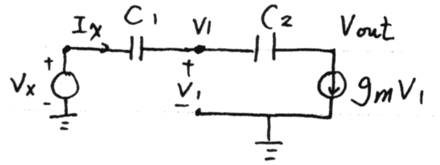
Chapter 6

6.1 (a)



g_m : transconductance of M_1

$$I_X = sC_1(V_X - V_1) = sC_2(V_1 - V_{out}) = g_m V_1$$

$$\therefore \quad sC_1 V_X = (g_m + sC_1)V_1 \Rightarrow V_1 = \left[\frac{sC_1}{g_m + sC_1}\right] V_X$$

$$\Rightarrow I_X = g_m V_1 = \left[g_m sC_1 / g_m + sC_1 \right] V_X$$

$$\Rightarrow Z_{in} = \frac{V_X}{I_X} = \frac{g_m + sC_1}{g_m sC_1}$$

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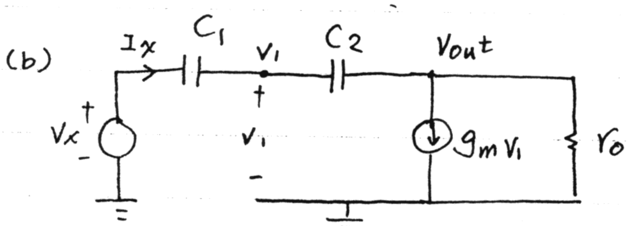
(b)



$g_m = g_{m1} + g_{m2}$

$r_o = r_{o1} \parallel r_{o2}$

g_{m1}, g_{m2} : transconductance for M_1, M_2

r_{o1}, r_{o2} : output resistance for M_1, M_2

$$\therefore \; I_X = \overbrace{sC_1(V_X - V_1) = sC_2(V_1 - V_{out})}^{②} \underbrace{= g_m V_1 + \frac{V_{out}}{r_o}}_{①}$$

from ① :

$$\frac{V_{out}}{V_1} = \frac{sC_2 - g_m}{sC_2 + \frac{1}{r_o}}$$

from ② :

$$(sC_1 + sC_2)V_1 = sC_1 V_X + sC_2 V_{out}$$

$$= sC_1 V_X + \frac{s^2C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} V_1$$

$$\Rightarrow \left[sC_1 + sC_2 - \frac{s^2C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_X$$

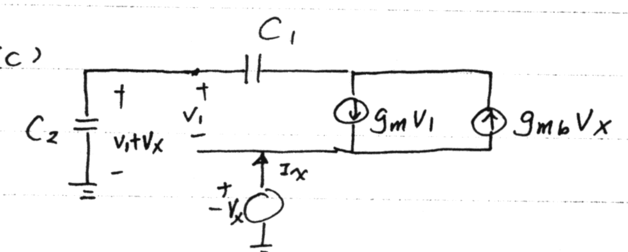
$$\Rightarrow \left[\frac{s^2 C_1 C_2 + \frac{sC_1}{r_o} + \frac{sC_2}{r_o} + g_m s C_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_x$$

$$\therefore V_1 = \left[\frac{sC_1C_2 + \frac{C_1}{r_o}}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$I_x = sC_1(V_x - V_1) = sC_1 \cdot \left[\frac{\frac{C_2}{r_o} + g_m C_2}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2}{sC_1C_2\left(\frac{1}{r_o} + g_m\right)}$$ #

(c)



$$sC_2(V_1 + V_x) + g_m V_1 = g_{mb} V_x$$

$$\Rightarrow (sC_2 + g_m)V_1 = (g_{mb} - sC_2)V_x$$

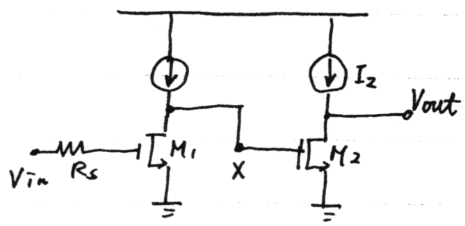
$$\frac{V_x}{V_1} = \frac{sC_2 + g_m}{g_{mb} - sC_2}$$

$$I_x = -g_m V_1 + g_{mb} V_x$$

$$= \left[-g_m \cdot \frac{g_{mb} - sC_2}{sC_2 + g_m} + g_{mb} \right] V_x = \left[\frac{(g_m + g_{mb})sC_2}{sC_2 + g_m} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_2 + g_m}{sC_2(g_m + g_{mb})}$$ #

6.2 (a)



There are three poles associated with this circuit.

