

Chapter 7, Solution 1.

Applying KVL to Fig. 7.1.

$$\frac{1}{C} \int_{-\infty}^t i \, dt + Ri = 0$$

Taking the derivative of each term,

$$\frac{i}{C} + R \frac{di}{dt} = 0$$

or
$$\frac{di}{i} = -\frac{dt}{RC}$$

Integrating,

$$\ln \left(\frac{i(t)}{I_0} \right) = \frac{-t}{RC}$$

$$i(t) = I_0 e^{-t/RC}$$

$$v(t) = Ri(t) = RI_0 e^{-t/RC}$$

or
$$\underline{v(t) = V_0 e^{-t/RC}}$$

Chapter 7, Solution 2.

$$\tau = R_{th} C$$

where R_{th} is the Thevenin equivalent at the capacitor terminals.

$$R_{th} = 120 \parallel 80 + 12 = 60 \, \Omega$$

$$\tau = 60 \times 0.5 \times 10^{-3} = \underline{\underline{30 \, \text{ms}}}$$

Chapter 7, Solution 3.

$$(a) \quad R_{Th} = 10 \parallel 10 = 5k\Omega, \quad \tau = R_{Th} C = 5 \times 10^3 \times 2 \times 10^{-6} = \underline{10 \, \text{ms}}$$

$$(b) \quad R_{Th} = 20 \parallel (5 + 25) + 8 = 20\Omega, \quad \tau = R_{Th} C = 20 \times 0.3 = \underline{6s}$$

Chapter 7, Solution 4.

$$\tau = R_{eq} C_{eq}$$

$$\text{where } C_{eq} = \frac{C_1 C_2}{C_1 + C_2}, \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\tau = \underline{\underline{\frac{R_1 R_2 C_1 C_2}{(R_1 + R_2)(C_1 + C_2)}}}$$

Chapter 7, Solution 5.

$$v(t) = v(4)e^{-(t-4)/\tau}$$

where $v(4) = 24$, $\tau = RC = (20)(0.1) = 2$

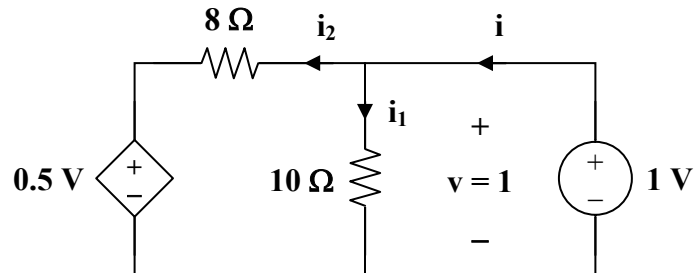
$$v(t) = 24e^{-(t-4)/2}$$
$$v(10) = 24e^{-6/2} = \underline{\underline{1.195 \text{ V}}}$$

Chapter 7, Solution 6.

$$v_o = v(0) = \frac{2}{10+2}(24) = 4 \text{ V}$$
$$v(t) = v_o e^{-t/\tau}, \quad \tau = RC = 40 \times 10^{-6} \times 2 \times 10^3 = \frac{2}{25}$$
$$v(t) = \underline{\underline{4e^{-12.5t} \text{ V}}}$$

Chapter 7, Solution 7.

$v(t) = v(0)e^{-t/\tau}$, $\tau = R_{th}C$
where R_{th} is the Thevenin resistance across the capacitor. To determine R_{th} , we insert a 1-V voltage source in place of the capacitor as shown below.



$$i_1 = \frac{1}{10} = 0.1, \quad i_2 = \frac{1-0.5}{8} = \frac{1}{16}$$
$$i = i_1 + i_2 = 0.1 + \frac{1}{16} = \frac{13}{80}$$
$$R_{th} = \frac{1}{i} = \frac{80}{13}$$
$$\tau = R_{th}C = \frac{80}{13} \times 0.1 = \frac{8}{13}$$
$$v(t) = \underline{\underline{20e^{-13t/8} \text{ V}}}$$

Chapter 7, Solution 8.

$$(a) \quad \tau = RC = \frac{1}{4}$$

$$-i = C \frac{dv}{dt}$$

$$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \underline{\underline{5 \text{ mF}}}$$

$$R = \frac{1}{4C} = \underline{\underline{50 \Omega}}$$

$$(b) \quad \tau = RC = \frac{1}{4} = \underline{\underline{0.25 \text{ s}}}$$

$$(c) \quad w_C(0) = \frac{1}{2} CV_0^2 = \frac{1}{2} (5 \times 10^{-3})(100) = \underline{\underline{250 \text{ mJ}}}$$

$$(d) \quad w_R = \frac{1}{2} \times \frac{1}{2} CV_0^2 = \frac{1}{2} CV_0^2 (1 - e^{-2t_0/\tau})$$

$$0.5 = 1 - e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$$

$$\text{or} \quad e^{8t_0} = 2$$

$$t_0 = \frac{1}{8} \ln(2) = \underline{\underline{86.6 \text{ ms}}}$$

Chapter 7, Solution 9.

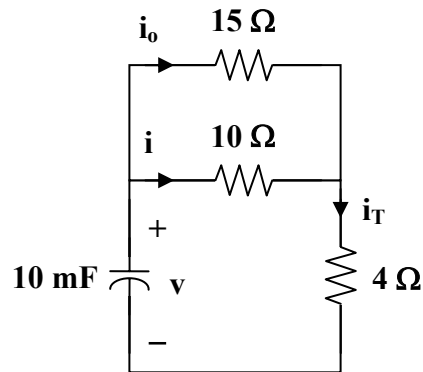
$$v(t) = v(0)e^{-t/\tau}, \quad \tau = R_{\text{eq}}C$$

$$R_{\text{eq}} = 2 + 8 \parallel 8 + 6 \parallel 3 = 2 + 4 + 2 = 8 \Omega$$

$$\tau = R_{\text{eq}}C = (0.25)(8) = 2$$

$$v(t) = \underline{\underline{20e^{-t/2} \text{ V}}}$$

Chapter 7, Solution 10.



$$15i_o = 10i \longrightarrow i_o = \frac{(10)(3)}{15} = 2 \text{ A}$$

i.e. if $i(0) = 3 \text{ A}$, then $i_o(0) = 2 \text{ A}$

$$i_T(0) = i(0) + i_o(0) = 5 \text{ A}$$

$$v(0) = 10i(0) + 4i_T(0) = 30 + 20 = 50 \text{ V}$$

across the capacitor terminals.

$$R_{th} = 4 + 10 \parallel 15 = 4 + 6 = 10 \Omega$$

$$\tau = R_{th}C = (10)(10 \times 10^{-3}) = 0.1$$

$$v(t) = v(0)e^{-t/\tau} = 50e^{-10t}$$

$$i_C = C \frac{dv}{dt} = (10 \times 10^{-3})(-500e^{-10t})$$

$$i_C = -5e^{-10t} \text{ A}$$

By applying the current division principle,

$$i(t) = \frac{15}{10+15}(-i_C) = -0.6i_C = \underline{\underline{3e^{-10t} \text{ A}}}$$

Chapter 7, Solution 11.

Applying KCL to the RL circuit,

$$\frac{1}{L} \int v dt + \frac{v}{R} = 0$$

Differentiating both sides,

$$\frac{v}{L} + \frac{1}{R} \frac{dv}{dt} = 0 \longrightarrow \frac{dv}{dt} + \frac{R}{L}v = 0$$

$$v = Ae^{-Rt/L}$$

If the initial current is I_0 , then

$$v(0) = I_0 R = A$$

$$v = I_0 R e^{-t/\tau}, \quad \tau = \frac{L}{R}$$

$$i = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

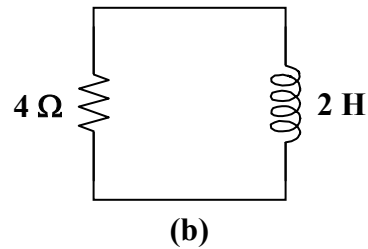
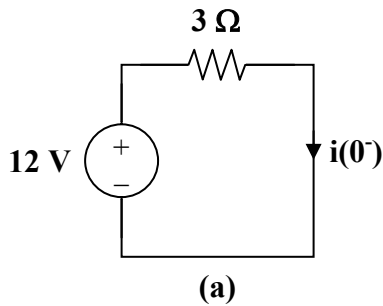
$$i = \frac{-\tau I_0 R}{L} e^{-t/\tau} \Big|_{-\infty}^t$$

$$i = -I_0 R e^{-t/\tau}$$

$$\underline{i(t) = I_0 e^{-t/\tau}}$$

Chapter 7, Solution 12.

When $t < 0$, the switch is closed and the inductor acts like a short circuit to dc. The 4Ω resistor is short-circuited so that the resulting circuit is as shown in Fig. (a).



$$i(0^-) = \frac{12}{3} = 4 \text{ A}$$

Since the current through an inductor cannot change abruptly,

$$i(0) = i(0^-) = i(0^+) = 4 \text{ A}$$

When $t > 0$, the voltage source is cut off and we have the RL circuit in Fig. (b).

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

Hence,

$$i(t) = i(0) e^{-t/\tau} = \underline{4 e^{-2t} \text{ A}}$$

Chapter 7, Solution 13.

$$\tau = \frac{L}{R_{th}}$$

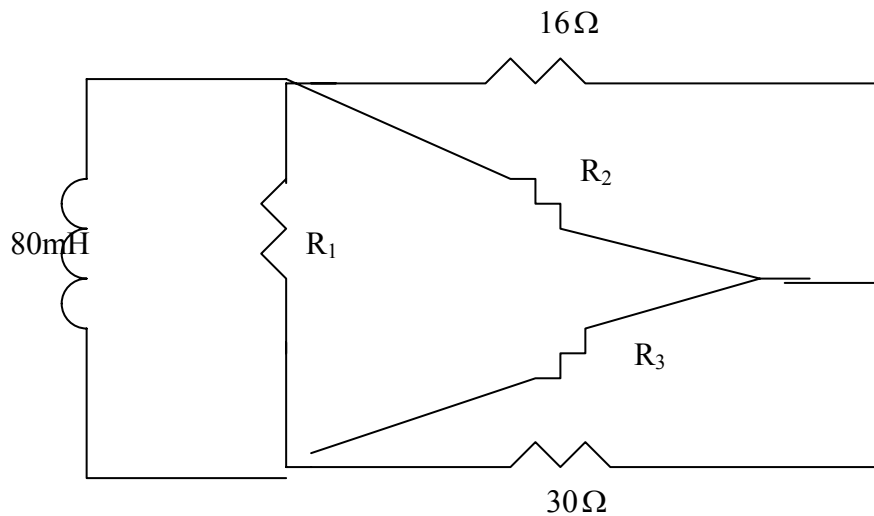
where R_{th} is the Thevenin resistance at the terminals of the inductor.

$$R_{th} = 70 \parallel 30 + 80 \parallel 20 = 21 + 16 = 37 \Omega$$

$$\tau = \frac{2 \times 10^{-3}}{37} = \underline{\underline{81.08 \mu s}}$$

Chapter 7, Solution 14

Converting the wye-subnetwork to delta gives



$$R_1 = \frac{10 \times 20 + 20 \times 50 + 50 \times 10}{20} = 1700 / 20 = 85 \Omega, \quad R_2 = \frac{1700}{50} = 34 \Omega, \quad R_3 = \frac{1700}{10} = 170 \Omega$$

$$30 \parallel 170 = (30 \times 170) / 200 = 25.5 \Omega, \quad 34 \parallel 16 = (34 \times 16) / 50 = 10.88 \Omega$$

$$R_{th} = 85 \parallel (25.5 + 10.88) = \frac{85 \times 36.38}{121.38} = 25.476 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{80 \times 10^{-3}}{25.476} = \underline{\underline{3.14 ms}}$$

Chapter 7, Solution 15

$$(a) R_{Th} = 12 + 10 // 40 = 20\Omega, \quad \tau = \frac{L}{R_{Th}} = 5 / 20 = \underline{0.25s}$$

$$(b) R_{Th} = 40 // 160 + 8 = 40\Omega, \quad \tau = \frac{L}{R_{Th}} = (20 \times 10^{-3}) / 40 = \underline{0.5 \text{ ms}}$$

Chapter 7, Solution 16.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$(a) \quad L_{eq} = L \text{ and } R_{eq} = R_2 + \frac{R_1 R_3}{R_1 + R_3} = \frac{R_2(R_1 + R_3) + R_1 R_3}{R_1 + R_3}$$

$$\tau = \frac{L(R_1 + R_3)}{R_2(R_1 + R_3) + R_1 R_3}$$

$$(b) \quad \text{where } L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \text{ and } R_{eq} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = \frac{R_3(R_1 + R_2) + R_1 R_2}{R_1 + R_2}$$

$$\tau = \frac{L_1 L_2 (R_1 + R_2)}{(L_1 + L_2)(R_3(R_1 + R_2) + R_1 R_2)}$$

Chapter 7, Solution 17.

$$i(t) = i(0)e^{-t/\tau}, \quad \tau = \frac{L}{R_{eq}} = \frac{1/4}{4} = \frac{1}{16}$$

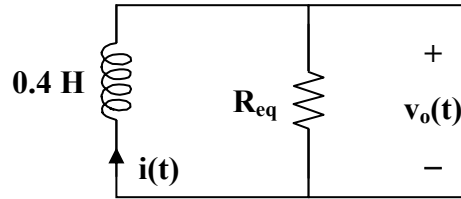
$$i(t) = 2e^{-16t}$$

$$v_o(t) = 3i + L \frac{di}{dt} = 6e^{-16t} + (1/4)(-16)2e^{-16t}$$

$$v_o(t) = \underline{-2e^{-16t} \text{ V}}$$

Chapter 7, Solution 18.

