3 Application of Distribution Transformers

Now that I'm almost up the ladder,
I should, no doubt, be feeling gladder,
It is quite fine, the view and such,
If just it didn't shake so much.

Richard Armour

3.1 INTRODUCTION

In general, distribution transformers are used to reduce primary system voltages (2.4–34.5 kV) to utilization voltages (120–600 V). Table 3.1 gives standard transformer capacity and voltage ratings according to ANSI Standard C57.12.20-1964 for single-phase distribution transformers. Other voltages are also available, for example, 2400 × 7200, which is used on a 2400-V system that is to be changed later to 7200 V.

Secondary symbols used are the letter Y, which indicates that the winding is connected or may be connected wye, and Gnd Y, which indicates that the winding has one end grounded to the tank or brought out through a reduced insulation bushing. Windings that are delta-connected or may be connected delta are designated by the voltage of the winding only.

In Table 3.1, further information is given by the order in which the voltages are written for low-voltage windings. To designate a winding with a mid-tap which will provide half the full-winding kilovoltampere rating at half the full-winding voltage, the full-winding voltage is written first, followed by a slant, and then the mid-tap voltage. For example, 240/120 is used for a three-wire connection to designate a 120-V mid-tap voltage with a 240-V full-winding voltage. A winding which is appropriate for series, multiple, and three-wire connections will have the designation of multiple voltage rating followed by a slant and the series voltage rating, for example, the notation 120/240 means that the winding is appropriate either for 120-V multiple connection, for 240-V series connection, and for 240/120 three-wire connection. When two voltages are separated by a cross (×), a winding is indicated which is appropriate for both multiple and series connection but not for three-wire connection. The notation 120 × 240 is used to differentiate a winding that can be used for 120-V multiple connection and for 240-V series connection, but not for a three-wire connection. Examples of all symbols used are given in Table 3.2.

To reduce installation costs to a minimum, small distribution transformers are made for pole mounting in overhead distribution. To reduce size and weight, preferred oriented steel is commonly used in their construction. Transformers 100 kVA and below are attached directly to the pole, and transformers larger than 100 up to 500 kVA are hung on cross-beams or support lugs. If three or more transformers larger than 100 kVA are used, they are installed on a platform supported by two poles.

In underground distribution, transformers are installed in street vaults, in manholes direct-buried, on pads at ground level, or within buildings. The type of transformer may depend on soil content, lot location, public acceptance, or cost.

The distribution transformers and any secondary-service junction devices are installed within elements, usually placed on either the front or the rear lot lines of the customer's premises. The installation of the equipment to either front or rear locations may be limited by customer preference, local ordinances, and landscape conditions, and so on. The rule of thumb requires that a transformer

TABLE 3.1
Standard Transformer Kilovoltamperes and Voltages

Kilovoltamperes		High Voltages		Low Voltages	
Single-Phase	Three-Phase	Single-Phase	Single-Phase	Single-Phase	Three-Phase
5	30	2400/4160 Y	2400	120/240	208 Y/120
10	45	4800/8320 Y	4160 Y/2400	240/480	240
15	75	4800 Y/8320 YX	4160 Y	2400	480
25	112½	7200/12,470 Y	4800	2520	480 Y/277
37½	150	12,470 Gnd Y/7200	8320 Y/4800	4800	240 × 480
50	225	7620/13,200 Y	8320 Y	5040	2400
75	300	13,200 Gnd Y/7620	7200	6900	4160 Y/2400
100	500	12,000	12,000	7200	4800
167		13,200/22,860 Gnd Y	12,470 Y/7200	7560	12,470 Y/7200
250		13,200	12,470 Y	7980	13,200 Y/7620
333		13,800 Gnd Y/7970	13,200 Y/7620		
500		13,800/23,900 Gnd Y	13,200 Y		
		13,800	13,200		
		14,400/24,940 Gnd Y	13,800		
		16,340	22,900		
		19,920/34,500 Gnd Y	34,400		
		22,900	43,800		
		34,400	67,000		
		43,800			
		67,000			

be centrally located with respect to the load it supplies in order to provide proper cable economy, voltage drop, and esthetic effect.

Secondary-service junctions for an underground distribution system can be of the pedestal, hand-hole, or direct-buried splice types. No junction is required if the service cables are connected directly from the distribution transformer to the user's apparatus.

TABLE 3.2
Designation of Voltage Ratings for Single- and Three-Phase Distribution Transformers

Single-Phase		Three-Phase	
Designation	Meaning	Designation	Meaning
120/240	Series, multiple, or three-wire connection	2400/4160 Y	Suitable for delta or wye connection
240/120	Series or three-wire connection only	4160 Y	Wye connection only (no neutral)
240 × 480	Series or multiple connection only	4160 Y2400	Wye connection only (with neutral available)
120/208 Y	Suitable for delta or wye connection three-phase	12,470 Gnd Y/7200	Wye connection only (with reduced insulation neutral available)
12,470 Gnd Y/7200	One end of winding grounded to tank or brought out through reduced insulation bushing	4160	Delta connection only

Secondary or service conductors can be either copper or aluminum. However, in general, aluminum conductors are mostly used due to cost savings. The cables are single-conductor or triplexed. Neutrals may be either bare or covered, installed separately, or assembled with the power conductors. All secondary or service conductors are rated 600 V, and their sizes differ from #6 AWG to 1000 kcmil.

3.2 TYPES OF DISTRIBUTION TRANSFORMERS

Heat is a limiting factor in transformer loading. Removing the coil heat is an important task. In liquid-filled types, the transformer coils are immersed in a smooth-surfaced, oil-filled tank. Oil absorbs the coil heat and transfers it to the tank surface which, in turn, delivers it to the surrounding air. For transformers 25 kVA and larger, the size of the smooth tank surface required to dissipate the heat becomes larger than that required to enclose the coils. Therefore the transformer tank may be corrugated to add surface, or external tubes may be welded to the tank. To further increase the heat disposal capacity, air may be blown over the tube surface. Such designs are known as forced-air-cooled, with respect to self-cooled types. Presently, however, all distribution transformers are built to be self-cooled.

Therefore, the distribution transformers can be classified as: (*i*) dry-type and (*ii*) liquid-filled-type. The dry-type distribution transformers are air-cooled and air-insulated. The liquid-filled-type distribution transformers can further be classified as (a) oil-filled and (b) inerteen-filled.

The distribution transformers employed in overhead distribution systems can be categorized as:

1. Conventional transformers
2. Completely self-protecting (CSP) transformers
3. Completely self-protecting for secondary banking (CSPB) transformers.

The conventional transformers have no integral lightning, fault, or overload protective devices provided as a part of the transformer. The CSP transformers are, as the name implies, self-protecting from lightning or line surges, overloads, and short circuits. Lightning arresters mounted directly on the transformer tank, as shown in Figure 3.1, protect the primary winding against the lightning and line surges. The overload protection is provided by circuit breakers inside the transformer tank. The transformer is protected against an internal fault by internal protective links located between the primary winding and the primary bushings. Single-phase CSP transformers (oil-immersed, pole-mounted, 65°C, 60 Hz, 10–500 kVA) are available for a range of primary voltages from 2400 to 34,400 V. The secondary voltages are 120/240 or 240/480/277 V. The CSPB distribution transformers are designed for banked secondary service. They are built similar to the CSP transformers, but they are provided with two sets of circuit breakers. The second set is used to sectionalize the secondary when it is needed.

The distribution transformers employed in underground distribution systems can be categorized as:

1. Subway transformers
2. Low-cost residential transformers
3. Network transformers.

Subway transformers are used in underground vaults. They can be conventional-type or current-protected-type. Low-cost residential transformers are similar to those conventional transformers employed in overhead distribution. Network transformers are employed in the secondary networks. They have the primary disconnecting and grounding switch and the network protector mounted integrally on the transformer. They can be either liquid-filled, ventilated dry-type, or sealed dry-type.

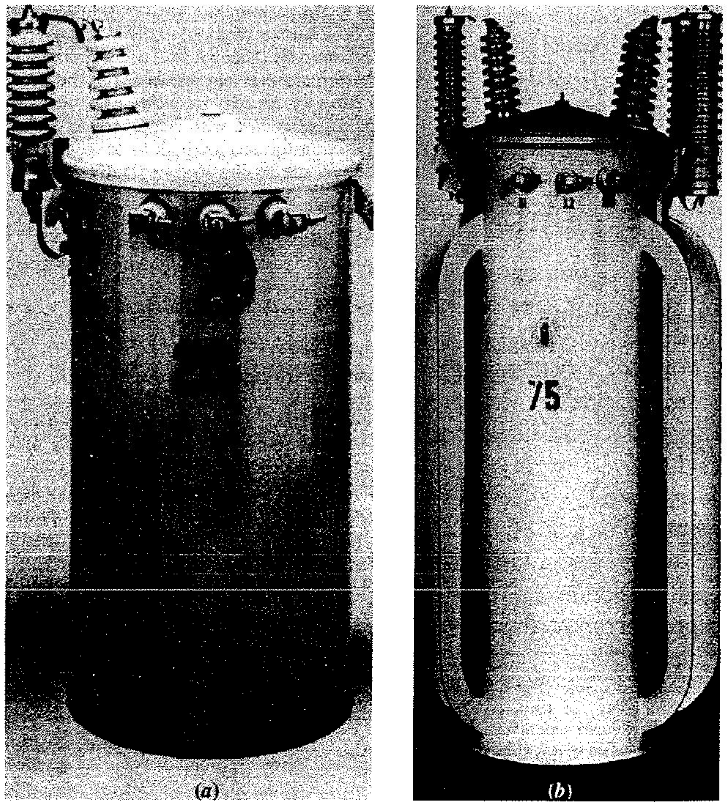

FIGURE 3.1 Overhead pole-mounted distribution transformers: (*a*) single-phase completely self-protecting (or conventional); (*b*) three phase. (From Westinghouse Electric Corporation. With permission.)

Figure 3.2 shows various types of transformers. Figure 3.2*a* shows a typical secondary-unit substation with the high- and low-voltage on opposite ends and full-length flanges for close coupling to high- and low-voltage switchgear. These units are normally made in sizes from 75 to 2500 kVA, three-phase, to 35-kV class. A typical single-phase pole-type transformer for a normal utility application is shown in Figure 3.2*b*. These are made from 10 to 500 kVA for delta and wye systems (one-bushing or two-bushing high voltage). Figure 3.2*c* shows a typical single-phase pad-mounted (minipad) utility-type transformer. These are made from 10 to 167 kVA. They are designed to do the same function as the pole type except that they are for the underground distribution system where all cables are below grade. A typical three-phase pad-mounted (stan-pad) transformer used by utilities as well as industrial and commercial applications is shown in Figure 3.2*d*. They are made from 45 to 2500 kVA normally, but have been made to 5000 kVA on special applications. They are also designed for underground service.

Figure 3.3*a* shows a typical three-phase subsurface-vault-type transformer used in utility applications in vaults below grade where there is no room to place the transformer elsewhere. These units are made for 75 to 2500 kVA and are made of a heavier gauge steel, special heavy corrugated radiators for cooling, and a special coal-tar type of paint.

A typical mobile transformer is shown in Figure 3.3*b*. These units are made for emergency applications and to allow utilities to reduce inventory. They are made typically for 500 to 2500 kVA. They can be used on underground service as well as overhead service. Normally they can have two or three primary voltages and two or three secondary voltages, so they may be used on any system

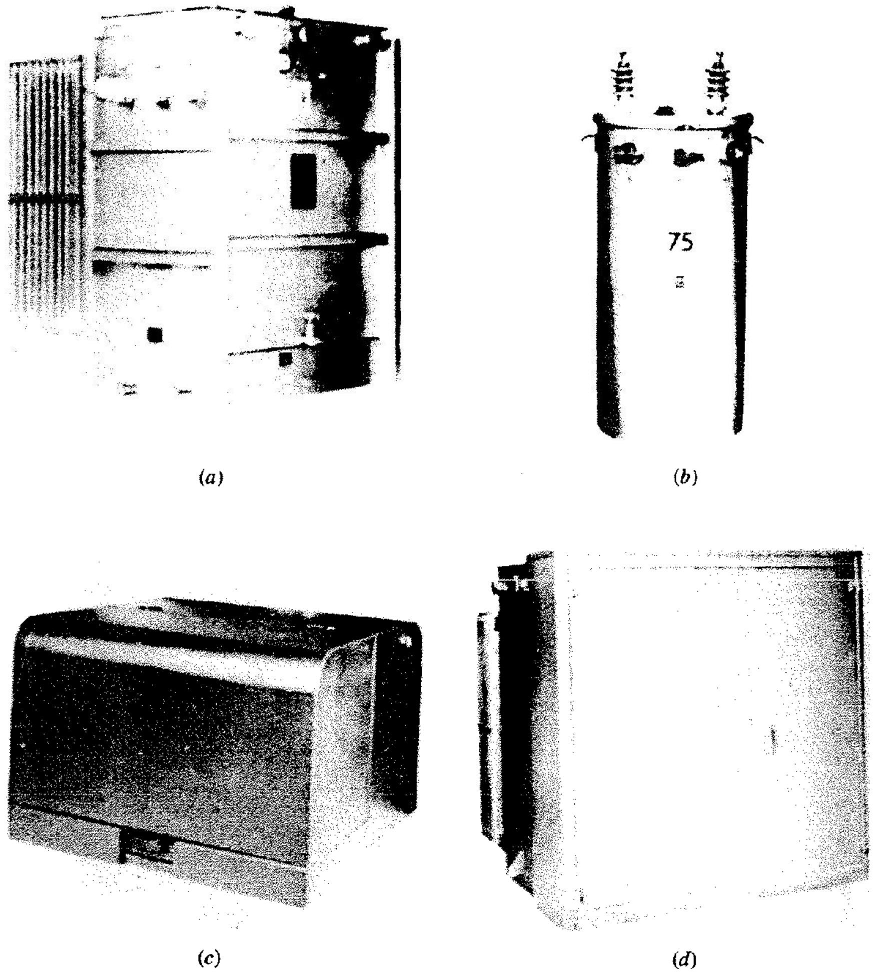

FIGURE 3.2 Various types of transformers. (From Balteau Standard Inc. With permission.)

the utility may have. For an emergency outage this unit is simply driven to the site, hooked up, and the power to the site is restored. This allows time to analyze and repair the failed unit. Figure 3.3*c* shows a typical power transformer. This class of unit is manufactured from 3700 kVA to 30 MVA up to about 138-kV class. The picture shows removable radiators to allow for a smaller size during shipment, and fans for increased capacity when required, including an automatic on-load tap changer which changes as the voltage varies.

Table 3.3 presents electrical characteristics of typical single-phase distribution transformers. Table 3.4 gives electrical characteristics of typical three-phase pad-mounted transformers. (For more accurate values, consult the individual manufacturer's catalogs.)

To find the resistance (R') and reactance (X') of a transformer of equal size and voltage, which has a different impedance value (Z') than the one shown in tables, multiply the tabulated percent values of R and X by the ratio of the new impedance value to the tabulated impedance value, that is, Z'/Z. Therefore, the resistance and the reactance of the new transformer can be found from

$$R' = R \times \frac{Z'}{Z} \tag{3.1}$$

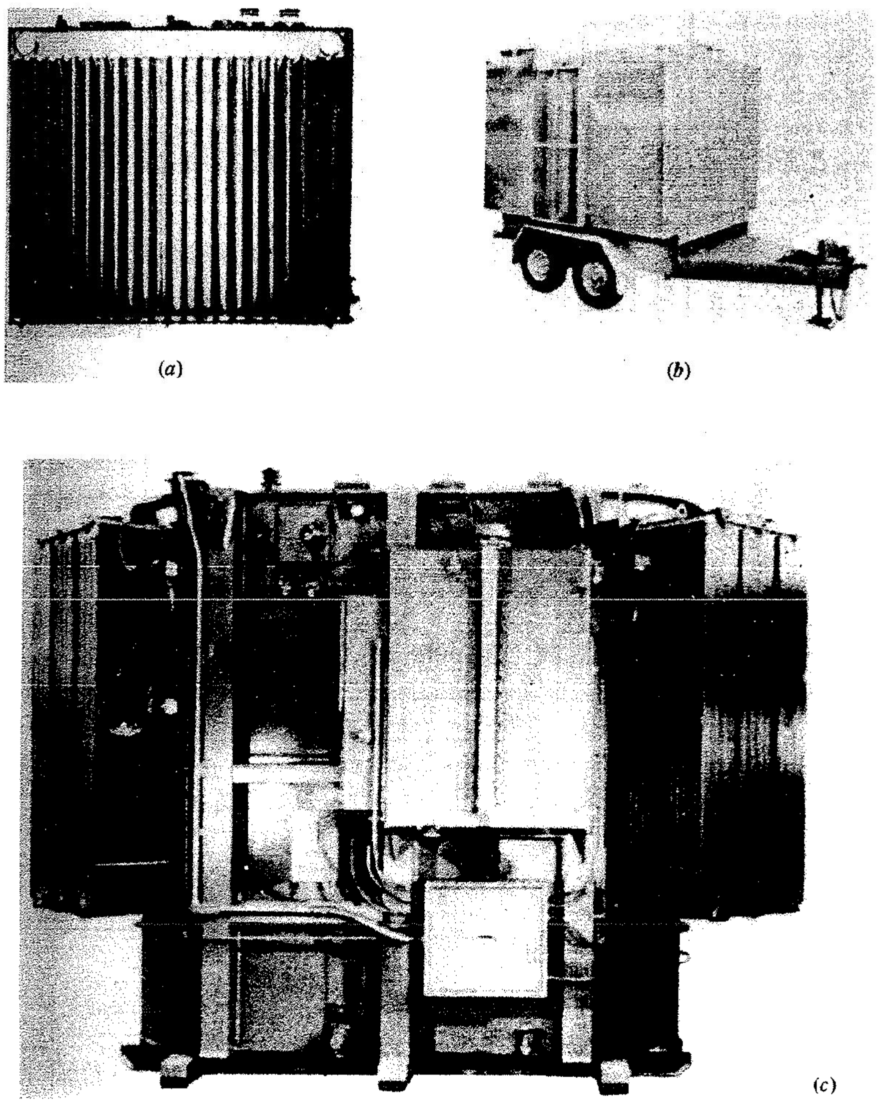

FIGURE 3.3 Various types of transformers. (From Balteau Standard Inc. With permission.)

and

$$X' = X \times \frac{Z'}{Z} \tag{3.2}$$

3.3 REGULATION

To calculate the transformer regulation for a kilovoltampere load of power factor $\cos\theta$, at rated voltage, any one of the following formulas can be used:

$$\%\,\text{regulation} = \frac{S_L}{S_T}\left[\%\,IR\cos\theta + \%\,IX\sin\theta + \frac{(\%\,IX\cos\theta - \%\,IR\sin\theta)^2}{200}\right] \tag{3.3}$$

TABLE 3.3
Electrical Characteristics of Typical Single-Phase Distribution Transformers*

kVA	Percent of Av. Excit. Curr.	Watts Loss No Load	Watts Loss Total	Watts Loss 1.0 PF	Watts Loss 0.8 PF	120/240-V Low Voltage % Regulation % Z	% R	% X	Watts Loss No Load	Watts Loss Total	Watts Loss 1.0 PF	Watts Loss 0.8 PF	240/480 and 277/480 Y V Low Voltage % Regulation % Z	% R	% X
						2400/4160 Y V High Voltage									
5	2.4	34	137	2.06	2.12	2.2	2.1	0.8							
10	1.6	68	197	1.30	1.68	1.7	1.3	1.1	68	202	1.35	1.69	1.7	1.3	1.0
15	1.4	84	272	1.27	1.59	1.6	1.3	1.0	84	277	1.30	1.60	1.6	1.3	1.1
25	1.3	118	385	1.10	1.65	1.7	1.1	1.1	118	390	1.11	1.65	1.7	1.1	1.3
38	1.1	166	540	1.00	1.55	1.6	1.0	1.3	166	550	1.04	1.54	1.6	1.0	1.2
50	1.0	185	615	0.88	1.58	1.7	0.9	1.5	185	625	0.90	1.58	1.7	0.9	1.5
75	1.3	285	910	0.85	1.41	1.5	0.8	1.2	285	925	0.86	1.33	1.4	0.9	1.1
100	1.2	355	1175	0.84	1.55	1.7	0.8	1.5	355	1190	0.85	1.49	1.6	0.8	1.4
167	1.0	500	2100	0.99	1.75	1.9	1.0	1.6	500	2000	0.90	1.57	1.7	0.9	1.4
250	1.0	610	3390	1.16	2.16	2.4	1.1	2.1	610	3280	1.11	2.02	2.2	1.1	1.9
333	1.0	840	4200	1.08	2.51	3.0	1.0	2.8	840	3690	0.88	1.90	2.2	0.9	1.9
500	1.0	1140	5740	0.97	2.50	3.1	0.9	3.0	1140	4810	0.95	2.00	2.3	0.7	2.2
						7200/12,470 Y V High Voltage									
5	2.4	41	144	2.07	2.11	2.2	2.1	0.8							
10	1.6	68	204	1.37	1.80	1.8	1.4	1.2	68	209	1.43	1.80	1.8	1.4	1.1
15	1.4	84	282	1.33	1.69	1.7	1.3	1.2	84	287	1.35	1.70	1.7	1.4	1.0
25	1.3	118	422	1.22	1.69	1.7	1.2	1.2	118	427	1.24	1.69	1.7	1.2	1.2
38	1.1	166	570	1.10	1.64	1.7	1.1	1.3	166	575	1.10	1.65	1.7	1.1	1.3
50	1.0	185	720	1.10	1.71	1.8	1.1	1.4	185	725	1.10	1.71	1.8	1.1	1.4
75	1.3	285	985	0.95	1.60	1.7	0.9	1.4	285	1000	0.97	1.52	1.6	1.0	1.3
100	1.2	355	1275	0.95	1.72	1.9	1.9	1.7	355	1290	0.95	1.60	1.7	1.9	1.4
167	1.0	500	2100	0.98	1.90	2.1	1.0	1.9	500	2000	0.91	1.70	1.9	0.9	1.7

continued

TABLE 3.3 (continued)
Electrical Characteristics of Typical Single-Phase Distribution Transformers*

	Percent of	Watts Loss				120/240-V Low Voltage % Regulation			Watts Loss				240/480 and 277/480 Y V Low Voltage % Regulation		
	Av. Excit.					%	%	%					%	%	%
kVA	Curr.	No Load	Total	1.0 PF	0.8 PF	*Z*	*R*	*X*	No Load	Total	1.0 PF	0.8 PF	*Z*	*R*	*X*
250	1.0	610	3490	1.22	2.45	2.8	1.2	2.6	610	3250	1.17	2.19	2.4	1.1	2.2
333	1.0	840	4255	1.07	2.50	3.0	1.0	2.8	840	3690	0.89	2.03	2.4	0.9	2.2
500	1.0	1140	5640	0.95	2.55	3.2	0.9	3.1	1140	4810	0.78	1.99	2.4	0.7	2.3
13,200/22,860 Gnd Y or 13,800/23,900 Gnd Y or 14,400/24,940 Gnd Y V High Voltage															
5	2.4	42	154	2.25	2.30	2.4	2.3	0.9							
10	1.6	73	215	1.45	1.89	1.9	1.4	1.3	73	220	1.49	1.89	1.9	1.5	1.2
15	1.4	84	305	1.48	1.80	1.8	1.5	1.0	84	310	1.52	1.80	1.8	1.5	1.0
25	1.3	118	437	1.29	1.79	1.8	1.3	1.3	118	442	1.30	1.78	1.8	1.3	1.2
38	1.1	166	585	1.15	1.72	1.8	1.1	1.4	166	590	1.16	1.72	1.8	1.1	1.4
50	1.0	185	735	1.14	1.81	1.9	1.1	1.4	185	740	1.15	1.81	1.9	1.1	1.5
75	1.4	285	1050	1.05	1.78	1.8	1.0	1.5	285	1065	1.06	1.78	1.8	1.0	1.5
100	1.3	355	1300	0.97	1.81	2.0	0.9	1.8	355	1310	0.98	1.74	1.9	1.0	1.6
167	1.0	500	2160	0.98	1.96	2.2	1.0	2.0	500	2060	0.95	1.80	2.0	0.9	1.8
250	1.0	610	3490	1.22	2.52	2.9	1.2	2.7	610	3285	1.11	2.16	2.5	1.1	2.3
333	1.0	840	4300	1.09	2.60	3.1	1.0	2.9	840	3750	0.91	2.05	2.4	0.9	2.2
500	1.0	1140	5640	0.95	2.55	3.2	1.1	3.0	1140	4760	0.76	1.98	2.4	0.7	2.3

TABLE 3.4
Electrical Characteristics of Typical Three-Phase Pad-Mounted Transformers

						208 Y/120 V Low Voltage % Regulation							480 Y/277 V Low Voltage % Regulation		
	Percent of Av. Excit.	Watts Loss				%	%	%	Watts Loss				%	%	%
kVA	Curr.	No Load	Total	1.0 PF	0.8 PF	*Z*	*R*	*X*	No load	Total	1.0 PF	0.8 PF	*Z*	*R*	*X*
						4160 Gnd Y/2400X12,470 Gnd Y/7200 V High Voltage									
75	1.5	360	1350	1.35	2.1	2.1	1.3	1.6	360	1350	1.35	2.1	2.1	1.3	1.6
112	1.0	530	1800	1.15	1.7	1.7	1.1	1.3	530	1800	1.15	1.7	1.7	1.1	1.3
150	1.0	560	2250	1.15	1.9	1.9	1.1	1.6	560	2250	1.15	1.9	1.9	1.1	1.6
225	1.0	880	3300	1.10	1.9	1.9	1.1	1.6	800	3300	1.10	1.9	1.9	1.1	1.6
300	1.0	1050	4300	1.10	1.9	2.0	1.1	1.7	1050	4100	1.05	1.8	1.8	1.0	1.5
500	1.0	1600	6800	1.15	2.2	2.3	1.0	2.1	1600	6500	1.10	2.0	2.0	1.0	1.7
750	1.0	1800	10,200	1.28	4.4	5.7	1.1	5.6	1800	9400	1.18	4.3	5.7	1.0	5.6
1000	1.0	2100	12,500	1.20	4.3	5.7	1.0	5.6	2100	10,900	1.04	4.2	5.7	0.9	5.7
1500	1.0	2900	19,400	1.26	4.3	5.7	1.1	5.6	3300	16,500	1.04	4.2	5.7	0.9	5.7
2500	1.0								4800	26,600	1.03	4.2	5.7	0.9	5.7
3750	1.0								6500	35,500	0.95	4.1	5.7	0.8	5.7
						12,470 Gnd Y/7200 V High Voltage									
75	1.5	360	1350	1.4	1.7	1.7	1.3	1.1	360	1350	1.4	1.5	1.5	1.3	0.8
112	1.0	530	1800	1.2	1.5	1.5	1.1	1.0	530	1800	1.2	1.3	1.3	1.1	0.7
150	1.0	560	2250	1.2	1.8	1.9	1.1	1.6	560	2250	1.2	1.7	1.7	1.1	1.3
225	1.0	880	3300	1.1	1.8	1.8	1.1	1.4	880	3300	1.1	1.6	1.6	1.1	1.2
300	1.0	1050	4300	1.1	1.6	1.6	1.1	1.2	1050	4100	1.1	1.4	1.4	1.0	1.0
500	1.0	1600	6800	1.1	1.7	1.7	1.0	1.4	1600	6500	1.1	1.4	1.4	1.0	1.0
750	1.0	1800	10,200	1.3	4.4	5.7	1.1	5.6	1800	9400	1.2	4.3	5.7	1.0	5.6
1000	1.0	2100	12,500	1.2	4.3	5.7	1.0	5.6	2100	10,900	1.0	4.2	5.7	0.9	5.7

continued

TABLE 3.4 (continued)
Electrical Characteristics of Typical Three-Phase Pad-Mounted Transformers

	Percent of Av. Excit.	Watts Loss				208 Y/120 V Low Voltage % Regulation			Watts Loss				480 Y/277 V Low Voltage % Regulation		
kVA	Curr.	No Load	Total	1.0 PF	0.8 PF	% *Z*	% *R*	% *X*	No Load	Total	1.0 PF	0.8 PF	% *Z*	% *R*	% *X*
1500	1.0	2900	19,400	1.3	4.3	5.7	1.1	5.6	3300	16,500	1.0	4.2	5.7	0.9	5.7
2500	1.0								4800	26,600	1.0	4.2	5.7	0.9	5.7
3750	1.0								6500	35,500	0.9	4.1	5.7	0.8	5.7
						2400/4160 Y/2400 V Low Voltage									
12,470 Delta V High Voltage															
1000	1.38	2443	11,480	1.06	4.09	5.56	0.89	5.49							
1500	1.33	3455	15,716	0.98	4.04	5.56	0.81	5.51							
2500	1.29	4956	23,193	0.92	3.97	5.56	0.73	5.52							
3750	1.37	6775	33,100	0.89	3.97	5.50	0.70	5.45							
5000	1.33	8800	42,125	0.86	3.94	5.50	0.67	5.45							
24, 940 Delta V High Voltage															
1000	1.42	2533	11.588	1.07	4.09	5.56	0.91	5.49							
1500	1.37	3625	15,213	0.96	4.03	5.56	0.80	5.50							
2500	1.31	5338	23,213	0.88	3.98	5.56	0.72	5.52							
3750	1.42	7075	33,700	0.90	3.97	5.50	0.71	5.44							
5000	1.33	8725	43,550	0.88	3.96	5.50	0.69	5.44							

or

$$\% \text{regulation} = \frac{I_{op}}{I_{ra}}\left[\% R\cos\theta + \% X\sin\theta + \frac{(\% X\cos\theta - \% R\sin\theta)^2}{200}\right] \quad (3.4)$$

or

$$\% \text{regulation} = V_R\cos\theta + V_X\sin\theta + \frac{(V_X\cos\theta - V_R\sin\theta)^2}{200} \quad (3.5)$$

where θ is the power factor angle of the load, V_R is the percent resistance voltage = copper loss/output × 100, S_L is the apparent load power, S_T is the rated apparent power of the transformer, I_{op} is the operating current, I_{ra} is the rated current, V_X is the percent leakage reactance voltage $(V_Z^2 - V_R^2)^{1/2}$, and V_Z is the percent impedance voltage.

