Chapter 10: Rotation

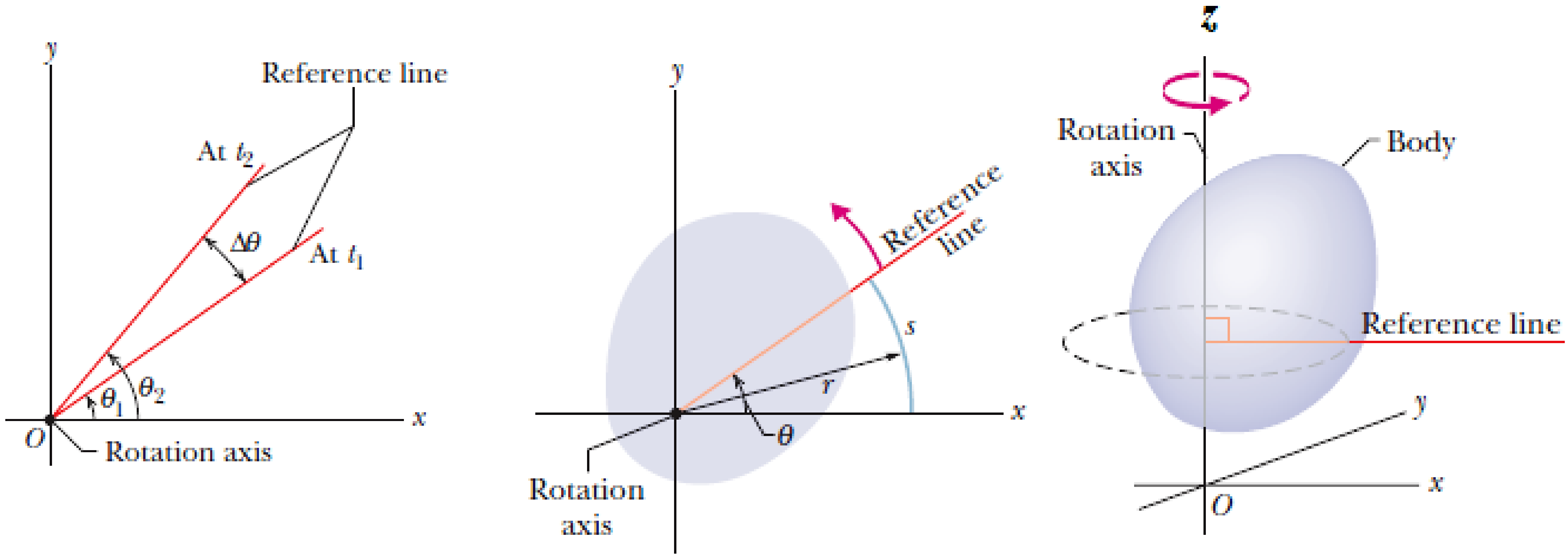
- ✓ Angular Position, Velocity and Acceleration
- ✓ Rotational Kinematics
- ✓ Kinetic Energy of Rotation
- ✓ Rotational Inertia
- ✓ Torque
- ✓ Energy Consideration in Rotational Motion

Chapter 10: Rotation

Session 20:

- ✓ **Angular Position, Velocity and Acceleration**
- ✓ **Rotational Kinematics**
- ✓ **Examples**

