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Distributed discrete-time event-triggered algorithm for economic dispatch problem

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ABSTRACT

In this paper, a distributed discrete-time event-triggered algorithm is proposed to deal with the economic dispatch problem with equality constraint. The communication can be reduced and the energy of systems can be saved by adopting event-triggered communication mechanism. The Zeno behavior is naturally excluded based on discrete iteration scheme. Moreover, the proposed algorithm has been proved to be exponentially convergent with the aid of convex optimization and Lyapunov stability theory that the optimal value is obtained with discrete exponential convergence rate. Finally, the effectiveness of the proposed algorithm is verified via a numerical example.

1. Introduction

Recently, many researchers have devoted to studying economic dispatch problem (EDP) in smart grids, which aims to handle the power allocation among generators under satisfying the conditions of total load [1,2,5–7,11–15,18]. Generally, EDP can be casted as a constrained optimization problem, that is, the total generation cost is minimized by allocating the generation power under the load constraint, i.e., the sum of active power generated by all generators equal to the total load demand. Traditional centralized method, such as Lagrange multiplier method, can solve the EDP, which demand powerful central processors to obtain global information and deal with a large number of data, see for example [8,13,16] and references therein. However, with the development of network science, renewable resources, clean energy and smart grids, network scale and data volume will continue to grow. The centralized method will be restricted, its processing capacity will be reduced, and the communication cost will be greatly increased. Therefore, distributed algorithm is a good choice to deal with EDP, which can increase processing capacity, efficiency and reliability. It also has good applications in large image data processing, machine learning and other fields [9,10].

Each generator does not need to obtain all variable information based on distributed economic dispatch algorithm. The core idea is to transform the centralized optimization problem into several local optimization problems. On the basis of local optimization, the global optimal result can be obtained by exchanging local information through the communication network. A class of important distributed algorithms for solving EDP are constructed based on the idea of consensus, which are consensus-based distributed economic dispatch approaches. For instance, by selecting the incremental cost of each generation unit as the consensus variable, the incremental cost consensus algorithm was able to solve EDP in a distributed manner in [20]. The authors in [2] studied not only the EDP with generator constraints but also the actual situation of transmission line losses in the system. In [19], a novel consensus based algorithm with a tactical initial setup was proposed to solve EDP in a distributed fashion under strongly connected communication graph. To avoid initialization, an initialization-free distributed algorithm was designed over a strongly connected, weight-balanced digraph in [7], starting from any initial power allocation. To solve the rapidity of economic dispatch, a distributed and fast economic dispatch algorithm was provided in [5], and the optimal value was achieved in the finite-time, which makes more

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sense in real applications. Moreover, considering that the increase of network scale will increase the communication burden, event-triggered communication mechanism can effectively reduce this communication burden. A distributed event-triggered algorithm was proposed in [22] to solve a convex quadratic optimization problem under undirected and connected topologies, which had appeared in the economic dispatch, the resources allocation, the optimal portfolio, and so on. The event-triggered communication-based scheme for EDP had been investigated in [12], where the economic dispatch procedure was involved in continuous-time optimization and discrete-time communication. To reduce required capacities for information exchanges in micro-grids, a novel distributed event-based algorithm was proposed in [21] for optimal energy management in a micro-grid with considering power losses. Continuous-time event-triggered algorithm must exclude the Zeno behavior to avoid continuous communication, that is, to avoid infinite communication in limited time. Because of this, the algorithm designed in this paper adopts discrete iterative scheme.

Motivated by the distributed algorithm in [4], a distributed discrete-time event-triggered algorithm is proposed to solve EDP in this paper. The burden of communication is reduced and the energy of systems is saved via event-triggered communication mechanism, and the Zeno behavior is naturally excluded based on discrete iteration scheme. The exponential convergence of the algorithm is proved with the aid of convex optimization and Lyapunov stability theory. The main contributions of this paper include: (1) A distributed discrete-time optimization algorithm is designed to solve a class of EDP based on event-triggered communication mechanism. The designed algorithm has the advantages to reduce the communication and save the energy of systems, and the Zeno behavior is naturally excluded; (2) The optimal value of the total generation cost is obtained with discrete exponential convergence rate.

The paper is organized as follows. Firstly, some preliminaries and problem formulation are given in Section **2**. Section **3** provides a distributed discrete-time event-triggered algorithm and convergence analysis. Numerical simulations are given in Section **4**. Section **5** concludes this paper.

