Chapter 2: Electric Field

- ✓ **Electric Field Due to a Point Charge**
- ✓ **Electric Fields Due to Multiple Charges**
- ✓ **Electric Field Lines**
- ✓ **Electric Field of a Continuous Charge Distribution**

Session 4:

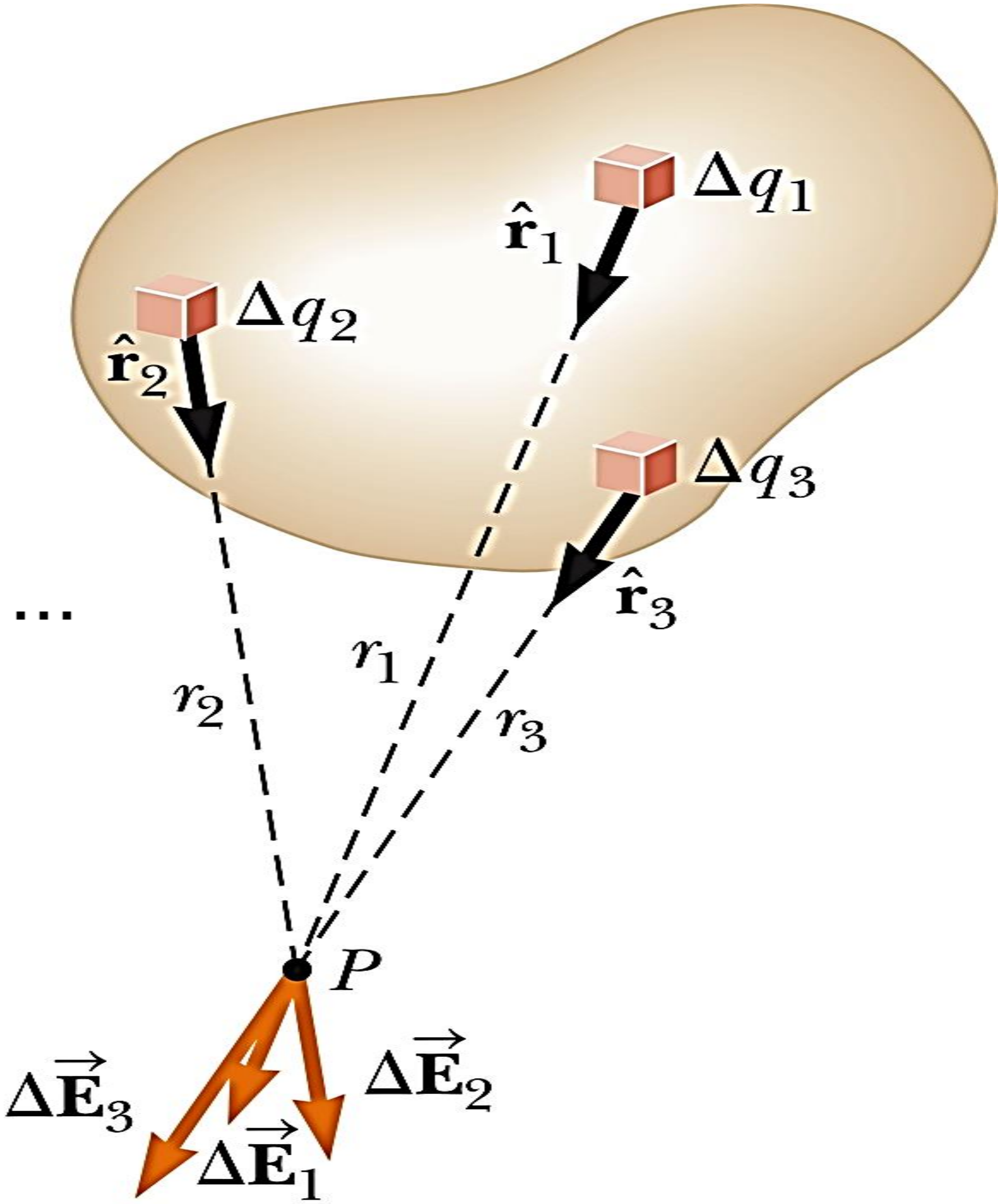
- ✓ **Electric Field of a Continuous Charge Distribution**
- ✓ **Examples**