Note that the percent regulation at unity power factor is

$$\% \text{regulation} = \frac{\text{copper loss}}{\text{output}} \times 100 + \frac{(\% \text{reactance})^2}{200}. \quad (3.6)$$

3.4 TRANSFORMER EFFICIENCY

The efficiency of a transformer can be calculated from

$$\% \text{efficiency} = \frac{\text{output in watts}}{\text{output in watts} + \text{total losses in watts}} \times 100. \quad (3.7)$$

The total losses include the losses in the electric circuit, magnetic circuit, and dielectric circuit. Stigant and Franklin [3] state that a transformer has its highest efficiency at a load at which the iron loss and copper loss are equal. Therefore, the load at which the efficiency is highest can be found from

$$\% \text{load} = \left(\frac{\text{iron loss}}{\text{copper loss}}\right)^{1/2} \times 100. \quad (3.8)$$

Figures 3.4 and 3.5 show nomograms for quick determination of the efficiency of a transformer. (For more accurate values, consult the individual manufacturer's catalogs.) With the cost of electric energy presently 5–6 cents/kWh and projected to double within the next 10–15 yr, as shown in Figure 3.6, the cost efficiency of transformers now shifts to align itself with energy efficiency.

Note that the iron losses (or core losses) include (i) hysteresis loss and (ii) eddy-current loss. The hysteresis loss is due to the power requirement of maintaining the continuous reversals of the elementary magnets (or individual molecules) of which the iron is composed as a result of the flux alternations in a transformer core. The eddy-current loss is the loss due to circulating currents in the core iron, caused by the time-varying magnetic fluxes within the iron. The eddy-current loss is proportional to the square of the frequency and the square of the flux density. The core is built up of thin laminations insulated from each other by an insulating coating on the iron to reduce the

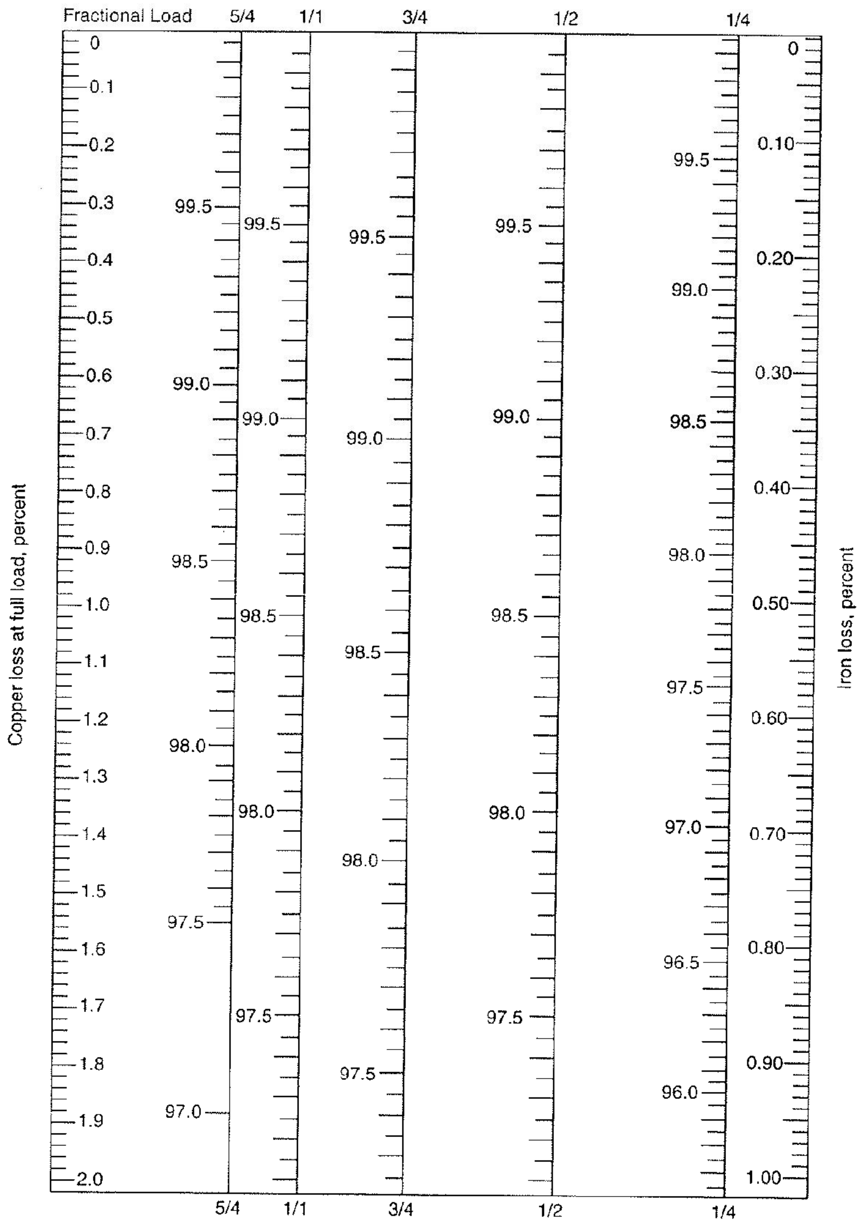

FIGURE 3.4 Transformer efficiency chart applicable only to the unity power factor condition. To obtain the efficiency at a given load, lay a straight edge across the iron and copper loss values and read the efficiency at the point where the straight edge cuts the required load ordinate. (From Stigant, S. A., and A. C. Franklin, *The J&P Transformer Book*, Butterworth, London, 1973.)

eddy-current loss. Also, in order to reduce the hysteresis loss and the eddy-current loss, special grades of steel alloyed with silicon are used. The iron or core losses are practically independent of the load. On the other hand, the copper losses are due to the resistance of the primary and secondary windings.

In general, the distribution transformer costs can be classified as: (*i*) the cost of the investment, (*ii*) the cost of lost energy due to the losses in the transformer, and (*iii*) the cost of demand lost

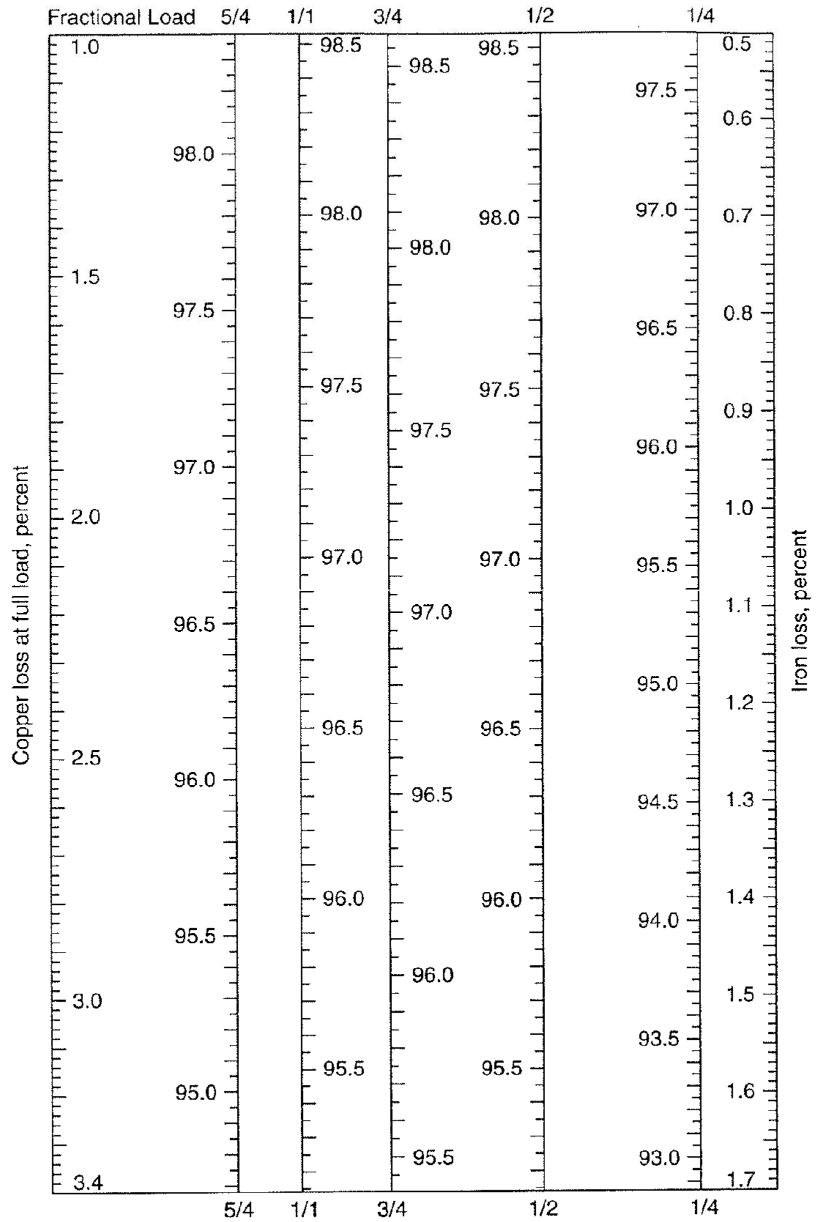

FIGURE 3.5 Transformer efficiency chart applicable only to the unity PF condition. To obtain the efficiency at a given load, lay a straightedge across the iron and copper loss values and read the efficiency at the point where the straightedge cuts the required load ordinate. (From Stigant, S. A., and A. C. Franklin, *The J&P Transformer Book*, Butterworth, London, 1973. With permission.)

(i.e., the cost of lost capacity) due to the losses in the transformer. Of course, the cost of investment is the largest cost component, and it includes the cost of the transformer itself and the costs of material and labor involved in the transformer installation.

Figure 3.7 shows the annual cost per unit load versus load level. At low-load levels, the relatively high costs result basically from the investment cost, whereas at high-load levels, they are due to the cost of additional loss of life of the transformer, the cost of lost energy, and the cost of

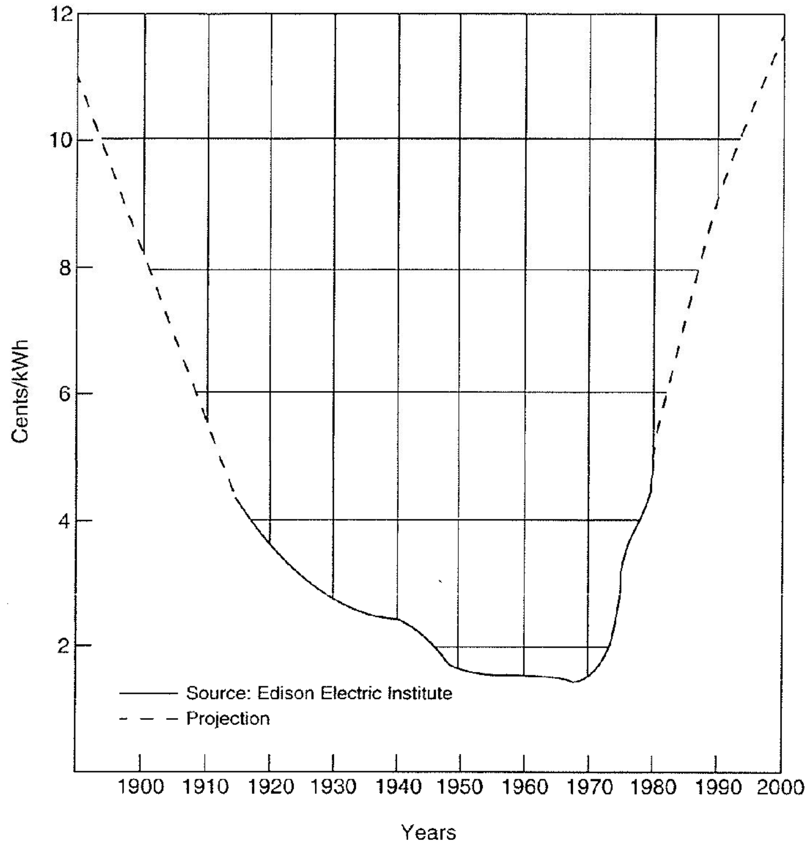

FIGURE 3.6 Cost of electric energy. (From Stigant, S. A., and A. C. Franklin, *The J&P Transformer Book*, Butterworth, London, 1973. With permission.)

demand loss in addition to the investment cost. Figure 3.7 indicates an operating range close to the bottom of the curve. Usually, it is economical to install a transformer at approximately 80% of its nameplate rating and to replace it later, at approximately 180%, by one with a larger capacity. However, presently, increasing costs of capital, plant and equipment, and energy tend to reduce these percentages.

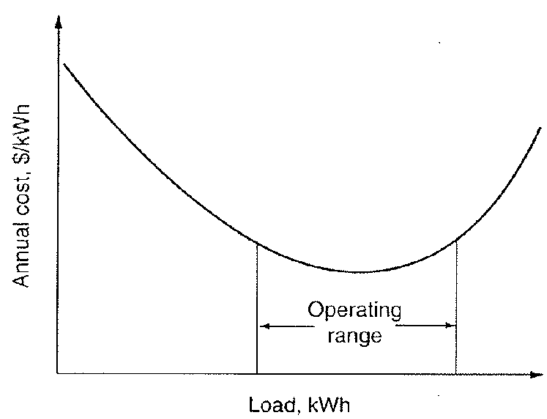

FIGURE 3.7 Annual cost per unit load versus load level.

3.5 TERMINAL OR LEAD MARKINGS

The terminals or leads of a transformer are the points to which external electric circuits are connected. According to NEMA and ASA standards, the higher-voltage winding is identified by HV or H, and the lower-voltage winding is identified by LV or x. Transformers with more than two windings have the windings identified as H, x, y, and z, in order of decreasing voltage. The terminal H_1 is located on the right-hand side when facing the high-voltage side of the transformer. On single-phase transformers the leads are numbered so that when H_1 is connected to x_1, the voltage between the highest-numbered H lead and the highest-numbered x lead is less than the voltage of the high-voltage winding.

On three-phase transformers, the terminal H_1 is on the right-hand side when facing the high-voltage winding, with the H_2 and H_3 terminals in numerical sequence from right to left. The terminal x_1 is on the left-hand side when facing the low-voltage winding, with the x_2 and x_3 terminals in numerical sequence from left to right.

3.6 TRANSFORMER POLARITY

Transformer-winding terminals are marked to show polarity, to indicate the high-voltage from the low-voltage side. Primary and secondary are not identified as such because which is which depends on input and output connections.

Transformer polarity is an indication of the direction of current flowing through the high-voltage leads with respect to the direction of current flowing through the low-voltage leads at any given instant. In other words, the transformer polarity simply refers to the relative direction of induced voltages between the high-voltage leads and the low-voltage terminals. The polarity of a single-phase distribution transformer may be additive or subtractive. With standard markings, the voltage from H_1 to H_2 is always in the same direction or in phase with the voltage from X_1 to X_2. In a transformer where H_1 and X_1 terminals are adjacent, as shown in Figure 3.8*a*, the transformer is said to have *subtractive* polarity. On the other hand, when terminals H_1 and X_1 are diagonally opposite, as shown in Figure 3.8*b*, the transformer is said to have *additive* polarity.

Transformer polarity can be determined by performing a simple test in which two adjacent terminals of the high- and low-voltage windings are connected together and a moderate voltage is applied to the high-voltage winding, as shown in Figure 3.9, and then the voltage between the high- and low-voltage winding terminals that are not connected together are measured. The polarity is subtractive if the voltage read is less than the voltage applied to the high-voltage winding, as shown in Figure 3.9*a*. The polarity is additive if the voltage read is greater than the applied voltage, as shown in Figure 3.9*b*.

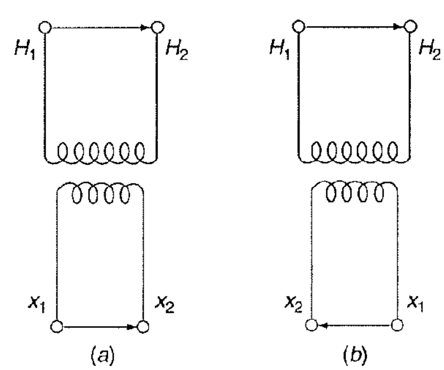

FIGURE 3.8 Additive and subtractive polarity connections: (*a*) subtractive polarity and (*b*) additive polarity.

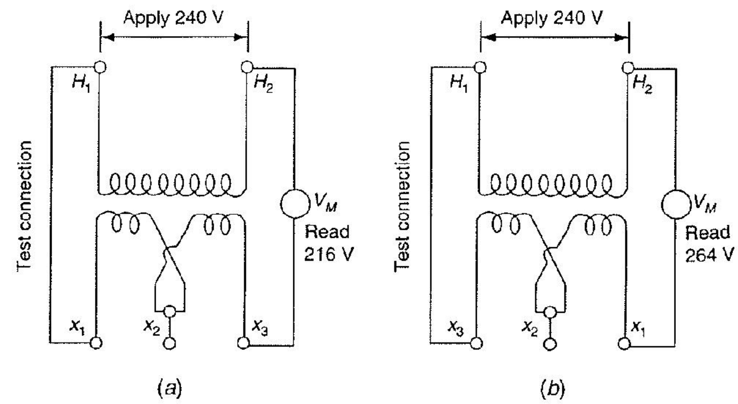

FIGURE 3.9 Polarity test: (*a*) subtractive polarity and (*b*) additive polarity.

By industry standards, all single-phase distribution transformers 200 kVA and smaller, having high voltages of 8660 V and below (winding voltages), have additive polarity. All other single-phase transformers have a subtractive polarity. Polarity markings are very useful when connecting transformers into three-phase banks.

3.7 DISTRIBUTION TRANSFORMER LOADING GUIDES

The rated kilovoltamperes of a given transformer is the output which can be obtained continuously at rated voltage and frequency without exceeding the specified temperature rise. Temperature rise is used for rating purposes rather than actual temperature, since the ambient temperature may vary considerably under operating conditions. The life of insulation commonly used in transformers depends on the temperature that the insulation reaches and the length of time that this temperature is sustained. Therefore, before the overload capabilities of the transformer can be determined, the ambient temperature, preload conditions, and the duration of peak loads must be known.

Based on Appendix C57.91 entitled *The Guide for Loading Mineral Oil-Immersed Overhead-Type Distribution Transformers with 55°C and 65°C Average Winding Rise* [4], which is an appendix to the ANSI Overhead Distribution Standard C57.12, 20 transformer insulation-life curves were developed. These curves indicate a minimum life expectancy of 20 yr at 95°C and 110°C hot-spot temperatures for 55°C and 65°C rise transformers. Previous transformer-loading guides were based on the so-called 8°C insulation life rule. For example, for transformers with class A insulation (usually oil-filled), the rate of deterioration doubles approximately with each 8°C increase in temperature. In other words, if a class A insulation transformer were operated 8°C above its rated temperature, its life would be reduced by half.

3.8 EQUIVALENT CIRCUITS OF A TRANSFORMER

It is possible to use several equivalent circuits to represent a given transformer. However, the general practice is to choose the simplest one which would provide the desired accuracy in calculations.

Figure 3.10 shows an equivalent circuit of a single-phase two-winding transformer. It represents a practical transformer with an iron core and connected to a load (L). When the primary winding is excited, a flux is produced through the iron core. The flux that links both primary and secondary is called the *mutual flux*, and its maximum value is denoted as ϕ_m. However, there are also leakage fluxes ϕ_{l1} and ϕ_{l2} that are produced at the primary and secondary windings,

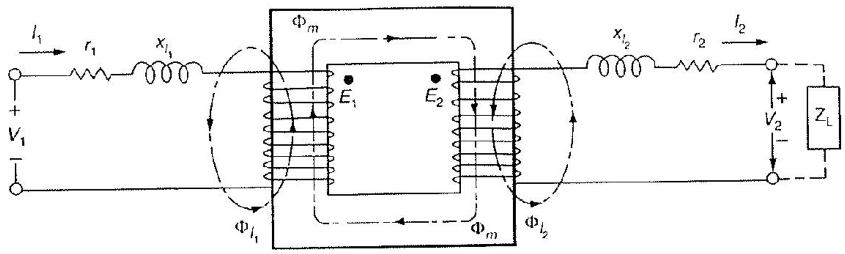

FIGURE 3.10 Basic circuit of a practical transformer.

respectively. In turn, the ϕ_{l1} and ϕ_{l2} leakage fluxes produce x_{l1} and x_{l2}, that is, primary and secondary inductive reactances, respectively. The primary and secondary windings also have their internal resistances of r_1 and r_2.

Figure 3.11 shows an equivalent circuit of a loaded transformer. Note that I_2' current is a primary-current (or load) component which exactly corresponds to the secondary current I_2, as it does for an ideal transformer. Therefore

$$I_2' = \frac{n_2}{n_1} \times I_2 \tag{3.9}$$

or

$$I_2' = \frac{I_2}{n} \tag{3.10}$$

where I_2 is the secondary current, n_1 is the number of turns in the primary winding, n_2 is the number of turns in the secondary winding, and n is the turns ratio = n_1/n_2.

The I_e current is the excitation current component of the primary current I_1 that is needed to produce the resultant mutual flux. As shown in Figure 3.12, the excitation current I_e also has two components, namely, (*i*) the magnetizing current component I_m and (*ii*) the core-loss component I_c. The r_c represents the equivalent transformer power loss due to (hysteresis and eddy current) iron losses in the transformer core as a result of the magnetizing current I_e. The x_m represents the inductive reactance components of the transformer with an open secondary.

Figure 3.13 shows an approximate equivalent circuit with combined primary and reflected secondary and load impedances. Note that the secondary current I_2 is seen by the primary side as I_2/n and that the secondary and load impedances are transferred (or *referred*) to the primary side

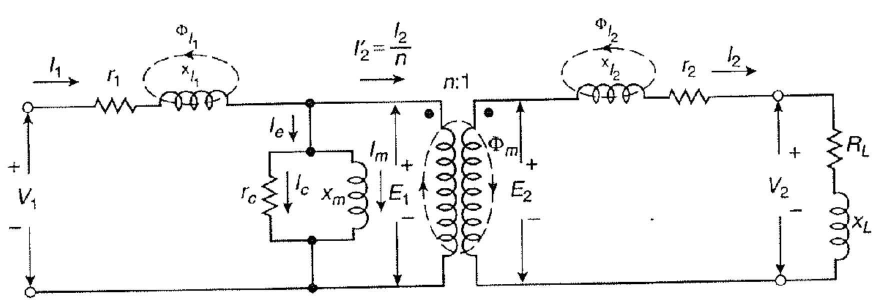

FIGURE 3.11 Equivalent circuit of a loaded transformer.

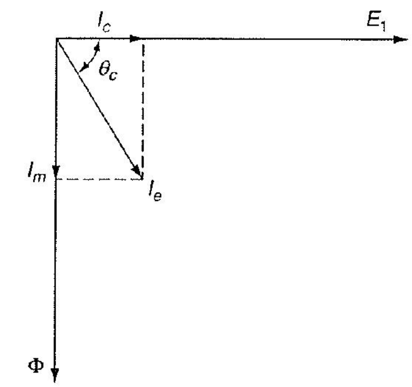

FIGURE 3.12 Phasor diagram corresponding to the excitation current components at no load.

as $n^2(r_2 + jx_{l2})$ and $n^2(R_L + jX_L)$, respectively. Also note that the secondary-side terminal voltage V_2 is transferred as nV_2.

