear quite similar within a factor. This will have a bearing on the ensuing formal tests of the tau-equivalence hypotheses, i.e., the assumption that the indicators measure the same latent variable in the same units of measurement, with regard to the indicators of Ability, Motivation, and Aspiration.

Covariance Matrix of ETA

	ABILITY	MOTIVATN	ASPIRATN
	- - - -	- - - - -	- - - - -
ABILITY	1.00		
MOTIVATN	0.64	1.00	
ASPIRATN	0.28	0.68	1.00

PSI

	ABILITY	MOTIVATN	ASPIRATN
ABILITY	1.00		
MOTIVATN	0.64 (0.05) 11.68	1.00	
ASPIRATN	0.28 (0.07) 3.76	0.68 (0.05) 12.58	1.00

From here one can infer that there are significant and medium-size relationships between Ability and Motivation (estimated at .64), as well as between Motivation and Aspiration (estimated at .68). Since the variances of the latent variables are fixed to 1, the COVARIANCE MATRIX OF ETA is identical to the PSI matrix in this example. Differences between these two matrices will emerge in the next chapter, where there will be explanatory relationships postulated between some of the constructs. Using the approximate confidence interval of the correlation between Ability and Aspiration, which at the 95% confidence level is $(.28 - 2 \times .07; .28 + 2 \times .07) = (.14; .42)$, we conclude that there is evidence suggesting that this correlation is non-zero in the studied population. As mentioned in earlier chapters, a significance test is also readily carried out by looking at the t -value associated with a given parameter in a structural equation model; for the presently considered correlation, since its t -value being 3.76 is outside the interval $(-2, +2)$, it is concluded that the correlation is significant (at the .05 level). We stress, however, that examining the earlier confidence interval provides to the researcher a whole range of plausible values for this unknown population correlation rather than only a test of its significance (i.e., a p -value associated with the pertinent null hypothesis).

THETA-EPS

ABILITY1	ABILITY2	ABILITY3	MOTIVN1	MOTIVN2	MOTIVN3
0.18 (0.02) 7.94	0.18 (0.03) 6.67	0.18 (0.02) 7.85	0.29 (0.03) 8.97	0.31 (0.04) 8.36	0.26 (0.03) 7.76

THETA-EPS

ASPIRN1	ASPIRN2
0.20 (0.04) 5.11	0.22 (0.04) 5.12

Squared Multiple Correlations for Y - Variables

ABILITY1	ABILITY2	ABILITY3	MOTIVN1	MOTIVN2	MOTIVN3
0.60	0.67	0.60	0.48	0.53	0.58

Squared Multiple Correlations for Y - Variables

ASPIRN1	ASPIRN2
0.65	0.65

Based on this output portion, apart from the first indicator of Motivation (MOTIVTN1), more than half of the variance in any of the remaining seven measures is explained in terms of latent individual differences on the corresponding factor.

Goodness of Fit Statistics

Degrees of Freedom = 17
 Minimum Fit Function Chi-Square = 20.58 (P = 0.25)
 Normal Theory Weighted Least Squares Chi-Square = 18.89 (P = 0.33)
 Estimated Non-centrality Parameter (NCP) = 1.89
 90 Percent Confidence Interval for NCP = (0.0 ; 16.78)

Minimum Fit Function Value = 0.083
 Population Discrepancy Function Value (F0) = 0.0076
 90 Percent Confidence Interval for F0 = (0.0 ; 0.067)
 Root Mean Square Error of Approximation (RMSEA) = 0.021
 90 Percent Confidence Interval for RMSEA = (0.0 ; 0.063)
 P-Value for Test of Close Fit (RMSEA < 0.05) = 0.84

Expected Cross-Validation Index (ECVI) = 0.23
 90 Percent Confidence Interval for ECVI = (0.22 ; 0.29)
 ECVI for Saturated Model = 0.29
 ECVI for Independence Model = 4.87

Chi-Square for Independence Model with 28 Degrees of Freedom = 1196.96
 Independence AIC = 1212.96
 Model AIC = 56.89
 Saturated AIC = 72.00
 Independence CAIC = 1249.13
 Model CAIC = 142.80
 Saturated CAIC = 234.77

Normed Fit Index (NFI) = 0.98
 Non-Normed Fit Index (NNFI) = 0.99
 Parsimony Normed Fit Index (PNFI) = 0.60
 Comparative Fit Index (CFI) = 1.00
 Incremental Fit Index (IFI) = 1.00
 Relative Fit Index (RFI) = 0.97

Critical N (CN) = 405.22

Root Mean Square Residual (RMR)	=	0.011
Standardized RMR	=	0.020
Goodness of Fit Index (GFI)	=	0.98
Adjusted Goodness of Fit Index (AGFI)	=	0.96
Parsimony Goodness of Fit Index (PGFI)	=	0.46

All of the goodness-of-fit indices indicate an acceptable model (cf. Jöreskog & Sörbom, 1993c). In particular, given normality of the data, the pertinent chi-square value denoted “minimum fit function chi-square” is of approximately the same magnitude as the model degrees of freedom, which is typical for well-fitting models (see also its *p*-value that is nonsignificant). The root mean square error of approximation (RMSEA), a highly popular fit measure in contemporary structural equation modeling literature, similarly indicates a well-fitting model. Its value is .021 that is lower than the suggested threshold of .05, and the left endpoint of its 90%-confidence interval is 0 and much smaller than that same threshold (e.g., Browne & Cudeck, 1993).

Modification Indices and Expected Change

Modification Indices for LAMBDA-Y			
	ABILITY	MOTIVATN	ASPIRATN
	— — — —	— — — —	— — — —
ABILITY1	— —	0.00	0.09
ABILITY2	— —	0.00	0.07
ABILITY3	— —	0.01	0.32
MOTIVN1	0.24	— —	0.02
MOTIVN2	0.96	— —	0.39
MOTIVN3	0.25	— —	0.52
ASPIRN1	0.11	0.11	— —
ASPIRN2	0.11	0.11	— —

Expected Change for LAMBDA-Y			
	ABILITY	MOTIVATN	ASPIRATN
	— — — —	— — — —	— — — —
ABILITY1	— —	0.00	0.01
ABILITY2	— —	0.00	0.01
ABILITY3	— —	-0.01	-0.02
MOTIVN1	0.03	— —	0.01
MOTIVN2	-0.07	— —	0.05
MOTIVN3	0.03	— —	-0.06
ASPIRN1	0.02	0.05	— —
ASPIRN2	-0.02	-0.06	— —

No Non-Zero Modification Indices for PSI

Modification Indices for THETA-EPS

	ABILITY1	ABILITY2	ABILITY3	MOTIVN1	MOTIVN2	MOTIVN3
ABILITY1	--					
ABILITY2	0.07	--				
ABILITY3	0.01	0.02	--			
MOTIVN1	0.11	0.30	2.09	--		
MOTIVN2	1.19	1.28	0.40	0.08	--	
MOTIVN3	1.13	1.66	0.00	0.18	0.49	--
ASPIRN1	2.81	1.47	0.39	1.80	2.09	7.48
ASPIRN2	4.30	0.79	1.88	1.13	1.23	4.67

Modification Indices for THETA-EPS

	ASPIRN1	ASPIRN2
ASPIRN1	--	
ASPIRN2	--	--

Expected Change for THETA-EPS

	ABILITY2	ABILITY3	MOTIVN1	MOTIVN2	MOTIVN3
ABILITY1	--				
ABILITY20.01	--				
ABILITY30.00	0.00	--			
MOTIVN1-0.01	-0.01	0.03	--		
MOTIVN20.02	-0.02	-0.01	-0.01	--	
MOTIVN3-0.02	0.03	0.00	-0.01	0.02	--
ASPIRN1-0.03	0.02	0.01	0.03	0.04	-0.06
ASPIRN20.04	-0.02	-0.02	-0.02	-0.03	0.05

Expected Change for THETA-EPS

	ASPIRN1	ASPIRN2
ASPIRN1	--	
ASPIRN2	--	--

Maximum Modification Index is 7.48 for Element (7, 6) of THETA-EPS

In Chap. 1, the topic of modification indices was discussed and it was suggested that all indices found to be larger than 5 could merit closer inspection. It was also indicated that any model changes based on modification indices should be justified on theoretical grounds and be consistent with available theory. Although there is a modification index larger than 5 in this output—the element (7,6) of the TE matrix—adding it to the proposed model cannot be theoretically justified since there does not appear to be a substantive reason for the error terms of ASPIRN1 and MOTIVN3, an Aspiration and a Motivation indicator, to correlate. In addition, since fit is already tenable no change really needs to be made to the model that is considered to be an acceptable means of data description.

Mplus Results

After echoing back the input file and providing the location of analyzed data and numerical details regarding method of estimation, the software presents information about model fit.

THE MODEL ESTIMATION TERMINATED NORMALLY

TESTS OF MODEL FIT

Chi-Square Test of Model Fit		
Value		20.664
Degrees of Freedom		17
P-Value		0.2417

Chi-Square Test of Model Fit for the Baseline Model

Value		804.276
Degrees of Freedom		28
P-Value		0.0000
CFI/TLI		
CFI		0.995
TLI		0.992

Loglikelihood

H0 Value		-1853.657
H1 Value		-1843.325

Information Criteria

Number of Free Parameters		19
Akaike (AIC)		3745.315
Bayesian (BIC)		3812.223
Sample-Size Adjusted	BIC	3751.991
(n* = (n + 2) / 24)		

RMSEA (Root Mean Square Error Of Approximation)

Estimate		0.029
90 Percent C.I.		0.000 0.067
Probability RMSEA <=		.05 0.778

SRMR (Standardized Root Mean Square Residual)

Value		0.020
-------	--	-------

We note that the fit indices that are essentially the same (within numerical procedure accuracy) as those provided by the preceding two programs for the current model and data, as are the following parameter estimates.

MODEL RESULTS

		Estimates	S.E.	Est./S.E.
F1	BY			
	ABILITY1	0.518	0.039	13.399
	ABILITY2	0.614	0.042	14.494
	ABILITY3	0.520	0.039	13.488
F2	BY			
	MOTIVN1	0.510	0.045	11.360
	MOTIVN2	0.592	0.048	12.230
	MOTIVN3	0.594	0.046	12.911
F3	BY			
	ASPIRN1	0.613	0.049	12.578
	ASPIRN2	0.634	0.050	12.571
F2	WITH			
	F1	0.636	0.054	11.700
F3	WITH			
	F1	0.276	0.073	3.770
	F2	0.681	0.054	12.604

We stress that all factor loadings are model parameters and that the latent covariances output are in fact the factor correlations that are therefore directly estimated and presented with a standard error and *t*-value.

Variances

F1	1.000	0.000	0.000
F2	1.000	0.000	0.000
F3	1.000	0.000	0.000

Residual Variances

ABILITY1	0.180	0.023	7.952
ABILITY2	0.181	0.027	6.687
ABILITY3	0.177	0.023	7.861
MOTIVN1	0.287	0.032	8.983
MOTIVN2	0.307	0.037	8.380
MOTIVN3	0.255	0.033	7.773
ASPIRN1	0.202	0.039	5.118
ASPIRN2	0.216	0.042	5.130

MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index 5.000

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
WITH Statements				
ASPIRN1 WITH MOTIVN3	7.511	-0.064	-0.064	-0.108

As seen from this section, the maximum modification index (and the only one higher than 5) is that associated with the covariance of the error terms pertaining to the first Aspiration and last Motivation indicators. As mentioned earlier in this section, when discussing the corresponding LISREL output part, adding this error covariance to the model cannot be theoretic-

cally justified since there does not appear to be a substantive reason for these error terms to correlate; further, and no less importantly, fit is tenable so no change really needs to be made to the model.

TESTING MODEL RESTRICTIONS: TRUE SCORE EQUIVALENCE

The model examined in the previous section focused on estimating the degree of interrelationship among Ability, Motivation, and Aspiration. In examining the results, however, it appeared that the loadings on each factor were quite similar within factor. Suppose a researcher hypothesized that the factor loadings of the Ability construct associated with the measures ABILITY1, ABILITY2, and ABILITY3 were indeed equal. In the psychometric literature, such a model is referred to as a model with true score equivalent, or alternatively tau-equivalent, measures. A *tau-equivalent indicator model* embodies the assumption that a latent variable's indicators assess the same construct in the same units of measurement. Specifically, this assumption is equivalent to a statement that the indicators in question evaluate the same true score (e.g., Lord & Novick, 1968; McDonald, 1999).

For each of the three latent variables included in the empirical study under consideration in this chapter—Ability, Motivation, and Aspiration—the assumption of true score equivalence can be tested by imposing appropriate restrictions on their indicators. To this end, the equality of loading constraint is introduced for the pertinent measures, and the resulting difference in chi-square values is evaluated (for a more detailed discussion of the chi-square difference test, see Chap. 3). This procedure is carried out in the present section using the three SEM software of concern in this text.

We begin by imposing equality constraints on the factor loadings of the Ability construct. This restriction is included in the EQS input file by simply adding a /CONSTRAINT section:

```
/CONSTRAINT
(V1,F1)=(V2,F1)=(V3,F1);
```

In LISREL, this constraint is accomplished by introducing the following equality line

```
EQ LY(1, 1) LY(2, 1) LY(3, 1)
```

In *Mplus*, one needs to indicate that the factor loadings are to be kept the same, which is achieved with the following extension of the corresponding model command line:

```
F1 BY ABILITY1*1 ABILITY2 ABILITY3 (1)
```

Note the added number within parentheses at the end of the line, whose effect is the restriction to identity of all preceding factor loadings (referred to in that line).

The result of introducing this restriction in the initial model is a nested model and an increase in the chi-square value up to $T = 25.74$ with $df = 19$. (Recall that the model without the true score equivalence assumption yielded a chi-square value of $T = 20.58$ with $df = 17$.) The difference in the chi-square values between this model and the original one (displayed in Fig. 10) is therefore $\Delta T = 5.16$ for $\Delta df = 2$, and is thus nonsignificant. (The critical value of the chi-square distribution with 2 degrees of freedom is 5.99 at the .05 significance level, and higher for any lower level.) One can therefore conclude that the imposed factor loading identity is plausible, and hence that the Ability measures are tau-equivalent. Because the restriction is found to be acceptable, it is retained in the model for the following analyses.

Next, we test whether the Motivation indicators are also true score equivalent by imposing the identity restriction on their factor loadings. This is achieved by including the line

$$(V4,F2)=(V5,F2)=(V6,F2);$$

as a second line of the /CONSTRAINTS section in the EQS input; by adding the line

$$EQ LY(4, 2) LY(5, 2) LY(6, 2)$$

as a second EQuality line in the LISREL input; and by modifying the second MODEL command line in the *Mplus* input to

$$F2 BY MOTIVN1*1 MOTIVN2 MOTIVN3 (2);$$

Note that in this *Mplus* line we use a different number placed within brackets at its end, since we are not willing to constrain to identity all six indicators of the Ability and Motivation constructs, but only the three loadings of Motivation. (That is, the equal Motivation loadings need not be the same as the loadings of the Ability latent variable.)

The newly-restricted model is nested in the preceding one and associated with a chi-square value of $T = 28.56$ with $df = 21$. The difference in chi-square values, compared to the preceding model is $\Delta T = 2.82$, with the difference in degrees of freedom being $\Delta df = 2$, and is thus nonsignificant as well. These results lead to the conclusion that there is not enough evidence in the data to warrant rejection of the restriction of identical factor loadings for the Motivation construct. Therefore one can consider these to be tau-equivalent measures of Motivation. Due to this nonsignificant finding, this loading equality is also retained in the model.

Finally, imposing the restriction of tau-equivalence on the Aspiration indicators is accomplished by adding in the /CONSTRAINTS section of the EQS input file the line

```
(V7,F3)=(V8,F3);
```

in the LISREL input file the line:

```
EQ LY(7, 3) LY(8, 3)
```

and by modifying the last line in the MODEL command in the *Mplus* input to

```
F3 BY ASPIRN1*1 ASPIRN2 (3);
```

The result of the last imposed restriction that leads to another nested model brings about only a slight increase in the chi-square value to $T = 28.71$ with $df = 22$. Hence, the change in chi-square is only $\Delta T = 0.15$, for a difference in degrees of freedom of $\Delta df = 1$, and is thus nonsignificant. (Recall that the critical value of the chi-square distribution with 1 degree of freedom is 3.84 at the .05 significance level, and higher for any lower level.) Based on these results, one can conclude that there is not enough evidence in the data to warrant rejection of the restriction of identical factor loadings for the Aspiration construct, and hence one can consider these indicators to be tau-equivalent measures of Aspiration.

To summarize, the above results indicate that there is not enough evidence in the data to disconfirm the true score (tau-) equivalence hypothesis for any of the three sets of measures respectively assessing Ability, Motivation, and Aspiration. One can suggest therefore that the indicators of Ability, Motivation, and Aspiration assess their underlying latent variables each in the same units of measurement.

By way of concluding this chapter, only the *Mplus* output file for the last, most restricted model is presented next, with comments inserted at appropriate places and repetitive material omitted (interested readers can easily verify that the results of the EQS and LISREL runs are essentially identical by submitting the appropriate input files).

TESTS OF MODEL FIT

Chi-Square	Test of Model Fit	
	Value	28.713
	Degrees of Freedom	22
	P-Value	0.1532
Chi-Square	Test of Model Fit for the Baseline Model	
	Value	804.276
	Degrees of Freedom	28
	P-Value	0.0000

CFI/TLI			
	CFI		0.991
	TLI		0.989
Loglikelihood			
	H0 Value		-1857.682
	H1 Value		-1843.325
Information Criteria			
	Number of Free Parameters		14
	Akaike (AIC)		3743.364
	Bayesian (BIC)		3792.664
	Sample-Size Adjusted BIC		3748.283
	(n* = (n + 2) / 24)		
RMSEA (Root Mean Square Error Of Approximation)			
	Estimate		0.035
	90 Percent C.I.	0.000	0.067
	Probability RMSEA <=	.05	0.747
SRMR (Standardized Root Mean Square Residual)			
	Value		0.042

We note that the fit indices clearly indicate a plausible model. Relative to the three models fitted in this section, this one has the smallest information criteria values. This corroborates further our preference for the present model.

MODEL RESULTS				
		Estimates	S.E.	Est./S.E.
F1	BY			
	ABILITY1	0.546	0.030	18.455
	ABILITY2	0.546	0.030	18.455
	ABILITY3	0.546	0.030	18.455
F2	BY			
	MOTIVN1	0.563	0.033	16.992
	MOTIVN2	0.563	0.033	16.992
	MOTIVN3	0.563	0.033	16.992
F3	BY			
	ASPIRN1	0.623	0.036	17.243
	ASPIRN2	0.623	0.036	17.243
F2	WITH			
	F1	0.640	0.055	11.712
F3	WITH			
	F1	0.275	0.074	3.743
	F2	0.683	0.054	12.613
Variances				
	F1	1.000	0.000	0.000
	F2	1.000	0.000	0.000
	F3	1.000	0.000	0.000

Residual Variances			
ABILITY1	0.172	0.021	7.992
ABILITY2	0.210	0.024	8.595
ABILITY3	0.168	0.021	7.928
MOTIVN1	0.273	0.031	8.752
MOTIVN2	0.316	0.035	9.107
MOTIVN3	0.269	0.031	8.714
ASPIRN1	0.192	0.030	6.470
ASPIRN2	0.226	0.032	7.161

The true relationship (latent correlation) between Ability and Aspiration is not impressive—being estimated at .28 means that just above 8% of individual true differences on Ability are explained in the present model in terms of these differences on Aspiration. On the contrary, the relationships between Ability and Motivation as well as that between Motivation and Aspiration are considerably stronger, being estimated in the mid to high .60s. Hence, more than 40% of true individual differences in motivation are explained in terms of such differences in Ability and alternatively in Aspiration.