The first pole @ Vout

$$\omega_{p,out} = \frac{1}{r_{O2} \cdot (C_{gd2} + C_{DB2})}$$

The pole @ the input

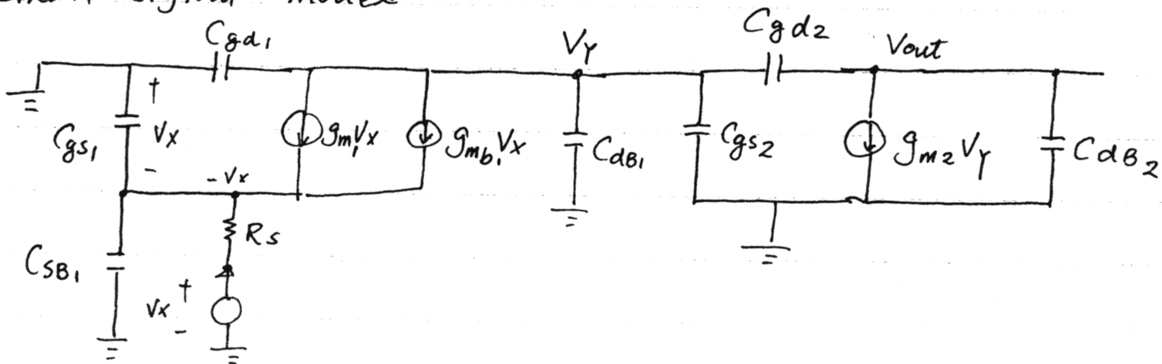
$$\omega_{p,in} = \frac{1}{R_S \cdot [(1+g_{m1} r_{O1}) C_{gd1} + C_{gs1}]}$$

The pole @ node X

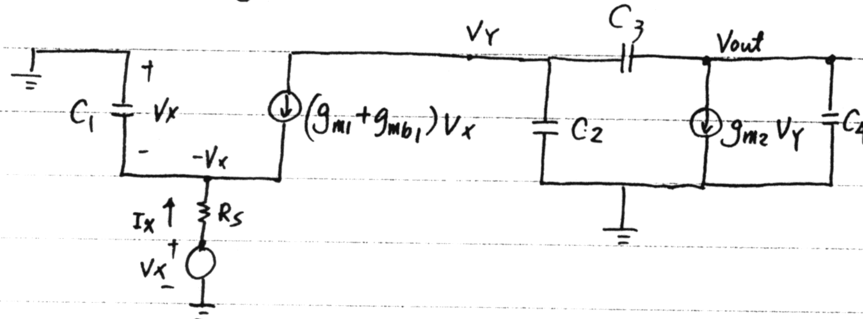
$$\omega_{p,X} = \frac{1}{r_{O1} \cdot [(C_{gd1} + C_{DB1} + C_{gs2}) + (1+g_{m2} r_{O2}) \cdot C_{gd2}]}$$

Please note that the above approximation is based on Miller effect. In order to get more accuracy approximation, transfer function has to be derived.

(b) Small signal model



Redraw small signal model



where,

$C_1 = C_{gs1} + C_{SB1}$

$C_2 = C_{gs2} + C_{dB1} + C_{gd1}$

$C_3 = C_{gd2}$

$C_4 = C_{dB2}$

KCL @ V_{out} : $sC_3(V_Y - V_{out}) = g_{m2}V_Y + sC_4 V_{out}$

$$\Rightarrow \frac{V_{out}}{V_Y} = \frac{-g_{m2} + sC_3}{s(C_3+C_4)}$$

KCL @ V_Y : $(g_{m1}+g_{mb1})V_X + sC_2V_Y + sC_3(V_Y - V_{out}) = 0$

$$(g_{m1}+g_{mb1})V_X = -V_Y\left(sC_2 + \frac{s^2C_3C_4 + sC_3 \cdot g_{m2}}{s(C_3+C_4)}\right)$$

$$\frac{V_Y}{V_X} = -\frac{g_{m1}+g_{mb1}}{[s(C_2C_3+C_2C_4+C_3C_4) + C_3g_{m2}]/(C_3+C_4)}$$