Since the excitation current I_e is very small with respect to I_2/n for a loaded transformer, the former may be ignored, as shown in Figure 3.14. Therefore, the equivalent impedance of the transformer referred to the primary is

$$\begin{aligned} Z_{eq} &= Z_1 + Z_2' \\ &= Z_1 + n^2 Z_2 \\ &= r_{eq} + jx_{eq} \end{aligned} \tag{3.11}$$

where

$$Z_1 = r_1 + jx_{l1} \tag{3.12}$$

$$Z_2 = r_2 + jx_{l2} \tag{3.13}$$

and therefore the equivalent resistance and reactance of the transformer referred to the primary are

$$r_{eq} = r_1 + jn^2 r_2 \tag{3.14}$$

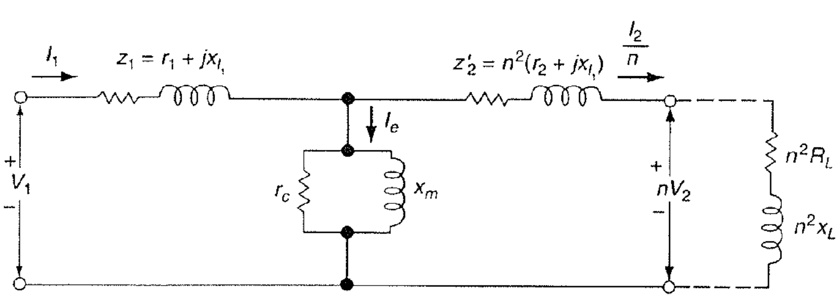

FIGURE 3.13 Equivalent circuit with the referred secondary values.

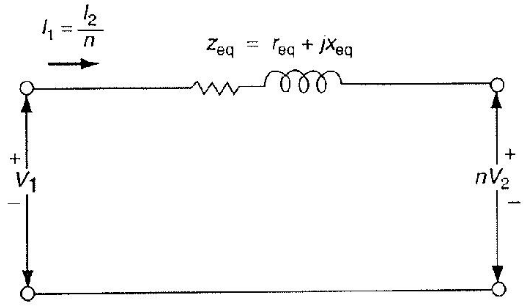

FIGURE 3.14 Simplified equivalent circuit assuming negligible excitation current.

and

$$x_{eq} = x_{l1} + n^2 x_{l2}. \tag{3.15}$$

As before in Figure 3.15, for large-size power transformers,

$$r_{eq} \longrightarrow 0$$

therefore the equivalent impedance of the transformer is

$$Z_{eq} = jx_{eq}. \tag{3.16}$$

3.9 SINGLE-PHASE TRANSFORMER CONNECTIONS

3.9.1 General

At the present time, the single-phase distribution transformers greatly outnumber the poly-phase ones. This is partially due to the fact that lighting and the smaller power loads are supplied at single-phase from single-phase secondary circuits. Also, most of the time, even poly-phase secondary systems are supplied by single-phase transformers which are connected as poly-phase banks.

Single-phase distribution transformers have one high-voltage primary winding and two low-voltage secondary windings which are rated at a nominal 120 V. Earlier transformers were built

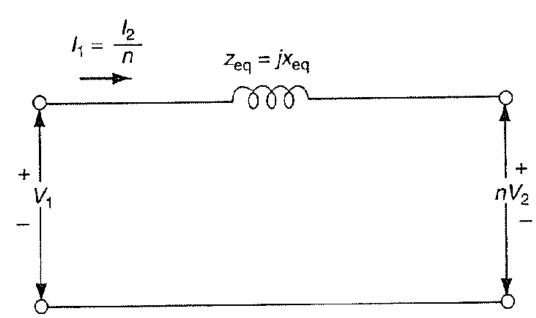

FIGURE 3.15 Simplified equivalent circuit for a large-sized power transformer.

with four insulated secondary leads brought out of the transformer tank, the series or parallel connection being made outside the tank. Presently, in modern transformers, the connections are made inside the tank, with only three secondary terminals being brought out of the transformer.

Single-phase distribution transformers have one high-voltage primary winding and two low-voltage secondary windings. Figure 3.16 shows various connection diagrams for single-phase transformers supplying single-phase loads. Secondary coils each rated at a nominal 120 V may be connected in parallel to supply a two-wire 120-V circuit, as shown in Figure 3.16*a* and *b*, or they may be connected in series to supply a three-wire 120/240-V single circuit, as shown in Figure 3.16*c* and *d*. The connections shown in Figure 3.16*a* and *b* are used where the loads are comparatively small and the length of the secondary circuits is short. It is often used for a single customer who requires only 120-V single-phase power. However, for modern homes, this connection usually is not considered adequate. If a mistake is made in polarity when connecting the two secondary coils in parallel (Figure 3.16*a*) so that the low-voltage terminal 1 is connected to terminal 4 and terminal 2 to terminal 3, the result will be a short-circuited secondary which will blow the fuses that are installed on the

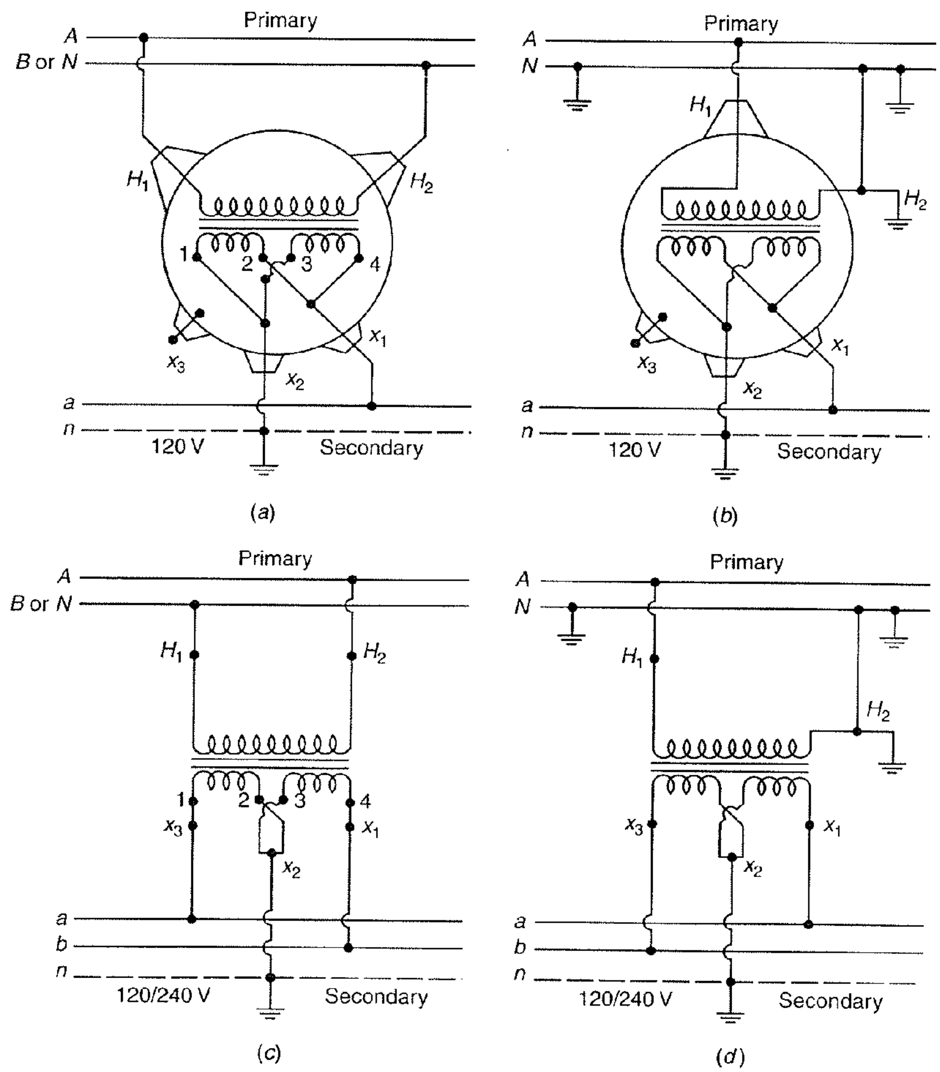

FIGURE 3.16 Single-phase transformer connections.

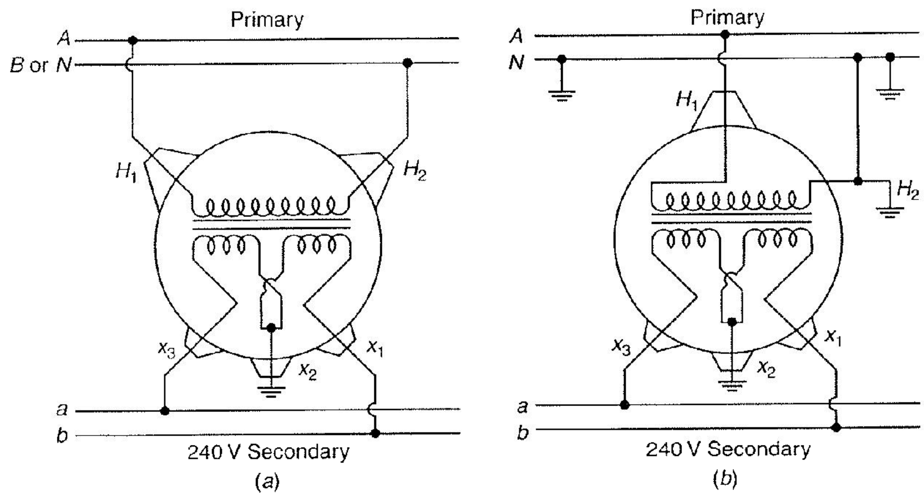

FIGURE 3.17 Single-phase transformer connections.

high-voltage side of the transformer (they are not shown in the figure). On the other hand, a mistake in polarity when connecting the coils in series (Fig. 3.16*c*) will result in the voltage across the outer conductors being zero instead of 240 V. Taps for voltage adjustment, if provided, are located on the high-voltage winding of the transformer. Figure 3.16*b* and d shows single-bushing transformers connected to a multigrounded primary. They are used on 12,470 GndY/7200-, 13,200 GndY/7620-, and 24,940 GndY/14,400-V multigrounded neutral systems. It is crucial that good and solid grounds are maintained on the transformer and on the system. Figure 3.17 shows single-phase transformer connections for single- and two-bushing transformers to provide customers who require only 240-V single-phase power. These connections are used for small industrial applications.

In general, however, the 120/240-V three-wire connection system is preferred since it has twice the load capacity of the 120-V system with only 12 times the amount of the conductor. Here, each 120-V winding has one-half the total kilovoltampere rating of the transformer. Therefore, if the connected 120-V loads are equal, the load is balanced and no current flows in the neutral conductor. Thus the loads connected to the transformer must be held as nearly balanced as possible to provide the most economical usage of the transformer capacity and to keep the regulation to a minimum. Normally, one leg of the 120-V two-wire system and the middle leg of the 240-V two-wire or 120/240-V three-wire system is grounded to limit the voltage to ground on the secondary circuit to a minimum.

3.9.2 Single-Phase Transformer Paralleling

When greater capacity is required in emergency situations, two single-phase transformers of the same or different kilovoltampere ratings can be connected in parallel. The single-phase transformers can be of either additive or subtractive polarity as long as the following conditions are observed and connected, as shown in Figure 3.18.

1. All transformers have the same turns ratio.
2. All transformers are connected to the same primary phase.
3. All transformers have identical frequency ratings.
4. All transformers have identical voltage ratings.
5. All transformers have identical tap settings.
6. Per unit (pu) impedance of one transformer is between 0.925 and 1.075 of the other in order to maximize capability.

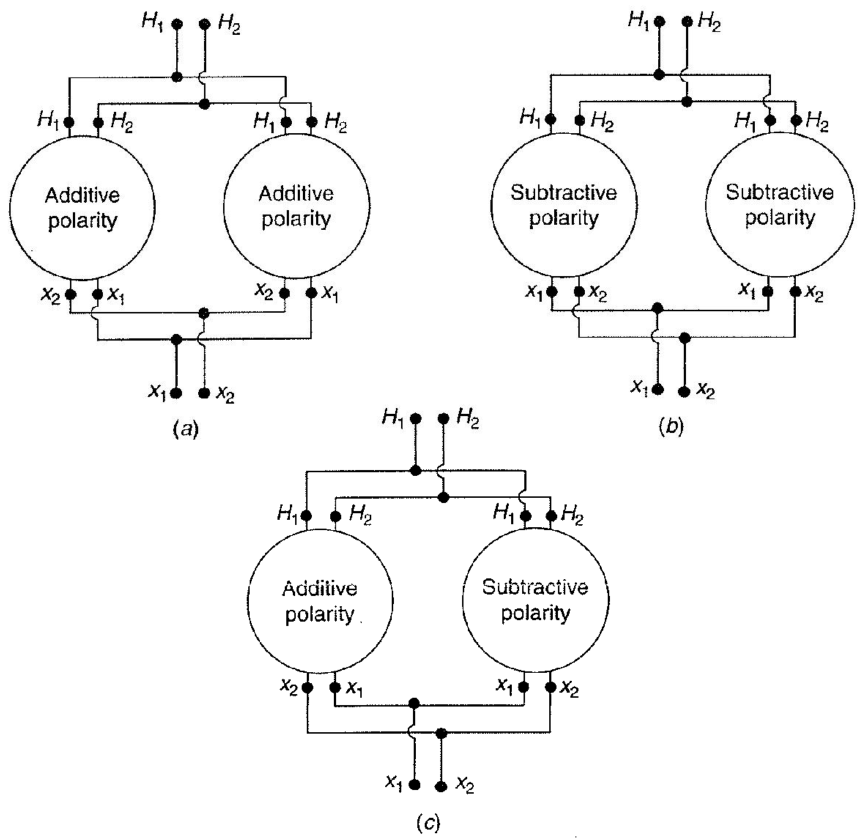

FIGURE 3.18 Single-phase transformer paralleling.

However, paralleling two single-phase transformers is not economical since the total cost and the losses of the two small transformers are much larger than one large transformer with the same capacity. Therefore, it should be used only as a temporary remedy to provide for increased demands for single-phase power in emergency situations. Figure 3.19 shows two single-phase transformers, each with two bushings, connected to a two-conductor primary to supply 120/240-V single-phase power on a three-wire secondary.

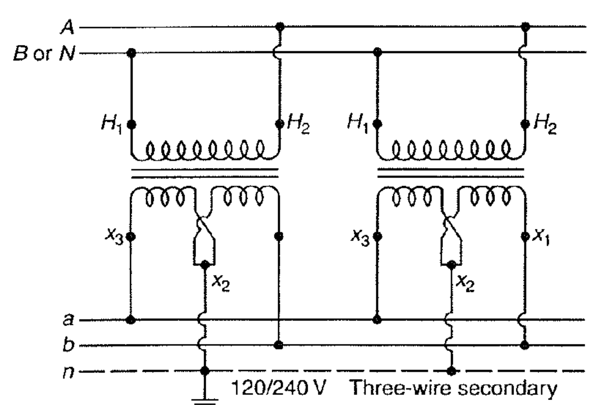

FIGURE 3.19 Parallel operation of two single-phase transformers.

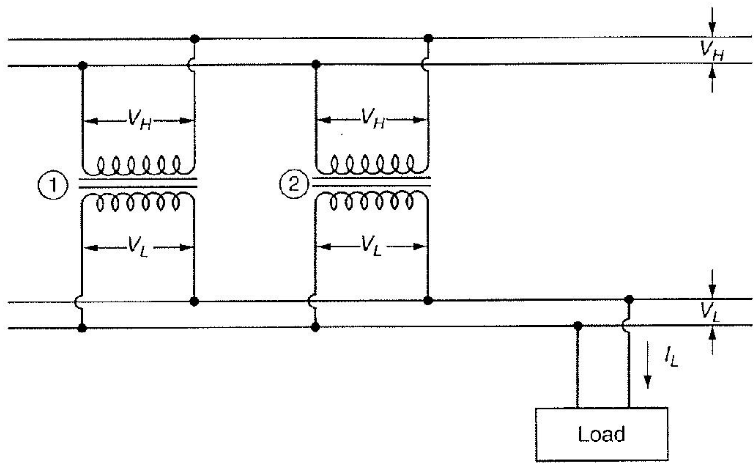

FIGURE 3.20 Two transformers connected in parallel and feeding a load.

To illustrate load division among the parallel-connected transformers, consider the two transformers connected in parallel and feeding a load, as shown in Figure 3.20. Assume that the aforementioned conditions for paralleling have already been met.

Figure 3.21 shows the corresponding equivalent circuit referred to as the low-voltage side. Since the transformers are connected in parallel, the voltage drop through each transformer must be equal.

Therefore,

$$I_1(Z_{\text{eq, T1}}) = I_2(Z_{\text{eq, T2}}) \tag{3.17}$$

from which

$$\frac{I_1}{I_2} = \frac{Z_{\text{eq, T2}}}{Z_{\text{eq, T1}}} \tag{3.18}$$

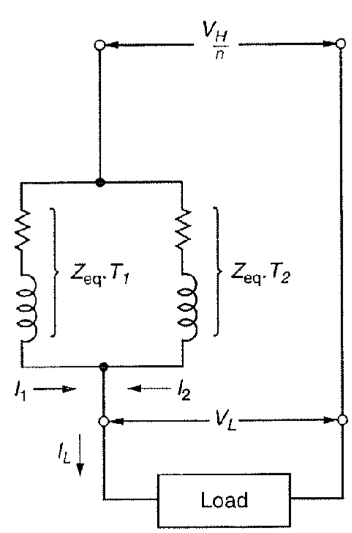

FIGURE 3.21 Equivalent circuit.

where I_1 is the secondary current of transformer 1, I_2 is the secondary current of transformer 2, I_L is the load current, $Z_{eq,1}$ is the equivalent impedance of transformer 1, and $Z_{eq,2}$ is the equivalent impedance of transformer 2.

From Equation 3.17 it can be seen that the load division is determined only by the relative ohmic impedance of the transformers. If the ohmic impedances in Equation 3.17 are replaced by their equivalent in terms of percent impedance, the following equation can be obtained.

$$\frac{I_1}{I_2} = \frac{(\%Z)_{T2}}{(\%Z)_{T1}} \frac{S_{T1}}{S_{T2}} \tag{3.19}$$

where $(\% Z)_{T1}$ is the percent impedance of transformer 1, $(\% Z)_{T2}$ is the percent impedance of transformer 2, S_{T1} is the kilovoltampere rating of transformer 1, and S_{T2} is the kilovoltampere rating of transformer 2.

Equation 3.19 can be expressed in terms of kilovoltamperes supplied by each transformer since the primary and the secondary voltages for each transformer are the same, respectively. Therefore,

$$\frac{S_{L1}}{S_{L2}} = \frac{(\%Z)_{T2}}{(\%Z)_{T1}} \frac{S_{T1}}{S_{T2}} \tag{3.20}$$

where S_{L1} is the kilovoltamperes supplied by transformer 1 to the load and S_{L2} is the kilovoltamperes supplied by transformer 2 to the load.

Example 3.1

Figure 3.22 shows an equivalent circuit of a single-phase transformer with three-wire secondary for three-wire single-phase distribution. The typical distribution transformer is rated as 25 kVA, 7200-120/240 V, 60 Hz, and has the following *pu** impedance based on the transformer ratings and based on the use of the entire low-voltage winding with zero neutral current:

$$R_T = 0.014 \text{ pu}$$

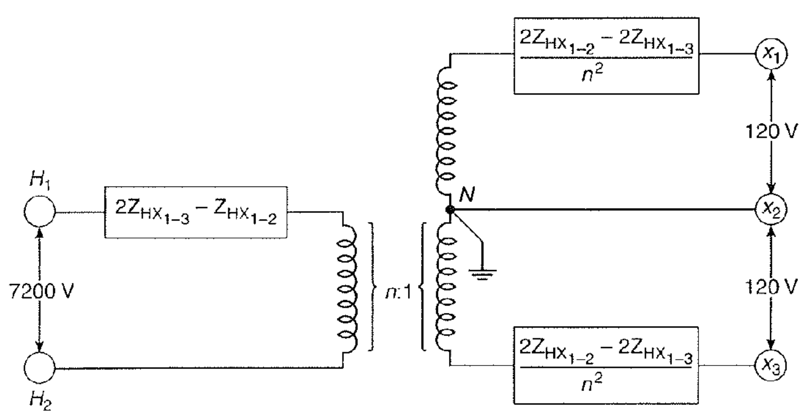

FIGURE 3.22 An equivalent circuit of a single-phase transformer with three-wire secondary. (From Westinghouse Electric Corporation, *Electric Utility Engineering Reference Book—Distribution Systems*, vol. 3, East Pittsburgh, PA, 1965.)

*Per unit systems are explained in Appendix D.

and

$$X_T = 0.012 \text{ pu}.$$

Here, the two halves of the low voltage may be independently loaded, and, in general, the three-wire secondary load will not be balanced. Therefore, in general, the equivalent circuit needed is that of a three-winding single-phase transformer as shown in Figure 3.22, when voltage drops and/or fault currents are to be computed. Thus use the meager amount of data (it is all that is usually available) and evaluate numerically all the impedances shown in Figure 3.22.

Solution

Figure 3.22 is based on the reference by Lloyd [1]. To determine $\bar{Z}_{HX_{1-2}}$ approximately, Lloyd gives the following formula:

$$\bar{Z}_{HX_{1-2}} = 1.5R_T + j1.2X_T \tag{3.21}$$

where $\bar{Z}_{HX_{1-2}}$ is the the transformer impedance referred to high-voltage winding when the section of the low-voltage winding between the terminals X_2 and X_3 is short-circuited.

From Figure 3.22, the turns ratio of the transformer is

$$n = \frac{V_H}{V_X} = \frac{7200\text{ V}}{120\text{ V}} = 60.$$

Since the given pu impedances of the transformer are based on the use of the entire low-voltage winding,

$$\begin{aligned}\bar{Z}_{HX_{1-3}} &= R_T + jX_T\\ &= 0.014 + j0.012\text{ pu}.\end{aligned}$$

Also, from Equation 3.21,

$$\begin{aligned}\bar{Z}_{HX_{1-2}} &= 1.5R_T + j1.2X_T\\ &= 1.5 \times 0.014 + j1.2 \times 0.012\\ &= 0.021 + j0.0144\text{ pu}.\end{aligned}$$

Therefore,

$$\begin{aligned}2\bar{Z}_{HX_{1-3}} - \bar{Z}_{HX_{1-2}} &= 2(0.014 + j0.012) - (0.021 + j0.0144)\\ &= 0.007 + j0.0096\text{ pu}\\ &= 14.515 + j19.906 = 24.637\angle 53.9^\circ\ \Omega\end{aligned}$$

and

$$\begin{aligned}\frac{2\bar{Z}_{HX_{1-2}} - 2\bar{Z}_{HX_{1-3}}}{n^2} &= \frac{2(0.021 + j0.0144) - 2(0.014 + j0.012)}{60^2}\\ &= 3.89 \times 10^{-6} + j1.334 \times 10^{-6}\text{ pu}\\ &= 0.008064 + j0.0028 = 8.525 \times 10^{-3}\angle 18.9^\circ\ \Omega.\end{aligned}$$

Example 3.2

Using the transformer equivalent circuit found in Example 3.1, determine the line-to-neutral (120 V) and line-to-line (240 V) fault currents in three-wire single-phase 120/240-V secondaries shown in Figures 3.23 and 3.24, respectively. In the figures, R represents the resistance of the service drop cable per conductor. Usually R is much larger than X for such cable and therefore X may be neglected.

Using the given data, determine the following:

(*a*) Find the symmetrical root-mean-square (RMS) fault currents in the high-voltage and low-voltage circuits for a 120-V fault if the R of the service drop cable is zero.
(*b*) Find the symmetrical RMS fault currents in the high-voltage and low-voltage circuits for a 240-V fault if the R of the service drop cable is zero.
(*c*) If the transformer is a CSPB type, find the minimum allowable interrupting capacity (in symmetrical RMS amperes) for a circuit breaker connected to the transformer's low-voltage terminals.

Solution

(*a*) When $R = 0$, from Figure 3.23, the line-to-neutral fault current in the secondary side of the transformer is

$$\bar{I}_{\text{f,LV}} = \frac{120}{8.525 \times 10^{-3}\angle 18.9° + \left(\frac{1}{60}\right)^2 (24.637\angle 53.9°)}$$
$$= 8181.7\angle -34.4° \text{ A}.$$

Thus, the fault current in the high-voltage side is

$$\bar{I}_{\text{f, HV}} = \frac{\bar{I}_{\text{f, LV}}}{n}$$
$$= \frac{8181.7}{60} = 136.4 \text{ A}.$$

Note that the turns ratio is found as

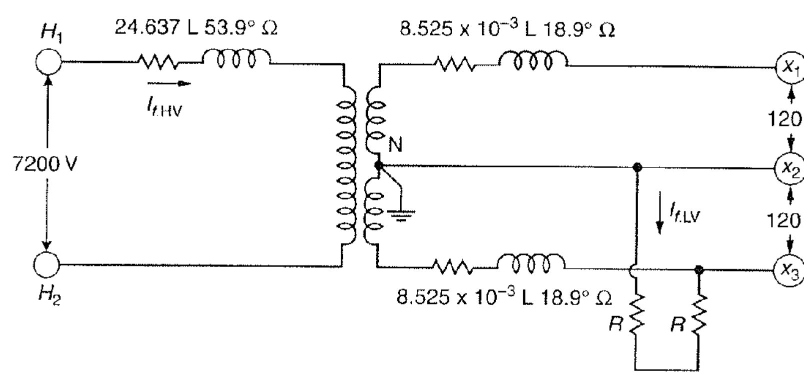

FIGURE 3.23 Secondary line-to-neutral fault.

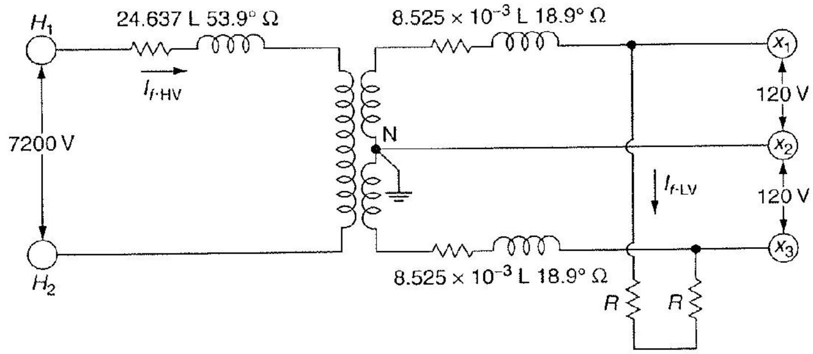

FIGURE 3.24 Secondary line-to-line fault.

$$n = \frac{7200\,\text{V}}{120\,\text{V}} = 60.$$

(*b*) When $R = 0$, from Figure 3.24, the line-to-line fault current in the secondary side of the transformer is

$$\bar{I}_{\text{f, LV}} = \frac{240}{2(8.525 \times 10^{-3}\angle 18.9^\circ) + \left(\frac{1}{30}\right)^2 (24.637\angle 53.9^\circ)}$$
$$= 5649\angle -40.6^\circ\,\text{A}.$$

Thus, the fault current in the high-voltage side is

$$\bar{I}_{\text{f, HV}} = \frac{\bar{I}_{\text{f, LV}}}{n} = \frac{5649}{30} = 188.3\,\text{A}.$$

Note that the turns ratio is found as

$$n = \frac{7200\,\text{V}}{240\,\text{V}} = 30.$$

(*c*) Therefore, the minimum allowable interrupting capacity for a circuit breaker connected to the transformer low-voltage terminals is 8181.7 A.