Electric Field of a Continuous Charge Distribution

$$\vec{\mathbf{E}} = \Delta\vec{\mathbf{E}}_1 + \Delta\vec{\mathbf{E}}_2 + \ldots = k_e \frac{\Delta q_1}{r_1^{\,2}}\hat{\mathbf{r}}_1 + k_e \frac{\Delta q_2}{r_2^{\,2}}\hat{\mathbf{r}}_2 + \ldots$$

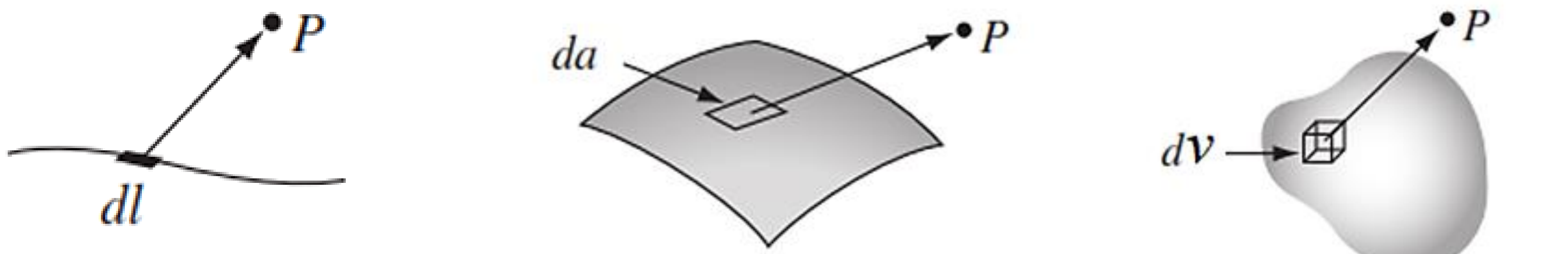
$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\boxed{\vec{\mathbf{E}} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$



Charge Densities

1. **Linear charge density: when a charge is distributed along a line**
   - $\lambda \equiv Q / \ell$ with units C/m
2. **Surface charge density: when a charge is distributed evenly over a surface area**
   - $\sigma \equiv Q / A$ with units C/m^2
3. **Volume charge density: when a charge is distributed evenly throughout a volume**
   - $\rho \equiv Q / V$ with units C/m^3



