Nonlinear Assignment 3

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1- Let P be the solution to

$$A^TP + PA = -I,$$

where A is an asymptotically stable matrix. Show that $G(s) = B^TP(sI - A)^{-1}B$ is passive. (*Hint.* Use the function $V(x) = x^TPx$.)

2- Consider the non-autonomous system:

$$a\ddot{y}(t) + p(t)\dot{y}(t) + e^{-t}y(t) = 0$$

a. If $a = 1$ Introduce a suitable time varying Lyapunov function and find some conditions of the function $p(t)$ that ensure the stability of the equilibrium 0 .

3- Consider the system

$$\dot{x}_1 = -x_1^3 + \alpha(t)x_2, \quad \dot{x}_2 = -\alpha(t)x_1 - x_2^3$$

where $\alpha(t)$ is a continuous, bounded function. Show that the origin is globally uniformly asymptotically stable. Is it exponentially stable?

4- Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 - (1 + b\cos t)x_2$$

Find $b^* > 0$ such that the origin is exponentially stable for all $|b| < b^*$.

5- Consider the system

$$\dot{x}_1 = x_2 - g(t)x_1(x_1^2 + x_2^2), \quad \dot{x}_2 = -x_1 - g(t)x_2(x_1^2 + x_2^2)$$

where $g(t)$ is continuously differentiable, bounded, and $g(t) \geq k > 0$ for all $t \geq 0$. Is the origin uniformly asymptotically stable? Is it exponentially stable?