KCL @ V_X : $\frac{V_{in}+V_X}{R_S} + sC_1V_X + (g_{m1}+g_{mb1})V_X = 0$

$$\frac{V_X}{V_{in}} = -\frac{1}{sC_1R_S + (1+(g_{m1}+g_{mb1})\cdot R_S)}$$

Thus, there are three poles

$\omega_{p0} = 0$

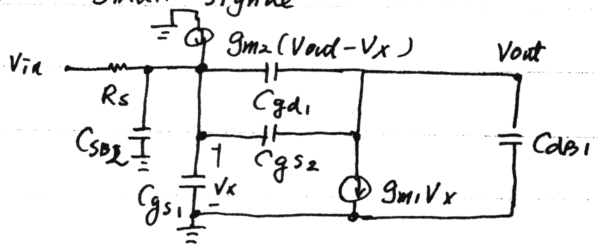
$$\omega_{p1} = \frac{-C_3g_{m2}}{C_2C_3+C_2C_4+C_3C_4} \quad ☆$$

$$\omega_{p2} = \frac{-(1+(g_{m1}+g_{mb1})\cdot R_S)}{C_1R_S} \quad ☆$$

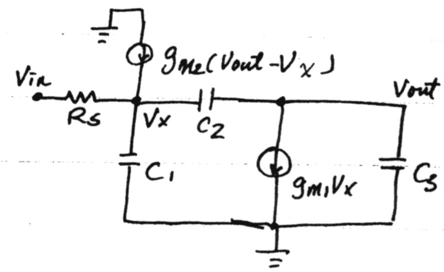
6.5

Pg 5

6.3 (a) Small signal



$\Rightarrow$



$C_1 = C_{gs1} + C_{SB2}$

$C_2 = C_{gd1} + C_{gs2}$

$C_3 = C_{DB1}$

KCL @ Vout :

$$sC_2(V_x - V_{out}) = g_{m1}V_x + sC_3 V_{out}$$

$$\therefore \frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2+C_3)} \quad \text{— (1)}$$

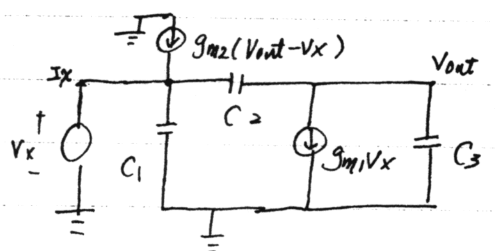
KCL @ Vx :

$$\frac{V_{in}-V_x}{R_s} + g_{m2}(V_{out}-V_x) = sC_1V_x + sC_2(V_x - V_{out})$$

$$\Rightarrow \frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m2} + sC_1 + sC_2\right) - (g_{m2}+sC_2)\cdot\left[\frac{sC_2 - g_{m1}}{s(C_2+C_3)}\right]V_x$$

$$= V_x \cdot \frac{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left(\frac{1}{R_s}(C_2+C_3)+g_{m1}C_2+g_{m2}C_3\right)+g_{m1}g_{m2}}{s(C_2+C_3)}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x}\cdot\frac{V_x}{V_{in}} = \frac{\frac{1}{R_s}(sC_2 - g_{m1})}{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left[\frac{1}{R_s}(C_2+C_3)+g_{m1}C_2+g_{m2}C_3\right]+g_{m1}g_{m2}} \quad *$$



$$I_x = sC_1V_x + sC_2(V_x - V_{out}) + g_{m2}(V_x - V_{out})$$

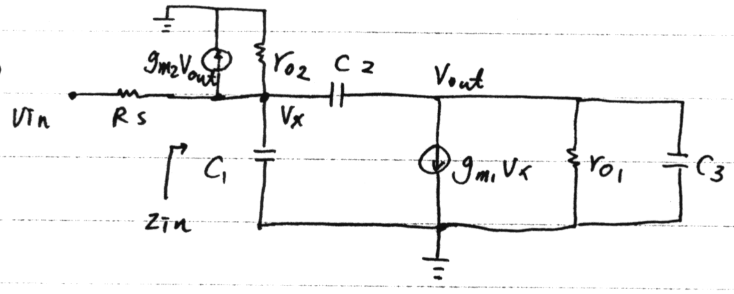
from (1): $V_x - V_{out} = \left(\frac{sC_3 + g_{m1}}{s(C_2+C_3)}\right)V_x$