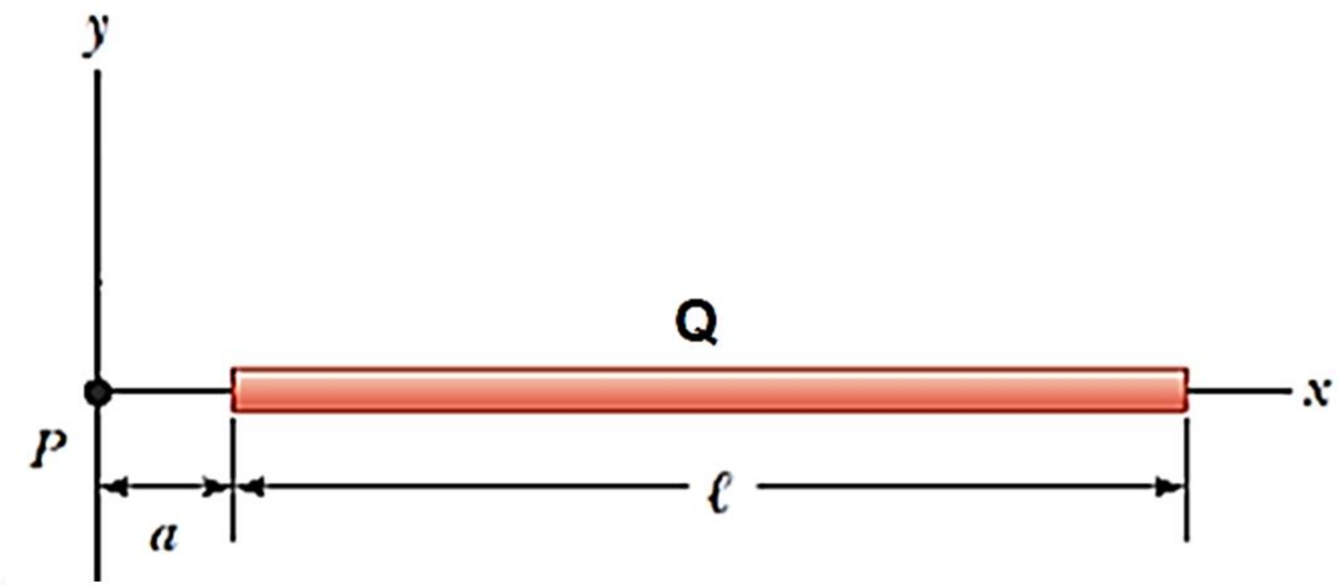
Line charge, λ   Surface charge, σ   Volume charge, ρ

$$\boxed{\vec{\mathbf{E}} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$

- **For the length element:** $dq = \lambda\, d\ell$
- **For the surface:** $dq = \sigma\, dA$
- **For the volume:** $dq = \rho\, dV$

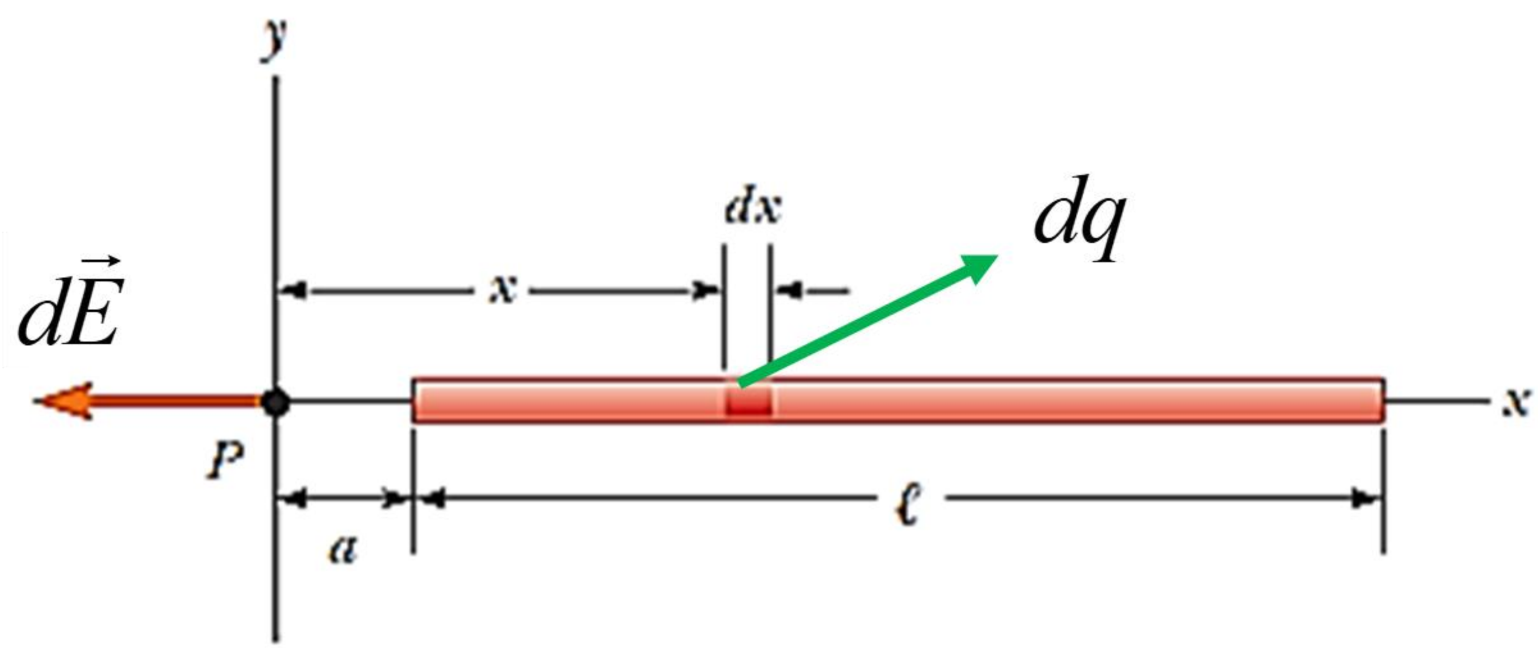
Ex 4 (Prob 22. 31). A rod of length **L= 25 cm**, has a uniform positive charge per unit length **λ** and a total charge **Q= 10 nC**. Calculate the electric field at a point P that is located along the long axis of the rod and a distance **a = 5 cm** from one end