If $v(t) = 0$, the circuit can be redrawn as shown below.

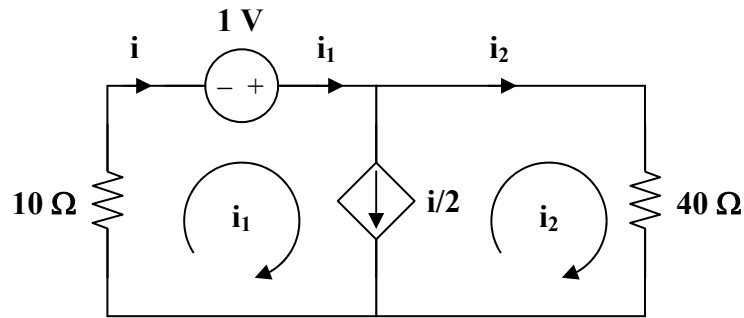


$$R_{eq} = 2 \parallel 3 = \frac{6}{5}, \quad \tau = \frac{L}{R} = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$i(t) = i(0)e^{-t/\tau} = e^{-3t}$$

$$v_o(t) = -L \frac{di}{dt} = \frac{-2}{5} (-3)e^{-3t} = \underline{\underline{1.2e^{-3t} \text{ V}}}$$

Chapter 7, Solution 19.



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$$10i_1 - 1 + 40i_2 = 0$$

But $i = i_2 + i/2$ and $i = i_1$

i.e. $i_1 = 2i_2 = 2i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30 \Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2 \text{ s}$$

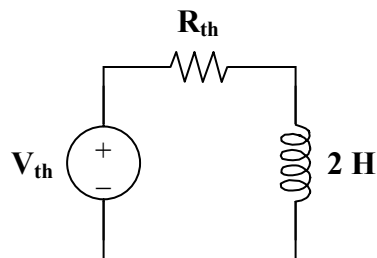
$$i(t) = \underline{\underline{2e^{-5t} \text{ A}}}$$

Chapter 7, Solution 20.

- (a). $\tau = \frac{L}{R} = \frac{1}{50} \longrightarrow R = 50L$
 $-v = L \frac{di}{dt}$
 $-150e^{-50t} = L(30)(-50)e^{-50t} \longrightarrow L = \underline{\underline{0.1 \text{ H}}}$
 $R = 50L = \underline{\underline{5 \Omega}}$
- (b). $\tau = \frac{L}{R} = \frac{1}{50} = \underline{\underline{20 \text{ ms}}}$
- (c). $w = \frac{1}{2}Li^2(0) = \frac{1}{2}(0.1)(30)^2 = \underline{\underline{45 \text{ J}}}$
- (d). Let p be the fraction
 $\frac{1}{2}LI_0 \cdot p = \frac{1}{2}LI_0(1 - e^{-2t_0/\tau})$
 $p = 1 - e^{-(2)(10)/50} = 1 - e^{-0.4} = 0.3296$
 i.e. $p = \underline{\underline{33\%}}$

Chapter 7, Solution 21.

The circuit can be replaced by its Thevenin equivalent shown below.



$$V_{th} = \frac{80}{80 + 40}(60) = 40 \text{ V}$$

$$R_{th} = 40 \parallel 80 + R = \frac{80}{3} + R$$

$$I = i(0) = i(\infty) = \frac{V_{th}}{R_{th}} = \frac{40}{80/3 + R}$$

$$w = \frac{1}{2}LI^2 = \frac{1}{2}(2)\left(\frac{40}{R + 80/3}\right)^2 = 1$$

$$\frac{40}{R + 80/3} = 1 \longrightarrow R = \frac{40}{3}$$

$$R = \underline{\underline{13.33 \Omega}}$$

Chapter 7, Solution 22.

$$i(t) = i(0)e^{-t/\tau}, \quad \tau = \frac{L}{R_{\text{eq}}}$$

$$R_{\text{eq}} = 5 \parallel 20 + 1 = 5 \Omega, \quad \tau = \frac{2}{5}$$

$$i(t) = \underline{\mathbf{10e^{-2.5t} \text{ A}}}$$

Using current division, the current through the 20 ohm resistor is

$$i_o = \frac{5}{5+20}(-i) = \frac{-i}{5} = -2e^{-2.5t}$$

$$v(t) = 20i_o = \underline{\mathbf{-40e^{-2.5t} \text{ V}}}$$

Chapter 7, Solution 23.

Since the 2 Ω resistor, 1/3 H inductor, and the (3+1) Ω resistor are in parallel, they always have the same voltage.

$$-i = \frac{2}{2} + \frac{2}{3+1} = 1.5 \longrightarrow i(0) = -1.5$$

The Thevenin resistance R_{th} at the inductor's terminals is

$$R_{\text{th}} = 2 \parallel (3+1) = \frac{4}{3}, \quad \tau = \frac{L}{R_{\text{th}}} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$i(t) = i(0)e^{-t/\tau} = -1.5e^{-4t}, \quad t > 0$$

$$v_L = v_o = L \frac{di}{dt} = -1.5(-4)(1/3)e^{-4t}$$

$$v_o = \underline{\mathbf{2e^{-4t} \text{ V}, \quad t > 0}}$$

$$v_x = \frac{1}{3+1}v_L = \underline{\mathbf{0.5e^{-4t} \text{ V}, \quad t > 0}}$$

Chapter 7, Solution 24.

$$(a) \quad v(t) = \underline{\mathbf{-5u(t)}}$$

$$(b) \quad i(t) = -10[u(t) - u(t-3)] + 10[u(t-3) - u(t-5)]$$

$$= \underline{\mathbf{-10u(t) + 20u(t-3) - 10u(t-5)}}$$

$$\begin{aligned}
 \text{(c)} \quad x(t) &= (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-3)] \\
 &\quad + (4-t)[u(t-3) - u(t-4)] \\
 &= (t-1)u(t-1) - (t-2)u(t-2) - (t-3)u(t-3) + (t-4)u(t-4) \\
 &= \underline{\underline{\mathbf{r(t-1) - r(t-2) - r(t-3) + r(t-4)}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad y(t) &= 2u(-t) - 5[u(t) - u(t-1)] \\
 &= \underline{\underline{\mathbf{2u(-t) - 5u(t) + 5u(t-1)}}}
 \end{aligned}$$

Chapter 7, Solution 25.

$$\underline{\underline{\mathbf{v(t) = [u(t) + r(t-1) - r(t-2) - 2u(t-2)] V}}}$$

Chapter 7, Solution 26.

$$\begin{aligned}
 \text{(a)} \quad v_1(t) &= u(t+1) - u(t) + [u(t-1) - u(t)] \\
 v_1(t) &= \underline{\underline{\mathbf{u(t+1) - 2u(t) + u(t-1)}}}
 \end{aligned}$$

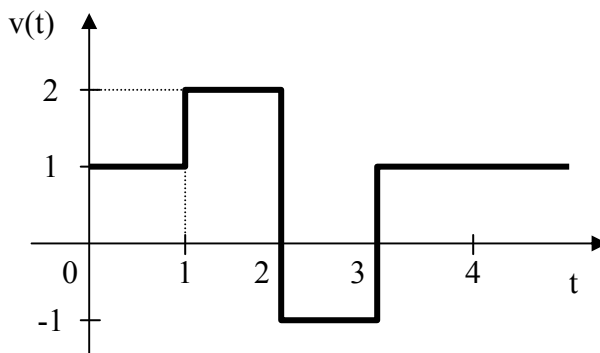
$$\begin{aligned}
 \text{(b)} \quad v_2(t) &= (4-t)[u(t-2) - u(t-4)] \\
 v_2(t) &= -(t-4)u(t-2) + (t-4)u(t-4) \\
 v_2(t) &= \underline{\underline{\mathbf{2u(t-2) - r(t-2) + r(t-4)}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad v_3(t) &= 2[u(t-2) - u(t-4)] + 4[u(t-4) - u(t-6)] \\
 v_3(t) &= \underline{\underline{\mathbf{2u(t-2) + 2u(t-4) - 4u(t-6)}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad v_4(t) &= -t[u(t-1) - u(t-2)] = -tu(t-1) + tu(t-2) \\
 v_4(t) &= (-t+1-1)u(t-1) + (t-2+2)u(t-2) \\
 v_4(t) &= \underline{\underline{\mathbf{-r(t-1) - u(t-1) + r(t-2) + 2u(t-2)}}}
 \end{aligned}$$

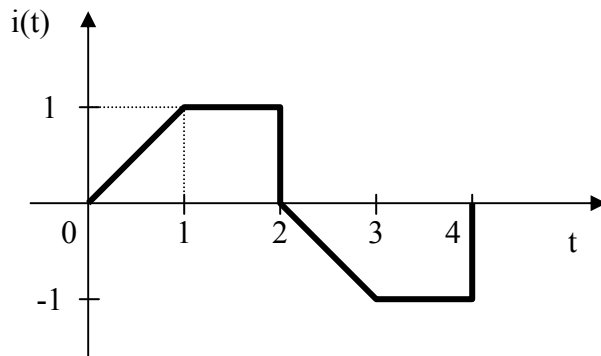
Chapter 7, Solution 27.

$v(t)$ is sketched below.



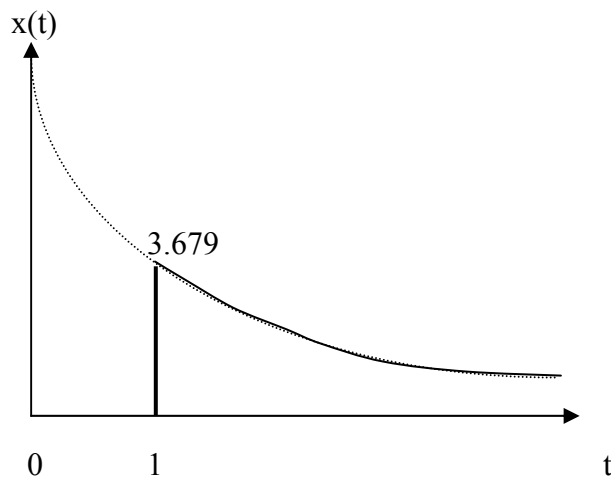
Chapter 7, Solution 28.

$i(t)$ is sketched below.

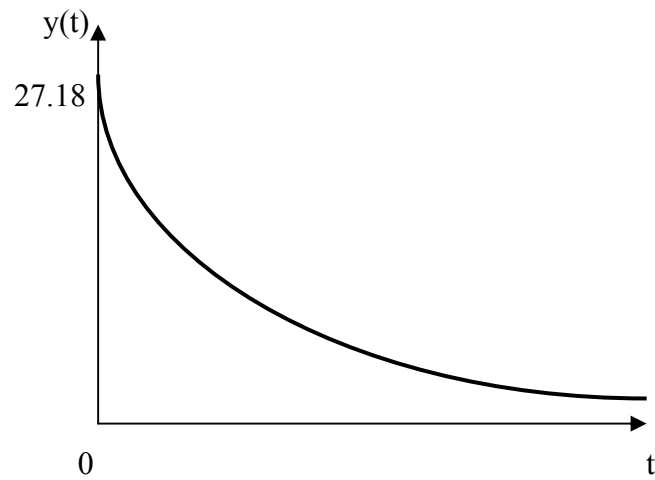


Chapter 7, Solution 29

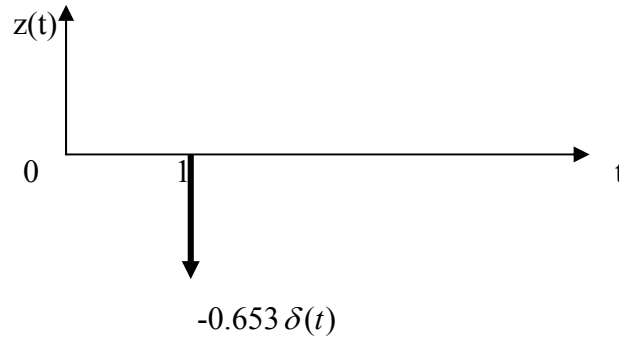
(a)



(b)



(c) $z(t) = \cos 4t \delta(t-1) = \cos 4\delta(t-1) = -0.6536\delta(t-1)$, which is sketched below.