$$\therefore I_x = \left[sC_1 + \frac{s^2C_2C_3 + g_{m1}sC_2 + g_{m2}sC_3 + g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$= \left[\frac{s^2(C_1C_2+C_2C_3+C_1C_3)+s(g_{m1}C_2+g_{m2}C_3)+g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2+C_3)}{s^2(C_1C_2+C_2C_3+C_1C_3)+s(g_{m1}C_2+g_{m2}C_3)+g_{m1}g_{m2}} \quad *$$

(b)



$C_1 = C_{gs1} + C_{dB2}$

$C_2 = C_{gd1} + C_{gd2}$

$C_3 = C_{dB1} + C_{gs2}$

KCL @ Vout : $sC_2(V_X - V_{out}) = g_{m1}V_X + V_{out}\left(\frac{1}{r_{o1}} + sC_3\right)$

$$\frac{V_{out}}{V_X} = \frac{sC_2 - g_{m1}}{s(C_2+C_3) + \frac{1}{r_{o1}}}$$

KCL @ Vx : $\frac{V_{in}-V_X}{R_S} = g_{m2}V_{out} + V_X\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_X - V_{out})$

$$\frac{V_{in}}{R_S} = \left(sC_1 + sC_2 + \frac{1}{r_{o2}} + \frac{1}{R_S}\right)V_X - \frac{(-g_{m2}+sC_2)(sC_2-g_{m1})}{s(C_2+C_3) + \frac{1}{r_{o1}}}\cdot V_X$$

$$= \frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)}{s(C_2+C_3) + \frac{1}{r_{o1}}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_S}(sC_2 - g_{m1})}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)} \quad \#$$

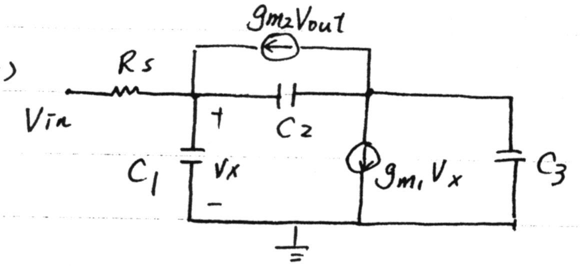
For Zin

$$I_X = g_{m2}V_{out} + V_X\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_X - V_{out})$$

$$= \left[\frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}{s(C_2+C_3) + \frac{1}{r_{o1}}}\right]V_X$$

$$Z_{in} = \frac{V_X}{I_X} = \frac{s(C_2+C_3) + \frac{1}{r_{o1}}}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 - g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}$$

(c)



$C_1 = C_{gs1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1}$

$C_3 = C_{dB1} + C_{SB2} + C_{gs2}$

KCL@ Vout : $sC_2(V_X - V_{out}) = g_{m1}V_X + sC_3V_{out} + g_{m2}V_{out}$

$$\Rightarrow \frac{V_{out}}{V_X} = \frac{sC_2 - g_{m1}}{s(C_2+C_3) + g_{m2}}$$

KCL@VX : $\frac{V_{in} - V_X}{R_S} + g_{m2}V_{out} = sC_1V_X + sC_2(V_X - V_{out})$

$$\frac{V_{in}}{R_S} = \left(\frac{1}{R_S} + sC_1 + sC_2\right)V_X - (g_{m2} + sC_2)V_{out} = \left(\frac{1}{R_S} + sC_1 + sC_2\right)V_X - \frac{(g_{m2} + sC_2)(sC_2 - g_{m1})}{s(C_2+C_3) + g_{m2}}$$

$$\frac{V_i}{V} = \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_S}(C_2+C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left(\frac{g_{m2}}{R_S} + g_{m1}g_{m2}\right)}{s(C_2+C_3) + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_X} \cdot \frac{V_X}{V_{in}}$$

$$= \frac{\frac{1}{R_S}\left[sC_2 - g_{m1}\right]}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_S}(C_2+C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left(\frac{g_{m2}}{R_S} + g_{m1}g_{m2}\right)}$$ #