$$d\vec{\mathbf{E}} = (-\hat{x})\, k_e \frac{dq}{x^2}$$



$$dq = \lambda\, dx = \frac{Q}{l} dx$$

$$d\vec{\mathbf{E}} = (-\hat{x})\, k_e \frac{Q}{l}\frac{dx}{x^2}$$

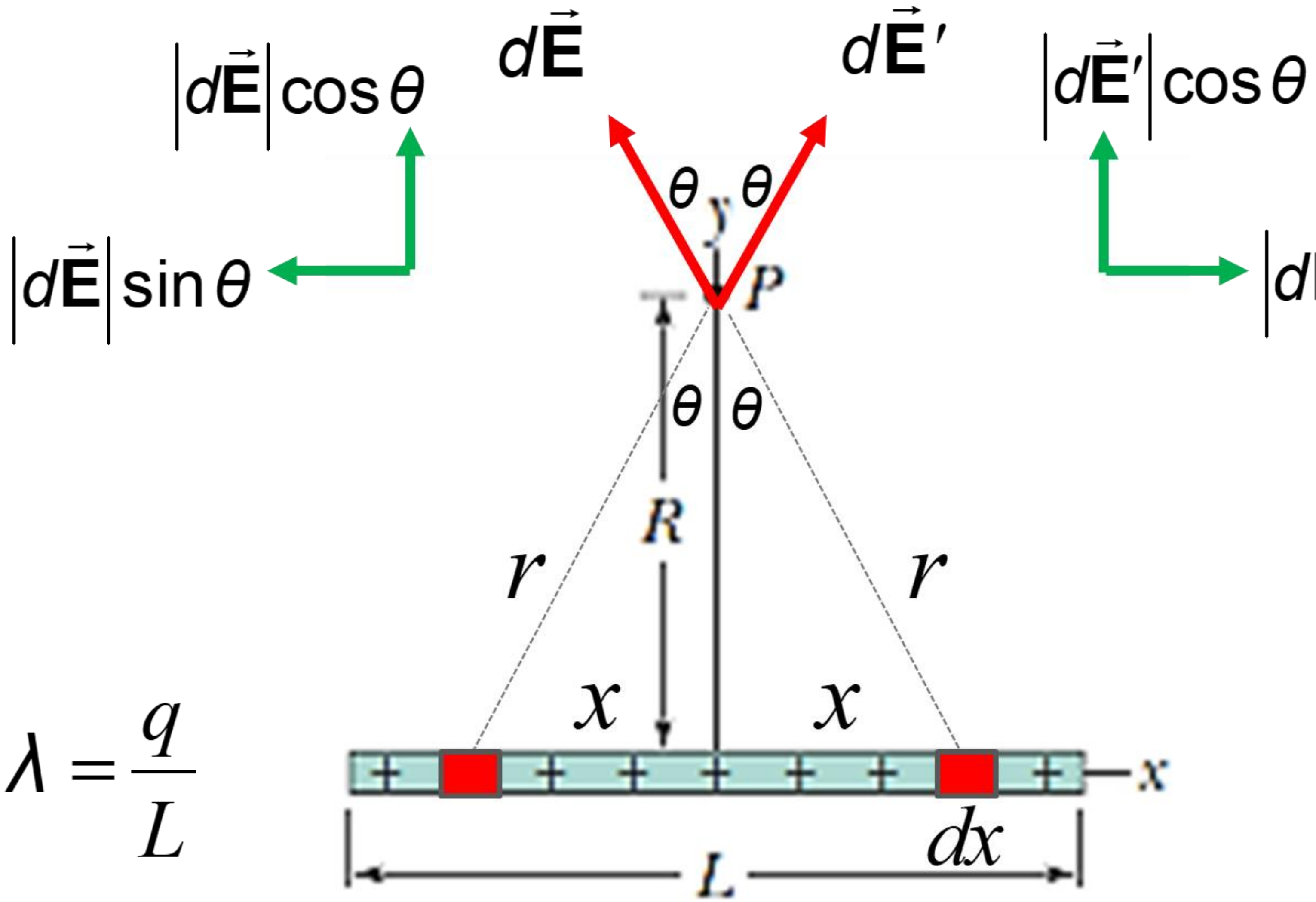


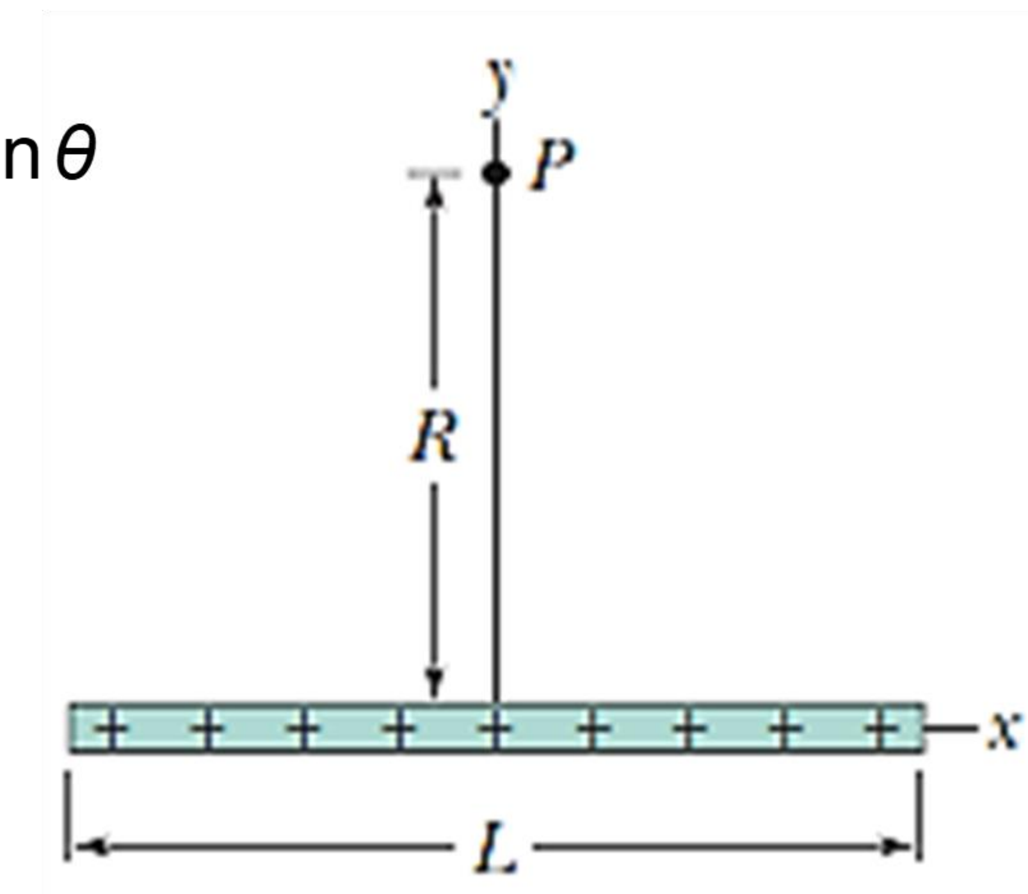
$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}} = (-\hat{x})\, k_e \frac{Q}{l}\int_a^{l+a} \frac{dx}{x^2}$$