APPENDIX TO CHAPTER 4

The general confirmatory factor analysis (CFA) model is a special case of the general LISREL model (see Appendix to Chap. 1). This special case is obtained when $B = 0$ is set in the second defining equation, (A1.2), of the general LISREL model (the structural part equation). That is, any CFA model is a LISREL model with vanishing explanatory relationship coefficients for its latent variables.

The particular generic model used for illustration purposes in this chapter, is a special case of the general CFA model and thus also a special case of the general LISREL model. Denoting by Y_1 through Y_8 the observed variables (three Ability, three Motivation, and two Aspiration measures), by ε_1 through ε_8 their pertinent error terms, and by λ 's factor loadings, the initial empirical model we were concerned with in this chapter was

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & \lambda_{73} \\ 0 & 0 & \lambda_{83} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{bmatrix}, \tag{A4.1}$$

with the additional assumption of normality for all latent variables and residual terms, zero mean residuals, and uncorrelated residuals with factors in pertinent equations; in that model, we also assumed that the covariance matrix of the eight error terms, ε_1 through ε_8 , was diagonal, and that the diagonal of the latent variable covariance matrix was fixed at 1.

The true score equivalence versions of this model added consecutively the assumptions

$$\lambda_{11} = \lambda_{21} = \lambda_{31} \tag{A4.2}$$

for the Ability indicators, then

$$\lambda_{42} = \lambda_{52} = \lambda_{62} \tag{A4.3}$$

for the Motivation measures, and finally

$$\lambda_{73} = \lambda_{83} \quad (\text{A4.4})$$

for the Aspiration measures.

Structural Regression Models

WHAT IS A STRUCTURAL REGRESSION MODEL?

In chapter 4, confirmatory factor analysis was presented as a method for examining interrelationships among latent constructs. No specific directional relationships were assumed among the constructs, only that they were correlated with one another. In many scientific fields, however, models that postulate specific explanatory relationships among constructs are often proposed and of interest to examine. To set these models apart from others discussed in this book, we will refer to them in the remainder as structural regression models.

Although structural regression models resemble confirmatory factor analysis models, they possess the noteworthy characteristic that some of their latent variables—that is, elements of the structure of the phenomenon under investigation—are regressed on other constructs. Hence, once the constructs have been assessed, structural regression models can be used to test the plausibility of hypothetical assertions about explanatory relationships among these latent dimensions. In terms of their path diagrams, structural regression models have at least one path (one-way arrow) leaving a putative explanatory latent variable and ending at another construct. Hence, structural regression models can also be viewed as extensions of path analysis models discussed in Chap. 3, with the qualification that instead of being conceived only in terms of observed variables they also include latent variables.

AN EXAMPLE STRUCTURAL REGRESSION MODEL

To demonstrate a structural regression model, consider the following example concerning mental ability. General mental ability is one of the most extensively studied constructs in the behavioral sciences. According to a popular theory (e.g., Horn, 1982), human intellectual capabilities can be roughly classified into two main clusters, fluid and crystallized intelligence. *Fluid intelligence* is the component of general intelligence that reflects an individual's ability to quickly process a potentially large amount of information in order to solve content-free tasks based on contexts that they are not familiar with from their education or prior socialization process. Metaphorically, fluid intelligence can be thought of as resembling a human brain's hardware, or the mechanics of our brain. Fluid intelligence does not include our abilities to retrieve knowledge obtained earlier in life through systems of acculturation or education, but instead refers to our ability to solve unfamiliar problems. *Crystallized intelligence*, on the other hand, is one's ability to retrieve knowledge likely obtained earlier in life through culture and education, and in this sense can be considered the software of our brains. In pragmatic terms, tests of fluid intelligence frequently contain series of context-free symbols arranged following a special rule that must be discovered and subsequently used in order to arrive at a correct solution. Alternatively, measures of crystallized intelligence typically contain items that assess subjects' levels of knowledge in certain areas.

The example structural regression model considered here focuses on two fluid intelligence subabilities, Induction and Figural relations (in a contrived empirical setting). Induction relates to one's capability to reason using analogies and rules of generalization to more comprehensive contexts. Figural relations pertains to our ability to see patterns of relationships between parts of figures, mentally rotate them, and also use forms of inductive reasoning with figural elements. A total of nine measures were collected from a sample of $N = 220$ high school students, with normality being plausible for the data. The following observed variables were used in the study:

1. Induction score 1 obtained in junior year (IND1).
2. Induction score 2 obtained in junior year (IND2).
3. Induction score 3 obtained in junior year (IND3).
4. Figural relations score 1 obtained in junior year (FR11).
5. Figural relations score 2 obtained in junior year (FR12).
6. Figural relations score 3 obtained in junior year (FR13).
7. Figural relations score 1 obtained in senior year (FR21).
8. Figural relations score 2 obtained in senior year (FR22).
9. Figural relations score 3 obtained in senior year (FR23).

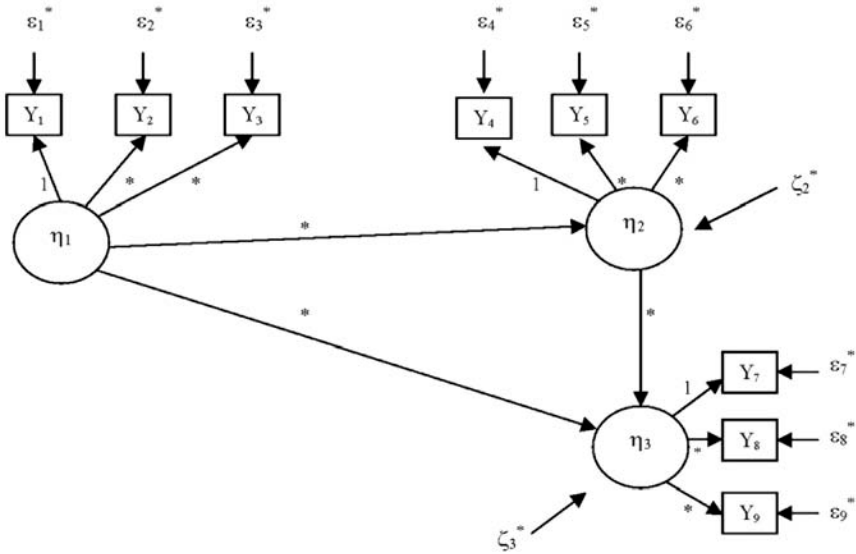


FIG. 11. Example structural regression model using LISREL notation.

The example structural regression model is presented in Fig. 11 and the observed covariance matrix is displayed in Table 2. The model is initially formulated in LISREL notation using Y_1 to Y_9 for the observed variables, ε_1 to ε_9 for the error terms associated with the observed variables, and η_1 to η_3 for the latent variables. The model assumes that Induction in junior year of high school (η_1) plays an explanatory role for Figural relations during both junior (η_2) and senior (η_3) years. In addition, the model posits that a student's junior-year Figural relations affects his or her senior-year Figural relations.

To determine the parameters of the model in Fig. 11, we can follow the six rules outlined in Chap. 1. According to Rule 1, all nine error term variances are model parameters and, according to Rule 3, the nine factor loadings are also model parameters. In addition, the variances of the structural regression disturbance terms ζ_2 and ζ_3 are also model parameters. These disturbances pertain to the latent variables η_2 to η_3 , respectively (i.e., junior- and senior-year Figural relations). Thereby, ζ_2 represents the part of junior-year Figural relations that is not accounted for in terms of its postulated (linear) explanatory relationship to Induction. Similarly, ζ_3 stands for the part of senior-year Figural relations that is not explained in terms of its assumed (linear) relationships to Induction and junior-year Figural relations. Note that Rule 2 is not applicable in this model because it does not

TABLE 2
Covariance matrix for the structural regression model example

<i>Variable</i>	<i>IND1</i>	<i>IND2</i>	<i>IND3</i>	<i>FR11</i>	<i>FR12</i>	<i>FR13</i>	<i>FR21</i>	<i>FR22</i>	<i>FR23</i>
IND1	56.21								
IND2	31.55	75.55							
IND3	23.27	28.30	44.45						
FR11	24.48	32.24	22.56	84.64					
FR12	22.51	29.54	20.61	57.61	78.93				
FR13	22.65	27.56	15.33	53.57	49.27	73.76			
FR21	33.24	46.49	31.44	67.81	54.76	54.58	141.77		
FR22	32.56	40.37	25.58	55.82	52.33	47.74	98.62	117.33	
FR23	30.32	40.44	27.69	54.78	53.44	59.52	96.95	84.87	106.35

Notes. $INDi$ = i th INDuction indicator at first assessment; $FR1i$ = i th Figural Relations indicator at first assessment, and $FR2i$ = i th Figural Relations indicator at second assessment ($i = 1, 2, 3$).

have any latent covariances. Indeed, the relationship between Induction and both Figural relation constructs are explained in terms of their structural regression coefficients, which, following Rule 4, are model parameters. Utilizing the LISREL notation discussed in Chap. 2, these coefficients can be denoted as β_{21} , β_{31} , and β_{32} . These parameters correspondingly relate Induction (η_1) to junior-year Figural relations (η_2) and senior-year Figural relations (η_3), and junior-year Figural relations (η_2) to senior-year Figural relations (η_3). Finally, Rule 6 requires that the scale of each latent variable be fixed. Since this study is focused on determining the explanatory role of the Induction and junior-year Figural relations latent dimensions, it is easier to achieve latent scale fixing by simply setting the loading of the first indicator on each latent variable to 1. Using this approach ensures that the Figural relations construct is assessed in the same metric on both occasions. Hence, as can be directly counted, the structural regression model in Fig. 11 has altogether 21 parameters that are symbolized by asterisks.

EQS, LISREL, AND *Mplus* COMMAND FILES

EQS Command File

The EQS input file includes two new definitions. The first deals with the specification of the equations that relate the latent variables to one another.

Since the model in Fig. 11 includes structural regression relationships, the following two equations must be introduced in the input file: (a) $F2 = *F1 + D2$, and (b) $F3 = *F1 + *F2 + D3$. The second definition concerns the two structural-disturbance terms in the model, i.e., $D2$ and $D3$ in these equations. (These terms denote the latent residuals ζ_2 and ζ_3 .) Given that $D2$ and $D3$ are independent variables, their variances are model parameters as indicated in the `/VARIANCE` section next. Thus, the following EQS command file results:

```

/TITLE
STRUCTURAL REGRESSION MODEL;
/SPECIFICATIONS
CASES=220; VARIABLES=9;
/LABELS
V1=IND1; V2=IND2; V3=IND3; V4=FR11; V5=FR12; V6=FR13;
V7=FR21; V8= FR22; V9=FR23; F1=INDUCTN; F2=FIGREL1;
F3=FIGREL2;
/EQUATIONS
V1= F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V4= F2+E4;
V5=*F2+E5;
V6=*F2+E6;
V7= F3+E7;
V8=*F3+E8;
V9=*F3+E9;
F2=*F1+D2;
F3=*F1+*F2+D3;
/VARIANCES
F1=*; D2 TO D3=*; E1 TO E9=*;
/MATRIX
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
/END;

```

LISREL Input File

The LISREL input file is constructed similarly following the guidelines outlined in Chap. 2. The file begins with a title command line succeeded by information about the number of analyzed variables, sample size and covariance matrix, and observed variable labels. Then, in the model command line, the matrices PS, TE, LY, and BE are defined, and subsequently specific elements of them appropriately declared. Hence, the following LISREL input file results:

```

STRUCTURAL REGRESSION MODEL
DA NI=9 NO=220
CM
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=DI,FR LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
OU

```

As can be seen, the covariance matrix of the Induction construct and the two structural disturbances associated with Figural relations are initially declared to be fixed (in order to fix their covariances), but that construct's variance and disturbance variances are freed in the last line before the OUTPUT command. The structural regression matrix is also declared to be fixed and only those elements of interest according to the model, that is β_{21} , β_{31} , and β_{32} , are defined as free two lines before the output command. This strategy of fixing model matrices first and then freeing only their elements of interest, is oftentimes a more economical way of setting up LISREL command

files. Similarly, the factor loading matrix LY is first declared to be fixed and in the next three lines only those of its elements are defined as free, which pertain to the six factor loadings that are model parameters. Subsequently, the latent variable scales are fixed by setting the pertinent first indicator loading to a value of 1. Finally, by declaring the error covariance matrix TE to be diagonal and free already in the model definition line, all its nine elements along the main diagonal are defined as parameters—they represent the nine error-term variances.

***Mplus* Command File**

The main difference in this file relative to the *Mplus* command file for the confirmatory factor analysis model in Chap. 4, is that here we need to declare explanatory relationships between latent variables, which as mentioned in Chap. 2 is achieved by correspondingly using the keyword “on”.

```
TITLE:          STRUCTURAL REGRESSION MODEL
DATA:          FILE IS EX5.1.COV;
               TYPE=COVARIANCE;
               NOBSERVATIONS=220;
VARIABLE:     NAMES ARE IND1 IND2 IND3 FR11 FR12 FR13 FR21
               FR22 FR23;
MODEL:        F1 BY IND1-IND3;
               F2 BY FR11-FR13;
               F3 BY FR21-FR23;
               F2 ON F1;
               F3 ON F1 F2;
```

We note that the default features of *Mplus* permit one in a very economical way to create the command file and in particular fix latent variable metrics.

MODELING RESULTS

EQS Results

The output produced by the EQS input file created in the previous section is presented next (interested readers can easily verify that they are essentially the same as those obtained with LISREL and *Mplus*, by submitting the above command files). As before, comments are inserted at appropriate places to clarify presented output portions. In addition, the echoed input file, analyzed covariance matrix, and recurring title line as well as method of estimation statement are omitted. (We also dispense with presenting the reliability related output section, since we are not dealing with scale construction in this chapter.)

PARAMETER ESTIMATES APPEAR IN ORDER,
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.

As indicated in previous chapters, this message is a reassurance that the program has not encountered problems stemming from lack of identification or other numerical difficulties, and that the model is technically sound.

RESIDUAL COVARIANCE MATRIX (S-SIGMA) :

	IND1 V1	IND2 V2	IND3 V3	FR11 V4	FR12 V5
IND1 V1	.000				
IND2 V2	-.552	.000			
IND3 V3	.790	-.210	.000		
FR11 V4	-.228	.905	.617	.000	
FR12 V5	-.165	.783	.472	.882	.000
FR13 V6	1.007	.111	-3.891	-.576	-.423
FR21 V7	-2.149	1.609	.011	2.579	-5.106
FR22 V8	1.487	.962	-2.016	-1.457	-.236
FR23 V9	-.889	.859	-.027	-2.747	.644
	FR13 V6	FR21 V7	FR22 V8	FR23 V9	
FR13 V6	.000				
FR21 V7	-2.561	.000			
FR22 V8	-2.433	1.631	.000		
FR23 V9	9.127	-.464	-.665	.000	

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = 1.1393
AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = 1.4242

STANDARDIZED RESIDUAL MATRIX:

	IND1 V1	IND2 V2	IND3 V3	FR11 V4	FR12 V5
IND1 V1	.000				
IND2 V2	-.008	.000			
IND3 V3	.016	-.004	.000		
FR11 V4	-.003	.011	.010	.000	
FR12 V5	-.002	.010	.008	.011	.000
FR13 V6	.016	.001	-.068	-.007	-.006
FR21 V7	-.024	.016	.000	.024	-.048
FR22 V8	.018	.010	-.028	-.015	-.002
FR23 V9	-.012	.010	.000	-.029	.007
	FR13 V6	FR21 V7	FR22 V8	FR23 V9	
FR13 V6	.000				
FR21 V7	-.025	.000			
FR22 V8	-.026	.013	.000		
FR23 V9	.103	-.004	-.006	.000	

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0134
AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0167


```

GOODNESS OF FIT SUMMARY FOR METHOD = ML

INDEPENDENCE MODEL CHI-SQUARE = 1177.363 ON 36 DEGREES OF FREEDOM

INDEPENDENCE AIC = 1105.36278 INDEPENDENCE CAIC = 947.19219
MODEL AIC = 4.09683 MODEL CAIC = -101.35023

CHI-SQUARE = 52.097 BASED ON 24 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .00076

THE NORMAL THEORY RLS CHI-SQUARE FOR THIS ML SOLUTION IS 48.276.

FIT INDICES
-----
BENTLER-BONETT NORMED FIT INDEX = .956
BENTLER-BONETT NON-NORMED FIT INDEX = .963
COMPARATIVE FIT INDEX (CFI) = .975

```

The goodness-of-fit indices do suggest that this is not a well-fitting model. For example, the chi-square value and its p value are unsatisfactory, and the Bentler-Bonett indices as well as the comparative fit index are not impressively high. Based on these results, one may suggest that there is evidence in the data pointing to the model not being a reasonably good means of data representation (at the level of analyzed variances and covariances). One therefore should not fully trust the output results. For this reason we do not present the rest of the output.

Since a considerable positive standardized residual corresponding to a particular part of the model was found, perhaps the model can be improved by making some modifications that relate to that model section. In particular, that residual suggests the model underpredicts the relationship between repeated assessments with the third Figural relation measure. When multiple measurements are conducted using the same set of indicators, it is possible and in fact likely that the residual terms associated with these measures contain some specific variance that is not explained by the latent variables involved. This seems to be the most likely explanation in the present case as well, with regard to the last indicator of Figural relations. Recall our discussion in the previous chapter on model modification, with the recommendation that such modifications should only be made in a model that can be theoretically justified. In the present empirical case, as just mentioned, there appears to be a good substantive reason for the error terms of the Figural relations indicators FR13 and FR23 to correlate, namely as pertaining to the repeated assessment with the same Figural relations measure. Therefore, in the next model version we free this particular error covariance that can be viewed as resulting from assessment method sources of variability specific to this fluid measure. In terms of software use for this purpose, we add the pertinent parameter to the EQS input file¹ with the following command:

¹ In the LISREL command file for this model with correlated errors, declare in the model definition line TE = SY, FI, and add the following line before the last one: FR (TE(1,1) TE(2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6) TE(7,7) TE(8,8) TE(9,9) TE(9,6) (see below in the present section of the chapter). In the *Mplus* command file for the same model, add as a final line the following one: FR13 WITH FR23.

/COVARIANCE

E9,E6=*;

The corresponding parts of the resulting output are presented next, beginning with the reassuring message that the model is technically sound and the underlying numerical optimization routine has been carried out uneventfully.

PARAMETER ESTIMATES APPEAR IN ORDER,
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.