Example 3.3

Using the data given in Example 3.2, determine the following:

(*a*) Estimate approximately the value of R, that is, the service drop cable's resistance, which will produce equal line-to-line and line-to-neutral fault currents.

(*b*) If the conductors of the service drop cable are aluminum, find the length of the service drop cable that would correspond to the resistance *R* found in part (*a*) in the case of (i) #4 AWG conductors with a resistance of 2.58 Ω/mi and (ii) #1/0 AWG conductors with a resistance of 1.03 Ω/mi.

Solution

(*a*) Since the line-to-line and the line-to-neutral fault currents are supposed to be equal to each other,

$$\frac{240}{2R+0.032256+j0.02765}=\frac{120}{2R+0.012096+j0.0083}$$

or

$$R\cong 0.0075\ \Omega.$$

(*b*) The length of the service drop cable is:

(i) If #4 AWG aluminum conductors with a resistance of 2.58 Ω/mi or 4.886×10^{-4} Ω/ft are used,

$$\begin{aligned}\text{Service drop length}&=\frac{R}{4.886\times10^{-4}}\\&=\frac{0.0075\ \Omega}{4.886\times10^{-4}\ \Omega/\text{ft}}\\&\cong 15.35\,\text{ft.}\end{aligned}$$

(ii) If #1/0 AWG aluminum conductors with a resistance of 1.03 Ω/mi or 4.886×10^{-4} Ω/ft are used,

$$\begin{aligned}\text{Service drop length}&=\frac{0.0075\ \Omega}{1.9508\times10^{-4}\ \Omega/\text{ft}}\\&\cong 38.45\,\text{ft.}\end{aligned}$$

EXAMPLE 3.4

Assume that a 250-kVA transformer with 2.4% impedance is paralleled with a 500-kVA transformer with 3.1% impedance. Determine the maximum load that can be carried without overloading either transformer. Assume that the maximum allowable transformer loading is 100% of the rating.

Solution

Designating the 250- and 500-kVA transformers as transformers 1 and 2, respectively, and using Equation 3.20,

$$\begin{aligned}\frac{S_{L1}}{S_{L2}}&=\frac{(\%Z)_{T2}}{(\%Z)_{T1}}\frac{S_{T1}}{S_{T2}}\\&=\frac{3.1}{2.4}\times\frac{250}{500}=0.6458.\end{aligned}$$

Assume a load of 500 kVA on the 500-kVA transformer. The preceding result shows that the load on the 250-kVA transformer will be 193.5 kVA when the load on the 500-kVA transformer is 500 kVA. Therefore, the 250-kVA transformer becomes overloaded before the 500-kVA transformer. The load on the 500-kVA tranformer when the 250-kVA transformer is carrying the rated load is

$$\begin{aligned} S_{L2} &= \frac{S_{L1}}{0.6458} \\ &= \frac{250}{0.6458} \\ &= 387.1 \text{ kVA}. \end{aligned}$$

Thus, the total load is

$$\begin{aligned} \sum_{i=1}^{2} S_{Li} &= S_{L1} + S_{L2} \\ &= 250 + 387.1 \\ &= 637.1 \text{ kVA}. \end{aligned}$$

3.10 THREE-PHASE CONNECTIONS

To raise or lower the voltages of three-phase distribution systems, either single-phase transformers can be connected to form three-phase transformer banks or three-phase transformers (having all windings in the same tank) are used.

Common methods of connecting three single-phase transformers for three-phase transformations are the delta-delta (Δ-Δ), wye-wye (Y-Y), wye-delta (Y-Δ), and delta-wye (Δ-Y) connections. Here, it is assumed that all transformers in the bank have the same kilovoltampere rating.

3.10.1 The Δ-Δ Transformer Connection

Figures 3.25 and 3.26 show the Δ-Δ connection formed by tying together single-phase transformers to provide 240-V service at 0 and 180° angular displacements, respectively.

This connection is often used to supply a small single-phase lighting load and three-phase power load simultaneously. To provide this type of service the mid-tap of the secondary winding of one of the transformers is grounded and connected to the secondary neutral conductor, as shown in Figure 3.27. Therefore, the single-phase loads are connected between the phase and neutral conductors. Thus, the transformer with the mid-tap carries two-thirds of the 120/240-V single-phase load and one-third of the 240-V three-phase load. The other two units each carry one-third of both the 120/240- and 240-V loads.

There is no problem from third-harmonic overvoltage or telephone interference. However, high circulating currents will result unless all three single-phase transformers are connected on the same regulating taps and have the same voltage ratios. The transformer bank rating is decreased unless all transformers have identical impedance values. The secondary neutral bushing can be grounded on only one of the three single-phase transformers, as shown in Figure 3.27.

Therefore, to get balanced transformer loading, the conditions include the following:

1. All transformers have identical voltage ratios
2. All transformers have identical impedance values
3. All transformers are connected on identical taps

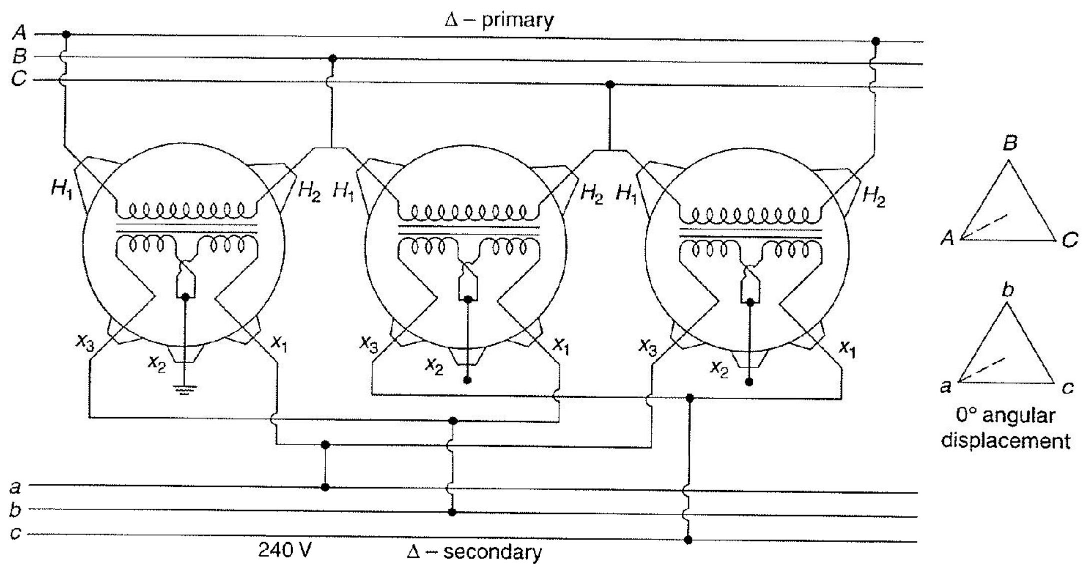

FIGURE 3.25 Delta-delta transformer bank connection with 0° angular displacement.

However, if two of the units have the identical impedance values and the third unit has an impedance value which is within, plus or minus, 25% of the impedance value of the like transformers, it is possible to operate the Δ-Δ bank, with a small unbalanced transformer loading, at reduced bank output capacity. Table 3.5 gives the permissible amounts of load unbalanced on the odd and like transformers. Note that ZZ_1 is the impedance of the odd transformer unit and Z_2 is the impedance of the like transformer units. Therefore, with unbalanced transformer loading, the load values have to be checked against the values of the table so that no one transformer is overloaded.

Assume that Figure 3.28 shows the equivalent circuit of a Δ-Δ-connected transformer bank referred to the low-voltage side. A voltage drop equation can be written for the low-voltage windings as

$$\bar{V}_{ba} + \bar{V}_{ac} + \bar{V}_{cb} = \bar{I}_{ba}\bar{Z}_{ab} + \bar{I}_{ac}\bar{Z}_{ca} + \bar{I}_{cb}\bar{Z}_{bc} \tag{3.22}$$

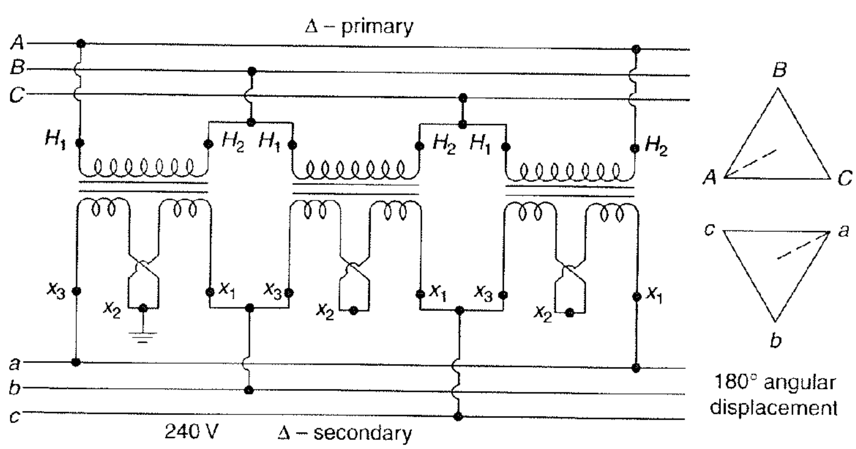

FIGURE 3.26 Delta-delta transformer bank connection with 180° angular displacement.

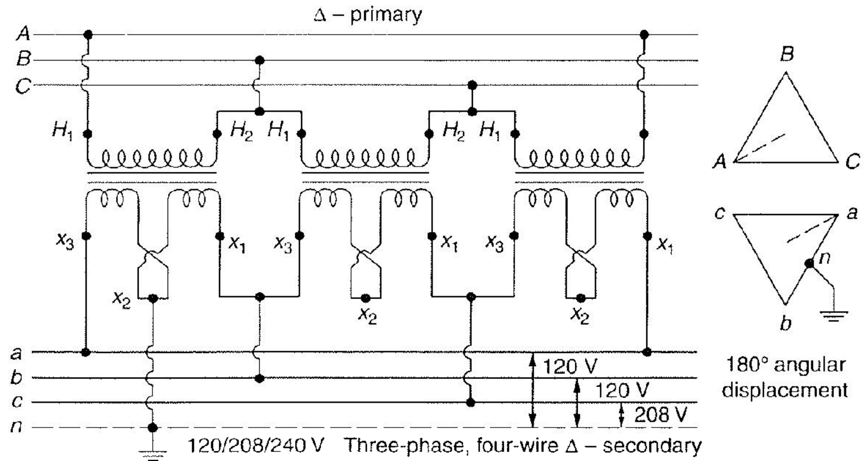

FIGURE 3.27 Delta-delta connection to provide 120/208/240 V three-phase four-wire service.

where

$$\bar{V}_{ba} + \bar{V}_{ac} + \bar{V}_{cb} = 0. \tag{3.23}$$

Therefore, Equation 3.22 becomes

$$\bar{I}_{ba}\bar{Z}_{ab} + \bar{I}_{ac}\bar{Z}_{ca} + \bar{I}_{cb}\bar{Z}_{bc} = 0. \tag{3.24}$$

For the Δ-connected secondary,

$$\bar{I}_a = \bar{I}_{ba} - \bar{I}_{ac} \tag{3.25}$$

$$\bar{I}_b = \bar{I}_{cb} - \bar{I}_{ba} \tag{3.26}$$

$$\bar{I}_c = \bar{I}_{ac} - \bar{I}_{cb}. \tag{3.27}$$

TABLE 3.5
The Permissible Percent Loading on Odd and Like Transformers as a Function of the Z_1/Z_2 Ratio

	Percent Load On	
Z_1/Z_2 Ratio	Odd Unit	Like Unit
0.75	109.0	96.0
0.80	107.0	96.5
0.85	105.2	97.3
0.90	103.3	98.3
1.10	96.7	102.0
1.15	95.2	102.2
1.20	93.8	103.1
1.25	92.3	103.9

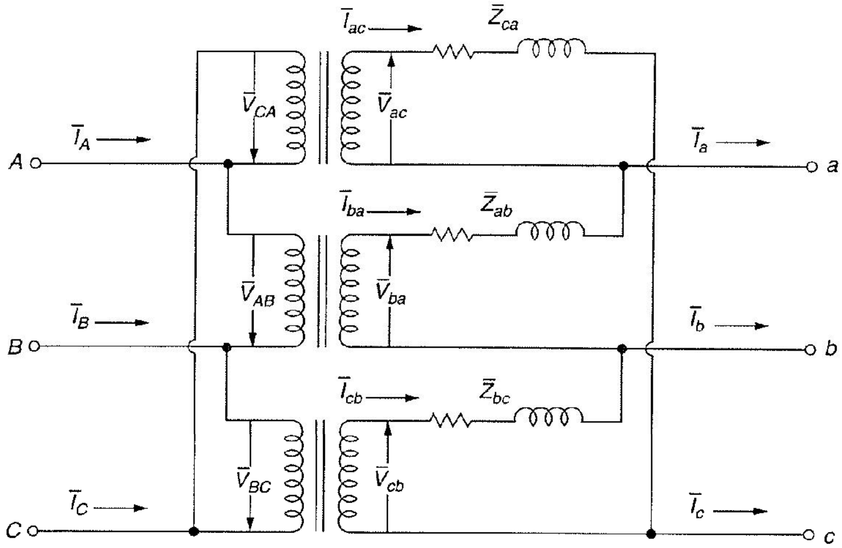

FIGURE 3.28 Equivalent circuit of a delta-delta-connected transformer bank.

From Equation 3.24,

$$\bar{I}_{ba}\bar{Z}_{ab} = -\bar{I}_{ac}\bar{Z}_{ca} - \bar{I}_{cb}\bar{Z}_{bc}. \tag{3.28}$$

Adding the terms of $\bar{I}_{ba}\bar{Z}_{bc}$ and $\bar{I}_{ba}\bar{Z}_{ca}$ to either side of Equation 3.28 and substituting Equation 3.25 into the resultant equation,

$$\bar{I}_{ba} = \frac{\bar{I}_a\bar{Z}_{ca} - \bar{I}_b\bar{Z}_{bc}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{3.29}$$

and similarly,

$$\bar{I}_{ac} = \frac{\bar{I}_c\bar{Z}_{bc} - \bar{I}_a\bar{Z}_{ab}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{3.30}$$

and

$$\bar{I}_{cb} = \frac{\bar{I}_b\bar{Z}_{ab} - \bar{I}_c\bar{Z}_{ca}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}}. \tag{3.31}$$

If the three transformers shown in Figure 3.28 have equal percent impedance and equal ratios of percent reactance to percent resistance, then Equations 3.29 through 3.32 can be expressed as

$$\bar{I}_{ba} = \frac{\dfrac{\bar{I}_a}{S_{T,ca}} - \dfrac{\bar{I}_b}{S_{T,bc}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3.32}$$

$$\bar{I}_{ac} = \frac{\dfrac{\bar{I}_c}{S_{T,bc}} - \dfrac{\bar{I}_a}{S_{T,ab}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3.33}$$

$$\bar{I}_{cb} = \frac{\dfrac{\bar{I}_b}{S_{T,ab}} - \dfrac{\bar{I}_c}{S_{T,ca}}}{\dfrac{1}{S_{T,ab}} + \dfrac{1}{S_{T,bc}} + \dfrac{1}{S_{T,ca}}} \tag{3.34}$$

where $S_{T,ab}$ is the kilovoltampere rating of the single-phase between phases a and b, $S_{T,bc}$ is the kilovoltampere rating between phases b and c, and $S_{T,ca}$ is the kilovoltampere rating between phases c and a.

Example 3.5

Three single-phase transformers are connected Δ-Δ to provide power for a three-phase Y-connected 200-kVA load with a 0.80 lagging power factor and a 80-kVA single-phase light load with a 0.90 lagging power factor, as shown in Figure 3.29.

Assume that the three single-phase transformers have equal percent impedance and equal ratios of percent reactance to percent resistance. The primary-side voltage of the bank is 7620/13,200 V and the secondary-side voltage is 240 V. Assume that the single-phase transformer connected between phases b and c is rated at 100 kVA and the other two are rated at 75 kVA. Determine the following:

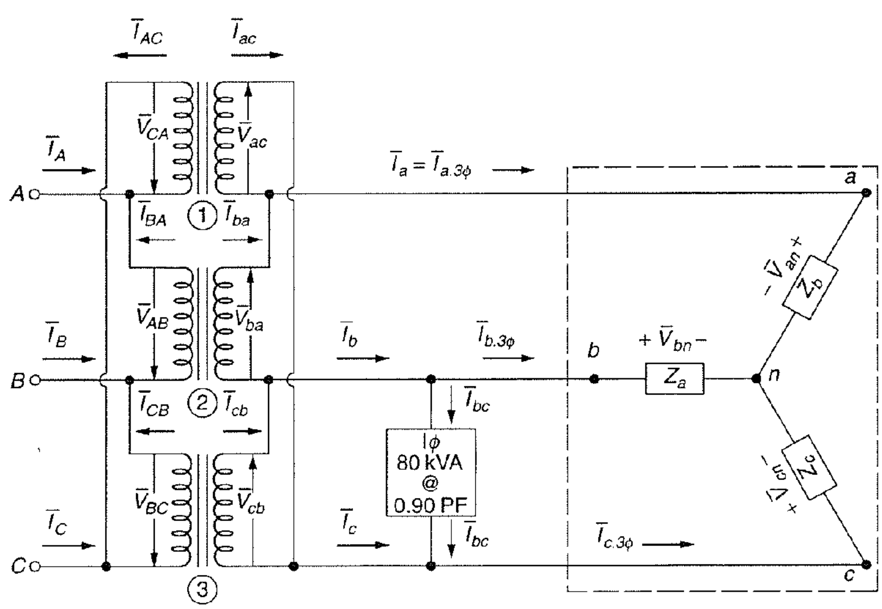

FIGURE 3.29 For Example 3.5.

(*a*) The line current flowing in each secondary-phase wire.
(*b*) The current flowing in the secondary winding of each transformer.
(*c*) The load on each transformer in kilovoltamperes.
(*d*) The current flowing in each primary winding of each transformer.
(*e*) The line current flowing in each primary-phase wire.

Solution

(*a*) Using the voltage drop $\bar{V}_{an}$ as the reference, the three-phase components of the line currents can be found as

$$\left|\bar{I}_{a,3\phi}\right| = \left|\bar{I}_{b,3\phi}\right| = \left|\bar{I}_{c,3\phi}\right| = \frac{S_{L,3\phi}}{\sqrt{3} \times V_{L-L}}$$
$$= \frac{200}{\sqrt{3} \times 0.240}$$
$$= 481.7\text{ A.}$$

Since the three-phase load has a lagging power factor of 0.80,

$$\bar{I}_{a,3\phi} = \left|\bar{I}_{a,3\phi}\right|(\cos\theta - j\sin\theta)$$
$$= 481.7(0.80 - j0.60)$$
$$= 385.36 - j289.02$$
$$= 481.7\ \angle -36.9°\text{ A.}$$

$$\bar{I}_{b,3\phi} = a^2\bar{I}_{a,3\phi}$$
$$= (1\ \angle 240°)481.7\ \angle -36.9°$$
$$= -443.08 - j188.99$$
$$= 481.7\ \angle 203.1°\text{ A.}$$

$$\bar{I}_{c,3\phi} = a\bar{I}_{a,3\phi}$$
$$= (1\ \angle 120°)481.7\ \angle -36.9°$$
$$= 57.87 + j478.21$$
$$= 481.7\ \angle 83.1°\text{ A.}$$

The single-phase component of the line currents can be found as

$$\left|\bar{I}_{bc}\right| = \frac{S_{L,1\phi}}{V_{L-L}} = \frac{80}{0.240} 333.33\text{ A.}$$

Since the single-phase load has a lagging power factor of 0.90, the current phasor $\bar{I}_{bc}$ lags its voltage phasor $\bar{V}_{bc}$ by −25.8°. Also, since the voltage phasor $\bar{V}_{bc}$ lags the voltage reference $\bar{V}_{an}$ by 90° (Fig. 3.26), the current phasor $\bar{I}_{bc}$ will lag the voltage reference $\bar{V}_{an}$ by −115.8° (= −25.8° − 90°). Therefore,

$$\begin{aligned}\bar{I}_{bc} &= 333.33\ \angle -115.8^\circ\\ &= -145.3 - j300\,\text{A}.\end{aligned}$$

Hence the line currents flowing in each secondary-phase wire can be found as

$$\begin{aligned}\bar{I}_a &= \bar{I}_{a,3\phi}\\ &= 481.7\angle -36.9^\circ\,\text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_b &= \bar{I}_{b,3\phi} + \bar{I}_{bc}\\ &= 481.7\ \angle 203.1^\circ + 333.33\ \angle -115.8^\circ\\ &= -588.38 - j488.99\\ &= 765.05\ \angle 219.7^\circ\,\text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_c &= \bar{I}_{c,3\phi} - \bar{I}_{bc}\\ &= 481.7\ \angle 83.1^\circ - 333.33\ \angle -115.8^\circ\\ &= -87.43 + j178.21\\ &= 198.5\ \angle -63.8^\circ\,\text{A}.\end{aligned}$$

(*b*) By using Equation 3.33, the current flowing in the secondary winding of transformer 1 can be found as

$$\begin{aligned}\bar{I}_{ac} &= \frac{\dfrac{\bar{I}_c}{S_{\text{T},bc}} - \dfrac{\bar{I}_a}{S_{\text{T},ab}}}{\dfrac{1}{S_{\text{T},ab}} + \dfrac{1}{S_{\text{T},bc}} + \dfrac{1}{S_{\text{T},ca}}}\\ &= \frac{\dfrac{198.5\ \angle -63.8^\circ}{100} - \dfrac{481.7\ \angle -36.9^\circ}{75}}{\dfrac{1}{75} + \dfrac{1}{100} + \dfrac{1}{75}}\\ &= \frac{1.985\ \angle -63.8^\circ - 6.4227\ \angle -36.9^\circ}{0.0367}\\ &= -116.07 + j56.55\\ &= 129.11\ \angle -33.1^\circ\,\text{A}.\end{aligned}$$

Similarly, by using Equation 3.32,

$$\bar{I}_{ba} = \frac{\dfrac{\bar{I}_a}{S_{\text{T},ca}} - \dfrac{\bar{I}_b}{S_{\text{T},bc}}}{\dfrac{1}{S_{\text{T},ab}} + \dfrac{1}{S_{\text{T},bc}} + \dfrac{1}{S_{\text{T},ca}}}$$

$$= \frac{\dfrac{481.7\angle -36.9^\circ}{75} - \dfrac{765.05\angle 219.7^\circ}{100}}{\dfrac{1}{75} + \dfrac{1}{100} + \dfrac{1}{75}}$$

$$= \frac{6.4227\angle -36.9^\circ - 7.6505\angle 219.7^\circ}{0.0367}$$

$$= 300.34 + j28.08$$

$$= 301.65\angle 5.3^\circ \text{ A}$$

and using Equation 3.34,

$$\bar{I}_{cb} = \frac{\dfrac{\bar{I}_b}{S_{\text{T},ab}} - \dfrac{\bar{I}_c}{S_{\text{T},ca}}}{\dfrac{1}{S_{\text{T},ab}} + \dfrac{1}{S_{\text{T},bc}} + \dfrac{1}{S_{\text{T},ca}}}$$

$$= \frac{\dfrac{765.05\angle 219.7^\circ}{75} - \dfrac{198.5\angle -63.8^\circ}{75}}{0.0367}$$

$$= -245.6 - j112.95$$

$$= 270.3\angle 204.7^\circ \text{ A}.$$

(*c*) The kilovoltampere load on each transformer can be found as

$$S_{\text{L},ab} = V_{ba} \times |\bar{I}_{ba}|$$
$$= 0.240 \times 301.65$$
$$= 72.4 \text{ kVA}.$$

$$S_{\text{L},bc} = V_{cb} \times |\bar{I}_{cb}|$$
$$= 0.240 \times 270.33$$
$$= 64.88 \text{ kVA}.$$

$$S_{\text{L},ca} = V_{ac} \times |\bar{I}_{ac}|$$
$$= 0.240 \times 129.11$$
$$= 30.99 \text{ kVA}.$$

(*d*) The current flowing in the primary winding of each transformer can be found by dividing the current flow in each secondary winding by the turns ratio. Therefore,

$$n = \frac{7620\,\text{V}}{240\,\text{V}} = 31.75$$

and hence

$$\begin{aligned}\bar{I}_{AC} &= \frac{\bar{I}_{ac}}{n}\\ &= \frac{129.11\angle -33.1^\circ}{31.75}\\ &= 4.07\angle -33.1^\circ\ \text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_{BA} &= \frac{\bar{I}_{ba}}{n}\\ &= \frac{301.65\angle 5.3^\circ}{31.75}\\ &= 9.5\angle 5.3^\circ\ \text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_{CB} &= \frac{\bar{I}_{cb}}{n}\\ &= \frac{270.3\angle 204.7^\circ}{31.75}\\ &= 8.51\angle 204.7^\circ\ \text{A}.\end{aligned}$$

(*e*) The line current flowing in each primary-phase wire can be found as

$$\begin{aligned}\bar{I}_A &= \bar{I}_{AC} - \bar{I}_{BA}\\ &= 4.07\angle -33.1^\circ - 9.5\angle 5.3^\circ\\ &= -6.05 - j3.1\\ &= 6.8\angle 270.1^\circ\ \text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_B &= \bar{I}_{BA} - \bar{I}_{CB}\\ &= 9.5\angle 5.3^\circ - 8.51\angle 204.7^\circ\\ &= 17.19 + j4.44\\ &= 17.76\angle 14.5^\circ\ \text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_C &= \bar{I}_{CB} - \bar{I}_{AC}\\ &= 8.51\angle 204.7^\circ - 4.07\angle -33.1^\circ\\ &= -11.14 - j1.34\\ &= 11.22\angle 186.8^\circ\ \text{A}.\end{aligned}$$

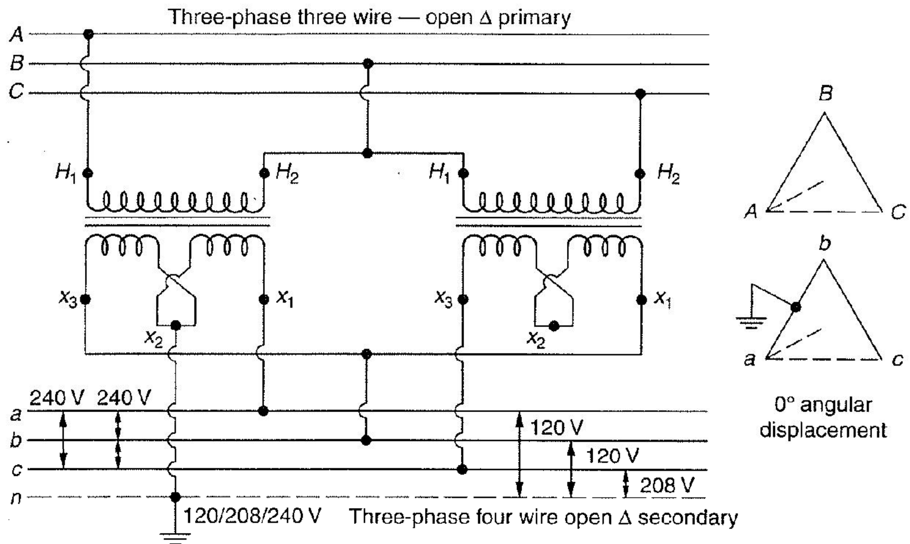

FIGURE 3.30 Three-phase four-wire open-delta connection. (Note that 3t-4W means a three-phase system made up of four wires.)