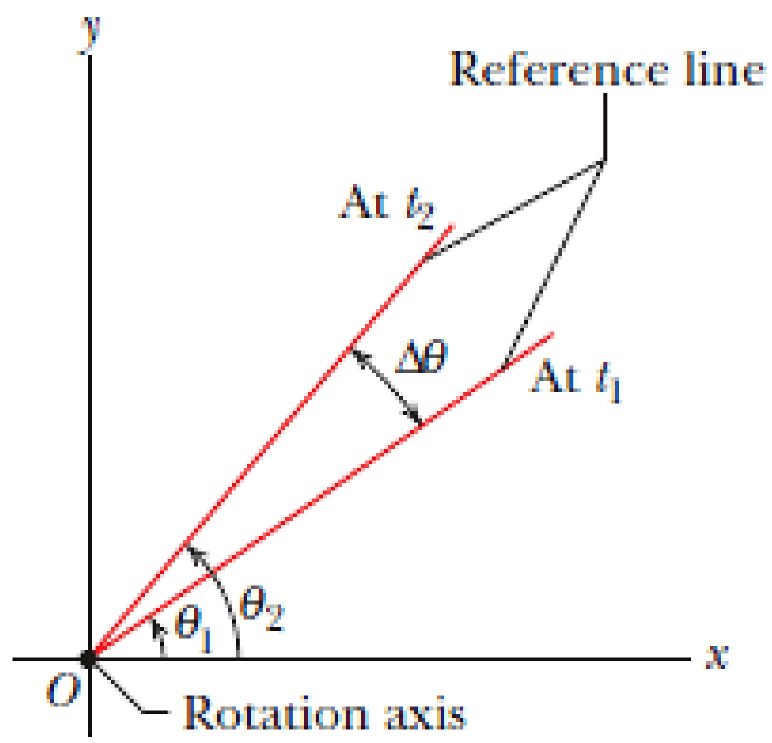
Angular Position and Displacement

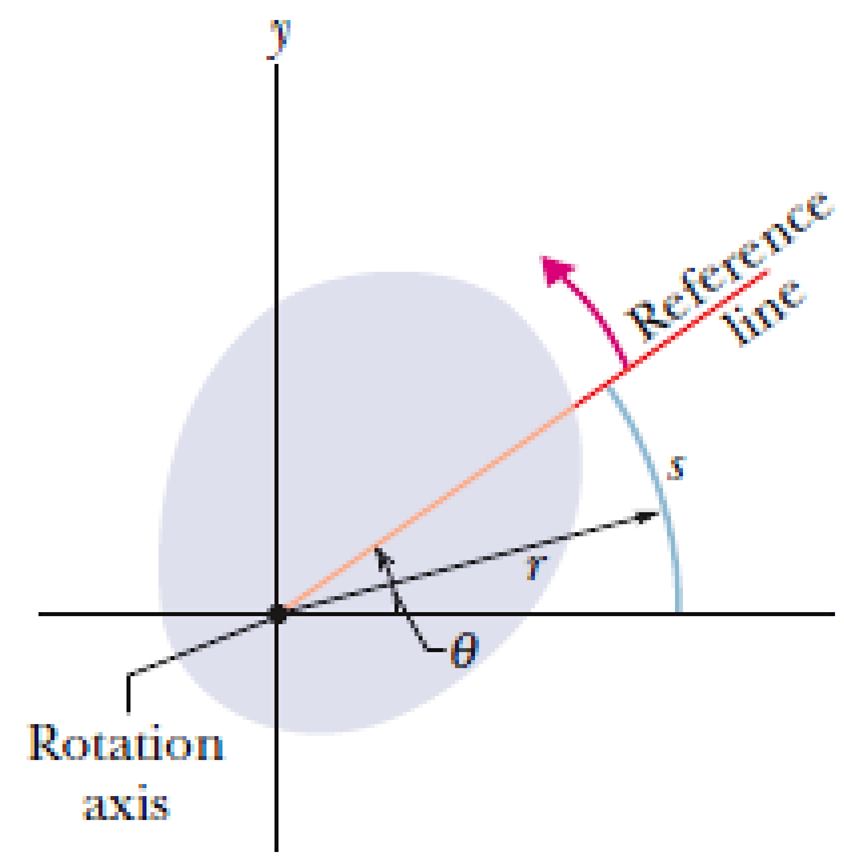


Angular Position : $\boxed{\theta = \dfrac{s}{r}}$ $\quad 1\,rad = 1\times\left(\dfrac{360^\circ}{2\pi}\right) = 57.3^\circ$

Angular Displacement : $\boxed{\Delta\theta = \theta_2 - \theta_1}$

Angular Speed and Acceleration





Angular speed : $$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$ $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$ **rad/s, rev/min**