Notation: In this paper, $\mathbf{R}^n$ denotes the set of $n \times 1$ real vectors; $\mathbf{R}^{n\times n}$ denotes the set of $n \times n$ real matrices; $\mathbf{0}_n/\mathbf{1}_n$ denotes an $n \times 1$ column vector of all zeros/ones; $\boldsymbol{I}_n$ denotes an n-dimensional identity matrix; $\|\cdot\|$ denotes the Euclidean norm; $\lfloor \chi \rfloor$ denotes the largest integer no more than χ; Let $\lambda_2(L)$ and $\lambda_{\max}(L)$ be the minimal positive and maximal eigenvalue of Laplacian matrix L, respectively. $\triangledown C : \mathbf{R}^n \to \mathbf{R}^n$ is the gradient of C; $\triangledown^2 C : \mathbf{R}^n \to \mathbf{R}^{n\times n}$ is the Hessian matrix of C.

2. Preliminaries and problem formulation

2.1. Algebraic graph theory

A communication network of n nodes is denoted by a weighted undirected graph $G = \{V, E, A\}$, in which $V = \{1, 2, \ldots, n\}$ denoting the network nodes and the edge set $E \subseteq V \times V$ representing the communication links. An undirected edge $(i, j) \in E$ indicates that node i and node j can exchange information with each other. Define the weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{n\times n}$ with $a_{ij} = a_{ji} > 0$ if $(i, j) \in E$ and $a_{ij} = 0$ otherwise. Note that G is said to be undirected and connected if $(j, i) \in E$ implies $(i, j) \in E$ for arbitrary i, $j \in V$. The set of communication neighbors of agent i is denoted by $N_i = \{j \in V \mid (j, i) \in E\}$. The Laplacian matrix $L \triangleq [l_{ij}] \in \mathbf{R}^{n\times n}$ of graph G is defined as $l_{ij} = -a_{ij}$ when $i \neq j$ and $l_{ii} = \sum_{j=1}^{n} a_{ij}$. For an undirected and connected graph, 0 is a simple eigenvalue of the Laplacian matrix L with the eigenvector $\mathbf{1}_n$, and all the other eigenvalues are positive.

2.2. Useful lemma

Lemma 1 ([17].). *For any vectors $x, y \in \mathbf{R}^n$ and one positive definite matrix $H \in \mathbf{R}^{n\times n}$, the following inequality holds*

$$2x^T y \leq x^T H x + y^T H^{-1} y. \tag{1}$$

2.3. Problem formulation

In this subsection, let us consider a smart grid of n generators. Let P_i denotes the active power generated by the generator i, where each generator has a generation cost $C_i(P_i)$, which is only known by generator i. The total generation cost $C(P)$ is the sum of the all local objective functions, i.e., $C(P) = \sum_{i=1}^{n} C_i(P_i)$, where $C_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i$. The target of economic dispatch is to minimize the total generation cost while satisfying the balance between demand and supply, i.e.

$$\begin{cases} \mathrm{Min} C(P) = \sum_{i=1}^{n} \left(\alpha_i P_i^2 + \beta_i P_i + \gamma_i\right), \\ s.t. \sum_{i=1}^{n} P_i = P_D, \end{cases} \tag{2}$$

where $P = [P_1, P_2, \ldots, P_n]^T \in \mathbf{R}^n$ is a decision vector; α_i, β_i and γ_i represent the cost coefficients; P_D is a global constraint variable.

The above economic dispatch problem can be solved by the conventional centralized method, that is, the Lagrange multiplier method. The optimal Lagrangian multiplier is obtained, i.e., $\lambda^* = \frac{P_D + \sum_{i=1}^{n} \frac{\beta_i}{2\alpha_i}}{\sum_{i=1}^{n} \frac{1}{2\alpha_i}}$, and the optimal solution $P_i^* = \frac{\lambda^* - \beta_i}{2\alpha_i}$. So centralized algorithm can solve this problem easily when all parameters can be processed centrally. The purposes of this work is to provide a distributed discrete-time algorithm to solve the above economic dispatch problem via event-triggered communication mechanism, which can reducing the communication burden on large scale networks.

Assumption 1. The objective function $C(P)$ is twice continuously differentiable, strongly convex with convexity parameters $\Theta \geq \sigma > 0$ such that $\sigma I_n \leq \nabla^2 C(P) \leq \Theta I_n$.

Assumption 2. The communication topology is undirected and connected.

Assumption 3. The communication environment is secure, reliable and without delay and disturbance in communication process.