RESIDUAL COVARIANCE MATRIX (S-SIGMA) :

	IND1	IND2	IND3	FR11	FR12
	V1	V2	V3	V4	V5
IND1 V1	.000				
IND2 V2	-.429	.000			
IND3 V3	.764	-.300	.000		
FR11 V4	-.649	.308	.087	.000	
FR12 V5	.199	1.188	.656	-.078	.000
FR13 V6	1.710	.950	-3.397	-.573	1.197
FR21 V7	-2.459	1.126	-.486	2.300	-3.406
FR22 V8	1.332	.686	-2.348	-1.487	1.448
FR23 V9	-.516	1.255	.113	-1.807	3.197
	FR13	FR21	FR22	FR23	
	V6	V7	V8	V9	
FR13 V6	.101				
FR21 V7	-.011	.000			
FR22 V8	-.015	.322	.000		
FR23 V9	.101	-.113	-.038	.007	

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = .8257
 AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = 1.0292

STANDARDIZED RESIDUAL MATRIX:

	IND1	IND2	IND3	FR11	FR12
	V1	V2	V3	V4	V5
IND1 V1	.000				
IND2 V2	-.007	.000			
IND3 V3	.015	-.005	.000		
FR11 V4	-.009	.004	.001	.000	
FR12 V5	.003	.015	.011	-.001	.000
FR13 V6	.027	.013	-.059	-.007	.016
FR21 V7	-.028	.011	-.006	.021	-.032
FR22 V8	.016	.007	-.033	-.015	.015
FR23 V9	-.007	.014	.002	-.019	.035
	FR13	FR21	FR22	FR23	
	V6	V7	V8	V9	
FR13 V6	.001				
FR21 V7	.000	.000			
FR22 V8	.000	.002	.000		
FR23 V9	.001	-.001	.000	.000	

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0102
 AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0127

LARGEST STANDARDIZED RESIDUALS:

NO.	PARAMETER	ESTIMATE	NO.	PARAMETER	ESTIMATE
1	V6, V3	-.059	11	V5, V2	.015
2	V9, V5	.035	12	V3, V1	.015
3	V8, V3	-.033	13	V8, V5	.015
4	V7, V5	-.032	14	V8, V4	-.015
5	V7, V1	-.028	15	V9, V2	.014
6	V6, V1	.027	16	V6, V2	.013
7	V7, V4	.021	17	V5, V3	.011
8	V9, V4	-.019	18	V7, V2	.011
9	V8, V1	.016	19	V4, V1	-.009
10	V6, V5	.016	20	V8, V2	.007

DISTRIBUTION OF STANDARDIZED RESIDUALS

					RANGE	FREQ	PERCENT
!			!				
40-			-				
!			!				
!			!				
!			!				
!			!				
30-			-				
!		*	!	1	-0.5 -	0	.00%
!		*	!	2	-0.4 -	0	.00%
!		*	!	3	-0.3 -	0	.00%
!		*	!	4	-0.2 -	0	.00%
20-		*	-	5	-0.1 -	0	.00%
!		* *	!	6	0.0 -	18	40.00%
!		* *	!	7	0.1 -	27	60.00%
!		* *	!	8	0.2 -	0	.00%
!		* *	!	9	0.3 -	0	.00%
10-		* *	-	A	0.4 -	0	.00%
!		* *	!	B	0.5 -	0	.00%
!		* *	!	C	++ -	0	.00%
!		* *	!				
!		* *	!				
					TOTAL	45	100.00%

1 2 3 4 5 6 7 8 9 A B C EACH "*" REPRESENTS 2 RESIDUALS

There are no ‘gaps’ in the magnitude of standardized residuals now, unlike the preceding fitted model, and none of these residuals is a cause for concern—as indicated previously, this is typically the case with well-fitting models. Specifically, the residuals listed in the LARGEST STANDARDIZED RESIDUALS section are much more uniform and smaller in magnitude than in the previous model version, and the distribution of standardized residuals is more symmetric. The overall fit indices further contribute to the impression of a well-fitting model and are displayed next.

GOODNESS OF FIT SUMMARY FOR METHOD = ML

INDEPENDENCE MODEL CHI-SQUARE = 1177.363 ON 36 DEGREES OF FREEDOM
 INDEPENDENCE AIC = 1105.36278 INDEPENDENCE CAIC = 947.19219
 MODEL AIC = -25.44764 MODEL CAIC = -126.50107

CHI-SQUARE = 20.552 BASED ON 23 DEGREES OF FREEDOM
 PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .60840

THE NORMAL THEORY RLS CHI-SQUARE FOR THIS ML SOLUTION IS 20.011.

FIT INDICES

 BENTLER-BONETT NORMED FIT INDEX = .983
 BENTLER-BONETT NON-NORMED FIT INDEX = 1.003
 COMPARATIVE FIT INDEX (CFI) = 1.000

These goodness-of-fit indices are quite satisfactory. Note in particular that the chi-square value is comparable to its degrees of freedom and the associated *p* value is well in excess of any reasonable significance level. In addition, the Bentler-Bonett normed and nonnormed fit indices, as well as the comparative fit index are all close to 1—the NNFI is actually slightly above 1, which sometimes happens with well-fitting models. We also notice that the information criteria values (AIC and CAIC) are considerably lower than in the preceding model, another indication of a better fit with the present one. We conclude, therefore, that this model is a reasonably good means of data description.

MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

IND1 =V1 = 1.000 F1 + 1.000 E1
 IND2 =V2 = 1.271*F1 + 1.000 E2
 .157
 8.073@
 IND3 =V3 = .894*F1 + 1.000 E3
 .116
 7.713@
 FR11 =V4 = 1.000 F2 + 1.000 E4
 FR12 =V5 = .888*F2 + 1.000 E5
 .064
 13.885@
 FR13 =V6 = .833*F2 + 1.000 E6
 .062
 13.465@
 FR21 =V7 = 1.000 F3 + 1.000 E7
 FR22 =V8 = .875*F3 + 1.000 E8
 .051
 17.202@

$$\begin{aligned} \text{FR23} = \text{V9} = & \quad .864 * \text{F3} \quad + 1.000 \text{ E9} \\ & \quad .047 \\ & \quad 18.391\text{e} \end{aligned}$$

CONSTRUCT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

$$\begin{aligned} \text{FIGREL1} = \text{F2} = & \quad .999 * \text{F1} \quad + 1.000 \text{ D2} \\ & \quad .150 \\ & \quad 6.676\text{e} \end{aligned}$$

$$\begin{aligned} \text{FIGREL2} = \text{F3} = & \quad .749 * \text{F2} \quad + \quad .671 * \text{F1} \quad + 1.000 \text{ D3} \\ & \quad .105 \quad \quad \quad .180 \\ & \quad 7.103\text{e} \quad \quad \quad 3.736\text{e} \end{aligned}$$

We note that all structural regression coefficients are significant—their t values are well outside the nonsignificance region (-2 ; $+2$). These results indicate that there are marked relationships between the Induction and Figural relations latent dimensions in junior and senior years. That is, the Induction construct (F_1) has marked explanatory power for individual differences in Figural relations at both assessment points (i.e., F_2 and F_3) because its regression coefficients, .999 and .671, are significant. Furthermore, these significance results indicate that in the presence of the Induction latent variable, Figural relations in junior year is also important for predicting individual differences in senior-year figural ability, as one might expect when measuring a latent construct at two consecutive assessments. Conversely, adding Induction in the equation for Figural relations in senior year explains significantly more variance in the latter construct than when Induction is not included in this equation.

VARIANCES OF INDEPENDENT VARIABLES

STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D
	---		---
E1 - IND1	31.045*I	D2 -FIGREL1	39.881*I
	3.880 I		6.264 I
	8.002@I		6.366@I
	I		I
E2 - IND2	34.912*I	D3 -FIGREL2	39.365*I
	5.052 I		6.050 I
	6.911@I		6.506@I
	I		I
E3 - IND3	24.322*I		I
	3.060 I		I
	7.948@I		I
	I		I
E4 - FR11	19.667*I		I
	3.352 I		I
	5.868@I		I
	I		I

E5 - FR12	27.709*I	I
	3.533 I	I
	7.842@I	I
	I	I
E6 - FR13	28.541*I	I
	3.463 I	I
	8.242@I	I
	I	I
E7 - FR21	29.401*I	I
	4.279 I	I
	6.871@I	I
	I	I
E8 - FR22	31.342*I	I
	3.952 I	I
	7.930@I	I
	I	I
E9 - FR23	22.501*I	I
	3.280 I	I
	6.860@I	I
	I	I

COVARIANCES AMONG INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	--		--	
E9 - FR23		12.264*I		I
E6 - FR13		2.460 I		I
		4.985@I		I
		I		

STANDARDIZED SOLUTION:		R-SQUARED
IND1 =V1	= .669 F1 + .743 E1	.448
IND2 =V2	= .733*F1 + .680 E2	.538
IND3 =V3	= .673*F1 + .740 E3	.453
FR11 =V4	= .876 F2 + .482 E4	.768
FR12 =V5	= .806*F2 + .593 E5	.649
FR13 =V6	= .783*F2 + .622 E6	.613
FR21 =V7	= .890 F3 + .455 E7	.793
FR22 =V8	= .856*F3 + .517 E8	.733
FR23 =V9	= .888*F3 + .460 E9	.788
FIGREL1 =F2	= .621*F1 + .783 D2	.386
FIGREL2 =F3	= .569*F2 + .317*F1 + .592 D3	.650

This output part presents the variance of the Induction latent variable and the variances of the disturbance terms (D_2 and D_3) for the Figural relations constructs. The covariance between error terms of the fluid measures FR13 and FR23 are also reported in the section entitled COVARIANCES AMONG INDEPENDENT VARIABLES and, as could be anticipated, this value is found to be significant (see last estimate before the standardized solution above).

CORRELATIONS AMONG INDEPENDENT VARIABLES			
	E		D
E9 - FR23	---	.484*I	I
E6 - FR13		I	I
		I	I
E N D O F M E T H O D			

The error term relationship of the Figural relations measures (FR13 and FR23) is given here in the form of a correlation coefficient. As indicated earlier, despite the fact that this correlation is only moderately strong in magnitude, it contributes to a substantial improvement in fit. This improvement can be easily judged by comparing the chi-square values for the two models fitted in this section—the model with the correlated error terms versus the model without their correlation. The difference in chi-square values of the two proposed models is found to be $\Delta T = 52.097 - 20.552 = 31.545$, with the difference in degrees of freedom being $\Delta df = 24 - 23 = 1$, and is therefore significant (as mentioned before, the cut-off value of the chi-square distribution with 1 *df* is 3.84 at significance level .05). Hence, including the correlated error term leads to a significant improvement in model fit.

FACTORIAL INVARIANCE ACROSS TIME IN REPEATED MEASURE STUDIES

An important question in studies that involve repeated measurements of latent variables concerns indicator invariance over time and pertains to the basic query about comparability of construct assessment across time. Imposing the condition of invariance deals with the requirement that the same construct be measured at all occasions, and in particular that its structure remains the same. Since a necessary condition for measuring the same constructs at repeated assessments is the identity of the associated factorial structure, the term factorial invariance is commonly used. Examination of factorial invariance in a given model is conducted by imposing an equality constraint on the factor loadings of the same indicators over time and testing the resulting difference in the chi-square values of the two involved nested models.²

A test of factorial invariance in our example concerning mental ability deals with examining whether the construct Figural relations is character-

²A stronger test of measurement invariance over time includes also the identity across assessment occasions of corresponding mean intercepts, which is not pursued in this introductory text (for details, see, e.g., Raykov, 2004).

ized by the same factor loadings on its measures in junior and senior high school years. This test is accomplished by introducing the loading equality restriction on these indicators across both assessments. The equality restraint can be included in the EQS input file by adding a /CONSTRAINT section; in the LISREL command file by adding an EQuality line; and in the *Mplus* input file by assigning the same integer number to the loadings of the same indicator over time. More specifically, this is accomplished with the following constraint command within the last EQS input file, leaving unchanged the rest:

```
/CONSTRAINTS
(V5,F2)=(V8,F3);
(V6,F2)=(V9,F3);
```

That is, the EQS command file for testing factorial invariance in the present empirical setting looks as follows (using three-letter abbreviations):

```
/TIT
TESTING FOR FACTORIAL INVARIANCE IN A STRUCTURAL
REGRESSION MODEL (WITH CORRELATED MEASUREMENT
ERRORS);
/SPE
CAS=220; VAR=9;
/LAB
V1=IND1; V2=IND2; V3=IND3; V4=FR11; V5=FR12; V6=FR13;
V7=FR21; V8= FR22; V9=FR23;
F1=INDUCTN; F2=FIGREL1; F3=FIGREL2;
/EQU
V1= F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V4= F2+E4;
V5=*F2+E5;
V6=*F2+E6;
V7= F3+E7;
V8=*F3+E8;
V9=*F3+E9;
F2=*F1+D2;
F3=*F1+*F2+D3;
/VAR
F1=*; D2 TO D3=*; E1 TO E9=*;
/COV
E6,E9=*;
```

```

/CON
(V5,F2)=(V8,F3);
(V6,F2)=(V9,F3);
/MAT
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
/END;

```

To accomplish the same goal in the LISREL input file for the last fitted model, we add the following equality restrictions:

```

EQ LY(5, 2) LY(8, 3)
EQ LY(6, 2) LY(9, 3)

```

Hence, the LISREL command file for testing factorial invariance in the present empirical context looks as follows (see also Note 1 to this chapter):

```

TESTING FACTORIAL INVARIANCE IN A STRUCTURAL REGRESSION
MODEL (WITH CORRELATED MEASUREMENT ERRORS)
DA NI=9 NO=220
CM
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=SY,FI LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)

```

```

FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
FR TE(1, 1) TE(2, 2) TE(3, 3) TE(4, 4) TE(5, 5) TE(6, 6) TE(7, 7)
FR TE(8, 8) TE(9, 9) TE(9, 6)
EQ LY(5, 2) LY(8, 3)
EQ LY(6, 2) LY(9, 3)
OU RS

```

In the *Mplus* command file for this model, we correspondingly define the measurement of the 2nd and 3rd constructs (Figural relations at the 1st and at 2nd occasions, respectively) as follows:

```

F2 BY FR11
FR12(1)
FR13(2);
F3 BY FR21
FR22(1)
FR23(2);

```

(Note that a semicolon comes only after all indicators are listed, for each construct in question.) That is, the *Mplus* command file with the factorial invariance constraint looks in the current empirical setting as follows (see also Note 1 to this chapter):

```

TITLE: THIS IS AN EXAMPLE STRUCTURAL EQUATION MODEL
WITH THE FACTORIAL INVARIANCE CONSTRAINT IMPOSED
DATA: FILE IS EX5.COV;
TYPE=COVARIANCE;
NOBSERVATIONS=220;
VARIABLE: NAMES ARE IND1 IND2 IND3 FR11 FR12 FR13
FR21 FR22 FR23;
MODEL: F1 BY IND1-IND3;
F2 BY FR11
FR12(1)
FR13(2);
F3 BY FR21
FR22(1)
FR23(2);
F3 ON F1 F2;
F2 ON F1;
FR13 WITH FR23;

```


The result of introducing the factorial invariance restriction is a chi-square value of $T = 20.937$, with $df = 25$ (recall that the model tested without the equality restriction resulted in a chi-square value of $T = 20.552$, with $df = 23$). The difference in chi-square values between the restricted model and the previous one is $\Delta T = 20.937 - 20.552 = 0.385$, with $\Delta df = 2$, and is non-significant (recall that the cut-off value of the chi-square distribution, at significance level .05, is 5.99). This indicates that the time-invariance restriction imposed on the loadings of the Figural relations indicators is plausible. Since this restriction is found to be acceptable, it is retained in the model.

We conclude this chapter by discussing the LISREL output file for the last fitted, restricted model, starting with the sections after the echoed back input, data, and model related details. (To request model residuals, given that this will be the only software output we will discuss in the rest of this section, we add RS to the final OUTput line; see Note 2 to Chap. 2.)

Parameter Specifications

LAMBDA-Y	INDUCTN	FIGREL1	FIGREL2
	---	---	---
IND1	0	0	0
IND2	1	0	0
IND3	2	0	0
FR11	0	0	0
FR12	0	3	0
FR13	0	4	0
FR21	0	0	0
FR22	0	0	3
FR23	0	0	4

As seen from this portion, all parameters constrained to be equal to one another are given the same number, whereas parameters that are fixed are not numbered. Accordingly, LISREL interpreted that this is a model with only four free factor loading parameters to be estimated. The remaining 16 parameters come next.

BETA	INDUCTN	FIGREL1	FIGREL2
	---	---	---
INDUCTN	0	0	0
FIGREL1	5	0	0
FIGREL2	6	7	0

PSI	INDUCTN	FIGREL1	FIGREL2
	---	---	---
	8	9	10

THETA-EPS

	IND1	IND2	IND3	FR11	FR12	FR13
	---	---	---	---	---	---
IND1	11					
IND2	0	12				
IND3	0	0	13			
FR11	0	0	0	14		
FR12	0	0	0	0	15	
FR13	0	0	0	0	0	16
FR21	0	0	0	0	0	0
FR22	0	0	0	0	0	0
FR23	0	0	0	0	0	19

THETA-EPS

	FR21	FR22	FR23
	---	---	---
FR21	17		
FR22	0	18	
FR23	0	0	20

Recall that one had to declare the error variance matrix TE as symmetric, in order to accommodate as a model parameter the error covariance pertaining to both assessments with the third Figural relations indicator (see Note 1 to this chapter).

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y

	INDUCTN	FIGREL1	FIGREL2
	---	---	---
IND1	1.000	--	--
IND2	1.271 (0.157) 8.073	--	--
IND3	0.894 (0.116) 7.712	--	--
FR11	--	1.000	--
FR12	--	0.879 (0.040) 22.130	--
FR13	--	0.855 (0.041) 20.678	--
FR21	--	--	1.000
FR22	--	--	0.879 (0.040) 22.130

FR23	--	--	0.855 (0.041) 20.678
------	----	----	----------------------------

In this factor loading matrix, notice in particular the restrained loadings that have identical estimates, standard errors, and *t* values.

BETA	INDUCTN	FIGREL1	FIGREL2
	-----	-----	-----
INDUCTN	--	--	--
FIGREL1	0.992 (0.146) 6.776	--	--
FIGREL2	0.674 (0.180) 3.747	0.758 (0.100) 7.617	--

Although the model parameter estimates have changed slightly due to the imposed restrictions, the same substantive conclusions are warranted with respect to the structural regression coefficients as in the earlier fitted model version without the factorial invariance constraint.

Covariance Matrix of ETA

	INDUCTN	FIGREL1	FIGREL2
	-----	-----	-----
INDUCTN	25.171		
FIGREL1	24.965	64.179	
FIGREL2	35.891	65.484	113.456

PSI

Note: This matrix is diagonal.

	INDUCTN	FIGREL1	FIGREL2
	-----	-----	-----
	25.179 (5.130) 4.908	39.426 (5.806) 6.790	39.625 (5.927) 6.686

In the LISREL output discussed in Chap. 4, for the confirmatory factor analysis model of interest there, the values in the COVARIANCE MATRIX OF ETA and those in the PSI matrix were identical. The results here, however, present quite different values for the diagonal of the COVARIANCE MATRIX OF ETA and the PSI matrices. The reason for this difference is that in this model explanatory relationships are assumed between the latent variables. As a result, the PSI matrix now contains the variance of the first latent variable, Induction (INDUCTN), and the residual variances of the other two latent

variables, Figural relations in junior and senior high school years (FIGREL1 and FIGREL2). Hence, only the first element on the diagonal of PSI is identical to the corresponding element of the COVARIANCE MATRIX OF ETA from that earlier model fitted in Chap. 4. This is because the variance of the first latent factor (Induction) is the common predictor for the other two constructs (Figural relations in junior and senior years) and is not regressed on any construct in either of the two models in question. We also note that the structural residual variances are both significant in the last fitted model (see second and third diagonal elements of the matrix PSI), which reflects the fact that the latent relationships assumed in this model are not capable of explaining all individual variability in junior- and senior-year Figural relations, the dependent variables in the structural part of this model.