Chapter 7, Solution 30.

$$(a) \int_0^{10} 4t^2 \delta(t-1) dt = 4t^2 \Big|_{t=1} = \underline{4}$$

$$(b) \int_{-\infty}^{\infty} \cos(2\pi t) \delta(t-0.5) dt = \cos(2\pi t) \Big|_{t=0.5} = \cos \pi = \underline{-1}$$

Chapter 7, Solution 31.

$$(a) \int_{-\infty}^{\infty} [e^{-4t^2} \delta(t-2)] dt = e^{-4t^2} \Big|_{t=2} = e^{-16} = \underline{112 \times 10^{-9}}$$

$$(b) \int_{-\infty}^{\infty} [5\delta(t) + e^{-t} \delta(t) + \cos 2\pi t \delta(t)] dt = (5 + e^{-t} + \cos(2\pi t)) \Big|_{t=0} = 5 + 1 + 1 = \underline{7}$$

Chapter 7, Solution 32.

$$(a) \int_1^t u(\lambda) d\lambda = \int_1^t 1 d\lambda = \lambda \Big|_1^t = \underline{t-1}$$

$$(b) \int_0^4 r(t-1) dt = \int_0^1 0 dt + \int_1^4 (t-1) dt = \frac{t^2}{2} - t \Big|_1^4 = \underline{4.5}$$

$$(c) \int_1^5 (t-6)^2 \delta(t-2) dt = (t-6)^2 \Big|_{t=2} = \underline{16}$$

Chapter 7, Solution 33.

$$i(t) = \frac{1}{L} \int_0^t v(t) dt + i(0)$$

$$i(t) = \frac{10^{-3}}{10 \times 10^{-3}} \int_0^t 20\delta(t-2) dt + 0$$

$$i(t) = \underline{\mathbf{2u(t-2) A}}$$

Chapter 7, Solution 34.

$$(a) \quad \frac{d}{dt} [u(t-1)u(t+1)] = \delta(t-1)u(t+1) + u(t-1)\delta(t+1) = \delta(t-1) \bullet 1 + 0 \bullet \delta(t+1) = \underline{\delta(t-1)}$$

$$(b) \quad \frac{d}{dt} [r(t-6)u(t-2)] = u(t-6)u(t-2) + r(t-6)\delta(t-2) = u(t-6) \bullet 1 + 0 \bullet \delta(t-2) = \underline{u(t-6)}$$

$$(c) \quad \frac{d}{dt} [\sin 4t u(t-3)] = 4 \cos 4t u(t-3) + \sin 4t \delta(t-3) \\ = 4 \cos 4t u(t-3) + \sin 4 \times 3 \delta(t-3) \\ = \underline{4 \cos 4t u(t-3) - 0.5366 \delta(t-3)}$$

Chapter 7, Solution 35.

$$(a) \quad v(t) = A e^{-5t/3}, \quad v(0) = A = -2 \\ v(t) = \underline{\mathbf{-2e^{-5t/3} V}}$$

$$(b) \quad v(t) = A e^{2t/3}, \quad v(0) = A = 5 \\ v(t) = \underline{\mathbf{5e^{2t/3} V}}$$

Chapter 7, Solution 36.

$$\begin{aligned} \text{(a)} \quad v(t) &= A + Be^{-t}, \quad t > 0 \\ A = 1, \quad v(0) = 0 = 1 + B & \quad \text{or} \quad B = -1 \\ v(t) &= \underline{1 - e^{-t} \text{ V}, \quad t > 0} \end{aligned}$$
$$\begin{aligned} \text{(b)} \quad v(t) &= A + Be^{t/2}, \quad t > 0 \\ A = -3, \quad v(0) = -6 = -3 + B & \quad \text{or} \quad B = -3 \\ v(t) &= \underline{-3(1 + e^{t/2}) \text{ V}, \quad t > 0} \end{aligned}$$

Chapter 7, Solution 37.

Let $v = v_h + v_p$, $v_p = 10$.

$$\dot{v}_h + \frac{1}{4}v_h = 0 \quad \longrightarrow \quad v_h = Ae^{-t/4}$$

$$v = 10 + Ae^{-0.25t}$$

$$v(0) = 2 = 10 + A \quad \longrightarrow \quad A = -8$$

$$v = 10 - 8e^{-0.25t}$$

$$\text{(a)} \quad \tau = \underline{4s}$$

$$\text{(b)} \quad v(\infty) = \underline{10 \text{ V}}$$

$$\text{(c)} \quad \underline{v = 10 - 8e^{-0.25t}}$$

Chapter 7, Solution 38

Let $i = i_p + i_h$

$$\dot{i}_h + 3i_h = 0 \quad \longrightarrow \quad i_h = Ae^{-3t}u(t)$$

$$\text{Let } i_p = ku(t), \quad \dot{i}_p = 0, \quad 3ku(t) = 2u(t) \quad \longrightarrow \quad k = \frac{2}{3}$$

$$i_p = \frac{2}{3}u(t)$$

$$i = (Ae^{-3t} + \frac{2}{3})u(t)$$

If $i(0) = 0$, then $A + 2/3 = 0$, i.e. $A = -2/3$. Thus

$$\underline{i = \frac{2}{3}(1 - e^{-3t})u(t)}$$

Chapter 7, Solution 39.

(a) Before $t = 0$,

$$v(t) = \frac{1}{4+1}(20) = \underline{4 \text{ V}}$$

After $t = 0$,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \quad v(\infty) = 20$$

$$v(t) = 20 + (8 - 20)e^{-t/8}$$

$$v(t) = \underline{20 - 12e^{-t/8} \text{ V}}$$

(b) Before $t = 0$, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12 \text{ V}$$

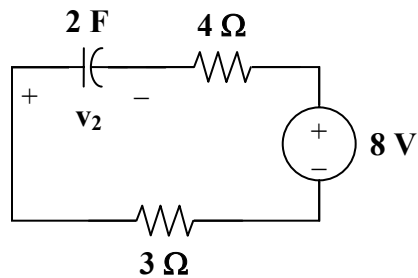
To get v_2 , transform the current source as shown in Fig. (a).

$$v_2 = -8 \text{ V}$$

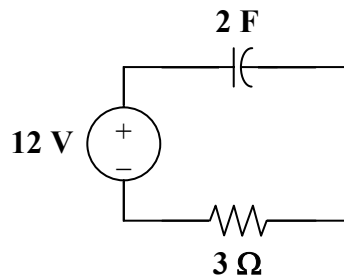
Thus,

$$v = 12 - 8 = \underline{4 \text{ V}}$$

After $t = 0$, the circuit becomes that shown in Fig. (b).



(a)



(b)

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

$$v(\infty) = 12, \quad v(0) = 4, \quad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)e^{-t/6}$$

$$v(t) = \underline{12 - 8e^{-t/6} \text{ V}}$$

Chapter 7, Solution 40.

(a) Before $t = 0$, $v = \underline{12 \text{ V}}$.

After $t = 0$, $v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$

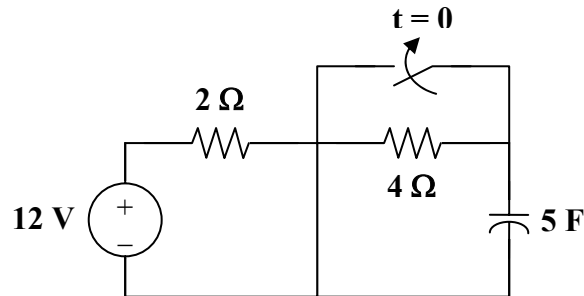
$$v(\infty) = 4, \quad v(0) = 12, \quad \tau = RC = (2)(3) = 6$$

$$v(t) = 4 + (12 - 4)e^{-t/6}$$

$$v(t) = \underline{4 + 8e^{-t/6} \text{ V}}$$

(b) Before $t = 0$, $v = \underline{12 \text{ V}}$.

After $t = 0$, $v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$
 After transforming the current source, the circuit is shown below.



$$v(0) = 12, \quad v(\infty) = 12, \quad \tau = RC = (2)(5) = 10$$

$$v = \underline{12 \text{ V}}$$

Chapter 7, Solution 41.

$$v(0) = 0, \quad v(\infty) = \frac{30}{16}(12) = 10$$

$$R_{\text{eq}}C = (6 \parallel 30)(1) = \frac{(6)(30)}{36} = 5$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

$$v(t) = 10 + (0 - 10)e^{-t/5}$$

$$v(t) = \underline{10(1 - e^{-0.2t}) \text{ V}}$$

Chapter 7, Solution 42.

$$(a) \quad v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)] e^{-t/\tau}$$

$$v_o(0) = 0, \quad v_o(\infty) = \frac{4}{4+2} (12) = 8$$

$$\tau = R_{eq} C_{eq}, \quad R_{eq} = 2 \parallel 4 = \frac{4}{3}$$

$$\tau = \frac{4}{3} (3) = 4$$

$$v_o(t) = 8 - 8e^{-t/4}$$

$$v_o(t) = \underline{\underline{8(1 - e^{-0.25t}) \text{ V}}}$$

$$(b) \quad \text{For this case, } v_o(\infty) = 0 \text{ so that}$$

$$v_o(t) = v_o(0) e^{-t/\tau}$$

$$v_o(0) = \frac{4}{4+2} (12) = 8, \quad \tau = RC = (4)(3) = 12$$

$$v_o(t) = \underline{\underline{8e^{-t/12} \text{ V}}}$$

Chapter 7, Solution 43.

Before $t = 0$, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.

$$0.5i = 2 - \frac{v_o}{40}, \quad i = \frac{v_o}{80}$$

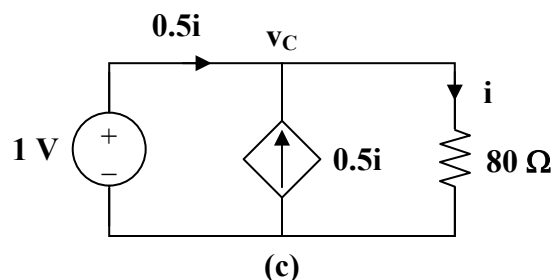
Hence, $\frac{1}{2} \frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\underline{0.8 \text{ A}}}$$

After $t = 0$, the circuit is as shown in Fig. (b).

$$v_C(t) = v_C(0) e^{-t/\tau}, \quad \tau = R_{th} C$$

To find R_{th} , we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



$$i = \frac{v_C}{80} = \frac{1}{80}, \quad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160 \Omega, \quad \tau = R_{th}C = 480$$

$$v_C(0) = 64 \text{ V}$$

$$v_C(t) = 64 e^{-t/480}$$

$$0.5i = -i_C = -C \frac{dv_C}{dt} = -3 \left(\frac{1}{480} \right) 64 e^{-t/480}$$

$$i(t) = \underline{\underline{0.8 e^{-t/480} \text{ A}}}$$

Chapter 7, Solution 44.

$$R_{eq} = 6 \parallel 3 = 2 \Omega, \quad \tau = RC = 4$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

Using voltage division,

$$v(0) = \frac{3}{3+6} (30) = 10 \text{ V}, \quad v(\infty) = \frac{3}{3+6} (12) = 4 \text{ V}$$

Thus,

$$v(t) = 4 + (10 - 4) e^{-t/4} = 4 + 6 e^{-t/4}$$

$$i(t) = C \frac{dv}{dt} = (2)(6) \left(\frac{-1}{4} \right) e^{-t/4} = \underline{\underline{-3 e^{-0.25t} \text{ A}}}$$

Chapter 7, Solution 45.

$$\text{For } t < 0, v_s = 5u(t) = 0 \longrightarrow v(0) = 0$$

$$\text{For } t > 0, v_s = 5, \quad v(\infty) = \frac{4}{4+12} (5) = \frac{5}{4}$$

$$R_{eq} = 7 + 4 \parallel 12 = 10, \quad \tau = R_{eq}C = (10)(1/2) = 5$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

$$v(t) = \underline{\underline{1.25(1 - e^{-t/5}) \text{ V}}}$$

$$i(t) = C \frac{dv}{dt} = \left(\frac{1}{2} \right) \left(\frac{-5}{4} \right) \left(\frac{-1}{5} \right) e^{-t/5}$$

$$i(t) = \underline{\underline{0.125 e^{-t/5} \text{ A}}}$$

Chapter 7, Solution 46.

$$\tau = R_{Th}C = (2 + 6) \times 0.25 = 2s, \quad v(0) = 0, \quad v(\infty) = 6i_s = 6 \times 5 = 30$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = \underline{30(1 - e^{-t/2})} \text{ V}$$

Chapter 7, Solution 47.

$$\text{For } t < 0, \quad u(t) = 0, \quad u(t-1) = 0, \quad v(0) = 0$$

$$\text{For } 0 < t < 1, \quad \tau = RC = (2 + 8)(0.1) = 1$$

$$v(0) = 0, \quad v(\infty) = (8)(3) = 24$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$v(t) = 24(1 - e^{-t})$$

$$\text{For } t > 1, \quad v(1) = 24(1 - e^{-1}) = 15.17$$

$$-6 + v(\infty) - 24 = 0 \quad \longrightarrow \quad v(\infty) = 30$$

$$v(t) = 30 + (15.17 - 30)e^{-(t-1)}$$

$$v(t) = 30 - 14.83e^{-(t-1)}$$

Thus,

$$v(t) = \begin{cases} 24(1 - e^{-t}) \text{ V}, & 0 < t < 1 \\ \underline{30 - 14.83e^{-(t-1)} \text{ V}}, & t > 1 \end{cases}$$

Chapter 7, Solution 48.