3.10.2 The Open-Δ Open-Δ Transformer Connection

The Δ-Δ connection is the most flexible of the various connection forms. One of the advantages of this connection is that if one transformer becomes damaged or is removed from service, the remaining two can be operated in what is known as the *open-delta* or *V connection*, as shown in Figure 3.30.

Assume that a balanced three-phase load with unity power factor is served by all three transformers of a Δ-Δ bank. The removal of one of the transformers from the service will result in having the currents in the other two transformers increase by a ratio of 1.73, although the output of the transformer bank is the same with a unity power factor as before. However, the individual transformers now function at a power factor of 0.866. One of the transformers delivers a leading load and the other a lagging load. To operate the remaining portion of the Δ-Δ transformer bank (i.e., the open-Δ open-Δ bank) safely, the connected load has to be decreased by the 57.7% which can be found as follows:

$$S_{\Delta\text{-}\Delta} = \frac{\sqrt{3}V_{L\text{-}L}I_L}{1000}\ \text{kVA} \tag{3.35}$$

and

$$S_{\angle\text{-}\angle} = \frac{\sqrt{3}V_{L\text{-}L}I_L}{\sqrt{3}\times 1000}\ \text{kVA}. \tag{3.36}$$

Therefore, by dividing Equation 3.35 by Equation 3.36, side by side,

$$\begin{aligned}\frac{S_{\angle\text{-}\angle}}{S_{\Delta\text{-}\Delta}} &= \frac{1}{\sqrt{3}} \\ &= 0.577 \text{ or } 57.7\%\end{aligned} \tag{3.37}$$

where $S_{\Delta-\Delta}$ is the kilovoltampere rating of the Δ-Δ bank, $S_{\angle-\angle}$ is the kilovoltampere rating of the open-Δ bank, V_{L-L} is the line-to-line voltage (V), and I_L is the line (or full load) current (A).

Note that the two transformers of the open-Δ bank make up 66.6% of the installed capacity of the three transformers of the Δ-Δ bank, but they can supply only 57.7% of the three. Here, the ratio of 57.7/66.6 = 0.866 is the power factor at which the two transformers operate when the load is at unity power factor. By being operated in this way, the bank still delivers three-phase currents and voltages in their correct phase relationships, but the capacity of the bank is reduced to 57.7% of what it was with all three transformers in service since it has only 86.6% of the rating of the two units making up the three-phase bank. Open-Δ banks are quite often used where the load is expected to grow, and when the load does grow, the third transformer may be added to complete a Δ-Δ bank.

Figure 3.31 shows an open-Δ connection for 240-V three-phase three-wire secondary service at 0° angular displacement. The neutral point *n* shown in the low-voltage phasor diagram exists only on the paper.

For the sake of illustration, assume that a balanced three-phase load, for example, an induction motor as shown in the figure, with a lagging power factor is connected to the secondary. Therefore the *a*, *b*, *c* phase currents in the secondary can be found as

$$\bar{I}_a = \frac{S_{3\phi}}{\sqrt{3}V_{L-L}} \angle\theta_{\bar{I}_a}, \tag{3.38}$$

$$\bar{I}_b = \frac{S_{3\phi}}{\sqrt{3}V_{L-L}} \angle\theta_{\bar{I}_b}, \tag{3.39}$$

$$\bar{I}_c = \frac{S_{3\phi}}{\sqrt{3}V_{L-L}} \angle\theta_{\bar{I}_c}. \tag{3.40}$$

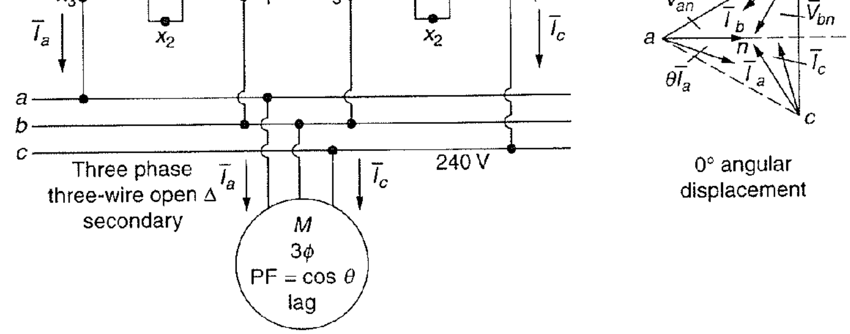

FIGURE 3.31 Three-phase three-wire open-delta connection.

The transformer kilovoltampere loads can be calculated as follows. The kilovoltampere load on the first transformer is

$$\begin{aligned} S_{\mathrm{T1}} &= V_{L-L} \times |\bar{I}_a| \\ &= V_{L-L} \times \frac{S_{3\phi}}{\sqrt{3} \times V_{L-L}} \\ &= \frac{S_{3\phi}}{\sqrt{3}} \text{ kVA} \end{aligned} \tag{3.41}$$

and the kilovoltampere load on the second transformer is

$$\begin{aligned} S_{\mathrm{T2}} &= V_{L-L} \times |\bar{I}_b| \\ &= V_{L-L} \times \frac{S_{3\phi}}{\sqrt{3} \times V_{L-L}} \\ &= \frac{S_{3\phi}}{\sqrt{3}} \text{ kVA.} \end{aligned} \tag{3.42}$$

Therefore, the total load that the transformer bank can be loaded to (or the total "effective" transformer bank capacity) is

$$\sum_{i=1}^{2} S_{\mathrm{T}_i} = \frac{2 \times S_{3\phi}}{\sqrt{3}} \tag{3.43}$$

and hence,

$$S_{3\phi} = \frac{\sqrt{3}}{2} \sum_{i=1}^{2} S_{\mathrm{T}_i} \text{ kVA.} \tag{3.44}$$

For example, if there are two 50-kVA transformers in the open-Δ bank, although the total transformer bank capacity appears to be

$$\sum_{i=1}^{2} S_{\mathrm{T}_i} = 100 \text{ kVA}$$

in reality the bank's "effective" maximum capacity is

$$S_{3\phi} = \frac{\sqrt{3}}{2} \times 100 = 86.6 \text{ kVA.}$$

If there are three 50-kVA transformers in the Δ-Δ bank, the bank's maximum capacity is

$$S_{3\phi} = \sum_{i=1}^{3} S_{\mathrm{T}_i} = 150 \text{ kVA}$$

which shows an increase of 73% over the 86.6-kVA load capacity.

Assume that the load power factor is cos θ and its angle can be calculated as

$$\theta = \theta_{\bar{V}_{an}} - \theta_{\bar{I}_a} \tag{3.45}$$

or using $\bar{V}_{an}$ as the reference,

$$\theta = 0° - \theta_{\bar{I}_a}. \tag{3.46}$$

If $\theta_{\bar{I}_a}$ is negative, then θ is positive which means it is the angle of a lagging load power factor. Also, it can be shown that

$$\theta = \theta_{\bar{V}_{bn}} - \theta_{\bar{I}_b} \tag{3.47}$$

or

$$\theta = -120° - \theta_{\bar{I}_b} \tag{3.48}$$

and

$$\theta = \theta_{\bar{V}_{cn}} - \theta_{\bar{I}_c} \tag{3.49}$$

or

$$\theta = +120 - \theta_{\bar{I}_c}. \tag{3.50}$$

The transformer power factors for transformers 1 and 2 can be calculated as

$$\cos\theta_{T_1} = \cos(\theta_{\bar{V}_{ab}} - \theta_{\bar{I}_a}) \tag{3.51}$$

or if

$$\theta_{\bar{I}_a} = -30°,$$

$$\cos\theta_{T_1} = \cos(\theta_{\bar{V}_{ab}} + 30°) \tag{3.52}$$

and

$$\cos\theta_{T_2} = \cos(\theta_{\bar{V}_{bc}} - \theta_{\bar{I}_c}) \tag{3.53}$$

or if

$$\theta_{\bar{I}_c} = +30°,$$

$$\cos\theta_{T_2} = \cos(\theta_{\bar{V}_{bc}} - 30°). \tag{3.54}$$

Therefore, the total real power output of the bank is

$$\begin{aligned} P_T &= P_{T_1} + P_{T_2} \\ &= V_{L-L}|\bar{I}_a|\cos(\theta + 30°) + V_{L-L}|\bar{I}_c|\cos(\theta - 30°) \\ &= \sqrt{3}V_{L-L}I_L\cos\theta \text{ kW} \end{aligned} \tag{3.55}$$

TABLE 3.6
The Effects of the Load Power Factor on the Transformer Power Factors

Load Power Factor		Transformer Power Factors	
$\cos\theta$	θ	$\cos\theta_{T_1} = \cos(\theta + 30°)$	$\cos\theta_{T_2} = \cos(\theta - 30°)$
0.866 lag	+30°	0.5 lag	1.0
1.0	0°	0.866 lag	0.866 lead

and, similarly, the total reactive power output of the bank is

$$\begin{aligned} Q_T &= Q_{T_1} + Q_{T_2} \\ &= V_{L-L}|\bar{I}_a|\sin(\theta + 30°) + V_{L-L}|\bar{I}_c|\sin(\theta - 30°) \\ &= \sqrt{3}V_{L-L}I_L \sin\theta \text{ kvar.} \end{aligned} \tag{3.56}$$

As shown in Table 3.6 when the connected bank load has a lagging power factor of 0.866, it has a 30° power factor angle and, therefore, transformer 1, from Equation 3.52, has a 0.5 lagging power factor and transformer 2, from Equation 3.54, has a unity power factor. However, when the bank load has a unity power factor, of course its angle is zero, and therefore transformer 1 has a 0.866 lagging power factor and transformer 2 has a 0.866 leading power factor.

3.10.3 The Y-Y Transformer Connection

Figure 3.32 shows three transformers connected Y-Y on a typical three-phase four-wire multigrounded system to provide for 120/208Y-V service at 0° angular displacement. This particular system provides a 208-V three-phase power supply for three-phase motors and a 120-V single-phase

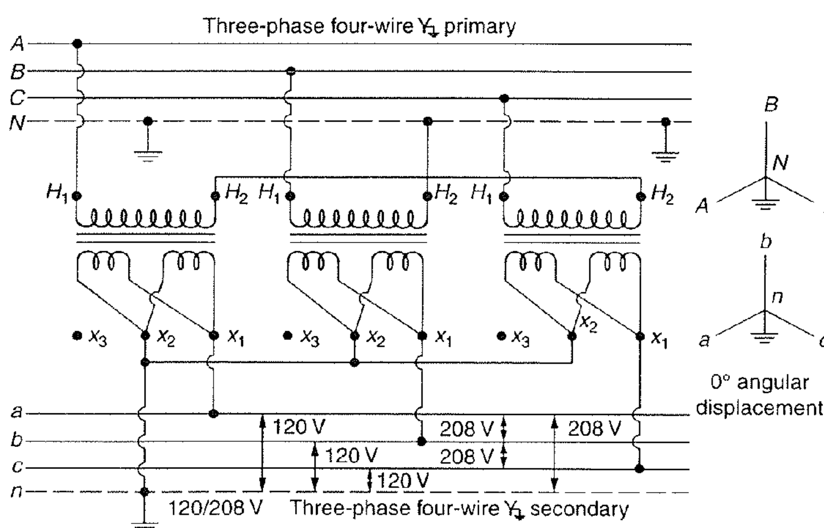

FIGURE 3.32 Wye-wye connection to provide a 120/208-V grounded-wye three-phase four-wire multigrounded service.

power supply for lamps and other small single-phase loads. An attempt should be made to distribute the single-phase loads reasonably equally among the three phases.

One of the advantages of the Y-Y connection is that when a system has changed from Δ to a four-wire Y to increase system capacity, existing transformers can be used. For example, assume that the old distribution system was 2.4-kV Δ and the new distribution system is 2.4/4.16 Y kV. Here the existing 2.4/4.16 Y-kV transformers can be connected in Y and used.

In the Y-Y transformer bank connection, only 57.7% (or 1/1.73) of the line voltage affects each winding, but full-line current flows in each transformer winding. Power distribution circuits supplied from a Y-Y bank often create series disturbances in communication circuits (e.g., telephone interference) in their immediate vicinity.

Also, the primary neutral point should be solidly grounded and tied firmly to the system neutral; otherwise, excessive voltages may be developed on the secondary side. For example, if the neutral of the transformer is isolated from the system neutral, an unstable condition results at the transformer neutral, caused primarily by third-harmonic voltages. If the transformer neutral is connected to the ground, the possibility of telephone interference is greatly enhanced and there is also a possibility of resonance between the line capacitance to the ground and the magnetizing impedance of the transformer.

3.10.4 The Y-Δ Transformer Connection

Figure 3.33 shows three single-phase transformers connected in Y-Δ on a three-phase three-wire ungrounded-Y, primary system to provide for 120/208/240-V three-phase four-wire Δ secondary service at 30° angular displacement.

Figure 3.34 shows three transformers connected in Y-Δ on a typical three-phase four-wire grounded-wye primary system to provide for 240-V three-phase three-wire Δ secondary service at 210° angular displacement.

The Y-Δ connection is advantageous in many cases because the voltage between the outside legs of the Y is 1.73 times the voltage of the neutral, so that higher distribution voltage can be gained by using transformers with primary winding of only the voltage between any leg and the neutral. For example, 2.4-kV primary single-phase transformers can be connected in Y on the primary to a 4.16-kV three-phase Y circuit.

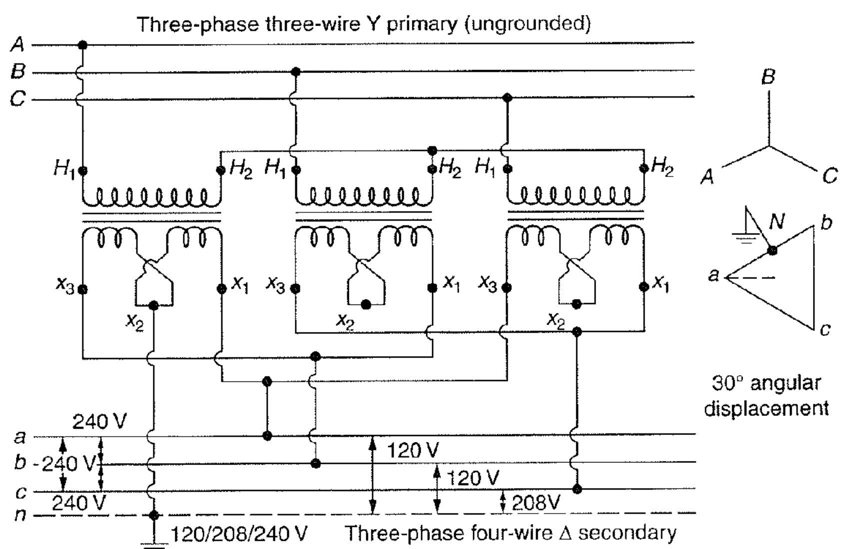

FIGURE 3.33 Wye-delta connection to provide a 120/208/240-V three-phase four-wire secondary service.

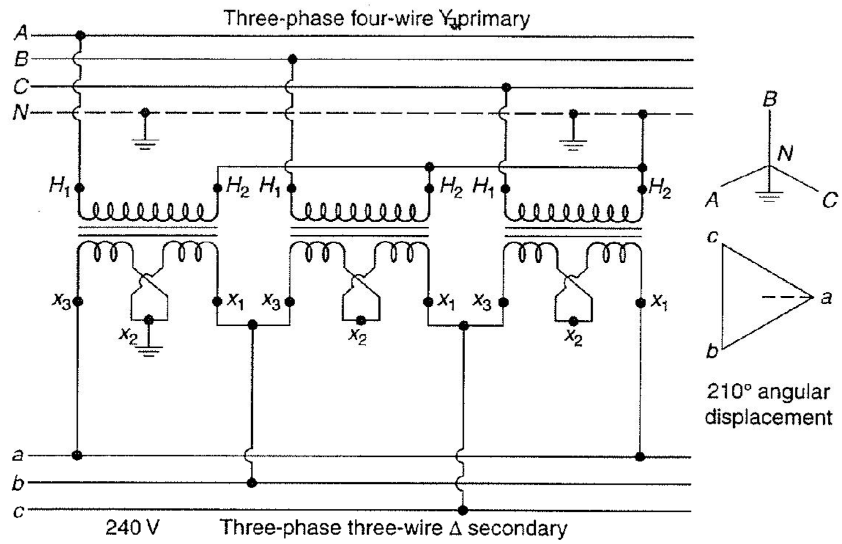

FIGURE 3.34 Wye-delta connection to provide a 240-V three-phase three-wire secondary service.

In the Y-Δ connection the voltage/transformation ratio of the bank is 1.73 times the voltage/transformation ratio of the individual transformers. When transformers of different capacities are used, the maximum safe bank rating is three times the capacity of the smallest transformers.

The primary supply, usually a grounded Y circuit, may be either three-wire or four-wire including a neutral wire. The neutral wire, running from the neutral of the Y-connected substation transformer bank supplying the primary circuit, may be completely independent of the secondary or may be united with the neutral of the secondary system. In the case of having the primary neutral independent of the secondary system, it is used as an isolated neutral and is grounded at the substation only.

In the case of having the same wire serving as both a primary neutral and the secondary neutral, it is grounded at many points, including each customer's service and is a multigrounded common neutral. However, in either case, the primary bank neutral is usually not connected to the primary circuit neutral since it is not necessary and prevents a burned-out transformer winding during phase-to-ground faults and extensive blowing of fuses throughout the system.

In the case of the Y-Y connection, neglecting the neutral on the primary side causes the voltages to be deformed from the sine-wave form. In the case of the Y-Δ connection, if the neutral is spared on the primary side the voltage waveform tends to deform, but this deformation causes circulating currents in the Δ, and these currents act as magnetizing currents to correct the deformation. Thus, there is no objection to neglecting the neutral. However, if the transformer supplies a motor load, a damaging overcurrent is produced in each three-phase motor circuit, causing an equal amount of current to flow in two wires of the motor branch circuit and the total of the two currents to flow in the third. If the highest of the three currents occurs in the unprotected circuit, motor burn-out will probably happen. This applies to ungrounded Y-Δ and Δ-Y banks.

If the transformer bank is used to supply three-phase and single-phase load, and if the bank neutral is solidly connected, disconnection of the large transformer by fuse operation causes an even greater overload on the remaining two transformers. Here, the blowing of a single fuse is hard to detect as no decrease in service quality is noticeable right away, and one of the two remaining transformers may be burned out by the overload. On the other hand, if the bank neutral is not connected to the primary circuit neutral, but left isolated, disconnection of one transformer results in a partial service interruption without danger of a transformer burn-out. The approximate rated capacity required in a Y-Δ-connected bank with an isolated bank neutral to serve a combined three-phase and single-phase load, assuming unity power factor, can be found as

$$\frac{2S_{1\phi}+S_{3\phi}}{3}$$

which is equal to rated transformer capacity across lighting phase, where $S_{1\phi}$ is the single-phase load (kVA) and $S_{3\phi}$ is the three-phase load (kVA).

In summary, when the primary-side neutral of the transformer bank is not isolated but connected to the primary circuit neutral, the Y-Δ transformer bank may burn-out due to the following reasons:

1. The transformer bank may act as a grounding transformer bank for unbalanced primary conditions and may supply fault current to any fault on the circuit to which it is connected, reducing its own capacity for connected load.
2. The transformer bank may be overloaded if one of the protective fuses opens on a line-to-ground fault, leaving the bank with only the capacity of an open-Y open-Δ bank.
3. The transformer bank causes circulating current in the Δ in an attempt to balance any unbalanced load connected to the primary line.
4. The transformer bank provides a Δ in which triple-harmonic currents circulate.

All the aforementioned effects can cause the transformer bank to carry current in addition to its normal load current, resulting in the burn-out of the transformer bank.

3.10.5 The Open-V Open-Δ Transformer Connection

As shown in Figure 3.35, in the case of having one phase of the primary supply opened, the transformer bank becomes open-Y open-Δ and continues to serve the three-phase load at a reduced capacity.

Example 3.6

Two single-phase transformers are connected open-Y open-Δ to provide power for a three-phase Y-connected 100-kVA load with a 0.80 lagging power factor and a 50-kVA single-phase load with a

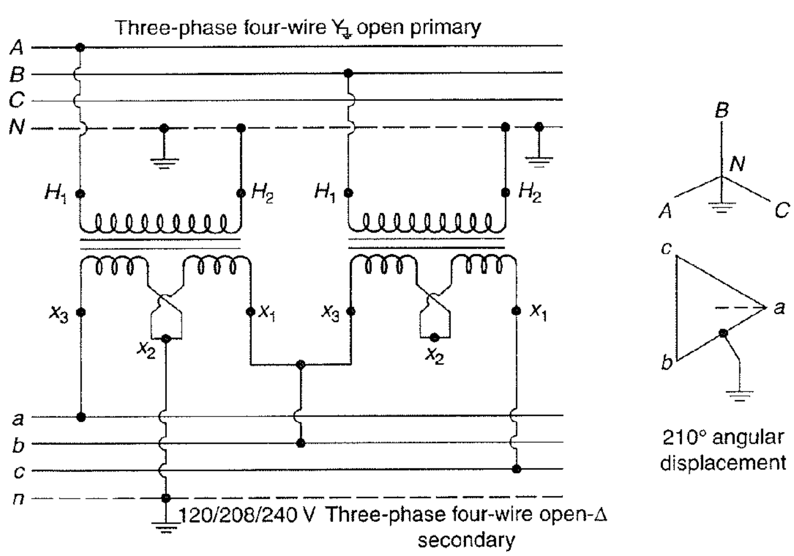

FIGURE 3.35 Open-wye open-delta connection.

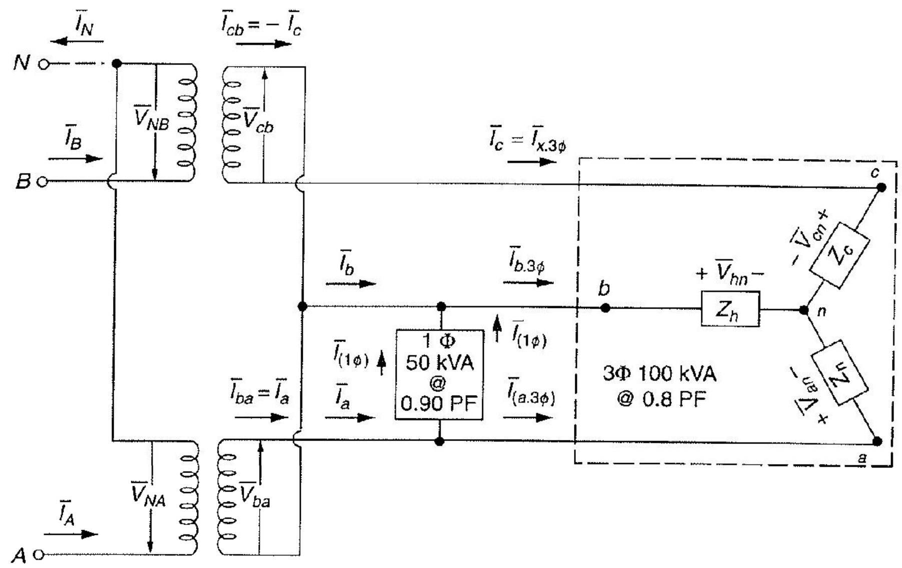

FIGURE 3.36 Open-wye open-delta connection for Example 3.6.

0.90 lagging power factor, as shown in Figure 3.36. Assume that the primary-side voltage of the bank is 7620/13,200 V and the secondary-side voltage is 240 V. Using the given information, calculate the following:

(*a*) The line current flowing in each secondary-phase wire.
(*b*) The current flowing in the secondary winding of each transformer.
(*c*) The kilovoltampere load on each transformer.
(*d*) The current flowing in each primary-phase wire and in the primary neutral.

