Topic 2 – Antenna Parameters and Figures of Merit (FOM)

EE-4382/5306 - Antenna Engineering

Outline

- Introduction
- Radiation Pattern
- Radiation Power Density
- Radiation Intensity
- Directivity
- Antenna Efficiency
- Gain, Realized Gain
- Bandwidth
- Polarization
- Input Impedance
- Radiation Efficiency
- Effective length and equivalent Area
- Friis Transmission Equation

Constantine A. Balanis, *Antenna Theory: Analysis and Design* 4th Ed., Wiley, 2016.
Stutzman, Thiele, *Antenna Theory and Design* 3rd Ed., Wiley, 2012.

Introduction

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What is a figures of merit for an antenna?

The figures of merit of an antenna are numbers that lets us describe the performance of a real antenna. Some of these parameters are compared to the ideal isotropic antenna, while others are not. Some of the parameters are interrelated, and not all of them need to be expressed to describe the full performance of an antenna.

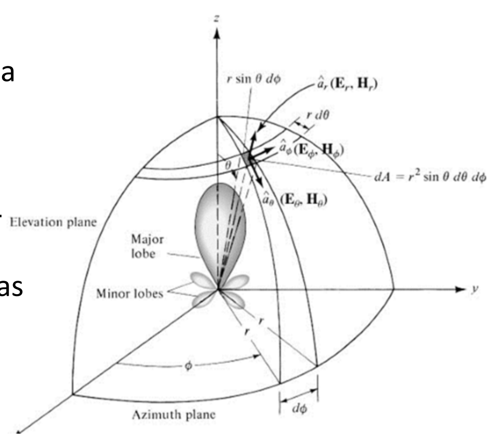
Radiation Pattern

What is the radiation pattern?

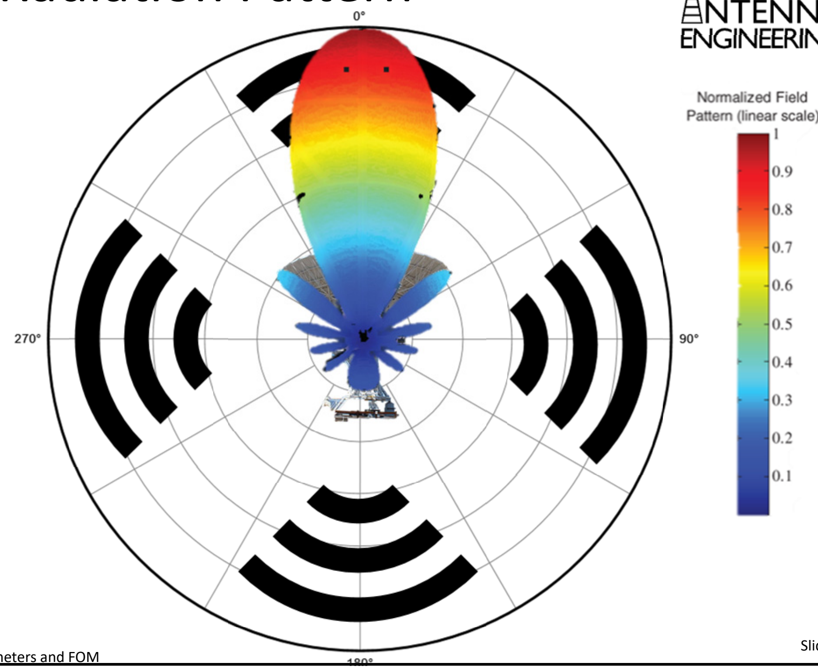
"A mathematical function or a graphical representation of the radiation properties of an antenna as a function of space"

Properties:

- Usually defined in the **far-field** region of the antenna
- Many other parameters, such as power density, directivity, radiation intensity, and others, are properties of the antenna radiation



2D Radiation Pattern



Antenna Parameters and FOM

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2D Radiation Pattern (circular)