Squared Multiple Correlations for Structural Equations

	INDUCTN	FIGREL1	FIGREL2			
	---	---	---			
	--	0.386	0.651			
THETA-EPS						
	IND1	IND2	IND3	FR11	FR12	FR13
	---	---	---	---	---	---
IND1	31.031 (3.879) 7.999					
IND2	--	34.901 (5.052) 6.908				
IND3	--	--	24.339 (3.061) 7.952			
FR11	--	--	--	19.854 (3.170) 6.262		
FR12	--	--	--	--	27.999 (3.433) 8.157	
FR13	--	--	--	--	--	28.191 (3.411) 8.266
FR21	--	--	--	--	--	--
FR22	--	--	--	--	--	--
FR23	--	--	--	--	--	12.225 (2.461) 4.967
THETA-EPS						
	FR21	FR22	FR23			
	---	---	---			
FR21	29.207 (4.210) 6.937					

FR22	- -	31.142 (3.921) 7.943				
FR23	- -	- -	22.742 (3.250) 6.998			
Squared Multiple Correlations for Y - Variables						
	IND1	IND2	IND3	FR11	FR12	FR13
	---	---	---	---	---	---
	0.448	0.538	0.452	0.764	0.639	0.625
Squared Multiple Correlations for Y - Variables						
	FR21	FR22	FR23			
	---	---	---			
	0.795	0.738	0.785			

Based on this output part dealing with squared multiple correlation coefficients, the model is less successful in predicting junior-year Figural relations than senior year Figural relations, and an obvious reason would be that the latter is explained here by two rather than a single construct, unlike the former latent variable. Regarding the explained variance in junior year Figural relations, one may suggest that a follow-up study could also consider other variables beyond Induction that may contribute to a better explanation of individual differences in Figural relations ability in junior high school year.

Goodness of Fit Statistics
Degrees of Freedom = 25

Minimum Fit Function Chi-Square = 20.937 (P = 0.696)
Normal Theory Weighted Least Squares Chi-Square = 20.310 (P = 0.730)
Estimated Non-centrality Parameter (NCP) = 0.0
90 Percent Confidence Interval for NCP = (0.0 ; 9.101)
Minimum Fit Function Value = 0.0956
Population Discrepancy Function Value (F0) = 0.0
90 Percent Confidence Interval for F0 = (0.0 ; 0.0416)
Root Mean Square Error of Approximation (RMSEA) = 0.0
90 Percent Confidence Interval for RMSEA = (0.0 ; 0.0408)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.981
Expected Cross-Validation Index (ECVI) = 0.297
90 Percent Confidence Interval for ECVI = (0.297 ; 0.338)
ECVI for Saturated Model = 0.411
ECVI for Independence Model = 5.458
Chi-Square for Independence Model with 36 Degrees of Freedom = 1177.363
Independence AIC = 1195.363
Model AIC = 60.310
Saturated AIC = 90.000
Independence CAIC = 1234.905
Model CAIC = 148.182
Saturated CAIC = 287.713
Normed Fit Index (NFI) = 0.982

Non-Normed Fit Index (NNFI) = 1.005
 Parsimony Normed Fit Index (PNFI) = 0.682
 Comparative Fit Index (CFI) = 1.000
 Incremental Fit Index (IFI) = 1.004
 Relative Fit Index (RFI) = 0.974
 Critical N (CN) = 464.534
 Root Mean Square Residual (RMR) = 1.462
 Standardized RMR = 0.0182
 Goodness of Fit Index (GFI) = 0.980
 Adjusted Goodness of Fit Index (AGFI) = 0.964
 Parsimony Goodness of Fit Index (PGFI) = 0.544

All goodness-of-fit indices point to the conclusion that the model represents a reasonable approximation to the data. In particular, note the low magnitude of both point estimate and left endpoint of the 90%-confidence interval of the RMSEA index, which are well below the widely followed threshold of .05.

Standardized Residuals

	IND1	IND2	IND3	FR11	FR12	FR13
IND1	- - -	- - -	- - -	- - -	- - -	- - -
IND2	-0.379	- -	- - -	- - -	- - -	- - -
IND3	0.663	-0.285	- -	- - -	- - -	- - -
FR11	-0.197	0.206	0.114	0.245	- - -	- - -
FR12	0.203	0.555	0.406	0.550	0.426	- - -
FR13	0.475	0.145	-1.556	-0.740	0.435	-0.526
FR21	-0.901	0.295	-0.244	1.023	-0.695	-0.373
FR22	0.345	0.092	-1.011	-0.518	0.599	-0.416
FR23	-0.152	0.560	0.113	-0.486	1.344	-0.296

Standardized Residuals

	FR21	FR22	FR23
FR21	-0.245	- - -	- - -
FR22	-0.448	-0.426	- - -
FR23	-0.054	-0.215	0.396

Summary Statistics for Standardized Residuals

Smallest Standardized Residual	=	-1.556
Median Standardized Residual	=	0.000
Largest Standardized Residual	=	1.344

Stemleaf Plot

```

- 1|6
- 1|0
- 0|977555
- 0|444443322221000
  0|1111222334444
  0|566667
  1|03
    
```

None of the standardized residuals is very large, which is another indication that the model also locally fits well, that is, beyond being a good overall means of data description as indicated by its goodness of fit indices mentioned earlier. One can therefore consider the last fitted model with factorial invariance to be an acceptable means of data description.

APPENDIX TO CHAPTER 5

The general structural regression (SR) model is subsumed under the general LISREL model (see Appendix to Chap. 1). Hence, all modeling developments presented in Chap. 1 and its Appendix apply to the general SR model (and all its special cases in relevance in a given empirical setting).

The particular generic model used for illustration purposes in the present chapter is a special case of the general SR model, and therefore of the general LISREL model. Indeed, denoting by Y_1 through Y_9 , the observed variables (three Induction test scores and altogether six Figural relations test scores), by ε_1 through ε_9 , their pertinent error terms, by β_{21} , β_{31} and β_{32} the structural regression coefficients of interest in this setting, and by λ 's pertinent factor loadings, the initial empirical model we dealt with in this chapter consists of the following measurement part

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & \lambda_{62} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{83} \\ 0 & 0 & \lambda_{93} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \\ \varepsilon_9 \end{bmatrix} \tag{A5.1}$$

and the following structural part

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \beta_{21} & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}, \tag{A5.2}$$

with the additional assumption of normality for all latent variables and residual terms, zero mean residuals, and uncorrelated residuals with predictors in pertinent equations as well as across the measurement and structural parts; in that model, we also assumed that the covariance matrix of the nine error terms, ε_1 through ε_9 , was diagonal.

In the second fitted model with correlated errors pertaining to the last (third) Figural relations indicator, we added the assumption

$$\text{Cov}(\varepsilon_6, \varepsilon_9) \neq 0. \quad (\text{A5.3})$$

In the last fitted model in this chapter, with the added restriction of invariance over time of the factorial structure for the Figural relations construct, we also assumed

$$\lambda_{52} = \lambda_{83} \quad \text{and} \quad (\text{A5.4})$$

$$\lambda_{62} = \lambda_{93} . \quad (\text{A5.5})$$

Latent Change Analysis

WHAT IS LATENT CHANGE ANALYSIS?

Repeated measurements are used quite often in research studies to examine temporal change processes in individuals or groups. One reason for their popularity in the social and behavioral disciplines is that repeated measurements allow researchers to investigate individual (intraindividual) development across time as well as between-individual (interindividual) differences and similarities in change patterns. For example, educational researchers may be interested in examining patterns of growth in ability exhibited by a group of subjects in longitudinally administered measures following some treatment program and relating them to various characteristics of students and environments they live in. The researchers may also be interested in comparing the rates of change in these variables across several student populations, or in studying the correlates and predictors of growth in ability over time in an effort to determine which students exhibited the fastest improvement in ability. Alternatively, developmental scientists may be interested in studying cognitive functioning decline in later life, comparing its pattern across gender, socio-economic, or education related subpopulations of elderly, as well as finding out the antecedents, correlates, and predictors of this decline.

A powerful methodology for addressing these type of questions exists within the traditional analysis of variance (ANOVA) and analysis of covariance (ANCOVA) frameworks. Some of the assumptions needed to use this methodology, however, can often be untenable, especially in social and behavioral research involving multiple assessments. Specifically, assumptions concerning the homogeneity of the variance-covariance matrix across levels

of the between-subjects factors, and particularly the assumption of sphericity (implying the same patterns of correlation for repeated assessments) can be problematic (e.g., Marcoulides & Hershberger, 1997; Raykov, 2001; Tabachnick & Fidell, 2001). Similarly, when studying correlates and predictors of growth or decline via ANCOVA, assumptions concerning the use of perfectly measured covariate(s) and regression homogeneity may also be questionable and their violation may result in misleading substantive conclusions (e.g., Huitema, 1980).

The structural equation modeling methodology offers a highly useful and readily applicable alternative framework allowing one to address comprehensively issues pertaining to studying change over time. This framework has been available and used for a number of years, and has become known under different though related names. Throughout this chapter, we will refer to this framework as latent change analysis (LCA), since its focus is the study of change over time in latent dimensions. LCA has been primarily formalized and popularized in the social and behavioral sciences over the past couple of decades as a general means for studying growth or decline at the unobserved variable level as well as their correlates and predictors. In several aspects, the traditional repeated measures analysis of variance models can be considered special cases of a more general LCA model (Meredith and Tisak, 1990) that provides in some ways more flexibility in the study of change than those traditional models. The underlying LCA model also resembles the confirmatory factor analysis model discussed in Chap. 4 with one qualification. Since the LCA model is based on data obtained from repeated measurements, the latent variables are generally interpreted as chronometric variables representing individual differences over time; their specific interpretation depends on the particular model and is developed within the extended conceptualization of latent variable that we discussed in Chap. 1.

The term latent change analysis (LCA) is used in this book as a general category for a large number of possible models that can be used in repeated-assessment studies. Many of these models have been referred to in the literature under alternative names, such as latent growth curve models, latent curve analysis models, or just growth curve models. We prefer the reference latent change analysis models, to emphasize that the models are equally applicable to all cases in which one is interested in studying growth or decline along unobservable dimensions, even those with a more complex pattern of change such as growth followed by decline or vice versa. Due to the introductory nature of this book, only two LCA models are discussed in this chapter, which however are widely applicable as demonstrated later: (a) the one-factor model; and (b) the level-and-shape (LS) model. For more extensive and advanced treatments of the subject, several excellent sources are available (e.g., Bollen & Curran, 2006; Duncan,

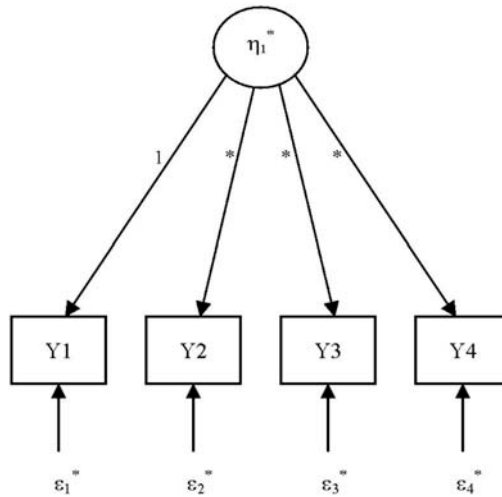


FIG. 12. Example one-factor LCA model.

Duncan, Strycker, Li, and Alpert, 1999; McArdle, 1998; Meredith & Tisak, 1990; Singer & Willett, 2003; Willett & Sayer, 1996).

ONE-FACTOR LATENT CHANGE ANALYSIS MODEL

To introduce the reader to latent change modeling, the discussion of LCA begins with a simple one-factor model. In this model, to prepare the ground for the following empirical example it is assumed that a set of $k = 4$ repeated measurements of cognitive ability give rise to a covariance matrix and means that can be explained in terms of a single latent variable (such a model is provided in Fig. 12). However, we stress that the following developments are immediately generalizable to any larger number, $k \geq 5$, of longitudinal assessments, and are equally well applicable even with 3 measurement points. This one-factor LCA model has been discussed in the literature on several earlier occasions, and McArdle (1988) has termed it a curve model because the latent variable can be interpreted as a time factor that governs the intraindividual latent change processes or curves (see also McArdle & Epstein, 1987). Alternatively, the time factor can be thought of as initial true status of the underlying ability that is being repeatedly measured. As indicated later, the loadings of the repeated measures on this factor can be interpreted as rates of mean change in the studied ability. Meredith and Tisak (1990) chose to term this model a monotonic stability model because, although significant changes in mean levels may occur in

the studied ability, the rank order of the observations tends to stay the same over repeated assessments (Duncan et al., 1999).

Analysis of Mean Structures

One important aspect in which LCA models differ from all preceding ones discussed in this book, is that the variable means and their development over time are taken into account in addition to variable variances and covariances. This is achieved by what is referred to as mean structure analysis (MSA). A MSA includes in the analysis not only the covariance matrix of the repeated measures, but also the manifest variable means. In order to accomplish this, a model must be fit to the so-called covariance/mean matrix, rather than only to the covariance matrix that has been used for modeling purposes up to this point (the slash in 'covariance/mean matrix' is meant to indicate extension rather than division). The covariance/mean matrix results after the observed variable means are added as a last row and column to the covariance matrix. That is, the covariance/mean matrix is the covariance matrix augmented by the manifest variable means as a last added row and column.

Why Is It Necessary to Include Variable Means into an Analysis of Change?

The inclusion of observed variable means and their development over time into a change analysis complies with the conceptual basis of the classical approaches to studying change. Accordingly, temporal development in the means of observed variables under investigation is of special importance. Indeed, one cannot imagine a repeated measures ANOVA, for example, which would exclude the information about change over time that is contained in the means. Because the use of LCA has the same goal of studying development, it is only natural to include variable means in the analysis. If the model were fitted only to the covariance matrix, however, one would omit the observed variable means and their dynamic from the analysis. The reason is that any covariance coefficient, and similarly any correlation coefficient, is based on the sum of cross-products around the means (e.g., Hays, 1994). That is, any covariance (as well as correlation) disregards the means of the two involved variables, and as a result the observed means and their development over time are inconsequential for the covariance matrix. Hence, different patterns of change over time (e.g., increase over time; decline; or growth followed by decline, or vice versa) can be equally consistent with a given covariance matrix resulting in a repeated measure context. Therefore, to fit a model only to the covariance matrix is tantamount to being wasteful of information about the development of the studied phenom-

enon over time, which information is contained in the observed variable means and their dynamic. Hence, in an analysis of change the observed means are to be included since without them one cannot achieve the goal of examining growth or decline over time.

As a result of this inclusion of variable means, there are more data points to which the model is fit. That is, in addition to the elements of the covariance matrix, there are also as many means to count as data points as there are observed variables. For example, with $k = 4$ repeated measurements assessments, there are $k(k + 1)/2 = 4(5)/2 = 10$ nonredundant elements of the covariance matrix plus $k = 4$ observed means that the model must also 'emulate'. As a result, in a mean structure analysis of $k = 4$ longitudinally administered measures there are altogether $q = k(k + 1)/2 + k = 4(5)/2 + 4 = 14$ pieces of empirical information to which the model is fitted. From this number q one needs to subtract the number of model parameters in order to obtain the model degrees of freedom. As shown later, conducting a MSA makes necessary the introduction of additional model parameters concerning the structure of the variable means and in fact may be unique to this structure; that is, these mean structure specific parameters may not have implications for the variable variances and covariances. This issue will be discussed further after details concerning LCA models are presented.

How Is a Model Fit to the Covariance/Mean Matrix?

SEM accomplishes the inclusion of observed variable means in the analysis—that is, achieves fitting a model to the sample covariance/mean matrix—by extending the fit function with a special term (see discussion of fit function in Chap. 1 and its Appendix). Hence, when fitting models to the covariance/mean matrix, one uses an extended fit function relative to the case when the model is fitted only to the covariance matrix (covariance structure) of a given model. This special term that extends the original fit function represents a weighted sum of the squared differences between the observed means and those reproduced by the model, with the weights being the corresponding elements of the inverse of the model-reproduced covariance matrix Σ (for the maximum likelihood method used throughout this book). Just as the model has certain implications for the variances and covariances of the analyzed variables, it also has consequences for the variable means. These consequences can easily be worked out by noting that the mean of any linear combination of variables, whether observed or latent or of both types, is simply the same linear combination of their means. In order to understand and conceptualize the LCA model implications and the reproduced covariance/mean matrix, as relevant for this introductory text, a fifth law is now added to those presented earlier in Chap. 1.

Law 5 of Variable Means

The relationship between means of a linear combination of variables and means of its constituents is given in the form of a specific law as follows, which for the purposes of this chapter is to be considered added to the four laws of variable variances and covariances discussed in Chap. 1:

Law 5. For any two random variables X and Y and any given constants a and b ,

$$M(aX + bY) = aM(X) + bM(Y),$$

where $M(\cdot)$ denotes mean.

The validity of this law follows directly from the additive property of the mean (e.g., Hays, 1994). Law 5 can obviously be generalized to any number of variables—the mean of a linear combination of variables, regardless of their number (and whether observed, latent or mixed), is identical to that same combination of their means.

Once advised to perform a mean structure analysis, SEM programs will automatically extend the fit function by the special term mentioned above, which evaluates model fit with regard to means. A minimization of this fit function therefore implies that when fitting a model to the mean structure one is seeking estimates of its parameters that render the reproduced covariance/mean matrix as close as possible to the observed covariance/mean matrix. Hence, at the final computed solution, the observed variances, covariances, and means are emulated as well as possible by the fitted model.

The One-Factor Latent Change Analysis Model

Returning to the one-factor model presented in Fig. 12, we note that the model path diagram is presented in LISREL notation and includes the four observed variables (Y 's), each indexed according to its measurement occasion. The model also proposes that each observed variable Y loads on a single latent construct η_1 , which represents the time factor underlying their change as indicated previously, and involves $k = 4$ successive measurements.

To determine the parameters of the model presented in Fig. 12, designated by asterisks there, we begin by following the six rules outlined in Chap. 1. According to Rule 1, all error-term variances and the variance of the factor η_1 are parameters. Following Rule 3, all factor loadings are also parameters. Rule 2 is not applicable to this model because there are no variable covariances—there are no two-way arrows in Fig. 12. Rule 6 requires that the scale of the latent variable be fixed, which can be accomplished by

fixing either the variance of η_1 or one of the factor loadings to a value of 1 (or another constant). Here, we set the scale of the latent variable by fixing the first measurement loading to a value of 1, that is, by setting the loading of Y_1 equal to 1. Using this approach effectively sets the metric of the first measurement as a baseline against which the subsequent change across the repeated assessments is estimated. This facilitates parameterization of the rates of change in means over time into the factor loadings (see below).

To demonstrate an analysis on the basis of this proposed model, consider the following data based partly on a cognitive intervention study by Baltes, Dittmann-Kohli, and Kliegl (1986). (We thank Drs. Baltes, Dittmann-Kohli, and Kliegl for making their data available.) The goal of their original study was to examine the extent of reserve capacity of older adults in test performance on repeatedly presented fluid-intelligence measures. As part of the study, $N = 161$ older adults were administered tutor-guided training in test-relevant skills that focused among other things on a specific component of fluid intelligence, Induction. (See Chap. 5 for a discussion concerning the construct of Fluid intelligence.) Four longitudinal assessments were carried out using Thurstone's Standard Induction test: (a) before the training program was administered (Y_1), and then (b) one week after (Y_2), (c) one month after (Y_3), and (d) 6 months after completion of training (Y_4). The covariance matrix and observed variable means are provided in Table 3. The means of the four repeated assessments are presented in the last column of Table 3 to emphasize that they are essential for achieving the goals of latent change analysis of concern to us in this chapter.

EQS, LISREL, AND *Mplus* COMMAND FILES FOR A ONE-FACTOR LCA MODEL

The input files for the EQS, LISREL, and *Mplus* software must indicate that the model should be fit to both the covariance matrix and means. Accom-

TABLE 3
Covariance Matrix and Means of the Four Consecutive Administrations of Thurstone's Standard Induction Test

	<i>THU_IND1</i>	<i>THU_IND2</i>	<i>THU_IND3</i>	<i>THU_IND4</i>	<i>Means</i>
<i>THU_IND1</i>	307.46				37.48
<i>THU_IND2</i>	296.52	377.21			53.30
<i>THU_IND3</i>	295.02	365.10	392.47		54.82
<i>THU_IND4</i>	291.02	355.88	358.25	376.84	52.63

Note. THU_INDi = Thurstone's Standard Induction test at i th assessment ($i = 1, 2, 3, 4$).

plishing this necessitates a slight modification of command files used when fitting a model only to the covariance matrix. The modification follows from an observation about model-reproduced means discussed in the previous section: Just as there is a model-reproduced covariance matrix $\Sigma(\gamma)$, so are there also model-reproduced means—denoted $\mu(\gamma)$ —that are obtained by applying Law 5 to the model-definition equations.