2.4. Event-triggered communication mechanism

Before proposing distributed event-triggered algorithm, we first make a brief introduction to event-triggered communication mechanism. $x_i(k)(i = 1, 2, \ldots, n, k = 0, 1, 2, \ldots)$ represents the transportable variable in the communication network. $\hat{x}_i(k) = x_i(k^i_{t_i}), k^i_{t_i} \leq k < k^i_{t_i+1}$ represents the information to be transmitted at event-triggered time instant $k^i_{t_i}$. $F_i(k, x_i(k))$ denotes trigger function for node i. Only when the trigger function $F_i(k, x_i(k)) > 0$ is satisfied, the node i transmits its current information to its neighboring nodes immediately. The sequence of event-triggered discrete instants $\{k^i_{t_i}\}$ for node i is defined iteratively as $k^i_{t_i+1} = \inf\{k : k > k^i_{t_i}, F_i(k, x_i(k)) > 0\}$ $(t_i = 0, 1, 2, \ldots)$. It is worth noting that the Zeno behavior is naturally excluded because of the discrete-time scheme.

3. Algorithm design and convergence analysis

3.1. Distributed discrete-time event-triggered algorithm

Decentralized trigger functions are defined as

$$F_i(k, x_i(k)) = e_i^2(k) - \gamma\alpha^{\lfloor\omega k\rfloor}, \tag{3}$$

$$G_i(k, y_i(k)) = \varsigma_i^2(k) - \mu\beta^{\lfloor\sigma k\rfloor}, \tag{4}$$

where $e_i(k) = \hat{x}_i(k) - x_i(k)$; $\hat{x}_i(k) = x_i(k_{t_i}^i), k_{t_i}^i \le k < k_{t_i+1}^i$; $\varsigma_i(k) = \hat{y}_i(k) - y_i(k)$; $\hat{y}_i(k) = y_i(k_{s_i}^i), k_{s_i}^i \le k < k_{s_i+1}^i$; α, β, ω, $\sigma \in (0, 1)$ and γ, $\mu > 0$ are design parameters.

Based on the above decentralized trigger functions (3)-(4) and Assumptions 1–3, the following distributed discrete-time event-triggered algorithm is proposed to solve economic dispatch problem (2)

$$\begin{cases} y_i(k+1) = y_i(k) + \varepsilon \sum\limits_{j\in N_i} a_{ij}\big(\hat{x}_i(k) - \hat{x}_j(k)\big), \\ P_i(k) = \sum\limits_{j\in N_i} a_{ij}\big(\hat{y}_j(k) - \hat{y}_i(k)\big) + P_i(0), \\ x_i(k) = \dfrac{\partial C_i(P_i(k))}{\partial P_i(k)}, \end{cases} \tag{5}$$

where $\varepsilon > 0$ is constant. We see that the neighboring nodes need to exchange the information of auxiliary variables $x_i(k)$ and $y_i(k)$.

Suppose that there is one node knows global constraint variable P_D, it can start from P_D while the others start from 0,then $\sum_{i=1}^n P_i(0) = P_D$. For undirected and connected graph G, $\sum_{i=1}^n P_i(k) = \sum_{i=1}^n \big(\sum\limits_{j\in N_i} a_{ij}\big(\hat{y}_j(k) - \hat{y}_i(k)\big)\big) + \sum_{i=1}^n P_i(0) = P_D$. Therefore, the economic dispatch problem (2) can be transformed into the following unconstrained convex optimization problem of variable $\hat{y}(k)$

$$\mathrm{Min}C(\hat{y}(k)) = \sum_{i=1}^n C_i\Big(\sum_{j\in N_i} a_{ij}\big(\hat{y}_j(k) - \hat{y}_i(k)\big) + P_i(0)\Big). \tag{6}$$

Remark 1. Note that the gradient of $C(P)$ can be described as $\nabla C(P) = x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T \in \mathbf{R}^n$. According to algorithm (5), we get the gradient of $C(\hat{y})$ as $\nabla C(\hat{y}(k)) = -L^T\nabla C(P) = -L^T x(k)$, where $\hat{y}(k) = [\hat{y}_1(k), \hat{y}_2(k), \ldots, \hat{y}_n(k)]^T \in \mathbf{R}^n$. It is obvious that the Hessian matrix of $C(\hat{y})$ satisfies $\nabla^2 C(\hat{y}) = L^T\nabla^2 C(P)L$. For the unconstrained convex optimization problem (6), the optimal value y^* can be obtained when $\nabla C(\hat{y}(k)) = \mathbf{0}_n (k \to \infty)$.