$$\text{For } t < 0, \quad u(-t) = 1, \quad v(0) = 10 \text{ V}$$

$$\text{For } t > 0, \quad u(-t) = 0, \quad v(\infty) = 0$$

$$R_{th} = 20 + 10 = 30, \quad \tau = R_{th}C = (30)(0.1) = 3$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$v(t) = \underline{10e^{-t/3}} \text{ V}$$

$$i(t) = C \frac{dv}{dt} = (0.1) \left(\frac{-1}{3} \right) 10e^{-t/3}$$

$$i(t) = \underline{\frac{-1}{3}e^{-t/3}} \text{ A}$$

Chapter 7, Solution 49.

$$\begin{aligned}\text{For } 0 < t < 1, \quad v(0) &= 0, & v(\infty) &= (2)(4) = 8 \\ R_{\text{eq}} &= 4 + 6 = 10, & \tau &= R_{\text{eq}}C = (10)(0.5) = 5 \\ v(t) &= v(\infty) + [v(0) - v(\infty)] e^{-t/\tau} \\ v(t) &= 8(1 - e^{-t/5}) \text{ V}\end{aligned}$$

$$\begin{aligned}\text{For } t > 1, \quad v(1) &= 8(1 - e^{-0.2}) = 1.45, & v(\infty) &= 0 \\ v(t) &= v(\infty) + [v(1) - v(\infty)] e^{-(t-1)/\tau} \\ v(t) &= 1.45e^{-(t-1)/5} \text{ V}\end{aligned}$$

Thus,

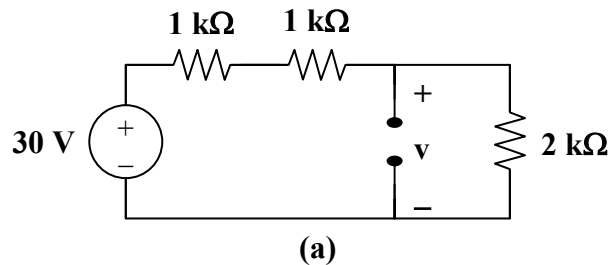
$$v(t) = \begin{cases} 8(1 - e^{-t/5}) \text{ V}, & 0 < t < 1 \\ 1.45e^{-(t-1)/5} \text{ V}, & t > 1 \end{cases}$$

Chapter 7, Solution 50.

For the capacitor voltage,

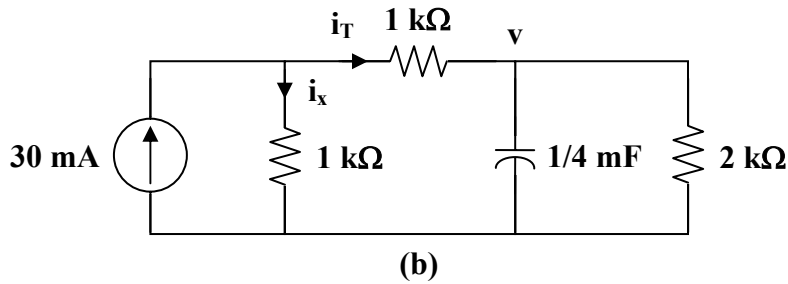
$$\begin{aligned}v(t) &= v(\infty) + [v(0) - v(\infty)] e^{-t/\tau} \\ v(0) &= 0\end{aligned}$$

For $t < 0$, we transform the current source to a voltage source as shown in Fig. (a).



$$\begin{aligned}v(\infty) &= \frac{2}{2+1+1} (30) = 15 \text{ V} \\ R_{\text{th}} &= (1+1) \parallel 2 = 1 \text{ k}\Omega \\ \tau &= R_{\text{th}}C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4} \\ v(t) &= 15(1 - e^{-4t}), \quad t > 0\end{aligned}$$

We now obtain i_x from $v(t)$. Consider Fig. (b).



$$i_x = 30 \text{ mA} - i_T$$

But
$$i_T = \frac{v}{R_3} + C \frac{dv}{dt}$$

$$i_T(t) = 7.5(1 - e^{-4t}) \text{ mA} + \frac{1}{4} \times 10^{-3} (-15)(-4)e^{-4t} \text{ A}$$

$$i_T(t) = 7.5(1 + e^{-4t}) \text{ mA}$$

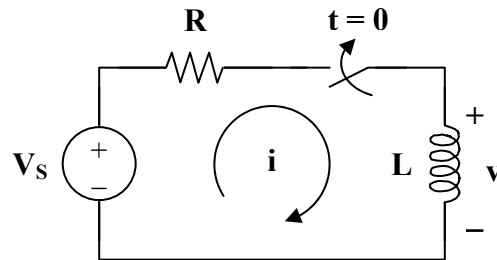
Thus,

$$i_x(t) = 30 - 7.5 - 7.5e^{-4t} \text{ mA}$$

$$i_x(t) = \underline{7.5(3 - e^{-4t}) \text{ mA}, \quad t > 0}$$

Chapter 7, Solution 51.

Consider the circuit below.



After the switch is closed, applying KVL gives

$$V_s = Ri + L \frac{di}{dt}$$

or
$$L \frac{di}{dt} = -R \left(i - \frac{V_s}{R} \right)$$

$$\frac{di}{i - V_s/R} = \frac{-R}{L} dt$$

Integrating both sides,

$$\ln\left(i - \frac{V_S}{R}\right)\Big|_{I_0}^{i(t)} = \frac{-R}{L}t$$

$$\ln\left(\frac{i - V_S/R}{I_0 - V_S/R}\right) = \frac{-t}{\tau}$$

or
$$\frac{i - V_S/R}{I_0 - V_S/R} = e^{-t/\tau}$$

$$\underline{i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right)e^{-t/\tau}}$$

which is the same as Eq. (7.60).

Chapter 7, Solution 52.

$$i(0) = \frac{20}{10} = 2 \text{ A}, \quad i(\infty) = 2 \text{ A}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$\underline{i(t) = 2 \text{ A}}$$

Chapter 7, Solution 53.

(a) Before $t = 0$, $i = \frac{25}{3+2} = \underline{5 \text{ A}}$

After $t = 0$, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \quad i(0) = 5$$

$$\underline{i(t) = 5e^{-t/2} \text{ A}}$$

- (b) Before $t = 0$, the inductor acts as a short circuit so that the 2Ω and 4Ω resistors are short-circuited.

$$\underline{i(t) = 6 \text{ A}}$$

After $t = 0$, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \quad \tau = \frac{L}{R} = \frac{3}{2}$$

$$\underline{i(t) = 6e^{-2t/3} \text{ A}}$$

Chapter 7, Solution 54.

- (a) Before $t = 0$, i is obtained by current division or

$$i(t) = \frac{4}{4+4} (2) = \underline{1 \text{ A}}$$

After $t = 0$,

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{\text{eq}}}, \quad R_{\text{eq}} = 4 + 4 \parallel 12 = 7 \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \quad i(\infty) = \frac{4 \parallel 12}{4 + 4 \parallel 12} (2) = \frac{3}{4+3} (2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right) e^{-2t}$$

$$i(t) = \underline{\frac{1}{7}(6 - e^{-2t}) \text{ A}}$$

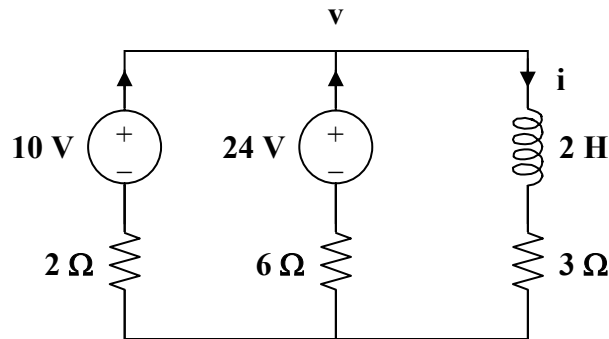
- (b) Before $t = 0$, $i(t) = \frac{10}{2+3} = \underline{2 \text{ A}}$

After $t = 0$, $R_{\text{eq}} = 3 + 6 \parallel 2 = 4.5$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at $t = \infty$ when the inductor becomes a short circuit,



$$\frac{10 - v}{2} + \frac{24 - v}{6} = \frac{v}{3} \longrightarrow v = 9$$

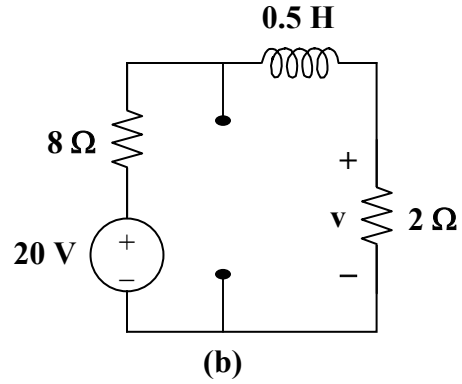
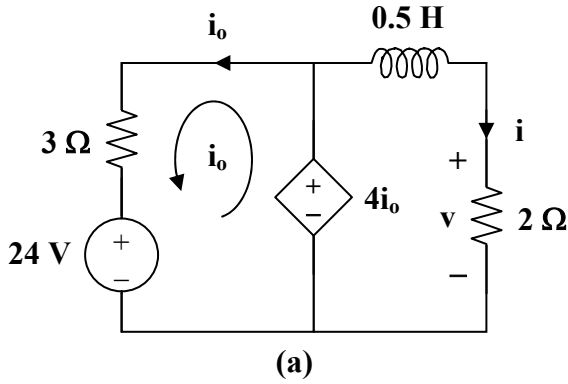
$$i(\infty) = \frac{v}{3} = 3 \text{ A}$$

$$i(t) = 3 + (2 - 3)e^{-9t/4}$$

$$i(t) = \underline{3 - e^{-9t/4} \text{ A}}$$

Chapter 7, Solution 55.

For $t < 0$, consider the circuit shown in Fig. (a).



$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$\underline{v(t) = 4i_o = 96 \text{ V}} \qquad i = \frac{v}{2} = 48 \text{ A}$$

For $t > 0$, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(0) = 48, \quad i(\infty) = \frac{20}{8+2} = 2 \text{ A}$$

$$R_{th} = 2 + 8 = 10 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{0.5}{10} = \frac{1}{20}$$

$$i(t) = 2 + (48 - 2)e^{-20t} = 2 + 46e^{-20t}$$

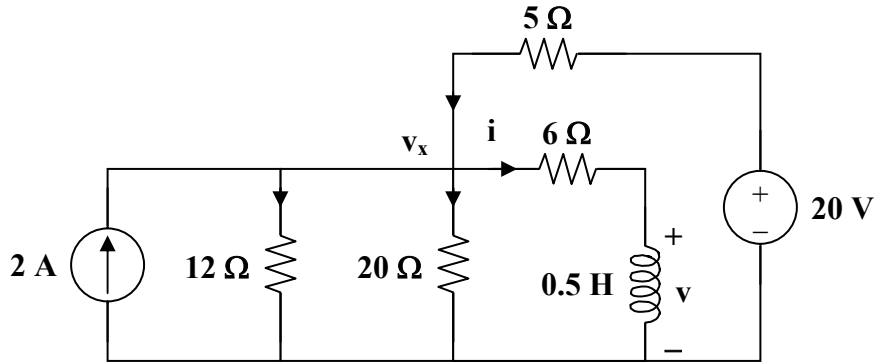
$$v(t) = 2i(t) = \underline{4 + 92e^{-20t} \text{ V}}$$

Chapter 7, Solution 56.

$$R_{eq} = 6 + 20 \parallel 5 = 10 \Omega, \quad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$i(0)$ is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2 \text{ A}$$

Since $20 \parallel 5 = 4$,

$$i(\infty) = \frac{4}{4 + 6} (4) = 1.6$$

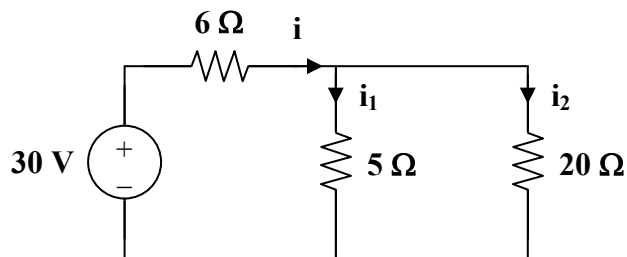
$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

$$v(t) = L \frac{di}{dt} = \frac{1}{2} (0.4)(-20)e^{-20t}$$

$$v(t) = \underline{-4e^{-20t} \text{ V}}$$

Chapter 7, Solution 57.

At $t = 0^-$, the circuit has reached steady state so that the inductors act like short circuits.



$$i = \frac{30}{6 + 5 \parallel 20} = \frac{30}{10} = 3, \quad i_1 = \frac{20}{25} (3) = 2.4, \quad i_2 = 0.6$$

$$i_1(0) = 2.4 \text{ A}, \quad i_2(0) = 0.6 \text{ A}$$

For $t > 0$, the switch is closed so that the energies in L_1 and L_2 flow through the closed switch and become dissipated in the 5Ω and 20Ω resistors.

$$i_1(t) = i_1(0) e^{-t/\tau_1}, \quad \tau_1 = \frac{L_1}{R_1} = \frac{2.5}{5} = \frac{1}{2}$$

$$i_1(t) = \underline{2.4 e^{-2t} \text{ A}}$$

$$i_2(t) = i_2(0) e^{-t/\tau_2}, \quad \tau_2 = \frac{L_2}{R_2} = \frac{4}{20} = \frac{1}{5}$$

$$i_2(t) = \underline{0.6 e^{-5t} \text{ A}}$$

Chapter 7, Solution 58.

$$\text{For } t < 0, \quad v_o(t) = 0$$

$$\text{For } t > 0, \quad i(0) = 10, \quad i(\infty) = \frac{20}{1+3} = 5$$

$$R_{th} = 1+3 = 4 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 5(1 + e^{-16t}) \text{ A}$$

$$v_o(t) = 3i + L \frac{di}{dt} = 15(1 + e^{-16t}) + \frac{1}{4}(-16)(5)e^{-16t}$$

$$v_o(t) = \underline{15 - 5e^{-16t} \text{ V}}$$

Chapter 7, Solution 59.