For example, consider the following equation for Y_2 in Fig. 12:

$$Y_2 = \lambda_2 \eta_1 + \varepsilon_2, \quad (5.1)$$

where λ_2 is the factor loading of Y_2 on η_1 and ε_2 is the corresponding error term. Applying Law 5 to Equation (5.1), the following mean value for Y_2 is obtained:

$$M(Y_2) = \lambda_2 M(\eta_1) + M(\varepsilon_2), \quad (5.2)$$

but because $M(\varepsilon_2) = 0$ as a residual mean, Equation (5.2) becomes

$$M(Y_2) = \lambda_2 M(\eta_1). \quad (5.3)$$

In other words, the one-factor model under consideration reproduces the mean of Y_2 as the product of the latent variable mean and that measure's factor loading. The same relationship applies to the means of all observed variables (keeping in mind that $\lambda_1 = 1$ due to the earlier decision to set the latent metric by fixing that loading to 1).

As mentioned earlier in this chapter, in order to analyze latent change it is necessary to analyze the mean structure of the observed variables, that is, fit the model to the covariance/mean matrix. This makes meaningful also the introduction of the latent variable mean $M(\eta_1)$ as a model parameter. Otherwise, if no latent variable mean is introduced one would be effectively assuming that its value is equal to zero. Obviously, this would lead to all the observed variable means being reproduced as zero, as implied from Equations (5.1) and (5.3), and in general would lead to a misfit of a proposed model. Hence, in the one-factor model under consideration in this section, one needs to include (at least) the latent mean as a separate parameter in order to ensure that the model is given a chance to fit the data as well as it possibly can.¹

Returning to the earlier discussion of parameters, the model under consideration in Fig. 12 has a total of nine parameters—four error variances,

¹There is a more general treatment of mean structure analysis models with additional mean structure parameters, the mean intercepts, which we will not deal with in this introductory text. We refer the reader to Jöreskog & Sörbom (1993b, 1999), Bentler (2004), Muthén & Muthén (2004), and Raykov (2004).

three factor loadings, one latent variance, and the latent variable mean. Since this is a LCA model, it is fit to the covariance/mean matrix as discussed previously. Therefore, with four repeated assessments corresponding to the four observed variables, it is fit to a total of $4(5)/2 + 4 = 14$ data points; these are the 10 nonredundant elements of the covariance matrix plus the four manifest means. Hence, the model has $14 - 9 = 5$ degrees of freedom.

Equation (5.3) also permits a direct interpretation of the factor loadings as rates of mean change. For example, if Law 5 is applied to the equation for the first occasion of measurement, $Y_1 = \eta_1 + \varepsilon_1$, the mean value, $M(Y_1) = M(\eta_1)$ is obtained; if one now inserts the value $M(Y_1)$ for $M(\eta_1)$ in Equation (5.3) and expresses the equation in terms of λ_2 , one obtains $\lambda_2 = M(Y_2)/M(Y_1)$. This implies that the value of the factor loading λ_2 , under the considered model, corresponds to the ratio of the means for that measurement occasion and the first assessment (that we used to fix the scale of the latent variable). The same relationship holds for the remaining factor loadings (as can be seen by simple analogy). Hence, the value of any factor loading included in the model is obtained as a ratio of two means—for that measurement occasion and the first assessment.

EQS Command File

The EQS input file for conducting a latent change analysis requires that the latent variable mean be introduced using a special auxiliary variable. The name of this variable is V999 and it can be thought of as a dummy variable with two features: (a) it takes on the value of 1 for all subjects in the sample, and (b) it is added internally by the program once it finds a reference to it in the input file. By regressing the latent variable on this dummy variable, one obtains in the resulting slope estimate the sample value of the latent mean. Indeed, this is equivalent to using a simple regression model in which the latent variable is the dependent variable, V999 is the single predictor, and no intercept term is included; then the estimate of the slope of V999 will obviously be the latent mean. This is, alternatively, equivalent to using a simple regression model with an intercept only, which will then evidently be estimated at the mean of the dependent variable—in this case the latent variable mean. Regressing the latent variable on the variable V999 implies also that a residual term would have to be associated with the factor. Recall the discussion in Chap. 1, in which it was emphasized that in general any dependent variable should be associated with a residual (disturbance) term. This requirement leads to an additional model equation for the latent mean in the model, and thus an added equation for the one factor in the model in Fig. 12. Specifically, the equation $F_1 = *V999 + D_1$ is used in the EQS command file to regress the factor on the dummy variable V999, and as a result the residual term D_1 receives the variance of the latent variable as a param-

ter. This is due to two reasons: (a) D_1 is formally an independent variable of the model and hence its variance is a model parameter, and (b) applying Law 4 for variances on the equation $F_1 = *V999 + D_1$ (see Chap. 1), one sees that its variance equals that of the latent factor; the latter follows from the fact that D_1 is assumed to be unrelated to F_1 and that $V999$ has no variance because it is a dummy, constant variable.

Since the command file being created is for an LCA model, EQS must also be provided with data to fit the model to a covariance/mean matrix. In order to add the means of the observed variables to the input file, the command /MEANS is used, typically included below the statement of the covariance matrix. (Variable means must appear in the same order as in the covariance matrix.) Once this is done, an analysis of the mean structure with EQS is invoked by using the keyword ANALYSIS=MOMENTS in the specification section of the input file (see also Chap. 2). This keyword, ANALYSIS=MOMENTS, signals to EQS that both the first-order and second-order moments of the observed variables are to be analyzed (recall from introductory statistics that means are first-order moments, and their variances and covariances are second-order moments). Hence, the following EQS command file is created:

```

/TITLE
ONE-FACTOR LATENT CHANGE ANALYSIS MODEL;
/SPECIFICATIONS
VARIABLES=4; CASES=161; ANALYSIS=MOMENTS;
/LABELS
V1=THU-IND1; V2=THU-IND2; V3=THU-IND3; V4=THU-IND4;
F1=TIME-FAC;
/EQUATIONS
V1= F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V4=*F1+E4;
F1=*V999+D1;
/VARIANCES
D1=*; E1 TO E4=*;
/MATRIX
307.46
296.52 377.21
295.02 365.10 392.47
291.02 355.88 358.25 376.84
/MEANS
37.48 53.30 54.82 52.63
/END;

```

LISREL Command File

The LISREL input file for conducting a LCA requires that the latent variable mean be introduced. Accordingly, latent variable means are referred to as part of an A matrix (the Greek letter *alpha*), denoted as AL in the LISREL syntax, and the same principles discussed in Chap. 2 apply when referring to its elements. The matrix A is actually a row vector—a special matrix that has only one row, but as many columns as there are latent variables in the model. Hence, the elements of the ALpha matrix in the current one-factor LCA example are AL(1), or simply AL(1, 1)—as there is only one latent variable mean. The default setting for the ALpha matrix in LISREL is that it is fixed, i.e., consists only of elements fixed at 0, unless the program is advised otherwise. Thus, once ALpha is mentioned in the model-definition line, LISREL is prepared for any instructions concerning latent means.

The LISREL command file for the present example is constructed following the guidelines outlined in Chap. 2 and includes the definition of the ALpha matrix.

```

ONE-FACTOR LATENT CHANGE ANALYSIS MODEL
DA NI=4 NO=161
CM
307.46
296.52 377.21
295.02 365.10 392.47
291.02 355.88 358.25 376.84
ME
 37.48 53.30 54.82 52.63
LA
THU-IND1 THU-IND2 THU-IND3 THU-IND4
MO NY=4 NE=1 AL=FR LY=FU, FR
LE
TIME-FAC
FI LY(1, 1)
VA 1 LY(1, 1)
OU

```

There are several points in this command file that are worth mentioning here. First, there is a new line immediately following the covariance matrix, which corresponds to the four means of the observed variables. This is the reason the command line uses the keyword ME (for MEans). The observed variables are subsequently labeled according to the four consecutive assessments using Thurstone's Induction test, i.e., THU-IND1 to THU-IND4. The MOdel definition line also includes the new command, AL=FR. By freeing

all elements of Alpha, this command tells the LISREL program that the latent means are to be considered free model parameters. Since the model being fitted contains only one factor, Alpha has only one element, and hence this latent mean is hereby declared a free parameter. Then, for simplicity, the matrix LY is declared to be full of free parameters. Because LY has here only one column containing all four factor loadings (since this model has just one latent variable), this in effect says that all of them are free model parameters. (The first loading is later fixed to 1 to set the scale metric.) Note that the PS matrix, here consisting of only one latent variance, has not been mentioned. This is because PS=DI, FR (i.e., PS having free elements along its diagonal) is the default option for it, which is exactly what the model under consideration entails. Similarly, the TE matrix containing all error variances has not been mentioned, because TE=DI, FR is also the default and just what is needed to specify all error variances as model parameters in this example. Finally, below the MModel definition line, a label TIME-FAC (for TIME-FACTOR) is provided for the single latent variable in the model.

***Mplus* Command File**

The *Mplus* input file has a few new features that we discuss in turn after presenting it.

```
TITLE:          ONE-FACTOR LATENT CHANGE MODEL
DATA:          FILE IS EX6.1.MCM;
               TYPE= MEANS COVARIANCE;
               NOBSEVATIONS=161;
VARIABLE:     NAMES ARE THU_IND1 THU_IND2 THU_IND3
               THU_IND4;
ANALYSIS:     TYPE=MEANSTRUCTURE;
MODEL:        F1 BY THU_IND1@1 THU_IND2-THU_IND4;
               [THU_IND1-THU_IND4@0];
               [F1];
```

The first feature to mention is that the file containing the data to be analyzed, in this case variable variances and covariances as well as means, consists of a first row containing the variable means, followed in the next rows by the covariance matrix. To indicate this, we mention in the TYPE subcommand of the DATA command that the data file represents both the covariance matrix and means. Since we do not point to raw data, we need to also mention the sample size subsequently. Given that we will analyze the mean structure, it is necessary that we indicate this detail with the command

ANALYSIS where the option TYPE states that analyzed will be the mean structure. The MODEL command indicates THU_IND1 through THU_IND4 as indicators of a latent factor, F1, whereby its first loading is fixed at 1, as discussed earlier. The default mean structure model in *Mplus* contains also the variable mean intercepts (see Note 1 to this chapter), which can each be thought of as the intercept in a formal regression model relating each manifest variable to the underlying latent factor. In the present one-factor model for growth, a main assumption is that the mean aspects of this change over time are explained entirely in terms of the latent mean and factor loadings. For this reason, none of the mean intercepts is a model parameter. (Note that there are four mean intercepts, and if they were all declared additional parameters the model would not be identified as it will have 15 parameters whereas there would be only 14 data points to which it could be fitted—10 variances and covariances plus 4 means, as mentioned above; thus, the model would fail the necessary condition for identification, see Chap. 1.) Therefore, the intercepts of the observed variables—referred to in the *Mplus* syntax by the name of the variable enclosed in brackets—need to be fixed at 0, as done in the 2nd line of the MODEL command. However, the latent mean is a model parameter and thus it needs to be declared as such. To this end, the factor mean is referred to in the last MODEL command line by the factor name enclosed in brackets.

MODELING RESULTS, ONE-FACTOR LCA MODEL

In this chapter, due to more extensive information provided in software outputs, we present analytic results of a single SEM program in turn across sections. In this section, we discuss the EQS results and note their essential identity to those obtained with alternative software used in this book.

EQS Program Results

After verifying from the echoed input file that the model fitted is identical to the one intended and that the data analyzed are those we wanted to fit the model to, we need to know whether the model is technically sound and whether EQS encountered any numerical difficulties while handling it.

```
PARAMETER ESTIMATES APPEAR IN ORDER,  
NO SPECIAL PROBLEMS WERE ENCOUNTERED DURING OPTIMIZATION.
```

This message is a reassurance that the results in the following output sections are trustworthy.

RESIDUAL COVARIANCE/MEAN MATRIX (S-SIGMA) :

	THU-IND1	THU-IND2	THU-IND3	THU-IND4	V999
	V1	V2	V3	V4	V999
THU-IND1 V1	49.719				
THU-IND2 V2	36.906	.396			
THU-IND3 V3	28.687	-7.147	-14.974		
THU-IND4 V4	34.865	-2.141	-9.037	-3.840	
V999 V999	-.658	-.004	.137	.036	.000

AVERAGE ABSOLUTE COVARIANCE RESIDUALS = 12.5697
 AVERAGE OFF-DIAGONAL ABSOLUTE COVARIANCE RESIDUALS = 11.9616

STANDARDIZED RESIDUAL MATRIX:

	THU-IND1	THU-IND2	THU-IND3	THU-IND4	V999
	V1	V2	V3	V4	V999
THU-IND1 V1	.162				
THU-IND2 V2	.108	.001			
THU-IND3 V3	.083	-.019	-.038		
THU-IND4 V4	.102	-.006	-.023	-.010	
V999 V999	-.037	.000	.007	.002	.000

AVERAGE ABSOLUTE STANDARDIZED RESIDUALS = .0399
 AVERAGE OFF-DIAGONAL ABSOLUTE STANDARDIZED RESIDUALS = .0388

LARGEST STANDARDIZED RESIDUALS:

NO.	PARAMETER	ESTIMATE	NO.	PARAMETER	ESTIMATE
1	V1, V1	.162	11	V4, V2	-.006
2	V2, V1	.108	12	V999, V4	.002
3	V4, V1	.102	13	V2, V2	.001
4	V3, V1	.083	14	V999, V2	.000
5	V3, V3	-.038	15	V999, V999	.000
6	V999, V1	-.037			
7	V4, V3	-.023			
8	V3, V2	-.019			
9	V4, V4	-.010			
10	V999, V3	.007			

DISTRIBUTION OF STANDARDIZED RESIDUALS

						RANGE	FREQ	PERCENT
!				!				
20-				-				
!				!				
!				!				
!				!				
!				!				
15-				-				
!				!	1	-0.5 -	0	.00%
!				!	2	-0.4 - -0.5	0	.00%
!				!	3	-0.3 - -0.4	0	.00%
!				!	4	-0.2 - -0.3	0	.00%
10-				-	5	-0.1 - -0.2	0	.00%
!				!	6	0.0 - -0.1	7	46.67%
!				!	7	0.1 - 0.0	5	33.33%
!				!	8	0.2 - 0.1	3	20.00%
!		*		!	9	0.3 - 0.2	0	.00%
!		*		!	A	0.4 - 0.3	0	.00%
5-		*	*	-	B	0.5 - 0.4	0	.00%
!		*	*	!	C	++ - 0.5	0	.00%
!		*	*	!				
!		*	*	!				
!		*	*	!				
!		*	*	!				
						TOTAL	15	100.00%

1 2 3 4 5 6 7 8 9 A B C

EACH "*" REPRESENTS 1 RESIDUALS

There seem to be no impressively large residuals, and the model appears to fit reasonably well locally. We note that the residuals for the means are presented in the last row of the matrix called "Residual covariance/mean matrix" above. We now want to assess overall model fit.

GOODNESS OF FIT SUMMARY FOR METHOD = ML

INDEPENDENCE MODEL CHI-SQUARE = 978.875 ON 10 DEGREES OF FREEDOM

INDEPENDENCE AIC = 958.87451 INDEPENDENCE CAIC = 918.06047
 MODEL AIC = .86043 MODEL CAIC = -19.54659

CHI-SQUARE = 10.860 BASED ON 5 DEGREES OF FREEDOM
 PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .05422

THE NORMAL THEORY RLS CHI-SQUARE FOR THIS ML SOLUTION IS 10.724.

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)

BENTLER-BONETT NORMED FIT INDEX = .990
 BENTLER-BONETT NON-NORMED FIT INDEX = .960
 COMPARATIVE FIT INDEX (CFI) = .992

Overall, the model seems to present a plausible means of data description. Note that the degrees of freedom for this model are five because this LCA model is fitted not only to the variance-covariance matrix of the four repeated assessments but also to their means.

ITERATIVE SUMMARY			
PARAMETER			
ITERATION	ABS CHANGE	ALPHA	FUNCTION
1	146.573300	1.00000	5.87053
2	182.487600	1.00000	.58563
3	11.850730	1.00000	.07043
4	1.159758	1.00000	.06788
5	.031294	1.00000	.06788
6	.001690	1.00000	.06788
7	.000085	1.00000	.06788

After an initially fairly large fit-function value, a subsequent clean convergence to the final solution is observed, which is what eventually matters most of the time for data analysis purposes.

MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

THU-IND1=V1 = 1.000 F1 + 1.000 E1

THU-IND2=V2 = 1.398*F1 + 1.000 E2
 .024
 57.505@

THU-IND3=V3 = 1.434*F1 + 1.000 E3
 .026
 55.685@

THU-IND4=V4 = 1.379*F1 + 1.000 E4
 .025
 55.042@

CONSTRUCT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

TIME-FAC=F1 = 38.138*V999 + 1.000 D1
 1.250
 30.504@

VARIANCES OF INDEPENDENT VARIABLES

 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

	E		D	
	---		---	
E1 -THU-IND1	71.993*I	D1 -TIME-FAC	185.748*I	
	8.435 I		21.977 I	
	8.535@I		8.452@I	
	I		I	
E2 -THU-IND2	13.958*I		I	
	3.067 I		I	
	4.551@I		I	
	I		I	

E3 -THU-IND3	25.563*I	I
	4.047 I	I
	6.317@I	I
	I	I
E4 -THU-IND4	27.429*I	I
	4.079 I	I
	6.725@I	I
	I	I

STANDARDIZED SOLUTION: R-SQUARED

THU-IND1=V1	=	.849 F1	+	.529 E1	.721
THU-IND2=V2	=	.981*F1	+	.192 E2	.963
THU-IND3=V3	=	.968*F1	+	.250 E3	.937
THU-IND4=V4	=	.963*F1	+	.268 E4	.928
TIME-FAC=F1	=	.000*V999	+	1.000 D1	.000

 E N D O F M E T H O D

We note first that the latent mean is estimated at 38.138. As discussed earlier for the EQS command file, this is the estimated slope of the regression of the latent factor on the dummy variable V999.

We next evaluate the factor loading estimates by taking note of their estimates and standard errors. This suggests that there is evidence of mean growth along the studied dimension at all the posttests (V_2 to V_4) compared to the pretest (V_1). For example, we can see it by comparing the confidence intervals (CI) of each posttest factor loading with that of the pretest, that is, the factor loading fixed at 1. Adding twice the standard errors to and subtracting twice the standard errors from each of the posttest loadings results in the following approximate 95% CI (rounded off to 2 decimals): (1.35; 1.45), (1.38; 1.50), and (1.33; 1.43). Because none of these includes the value of the factor loading at pretest, namely 1, it is suggested that each of the posttest loadings is markedly higher than the pretest loading. Given the earlier discussed interpretation of factor loadings as ratios of means to initial assessment mean, it appears that at each posttest there is considerable growth in performance compared to pretest.

We emphasize, however, that using this confidence-interval approach is not equivalent to formal hypothesis testing. Specifically, it is possible that conclusions arrived at with this method may not be confirmed by a formal hypothesis test, especially when more than two parameters are simultaneously compared. It is therefore recommended that in general this confidence-interval based approach be used only as a rough explorative method to informally examine parameter relationships.

