Chapter 3: Gauss's Law

✓ **Electric Flux**

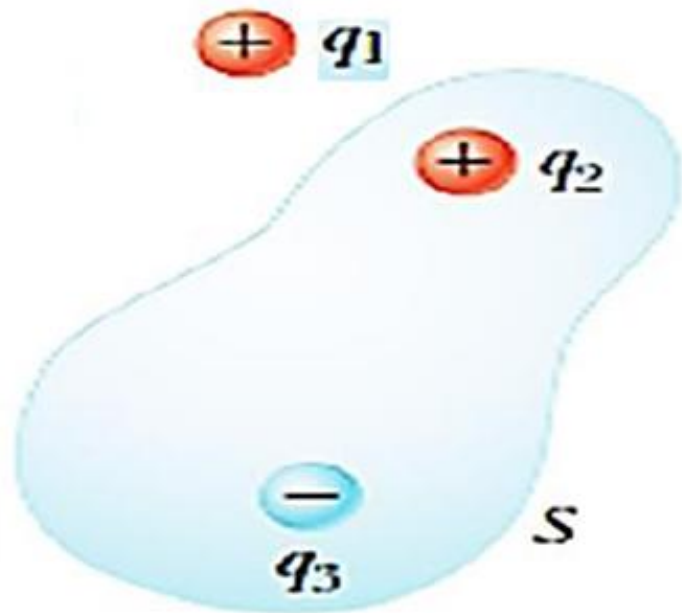✓ **Gauss's Law**

✓ **Applying Gauss's Law**

Session 7:

✓ **Applying Gauss's Law**

✓ **Examples**

Gauss's Law

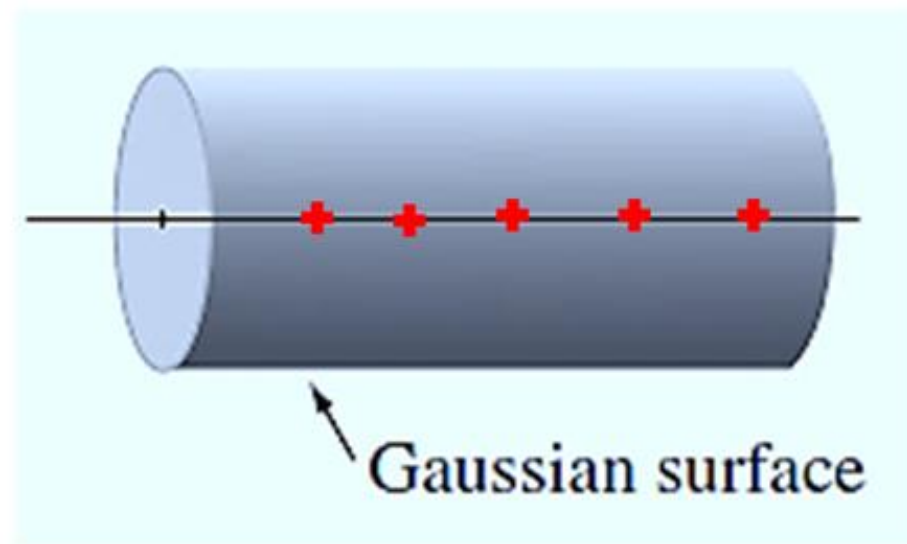❖ The **mathematical form of Gauss's law** states:

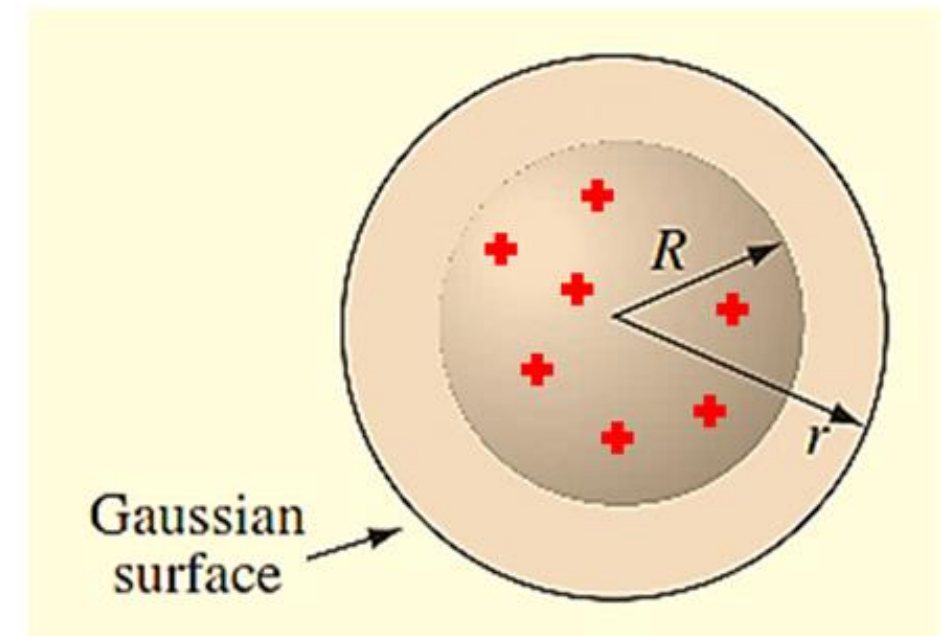$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

q_{in} :
$$\text{point charge} : \pm q$$

$$\text{Linear} : \int \lambda \, dl \xrightarrow{\lambda = \text{constant}} \lambda L$$

$$\text{surface} : \iint \sigma \, dA \xrightarrow{\sigma = \text{constant}} \sigma A$$

$$\text{Volume} : \iiint \rho \, dv \xrightarrow{\rho = \text{constant}} \rho V$$

$$q_{in} = q_2 + q_3$$

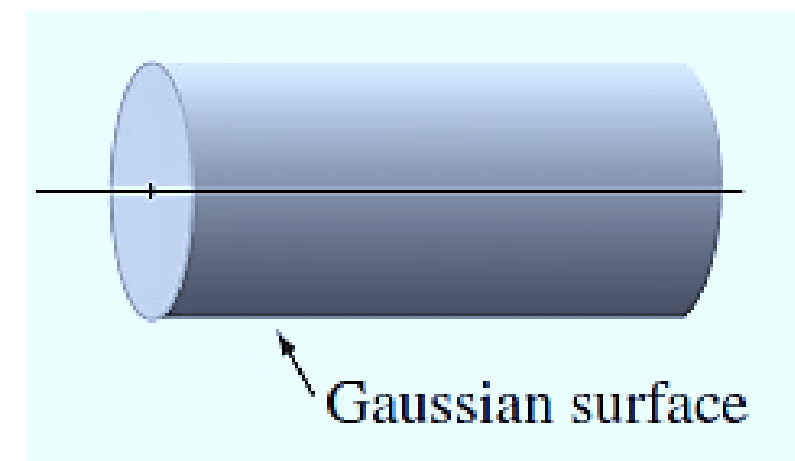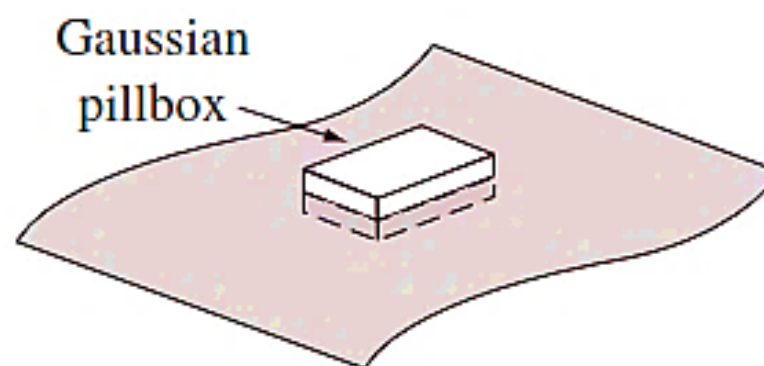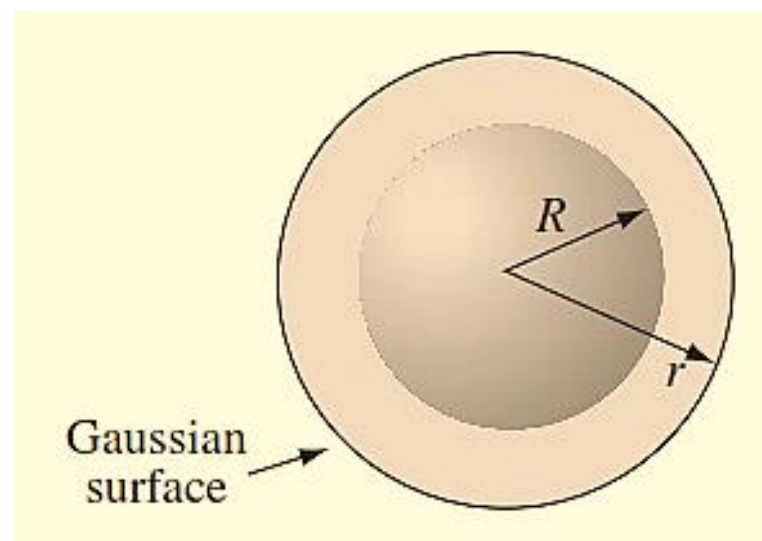$$q_{in} = \lambda L$$