Let I be the current through the inductor.

$$\text{For } t < 0, \quad v_s = 0, \quad i(0) = 0$$

$$\text{For } t > 0, \quad R_{eq} = 4 + 6 \parallel 3 = 6, \quad \tau = \frac{L}{R_{eq}} = \frac{1.5}{6} = 0.25$$

$$i(\infty) = \frac{2}{2+4} (3) = 1$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 1 - e^{-4t}$$

$$v_o(t) = L \frac{di}{dt} = (1.5)(-4)(-e^{-4t})$$

$$v_o(t) = \underline{6e^{-4t} \text{ V}}$$

Chapter 7, Solution 60.

Let i be the inductor current.

$$\text{For } t < 0, \quad u(t) = 0 \longrightarrow i(0) = 0$$

$$\text{For } t > 0, \quad R_{\text{eq}} = 5 \parallel 20 = 4 \Omega, \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{8}{4} = 2$$

$$i(\infty) = 4$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

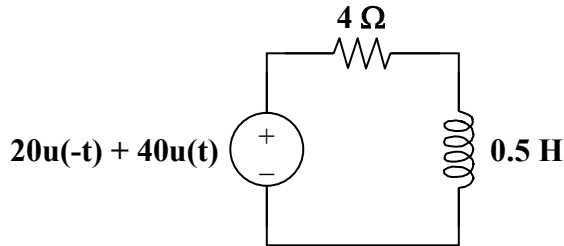
$$i(t) = 4(1 - e^{-t/2})$$

$$v(t) = L \frac{di}{dt} = (8)(-4) \left(\frac{-1}{2} \right) e^{-t/2}$$

$$v(t) = \underline{\underline{16e^{-0.5t} \text{ V}}}$$

Chapter 7, Solution 61.

The current source is transformed as shown below.



$$\tau = \frac{L}{R} = \frac{1/2}{4} = \frac{1}{8}, \quad i(0) = 5, \quad i(\infty) = 10$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = \underline{\underline{10 - 5e^{-8t} \text{ A}}}$$

$$v(t) = L \frac{di}{dt} = \left(\frac{1}{2} \right) (-5)(-8) e^{-8t}$$

$$v(t) = \underline{\underline{20e^{-8t} \text{ V}}}$$

Chapter 7, Solution 62.

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{2}{3 \parallel 6} = 1$$

For $0 < t < 1$, $u(t-1) = 0$ so that

$$i(0) = 0, \quad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}(1 - e^{-t})$$

For $t > 1$, $i(1) = \frac{1}{6}(1 - e^{-1}) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946e^{-(t-1)}$$

Thus,

$$i(t) = \begin{cases} \frac{1}{6}(1 - e^{-t}) \text{ A} & 0 < t < 1 \\ 0.5 - 0.3946e^{-(t-1)} \text{ A} & t > 1 \end{cases}$$

Chapter 7, Solution 63.

For $t < 0$, $u(-t) = 1$, $i(0) = \frac{10}{5} = 2$

For $t > 0$, $u(-t) = 0$, $i(\infty) = 0$

$$R_{th} = 5 \parallel 20 = 4 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = \underline{2e^{-8t} \text{ A}}$$

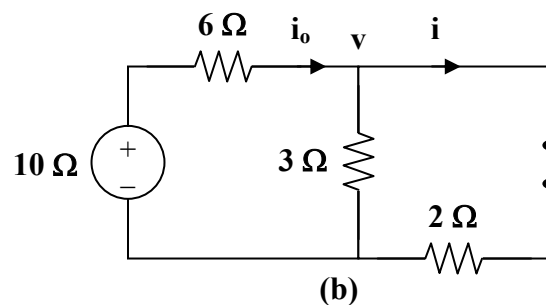
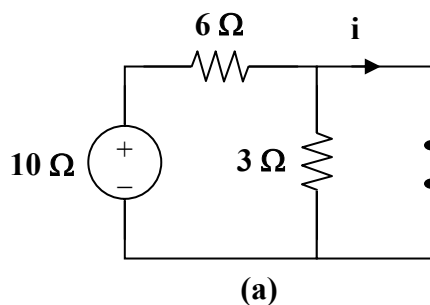
$$v(t) = L \frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)e^{-8t}$$

$$v(t) = \underline{-8e^{-8t} \text{ V}}$$

Chapter 7, Solution 64.

Let i be the inductor current.

For $t < 0$, the inductor acts like a short circuit and the 3Ω resistor is short-circuited so that the equivalent circuit is shown in Fig. (a).



$$i = i(0) = \frac{10}{6} = 1.667 \text{ A}$$

$$\text{For } t > 0, \quad R_{\text{th}} = 2 + 3 \parallel 6 = 4 \Omega, \quad \tau = \frac{L}{R_{\text{th}}} = \frac{4}{4} = 1$$

To find $i(\infty)$, consider the circuit in Fig. (b).

$$\frac{10 - v}{6} = \frac{v}{3} + \frac{v}{2} \longrightarrow v = \frac{10}{6}$$

$$i = i(\infty) = \frac{v}{2} = \frac{5}{6}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = \frac{5}{6} + \left(\frac{10}{6} - \frac{5}{6} \right) e^{-t} = \frac{5}{6} (1 + e^{-t}) \text{ A}$$

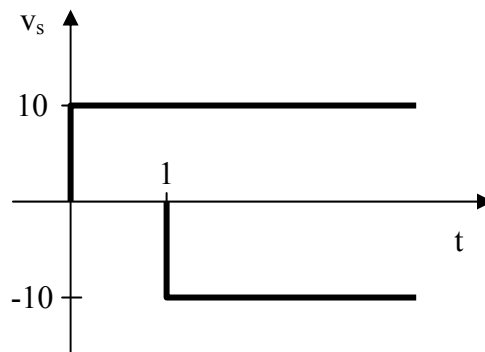
v_o is the voltage across the 4 H inductor and the 2 Ω resistor

$$v_o(t) = 2i + L \frac{di}{dt} = \frac{10}{6} + \frac{10}{6} e^{-t} + (4) \left(\frac{5}{6} \right) (-1) e^{-t} = \frac{10}{6} - \frac{10}{6} e^{-t}$$

$$v_o(t) = \underline{\underline{1.667(1 - e^{-t}) \text{ V}}}$$

Chapter 7, Solution 65.

Since $v_s = 10[u(t) - u(t-1)]$, this is the same as saying that a 10 V source is turned on at $t = 0$ and a -10 V source is turned on later at $t = 1$. This is shown in the figure below.



$$\text{For } 0 < t < 1, \quad i(0) = 0, \quad i(\infty) = \frac{10}{5} = 2$$

$$R_{\text{th}} = 5 \parallel 20 = 4, \quad \tau = \frac{L}{R_{\text{th}}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 2(1 - e^{-2t}) \text{ A}$$

$$i(1) = 2(1 - e^{-2}) = 1.729$$

For $t > 1$, $i(\infty) = 0$ since $v_s = 0$

$$i(t) = i(1)e^{-(t-1)/\tau}$$

$$i(t) = 1.729e^{-2(t-1)} \text{ A}$$

Thus,

$$i(t) = \begin{cases} 2(1 - e^{-2t}) \text{ A} & 0 < t < 1 \\ 1.729e^{-2(t-1)} \text{ A} & t > 1 \end{cases}$$

Chapter 7, Solution 66.

Following Practice Problem 7.14,

$$v(t) = V_T e^{-t/\tau}$$

$$V_T = v(0) = -4, \quad \tau = R_f C = (10 \times 10^3)(2 \times 10^{-6}) = \frac{1}{50}$$

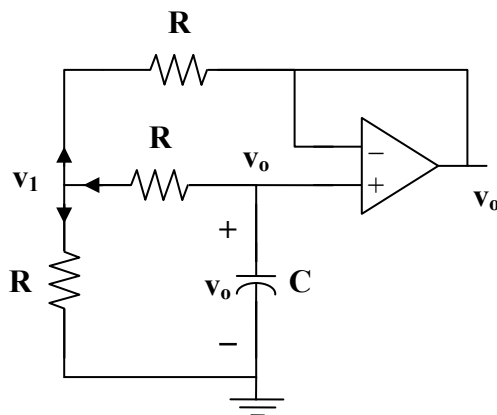
$$v(t) = -4e^{-50t}$$

$$v_o(t) = -v(t) = 4e^{-50t}, \quad t > 0$$

$$i_o(t) = \frac{v_o(t)}{R_o} = \frac{4}{10 \times 10^3} e^{-50t} = \underline{0.4e^{-50t} \text{ mA}, \quad t > 0}$$

Chapter 7, Solution 67.

The op amp is a voltage follower so that $v_o = v$ as shown below.



At node 1,

$$\frac{v_o - v_1}{R} = \frac{v_1 - 0}{R} + \frac{v_1 - v_o}{R} \longrightarrow v_1 = \frac{2}{3}v_o$$

At the noninverting terminal,

$$C \frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0$$

$$-RC \frac{dv_o}{dt} = v_o - v_1 = v_o - \frac{2}{3}v_o = \frac{1}{3}v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC}$$

$$v_o(t) = V_T e^{-t/3RC}$$

$$V_T = v_o(0) = 5 \text{ V}, \quad \tau = 3RC = (3)(10 \times 10^3)(1 \times 10^{-6}) = \frac{3}{100}$$

$$v_o(t) = \underline{\underline{5e^{-100t/3} \text{ V}}}$$

Chapter 7, Solution 68.

This is a very interesting problem and has both an important ideal solution as well as an important practical solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero which means that the voltage at both terminals input of the op amp are each zero. As soon as the switch closes, the output tries to go to a voltage such that the input to the op amp both go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all $t > 0$.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

$$\begin{aligned} v_C(t) &= V_{\text{op amp max}}(1 - e^{-t/(R_{\text{out}}C)}) \text{ volts, for all values of } v_C \text{ less than } 8 \text{ V,} \\ &= 8 \text{ V when } t \text{ is large enough so that the } 8 \text{ V is reached.} \end{aligned}$$

Chapter 7, Solution 69.

Let v_x be the capacitor voltage.

$$\text{For } t < 0, \quad v_x(0) = 0$$

For $t > 0$, the $20\text{ k}\Omega$ and $100\text{ k}\Omega$ resistors are in series since no current enters the op amp terminals. As $t \rightarrow \infty$, the capacitor acts like an open circuit so that

$$v_x(\infty) = \frac{20+100}{20+100+10} (4) = \frac{48}{13}$$

$$R_{th} = 20 + 100 = 120\text{ k}\Omega, \quad \tau = R_{th}C = (120 \times 10^3)(25 \times 10^{-3}) = 3000$$

$$v_x(t) = v_x(\infty) + [v_x(0) - v_x(\infty)] e^{-t/\tau}$$

$$v_x(t) = \frac{48}{13} (1 - e^{-t/3000})$$

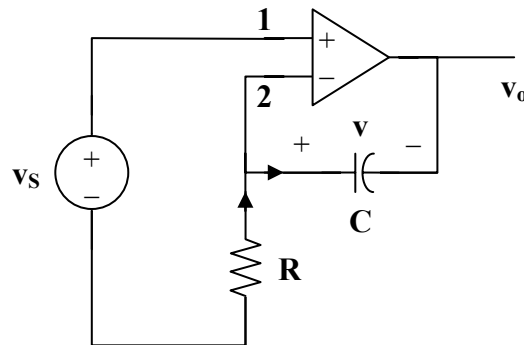
$$v_o(t) = \frac{100}{120} v_x(t) = \underline{\underline{\frac{40}{13} (1 - e^{-t/3000}) \text{ V}}}}$$

Chapter 7, Solution 70.

Let v = capacitor voltage.

For $t < 0$, the switch is open and $v(0) = 0$.