$$\vec{\mathbf{E}} = (-\hat{x}) k_e \frac{Q}{l}\left(-\frac{1}{x}\Big|_a^{l+a}\right) = (-\hat{x}) k_e \frac{Q}{l}\left(\frac{1}{a} - \frac{1}{l+a}\right)$$

$$\boxed{\vec{\mathbf{E}} = (-\hat{x}) k_e \frac{Q}{a(l+a)} = (-\hat{x})\ 6\times 10^3\ \ N/C}$$

Ex 5. (Prob 22. 32) In Fig. 22-55, positive charge $q = 7.81$ pC is spread uniformly along a thin nonconducting rod of length $L = 14.5$ cm. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at point P, at distance $R = 6.00$ cm from the rod along its perpendicular bisector?





$$\lambda = \frac{q}{L}$$

$$\left|d\vec{\mathbf{E}}\right| = \left|d\vec{\mathbf{E}}'\right| = k_e\frac{dq}{r^2} = k_e\frac{\lambda\,dx}{(R^2+x^2)}$$

$$d\vec{\mathbf{E}} = \left|d\vec{\mathbf{E}}\right|\sin\theta(\hat{x}) + \left|d\vec{\mathbf{E}}\right|\cos\theta(\hat{y})$$

$$\vec{\mathbf{E}} = \underbrace{\int\left|d\vec{\mathbf{E}}\right|\sin\theta(\hat{x})}_{0} + \int\left|d\vec{\mathbf{E}}\right|\cos\theta(\hat{y})$$

$$\vec{\mathbf{E}} = (\hat{y})\int \left|d\vec{\mathbf{E}}\right| \cos\theta = (\hat{y})\int_{-\frac{L}{2}}^{+\frac{L}{2}} k_e \frac{\lambda\, dx}{(R^2 + x^2)} \frac{R}{\sqrt{R^2 + x^2}} = (\hat{y}) k_e \lambda R \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{(R^2 + x^2)^{\frac{3}{2}}}$$

$$\tan\theta = \frac{x}{R} \Rightarrow x = R\tan\theta \Rightarrow dx = R\sec^2\theta\, d\theta$$

$$(R^2 + x^2)^{\frac{3}{2}} = (R^2 + R^2\tan^2\theta)^{\frac{3}{2}} = R^3\sec^3\theta$$

$$\vec{\mathbf{E}} = (\hat{y}) k_e \lambda R \int \frac{R\sec^2\theta\, d\theta}{R^3\sec^3\theta} = (\hat{y})\frac{k_e\lambda}{R}\int \frac{d\theta}{\sec\theta} = (\hat{y})\frac{k_e\lambda}{R}\int \cos\theta d\theta = (\hat{y})\frac{k_e\lambda}{R}\sin\theta$$