The three CIs considered here also overlap to a considerable degree, suggesting informally that the posttest factor loadings may be fairly similar

in magnitude. To examine this suggestion formally, that is, via a statistical test, one can impose a restriction on this LCA model and add the following CONSTRAINT command in the EQS input file:

```
/CONSTRAINT
(V2,F1)=(V3,F1)=(V4,F1);
```

This equality constraint leads to a substantial decrement in model fit and is associated with a chi-square value of $T = 25.757$ with $df = 7$. The difference in chi-square values compared to the original model is $\Delta T = 14.897$ with $\Delta df = 2$, and hence significant (as the cut-off for the pertinent chi-square distribution with 2 degrees of freedom is 5.99 at significance level .05). This finding indicates that the rate of change in means is not the same over all posttests.

LEVEL AND SHAPE MODEL

The one-factor LCA model presented in the previous sections is a rather restricted means of longitudinal data description because only one factor is postulated to account for change over time. A substantially less restrictive model, called the level and shape model (LS), was described and popularized by McArdle and colleagues as a very useful two-factor LCA model in longitudinal research (McArdle, 1988; McArdle & Anderson, 1990). The LS model was specifically developed to study two important aspects of the process of latent change: (a) initial true status at the beginning of a study, referred to as Level factor; and (b) the change (increase or decrease) along the underlying latent dimension of interest across all repeated assessments, called Shape factor. An example path diagram of a LS model with four successive measurement occasions (denoted by Y 's) is presented in Fig. 13 using LISREL notation.

As can be seen by examining the model in Fig. 13, each observed variable Y loads on two factors (except that for Y_1 the loading on the second factor is set to 0 – see discussion below), the Level and the Shape factors, denoted by η_1 and η_2 , respectively. Interpretation of the Level and Shape factors as indicated above is accomplished by fixing the majority of the factor loadings to a value of 1, as seen from Fig. 13 where 1s are attached to most of the paths. Specifically, fixing the loadings of the four assessment occasions on the Level factor to 1 ensures that it is interpreted as an initial true (i.e., error-free) status, that is, as a baseline level of the underlying developmental process under investigation. Fixing the loading of the last assessment occasion on the Shape factor to 1 as well and that of the first assessment occasion on it to 0, ensures that this factor is interpreted as a true overall change factor, i.e., shape of the change process along the studied latent dimension. Freeing the loadings of the second and third assessment occasions on the Shape factor implies that they denote the part of overall true change that occurs between the first and each of these later two measurement occasions (McArdle & Anderson, 1990;

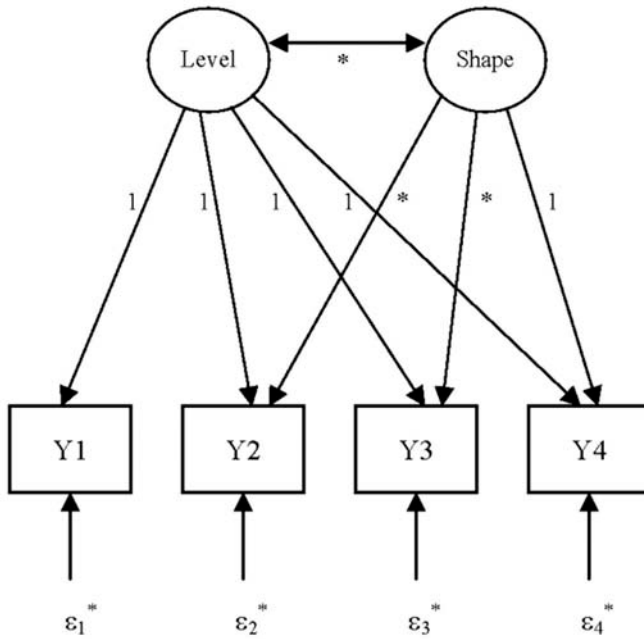


FIG. 13. Example two-factor LCA model.

Duncan et al., 1999). Finally, the Level and Shape factors are assumed to be correlated, although their degree of interrelationship need not necessarily be strong or even notable in an empirical setting.

We note that this interpretation of the Level factor and especially of the Shape factor does not really fall within the boundaries of the traditional, ‘psychometric’ conceptualization of latent constructs, which we discussed in some detail in Chap. 1. Specifically, here we interpret the Level and Shape constructs as containing information about the individual development on the studied, unobserved latent dimension of interest. Accordingly, the Level factor represents the individual true scores of starting position, i.e., the scores on the underlying latent dimension of the entire group of studied subjects at the beginning of the investigation. The Shape factor, on the other hand, represents the individual scores of true change across the repeated assessments of the study. These scores, on both the Level or Shape variables, are not observed but are associated with each individual in an available sample. Therefore, in a sense these individual-specific scores can be considered realizations, although unobserved, of random variables. The scores describe the pattern of change over time along the underlying dimension, by representing the individual locations on the latter at the start of the study and the amount of change that each individual undergoes across the period of repeated assessment. That is, the Level and Shape

latent variables describe specific aspects of the individual trajectories over time, rather than capture proper, unobserved traits that would be of separate substantive interest per se (like intelligence, motivation, aptitude, etc.) as would traditionally conceptualized latent variables. This distinction between the conventional, 'psychometric' conceptualization of the notion of latent variable and that of unobserved variable containing scores on specific characteristics of individual developmental trajectories over time becomes even more pronounced in more complex latent change models (see Appendix to this chapter for further discussion), but we stress that both 'types' of latent variables can be included in a single model as done later in this chapter when studying correlates and predictors of change.

In terms of analytic opportunities, the LS model achieves in general more than the simple one-factor model presented earlier in this chapter. In particular, with this two-factor modeling approach individual differences in change over the longitudinal occasions are explicitly captured in the Shape factor. The reason this is not possible in the one-factor model, is that the latter cannot effectively separate initial status from change in a studied construct. This limitation becomes particularly important when one is interested in studying correlates and predictors of growth or decline. In such instances, as shown in a later section, the LS model advantages come to the fore.

The LS model has also certain advantages over a particularly popular growth curve model, the so-called Intercept-and-Slope (IS) model (e.g., Duncan et al., 1999). This model is obtained as a special case of the LS model, when the loadings of the Shape factor in the LS model are fixed at the times at which the repeated assessments occur. In the empirical case under consideration in this chapter, with $k = 4$ measurements, the IS model would be obtained directly from the LS model by setting the 2nd loading of the Shape factor equal to a value of 2, the third to a 3, and the last one to 4 rather than 1 as in the LS model. (To keep also this detail of the LS model unchanged, fix the 2nd Shape loading to $1/3$ and its 3rd loading to $2/3$.) A main assumption of the IS model is that latent change specifically progresses in a linear fashion. This assumption may oftentimes not be fulfilled in social and behavioral research, however. In those cases, the more relaxed LS model would be expected to fit the data better, since it does not impose any specific form upon the pattern of latent change. In fact, the LS model is equally applicable when change progresses in a nonlinear (as well as linear) manner, and also when it is not monotonic—e.g., growth followed by decline or vice versa, or with intermediate or initial/final periods of no change. The LS model can be alternatively looked at as obtained from the IS model when the fixing of all but first and last loadings on the Slope factor is given up. Hence, the LS model is more general than the IS model in that it allows nonlinear change as well, with the IS model being a special case of the LS model. This relationship will become of particular relevance when discuss-

ing the *Mplus* input file for the LS model. Since the IS model is a special case of the LS model, the rest of this chapter is developed around the more general model, the Level-and-Shape model. Hence, all discussion and developments in the remainder of the chapter are applicable to the IS model as well, after the corresponding fixing (see above in this paragraph) of intermediate loadings on the Shape factor is carried out in the LS model.

To demonstrate a latent change analysis based on the LS model, consider again the data from the cognitive intervention study that we used with the one-factor model. Recall that four repeated assessments on $N = 161$ older adults were carried out in that study using Thurstone's Standard Induction test. To determine the parameters of the LS model in Fig. 13, we follow the six rules outlined in Chap. 1. Rules 1 and 2 imply that all error variances and latent variances and covariances are parameters. Rule 3 suggests that all factor loadings are parameters (except of course those that are already specified as fixed). Rule 4 is not applicable here because there are no explanatory relationships among latent variables. Rule 6 has already been taken care of in this model because, by fixing most of the factor loadings to 1, the metric of each latent variable is already set.

EQS, LISREL, AND *Mplus* COMMAND FILES, LEVEL AND SHAPE MODEL

In order to set up the software input files, we follow guidelines discussed in Chap. 2, and in the case of *Mplus* take advantage of some additional default features. Given that the same developmental process is followed at the four repeated assessments, we also incorporate into the LS model the reasonable assumption of equal measurement error impact upon subject performance over all measurement occasions.

EQS Command File

```

/TITLE
LEVEL AND SHAPE MODEL;
/SPECIFICATIONS
VARIABLES=4; CASES=161; ANALYSIS=MOMENTS;
/LABELS
V1=THU-IND1; V2=THU-IND2; V3=THU-IND3; V4=THU-IND4;
F1=LEVEL; F2=SHAPE;
/EQUATIONS
V1=F1+E1;
V2=F1+*F2+E2;
V3=F1+*F2+E3;
V4=F1+F2+E4;

```

```

F1=*V999+D1;
F2=*V999+D2;
/VARIANCES
D1 TO D2=*; E1 TO E4=*;
/COVARIANCES
D1,D2=*;
/CONSTRAINTS
(E1,E1)=(E2,E2)=(E3,E3)=(E4,E4);
/MATRIX
307.46
296.52      377.21
295.02      365.10      392.47
291.02      355.88      358.25      376.84
/MEANS
37.48      53.30      54.82      52.63
/END;

```

We note that error impact identity over time is accomplished by setting all error variances equal to one another with the CONSTRAINT command. The command ANALYSIS=MOMENTS ensures that a covariance and mean structure matrix is to be analyzed.

LISREL Command File

```

LEVEL AND SHAPE MODEL
DA NI=4 NO=161
CM
307.46
296.52      377.21
295.02      365.10      392.47
291.02      355.88      358.25      376.84
ME
37.48      53.30      54.82      52.63
LA
THU-IND1 THU-IND2 THU-IND3 THU-IND4
MO NY=4 NE=2 AL=FR PS=SY,FR
LE
LEVEL SHAPE
VA 1 LY(1, 1) LY(2, 1) LY(3, 1) LY(4, 1)
VA 1 LY(4, 2)
FR LY(2, 2) LY(3, 2)
EQ TE(1)-TE(4)
OU

```

We note the use of dash to connect the simpler references to first and last assessment error variances in the last line before the output command, which readily accomplishes introduction of the error variance constancy over time.

***Mplus* Command File**

```
TITLE:          LEVEL AND SHAPE MODEL
DATA:          FILE IS EX6.2.MCM;
              TYPE IS MEANS COVARIANCE;
              NOOBSERVATIONS=161;
VARIABLE:     NAMES ARE THU_IND1 THU_IND2 THU_IND3
              THU_IND4;
ANALYSIS:     TYPE IS MEANSTRUCTURE;
MODEL:        I S | THU_IND1@0 THU_IND2*.9 THU_IND3*1
              THU_IND4@1;
              THU_IND1-THU_IND4 (1);
OUTPUT:       RESIDUAL;
```

Relative to the *Mplus* command file for the one-factor latent change model in the preceding section, the only new part in this file is found in the MODEL command. The first line of this command begins with a reference to the Intercept-and-Slope model (that as discussed earlier is a special case of the more general Level-and-Shape model of concern to us in this chapter), stating formally the initials of its two factors, Intercept and Slope. Subsequently in the same line, however after the vertical bar to separate I and S from the rest in that line, we free the intermediate two loadings on the Slope factor. The effect of this, as indicated in an earlier section of this chapter, is to generalize the IS model to the LS model that is thereby completely defined as a generic model. In the second line of the MODEL command, we fix the error variances of the observed variables to be the same, by just listing their names and attaching an integer number (e.g., 1) within parentheses at the end of that line. (We include the request for model residuals in the last command line since the *Mplus* output will be the only one for this model, which we will discuss shortly.)

MODELING RESULTS FOR THE LEVEL AND SHAPE MODEL

In this section, in continuation of the last part of the preceding subsection, we discuss the output obtained with *Mplus* (which is essentially identical to those furnished by EQS and LISREL as can be seen by submitting their above provided inputs).

Mplus Results

After dispensing with the echoed back command file and technical details on the execution of the numerical optimization routine, we look for a reassurance that *Mplus* has not had problems translating our command file.

```
INPUT READING TERMINATED NORMALLY
THE MODEL ESTIMATION TERMINATED NORMALLY
```

The above two statements confirm that compiling the input was not problematic and that the fit function minimization process was uneventful.

```
TESTS OF MODEL FIT
  Chi-Square Test of Model Fit
    Value                6.836
    Degrees of Freedom    6
    P-Value              0.3361

  Chi-Square Test of Model Fit for the Baseline Model
    Value                984.992
    Degrees of Freedom    6
    P-Value              0.0000

  CFI/TLI
    CFI                 0.999
    TLI                 0.999

  Loglikelihood
    H0 Value            -2319.729
    H1 Value            -2316.311

  Information Criteria
    Number of Free Parameters      8
    Akaike (AIC)                  4655.458
    Bayesian (BIC)                 4680.109
    Sample-Size Adjusted BIC       4654.783
      (n* = (n + 2) / 24)

  RMSEA (Root Mean Square Error Of Approximation)
    Estimate                    0.029
    90 Percent C.I.             0.000 0.110
    Probability RMSEA <= .05    0.571

  SRMR (Standardized Root Mean Square Residual)
    Value                       0.004
```

The fit indices of the model point to it being a plausible overall means of data description and explanation. (That the model fits well also locally is seen by an examination of the covariance and means residuals presented later in the output and discussed below.) We now proceed to interpreting its parameter estimates.

MODEL RESULTS

	Estimates	S.E.	Est./S.E.
I			
THU_IND1	1.000	0.000	0.000
THU_IND2	1.000	0.000	0.000
THU_IND3	1.000	0.000	0.000
THU_IND4	1.000	0.000	0.000
S			
THU_IND1	0.000	0.000	0.000
THU_IND2	1.034	0.033	31.596
THU_IND3	1.138	0.035	32.893
THU_IND4	1.000	0.000	0.000
S WITH			
I	8.682	12.164	0.714

There is evidence of growth from pretest to first posttest, as well as from pretest to 2nd posttest. Indeed, the second and third Shape loadings are significant; in addition, their approximate 95%-confidence intervals (obtained by subtracting and adding twice the standard error to corresponding loading estimate) are respectively (.968, 1.100) and (1.068, 1.208), and thus completely to the right of the value of 0 that this factor has as a loading on pretest. At delayed posttest (4th assessment), elderly are on average at about the same position as at 1st posttest, since the confidence interval for the second Shape loading covers 1, its loading at the last measurement point. Given that the confidence interval for 3rd assessment is completely above 1, it is suggested that on average subjects perform at 3rd posttest better than they do at delayed posttest (last assessment occasion, at the end of the study). Hence, there seems to be some indication of a drop in performance at the last measurement point relative to the best average achievement in the study, which is at 3rd assessment.

Means

I	37.476	1.377	27.213
S	15.238	0.756	20.152

The significance of the mean of the Shape factor, as seen from its *t*-value being 20.152 and thus outside the nonsignificance interval (-2, 2), indicates that overall there has been an improvement in average performance. This suggests average ability growth across the six months of this study.

Intercepts

THU_IND1	0.000	0.000	0.000
THU_IND2	0.000	0.000	0.000
THU_IND3	0.000	0.000	0.000
THU_IND4	0.000	0.000	0.000

The intercept parameters are kept at 0 since they are not relevant in the context of modeling repeated assessments along a given dimension using a sin-

gle indicator of the latter; this empirical setting is precisely the one we are dealing with in the present example.

Variances			
I	283.233	34.058	8.316
S	50.070	8.976	5.578

The significant Level and Shape variances (*t*-values being 8.316 and 5.578, respectively) indicate marked individual differences in starting position as well as in the amount of ability change over the course of the study.

Residual Variances			
THU_IND1	22.147	1.745	12.689
THU_IND2	22.147	1.745	12.689
THU_IND3	22.147	1.745	12.689
THU_IND4	22.147	1.745	12.689

All error variances are significant, suggesting considerable portions of observed variance in the Induction measure as being a result of error of measurement with stable variance.

RESIDUAL OUTPUT

ESTIMATED MODEL AND RESIDUALS (OBSERVED - ESTIMATED)

Model Estimated Means/Intercepts/Thresholds				
	THU_IND1	THU_IND2	THU_IND3	THU_IND4
1	37.476	53.227	54.812	52.714
Residuals for Means/Intercepts/Thresholds				
	THU_IND1	THU_IND2	THU_IND3	THU_IND4
1	0.004	0.073	0.008	-0.084
Model Estimated Covariances/Correlations/Residual Correlations				
	THU_IND1	THU_IND2	THU_IND3	THU_IND4
THU_IND1	305.379			
THU_IND2	292.207	376.827		
THU_IND3	293.111	360.968	389.944	
THU_IND4	291.915	352.645	358.757	372.814
Residuals for Covariances/Correlations/Residual Correlations				
	THU_IND1	THU_IND2	THU_IND3	THU_IND4
THU_IND1	0.171			
THU_IND2	2.471	-1.960		
THU_IND3	0.077	1.865	0.089	
THU_IND4	-2.703	1.024	-2.732	1.685

These mean and covariance residuals indicate that the model fits well the data also locally, i.e., with regard to all means, variances, and covariances. Specifically, the relatively low magnitude of the mean and covariance residuals (compare their magnitude to those of observed means and of the variances and covariances, respectively) demonstrates that the model has been in a position to reproduce quite well the analyzed data. Together with the above discussed finding of satisfactory overall fit, these results allow one to place trust in the provided interpretations of model parameters in this section.

STUDYING CORRELATES AND PREDICTORS OF LATENT CHANGE

The LS model we have been dealing with in the preceding section is also very useful for purposes of examining correlates and predictors of change. These type of concerns arise frequently in empirical research when a scientist is interested in determining those characteristics of individuals, which are related to patterns of change over time. For example, in the cognitive intervention study of older adults presented earlier, there may be an interest in finding out which individuals exhibit the most salient improvement or least pronounced decline along the studied dimensions. The special property of the LS model that makes it a very useful means for addressing these kinds of queries is the fact that it parameterizes overall ability change in one of its latent variables, the Shape factor.

To permit the study of correlates and predictors of change, we extend the LS model to include putative covariates and relate them to the Level and Shape factors. Specifically, by focusing on the correlations of the Shape factor with presumed predictors, one obtains estimates of the degrees of interrelationship between the latter and overall latent change. For example, if these correlations are notable and positive like the mean of this factor, individuals with high values on the correlates tend to be among those who improve most, and individuals with low values tend to improve the least. Conversely, if the correlations are negative then, individuals with high correlate values tend to be among those who improve least, and individuals with low correlate values tend to improve the most. Alternatively, when studying correlates of decline (and the average of the Shape factor is negative), if these correlations are marked and positive then individuals with high correlate values tend to be among those who decline least, and individuals with low correlate values tend to decline the most. If the correlations are negative then, individuals with high correlate values tend to be among those who decline the most, and individuals with low correlate values tend to decline the least.

This extension of the LS model is displayed graphically in Fig. 14 for the case of a latent covariate (η_3) with two indicators (Y_5 and Y_6) using the cognitive intervention study on older adults presented earlier, which is examined

further in the next section. The only difference between the LS models presented in Figures 13 and 14 is the added latent covariate. We emphasize that for the purposes of studying correlates of change one can include any number of covariates in an appropriately extended LS model. In fact, the covariates may be (a) manifest or latent, (b) perfectly measured or fallible, or (c) once or repeatedly assessed. In case (c), one may also consider modeling the repeated assessments of the covariates themselves in terms of a pertinent LS model (e.g., Raykov, 1995).