For $t > 0$, the switch is closed and the circuit becomes as shown below.



$$v_1 = v_2 = v_s \tag{1}$$

$$\frac{0 - v_s}{R} = C \frac{dv}{dt} \tag{2}$$

$$\text{where } v = v_s - v_o \longrightarrow v_o = v_s - v \tag{3}$$

From (1),

$$\frac{dv}{dt} = \frac{v_s}{RC} = 0$$

$$v = \frac{-1}{RC} \int v_s dt + v(0) = \frac{-t v_s}{RC}$$

Since v is constant,

$$RC = (20 \times 10^3)(5 \times 10^{-6}) = 0.1$$

$$v = \frac{-20t}{0.1} \text{ mV} = -200t \text{ mV}$$

From (3),

$$v_o = v_s - v = 20 + 200t$$

$$v_o = \underline{\underline{20(1 + 10t) \text{ mV}}}$$

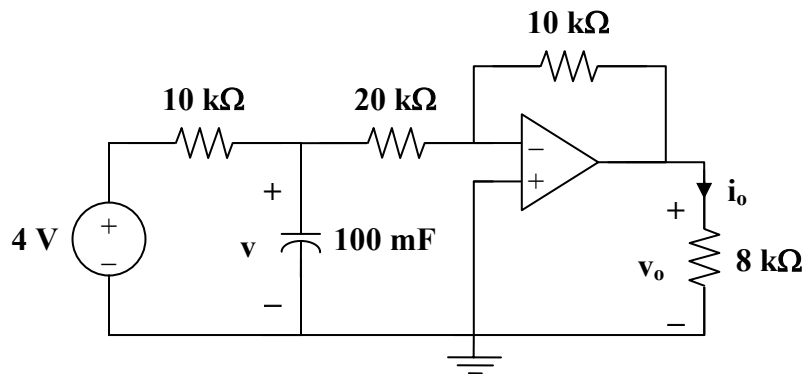
Chapter 7, Solution 71.

Let v = voltage across the capacitor.

Let v_o = voltage across the $8 \text{ k}\Omega$ resistor.

For $t < 2$, $v = 0$ so that $v(2) = 0$.

For $t > 2$, we have the circuit shown below.



Since no current enters the op amp, the input circuit forms an RC circuit.

$$\tau = RC = (10 \times 10^3)(100 \times 10^{-3}) = 1000$$

$$v(t) = v(\infty) + [v(2) - v(\infty)] e^{-(t-2)/\tau}$$

$$v(t) = 4(1 - e^{-(t-2)/1000})$$

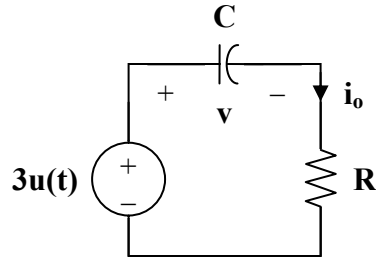
As an inverter,

$$v_o = \frac{-10\text{k}}{20\text{k}} v = 2(e^{-(t-2)/1000} - 1)$$

$$i_o = \frac{v_o}{8} = \underline{\underline{0.25(e^{-(t-2)/1000} - 1) \text{ A}}}$$

Chapter 7, Solution 72.

The op amp acts as an emitter follower so that the Thevenin equivalent circuit is shown below.



Hence,

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

$$v(0) = -2 \text{ V}, \quad v(\infty) = 3 \text{ V}, \quad \tau = RC = (10 \times 10^3)(10 \times 10^{-6}) = 0.1$$

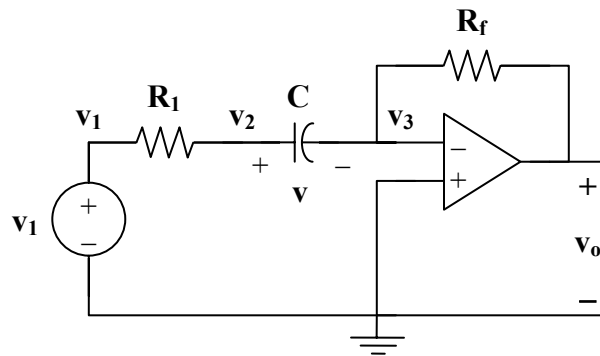
$$v(t) = 3 + (-2 - 3)e^{-10t} = 3 - 5e^{-10t}$$

$$i_o = C \frac{dv}{dt} = (10 \times 10^{-6})(-5)(-10)e^{-10t}$$

$$i_o = \underline{\underline{0.5e^{-10t} \text{ mA}, \quad t > 0}}$$

Chapter 7, Solution 73.

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C \frac{dv}{dt} \tag{1}$$

At node 3,

$$C \frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \quad (2)$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C \frac{dv}{dt}$$

$$v_1 - v = R_1 C \frac{dv}{dt}$$

or
$$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + (v_T - v_1)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10 \times 10^3)(20 \times 10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

From (2),

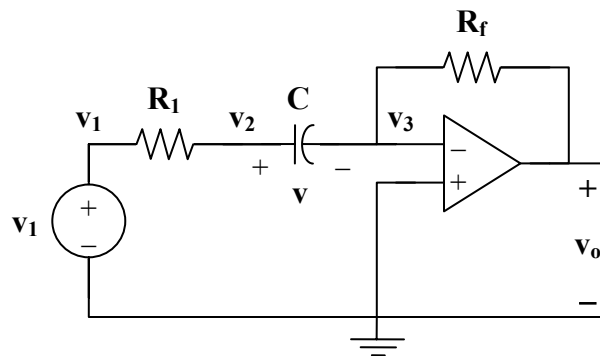
$$v_o = -R_f C \frac{dv}{dt} = (20 \times 10^3)(20 \times 10^{-6})(15e^{-5t})$$

$$v_o = -6e^{-5t}, \quad t > 0$$

$$v_o = \underline{\underline{-6e^{-5t} \text{ u}(t) \text{ V}}}$$

Chapter 7, Solution 74.

Let v = capacitor voltage.



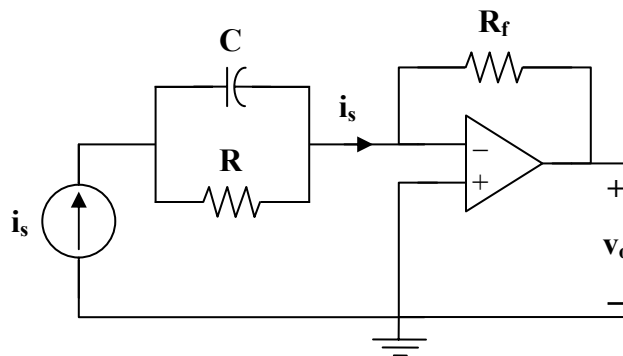
For $t < 0$, $v(0) = 0$
 For $t > 0$, $i_s = 10 \mu\text{A}$. Consider the circuit below.

$$i_s = C \frac{dv}{dt} + \frac{v}{R} \quad (1)$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau} \quad (2)$$

It is evident from the circuit that

$$\tau = RC = (2 \times 10^{-6})(50 \times 10^3) = 0.1$$



At steady state, the capacitor acts like an open circuit so that i_s passes through R . Hence,

$$v(\infty) = i_s R = (10 \times 10^{-6})(50 \times 10^3) = 0.5 \text{ V}$$

Then,

$$v(t) = 0.5(1 - e^{-10t}) \text{ V} \quad (3)$$

But $i_s = \frac{0 - v_o}{R_f} \longrightarrow v_o = -i_s R_f \quad (4)$

Combining (1), (3), and (4), we obtain

$$v_o = \frac{-R_f}{R} v - R_f C \frac{dv}{dt}$$

$$v_o = \frac{-1}{5} v - (10 \times 10^3)(2 \times 10^{-6}) \frac{dv}{dt}$$

$$v_o = -0.1 + 0.1e^{-10t} - (2 \times 10^{-2})(0.5)(-10e^{-10t})$$

$$v_o = 0.2e^{-10t} - 0.1$$

$$v_o = \underline{\underline{0.1(2e^{-10t} - 1) \text{ V}}}$$

Chapter 7, Solution 75.

Let v_1 = voltage at the noninverting terminal.

Let v_2 = voltage at the inverting terminal.

For $t > 0$, $v_1 = v_2 = v_s = 4$

$$\frac{0 - v_s}{R_1} = i_o, \quad R_1 = 20 \text{ k}\Omega$$

$$v_o = -i_o R \quad (1)$$

Also, $i_o = \frac{v}{R_2} + C \frac{dv}{dt}$, $R_2 = 10 \text{ k}\Omega$, $C = 2 \text{ }\mu\text{F}$

$$\text{i.e.} \quad \frac{-v_s}{R_1} = \frac{v}{R_2} + C \frac{dv}{dt} \quad (2)$$

This is a step response.

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}, \quad v(0) = 1$$

$$\text{where } \tau = R_2 C = (10 \times 10^3)(2 \times 10^{-6}) = \frac{1}{50}$$

At steady state, the capacitor acts like an open circuit so that i_o passes through R_2 . Hence, as $t \rightarrow \infty$

$$\frac{-v_s}{R_1} = i_o = \frac{v(\infty)}{R_2}$$

$$\text{i.e.} \quad v(\infty) = \frac{-R_2}{R_1} v_s = \frac{-10}{20} (4) = -2$$

$$v(t) = -2 + (1 + 2) e^{-50t}$$

$$v(t) = -2 + 3 e^{-50t}$$

But $v = v_s - v_o$

$$\text{or} \quad v_o = v_s - v = 4 + 2 - 3 e^{-50t}$$

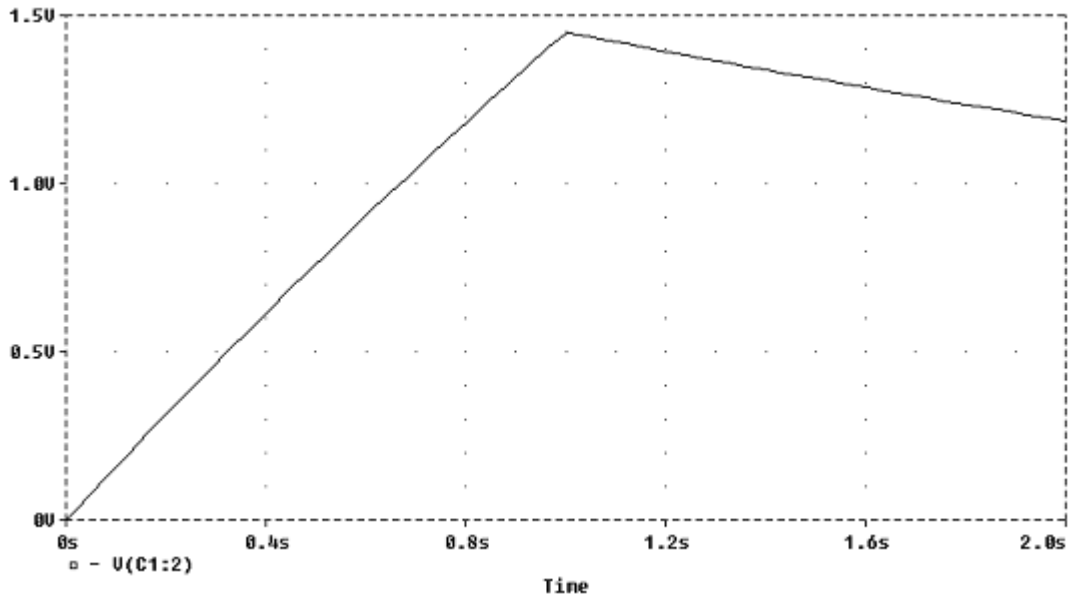
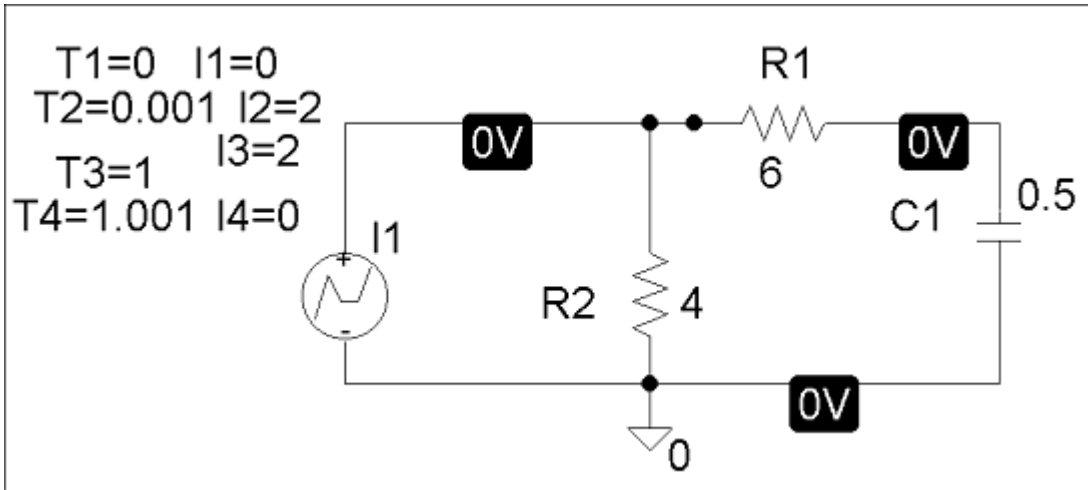
$$v_o = \underline{\underline{6 - 3 e^{-50t} \text{ V}}}$$

$$i_o = \frac{-v_s}{R_1} = \frac{-4}{20\text{k}} = -0.2 \text{ mA}$$

$$\text{or} \quad i_o = \frac{v}{R_2} + C \frac{dv}{dt} = \underline{\underline{-0.2 \text{ mA}}}$$