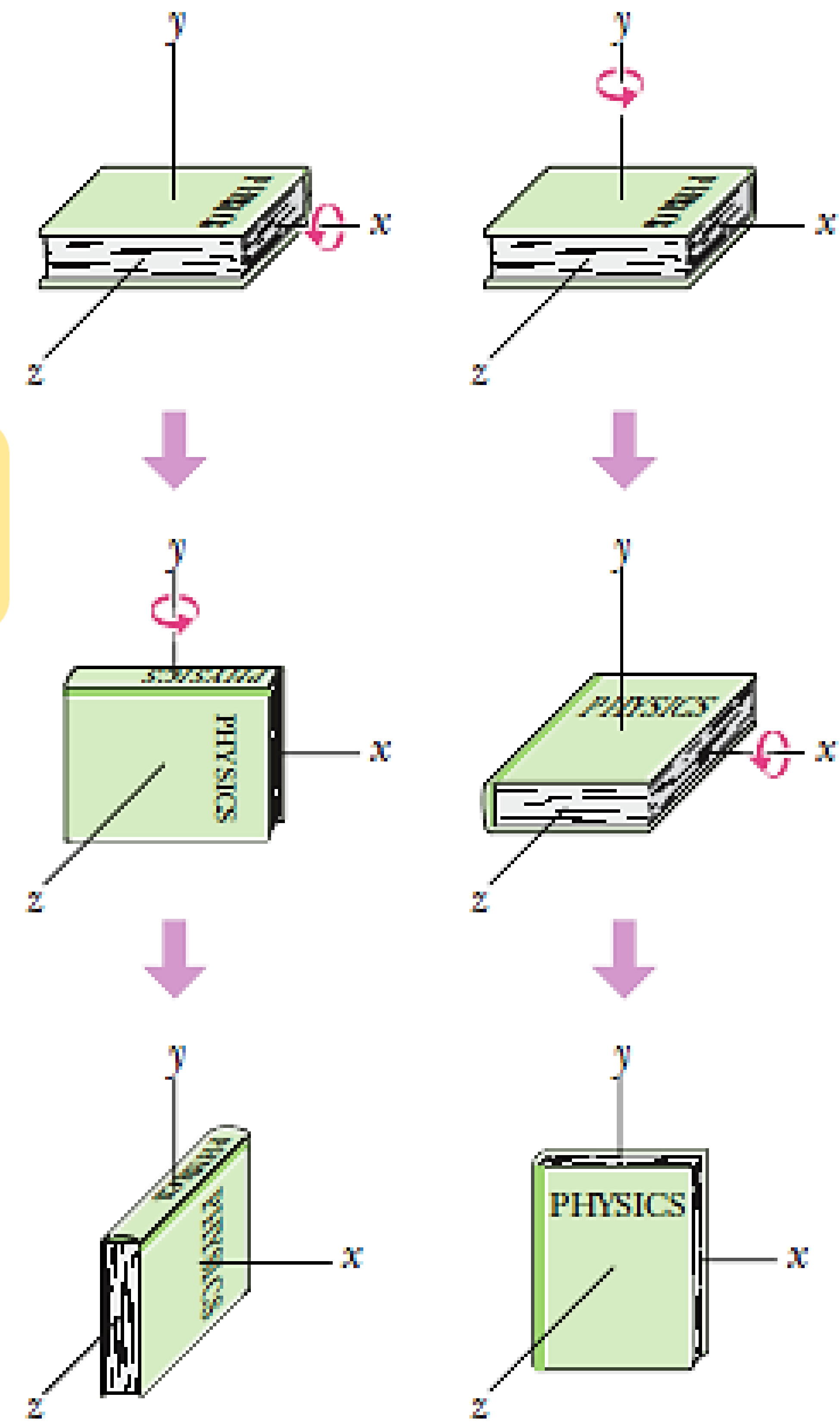
ω > 0 if θ is increasing (**counterclockwise**), **ω < 0** if θ is decreasing (**clockwise**)

Angular Acceleration : $$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$ $$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$ **rad/s²**

α > 0 if an object **rotating counterclockwise** (clockwise) is **speeding up** (slowing down)