Before proceeding to the analyses of the cognitive intervention data based on the LS model in Fig. 14, a useful generalization of the modeling approach for studying multiple groups, called a multisample analysis, is introduced.

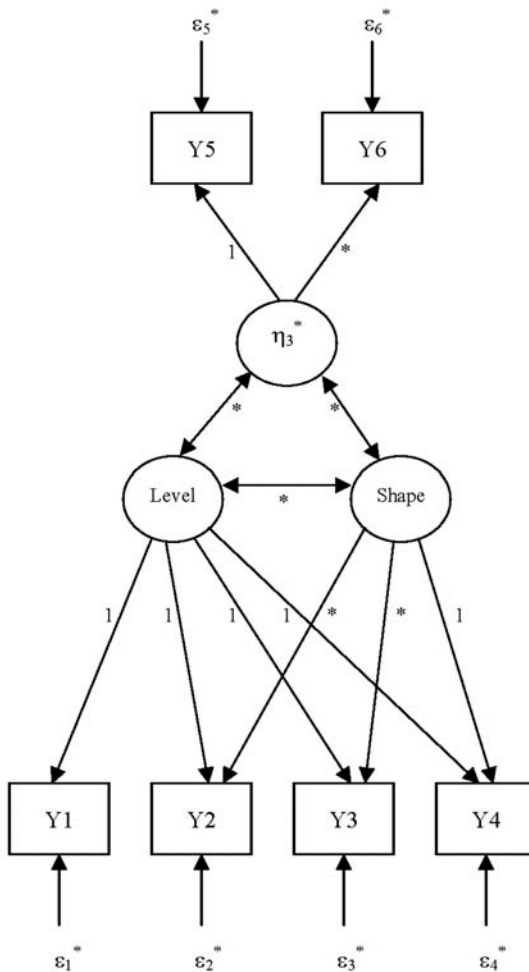


FIG. 14. Example latent covariate LCA model.

Multisample Analysis

Many studies, especially in the behavioral and social sciences, examine differences or similarities between two or more groups in the structure of a phenomenon under investigation. For example, groups may differ in terms of age, educational level, nationality, ethnicity, religious or political affiliation. When the same phenomenon is studied in all groups, it is important to have a methodological means that allows researchers to compare the groups along special dimensions of interest as well as to pinpoint their similarities or lack thereof. SEM offers a widely and readily applicable method for conducting these types of comparisons using a modification of its general model-fitting approach. This generalization accounts for the fact that group comparisons necessitate the simultaneous estimation of models in all samples involved. The generalization is applicable both to models fitted only to the covariance matrix of a set of observed variables (i.e., in analyses of their covariance structure only), and to models fitted to their covariance/mean matrix (i.e., in analyses of their mean structure).

To accomplish this generalization, the model(s) of interest are postulated within each of the groups and then their simultaneous estimation is carried out. This is accomplished by minimizing a compound fit function that results by adding the fit functions across all study groups, thereby weighting them proportionately to the sizes of the available samples. This permits the simultaneous estimation of all parameters of the models in all groups. At the minimum of the compound fit function, a test of the overall model is possible, just as in the case of a single group or population (see Chap. 1). It is also possible to impose restrictions on model parameters within as well as across groups, and to estimate and test the overall model subject to these constraints. This is achieved in a way that is very similar to fitting a model to a single group, and accounts for the fact that one is dealing with an extended simultaneous model across all groups in which the (cross-group) restricted parameters satisfy the imposed constraint(s).

Correlates of Change in a Cognitive Intervention Study

In this section, the multisample approach is demonstrated in an example study of correlates of change with the extended LS model presented in Fig. 14. The approach is applied to data based partly on the Baltes et al. (1986) investigation, which was originally conceptualized and conducted as a two-group study. In the previous sections, only the data from the experimental group ($N = 161$) was analyzed. In fact, there was also a no contact control group in that investigation, with $N = 87$ older adults, who did not receive any cognitive training nor feedback on how they did on the intelligence tests administered to them. All elderly in the control group were

given the same test battery at the same assessment points as the experimental group. Now a two-group comparison can be conducted to determine whether the Figural relations construct can be considered to be a covariate of performance improvement. Two measures are used next as indicators of the Figural relations construct, a figural relations test and a culture-fair test, which were administered to both groups of elders before the training was administered in the experimental group. By conducting the multisample analysis, we will be able to accomplish the following: (a) ascertain if there is performance improvement in the control and experimental groups, (b) examine group similarities and differences in the pattern of change across the six months of the study, (c) see whether the construct of Figural relations is predictive of subsequent improvement in Induction test performance, and (d) explore whether the two groups differ with respect to the prediction mentioned in point (c).

For the demonstration purposes of this section, we use the LISREL program that we must advise of our request to carry out a multisample analysis. In order to convey this information, a number of new commands lines are introduced in the following input file.

```

STUDYING COVARIATES AND PREDICTORS OF CHANGE * EXP. GR
DA NI=6 NO=161 NG=2
CM
307.46
296.52  377.21
295.02  365.10  392.47
291.02  355.88  358.25  376.84
203.47  229.25  221.49  229.92  260.76
169.40  180.76  185.95  181.70  155.22  159.38
ME
37.48  53.30  54.82  52.63  49.48  48.39
LA
THU-IND1 THU-IND2 THU-IND3 THU-IND4 FR11 FR12
MO NY=6 NE=3 AL=FR PS=SY,FR
LE
LEVEL SHAPE FIG-REL1
VA 1 LY(1, 1) LY(2, 1) LY(3, 1) LY(4, 1)
VA 1 LY(4, 2)
FR LY(2, 2) LY(3, 2)
VA 1 LY(5, 3)
FR LY(6, 3)
EQ TE(1)-TE(4)
ST .1 ALL

```

```

ST 100 PS(1, 1) PS(2, 2) PS(3, 3) TE(1)-TE(6)
OU NS RS
* CONTROL GROUP
DA NI=6 NO=87
CM
319.22
326.42 410.25
318.80 378.98 409.07
333.06 388.68 390.59 432.70
171.01 209.41 215.96 213.76 262.79
149.82 192.75 193.69 200.82 148.86 161.98
ME
38.62 44.96 47.88 48.01 51.97 49.81
LA
THU-IND1 THU-IND2 THU-IND3 THU-IND4 FR11 FR12
MO LY=PS PS=PS TE=PS AL=PS
LE
LEVEL SHAPE FIG-REL1
OU

```

The LISREL command file has two main parts that pertain to the model specification for each of the two groups under consideration. Note that in the title line (which will recur as a page title throughout the output for the first group) there is an indication that the first portion of the input contains the information for the experimental group. The keyword `NG=2` on the following line in the input advises the program that a two-group analysis is to be performed (`NG` stands there for “Number of Groups”). Also note that the number of analyzed variables is now six because the earlier `LS` model has been extended to include the two indicators of the covariate factor, as presented in Fig. 14. The remaining portion of the first part of the input file is largely the same as that for the single-group `LS` model in the previous section; this is because the portion defines exactly the same model for the experimental group. The only difference is that here there are three latent variables—in addition to the Level and Shape factors, that of Figural relations is also included in the model.

The three latent means are next declared to be model parameters by the keyword `AL=FR`. Then the latent covariance matrix is defined as having all its elements as parameters by the keyword `PS=SY,FR`, that is, as being symmetric and consisting of free parameters only. The remaining three matrices (`LY`, `BE`, and `TE`) are not mentioned in this line because they being at their defaults serve exactly our purposes. (At the default settings, `LY` is full and fixed; `BE` is a zero matrix, i.e., there are no latent explanatory relationships; and `TE` is diagonal and free, i.e., having model parameters along its

diagonal that are all error variances and parameters.) The free factor loadings are then declared: one for a Figural relations indicator, using the keyword `FR LY(5, 3)`; and two pertaining to the repeated assessments with the Induction test. Next the scale of the Figural relations construct is set by fixing its other factor loading to 1.

After imposing the time-invariance constraint on the corresponding four error variances, one provides start values for all the model parameters. This activity is sometimes needed in empirical research because SEM programs are not always able to come up with their own good start values. Hence they need an initial jump start to find the way toward the final solution. To achieve this, first start all parameters at a value of 0.1 and then, with the next line, override this choice only for the variances by starting them at a value of 100. This choice of large start values for all variance parameters (pertaining to both latent and observed variables) and small values for all the other model parameters (including factor loadings and independent variable covariances) is in general a good way of providing initial values for model parameters if needed. The choice contributes to the stability of the numerical subroutine during the fit function minimization process, and especially at its start. The last line pertaining to the experimental group signals the end of the input by the keyword `OU`, and asks the program to use the supplied start values instead of computing its own, as it would otherwise. This is achieved by the added keyword `NS` (for “No Start values”).

The next input line provides a title for the output portions pertaining to the results for the second group, i.e., the control group. This title line will also appear at the beginning of any output page that displays results of analyses for the control group. Hence, it is advisable to always have a title line to label each group in a multisample analysis. The next data line advises the program of the number of variables underlying the following covariance matrix and means row, as well as sample size in the control group. The model line then notifies the program that in this group a model is being defined in which all matrices have the same pattern and start values as those of the preceding group, the experimental group. This is achieved by stating each matrix name followed by the keyword `PS` (for “Same Pattern and Same Start values”; matrices not mentioned in this line are assumed to be invariant across groups). The effect of this is that exactly the same model is defined in the control group as in the experimental one, and its parameters are assigned the same initial values in both groups.

The last line `OU` signals the end of the input file for the current group, and hence for the whole model because it is being fitted to two groups.

This LISREL command file generates the following output (again, omitting irrelevant or redundant sections and inserting comments where appropriate).

EXPERIMENTAL GROUP

LAMBDA-Y			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
THU-IND1	0	0	0
THU-IND2	0	1	0
THU-IND3	0	2	0
THU-IND4	0	0	0
FR11	0	0	0
FR12	0	0	3

PSI			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	4		
SHAPE	5	6	
FIG-REL1	7	8	9

THETA-EPS						
	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	---	---	---	---	---	---
	10	10	10	10	11	12

ALPHA			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	13	14	15

CONTROL GROUP

LAMBDA-Y			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
THU-IND1	0	0	0
THU-IND2	0	16	0
THU-IND3	0	17	0
THU-IND4	0	0	0
FR11	0	0	0
FR12	0	0	18

PSI			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	19		
SHAPE	20	21	
FIG-REL1	22	23	24

THETA-EPS						
	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	---	---	---	---	---	---
	25	25	25	25	26	27

ALPHA			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	28	29	30

First, make sure that the numbering and positions of the model parameters are correct for both groups. Note that the numbering of model parameters begins in the experimental group and continues in the control group, rather than starts anew there. This is due to the fact that we requested a simultaneous two-group analysis rather than two separate analyses. This feature will become especially important in the next fitted model, when cross-group parameter constraints are introduced.

Before beginning the parameter estimate interpretation, inspect the global goodness-of-fit section of the output.

```

Global Goodness of Fit Statistics

Degrees of Freedom = 24
Minimum Fit Function Chi-Square = 27.33 (P = 0.29)
Normal Theory Weighted Least Squares Chi-Square = 26.14 (P = 0.35)
  Estimated Non-centrality Parameter (NCP) = 2.14
  90 Percent Confidence Interval for NCP = (0.0 ; 18.80)
  Minimum Fit Function Value = 0.11
  Population Discrepancy Function Value (F0) = 0.0087
  90 Percent Confidence Interval for F0 = (0.0 ; 0.076)
  Root Mean Square Error of Approximation (RMSEA) = 0.027
  90 Percent Confidence Interval for RMSEA = (0.0 ; 0.080)
  P-Value for Test of Close Fit (RMSEA < 0.05) = 0.70
  Expected Cross-Validation Index (ECVI) = 0.35
  90 Percent Confidence Interval for ECVI = (0.29 ; 0.37)
  ECVI for Saturated Model = 0.17
  ECVI for Independence Model = 9.61
Chi-Square for Independence Model with 30 Degrees of Freedom = 2351.89
  Independence AIC = 2375.89
  Model AIC = 86.14
  Saturated AIC = 84.00
  Independence CAIC = 2430.05
  Model CAIC = 221.55
  Saturated CAIC = 273.56
  Normed Fit Index (NFI) = 0.99
  Non-Normed Fit Index (NNFI) = 1.00
  Parsimony Normed Fit Index (PNFI) = 0.79
  Comparative Fit Index (CFI) = 1.00
  Incremental Fit Index (IFI) = 1.00
  Relative Fit Index (RFI) = 0.99
  Critical N (CN) = 387.88

```

All indices point to an acceptable fit of the overall two-group model. Note the low magnitude of the chi-square value compared to the model degrees of freedom (observe its p value), as well as the RMSEA index, which is well below the proposed threshold level of .05. Furthermore, the left endpoints of the confidence intervals for RMSEA, the noncentrality parameter, and the population minimal fit-function value are all at their best value of 0. Observe that the degrees of freedom are twice those for the extended LS model when it is fitted only to a single group. The same holds true for the model parameters—their number is twice that of the extended LS model when fitted to only one group. This result will always be true whenever the same

model is postulated in two distinct groups and no cross-group parameter constraints are imposed—both the degrees of freedom and number of parameters in the overall two-group model are exactly twice those if the model were fitted to either of the groups separately.

Given the satisfactory fit indices of the overall two-group model, one can now continue with parameter interpretation.

EXPERIMENTAL GROUP

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y	LEVEL	SHAPE	FIG-REL1
	---	---	-----
THU-IND1	1.00	- -	- -
THU-IND2	1.00	1.03 (0.03) 31.43	- -
THU-IND3	1.00	1.14 (0.03) 32.72	- -
THU-IND4	1.00	1.00	- -
FR11	- -	- -	1.00
FR12	- -	- -	0.97 (0.02) 61.34

The first four lines of the factor loading matrix contain essentially the same results as those in the preceding section in which only data from the experimental group data was analyzed with the LS model, and hence warrant the same interpretations (see also the significantly positive Shape factor mean below). The last two lines of the factor loadings matrix pertain to the added covariate indicators, and it is clear that they load quite similarly on the Figural relations factor—observe the standard error of the free factor loading and its readily obtained approximate confidence interval (0.93; 1.01), which includes 1 that is the factor loading of the other latent covariate indicator.

Covariance Matrix of ETA

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	285.00		
SHAPE	8.74	50.32	
FIG-REL1	182.51	15.44	150.63

Mean Vector of Eta-Variables

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	37.48	15.23	49.78

PSI

	LEVEL	SHAPE	FIG-REL1
LEVEL	285.00 (34.38) 8.29		
SHAPE	8.74 (12.27) 0.71	50.32 (9.05) 5.56	
FIG-REL1	182.51 (23.32) 7.83	15.44 (9.17) 1.68	150.63 (19.92) 7.56

From the last set of lines in the PSI matrix one can infer that there is a weak (linear) relationship between Figural relations as a putative correlate of change along the Induction dimension in the experimental group. The covariance between the Shape factor and the Figural relations construct falls short of statistical significance: estimate = 15.44, standard error = 9.17, and *t* value = 1.68. Although this result may be a consequence of the limited sample size (as mentioned before), it appears that although the Figural relations construct represents a correlate of the initial status in inductive-reasoning ability, it has a limited capability for predicting the amount of improvement along the Induction dimension in this group.

THETA-EPS

THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
22.29 (1.76) 12.65	22.29 (1.76) 12.65	22.29 (1.76) 12.65	22.29 (1.76) 12.65	80.44 (10.98) 7.32	28.07 (6.71) 4.18

Squared Multiple Correlations for Y - Variables

THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
0.93	0.94	0.94	0.94	0.65	0.83

ALPHA

LEVEL	SHAPE	FIG-REL1
37.48 (1.39) 27.04	15.23 (0.76) 20.03	49.78 (1.19) 41.66

Group Goodness of Fit Statistics

Contribution to Chi-Square = 22.42
 Percentage Contribution to Chi-Square = 82.05

Root Mean Square Residual (RMR) = 14.38
 Standardized RMR = 0.055
 Goodness of Fit Index (GFI) = 0.96

The goodness-of-fit statistics represent the chi-square contribution by the experimental group to the overall fit of the model. Although it is rather difficult to judge this single-group contribution quantity in a simultaneous modeling analysis, one can always relate it to the degrees of freedom for a model involving no group constraints. In this case, the degrees of freedom are 12 (half these of the overall model) and so this chi-square value does not seem excessively large. Thus, informally, in the experimental group there is no indication of serious model misfit.

CONTROL GROUP

LAMBDA-Y	LEVEL	SHAPE	FIG-REL1
THU-IND1	1.00	- -	- -
THU-IND2	1.00	0.70 (0.07) 9.56	- -
THU-IND3	1.00	0.98 (0.08) 12.03	- -
THU-IND4	1.00	1.00	- -
FR11	- -	- -	1.00
FR12	- -	- -	0.96 (0.02) 43.40

Although the results for the control group share some similarities with those observed in the experimental group (i.e., the loadings of the covariates on the Figural relations construct are quite similar to one another), there are some differences. Despite the fact that there is also growth in mean performance for the control group (i.e., the Shape factor loadings are significant; see also the significant and positive Shape factor mean below in this section), looking closely at the approximate 95%-confidence intervals for the third Induction assessment (0.82; 1.14) reveals that performance at that measurement point is the same as at delayed posttest (i.e., at final assessment). This is an aspect in which the two groups differ. Although in the experimental group there is a drop at the last assessment occasion, in the control group there is an indication of steady mean performance at the last two assessment occasions. (See also the estimate and confidence interval for the 2nd assessment loading on the Shape factor, whereby this interval is completely to the left of 1 that is the last loading on this factor.)

Covariance Matrix of ETA

	LEVEL	SHAPE	FIG-REL1
LEVEL	293.33		
SHAPE	38.71	27.51	
FIG-REL1	161.28	49.40	153.54

Mean Vector of Eta-Variables

	LEVEL	SHAPE	FIG-REL1
	38.57	9.40	52.02

PSI

	LEVEL	SHAPE	FIG-REL1
LEVEL	293.33 (49.23) 5.96		
SHAPE	38.71 (17.13) 2.26	27.51 (12.61) 2.18	
FIG-REL1	161.28 (31.00) 5.20	49.40 (13.53) 3.65	153.54 (27.76) 5.53

Unlike the experimental group, in the control group there is a significant (linear) relationship between Figural relations as a correlate of change along the Induction dimension. The covariance between the Shape factor and the Figural relations construct is significant here: parameter estimate = 49.40, standard error = 13.53, and t value = 3.65. This difference in significance of covariance coefficients across groups may be explainable with the fact that there was no training in the control group, whereas in the experimental group its effect may have capitalized on the particular modules of Induction-related tutor-guided instructions and part of them may have been forgotten by the elderly six months after training.

THETA-EPS

	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	30.72 (3.31) 9.27	30.72 (3.31) 9.27	30.72 (3.31) 9.27	30.72 (3.31) 9.27	104.14 (18.69) 5.57	22.36 (9.66) 2.32

Squared Multiple Correlations for Y - Variables

	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	0.91	0.92	0.93	0.93	0.60	0.86

ALPHA	LEVEL	SHAPE	FIG-REL1
	---	---	---
	38.57	9.40	52.02
	(1.94)	(1.00)	(1.72)
	19.88	9.37	30.24

As in the experimental group, overall mean performance improvement is significant in the control group. This is reflected in the mean of the Shape factor: estimate = 9.40, standard error = 1.00, and a significant t value = 9.37. This is perhaps a reflection of the practice effects at work in both groups, due to the repeated assessment with the same instruments. Nevertheless, it is interesting to compare the mean overall growth across groups, to find out if there is significant training gain in the experimental group over and above these practice effects. As shown next, this comparison can be accomplished by examining restricted versions of the model. Another interesting comparison involves the Shape mean across the control and experimental groups. Because the two groups were randomly formed at pretest, their means were comparable then. Regarding the Shape mean (control = 9.40 and experimental = 15.23), one can explore informally if the overall change was higher in the experimental group by comparing their confidence intervals. As it turns out, the approximate 95% CI for the Shape mean in the control group is (7.40; 11.40) and to the left of that in the experimental group, (13.71; 16.76), which suggests informally that mean overall change is considerably more pronounced in the experimental than the control group. We return to this issue in the next subsection where we examine it more precisely.