$$q_{in} = \rho V$$

Applying Gauss's Law

❖ **Calculating Electric Field of Highly Symmetric Charge Distribution:**

Spherical Symmetry

Cylindrical Symmetry

Planar Symmetry

Gaussian surface

Gaussian pillbox

Gaussian surface

Choose a **Gaussian surface** that satisfies one or more of these conditions:

$\left| \vec{E} \right|$ constant over the surface.

$\vec{E} \cdot d\vec{A} = \left| \vec{E} \right| dA$, $\vec{E} \parallel d\vec{A}$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$\vec{E} \cdot d\vec{A} = 0$ over the portion of the surface.

Spherical Symmetry

Ex 2. An insulating shell of radius **R** has a uniform surface charge density **σ** and total positive charge **Q**. Find the magnitude of the electric field at a point **a)** outside and, **b)** inside the shell.

$$\sigma = \frac{Q}{4\pi R^2}$$

$$r > R : \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o} \implies \oint E \, dA = E \oint dA = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}}$$

$$r < R : \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_o} \implies \oint E \, dA = E \oint dA = E(4\pi r^2) = 0$$
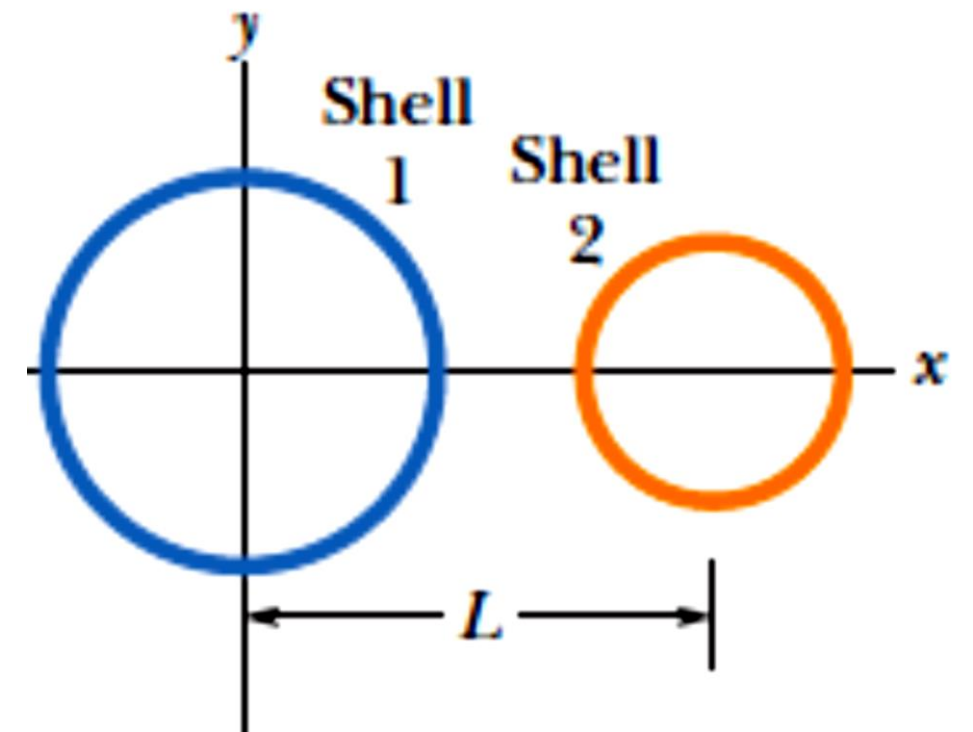
$$\boxed{\vec{E} = 0}$$