3.2. Convergence analysis

Theorem 1. *Under Assumptions 1–3, if the parameter ε satisfying $0 < \varepsilon < \frac{1}{1+2\Theta\hat{\lambda}}$, the distributed discrete-time event-triggered algorithm (5) solves economic dispatch problem (2)with trigger functions (3) and (4), and the Zeno behavior is naturally excluded, where $\hat{\lambda} = \lambda_{\max}^2(L)$, $\lambda_{\max}(L)$ denotes the largest eigenvalue of the Laplacian matrix L, and $\Theta > 0$ is the convexity parameter in Assumption 1.*

Proof. For the convenience of proof, the algorithm (5) can be written in matrix form

$$\begin{cases} y(k+1) = y(k) + \varepsilon L\hat{x}(k), \\ P(k) = -L\hat{y}(k) + P(0), \\ x(k) = \nabla C(P(k)), \end{cases} \tag{7}$$

where $\hat{x}(k) = x(k) + e(k)$, $\hat{y}(k) = y(k) + \varsigma(k)$, $\hat{x}(k) = [\hat{x}_1(k), \hat{x}_2(k), \ldots, \hat{x}_n(k)]^T$, $y(k) = [y_1(k), y_2(k), \ldots, y_n(k)]^T$, $P(k) = [P_1(k), P_2(k), \ldots, P_n(k)]^T$. □

Suppose that the unique optimal solution for the unconstrained convex optimization problem (6) is denoted by y^*, then $C(y^*) \le C(\hat{y}(k))$. The discrete-time Lyapunov function is selected as

$$V(k) = C(\hat{y}(k)) - C(y^*). \tag{8}$$

The difference of $V(k)$ is given by $\triangle V(k+1) = V(k+1) - V(k) = C\big(\hat{y}(k+1)\big) - C\big(\hat{y}(k)\big)$. It follows from the strong convexity property [3] that

$$\begin{aligned} &C\big(\hat{y}(k+1)\big) - C\big(\hat{y}(k)\big) \\ &= \nabla^T C(\hat{y}(k))\big(\hat{y}(k+1) - \hat{y}(k)\big) \\ &+ \frac{1}{2}\big(\hat{y}(k+1) - \hat{y}(k)\big)^T\nabla^2 C(\bar{y})\big(\hat{y}(k+1) - \hat{y}(k)\big), \end{aligned} \tag{9}$$

where $\bar{y} = \hat{y}(k) + \theta\big(\hat{y}(k+1) - \hat{y}(k)\big)$ with $\theta \in (0, 1)$. In addition, according to algorithm (7), we have

$$\begin{aligned} \triangle V(k+1) = &-x^T(k)L\Big(\varepsilon L(x(k) + e(k)) + \varsigma(k+1) - \varsigma(k)\Big) \\ &+ \frac{1}{2}\Big(\varepsilon L(x(k) + e(k)) + \varsigma(k+1) - \varsigma(k)\Big)^T L^T\nabla^2 C(\bar{P}) \\ &\times L\Big(\varepsilon L(x(k) + e(k)) + \varsigma(k+1) - \varsigma(k)\Big). \end{aligned} \tag{10}$$

For undirected and connected graph G, one gets $L^T = L$. Moreover, by applying Lemma 1, one has

$$\begin{aligned} \triangle V(k+1) \le &-\varepsilon x^T(k)LL^T x(k) + \frac{\varepsilon^2}{2}x^T(k)LL^T x(k) + \frac{\hat{\lambda}}{2}e^T(k)e(k) \\ &+ \frac{\varepsilon^2}{2}x^T(k)LL^T x(k) + \frac{1}{2\varepsilon^2}\parallel \varsigma(k+1) - \varsigma(k)\parallel^2 \\ &+ \frac{\Theta\hat{\lambda}}{2}\parallel \varepsilon L^T(x(k) + e(k)) + \varsigma(k+1) - \varsigma(k)\parallel^2 \\ \le &-\big(\varepsilon - (1 + 2\Theta\hat{\lambda})\varepsilon^2\big)x^T(k)LL^T x(k) + \Big(\frac{\hat{\lambda}}{2} + 2\Theta\hat{\lambda}^2\varepsilon^2\Big) \\ &\times e^T(k)e(k) + \Big(\frac{1}{2\varepsilon^2} + \Theta\hat{\lambda}\Big)\parallel \varsigma(k+1) - \varsigma(k)\parallel^2, \end{aligned} \tag{11}$$