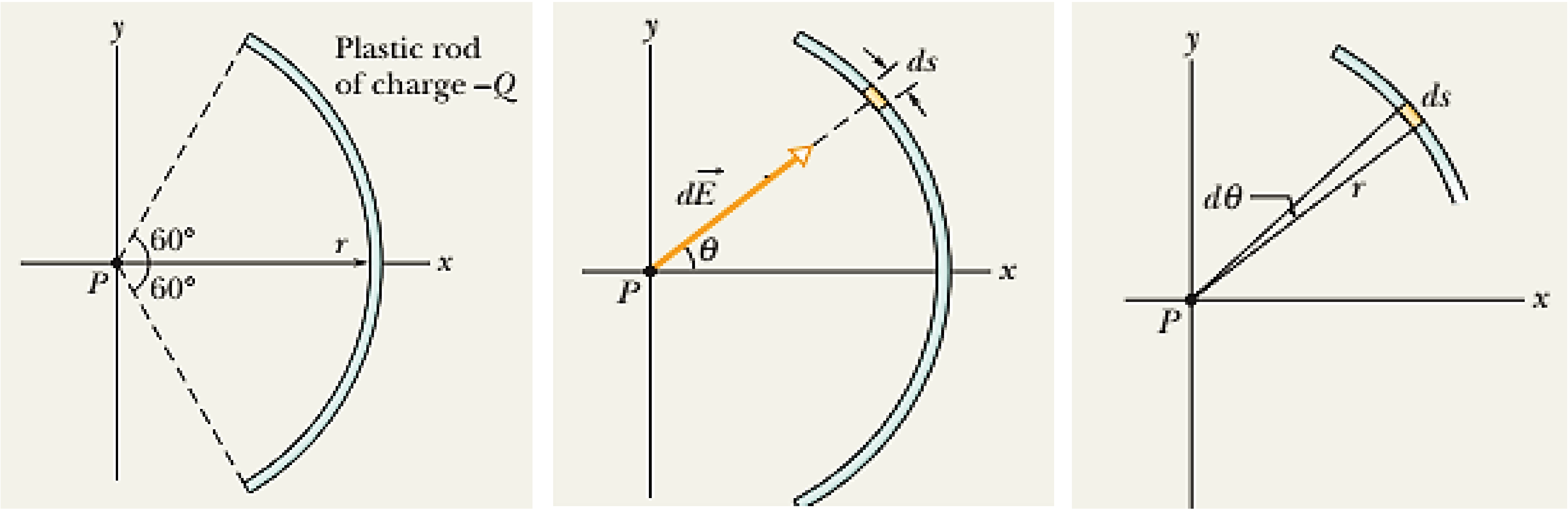
$$\boxed{\vec{\mathbf{E}} = (\hat{y})\frac{k_e\lambda}{R}\left.\frac{x}{\sqrt{R^2 + x^2}}\right|_{-\frac{L}{2}}^{+\frac{L}{2}} = (\hat{y})\frac{2k_e\lambda}{R}\frac{(\frac{L}{2})}{\sqrt{R^2 + (\frac{L}{2})^2}} \approx (\hat{y})\ 12.5\ \frac{N}{C}}$$