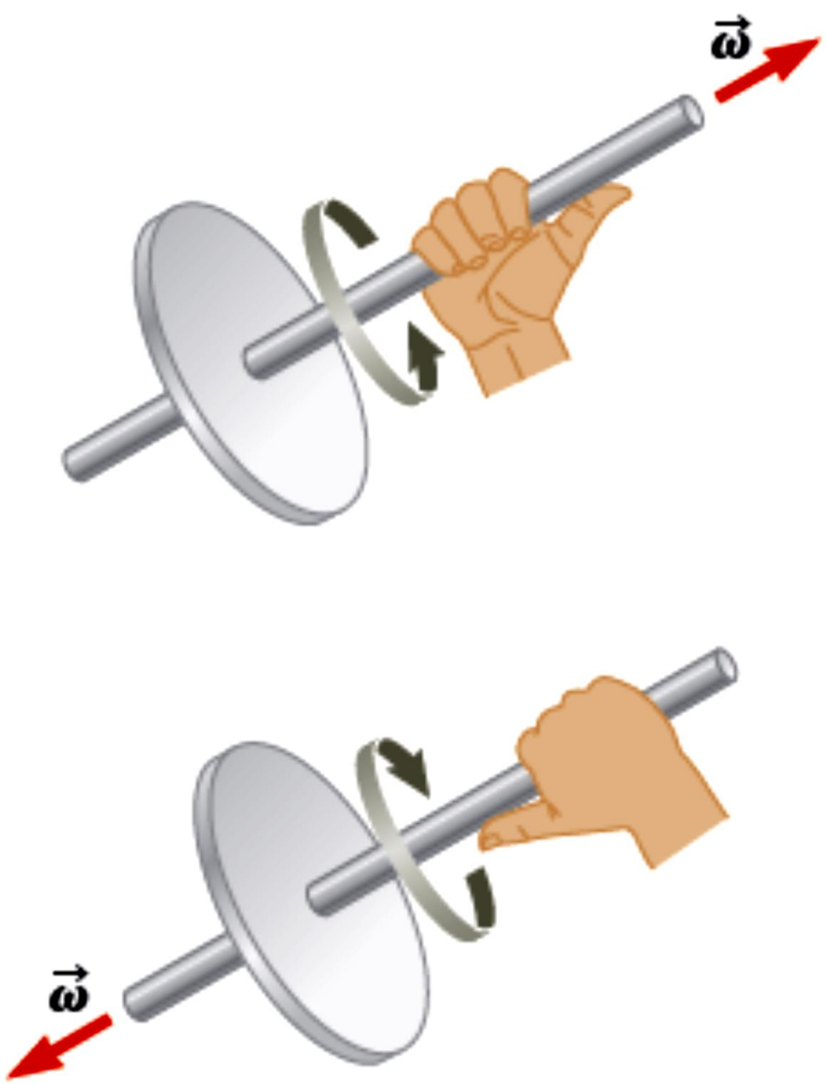
Angular Speed and Acceleration

Angular displacements cannot be treated as vectors



Directions of $\vec{\omega}$ and $\vec{\alpha}$ are along the axis of rotation

The directions are actually given by the right-hand rule.



Rotational Kinematics

Translational Motion: (x, v, a)

$$v = \frac{dx}{dt} \Rightarrow x_f - x_i = \int_{t_i}^{t_f} v\, dt$$

$$a = \frac{dv}{dt} \Rightarrow v_f - v_i = \int_{t_i}^{t_f} a\, dt$$

if $a =$ constant

$$\begin{cases} v(t) = v_0 + a\,t \\ x = x_0 + v_0\,t + \dfrac{1}{2} a\,t^2 \\ x = x_0 + v_{avg} t \\ v_{avg} = \dfrac{v_0 + v(t)}{2} \\ v^2 - v_0{}^2 = 2a(x - x_0) \end{cases}$$

Rotational Motion: (θ, ω, α)

$$\omega = \frac{d\theta}{dt} \Rightarrow \theta_f - \theta_i = \int_{t_i}^{t_f} \omega\, dt$$

$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega_f - \omega_i = \int_{t_i}^{t_f} \alpha\, dt$$

if $\alpha =$ constant

$$\begin{cases} \omega(t) = \omega_0 + \alpha\,t \\ \theta = \theta_0 + \omega_0\,t + \dfrac{1}{2} \alpha\,t^2 \\ \theta = \theta_0 + \omega_{avg} t \\ \omega_{avg} = \dfrac{\omega_0 + \omega(t)}{2} \\ \omega^2 - \omega_0{}^2 = 2\alpha(\theta - \theta_0) \end{cases}$$