where $\hat{\lambda} = \lambda_{\max}^2(L)$, $\lambda_{\max}(L)$ denotes the largest eigenvalue of the Laplacian matrix L, and $\Theta > 0$ is the convexity parameter in Assumption 1. Moreover, according to trigger functions (3) and (4), we have $e^T(k)e(k) \le n\gamma\alpha^{\lfloor\omega k\rfloor}$ and $\parallel \varsigma(k+1) - \varsigma(k)\parallel^2 \le 4n\mu\beta^{\lfloor\sigma k\rfloor}$. Substituting these two inequalities back into (11) yields

$$\begin{aligned} \triangle V(k+1) &\le -\xi x^T(k)LL^T x(k) + \mu_1\alpha^{\lfloor\omega k\rfloor} + \mu_2\beta^{\lfloor\sigma k\rfloor}, \\ &= -\xi \parallel \nabla C(\hat{y}(k))\parallel^2 + \mu_1\alpha^{\lfloor\omega k\rfloor} + \mu_2\beta^{\lfloor\sigma k\rfloor}, \end{aligned} \tag{12}$$

where $\xi = \varepsilon - (1 + 2\Theta\hat{\lambda})\varepsilon^2 > 0$, $\mu_1 = n\gamma\big(\frac{\hat{\lambda}}{2} + 2\Theta\hat{\lambda}^2\varepsilon^2\big)$ and $\mu_2 = 4n\mu\big(\frac{1}{2\varepsilon^2} + \Theta\hat{\lambda}\big)$.

According to the strong convexity property [3] and Ref. [4], it is easy to get $\parallel \nabla C(\hat{y}(k))\parallel^2 \ge \frac{1}{2}\sigma\lambda_2^2\big(C(\hat{y}(k)) - C(y^*)\big) = \frac{1}{2}\sigma\lambda_2^2 V(k)$, where σ is the convexity parameter of function C in Assumption 1, and λ_2 represents the smallest positive eigenvalue of the Laplacian matrix L.

In addition, $\lfloor\omega k\rfloor$ denotes the largest integer no more than ωk with $\omega \in (0, 1)$, thus, $\alpha^{\lfloor\omega k\rfloor} \le \alpha^{\omega k - 1} = \frac{1}{\alpha}(\alpha^\omega)^k$. Meanwhile, $\beta^{\lfloor\sigma k\rfloor} \le \beta^{\sigma k - 1} = \frac{1}{\beta}(\beta^\sigma)^k$. Then, one gets

$$\triangle V(k+1) \le -\frac{1}{2}\xi\sigma\lambda_2^2 V(k) + \frac{\mu_1}{\alpha}(\alpha^\omega)^k + \frac{\mu_2}{\beta}(\beta^\sigma)^k. \tag{13}$$

Let $\nu = 1 - \frac{1}{2}\xi\sigma\lambda_2^2 \in (0, 1)$, one has

$$\begin{aligned} V(k) &\le \nu V(k-1) + \frac{\mu_1}{\alpha}(\alpha^\omega)^{k-1} + \frac{\mu_2}{\beta}(\beta^\sigma)^{k-1} \\ &\ \ \vdots \\ &\le \nu^k V(0) + \frac{\mu_1}{\alpha}\sum_{i=1}^k \nu^{k-i}(\alpha^\omega)^{i-1} + \frac{\mu_2}{\beta}\sum_{i=1}^k \nu^{k-i}(\beta^\sigma)^{i-1} \\ &= \nu^k V(0) + \frac{\mu_1}{\alpha}\frac{\nu^k - (\alpha^\omega)^k}{\nu - (\alpha^\omega)} + \frac{\mu_2}{\beta}\frac{\nu^k - (\beta^\sigma)^k}{\nu - (\beta^\sigma)} \\ &= \Big(V(0) - \frac{\mu_1}{\alpha(\alpha^\omega - \nu)} - \frac{\mu_2}{\beta(\beta^\sigma - \nu)}\Big)\nu^k \\ &\quad + \frac{\mu_1}{\alpha(\alpha^\omega - \nu)}(\alpha^\omega)^k + \frac{\mu_2}{\beta(\beta^\sigma - \nu)}(\beta^\sigma)^k. \end{aligned} \tag{14}$$

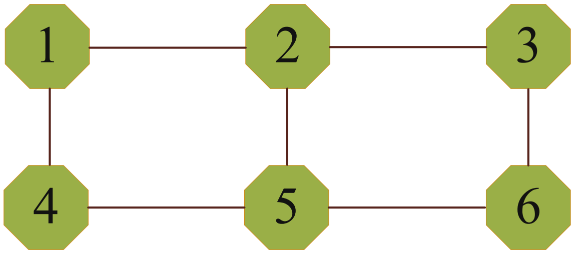

Fig. 1. Communication network.