Studying Plausibility of Group Restrictions

As discussed in chapter 1, in general one can make more precise statements about parameter comparability across and within groups with more restrictive versions of a proposed model, as long as the restricted model is found tenable as a means of data description. This is due to the fact that parameter estimates in a more restrictive model are generally associated with smaller standard errors and hence shorter confidence intervals that lead to more accurate statements. This is part of the reason for looking for more parsimonious models that in addition have higher degrees of freedom and are associated with more dimensions along which they could be disconfirmed.

In the two-group analyses of the cognitive intervention data of interest in this section, one can further gain in model parsimony by observing that the groups were randomly formed in that study after the pretest was conducted. Therefore, no differences are expected across groups in any of the

variable characteristics at pretest. That is, one would expect that if group equality restrictions were imposed on variances, covariances, means, and covariate factor loadings at pretest, the fit of the model would not deteriorate markedly. This is what the nested two-group model fitted next will do. In this constrained model all details pertaining to the pretests of Induction and Figural relations will be fixed for equality across groups—the factor loadings of the two covariate indicators, the variances and covariance of their construct and of the Level factor, as well as the means of these two factors. This is accomplished by adding the following LISREL command lines (one per constrained parameter) immediately before the last line of the whole input file.

```
EQ PS(1, 1, 1) PS(1, 1)
EQ PS(1, 3, 3) PS(3, 3)
EQ PS(1, 3, 1) PS(3, 1)
EQ LY(1, 6, 3) LY(6, 3)
EQ AL(1, 1) AL(1)
EQ AL(1, 3) AL(3)
```

In each of these new command lines, a parameter from Group 1 (the experimental group) is fixed to equal the same parameter in Group 2 (the control group). In LISREL notation, this is accomplished by having three indices, where the first refers to the group number and the following two refer to parameter location. Thus, PS(1, 1, 1) refers to element 1,1 of the PSI matrix in Group 1. Hence, the equality constraint EQ PS(1, 1, 1) PS(1, 1) sets that parameter of the current (control) group, which is listed second, equal to the same parameter in the experimental group (i.e., in Group 1, the experimental group) which is listed first in this constraint line. Using this notation, the first two command lines fix for group equality the variances of the Level and Figural relations factors (first and third factors), the third line fixes their covariance, and the fourth line fixes the factor loading of the second indicator of Figural relations. Finally, the means of the first and third factors (the Level and Figural relations factors, respectively) are set identical in both groups, as they similarly are not expected to differ across groups. (Note the reference by only two indices because the pertinent matrix ALpha is actually only a row vector and hence one needs to refer just to the position of the parameter of interest within it.) Each of the six parameters fixed in this way pertains to a characteristic of a variable assessed at pretest. Because the groups were randomly formed after pretest, it seems reasonable to assume that these characteristics are identical across groups; this is being examined by testing these six restrictions. (We also note as an aside that fitting this restricted model has also the added benefit that if the results fail to

support lack of differences in this way, that is, if the chi-square deteriorates significantly, one may actually question the efficacy of the randomization procedure performed at the first assessment occasion of the study.)

This constrained model is associated with a chi-square value $T = 33.04$ with $df = 30$, $p = .32$, and an RMSEA = .025 with a 90% confidence interval (0; .074). These indices suggest an acceptable model fit. For future reference, we will call this model M_1 and the one fitted immediately before it M_0 . M_1 is nested in M_0 because it can be obtained from the former by imposing the proposed six parameter restrictions. Thus, M_1 has six more degrees of freedom because it has six fewer parameters than M_0 .

The difference in chi-squares between the two nested models M_1 and M_0 , is $\Delta T = 33.04 - 27.33 = 5.71$ with $\Delta df = 6$. Because the cut-off of the pertinent chi-square distribution with 6 degrees of freedom is 12.59 (at significance level .05, and obviously higher for any smaller level), this difference is nonsignificant. This implies that the imposed constraints are plausible, and can be retained in the ensuing analyses.

We now also examine whether there may be evidence in this data favoring group invariance of the relationships among the three latent variables. To obtain the input for this model that will be nested in M_1 (and thus in M_0 as well), one only needs to change the keyword PS=PS to PS=IN (for "group INvariance") in the second-group model command line of the input. This constraint yields an increase in the chi-square value up to $T = 44.54$ with $df = 33$. Hence, the resulting difference in chi-squares is $\Delta T = 44.54 - 33.04 = 11.50$ with $\Delta df = 3$, and is significant (even at a significance level .01 that one may want to adopt for protection in this multiple testing procedure with an unknown overall significance level). Thus, one can conclude that it is not likely that the same latent correlation matrix applies for both groups in the population. That is, the correlation structure among the Level, Shape, and Figural relation factors is not group invariant. Interestingly, even adding the constraint of identical latent means instead of the last imposed group restriction, accomplished by changing the keyword AL=PS to AL=IN in the control group, yields a similar outcome. The chi-square value once again goes up to $T = 62.05$ with $df = 31$, and the chi-square difference test yields $\Delta T = 62.05 - 33.04 = 29.01$ with $\Delta df = 1$, which is also significant. One can therefore conclude that the latent mean of the Shape factor is not the same across groups (because this is the only latent mean not already set as group invariant in model M_1). Finally, restraining instead the factor loadings of the Shape factor across the groups (by changing the keyword LY=PS to LY=IN in the control group) yields $\Delta T = 46.68 - 33.04 = 13.64$ with $\Delta df = 2$, which is similarly significant and indicates that these loadings are not group invariant. Hence, the two groups differ also in the proportion of overall ability change that occurred from pretest to first and

second assessment occasions. (We note however that these proportions are out of different overall changes, as found earlier; thus, the last group difference statement refers only to relative, or proportionate, rather than absolute change.)

We present now relevant portions of the output of model M_1 , the most restrictive and tenable of the models fitted in this section.

EXPERIMENTAL GROUP

Parameter Specifications

LAMBDA-Y						
	LEVEL	SHAPE	FIG-REL1			
	---	---	---	---	---	---
THU-IND1	0	0				0
THU-IND2	0	1				0
THU-IND3	0	2				0
THU-IND4	0	0				0
FR11	0	0				0
FR12	0	0				3
PSI						
	LEVEL	SHAPE	FIG-REL1			
	---	---	---	---	---	---
LEVEL	4					
SHAPE	5	6				
FIG-REL1	7	8	9			
THETA-EPS						
	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	---	---	---	---	---	---
	10	10	10	10	11	12
ALPHA						
	LEVEL	SHAPE	FIG-REL1			
	---	---	---	---	---	---
	13	14	15			

CONTROL GROUP

Parameter Specifications

LAMBDA-Y			
	LEVEL	SHAPE	FIG-REL1
	---	---	---
THU-IND1	0	0	0
THU-IND2	0	16	0
THU-IND3	0	17	0
THU-IND4	0	0	0
FR11	0	0	0
FR12	0	0	3

PSI						
	LEVEL	SHAPE	FIG-REL1			
	---	---	-----			
LEVEL	4					
SHAPE	18	19				
FIG-REL1	7	20	9			
THETA-EPS						
	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	-----	-----	-----	-----	-----	-----
	21	21	21	21	22	23
ALPHA						
	LEVEL	SHAPE	FIG-REL1			
	---	---	-----			
	13	24	15			

Note the identical numbers attached to the parameters constrained for equality across the two groups. Thus, model M_1 has altogether 24 parameters in both groups, six of which relate to the pretest assessment of the two groups obtained randomly after the first measurement occasion, and are set identical for experimental and control subjects. We turn now to the overall model fit statistics to evaluate whether this two-group model represents a plausible means of description and explanation of the analyzed data.

Global Goodness of Fit Statistics

Degrees of Freedom = 30
 Minimum Fit Function Chi-Square = 33.04 (P = 0.32)
 Normal Theory Weighted Least Squares Chi-Square = 32.35 (P = 0.35)
 Estimated Non-centrality Parameter (NCP) = 2.35
 90 Percent Confidence Interval for NCP = (0.0 ; 20.33)

Minimum Fit Function Value = 0.13
 Population Discrepancy Function Value (F0) = 0.0096
 90 Percent Confidence Interval for F0 = (0.0 ; 0.083)
 Root Mean Square Error of Approximation (RMSEA) = 0.025
 90 Percent Confidence Interval for RMSEA = (0.0 ; 0.074)
 P-Value for Test of Close Fit (RMSEA < 0.05) = 0.75

Expected Cross-Validation Index (ECVI) = 0.33
 90 Percent Confidence Interval for ECVI = (0.27 ; 0.35)
 ECVI for Saturated Model = 0.17
 ECVI for Independence Model = 9.61

Chi-Square for Independence Model with 30 Degrees of Freedom = 2351.89
 Independence AIC = 2375.89
 Model AIC = 80.35
 Saturated AIC = 84.00
 Independence CAIC = 2430.05
 Model CAIC = 188.67
 Saturated CAIC = 273.56

Normed Fit Index (NFI) = 0.99
 Non-Normed Fit Index (NNFI) = 1.00
 Parsimony Normed Fit Index (PNFI) = 0.99
 Comparative Fit Index (CFI) = 1.00
 Incremental Fit Index (IFI) = 1.00
 Relative Fit Index (RFI) = 0.99
 Critical N (CN) = 379.87

All overall indices point to a tenable model so we can move on to interpretation of its parameter estimates. (Examination of the model residuals within each group that are not included here, none of which is particularly large, corroborates this conclusion.)

EXPERIMENTAL GROUP

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y	LEVEL	SHAPE	FIG-REL1
	---	---	-----
THU-IND1	1.00	- -	- -
THU-IND2	1.00	1.03 (0.03) 31.84	- -
THU-IND3	1.00	1.14 (0.03) 33.15	- -
THU-IND4	1.00	1.00	- -
FR11	- -	- -	1.00
FR12	- -	- -	0.97 (0.01) 75.07

The factor loading estimates and standard errors are essentially the same as those discussed earlier for model M_0 , and hence lead to the same interpretations. Only the standard error of the second Figural relations indicator loading is slightly smaller (due to this model M_1 being a more parsimonious one than M_0), but this is a finding of marginal relevance in these analyses that are primarily concerned with studying change over time and latent variable relationships.

Covariance Matrix of ETA

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	287.41		
SHAPE	5.70	51.73	
FIG-REL1	175.72	16.45	151.08

Mean Vector of Eta-Variables

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	37.87	15.35	50.52
PSI			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	287.41 (28.16) 10.21		
SHAPE	5.70 (12.15) 0.47	51.73 (9.16) 5.65	
FIG-REL1	175.72 (18.64) 9.43	16.45 (9.04) 1.82	151.08 (16.16) 9.35

Note again that in the experimental group there does seem to be only a weak (linear) relationship between the Level and Shape factors—the estimate of their covariance is 5.70, with a standard error = 12.15 and a t value = 0.47 that is nonsignificant. That is, for experimental subjects starting position on Induction is not predictive of amount of change along that dimension. Similarly, there is a weak evidence of relatedness between the Shape and Figural relations factors—their covariance is estimated at 16.45, with a standard error = 9.04 and a t value = 1.82 that is not significant either. Hence, in the experimental group there is a weak (linear) relationship between starting position on Figural relations and change along the Induction dimension. As mentioned before, this may have been related to the specific modules of the cognitive training. Nevertheless, in this model, M_1 , the substantive statements just made are more precise due to the model having more degrees of freedom than M_0 . Specifically, the standard errors of all covariance estimates are on average markedly lower in M_1 than in M_0 , which generally allows one to arrive at more accurate conclusions.

THETA-EPS

THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
---	---	---	---	---	---
22.08	22.08	22.08	22.08	81.18	24.77
(1.74)	(1.74)	(1.74)	(1.74)	(11.19)	(6.51)
12.68	12.68	12.68	12.68	7.25	3.81

Squared Multiple Correlations for Y - Variables

THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
---	---	---	---	---	---
0.93	0.94	0.94	0.94	0.65	0.85

ALPHA			
	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	37.87	15.35	50.52
	(1.13)	(0.75)	(0.98)
	33.60	20.43	51.50

The findings about mean change with this model are essentially the same as those obtained with M_0 in the experimental group. Further, as can be seen in the control group output, the results in it are also quite similar to the ones with model M_0 discussed previously. We therefore present next only the control group results regarding latent variable relationships, in which the two groups were found with M_0 to differ. The question now is whether a similar interpretation would be warranted with the more parsimonious model M_1 that as such allows more precise statements.

Covariance Matrix of ETA

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	287.41		
SHAPE	48.14	23.82	
FIG-REL1	175.72	47.44	151.08

Mean Vector of Eta-Variables

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
	37.87	9.02	50.52

PSI

	LEVEL	SHAPE	FIG-REL1
	---	---	-----
LEVEL	287.41 (28.16) 10.21		
SHAPE	48.14 (15.17) 3.17	23.82 (12.05) 1.98	
FIG-REL1	175.72 (18.64) 9.43	47.44 (11.71) 4.05	151.08 (16.16) 9.35

THETA-EPS

	THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
	---	---	---	---	---	---
	31.46 (3.37) 9.34	31.46 (3.37) 9.34	31.46 (3.37) 9.34	31.46 (3.37) 9.34	105.00 (18.61) 5.64	29.34 (9.17) 3.20

Squared Multiple Correlations for Y - Variables

THU-IND1	THU-IND2	THU-IND3	THU-IND4	FR11	FR12
0.90	0.92	0.93	0.93	0.59	0.83

ALPHA

LEVEL	SHAPE	FIG-REL1
37.87	9.02	50.52
(1.13)	(0.91)	(0.98)
33.60	9.90	51.50

Unlike the experimental group, in the control group one again finds evidence of a significant relationship between the Level and Shape factors—their covariance is estimated at 48.14, with a standard error = 15.17 and a t value = 3.17 that is significant. The correlation of the two factors using the values of the variance and covariance estimates in the PSI matrix is readily computed as .58, indicating a moderately strong (linear) relationship. This suggests that in the control group, in which subjects did not receive any information about how well they were doing, individuals starting high on Induction tended to be among those exhibiting the most improvement along this dimension. Furthermore, the relationship between pretest Figural relations and change along the Induction dimension is also significant in the control group—their covariance is estimated at 47.44, with a standard error = 11.71 and t value = 4.05 that is significant. This leads to a correlation estimated at .79, indicating a close to strong relationship. Hence, individuals in the control group that start high on Figural relations tend to be among those improving most along the Induction dimension.

In conclusion, as illustrated in this section, the Level-and-Shape model represents a highly useful and at the same time readily and widely applicable approach to the analysis of change patterns, including the study of correlates and predictors of growth or decline, in longitudinal studies with at least three assessment occasions. The model does not impose the restriction of linear growth over time, unlike the Intercept-and-Slope model, and similarly to the latter assumes that all subjects are measured at the same time points.

APPENDIX TO CHAPTER 6

The one-factor latent change model as well as the Level-and-Shape model (and its more restrictive version, the Intercept-and-Slope model) are special cases of the general LISREL model. For the one-factor latent change model, this inclusive relationship immediately follows from the discussion in the main text. With regard to the LS model, we first note that at the individual subject level it assumes the following relations between the repeated assessments Y_{it} of a given characteristics:

$$Y_{it} = I_i + \lambda_t s_i + \varepsilon_{it}, \quad (\text{A6.1})$$

where $t = 1, 2, \dots, T$ denotes measurement occasion ($T > 2$; see Chap. 1 for the case $T = 2$), I_i is the true level of the i th subject (i.e., his/her true starting position on the measured characteristic), s_i his/her shape or true change over the entire set of repeated assessments, λ_t the part of overall mean true change that occurs between first and t th occasion (with $\lambda_1 = 0$ and $\lambda_T = 1$), and i is the subject index ($i = 1, 2, \dots, n$; n being sample size).

We next rewrite (A6.1) as follows:

$$\begin{aligned} Y_{it} &= I_i + \lambda_t s_i + \varepsilon_{it} \\ &= 1 I_i + \lambda_t s_i + \varepsilon_{it} \quad (i = 1, 2, \dots, n, t = 1, 2, \dots, T). \end{aligned} \quad (\text{A6.2})$$

The first line of Equation (A6.2), when considered across all assessment occasions, is readily seen as describing the individual trajectory, on the studied variable, of the i th individual across the T repeated assessments ($i = 1, 2, \dots, n$); while the second line is seen as representing a restricted CFA model. In the latter, according to the extended conceptualization of latent variable (see Chap. 1 and main text of present chapter), (a) the individual starting positions are viewed as scores on the first factor, F1, which loads 1 on each consecutive assessment; whereas the individual true changes across the study period are considered scores on the second factor, F2, with time-specific loadings on each repeated assessment.

Note that in discussing the components of Equation (A6.2) we depart from the traditional, 'psychometric' conceptualization of a latent variable. Specifically, the two factors of this CFA model, F1 and F2, are not viewed as proper behavioral traits that are of separate interest on their own. Rather, $F1 = I$ is a latent variable with individual scores being the subject-specific starting positions, while $F2 = s$ is a latent variable whose individual scores are the subject true change across the study period. Thus, F1 and F2 can be considered unobserved random variables whose individual realizations in a given study (i) are similarly not observed (as in the entire studied popula-

tion, for the same matter), and (ii) represent theoretically and empirically motivated aspects of the underlying developmental process of concern. In this feature, F1 and F2 can be considered extensions of the traditional behavioral conceptualization of latent variable, with means, variances and covariance that are of special interest in longitudinal analyses using the LS model.

This discussion shows that the LS model (A6.2) is a special case of a confirmatory factor analysis model, which itself is a special case of the general LISREL model. The same inclusive relationship holds for more general latent change analysis models (also referred to as latent curve analysis models), as shown in Meredith & Tisak (1990).

Epilogue

With this text, we aimed to introduce readers to the basics of the increasingly popular in the educational, medical, social, and behavioral sciences structural equation modeling (SEM) methodology. We discussed a number of issues that in our view are of special relevance for a newcomer to this field or someone having limited experience with this analytic approach. Our view is that while the SEM methodology can nowadays be readily and quickly applied on one's data, at times with minimal effort, there are nonetheless a multitude of fundamentals of this method whose understanding and mastery will greatly help its user benefit much more significantly from it.

Due to the introductory nature of this book, whose goal was to offer a first exposure to a number of important issues underlying much of the theory and practice of SEM, we could not cover many more advanced topics. We therefore caution the reader that having mastered the material presented in this book she or he only has what we hope to be a sound basis in this field, and thus should feel prepared for a study of those advanced topics; such a study needs to be undertaken before dealing with more complicated structural equation models. These issues include handling of missing data, analysis of categorical data, modeling with non-normal data, analyses of nonlinear relationships, and the more general framework of latent variable modeling with all its many ramifications and opportunities to analyze data and designs not touched upon here. A number of highly instructive books and treatments of these issues have appeared in the last decade or so (some of which can be found at <http://www.erlbaum.com>), and we recommend the reader study them for her/his dealing with advanced structural equation and latent variable models. We hope our text has provided the reader with most of what may be needed for this worthwhile journey ahead of them.

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