For Zin

$I_X = sC_1V_X - g_{m2}V_{out} + sC_2(V_X - V_{out})$

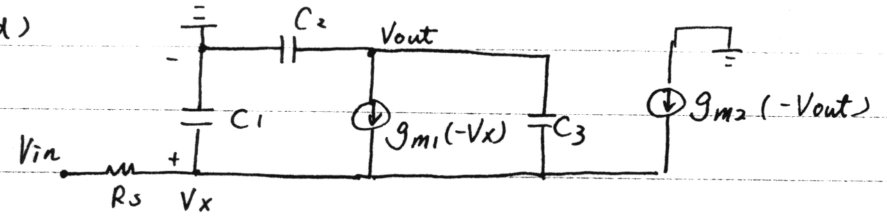
$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}{s(C_2+C_3) + g_{m2}}\right]V_X$$

$$Z_{in} = \frac{s(C_2+C_3) + g_{m2}}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}$$ #

6.8

Pg 8

(d)



$C_1 = C_{gs1} + C_{SB1} + C_{DB2}$

$C_2 = C_{gd1} + C_{gs2} + C_{dB1}$

$C_3 = C_{gd2}$

KCL @ Vout : $-sC_2 V_{out} = -g_{m1}V_x + sC_3(V_{out} - V_x)$

$$\frac{V_{out}}{V_x} = \frac{sC_3 + g_{m1}}{s(C_2 + C_3)}$$

KCL @ Vx : $\dfrac{V_{in} - V_x}{R_s} = sC_1V_x + g_{m1}V_x + sC_3(V_x - V_{out}) + g_{m2}V_{out}$

$$\frac{V_{in}}{R_s} = \left[\frac{1}{R_s} + s(C_1 + C_3) + g_{m1}\right]V_x + \frac{(sC_3 + g_{m1})(g_{m2} - sC_3)}{s(C_2 + C_3)}V_x$$

$$= V_x\left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1}C_2 + g_{m2}C_3\right] + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{sC_3 + g_{m1}}{s^2R_s(C_1C_2 + C_2C_3 + C_1C_3) + s[C_2 + C_3 + R_s(g_{m1}C_2 + g_{m2}C_3)] + g_{m1}g_{m2}R_s}$$ #

For Zin

$I_x = sC_1V_x + g_{m1}V_x + sC_3(V_x - V_{out}) - g_{m2}V_{out}$

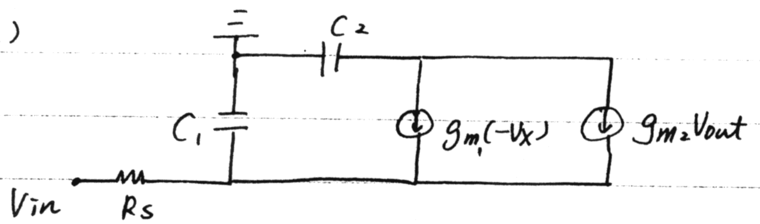
$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]V_x$$

$$\therefore Z_{in} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}$$ #

6.9

P9

(e)



$C_1 = C_{gs1} + C_{SB1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1} + C_{SB2} + C_{gs2} + C_{dB1}$

KCL @ Vout $\Rightarrow -sC_2 V_{out} = -g_{m1} V_X + g_{m2} V_{out}$

$$\Rightarrow \frac{V_{out}}{V_X} = \frac{g_{m1}}{sC_2 + g_{m2}}$$

KCL @ $V_X \Rightarrow \dfrac{V_{in} - V_X}{R_S} = sC_1 V_X + g_{m1} V_X - g_{m2} V_{out}$

$$\Rightarrow \frac{V_{in}}{R_S} = \left[\frac{1}{R_S} + sC_1 + g_{m1}\right] V_X - \frac{g_{m1} g_{m2}}{(sC_2 + g_{m2})} V_X$$

$$= \frac{s^2 C_1 C_2 + s\left[\left(\frac{1}{R_S} + g_{m1}\right) C_2 + g_{m2} C_1\right] + \frac{g_{m2}}{R_S}}{sC_2 + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{s^2 R_S C_1 C_2 + s\left[(1 + g_{m1} R_S) C_2 + g_{m2} R_S C_1\right] + g_{m2}}$$