Table 1
Parameters α_i, β_i, γ_i of C_i.

C_i	1	2	3	4	5	6
α_i	0.096	0.072	0.105	0.082	0.078	0.090
β_i	1.22	3.41	2.53	4.02	2.90	2.72
γ_i	51	31	78	42	57	49

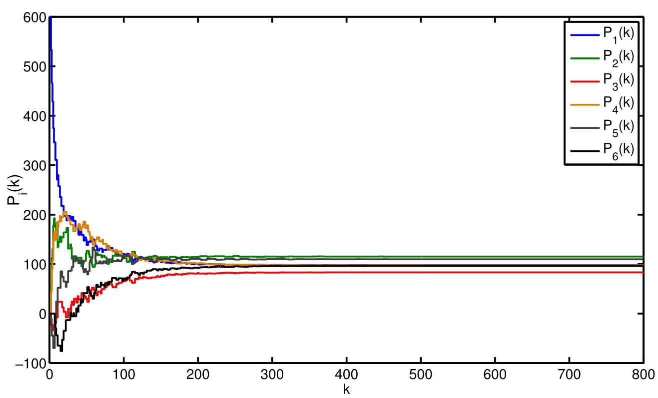

Fig. 2. Trajectories of $P_i(t)$.

Because ν, α^{ω}, β^{σ} $\in$ (0, 1), $V(k)$ exponentially converge to 0. Thus, the function $C(\hat{y}(k))$ exponentially converges to $C(y^*)$. In other words, the optimal value y^* can be obtained by the designed distributed discrete-time event-triggered algorithm (5). In that way, the optimal value P^* of the original economic dispatch problem (2) can also be obtained, i.e., $P^* = -Ly^* + P(0)$. Moreover, the Zeno behavior is naturally excluded under discrete iterative scheme. ■

4. Simulation results

In this section, an example is investigated to illustrate the distributed discrete-time event-triggered algorithm proposed in this paper. The corresponding communication network is a undirected and connected graph as shown in Fig. 1. The parameters of each cost function $C_i(P_i)$ are listed in Table 1([22]).

Case 1: Algorithm Simulation

The parameters in our algorithm are set as follows: $P_D = 600$, $\varepsilon = 0.1$, $\gamma = 50$, $\mu = 100$, $\alpha = 0.85$, $\beta = 0.79$, $\omega = 0.19$, $\sigma = 0.13$. The initial values are set as $P(0) = [P_D, 0, 0, 0, 0, 0]^T$ and $y(0) = [0, 0, 0, 0, 0, 0]^T$. For the communication topology in Fig. 1, $\lambda_2 = 1$ and $\lambda_{\max} = 5$. For the $C(P) = \sum_{i=1}^{n} C_i(P_i)$, two convexity parameters are chosen as $\Theta = 0.105$ and $\sigma = 0.072$. The iterative number of each generator is 800. The simulation results under our algorithm (5) are shown in Figs. 2–7.

Fig. 2 shows that $P_i(k), i = 1, 2, \ldots, 6$, converge to their corresponding optimal values: $P_1^* = 97.9219, P_2^* = 115.3539, P_3^* = 83.2905, P_4^* = 97.5672, P_5^* = 109.7500$ and $P_6^* = 96.1165$.

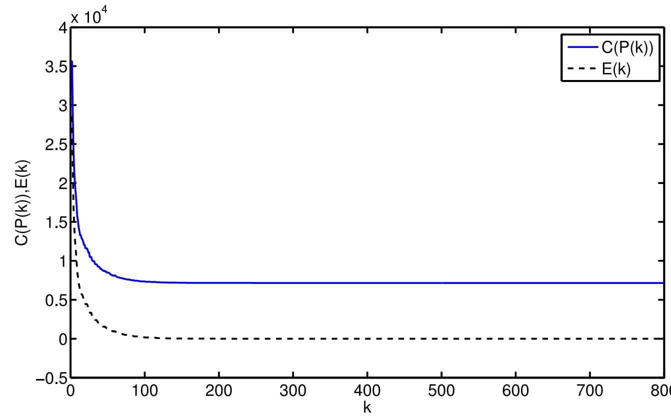

Fig. 3. Trajectories of $C(P(k))$ and $E(k)$.