Solution

(*a*) Using the voltage drop $\bar{V}_{an}$ as the reference, the three-phase components of the line currents can be found as

$$\begin{aligned}|\bar{I}_{a,3\phi}| &= |\bar{I}_{b,3\phi}| \\ &= |\bar{I}_{c,3\phi}| \\ &= \frac{S_{L,3\phi}}{\sqrt{3}\times V_{L-L}} \\ &= \frac{100}{\sqrt{3}\times 0.240} = 240.8\text{ A.}\end{aligned} \tag{3.57}$$

Since the three-phase load has a lagging power factor of 0.80,

$$\begin{aligned}\bar{I}_{a,3\phi} &= |\bar{I}_{a,3\phi}|(\cos\theta - j\sin\theta) \\ &= 240.8(0.80 - j0.60) \\ &= 192.68 - j144.5 \\ &= 240.8\angle -36.9^\circ\text{ A.}\end{aligned} \tag{3.58}$$

$$\begin{aligned}\bar{I}_{b,3\phi} &= a^2\bar{I}_{a,3\phi}\\ &= (1\angle 240^\circ)(240.8\angle -36.9^\circ)\\ &= 240.8\angle 203.1^\circ\\ &= -221.5 - j94.5\,\text{A}.\end{aligned} \tag{3.59}$$

$$\begin{aligned}\bar{I}_{c,3\phi} &= a\bar{I}_{a,3\phi}\\ &= (1\angle 120^\circ(240.8\angle -36.9^\circ)\\ &= 240.8\angle 83.1^\circ\\ &= 28.9 + j239.1\,\text{A}.\end{aligned} \tag{3.60}$$

The single-phase component of the line currents can be found as

$$\begin{aligned}\left|\bar{I}_{1\phi}\right| &= \frac{S_{L,1\phi}}{V_{L-L}}\\ &= \frac{50}{0.240} = 208.33\,\text{A}\end{aligned} \tag{3.61}$$

therefore

$$\begin{aligned}\bar{I}_{1\phi} &= \left|\bar{I}_{1\phi}\right|[\cos(30^\circ - \theta_1) + j\sin(30^\circ - \theta_1)]\\ &= 208.33[\cos(30^\circ - 25.8^\circ) + j\sin(30^\circ - 25.8^\circ)]\\ &= 207.78 + j15.26\,\text{A}.\end{aligned} \tag{3.62}$$

Hence, the line currents flowing in each secondary-phase wire can be found as

$$\begin{aligned}\bar{I}_a &= \bar{I}_{a,3\phi} + \bar{I}_{1\phi}\\ &= 192.68 - j144.5 + 207.78 + j15.26\\ &= 400.46 - j129.24\\ &= 420.8\angle -17.9^\circ\,\text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_b &= \bar{I}_{b,3\phi} - \bar{I}_{1\phi}\\ &= -221.5 - j94.5 - 207.78 - j15.26\\ &= 429.28 - j109.76\\ &= 442.8\angle -165.7^\circ\,\text{A}.\end{aligned}$$

$$\begin{aligned}\bar{I}_c &= I_{c,3\phi}\\ &= 240.8\angle 83.1^\circ\,\text{A}.\end{aligned}$$

(*b*) The current flowing in the secondary winding of each transformer is

$$\begin{aligned}\bar{I}_{ba} &= \bar{I}_a\\ &= 420.8\angle -17.9^\circ\,\text{A}.\end{aligned}$$

$$
\begin{aligned}
\bar{I}_{cb} &= -\bar{I}_c \\
&= -240.8\angle 83.1^\circ \\
&= 240.8\angle 83.1^\circ + 180^\circ \\
&= 240.8\angle 263.1^\circ \,\text{A}.
\end{aligned}
$$

(*c*) The kilovoltampere load on each transformer can be found as

$$
\begin{aligned}
S_{\text{L},\,ba} &= V_{ba} \times |\bar{I}_{ba}| \\
&= 0.240 \times 420.8 \\
&= 101\,\text{kVA}.
\end{aligned}
\tag{3.63a}
$$

$$
\begin{aligned}
S_{\text{L},\,cb} &= V_{cb} \times |\bar{I}_{cb}| \\
&= 0.240 \times 240.8 \\
&= 57.8\,\text{kVA}.
\end{aligned}
\tag{3.63b}
$$

(*d*) The current flowing in each primary-phase wire can be found by dividing the current flow in each secondary winding by the turns ratio. Therefore,

$$
n = \frac{7620\,\text{V}}{240\,\text{V}} = 31.7
$$

and hence

$$
\begin{aligned}
\bar{I}_A &= \frac{\bar{I}_{ba}}{n} \\
&= \frac{420.8\angle -17.9^\circ}{31.75} \\
&= 12.6 - j4.07 \\
&= 13.25\angle -17.9^\circ \text{A}.
\end{aligned}
\tag{3.64a}
$$

$$
\begin{aligned}
\bar{I}_B &= \frac{\bar{I}_{cb}}{n} \\
&= \frac{240.8\angle 263.1^\circ}{31.75} \\
&= -0.91 - j7.52 \\
&= 7.58\angle 263.1^\circ \text{A}.
\end{aligned}
\tag{3.64b}
$$

Therefore, the current in the primary neutral is

$$
\begin{aligned}
\bar{I}_N &= \bar{I}_A + \bar{I}_B \\
&= 13.25\angle -17.9^\circ + 7.58\angle 263.1^\circ \\
&= 11.69 - j11.6 \\
&= 16.47\angle -44.8^\circ \,\text{A}.
\end{aligned}
\tag{3.65}
$$

3.10.6 The Δ-Y Transformer Connection

Figures 3.37 and 3.38 show three single-phase transformers connected in Δ-Y to provide for 120/208-V three-phase four-wire grounded-Y service at 30° and 210° angular displacements, respectively.

In the previously mentioned transformer banks the single-phase lighting load is all in one phase, resulting in unbalanced primary currents in any one bank. To eliminate this difficulty, the Δ-Y system finds many uses. Here the neutral of the secondary three-phase system is grounded and single-phase loads are connected between the different phase wires and the neutral while the three-phase loads are connected to the phase wires. Therefore, the single-phase loads can be balanced on three phases in each bank, and banks may be paralleled if desired.

When transformers of different capacities are used, maximum safe transformer bank rating is three times the capacity of the smallest transformer. If one transformer becomes damaged or is removed from service, the transformer bank becomes inoperative.

With both the Y-Y and the Δ-Δ connections, the line voltages on the secondaries are in phase with the line voltages on the primaries, but with the Y-Δ or the Δ-Y connections, the line voltages on the secondaries are at 30° to the line voltages on the primaries. Consequently a Y-Δ or Δ-Y transformer bank cannot be operated in parallel with a Δ-Δ or Y-Y transformer bank. Having the identical angular displacements becomes especially important when three-phase transformers are interconnected into the same secondary system or paralleled with three-phase banks of single-phase transformers. The additional conditions to successfully parallel three-phase distribution transformers are the following:

1. All transformers have identical frequency ratings.
2. All transformers have identical voltage ratings.
3. All transformers have identical tap settings.
4. Per unit impedance of one transformer is between 0.925 and 1.075 of the other.

The Δ-Y step-up and Y-Δ step-down connections are especially suitable for high-voltage transmission systems. They are economical in cost, and they supply a stable neutral point to be solidly grounded or grounded through resistance of such value so as to damp the system critically and prevent the possibility of oscillation.

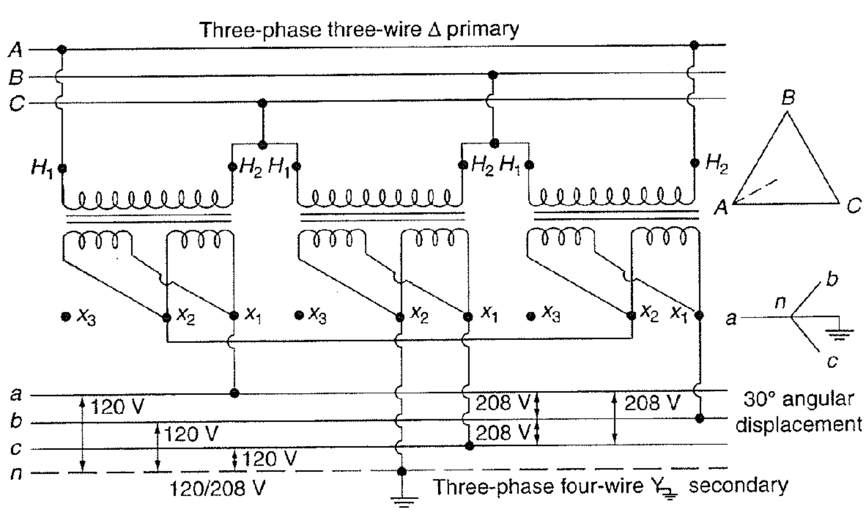

FIGURE 3.37 Delta-wye connection with 30° angular displacement.

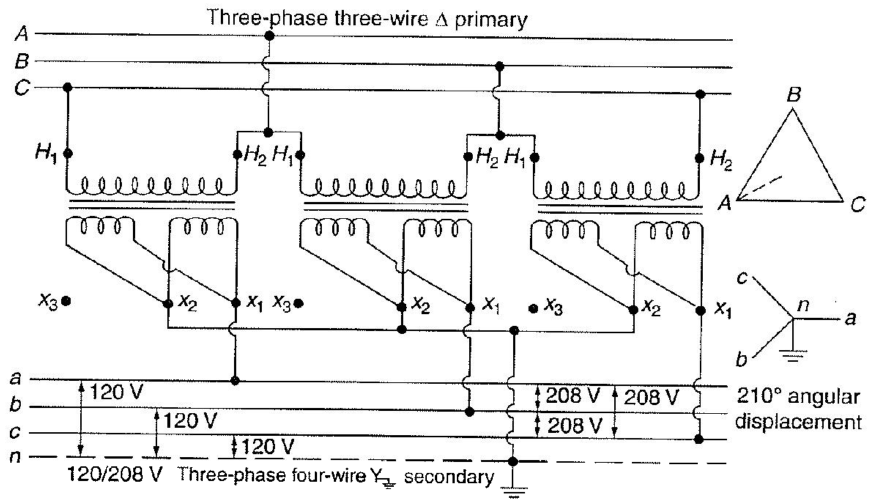

FIGURE 3.38 Delta-wye connection with 210° angular displacement.

3.11 THREE-PHASE TRANSFORMERS

Three-phase voltages may be transformed by means of three-phase transformers. The core of a three-phase transformer is made with three legs, a primary and secondary winding of one phase being placed on each leg. It is possible to construct the core with only three legs since the fluxes established by the three windings are 120° apart in time phase. Two core legs act as the return for the flux in the third leg. For example, if flux is at a maximum value in one leg at some instant, the flux is half that value and in the opposite direction through the other two legs at the same instant.

The three-phase transformer takes less space than does the three single-phase transformers having the same total capacity rating since the three windings can be placed together on one core. Furthermore, three-phase transformers are usually more efficient and less expensive than the equivalent single-phase transformer banks. This is especially noticeable at the larger ratings. On the other hand, if one phase winding becomes damaged the entire three-phase transformer has to be removed from the service. Three-phase transformers can be connected in any of the aforementioned connection types. The difference is that all connections are made inside the tank.

Figures 3.39 through 3.43 show various connection diagrams for three-phase transformers. Figure 3.39 shows a Δ-Δ connection for 120/208/240-V three-phase four-wire secondary service at 0° angular displacement. It is used to supply 240-V three-phase loads with small amounts of 120-V single-phase load. Usually, transformers with a capacity of 150 kVA or less are built in such a design that when 5% of the rated kilovoltamperes of the transformer is taken from the 120-V tap on the 240-V connection, the three-phase capacity is decreased by 25%.

Figure 3.40 shows a three-phase open-Δ connection for 120/240-V service. It is used to supply large 120- and 240-V single-phase loads simultaneously with small amounts of three-phase load. The two sets of windings in the transformer are of different capacity sizes in terms of kilovoltamperes. The transformer efficiency is low especially for three-phase loads. The transformer is rated only 86.6% of the rating of the two sets of windings when they are equal in size, and less than this when they are unequal.

Figure 3.41 shows a three-phase Y-Δ connection for 120/240-V service at 30° angular displacement. It is used to supply three-phase 240-V loads and small amounts of 120-V single-phase loads. Figure 3.42 shows a three-phase open-Y open-Δ connection for 120/240-V service at 30° angular displacement. The statements on efficiency and capacity for three-phase open-Δ connection are also

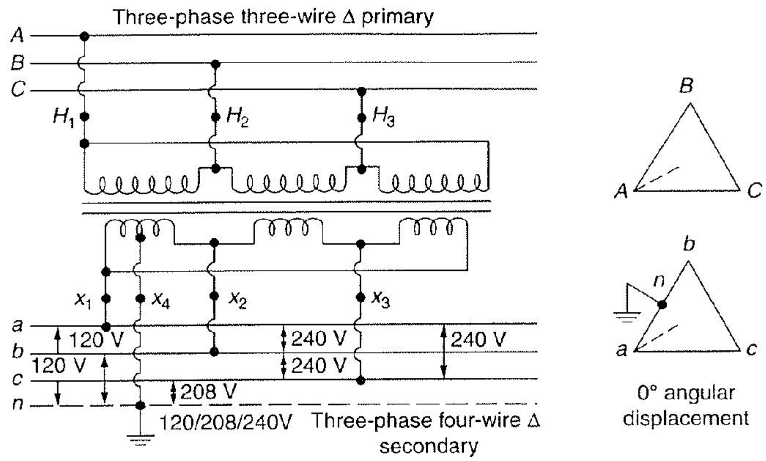

FIGURE 3.39 Three-phase transformer connected in delta-delta.

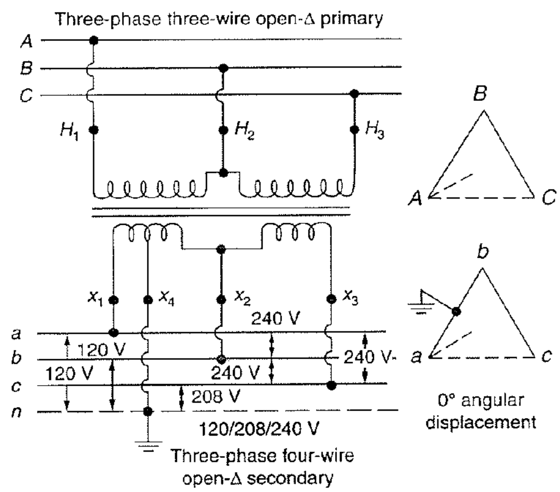

FIGURE 3.40 Three-phase transformer connected in open-delta.

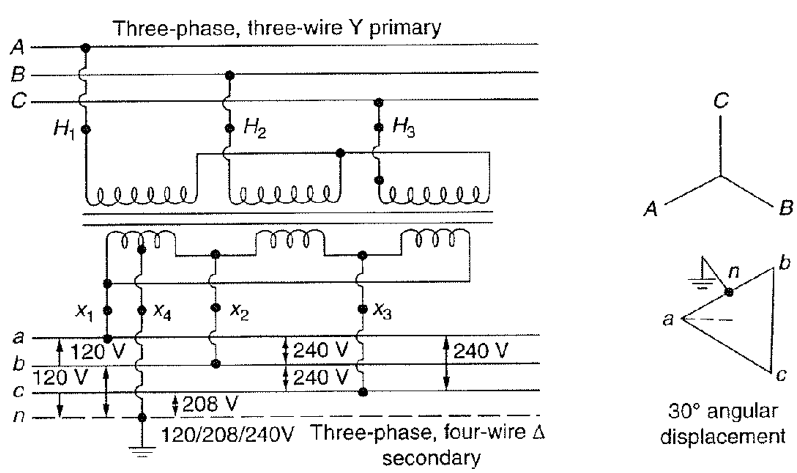

FIGURE 3.41 Three-phase transformer connected in wye-delta.

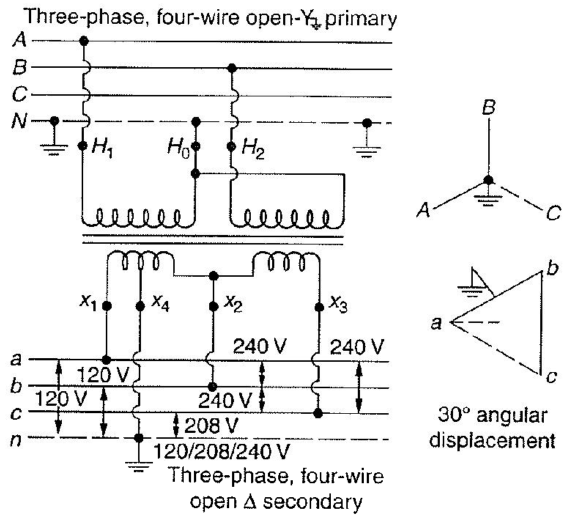

FIGURE 3.42 Three-phase transformer connected in open-wye open-delta.

applicable for this connection. Figure 3.43 shows a three-phase transformer connected in Y-Y for 120/208Y-V service. The connection allows single-phase loads to balance among the three phases.

3.12 THE T OR SCOTT CONNECTION

In some localities, two-phase is required from a three-phase system. The T or Scott connection, which employs two transformers, is the most frequently used connection for three-phase to two-phase (or even three-phase) transformations. In general, the T connection is primarily used for getting a three-phase transformation, whereas the Scott connection is mainly used for getting a two-phase transformation. In either connection type, the basic design is the same. Figures 3.44 through 3.46 show various types of the Scott connection. This connection type requires two single-phase transformers with Scott taps. The first transformer is called the main transformer and

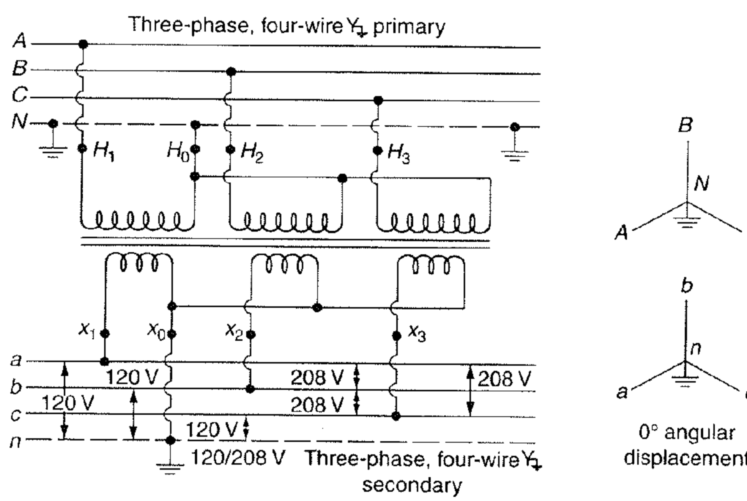

FIGURE 3.43 Three-phase transformer connected in grounded wye-wye.

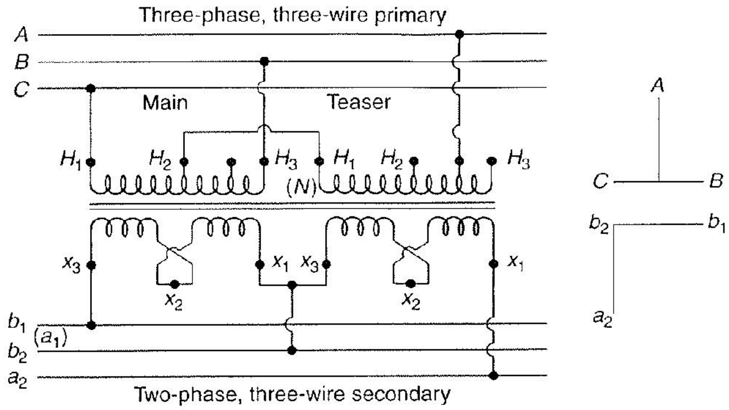

FIGURE 3.44 The T or Scott connection for three-phase to two-phase three-wire, transformation.

connected from line-to-line, and the second one is called the teaser transformer and connected from the midpoint of the first transformer to the third line. It dictates that the midpoints of both primary and secondary windings be available for connections. The secondary may be either three-, four-, or five-wire, as shown in the figures.

In either case, the connection needs specially wound, single-phase transformers. The main transformer has a 50% tap on the primary-side winding, whereas the teaser transformer has an 86.6% tap. (In usual design practice, both transformers are built to be identical so that both have a 50% and an 86.6% tap in order to be used interchangeably as main and teaser transformers.) Although only two single-phase transformers are required, their total rated kilovoltampere capacity must be 15.5% greater if the transformers are interchangeable, or 7.75% greater if noninterchangeable, than the actual load supplied (or than the standard single-phase transformer of the same kilovoltampere and voltage). It is very important to keep the relative phase sequence of the windings the same so that the impedance between the two half windings is a minimum to prevent excessive voltage drop and the resultant voltage unbalance between phases.

The T or Scott connections change the number of phases but not the power factor, which means that a balanced load on the secondary will result in a balanced load on the primary. When the two-phase load at the secondary has a unity power factor, the main transformer operates at 86.6% power

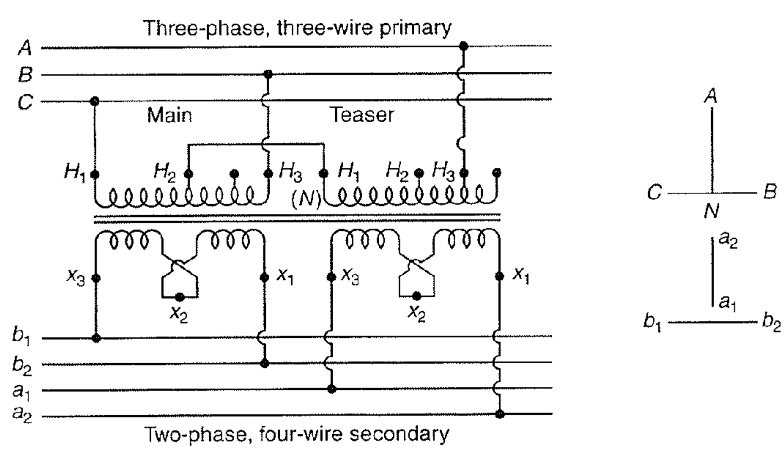

FIGURE 3.45 The T or Scott connection for three-phase to two-phase, four-wire, transformation.

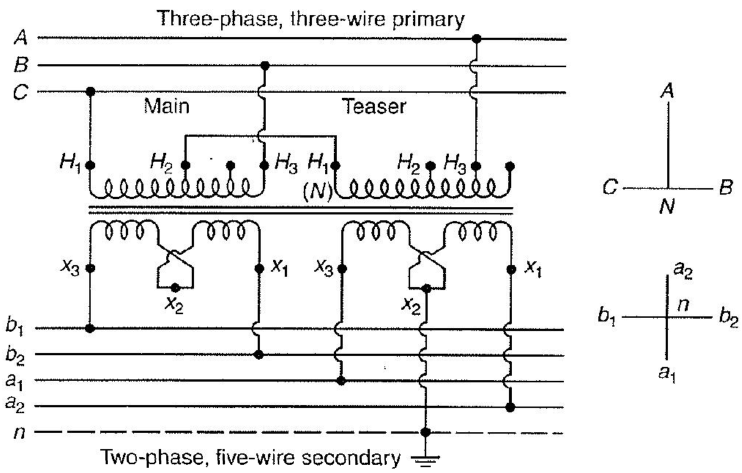

FIGURE 3.46 The T or Scott connection for three-phase to two-phase, five-wire, transformation.

factor and the teaser transformer operates at unity power factor. These connections can transform power in either direction, that is, from three-phase to two-phase or from two-phase to three-phase.

Example 3.7

Two transformer banks are sometimes used in distribution systems, as shown in Figure 3.47, especially to supply customers having large single-phase lighting loads and small three-phase (motor) loads.

The low-voltage connections are three-phase four-wire 120/240-V open-Δ. The high-voltage connections are either open-Δ or open-Y. If it is open-Δ, the transformer-rated high voltage is the primary line-to-line voltage. If it is open-Y, the transformer-rated high voltage is the primary line-to-neutral voltage.

In preparing wiring diagrams and phasor diagrams, it is important to understand that all odd-numbered terminals of a given transformer, that is, H_1, x_1, x_3, and so on, have the same instantaneous voltage polarity. For example, if all the odd-numbered terminals are positive (+) at a particular

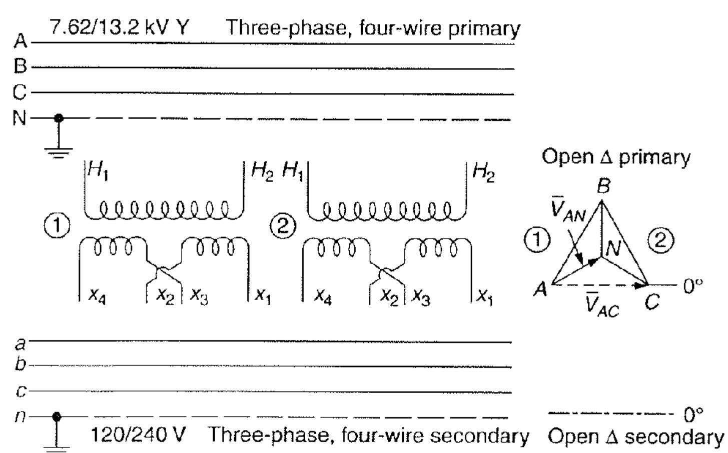

FIGURE 3.47 For Example 3.7.

instant of time, then all the even-numbered terminals are negative (–) at the same instant. In other words, the no-load phasor voltages of a given transformer, for example, $\bar{V}_{H_1H_2}$, $\bar{V}_{x_1x_2}$, and $\bar{V}_{x_3x_4}$, are all in phase.

Assume that ABC phase sequence is used in the connections for both high- and low voltages and the phasor diagrams and

$$\bar{V}_{AC} = 13,200\angle 0°\,\text{V}$$

and

$$\bar{V}_{AN} = 7620\angle 30°\,\text{V}.$$

Also assume that the left-hand transformer is used for lighting. To establish the two-transformer bank with open-Δ primary and open-Δ secondary:

(a) Draw and/or label the voltage phasor diagram required for the open-Δ primary and open-Δ secondary on the 0° references given.
(b) Show the connections required for the open-Δ primary and open-Δ secondary.

Solution

Figure 3.48 illustrates the solution. Note that, because of Kirchhoff's voltage law, there are $\bar{V}_{AC}$ and $\bar{V}_{ac}$ voltages between A and C and between a and c, respectively. Also note that the midpoint of the left-hand transformer is grounded to provide the 120 V for lighting loads.

Example 3.8

Figure 3.49 shows another two-transformer bank which is known as the T-T connection. Today, some of the so-called three-phase distribution transformers now marketed contain two single-phase cores and coils mounted in one tank and connected T-T. The performance is substantially like banks of three identical single-phase transformers or classical core- or shell-type three-phase

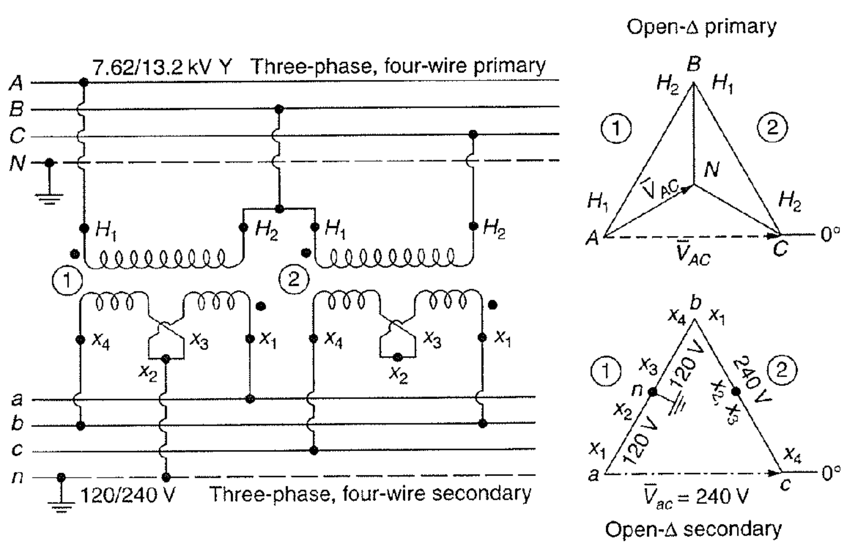

FIGURE 3.48 For Example 3.7.