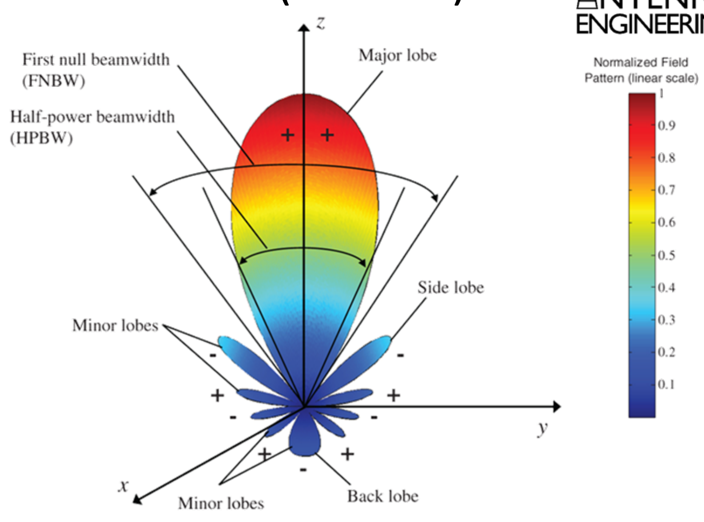

Fig. 2.3a

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2D Radiation Pattern (linear)

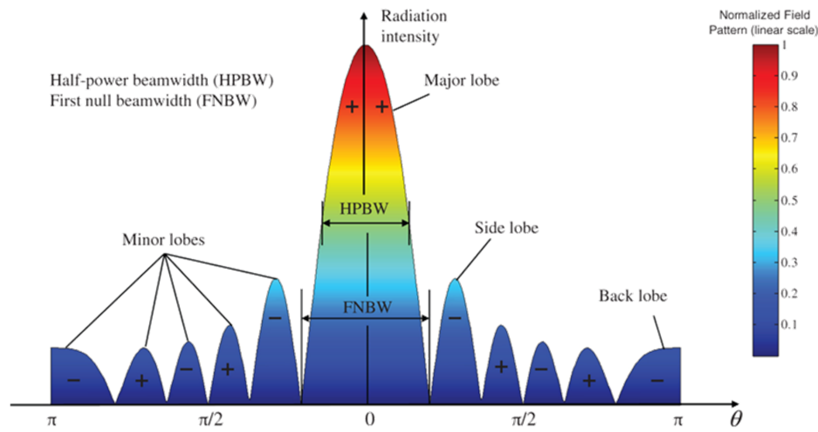

Fig. 2.3b

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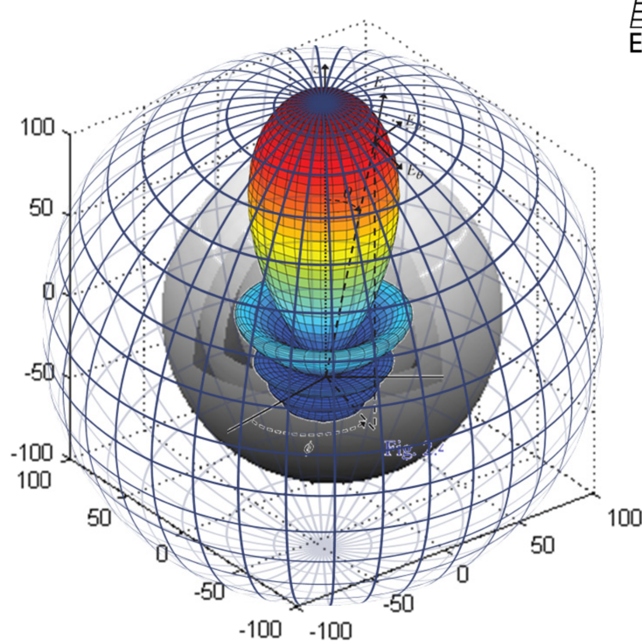
3D Radiation Pattern