Ex 1: A wheel rotates with a constant angular acceleration of **3.5 rad/s²**. (a) If the angular speed of the wheel is **2 rad/s** at **$t_i = 0$**, through what angular displacement does the wheel rotate in **2 s**? (b) Through how many revolutions has the wheel turned during this time interval? (c) What is the angular speed of the wheel at **t = 2 s**?

$$\alpha = 3.5\ (rad/s^2) = \text{constant} \Rightarrow \Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (2)(2) + \frac{1}{2}(3.5)(2)^2 = 11\ rad = 11 \times \left(\frac{360^\circ}{2\pi\ rad}\right) = 630^\circ$$

$$\Delta\theta = 630^\circ \times \left(\frac{1\ rev}{360^\circ}\right) = 1.75\ rev$$

$$\omega = \omega_0 + \alpha t \Rightarrow \omega = 2 + (3.5)(2) = 9\ (rad/s)$$

Ex 2: (Problem 10. 12 Halliday)

The angular speed of an automobile engine is increased at a constant rate from **1200 rev/min** to **3000 rev/min** in **12 s.** (a) What is its angular acceleration in revolutions per minute-squared? (b) How many revolutions does the engine make during this **12** s interval?

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \boxed{\alpha = \frac{\omega - \omega_0}{t} = \frac{3000 - 1200}{12\left(\frac{1\ \text{min}}{60}\right)} = 9000\ (rev/\text{min}^2)}$$

$$\Delta\theta = \omega_{avg} t = \left(\frac{\omega_0 + \omega}{2}\right)t \quad \Rightarrow \quad \boxed{\Delta\theta = \left(\frac{1200 + 3000}{2}\right)\left(\frac{12}{60}\right) = 420\ rev}$$

Ex 3: (Problem 10. 8 Halliday)

The angular acceleration of a wheel is $\alpha = 6t^4 - 4t^2$ with **α** in radians per second-squared and **t** in seconds. At time **t = 0**, the wheel has an angular velocity of **+2 rad/s** and an angular position of **+1 rad**. Write expressions for (a) the angular velocity (rad/s) and (b) the angular position (rad) as functions of time (s).

$$\omega_f - \omega_i = \int_{t_i}^{t_f} \alpha\, dt \quad \Rightarrow \quad \omega(t) - 2 = \int_0^t (6t^4 - 4t^2)\,dt = \frac{6}{5}t^5 - \frac{4}{3}t^3$$

$$\boxed{\omega(t) = \frac{6}{5}t^5 - \frac{4}{3}t^3 + 2}$$

$$\theta_f - \theta_i = \int_{t_i}^{t_f} \omega\, dt \quad \Rightarrow \quad \theta(t) - 1 = \int_0^t \left(\frac{6}{5}t^5 - \frac{4}{3}t^3 + 2\right)dt = \frac{t^6}{5} - \frac{t^4}{3} + 2t$$

$$\boxed{\theta(t) = \frac{t^6}{5} - \frac{t^4}{3} + 2t + 1}$$

Relationship Between Angular and Linear Quantities

Position:

$$\boxed{s = r\,\theta}$$

Velocity:

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$$

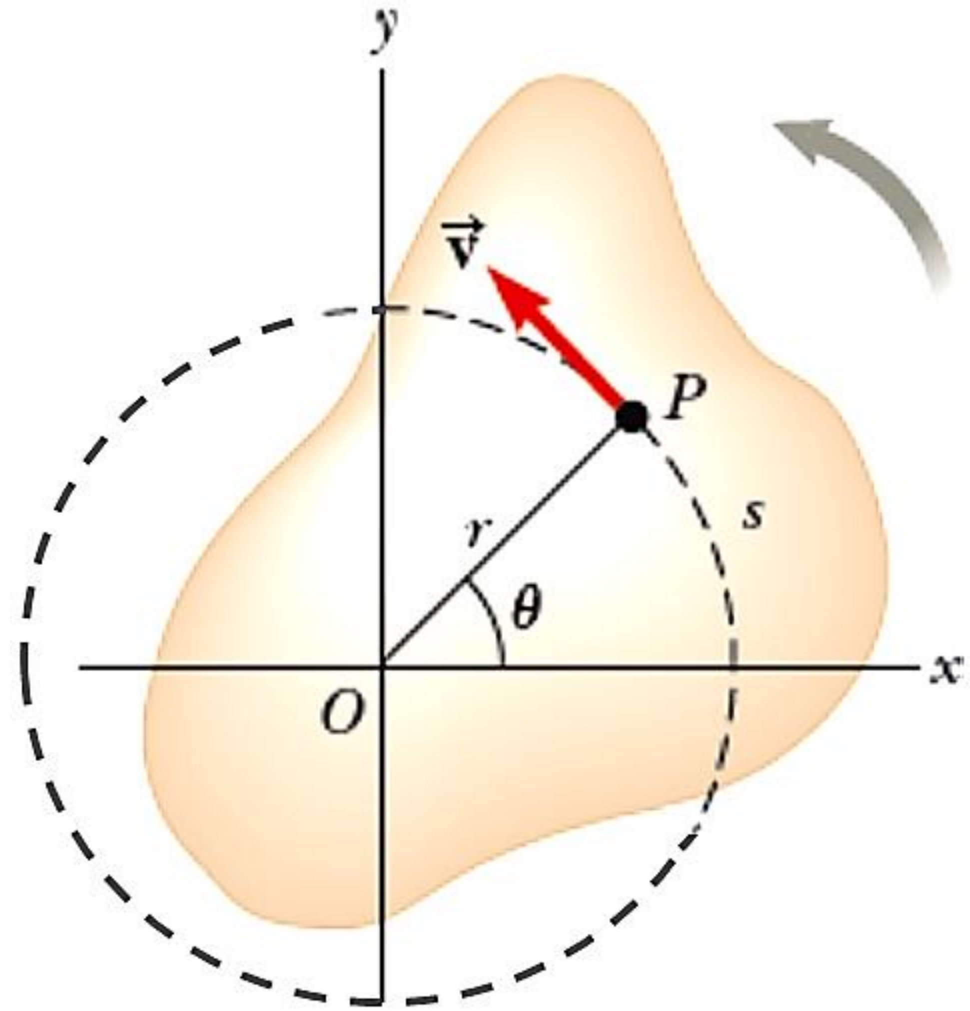
$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}}$$

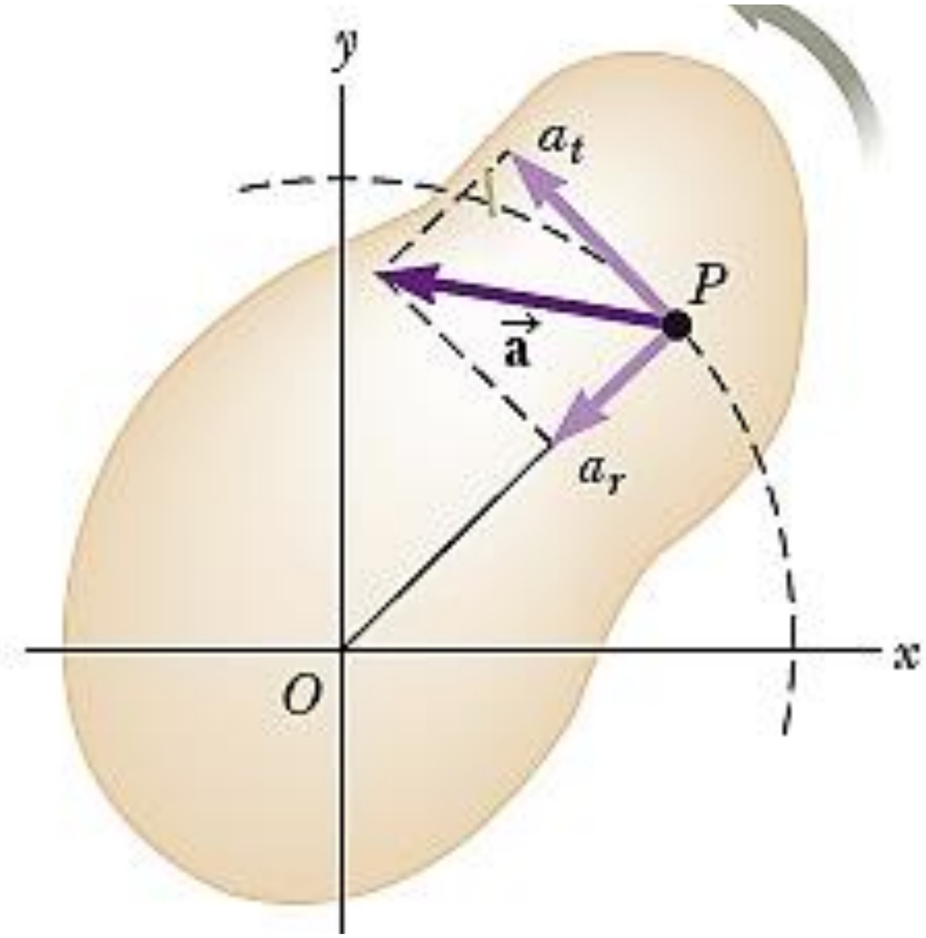
Acceleration:

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow \boxed{a_t = r\alpha}$$

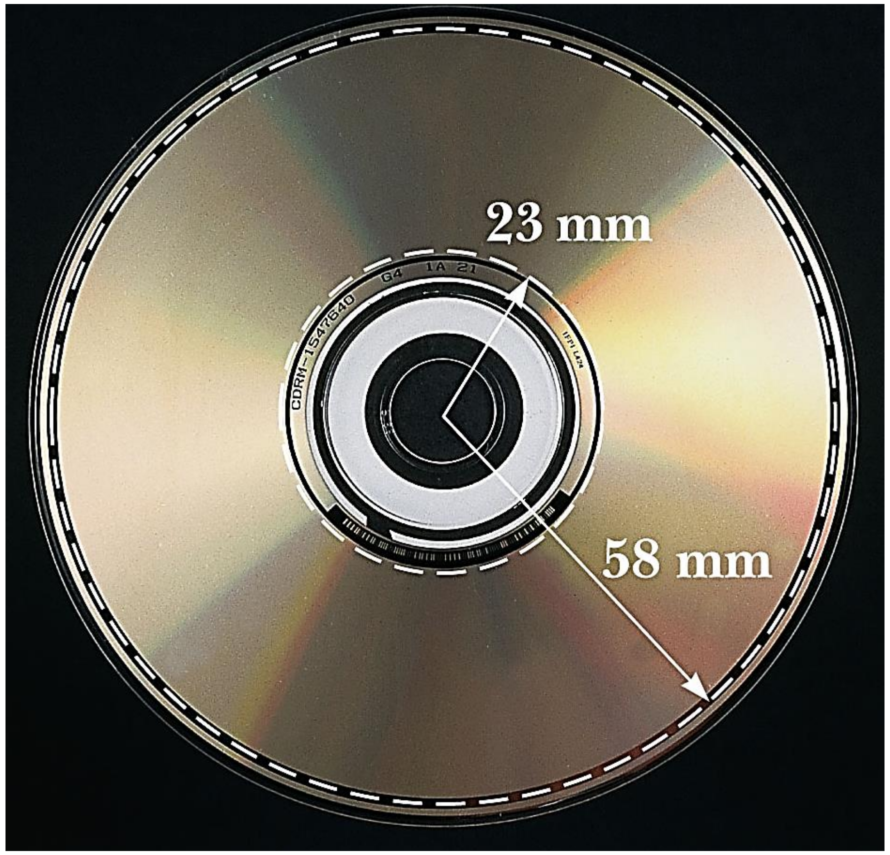
$$\boxed{a_r = \frac{v^2}{r} = r\omega^2}$$

$$\boxed{a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}}$$





Ex 4: In a typical CD player, the constant speed of the surface at the point of the laser-lens system is **1.3 m/s**. (a) Find the angular speed of the disc in revolutions per minute when information is being read from the innermost first track (**r = 23 mm**) and the outermost final track (**r = 58 mm**). (b) The maximum playing time of a standard music disc is **74 min** and **33 s**. How many revolutions does the disc make during that time? (c) What is the angular acceleration of the compact disc over the **4473-s** time interval?