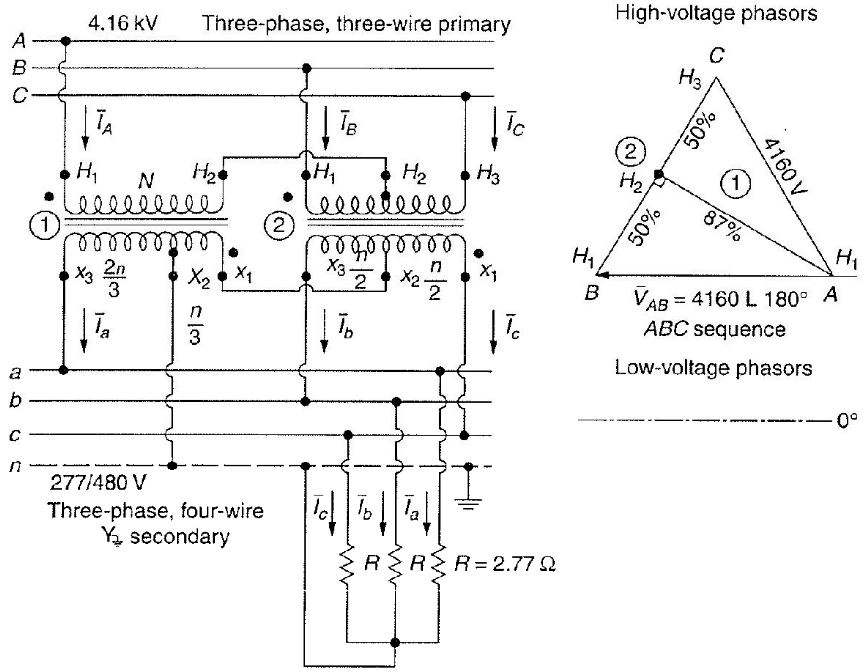

FIGURE 3.49 A particular T-T connection.

transformers. However, perfectly balanced secondary voltages do not occur although the load and the primary voltages are perfectly balanced. In spite of that, the unbalance in secondary voltages is small.

Figure 3.49 shows a particular T-T connection diagram and an arbitrary set of balanced three-phase primary voltages. Assume that the no-load line-to-line and line-to-neutral voltages are 480 and 277 V, respectively, exactly like Y circuitry, and *abc* sequence.

Based on the given information and Figure 3.49, determine the following:

(*a*) Draw the low-voltage phasor diagram, correctly oriented on the 0° reference shown.
(*b*) Find the value of the $\bar{V}_{ab}$ phasor.
(*c*) Find the magnitudes of the following rated winding voltages:

- (i) The voltage $V_{H_1H_2}$ on transformer 1.
- (ii) The voltage $V_{x_1x_2}$ on transformer 1.
- (iii) The voltage $V_{x_2x_3}$ on transformer 1.
- (iv) The voltage $V_{H_1H_2}$ on transformer 2.
- (v) The voltage $V_{H_2H_3}$ on transformer 2.
- (vi) The voltage $V_{x_1x_2}$ on transformer 2.
- (vii) The voltage $V_{x_2x_3}$ on transformer 2.

(*d*) Would it be possible to parallel a T-T transformer bank with:

- (i) A Δ-Δ bank?
- (ii) A Y-Y bank?
- (iii) A Δ-Y bank?

Solution

(*a*) Figure 3.50 shows the required low-voltage phasor diagram. Note the 180° phase shift among the corresponding phasors.

(b) The value of the voltage phasor is

$$\bar{V}_{ab} = 480\angle 0^\circ \text{ V}.$$

(*c*) The magnitudes of the rated winding voltages:

(i) From the high-voltage phasor diagram shown in Figure 3.49,

$$\left|\bar{V}_{H_1H_2}\right| = (4160^2 - 2080^2)^{1/2}$$
$$= 3600 \text{ V}.$$

(ii) From Figures 3.49 and 3.50,

$$\left|\bar{V}_{X_1X_2}\right| = \frac{1}{2}(480^2 - 240^2)^{1/2}$$
$$= 139 \text{ V}.$$

(iii) From Figures 3.49 and 3.50,

$$\left|\bar{V}_{X_2X_3}\right| = \frac{2}{3}(480^2 - 240^2)^{1/2}$$
$$= 277 \text{ V}.$$

(iv) From Figure 3.49,

$$\left|\bar{V}_{H_1H_2}\right| = 50\%(4160\text{V})$$
$$= 2080 \text{ V}.$$

(v) From Figure 3.49,

$$\left|\bar{V}_{H_2H_3}\right| = 2080 \text{ V}$$

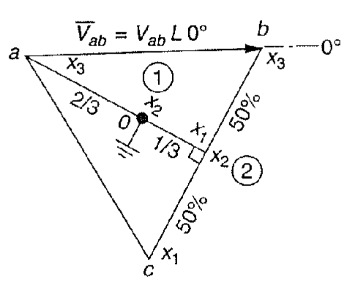

FIGURE 3.50 The required low-voltage phasor diagram.

(vi) From Figure 3.50,

$$\left|\bar{V}_{x_2x_3}\right| = 240\,\text{V}.$$

(*d*) (i) No; (ii) no; (iii) yes.

EXAMPLE 3.9

Assume that the T-T transformer bank of Example 3.8 is to be loaded with the balanced resistors ($R = 2.77\ \Omega$) shown in Figure 3.49. Also assume that the secondary voltages are to be perfectly balanced and that the necessary high-voltage applied voltages then are not perfectly balanced. Determine the following:

(*a*) The low-voltage current phasors.
(*b*) The low-voltage current phasor diagram.
(*c*) At what power factor does the transformer operate?
(*d*) What power factor is seen by winding x_2x_3 of transformer 2?
(*e*) What power factor is seen by winding x_1x_2 of transformer 2?

Solution

(*a*) The low-voltage phasor diagram of Figure 3.50 can be redrawn as shown in Figure 3.51*a*. Therefore, from Figure 3.51*a*, the low-voltage current phasors are:

$$\begin{aligned}\bar{I}_a &= \frac{\bar{V}_{a0}}{R}\\ &= \frac{277\angle -30^\circ}{2.77}\\ &= 100\angle -30^\circ\,\text{A}\end{aligned}$$

$$\begin{aligned}\bar{I}_b &= \frac{\bar{V}_{b0}}{R}\\ &= \frac{-277\angle +30^\circ}{2.77}\\ &= -100\angle +30^\circ\\ &= 100\angle -150^\circ\,\text{A}.\end{aligned}$$

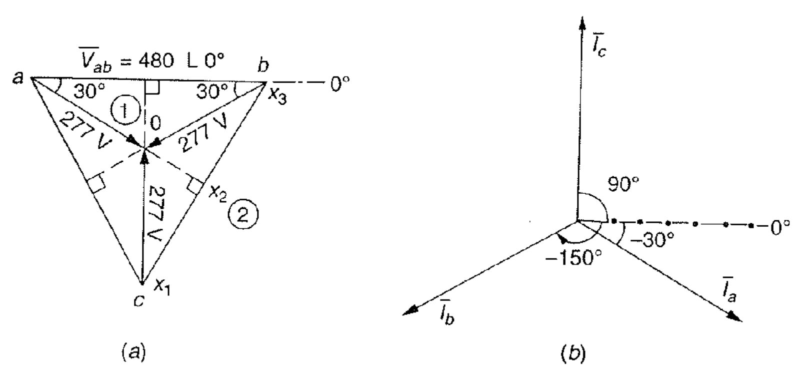

FIGURE 3.51 Phasor diagrams for Example 3.9.

$$\bar{I}_c = \frac{\bar{V}_{c0}}{R}$$
$$= \frac{277\angle 90^\circ}{2.77}$$
$$= 100\angle 90^\circ \text{ A.}$$

(*b*) Figure 3.51*b* shows the low-voltage current phasor diagram.
(*c*) From part (*a*), the power factor of transformer 1 can be found as

$$\cos\theta_{T1} = \cos(\theta_{\bar{V}_{a0}} - \theta_{\bar{I}_a})$$
$$= \cos[(-30^\circ) - (-30^\circ)]$$
$$= 1.0.$$

(*d*) The power factor seen by winding x_3x_2 of transformer 2 is 0.866, lagging.
(*e*) The power factor seen by winding x_1x_2 of transformer 2 is 0.866, leading.

Example 3.10

Consider Example 3.9 and Figure 3.50, and determine the following

(*a*) The necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 1.
(*b*) The necessary voltampere rating of the x_2x_1 low-voltage winding of transformer 1.
(*c*) Total voltampere output from transformer 1.
(*d*) The necessary voltampere rating of the x_1x_2 low-voltage winding of transformer 2.
(*e*) The necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 2.
(*f*) Total voltampere output from transformer 2.
(*g*) The ratio of total voltampere rating of all low-voltage windings in the transformer bank to maximum continuous voltampere output from the bank.

Solution

(a) From Figure 3.50, the necessary voltampere rating of the x_2x_3 low-voltage winding of transformer 1 is

$$S_{x_2x_3} = \frac{2}{3}\left(\frac{\sqrt{3}}{2}V\right)I$$
$$= \frac{VI}{\sqrt{3}} \text{ VA.}$$

(*b*) Similarly,

$$S_{x_2x_1} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}V\right)I$$
$$= \frac{VI}{2\sqrt{3}} \text{ VA.}$$

(*c*) Therefore, total voltampere output rating from transformer 1 is

$$\sum S_{T_1} = S_{x_2x_1} + S_{x_2x_3}$$
$$= \frac{\sqrt{3}}{2} VI \text{ VA.}$$

(*d*) From Figure 3.50, the necessary voltampere rating of the x_1x_2 low-voltage winding of transformer 2 is

$$S_{x_1x_2} = \frac{V}{2} \times I \text{ VA.}$$

(*e*) Similarly,

$$S_{x_2x_3} = \frac{V}{2} \times I \text{ VA.}$$

(*f*) Therefore, total voltampere output rating from transformer 2 is

$$\sum S_{T_2} = S_{x_1x_2} + S_{x_2x_3}$$
$$= VI \text{ VA.}$$

(*g*) The ratio is

$$\frac{\sum \text{Installed core and coil capacity}}{\text{Max continuous output}} = \frac{\left(\sqrt{3}/2\right)+1}{\sqrt{3}} = 1.078.$$

The same ratio for two-transformer banks connected in open-Δ high-voltage open-Δ low-voltage, or open-Y high-voltage open-Δ low-voltage is 1.15.

Example 3.11

In general, except for unique unbalanced loads, two-transformer banks do not deliver balanced three-phase low-voltage terminal voltages even when the applied high-voltage terminal voltages are perfectly balanced. Also the two transformers do not, in general, operate at the same power factor or at the same percentages of their rated kilovoltamperes. Hence, the two transformers are likely to have unequal percentages of voltage regulation.

Figure 3.52 shows two single-phase transformers connected in open-Y high-voltage and open-Δ low-voltage. The two-transformer bank supplies a large amount of single-phase lighting and some small amount of three-phase power loads. Both transformers have 7200/120–240-V ratings and have equal transformer impedance of $\bar{Z}_T = 0.01 + j0.03$ pu based on their ratings. Here, neglect transformer magnetizing currents.

Figure 3.53 shows the low-voltage phasor diagram. In this problem the secondary voltages are to be assumed to be perfectly balanced and the primary voltages are then unbalanced as required. Note that, in Figure 3.53, the 0 indicates the three-phase neutral point. Based on the given information, determine the following:

(*a*) Find the phasor currents $\bar{I}_a$, $\bar{I}_b$, and $\bar{I}_c$.
(*b*) Select suitable standard kilovoltampere ratings for both transformers. Overloads, as much as 10%, will be allowable as an arbitrary criterion.
(*c*) Find the pu kilovoltampere load on each transformer.

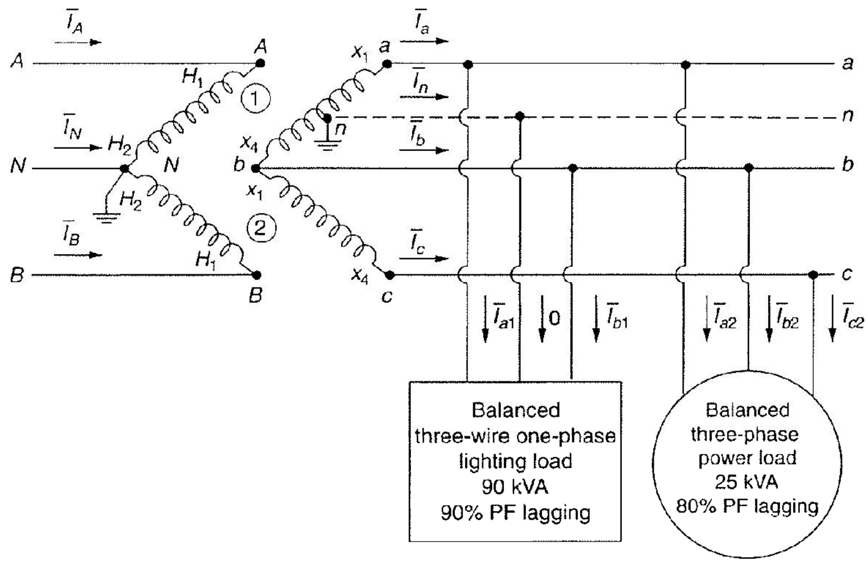

FIGURE 3.52 Two single-phase transformers connected in open-Y and open-Δ.

(d) Find the power factor of the output of each transformer.
(e) Find the phasor currents $\bar{I}_A$, $\bar{I}_B$, and $\bar{I}_N$ in the high-voltage leads.
(f) Find the high-voltage terminal voltages $\bar{V}_{AN}$ and $\bar{V}_{BN}$. Therefore, this part of the question can indicate the amount of voltage unbalance that may be encountered with typical equipment and typical loading conditions.
(g) Also write the necessary codes to solve the problem in MATLAB.

Solution

(a) For the three-wire single-phase balanced lighting load, cos $\theta = 0.90$ lagging or $\theta = 25.8°$ therefore, using the symmetrical components theory,

$$\begin{aligned}\bar{I}_{a1} &= \frac{90\,\text{kVA}}{0.240\,\text{kV}} \angle \theta_{\bar{V}_{ab}} - \theta \\ &= 375\angle 30° - 25.8° \\ &= 375(\cos 4.2° + j \sin 4.2°) \\ &= 374 + j27.5 \\ &= 375\angle 4.2°\,\text{A}.\end{aligned}$$

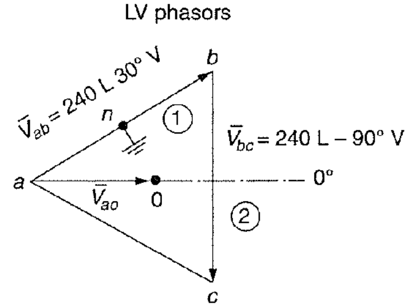

FIGURE 3.53 The low-voltage phasor diagram for Example 3.11.

Also

$$\bar{I}_{b1} = -\bar{I}_{a1}$$
$$= -374 - j27.5$$
$$= -375\angle 4.2°\text{ A}.$$

For the three-phase balanced power load, cos $\theta = 0.80$ lagging or $\theta = 36.8°$; therefore,

$$\bar{I}_{a2} = \frac{25\,\text{kVA}}{\sqrt{3}\times 0.240\,\text{kV}}\angle\theta_{\bar{V}_{ao}} - \theta$$
$$= 60.2\angle 0° - 36.8°$$
$$= 60.2\angle -36.8°\text{ A}.$$

Also

$$\bar{I}_{b_2} = a^2\bar{I}_{a_2}$$
$$= 1\angle 240° \times 60.2\angle -36.8°$$
$$= 60.2\angle 203.2°\text{ A}$$

and

$$\bar{I}_{c_2} = a\bar{I}_{a_2}$$
$$= 1\angle 120° \times 60.2\angle -36.8°$$
$$= 60.2\angle 83.2°\text{ A}.$$

Therefore, the phasor currents in the transformer secondary are

$$\bar{I}_a = \bar{I}_{a1} + \bar{I}_{a2}$$
$$= 375\angle 4.2° + 60.2\angle -36.8$$
$$= 422.04 - j8.44$$
$$= 422.12\angle -1.15°\text{ A}.$$

$$\bar{I}_b = \bar{I}_{b1} + \bar{I}_{b2}$$
$$= -375\angle 4.2° + 60.2\angle 203.2°$$
$$= -429.33 - j51.22$$
$$= 432.37\angle -173.2°\text{ A}.$$

$$\bar{I}_c = \bar{I}_{c1} + \bar{I}_{c2}$$
$$= 0 + 60.2\angle 83.2°$$
$$= 60.2\angle 83.2°\text{A}.$$

(*b*) For transformer 1,

$$\begin{aligned} S_{T_1} &= 0.240\,\text{kV} \times I_a \\ &= 0.240 \times 422.12 \\ &= 101.3\,\text{kVA}. \end{aligned}$$

If a transformer with 100 kVA is selected, S_{T_1} = 1.013 pu kVA with an overload of 1.3%. For transformer 2,

$$\begin{aligned} S_{T_2} &= 0.240\,\text{kV} \times I_c \\ &= 0.240 \times 60.2 = 14.4\,\text{kVA}. \end{aligned}$$

If a transformer with 15 kVA is selected,

$$S_{T2} = 0.96\ \text{pu kVA}$$

with a 4% excess capacity.

(*c*) From part (*b*),

$$S_{T1} = 1.013\,\text{pu kVA}.$$

$$S_{T_2} = 0.96\,\text{pu kVA}.$$

(*d*) Since the power factor that a transformer sees is not the power factor that the load sees, for transformer 1,

$$\begin{aligned} \cos\theta_{T_1} &= \cos\left(\theta_{\bar{V}_{ab}} - \theta_{\bar{I}_a}\right) \\ &= \cos[30° - (-1.15°)] \\ &= \cos 31.15° \\ &= 0.856\,\text{lagging} \end{aligned}$$

and for transformer 2,

$$\begin{aligned} \cos\theta_{T_2} &= \cos\left(\theta_{\bar{V}_{cb}} - \theta_{\bar{I}_c}\right) \\ &= \cos[90° - 83.2°] \\ &= \cos 6.8° \\ &= 0.993\,\text{lagging}. \end{aligned}$$

(*e*) The turns ratio is

$$n = \frac{7200\,\text{V}}{240\,\text{V}} = 30$$

therefore,

$$\bar{I}_A = \frac{\bar{I}_a}{n} = \frac{422.12\angle -1.15^\circ}{30} = 14.07\angle -1.15^\circ \text{ A}$$

and

$$\bar{I}_B = -\frac{\bar{I}_c}{n} = -\frac{60.2\angle 83.2^\circ}{30} \cong -2\angle 83.2^\circ \text{A}.$$

Thus,

$$\bar{I}_N = -(\bar{I}_A + \bar{I}_B) = -(14.07\angle -1.15^\circ - 2\angle 83.2^\circ) = -14.02\angle -9.3^\circ \text{ A}.$$

(*f*) In pu,

$$\bar{V}_{AN,\text{ pu}} = \bar{V}_{ab,\text{ pu}} + \bar{I}_{a,\text{ pu}} \times \bar{Z}_{\text{T, pu}}$$

where

$$\bar{I}_{\text{base, LV}} = \frac{100 \text{ kVA}}{0.240 \text{ kV}} = 416.67 \text{ A}.$$

$$\bar{I}_{a,\text{ pu}} = \frac{\bar{I}_a}{I_{\text{base, LV}}} = \frac{422.12\angle -1.15^\circ}{416.67} = 1.013\angle -1.15^\circ \text{pu A}.$$

$$\bar{V}_{ab,\text{ pu}} = \frac{\bar{V}_{ab}}{V_{\text{base, LV}}} = \frac{0.240\angle 30^\circ}{0.240} = 1.0\angle 30^\circ \text{pu V}.$$

$$\bar{Z}_{\text{T, pu}} = 0.01 + j0.03 \text{ pu } \Omega.$$

Therefore,

$$\bar{V}_{AN,\text{pu}} = 1.0\angle 30^\circ + (1.013\angle -1.15^\circ)(0.01 + j0.03) = 1.024\angle 31.15^\circ \text{puV}$$

r

$$\begin{aligned}\bar{V}_{AN} &= \bar{V}_{AN,\,pu} \times V_{base,\,HV}\\ &= (1.024\angle 31.15°)(7200\,V)\\ &= 7372.8\angle 31.15°\,V.\end{aligned}$$

lso

$$\bar{V}_{BN,\,pu} = \bar{V}_{bc,\,pu} - \bar{I}_{c,\,pu} \times \bar{Z}_{T,\,pu}$$

where

$$\begin{aligned}\bar{I}_{c,\,pu} &= \frac{\bar{I}_c}{I_{base,LV}}\\ &= \frac{60.2\angle 83.2°}{416.67} = 0.144\angle 83.2°\,pu\,A.\end{aligned}$$

$$\begin{aligned}\bar{V}_{bc,\,pu} &= \frac{\bar{V}_{bc}}{V_{base,\,LV}}\\ &= \frac{0.240\angle -90°}{0.240} = 1.0\angle -90° pu\,V.\end{aligned}$$

Therefore,

$$\begin{aligned}\bar{V}_{BN,\,pu} &= 1.0\angle -90° + (0.144\angle 83.2°)(0.01 + j0.03)\\ &= 1.00195\angle -89.76° pu\ V\end{aligned}$$

or

$$\begin{aligned}\bar{V}_{BN} &= \bar{V}_{BN,\,pu} \times V_{base,\,HV}\\ &= (1.00195\angle -89.76°)(7200\,V)\\ &= 7214.04\angle -89.76°\,V.\end{aligned}$$

Note that the difference between the phase angles of the $\bar{V}_{AN}$ and $\bar{V}_{BN}$ voltages is almost 120° and the difference between their magnitudes is almost 80 V.

(*g*) Here is the MATLAB script:

~~~~~~

```
clc
clear

% System parameters
ZT = 0.01 + j*0.03;
PF11 = 0.9;
Smag11 = 90; % kVA
```
~~~~~~

```
PFpl = 0.8;
Smagpl = 25; % kVA
kVa = 0.24;
thetaVab = (pi*30)/180;
thetaVcb = (pi*90)/180;
thetaVa0 = 0;
a = -0.5 + j*0.866;
n = 7200/240; % turns ratio

% Solution for part a

% Phasor currents Ia, Ib and Ic
Ia1 = (Smagll/kVa)*(cos(thetaVab - acos(PFll)) + j*sin(thetaVab
- acos(PFll)))
Ia2 = (Smagpl/(sqrt(3)*kVa))*(cos(thetaVa0 - acos(PFpl)) +
j*sin(thetaVa0 - acos(PFpl)))
Ib1 = -Ia1
Ib2 = a^2*Ia2
Ic2 = a*Ia2

Ia = Ia1 + Ia2
Ib = Ib1 + Ib2
Ic = Ic2

% Solution for part b and part c

% For transformer 1
ST1 = kVa*abs(Ia)
ST1pu100kVA = ST1/100

% For transformer 2
ST2 = kVa*abs(Ic)
ST2pu15kVA = ST2/15

% Solution for part d
PFT1 = cos(thetaVab - atan(imag(Ia)/real(Ia)))
PFT2 = cos(thetaVcb - atan(imag(Ic)/real(Ic)))

% Solution for part e
IA = Ia/n
IB = Ib/n
IN = -(IA + IB)
% Solution for part f
IbaseLV = 100/kVa
Iapu = Ia/IbaseLV
Vabpu = (kVa*(cos(thetaVab) + j*sin(thetaVab)))/kVa
VANpu = Vabpu + Iapu*ZT
VAN = VANpu*7200

Icpu = Ic/IbaseLV
Vbcpu = (kVa*(cos(-thetaVcb) + j*sin(-thetaVcb)))/kVa
VBNpu = Vbcpu - Icpu*ZT
```

```
VBN = VBNpu*7200
Vmagdiff = abs(VAN) - abs(VBN)
Thetadiff = 180*(atan(imag(VAN)/real(VAN)) - atan(imag(VBN)/
real(VBN)))/pi
```

3.13 THE AUTOTRANSFORMER

The usual transformer has two windings (not including a tertiary, if there is any) which are not connected to each other, whereas an autotransformer is a transformer in which one winding is connected in series with the other as a single winding. In this sense, an autotransformer is a normal transformer connected in a special way. It is rated on the basis of output kilovoltamperes rather than the transformer's kilovoltamperes. It has lower leakage reactance, lower losses, smaller excitation current requirements, and, most of all, it is cheaper than the equivalent two-winding transformer (especially when the voltage ratio is 2:1 or less).

Figure 3.54 shows the wiring diagram of a single-phase autotransformer. Note that S and C denote the series and common portions of the winding. There are two voltage ratios, namely, circuit and winding ratios. The circuit ratio is

$$\begin{aligned}\frac{V_H}{V_x} &= n \\ &= \frac{n_1+n_2}{n_1} \\ &= 1+\frac{n_2}{n_1}\end{aligned} \tag{3.66}$$

where V_H is the voltage on the high-voltage side, V_x is the voltage on the low-voltage side, n is the turns ratio of the autotransformer, n_1 is the number of turns in the common winding, and n_2 is the number of turns in the series winding.

As can be observed from Equation 3.66, the circuit ratio is always larger than 1.

On the other hand, the winding-voltage ratio is

$$\begin{aligned}\frac{V_S}{V_C} &= \frac{n_2}{n_1} \\ &= n-1\end{aligned} \tag{3.67}$$

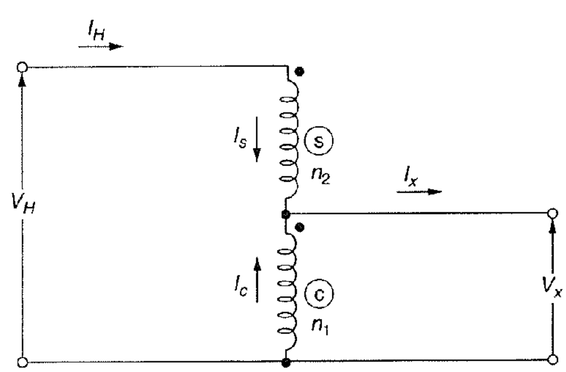

FIGURE 3.54 Wiring diagram of a single-phase autotransformer.

where V_S is the voltage across the series winding and V_C is the voltage across the common winding. Similarly, the current ratio is

$$\begin{aligned}\frac{I_C}{I_S} &= \frac{I_C}{I_H}\\ &= \frac{I_x - I_H}{I_H}\\ &= n-1\end{aligned} \tag{3.68}$$

where I_C is the current in the common winding, I_S is the current in the series winding, I_x is the output current at the low-voltage side, and I_H is the input current at the high-voltage side.