3D Radiation Pattern

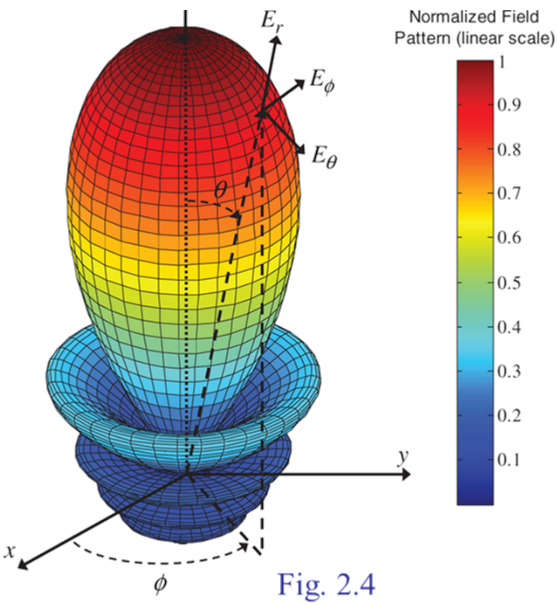


Fig. 2.4

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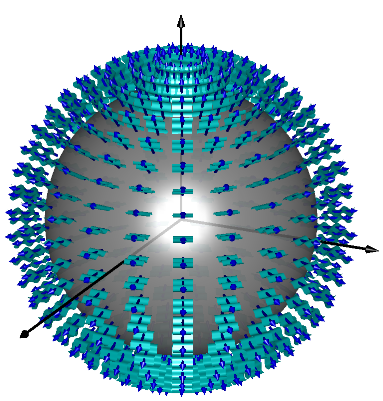
The isotropic radiator

Hypothetical lossless antenna that emits radiation <u>equally</u> in <u>all</u> directions



<u>dBi</u>

Units for antenna gain, directivity, etc. relative to the isotropic radiator.

Radiation Power Density

Radiation Power Density

The power associated by an electromagnetic wave is given by the average Poynting Vector (in W/m^2) or average/radiated power density

Average Poynting Vector $\left(\frac{W}{m^2}\right)$ ← $\mathbf{W} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]$

Peak Magnetic Field (A/m)

Peak Electric Field (V/m)

The average power radiated by an antenna is thus the total power density crossing through the closed surface of a sphere placed at the far-field.

$$P_{rad} = P_{av} = \oiint_S \boldsymbol{W}_{rad} \cdot d\boldsymbol{s} = \oiint_{\boldsymbol{S}} \boldsymbol{W}_{rad} \cdot \hat{\boldsymbol{n}} da = \frac{1}{2}\oiint_S \mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot d\boldsymbol{s}$$

Radiation Power Density

Example 2.2 (page 37 of Balanis)

The radial component of the radiated power density of an antenna is given by

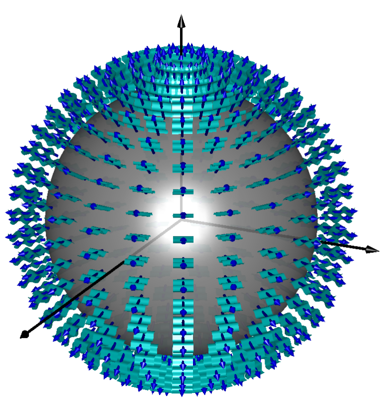
$$\mathbf{W}_{rad} = \hat{\boldsymbol{a}}_r W_r = \hat{\boldsymbol{a}}_r A_0 \frac{\sin(\theta)}{r^2} \quad \left(\frac{W}{m^2}\right)$$

Solution:

Where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\boldsymbol{a}}_r$ is the radial unit vector. Determine the total radiated power.

$$P_{rad} = \oint\!\!\!\oint_S \mathbf{W}_{rad} \cdot \hat{\boldsymbol{n}} da = \int_0^{2\pi} \int_0^{\pi} \hat{\boldsymbol{a}}_r A_0 \frac{\sin(\theta)}{r^2} \cdot \hat{\boldsymbol{a}}_r r^2 \sin(\theta)\, d\theta d\phi = \boxed{\pi^2 A_0 \ (W)}$$

Power Density of Isotropic Radiator



Power Density

$$W_0 = \frac{P_{\text{rad}}}{4\pi r^2} \quad \left(\frac{\text{W}}{\text{m}^2}\right)$$

$P_{\text{rad}} \equiv$ total power radiated by source

This means that for an isotropic radiator, the power density at any distance r is equal to the amount of power radiated divided by the surface area of the sphere at that r. The power density is uniform in all directions.