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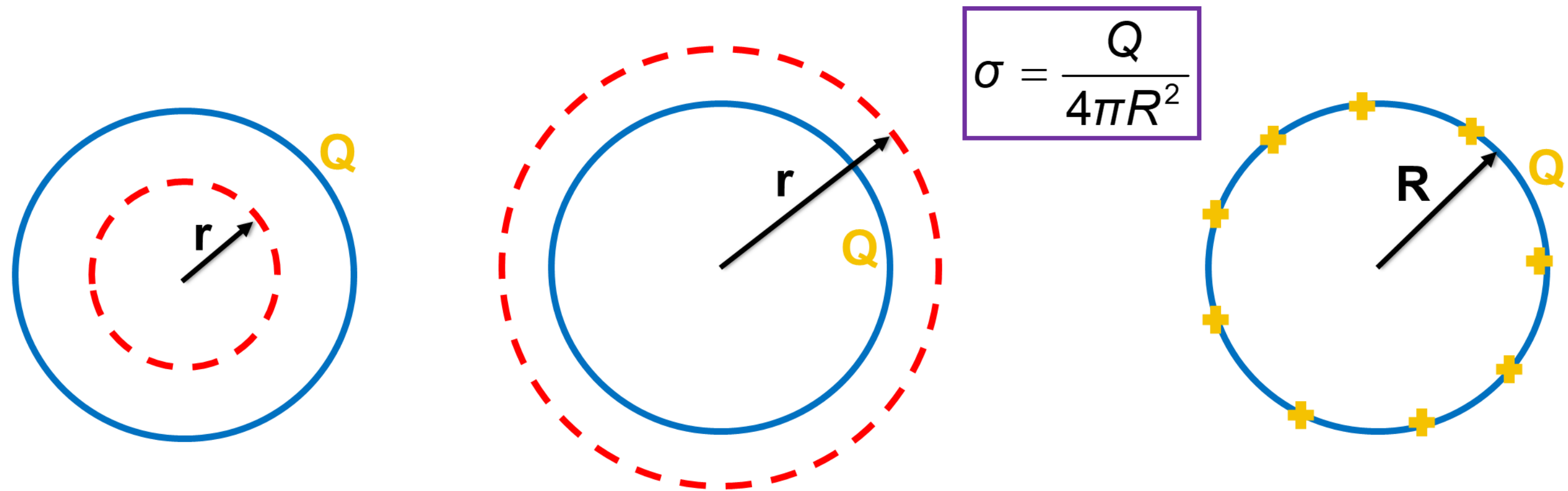
Ex 3. (Prob 23.12) Figure 23-36 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density **+6.0 μC/m²** on its outer surface and radius **3.0 cm**; shell 2 has uniform surface charge density **+4.0 μC/m²** on its outer surface and radius **2.0 cm**; the shell centers are separated by **L =10 cm**. In unit-vector notation, what is the net electric field at $x = 2.0$ **cm**?