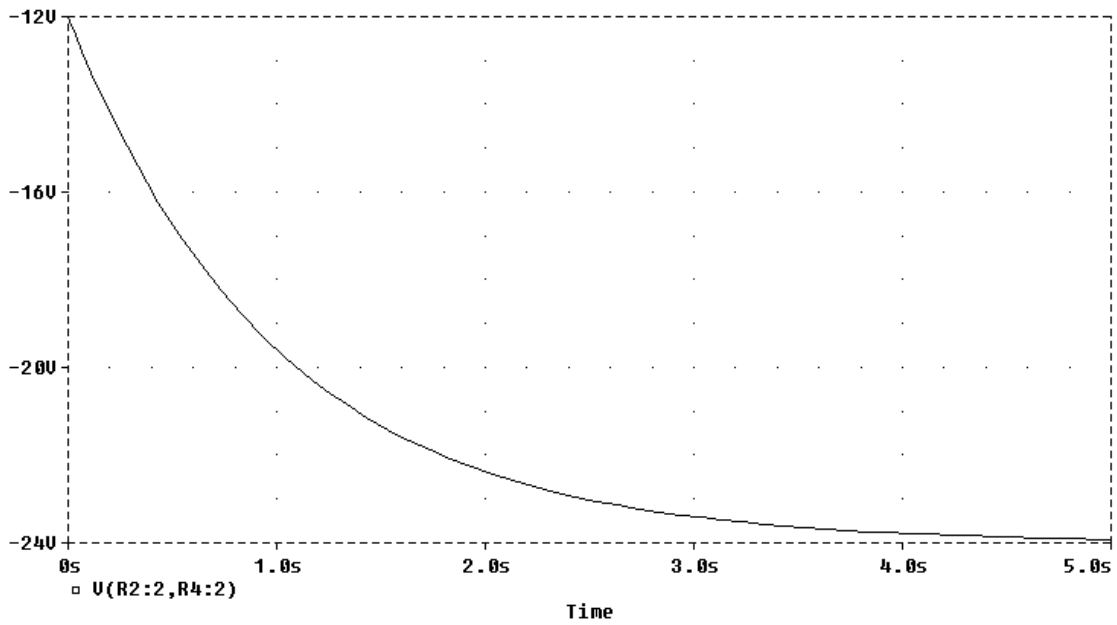
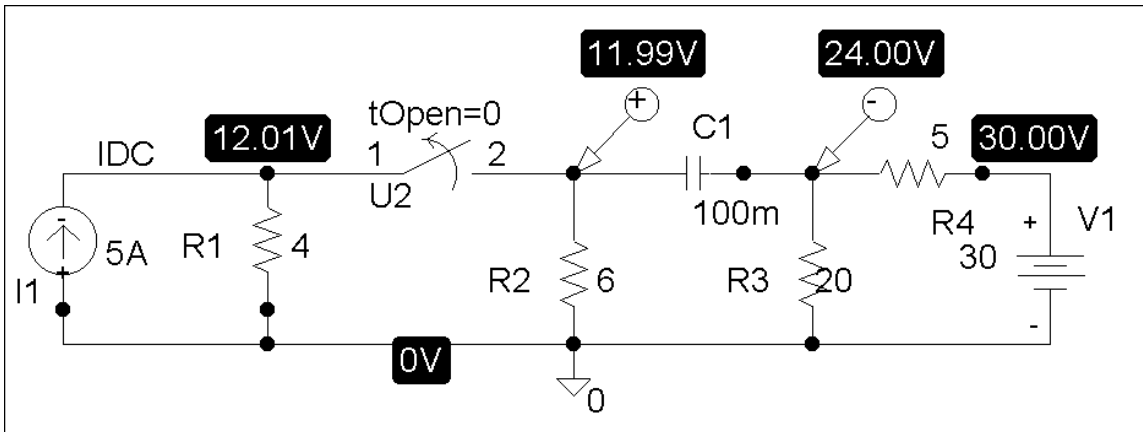
Chapter 7, Solution 76.

The schematic is shown below. For the pulse, we use IPWL and enter the corresponding values as attributes as shown. By selecting Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s since the width of the input pulse is 1 s. After saving and simulating the circuit, we select Trace/Add and display $-V(C1:2)$. The plot of $V(t)$ is shown below.



Chapter 7, Solution 77.

The schematic is shown below. We click Marker and insert Mark Voltage Differential at the terminals of the capacitor to display V after simulation. The plot of V is shown below. Note from the plot that $V(0) = 12 \text{ V}$ and $V(\infty) = -24 \text{ V}$ which are correct.

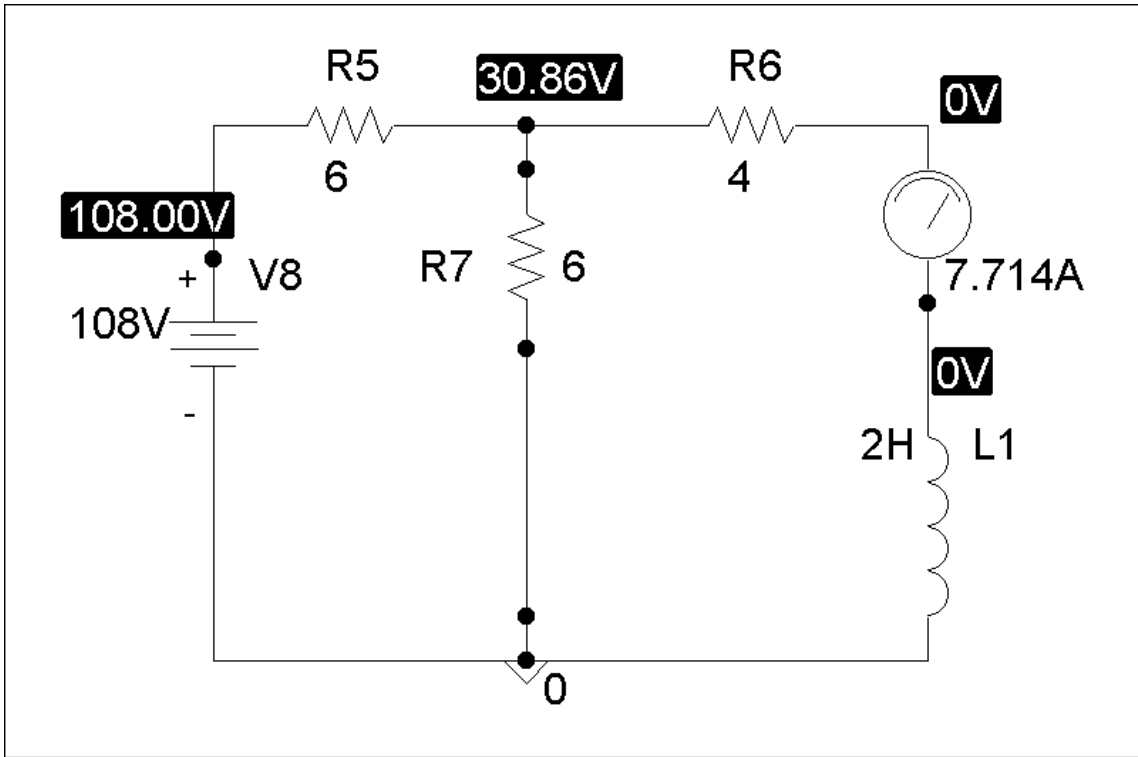


Chapter 7, Solution 78.

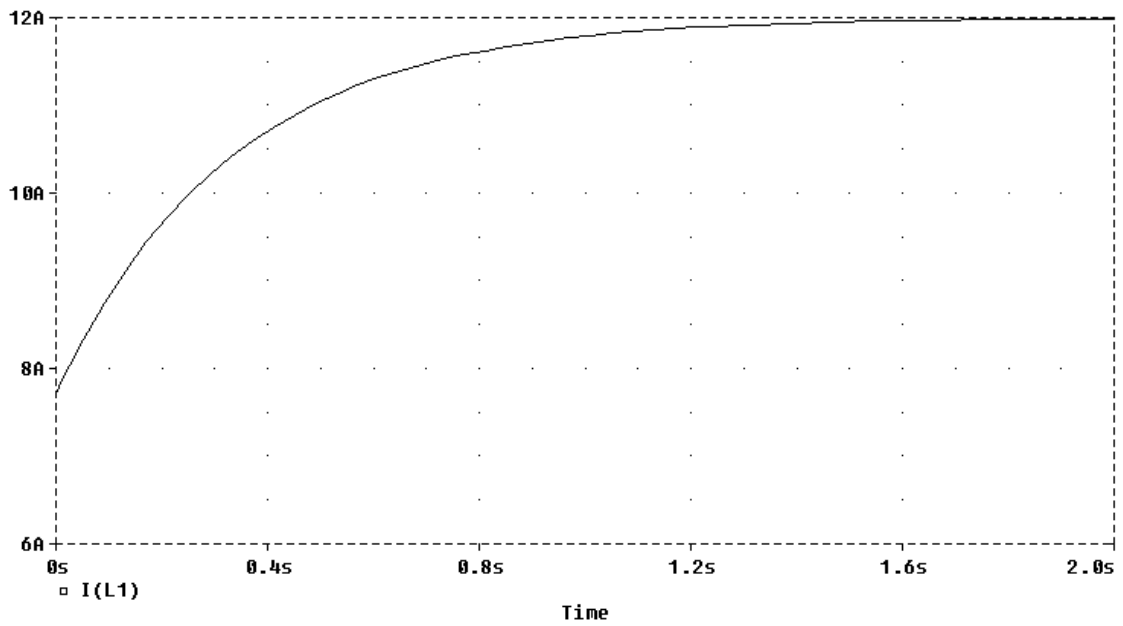
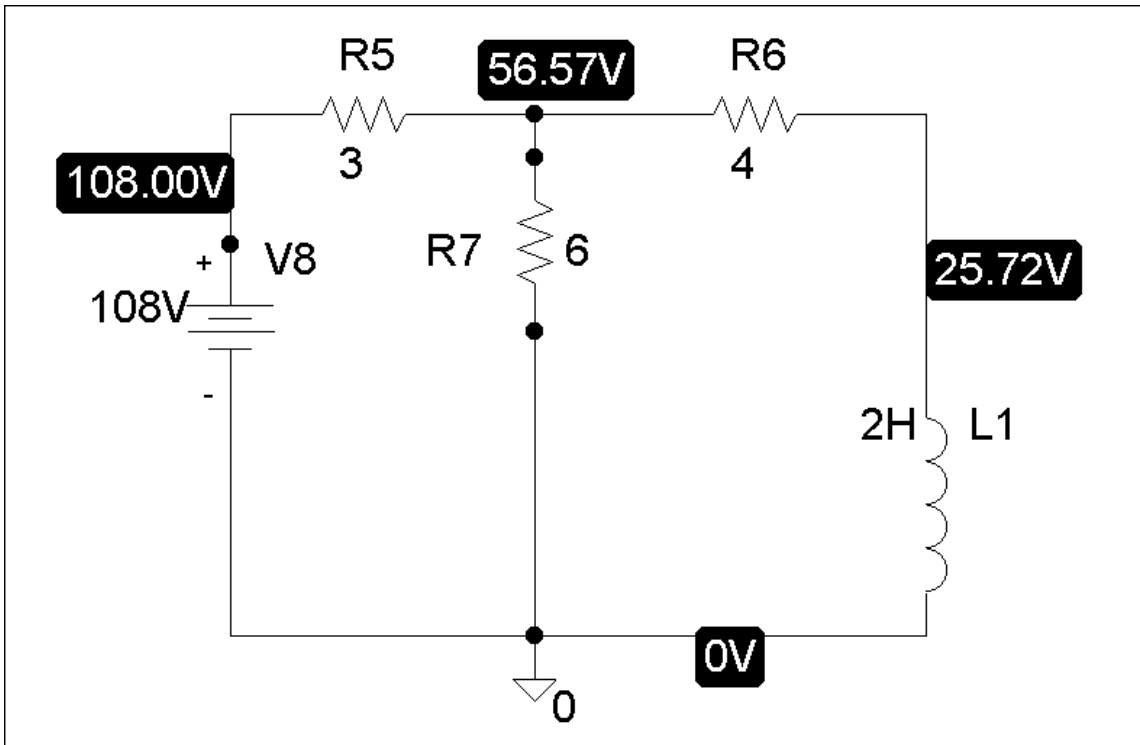
- (a) When the switch is in position (a), the schematic is shown below. We insert IPROBE to display i . After simulation, we obtain,

$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position (b), the schematic is as shown below. For inductor I1, we let $I_C = 7.714$. By clicking Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click Trace/Add in the probe menu and display I(L1) as shown below. Note that $i(\infty) = 12A$, which is correct.



Chapter 7, Solution 79.

When the switch is in position 1, $i_o(0) = 12/3 = 4\text{A}$. When the switch is in position 2,

$$i_o(\infty) = -\frac{4}{5+3} = -0.5\text{A}, \quad R_{Th} = (3+5) // 4 = 8/3, \quad \tau = \frac{R_{Th}}{L} = 80/3$$

$$i_o(t) = i_o(\infty) + [i_o(0) - i_o(\infty)]e^{-t/\tau} = \underline{-0.5 + 4.5e^{-3t/80}\text{A}}$$

Chapter 7, Solution 80.

- (a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0) = 30/10 = 3\text{A}$.