Radiation Power Intensity

Radiation Power Intensity

“The power radiated from an antenna per unit solid angle (steradians)”

Radiation intensity $\left(\frac{W}{solid\ angle}\right)$ ← $\mathbf{U} = \mathrm{r}^2\mathrm{W}_{\mathrm{rad}}$

Radiation Density $\left(\frac{W}{m^2}\right)$

For an isotropic radiator, the power intensity is independent of angle $(\theta\ and\ \phi)$

$$U_o = \frac{\mathrm{P}_{\mathrm{rad}}}{4\pi}$$

Power Radiated (W)

Complete sphere in steradians

Directivity

Directivity

"The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions."

- Average Radiation Intensity: Total power radiated by the antenna divided by 4π

Directivity is the ratio of the radiation intensity in a given direction relative to an isotropic source.

Radiation intensity from antenna

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

Radiation intensity from isotropic source

Maximum Directivity:

$$D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

Directivity Examples

Example 2.5(page 42 of Balanis)

Find the directivity of the antenna with the following radiation intensity:

$$\mathrm{W_{rad}} = \hat{\boldsymbol{a}}_r W_{rad} = \hat{\boldsymbol{a}}_r A_0 \frac{\sin(\theta)}{r^2}$$

Solution:

$$\mathrm{U} = r^2 W_{rad} = A_0 \sin(\theta)$$

Radiation is maximum at $\theta = \frac{\pi}{2}$, so

$$U_{max} = A_0$$

From example 2.2, $\mathrm{P_{rad}} = \pi^2 A_0$

So we can calculate the maximum directivity:

$$\mathrm{D_0} = \mathrm{D_{max}} = \frac{4\pi \mathrm{U_{max}}}{\mathrm{P_{rad}}} = \frac{4}{\pi} = \boxed{1.27}$$

$$\boxed{\mathrm{D} = 1.27\sin(\theta)}$$

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Directivity Examples

Example 2.6(page 43 of Balanis)

Find the maximum directivity of an infinitesimal linear dipole ($l \ll \lambda$) with radiation intensity:

$$\mathrm{W_{ave}} = \hat{\boldsymbol{a}}_r W_{rad} = \hat{\boldsymbol{a}}_r A_0 \frac{\sin^2(\theta)}{r^2}$$

Solution:

$$\mathrm{U} = r^2 W_{rad} = A_0 \sin^2(\theta)$$

Radiation is maximum at $\theta = \frac{\pi}{2}$, so $\mathrm{U_{max}} = A_0$

We need to calculate $\mathrm{P_{rad}}$:

Solid Angle

$$P_{rad} = \oiint_{\Omega} \mathrm{U} d\Omega = \int_0^{2\pi} \int_0^{\pi} A_0 \sin^2(\theta) \sin(\theta)\, d\theta d\phi = A_0 \left(\frac{8\pi}{3}\right)$$

Integrate over full sphere

Now we can obtain the maximum directivity:

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{\frac{8\pi}{3} A_0} = \boxed{1.5}$$

Maximum Directivity of Infinitesimal Dipole

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Two-Dimensional Directivity Pattern