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Shell 1
Shell 2

$x = 2\,cm < 3\,cm$; inside shell 1 $\Rightarrow$ $\boxed{\vec{E}_1 = 0}$
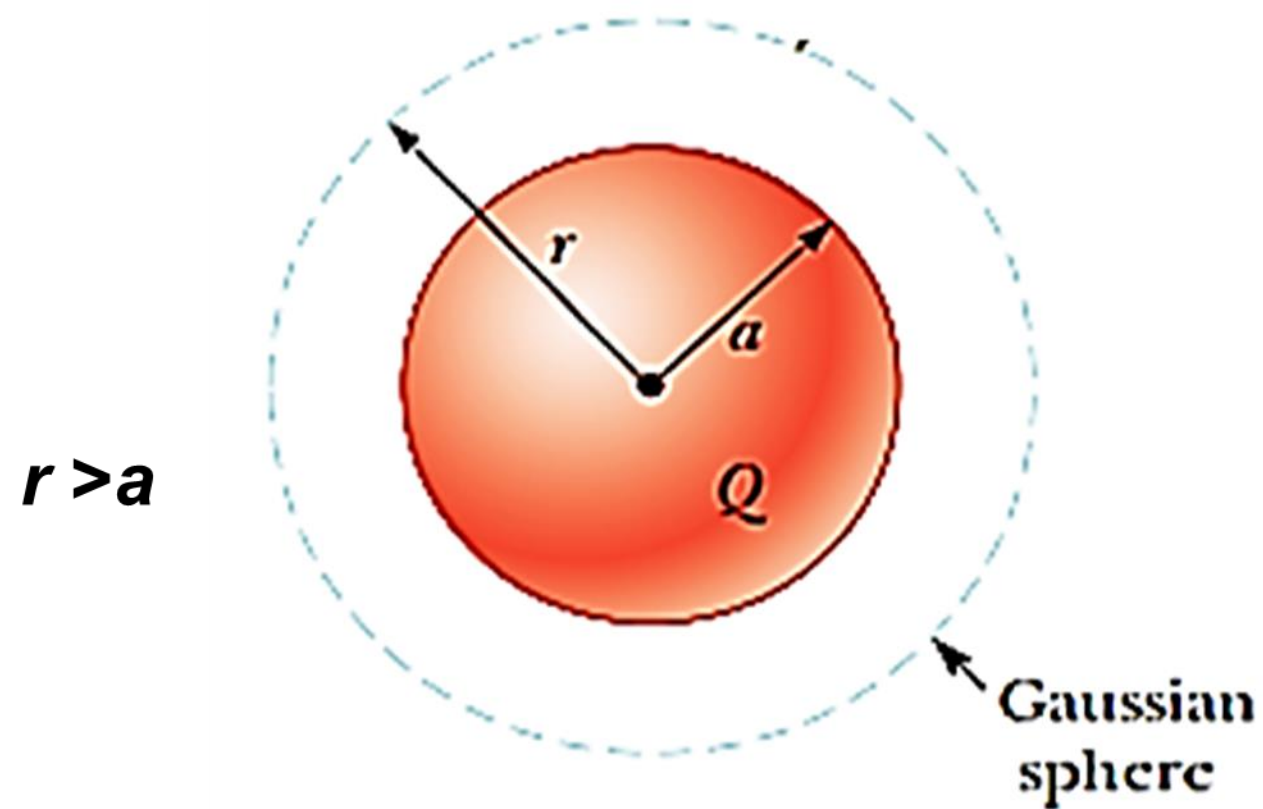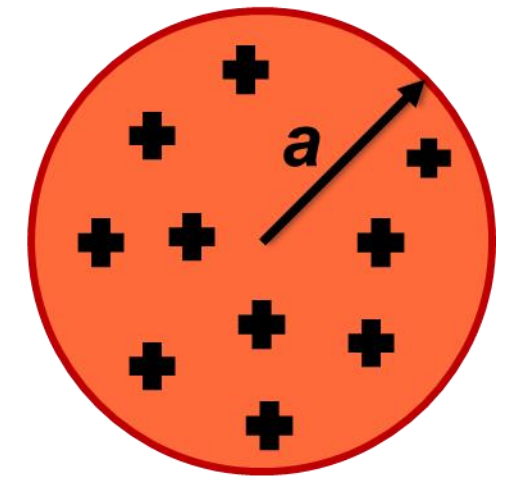
$x = 2\,cm$; outside shell 2: $r = 8\,cm$

$$\boxed{\vec{E}_2 = \frac{Q_b}{4\pi\varepsilon_o r^2}(-\hat{i}) = \frac{(4\times10^{-6})(4\pi(2\times10^{-2})^2)}{4\pi\varepsilon_o(8\times10^{-2})^2}(-\hat{i}) = 2.8\times10^4(-\hat{i})}$$