- (b) When the switch is in position B,

$$R_{Th} = 3 // 6 = 2\Omega, \quad \tau = \frac{R_{Th}}{L} = 2/4 = 0.5$$

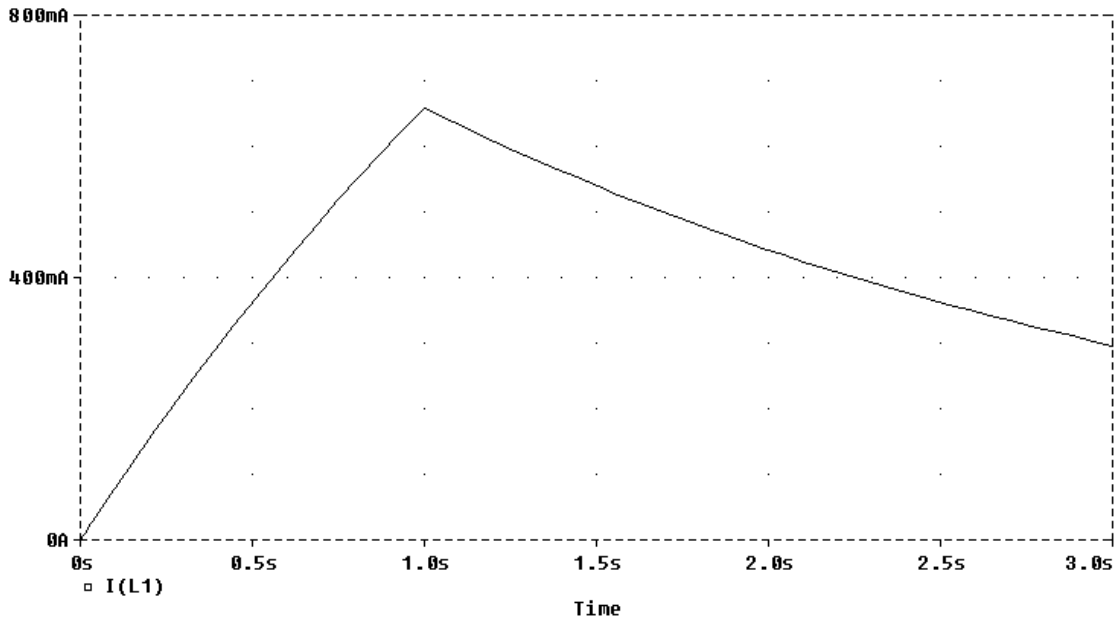
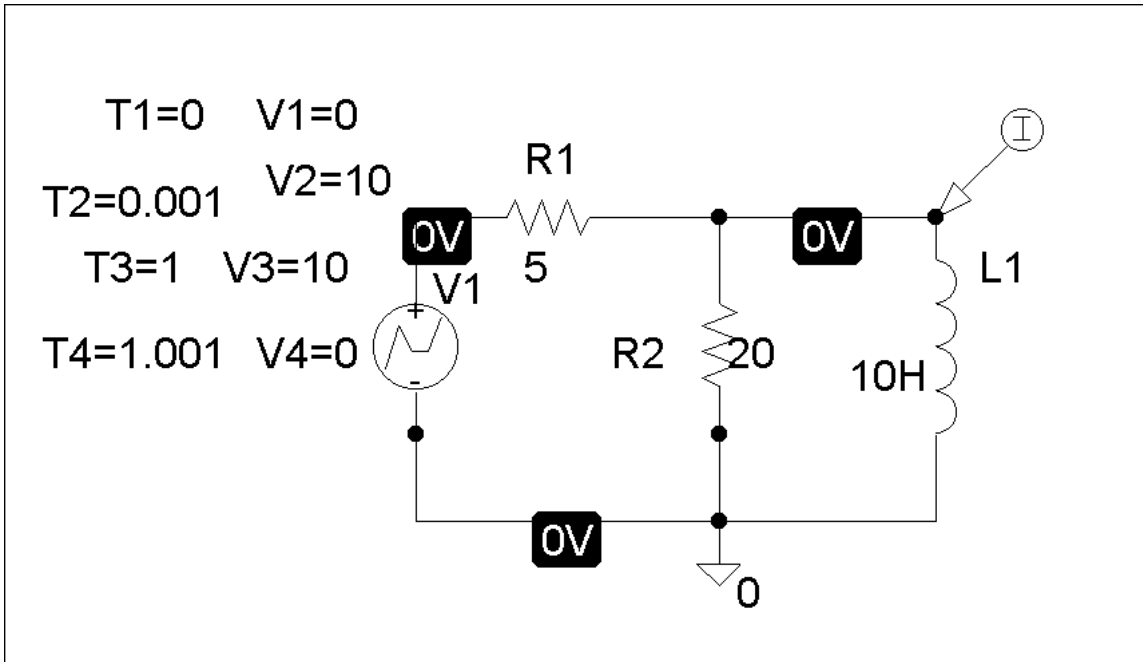
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/0.5} = \underline{3e^{-2t}\text{A}}$$

(c) $i_1(\infty) = \frac{30}{10+5} = \underline{2\text{A}}, \quad i_2(\infty) = -\frac{3}{9}i_L(\infty) = \underline{0\text{A}}$

$$v_o(t) = L \frac{di_L}{dt} \quad \longrightarrow \quad \underline{v_o(\infty) = 0\text{V}}$$

Chapter 7, Solution 81.

The schematic is shown below. We use VPWL for the pulse and specify the attributes as shown. In the Analysis/Setup/Transient menu, we select Print Step = 25 ms and final Step = 3 S. By inserting a current marker at one terminal of LI, we automatically obtain the plot of i after simulation as shown below.



Chapter 7, Solution 82.

$$\tau = RC \longrightarrow R = \frac{\tau}{C} = \frac{3 \times 10^{-3}}{100 \times 10^{-6}} = \underline{\underline{30 \Omega}}$$

Chapter 7, Solution 83.

$$v(\infty) = 120, \quad v(0) = 0, \quad \tau = RC = 34 \times 10^6 \times 15 \times 10^{-6} = 510 \text{ s}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \quad \longrightarrow \quad 85.6 = 120(1 - e^{-t/510})$$

Solving for t gives

$$t = 510 \ln 3.488 = 637.16 \text{ s}$$

$$\text{speed} = 4000 \text{ m} / 637.16 \text{ s} = \underline{6.278 \text{ m/s}}$$

Chapter 7, Solution 84.

Let I_o be the final value of the current. Then

$$i(t) = I_o(1 - e^{-t/\tau}), \quad \tau = R/L = 0.16/8 = 1/50$$

$$0.6I_o = I_o(1 - e^{-50t}) \quad \longrightarrow \quad t = \frac{1}{50} \ln \frac{1}{0.4} = \underline{18.33 \text{ ms.}}$$

Chapter 7, Solution 85.

$$(a) \quad \tau = RC = (4 \times 10^6)(6 \times 10^{-6}) = 24 \text{ s}$$

$$\text{Since } v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$v(t_1) - v(\infty) = [v(0) - v(\infty)]e^{-t_1/\tau} \quad (1)$$

$$v(t_2) - v(\infty) = [v(0) - v(\infty)]e^{-t_2/\tau} \quad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau \ln \left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} \right)$$

$$t_0 = 24 \ln \left(\frac{75 - 120}{30 - 120} \right) = 24 \ln(2) = \underline{16.63 \text{ s}}$$

(b) Since $t_0 < t$, the light flashes repeatedly every

$$\tau = RC = \underline{24 \text{ s}}$$

Chapter 7, Solution 86.

$$\begin{aligned}v(t) &= v(\infty) + [v(0) - v(\infty)] e^{-t/\tau} \\v(\infty) &= 12, \quad v(0) = 0 \\v(t) &= 12(1 - e^{-t/\tau}) \\v(t_0) &= 8 = 12(1 - e^{-t_0/\tau}) \\ \frac{8}{12} &= 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{1}{3} \\t_0 &= \tau \ln(3)\end{aligned}$$

For $R = 100 \text{ k}\Omega$,

$$\begin{aligned}\tau &= RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2 \text{ s} \\t_0 &= 0.2 \ln(3) = 0.2197 \text{ s}\end{aligned}$$

For $R = 1 \text{ M}\Omega$,

$$\begin{aligned}\tau &= RC = (1 \times 10^6)(2 \times 10^{-6}) = 2 \text{ s} \\t_0 &= 2 \ln(3) = 2.197 \text{ s}\end{aligned}$$

Thus,

$$\underline{\underline{0.2197 \text{ s} < t_0 < 2.197 \text{ s}}}$$

Chapter 7, Solution 87.

Let i be the inductor current.

$$\text{For } t < 0, \quad i(0^-) = \frac{120}{100} = 1.2 \text{ A}$$

For $t > 0$, we have an RL circuit

$$\begin{aligned}\tau &= \frac{L}{R} = \frac{50}{100 + 400} = 0.1, \quad i(\infty) = 0 \\i(t) &= i(\infty) + [i(0) - i(\infty)] e^{-t/\tau} \\i(t) &= 1.2 e^{-10t}\end{aligned}$$

At $t = 100 \text{ ms} = 0.1 \text{ s}$,

$$i(0.1) = 1.2 e^{-1} = \underline{\underline{0.441 \text{ A}}}$$

which is the same as the current through the resistor.

Chapter 7, Solution 88.

$$(a) \quad \tau = RC = (300 \times 10^3)(200 \times 10^{-12}) = 60 \mu\text{s}$$

As a differentiator,

$$T > 10\tau = 600 \mu\text{s} = 0.6 \text{ ms}$$

$$\text{i.e.} \quad T_{\min} = \underline{\mathbf{0.6 \text{ ms}}}$$

$$(b) \quad \tau = RC = 60 \mu\text{s}$$

As an integrator,

$$T < 0.1\tau = 6 \mu\text{s}$$

$$\text{i.e.} \quad T_{\max} = \underline{\mathbf{6 \mu\text{s}}}$$

Chapter 7, Solution 89.

Since $\tau < 0.1T = 1 \mu\text{s}$

$$\frac{L}{R} < 1 \mu\text{s}$$

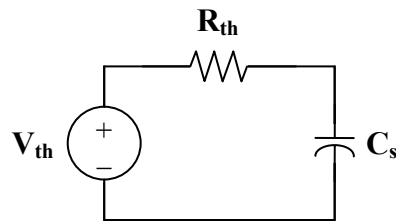
$$L < R \times 10^{-6} = (200 \times 10^3)(1 \times 10^{-6})$$

$$\underline{\mathbf{L < 200 \text{ mH}}}$$

Chapter 7, Solution 90.

We determine the Thevenin equivalent circuit for the capacitor C_s .

$$v_{\text{th}} = \frac{R_s}{R_s + R_p} v_i, \quad R_{\text{th}} = R_s \parallel R_p$$



The Thevenin equivalent is an RC circuit. Since

$$v_{\text{th}} = \frac{1}{10} v_i \longrightarrow \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

$$R_s = \frac{1}{9} R_p = \frac{6}{9} = \underline{\underline{\mathbf{\frac{2}{3} \text{ M}\Omega}}}$$

Also,

$$\tau = R_{th} C_s = 15 \mu s$$

$$\text{where } R_{th} = R_p \parallel R_s = \frac{6(2/3)}{6 + 2/3} = 0.6 \text{ M}\Omega$$

$$C_s = \frac{\tau}{R_{th}} = \frac{15 \times 10^{-6}}{0.6 \times 10^6} = \underline{\underline{25 \text{ pF}}}$$

Chapter 7, Solution 91.

$$i_o(0) = \frac{12}{50} = 240 \text{ mA}, \quad i(\infty) = 0$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 240 e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{R}$$

$$i(t_0) = 10 = 240 e^{-t_0/\tau}$$

$$e^{t_0/\tau} = 24 \longrightarrow t_0 = \tau \ln(24)$$

$$\tau = \frac{t_0}{\ln(24)} = \frac{5}{\ln(24)} = 1.573 = \frac{2}{R}$$

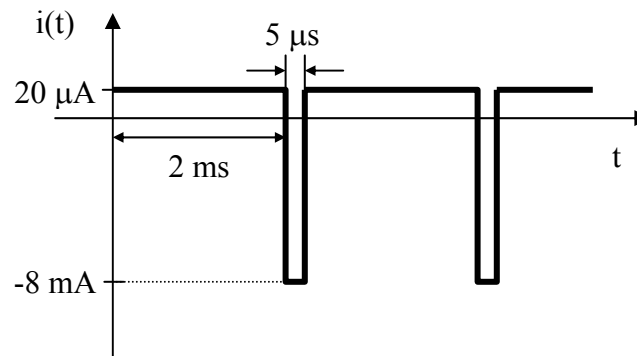
$$R = \frac{2}{1.573} = \underline{\underline{1.271 \Omega}}$$

Chapter 7, Solution 92.

$$i = C \frac{dv}{dt} = 4 \times 10^{-9} \cdot \begin{cases} \frac{10}{2 \times 10^{-3}} & 0 < t < t_R \\ \frac{-10}{5 \times 10^{-6}} & t_R < t < t_D \end{cases}$$

$$i(t) = \begin{cases} 20 \mu\text{A} & 0 < t < 2 \text{ ms} \\ -8 \text{ mA} & 2 \text{ ms} < t < 2 \text{ ms} + 5 \mu\text{s} \end{cases}$$

which is sketched below.



(not to scale)