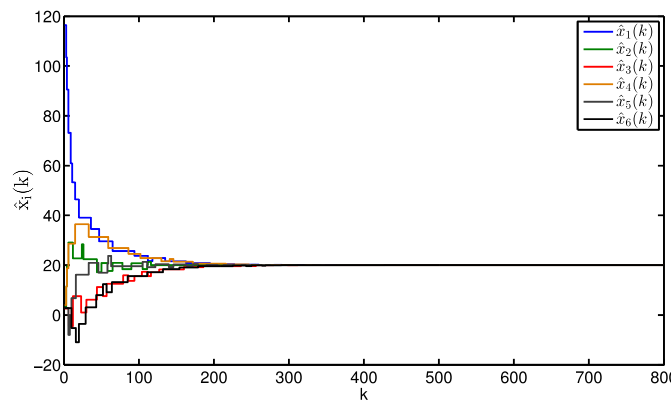

Fig. 4. Trajectories of $\hat{x}(k)$.

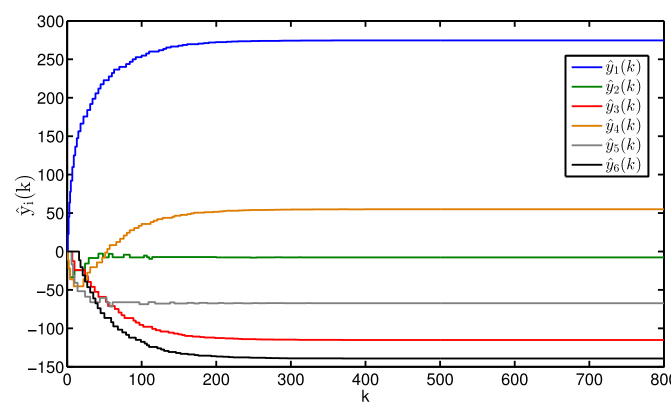

Fig. 5. Trajectories of $\hat{y}(k)$.

Fig. 3 shows the trajectories of the total cost function $C(P(k))$ and error function $E(k)$, where $E(k) = C(P(k)) - C(P^*)$, $C(P^*) = 7162.03705$. Figs. 4 and 5 show the trajectories of $\hat{x}(k)$ and $\hat{y}(k)$, which are transmitted in the communication network. They are also the values at the event-triggered instant. The event instants of each generator, i.e. $\{k_{t_i}^i\}, i = 1, 2, \ldots, 6$ and $\{k_{s_i}^i\}, i = 1, 2, \ldots, 6$ are shown in Figs. 6 and 7.

Moreover, iterations, events and the ratios of communication to iteration for all generators are listed in Table 2. The information transmission in the network does not occur at every iteration time, only at the moment of the events. The maximum ratio is only 9.75%, which shows that event-triggered communication mecha-

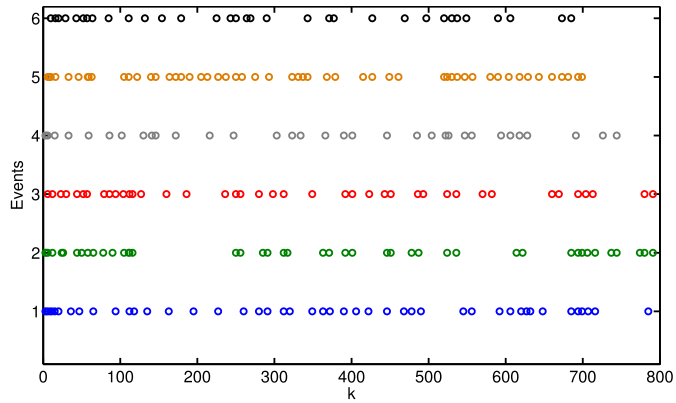

Fig. 6. Events of 6 generators with variable $x(k)$.

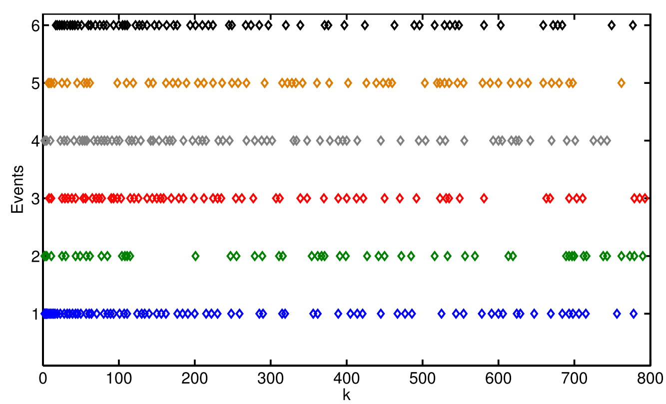

Fig. 7. Events of 6 generators with variable $y(k)$.