$$\boxed{\vec{E} = 2.8\times10^4(-\hat{i})}$$

Ex 4. An insulating solid sphere of radius *a* has a uniform volume charge density *ρ* and carries a total positive charge *Q*. Find the magnitude of the electric field at a point **a)** outside and, **b)** inside the sphere.
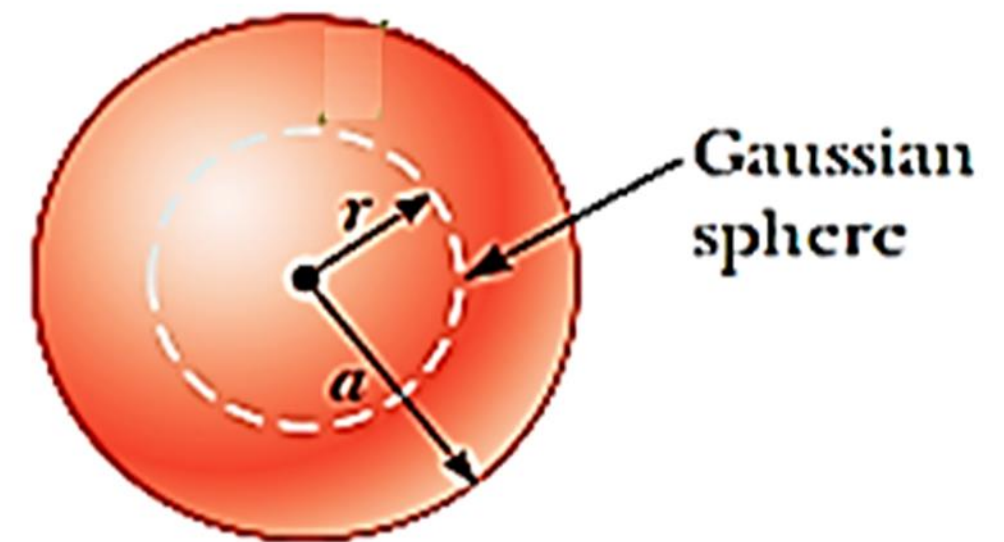

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

Gaussian sphere

$r > a$

$$r > R: \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o} \Rightarrow \oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

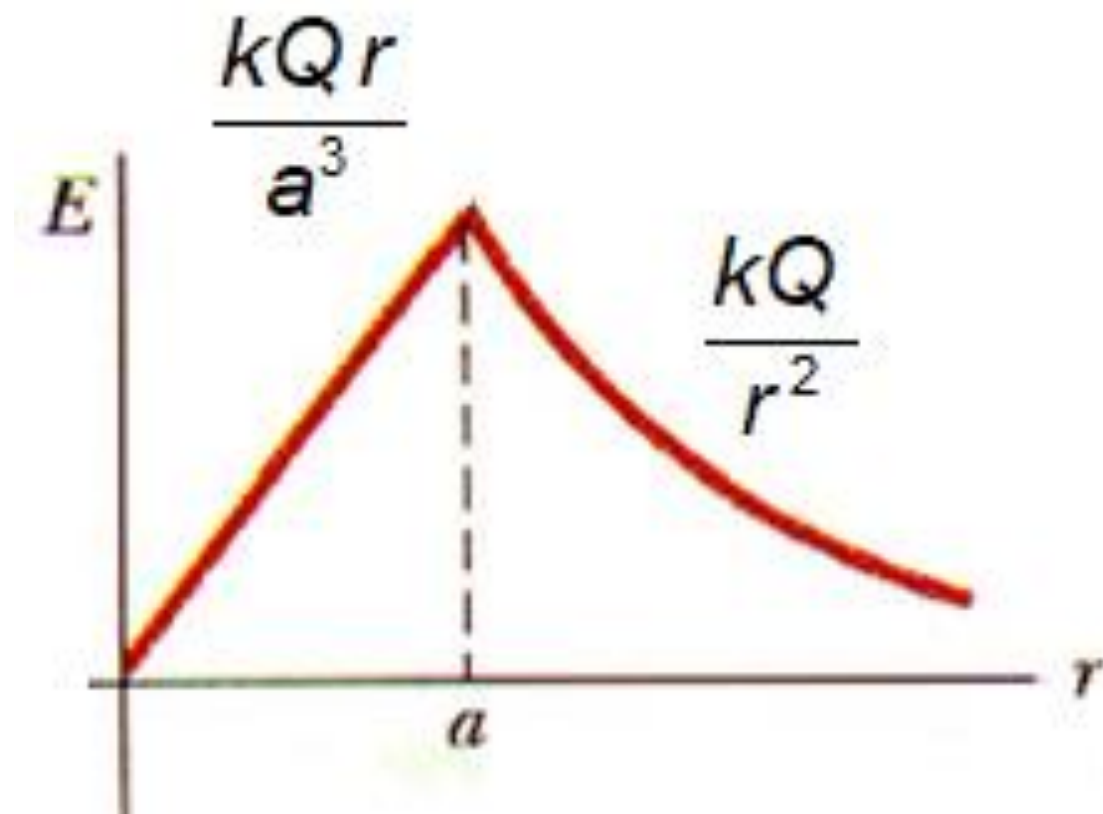$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$$