Therefore, the circuit's voltampere rating for an ideal autotransformer is

$$\begin{aligned}\text{Circuit's VA rating} &= V_H I_H\\ &= V_x I_x\end{aligned} \tag{3.69}$$

and the winding's voltampere rating is

$$\begin{aligned}\text{Winding's VA rating} &= V_S I_S\\ &= V_C I_C\end{aligned} \tag{3.70}$$

which describes the capacity of the autotransformer in terms of core and coils.

Therefore, the capacity of an autotransformer can be compared with the capacity of an equivalent two-winding transformer (assuming that the same core and coils are used) as

$$\begin{aligned}\frac{\text{Capacity as autotransformer}}{\text{Capacity as two-winding transformer}} &= \frac{V_H I_H}{V_S I_S}\\ &= \frac{V_H I_H}{(V_H - V_x) I_H}\\ &= \frac{V_H / V_x}{(V_H - V_x)/V_x}\\ &= \frac{n}{n-1}.\end{aligned} \tag{3.71}$$

For example, if n is given as 2, the ratio, given by Equation 3.71, is 2, which means that

Capacity as autotransformer = 2 × capacity as two-winding transformer.

Therefore, one can use a 500-kVA autotransformer instead of using a 1000-kVA two-winding transformer. Note that as n approaches 1, which means that the voltage ratios approach 1, such as 7.2 kV/6.9 kV, then the savings, in terms of the core and coil sizes of autotransformer, increases. An interesting case happens when the voltage ratio (or the turns ratio) is unity: the maximum savings is achieved but then there is no need for any transformer since the high- and low voltages are the same.

Figure 3.55 shows a single-phase autotransformer connection used in distribution systems to supply 120/240-V single-phase power from an existing 208Y/120-V three-phase system, most economically.

Figure 3.56 shows a three-phase autotransformer Y-Y connection used in distribution systems to increase voltage at the ends of feeders or where extensions are being made to existing feeders. It is

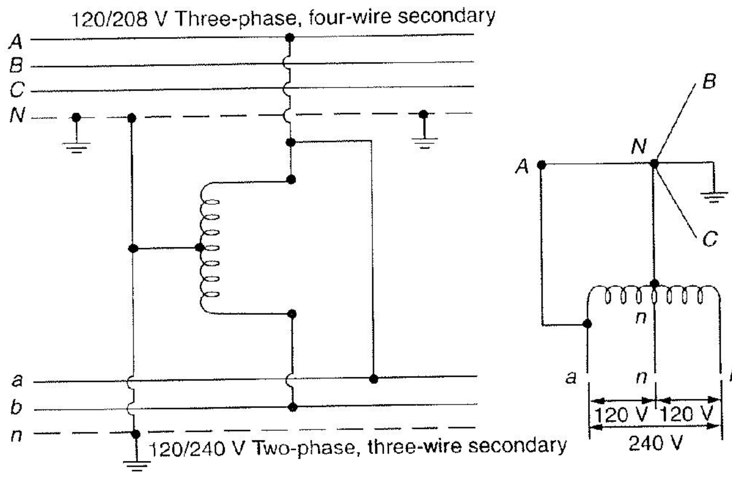

FIGURE 3.55 Single-phase autotransformer.

ılso the most economical way of stepping down the voltage. It is necessary that the neutral of the ıutotransformer bank be connected to the system neutral to prevent excessive voltage development ɔn the secondary side. Also, the system impedance should be large enough to restrict the short-circuit current to about 20 times the rated current of the transformer to prevent any transformer burn-outs.

3.14 THE BOOSTER TRANSFORMERS

Booster transformers are also called the *buck-and-boost transformers* and provide a fixed buck or boost voltage to the primary of a distribution system when the line voltage drop is excessive. The transformer connection is made in such a way that the secondary is in series and in phase with the main line.

Figure 3.57 shows a single-phase booster transformer connection. The connections shown in Figure 3.57*a* and *b* boost the voltage 5% and 10%, respectively. In Figure 3.57*a*, if the lines to the low-voltage bushings x_3 and x_1 are interchanged, a 5% buck in the voltage results. Figure 3.58 shows a three-phase three-wire booster transformer connection using two single-phase booster transformers. Figure 3.59 shows a three-phase four-wire booster transformer connection using

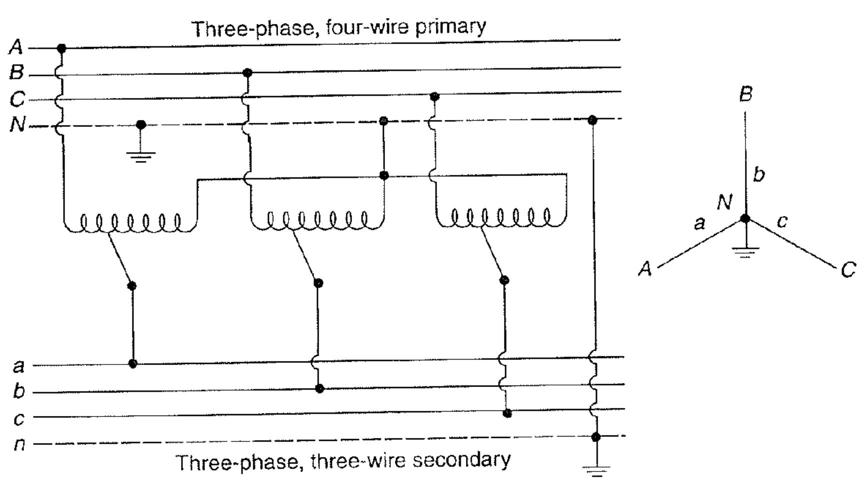

FIGURE 3.56 Three-phase autotransformer.

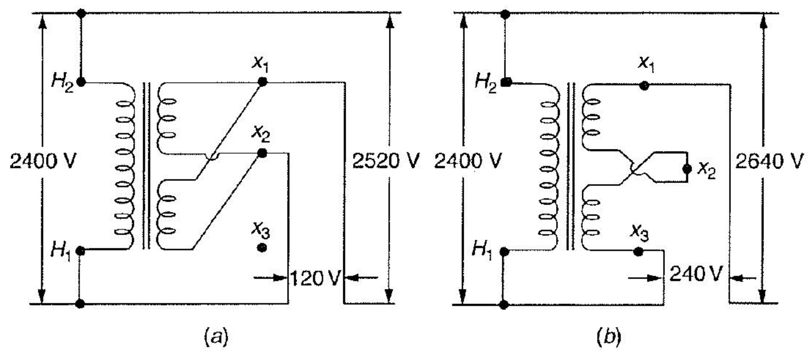

FIGURE 3.57 Single-phase booster transformer connection: (*a*) for 5% boost and (*b*) for 10% boost.

three single-phase booster transformers. Both low- and high-voltage windings and bushings have the same level of insulation. To prevent harmful voltage induction by the series winding, the transformer primary must never be open under any circumstances before opening or unloading the secondary. Also, the primary side of the transformer should not have any fuses or disconnecting devices. Boosters are often used in distribution feeders where the cost of tap-changing transformers is not justified.

3.15 AMORPHOUS METAL DISTRIBUTION TRANSFORMERS

The continuing importance of distribution system efficiency improvement and its economic evaluation has focused greater attention on designing equipment with exceptionally high efficiency levels. For example, because of extremely low magnetic losses, amorphous metal offers the opportunity to reduce the core loss of distribution transformers by approximately 60% and thereby reduce operating

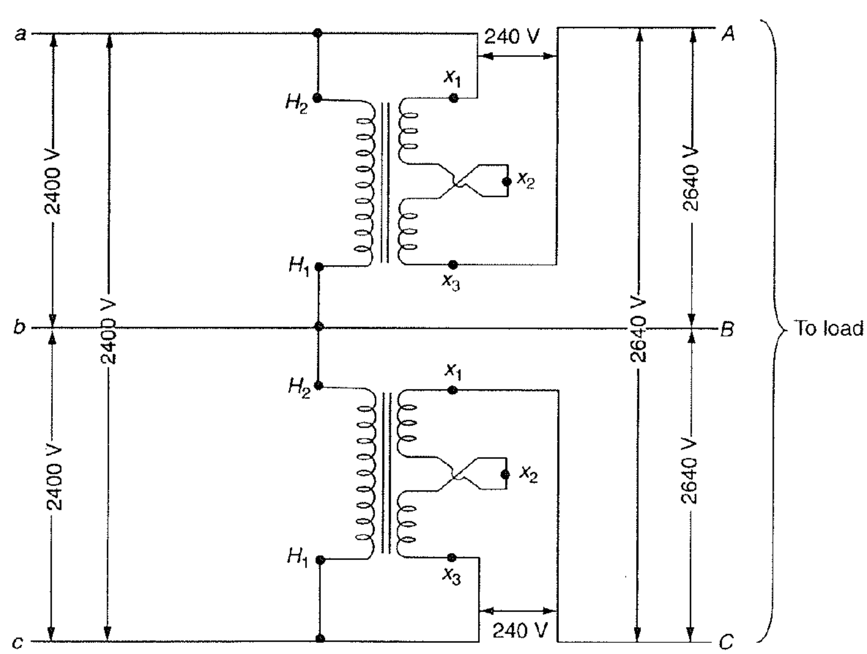

FIGURE 3.58 Three-phase three-wire booster transformer connection using two single-phase booster transformers.

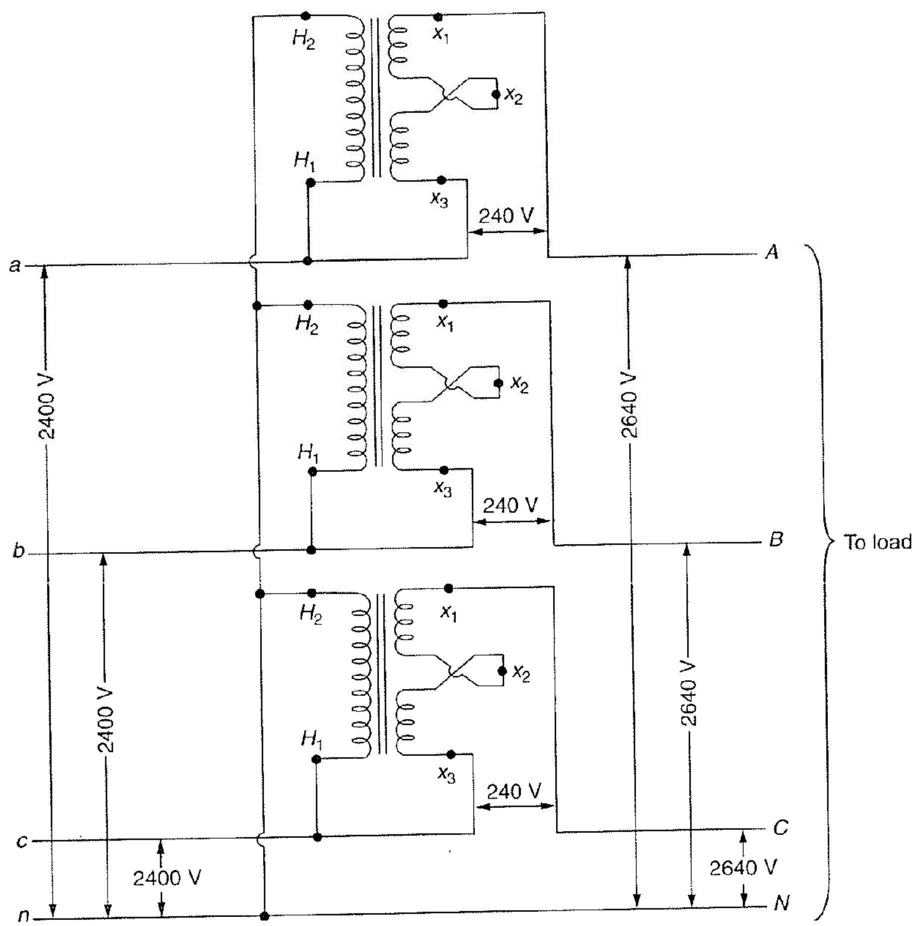

FIGURE 3.59 Three-phase four-wire booster transformer connection using three single-phase booster transformers.

costs. For example, core loss of a 25 kVA, 7200/12,470Y-120/240 V silicon steel transformer is 86 W, whereas it is only 28 W for an amorphous transformer. In addition, it is quieter (with 38 db) than its equivalent silicon steel transformer (with 48 db). There are more than 25 million distribution transformers installed in this country. Replacing then with amorphous units could result in an energy savings of nearly 15 billion kWh per year. Nationally, this could represent a savings of more than $700 million which is annually equivalent to the energy consumed by a city of 4 million people. Each year, approximately 1 million distribution transformers are installed on utility systems in United States. Application of amorphous metal transformers is a substantial opportunity to reduce utility-operating costs and defer generating capacity additions.

PROBLEMS

3.1 Repeat Example 3.7, assuming an open-Y primary and an open-Δ secondary and using the 0° references given in Figure P3.1.

1. Also, determine:

   (*a*) The value of the open-Δ high-voltage phasor between *A* and *B*, that is, $\bar{V}_{AB}$.

   (*b*) The value of the open-Y high-voltage phasor between *A* and *N*, that is, $\bar{V}_{AN}$.

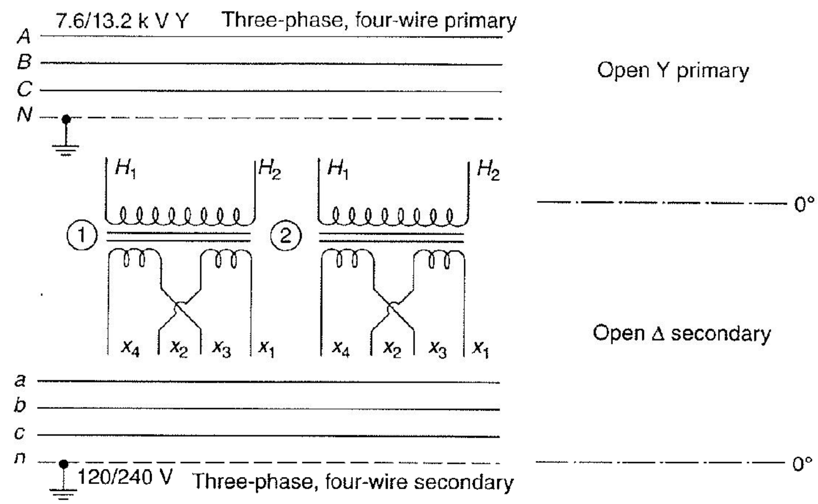

FIGURE P3.1 For Problem 3.1.

3.2 Repeat Example 3.10, if the low-voltage line current I is 100 A and the line-to-line low voltage is 480 V.

3.3 Consider the T-T connection given in Figure P3.3 and determine the following:

(*a*) Draw the low-voltage diagram, correctly oriented on the 0° reference shown.

(*b*) Find the value of the $\bar{V}_{ab}$ and $\bar{V}_{an}$ phasors.

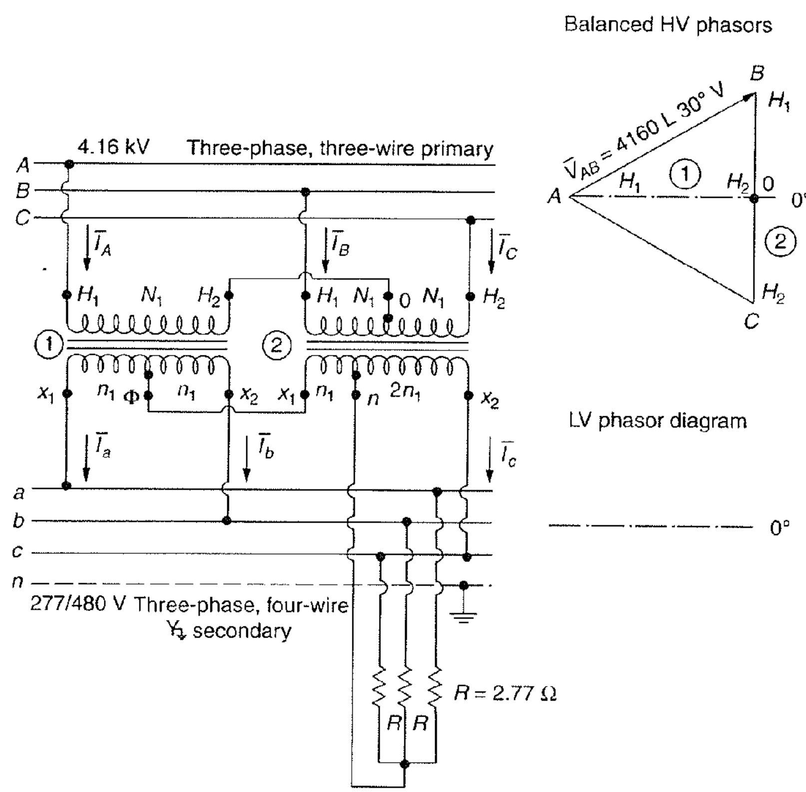

FIGURE P3.3 A T-T connection.

(c) Find the magnitudes of the following rated winding voltages:

(i) The voltage $V_{H_1H_2}$ on transformer 1.
(ii) The voltage $V_{x_1\phi}$ on transformer 1.
(iii) The voltage $V_{\phi x_2}$ on transformer 1.
(iv) The voltage V_{H_10} on transformer 2.
(v) The voltage V_{0H} on transformer 2.
(vi) The voltage V_{x_1n} on transformer 2.
(vii) The voltage V_{nx} on transformer 2.

3.4 Assume that the T-T transformer bank given in Problem 3.3 is loaded with the balanced resistors given. Assume that the secondary voltages are perfectly balanced; the necessary high voltages applied then are not perfectly balanced. Use secondary voltages of 480 V and neglect magnetizing currents. Determine the following:

(*a*) The low-voltage current phasors.
(*b*) The high-voltage current phasors.

3.5 Use the results of Problems 3.3 and 3.4 and apply the complex power formula $S = P + jQ = \overline{VI}^*$ four times, once for each low-voltage winding, for example, a part of the output of transformer 1 is $\bar{V}_{X_1X_2}\bar{I}^*$. Based on these results, find:

(*a*) Total complex power output from the T-T bank. (Does your result agree with that which is easily computed as input to the resistors?)
(*b*) The necessary kilovoltampere ratings of both low-voltage windings of both the transformers.
(*c*) The ratio of total kilovoltampere ratings of all low-voltage windings in the transformer bank to total kilovoltampere output from the bank.

3.6 Consider Figure P3.6 and assume that the motor is rated 25 hp and is mechanically loaded so that it draws 25.0-kVA three-phase input at $\cos\theta = 0.866$ lagging power factor.

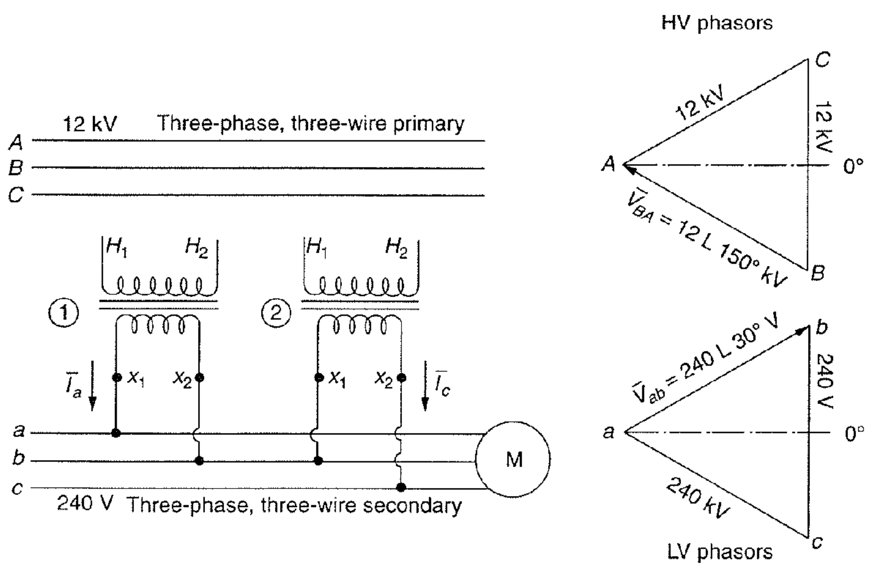

FIGURE P3.6 For Problem 3.6.

(*a*) Draw the necessary high-voltage connections so that the low voltages shall be as shown, that is, of *abc* phase sequence.
(*b*) Find the power factors cosqΓ$_1$, and cosqΓ$_2$ at which each transformer operates.
(*c*) Find the ratio of voltampere load on one transformer to total voltamperes delivered to the load.

3.7 Consider Figure P3.7 and assume that the two-transformer T-T bank delivers 120/208 V three-phase four-wire service from a three-phase three-wire 4160 V primary line. The problem is to determine if this bank can carry unbalanced loads although the primary neutral terminal N is not connected to the source neutral. (If it can, the T-T performance is quite different from the three-transformer Y-grounded Y bank.) Use the ideal transformer theory and pursue the question as follows:

(*a*) Load phase an with $R = 1.20\ \Omega$ resistance and then find the following six complex currents numerically: $\bar{I}_a$, $\bar{I}_b$, $\bar{I}_c$, $\bar{I}_A$, $\bar{I}_B$, and $\bar{I}_C$.
(*b*) Find the following complex powers of windings by using the $S = P + jQ = \bar{V}\bar{I}^*$ equation numerically:

$S_{T_{1(x1-n)}}$ = complex power of $x_1 - n$ portion of transformer 1.

$S_{T_{1(H_1-H_2)}}$ = complex power of $H_1 - H_2$ portion of transformer 1,

$S_{T_{2(H_2-0)}}$ = complex power of $H_1 - 0$ portion of transformer 2,

$S_{T_{2(H_2-0)}}$ = complex power of $H_2 - 0$ portion of transformer 2.

(*c*) Do your results indicate that this bank will carry unbalanced loads successfully? Why?

3.8 Figure P3.8 shows two single-phase transformers, each with a 7620-V high-voltage winding and two 120-V low-voltage windings. The diagram shows the proposed connections for an open-Y to open-Δ bank and the high-voltage-applied phasor voltage drops. Here, *abc* phase sequence at low-voltage and high-voltage sides and 120/240 V are required.

(*a*) Sketch the low-voltage phasor diagram, correctly oriented on the 0° reference line. Label it adequately with *x*s (1), and (2), and so on, to identify.
(*b*) State whether or not the proposed connections will output the required three-phase four-wire 120/240-V Δ low voltage.

3.9 A large number of 25-kVA distribution transformers are to be purchased. Two competitive bids have been received. The bid data are tabulated as follows.

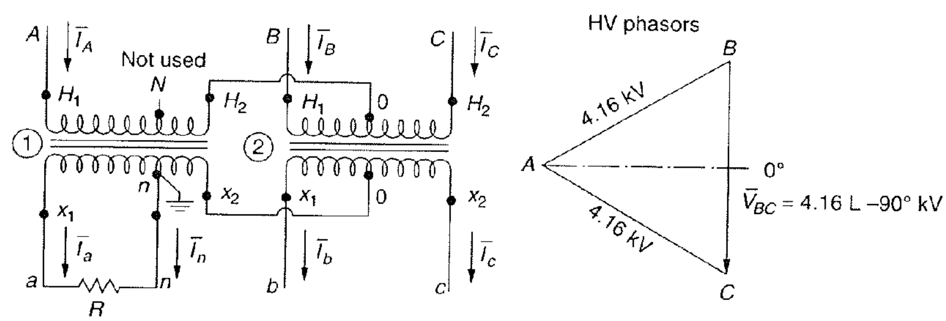

FIGURE P3.7 For Problem 3.7.

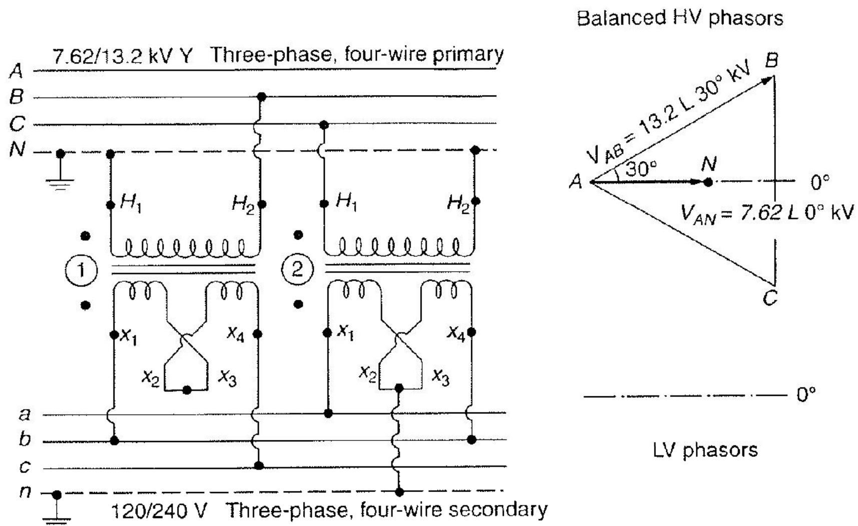

FIGURE P3.8 For Problem 3.8.

Transformer	Cost of Transformer Delivered to NL&NP's Warehouse	Core Loss at Rated Load	Copper Loss at Rated Voltage and Frequency	Per-Unit Exciting Current
A	\$355	360 W	130 W	0.015
B	\$345	380 W	150 W	0.020

Evaluate the bids on the basis of total annual cost (TAC) and recommend the purchase of the one having the least TAC. The cost of installing a transformer is not to be included in this study. The following system data are given:

Annual peak load on transformer = 35 kVA
Annual loss factor = 0.15
Per unit annual fixed charge rate = 0.15
Installed cost of shunt capacitors = \$10/kvar
Incremental cost of off-peak energy = \$0.01/kWh
Incremental cost of on-peak energy = \$0.012/kWh
Investment cost of power system upstream from distribution transformers = \$300/kVA.

Calculate the TAC of owning and operating one such transformer, and state which transformer should be purchased. (*Hint*: Study the relevant equations in Chapter 6 before starting to calculate.)

3.10 Assume that a 250-kVA distribution transformer is used for single-phase pole mounting. The transformer is connected phase-to-neutral 7200 V on the primary, and 2520 V phase-to-neutral on the secondary side. The leakage impedance of the transformer is 3.5%. Based on the given information, determine the following:

(*a*) Assume that the transformer has 0.7 pu A in the high-voltage winding. Find the actual current values in the high- and low-voltage windings. What is the value of the current in the low-voltage winding in per units?

(*b*) Find the impedance of the transformer as referred to the high- and low-voltage windings in ohms.

(*c*) Assume that the low-voltage terminals of the transformer are short-circuited and 0.22 pu V is applied to the high-voltage winding. Find the high- and low-voltage winding currents that exist as a result of the short circuit in pu and amperes.

(*d*) Determine the internal voltage drop of the transformer, due to its leakage impedance, if a 1.2 pu current flows in the high-voltage winding. Give the result in pu and volts.

3.11 Resolve Example 3.11 by using MATLAB. Assume that all the quantities remain the same.

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