Table 2
The number of communication events.

Generators	1	2	3	4	5	6
Iterations	800	800	800	800	800	800
Events-$x(k)$	46	44	41	34	53	33
Ratios(%)	5.75	5.5	5.125	4.25	6.625	4.125
Events-$y(k)$	78	52	61	72	59	64
Ratios(%)	9.75	6.5	7.625	9	7.375	8

nism can reduce the burden of communication and save the energy of systems.

From the above simulation results, the optimal active powers generated by generators can be obtained based on distributed discrete-time event-triggered algorithm (5), and the number of communication is significantly reduced based on the event-triggered communication mechanism, which is very beneficial to save network bandwidth.

Case 2: Algorithm Comparison

The main advantage of the algorithm proposed in this paper is to save communication, and it does not need to exchange information in each iteration process. Based on event-triggered communication mechanism, information exchange only occurs when the trigger function is triggered. A excellent consensus based algorithm [19] was proposed to solve EDP in a distributed fashion. For the convenience of statement, we use A1 and A2 to respectively denote the distributed discrete-time algorithm [19] and our distributed discrete-time event-triggered algorithm (5). Algorithm

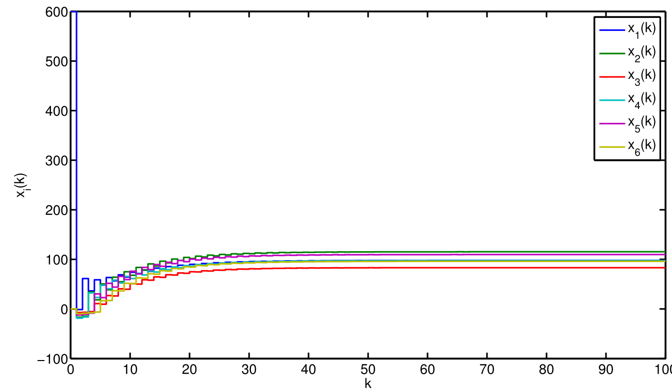

Fig. 8. Trajectories of $x_i(t)$.

Table 3
The number of communication.

Node	1	2	3	4	5	6
Algorithm A1	200	200	200	200	200	200
Algorithm A2	124	96	102	106	112	97

A1 is described as

$$\begin{cases} \lambda_i(k+1) = \sum\limits_{j \in N_i^+} p_{ij}\lambda_j(k) + \epsilon y_i(k), \\ x_i(k+1) = b_i\lambda_i(k+1) + a_i, \\ y_i(k+1) = \sum\limits_{j \in N_i^+} q_{ij}y_j(k) - (x_i(k+1) - x_i(k)), \end{cases}$$

where $\epsilon = 0.02$, $a_i = -\frac{\beta_i}{2\alpha_i}$, $b_i = \frac{1}{2\alpha_i}$; p_i and q_i see [19] for details. Here, $x_i(k)$ denotes the active power generated. That is, $P_i(k)$ in our algorithm. Fig. 8 shows that the optimal values are obtained only after 100 iterations by using algorithm A1. However, the variables x and y need to be transmitted in each iteration process. Therefore, the number of communication of each node are 200. It can be seen from Table 3 that the number of communication by using our distributed discrete-time event-triggered algorithm (5) (Algorithm A2) are less than that of Algorithm A1. It is undeniable that Algorithm A1 is still an excellent algorithm.

5. Conclusion

The target of economic dispatch in smart grids is to minimize the total generation cost while satisfying the balance between demand and supply. In this paper, a distributed discrete-time event-triggered algorithm is designed to solve the economic dispatch problem. The optimal value of the total generation cost is obtained with discrete exponential convergence rate. The designed algorithm has the advantages to reduce the communication and save the energy of systems, and naturally exclude the Zeno behavior. The simulation results have validated the correctness of the theoretical results and shown the advantages of the algorithm. Considering the fast convergence of the algorithm, the finite-step or fixed-step iterative algorithm will be further studied in the future work. Moreover, under the unreliable communication environment, how to design a fast iterative algorithm is also very meaningful.

Declaration of Competing Interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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