$$q_{in} = \rho V = \frac{Q}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi r^3\right) = Q\left(\frac{r^3}{a^3}\right)$$

r < a

$$r < a: \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o} \implies \oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{Q\left(\frac{r^3}{a^3}\right)}{\varepsilon_o}$$

$$\boxed{\vec{\mathbf{E}} = \frac{Q\,r}{4\pi\varepsilon_o a^3}\,\hat{r}}$$

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Ex 5. (Prob 23.54) Figure 23-58 shows, in cross section, two **solid spheres** with **uniformly distributed charge throughout their volumes**. Each has radius **R**. Point P lies on a line connecting the centers of the spheres, at radial distance **R/2.00** from the center of sphere 1. If the net electric field at point P is zero, what is the ratio **q_2/q_1** of the total charges?

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 = 0 \implies \left|\vec{E}_1\right| = \left|\vec{E}_2\right|$$

p is inside sphere 1:
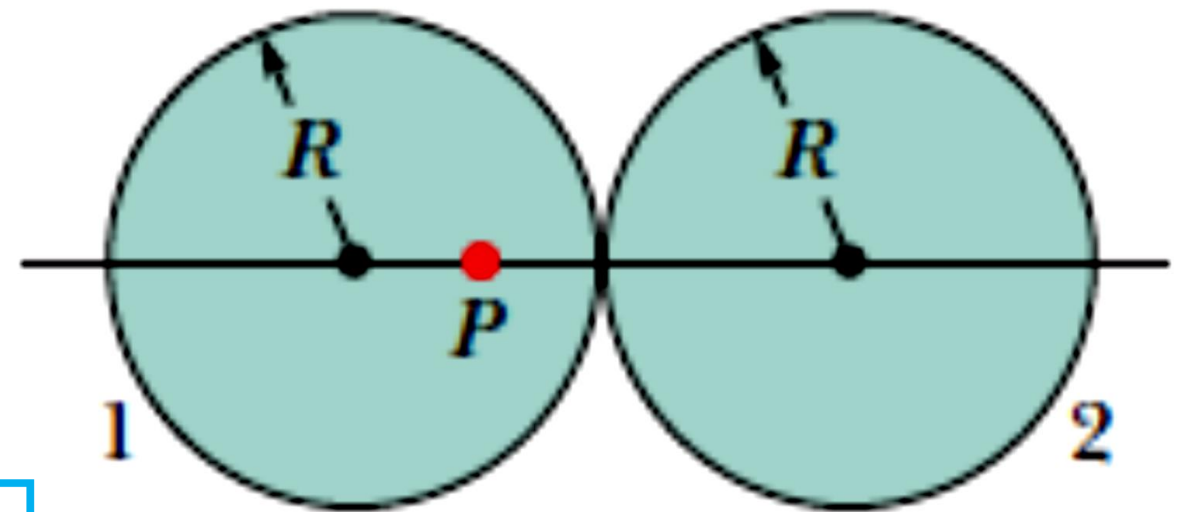
$$\vec{E}_1 = \frac{k\,q_1\,r_1}{R^3}\hat{i} = \frac{k\,q_1(\frac{R}{2})}{R^3}\hat{i} = \frac{1}{2}\frac{k\,q_1}{R^2}\hat{i}$$

$$\frac{1}{2}\frac{k\,q_1}{R^2} = \frac{4}{9}\frac{k\,q_2}{R^2}$$

P is outside sphere 2:

$$\vec{E}_2 = \frac{k\,q_2}{r_2^2}(-\hat{i}) = \frac{k\,q_2}{(\frac{3R}{2})^2}(-\hat{i}) = \frac{4}{9}\frac{k\,q_2}{R^2}(-\hat{i})$$

$$\frac{q_2}{q_1} = \frac{9}{8} = 1.125$$

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