Infinite Line Charge

$L \gg R$

$$\boxed{\vec{\mathbf{E}} = (\hat{y})\frac{2k_e\lambda}{R}}$$

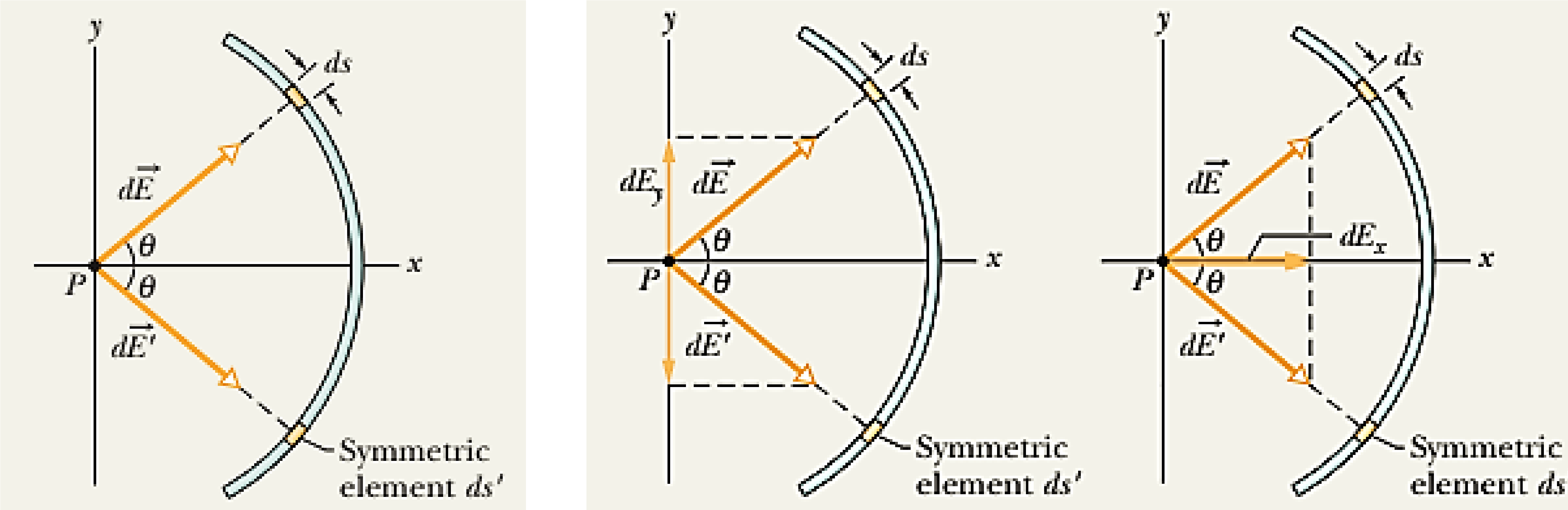
Ex 6. A plastic rod with a uniform charge **-Q** is bent in a **120°** circular arc of radius **r** and symmetrically paced across an x axis with the origin at the center of curvature **P** of the rod. In terms of **Q** and **r**, what is the electric field due to the rod at point P?



$$\lambda = \frac{Q}{l} = \frac{Q}{\frac{1}{3}(2\pi r)} = (0.477)\frac{Q}{r}$$

$$ds = r d\theta$$

$$\left|\mathbf{d\vec{E}}\right| = k_e \frac{dq}{r^2} = k_e \frac{\lambda ds}{r^2} = k_e \frac{\lambda r d\theta}{r^2} = k_e \frac{\lambda d\theta}{r}$$



$$\mathbf{dE}_x = \left|\mathbf{d\vec{E}}\right| \cos\theta = k_e \frac{\lambda \cos\theta \, d\theta}{r}$$

$$\mathbf{E}_x = \int \mathbf{dE}_x = \int_{-60}^{+60} k_e \frac{\lambda \cos\theta \, d\theta}{r} = k_e \frac{\lambda}{r} \int_{-60}^{+60} \cos\theta \, d\theta = k_e \frac{\lambda}{r} \underbrace{(\sin 60 - \sin(-60))}_{2\sin 60 = \sqrt{3}}$$

$$\mathbf{E}_y = \int \left|\mathbf{d\vec{E}}\right| \sin\theta = k_e \frac{\lambda}{r} \int_{-60}^{+60} \sin\theta \, d\theta = -k_e \frac{\lambda}{r} \underbrace{(\cos 60 - \cos(-60))}_{0} = 0$$

$$\boxed{\mathbf{E}_x = \sqrt{3} k_e \frac{\lambda}{r} = (0.83) k_e \frac{Q}{r^2}}$$