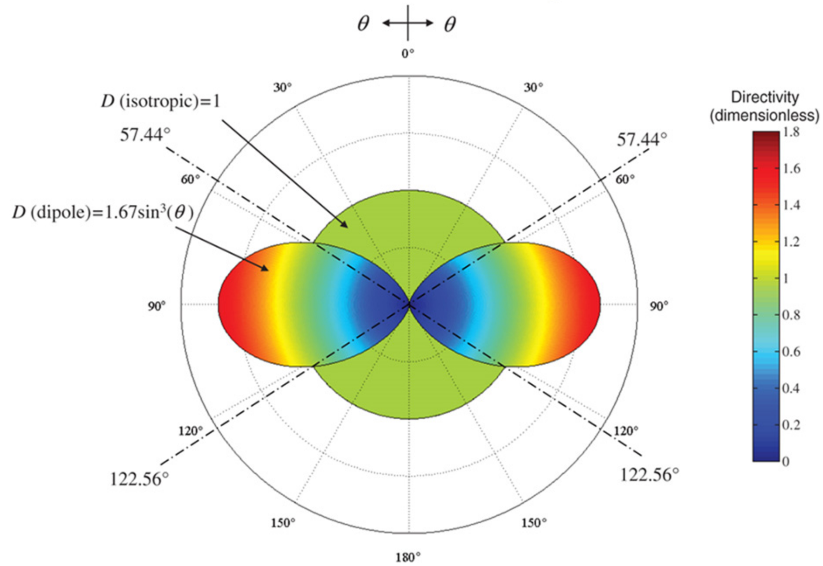

Fig. 2.13(a)

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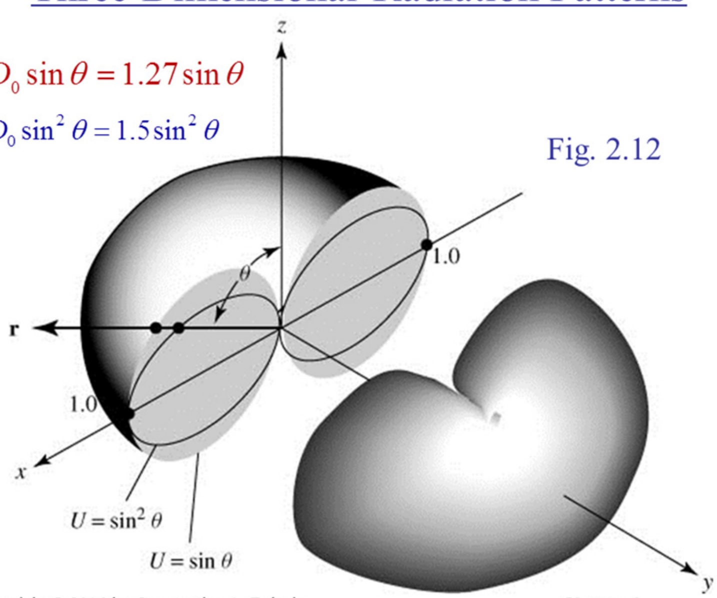
Three-Dimensional Radiation Patterns

$D = D_0 \sin\theta = 1.27 \sin\theta$

$D = D_0 \sin^2\theta = 1.5 \sin^2\theta$

Fig. 2.12



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Directivity of various antennas

For an isotropic radiator, the directivity is 1 (radiates equally in all directions), or 0 dBi.

$$10\log_{10}(1) = 0$$

For the small dipole, the directivity is 1.5, or 1.76 dBi

$$10\log_{10}(1.5) = 1.76$$

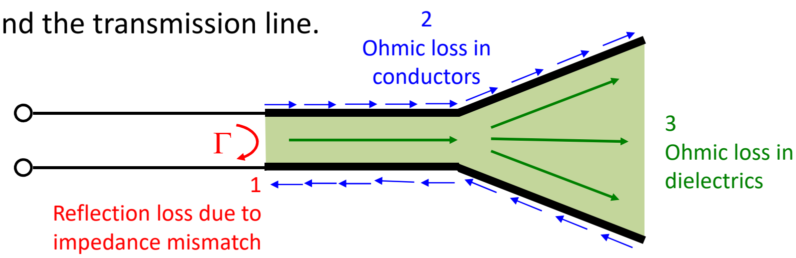
For the half-wave dipole, the directivity is 1.67, or 2.23 dBi

$$10\log_{10}(1.67) = 2.23$$

Antenna Efficiency

Antenna Efficiency e_0

Takes into account the physical losses of the antenna such as metal and dielectric losses, as well as mismatch between the connection of the antenna and the transmission line.