Angular speed must vary to keep the tangential speed constant

$$v = r\omega$$

$$\omega_i = \frac{v}{r_i} = \frac{1.3}{0.023} = 57\ (rad/s) = 5.4\times 10^2\ (rev/\min)$$

$$\omega_f = \frac{v}{r_f} = \frac{1.3}{0.058} = 22\ (rad/s) = 2.1\times 10^2\ (rev/\min)$$

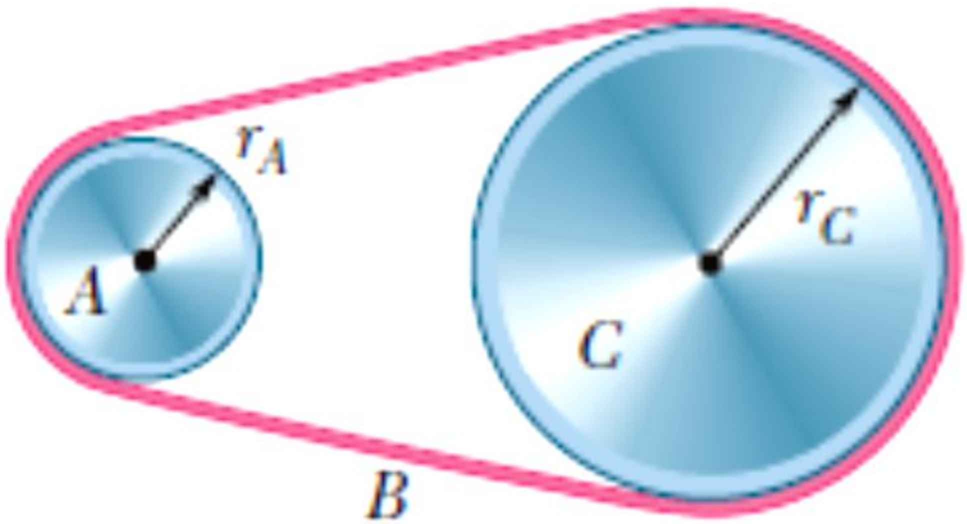
$$\Delta\theta = \omega_{avg} t = (\frac{\omega_0 + \omega}{2})t \quad \Rightarrow \quad \Delta\theta = (\frac{57+22}{2})(4473) = 1.8\times 10^5\ rad = 2.8\times 10^4\ rev$$

$$\alpha = \frac{\omega - \omega_0}{t} \quad \Rightarrow \quad \alpha = \frac{22-57}{4473} = -7.6\times 10^{-3}\ (rad/s^2)$$

Ex 5: **(Problem 10.28 Halliday)**

Wheel **A** of radius **r_A =10 cm** is coupled by **belt B** to wheel **C** of radius **r_C =25 cm**. The angular speed of wheel **A** is increased from **rest** at a constant rate of **1.6 rad/s²**. Find the time needed for wheel **C** to reach an angular speed of **100 rev/min**, assuming the **belt does not slip**. (Hint: If the belt does not slip, the **linear speeds at the two rims must be equal**.)

$$\boxed{v_A = v_B = v_C}$$



$$r_A\,\omega_A = r_C\,\omega_C \Rightarrow r_A\,\alpha_A = r_C\,\alpha_C \Rightarrow \alpha_C = \frac{r_A}{r_C}\alpha_A = \frac{10}{25}(1.6) = 0.64\ rad/s^2$$

$$\omega_C = \underbrace{\omega_{0C}}_{0} + \alpha_C\,t \Rightarrow t = \frac{\omega_C}{\alpha_C} = \frac{100(\frac{2\pi}{60})}{0.64} = 16.36\ s$$

$$\omega_A = \frac{r_C}{r_A}\omega_C = 2.5\,\omega_C = 250\ (rev/\min)$$