Total efficiency → $\mathrm{e}_0 = \mathrm{e}_r \mathrm{e}_c \mathrm{e}_d$ ← Dielectric efficiency

Reflection efficiency Conductor efficiency

$\mathrm{e}_{cd} = \mathrm{e}_c \mathrm{e}_d$ $\mathrm{e}_r = (1 - |\Gamma|)^2$ $\boxed{\mathrm{e}_0 = (1 - |\Gamma|)^2 \mathrm{e}_{cd}}$

Reflection coefficient $\left(\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right)$

Antenna Gain/Realized Gain

Antenna Gain

The ratio of intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna was radiated isotropically. The gain is usually taken in the direction of maximum radiation.

The radiation intensity for the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π.

$$\mathrm{G} = \frac{4\pi \mathrm{U}(\theta,\phi)}{\mathrm{P_{in}}(lossless\ isotropic\ source)}$$

The input power is related to the output power by

$$\mathrm{P_{rad}} = \mathrm{e_{cd}}\mathrm{P_{in}}$$

This means that the gain does <u>not</u> take into account mismatch losses when connected to a transmission line. <u>Realized Gain</u> does.

Antenna Gain

Using the two equations in the previous slide we obtain:

$$\mathrm{G}(\theta,\phi) = \mathrm{e_{cd}}\left[4\pi\frac{\mathrm{U}(\theta,\phi)}{P_{\mathrm{rad}}}\right]$$

Gain and Directivity are related by

$$\mathrm{G}(\theta,\phi) = \mathrm{e_{cd}}\mathrm{D}(\theta,\phi)$$

So maximum Gain is equal to

$$\boxed{\mathrm{G_0} = \mathrm{e_{cd}}\mathrm{D}(\theta,\phi)\Big|_{max} = \mathrm{e_{cd}}\mathrm{D_0}}$$

Maximum Realized Gain is equal to

$$\boxed{\mathrm{G}_{re_0} = \mathrm{e_0}\mathrm{D}(\theta,\phi)\Big|_{max} = \mathrm{e_0}\mathrm{D_0}}$$

Antenna Gain – Example

A lossless resonant half-wave $\left(\frac{\lambda}{2}\right)$ dipole with an impedance of 73Ω is connected to a transmission line with characteristic impedance of 50Ω. The radiation pattern is given by

$$U = B_0 \sin^3 \theta$$

Find the maximum realized gain of this antenna.

Solution: First we need to find the maximum directivity of the antenna.

$$U_{max} = B_0, \qquad P_{rad} = \oiint_{\Omega} \mathrm{U} d\Omega = \int_0^{2\pi} \int_0^{\pi} B_0 \sin^3(\theta) \sin(\theta)\, d\theta d\phi = B_0 \left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} = 1.697$$

Since the antenna is lossless, $e_{cd} = 1$

Antenna Gain - Example

We can now compute the maximum gain

$$G_0 = e_{cd} D_0 = 1.697$$
$$G_0(dB) = 10 \log_{10}(1.697) = 2.297$$

Here, the gain is equal to the directivity because the antenna is lossless.

With the realized gain, we can take into account the mismatch losses between the transmission line and the antenna impedance.

$$e_r = (1 - |\Gamma|)^2 = \left(1 - \left|\frac{73 - 50}{73 + 50}\right|^2\right) = 0.965, \qquad e_r(dB) = -0.155$$

Since the antenna is lossless, this is also the overall efficiency of the antenna.

$$e_0 = e_r e_{cd} = 0.965, \qquad e_r(dB) = -0.155$$

Now the maximum realized gain is equal to

$$\boxed{G_{re0} = e_0 D_0 = (0.965)(1.697) = , 1.6376 \qquad G_{re0}(dB) = 2.142}$$

Bandwidth

It is related to the frequency response of the antenna. It is the range of frequencies in which the antenna, with respect to some metric, conforms to a specified standard.

There is usually a band of frequencies in which the extreme values (upper and lower) are measured, and a center frequency, which usually is the resonant frequency of the antenna. From this, the antenna characteristics are at an acceptable value relative to the center frequency.

- For broadband antennas, the bandwidth is a ratio from upper to lower frequencies (10:1, 2:1, etc.)
- For narrowband antennas, bandwidth is usually a percentage of the frequency difference over the center frequency $\left(\frac{f_2 - f_1}{f_c}\right)$