#

For Zin

$$I_X = sC_1 V_X + g_{m1} V_X - g_{m2} V_{out}$$

$$= \left[\frac{s^2 C_1 C_2 + s\left[g_{m1} C_2 + g_{m2} C_1\right]}{sC_2 + g_{m2}}\right]$$

$$\therefore Z_{in} = \frac{sC_2 + g_{m2}}{s^2 C_1 C_2 + s(g_{m1} C_2 + g_{m2} C_1)}$$

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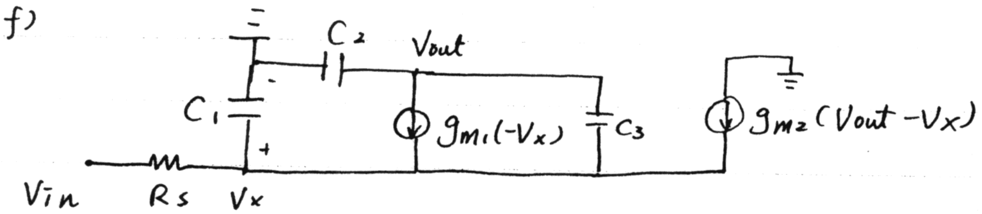
6.10

Pg 9+

(f)



$C_1 = C_{SB1} + C_{SB2} + C_{gs1}$

$C_2 = C_{gd1} + C_{gd2} + C_{dB1}$

$C_3 = C_{gs2}$

KCL @ Vout : $sC_2(-V_{out}) = g_{m1}(-V_x) + sC_3(V_{out} - V_x)$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1} + sC_3}{s(C_2 + C_3)}$$

KCL @ V_x: $\dfrac{V_{in} - V_x}{R_s} + g_{m1}(-V_x) + g_{m2}(V_{out} - V_x) = sC_1 V_x + sC_3(V_x - V_{out})$

$$\frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m1} + g_{m2} + sC_1 + sC_3\right) - \frac{(g_{m2} + sC_3)(g_{m1} + sC_3)}{s(C_2 + C_3)} \cdot V_x$$

$$= \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2}}{s(C_2 + C_3)}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} + sC_3}{s^2R_s(C_1C_2 + C_2C_3 + C_1C_3) + s[C_2 + C_3 + R_s(g_{m1}C_2 + g_{m2}C_2)] - g_{m1}g_{m2}R_s}$$

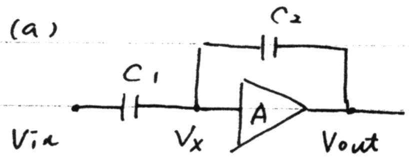
For Zin

$I_x = g_{m1}V_x + g_{m2}(V_x - V_{out}) + sC_1V_x + sC_3(V_x - V_{out})$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s[g_{m1}C_2 + g_{m2}C_2] - g_{m1}g_{m2}}{s(C_2 + C_3)}\right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_2) - g_{m1}g_{m2}}$$

6.4 (a)



$A = -\infty$

(i) At low frequency, V_x is like virtual ground

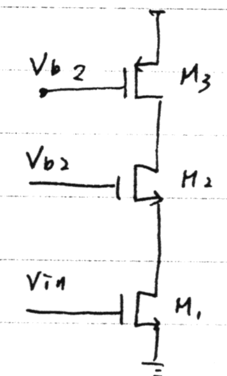
$sC_1 V_{in} = -sC_2 V_{out}$

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}$$

(ii) At high frequency, C_1, C_2 is like a short circuit

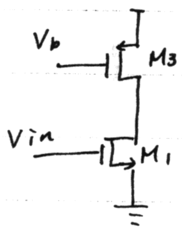
$$\frac{V_{out}}{V_{in}} = 1$$

(b) At low frequency, the equivalent circuit is shown as



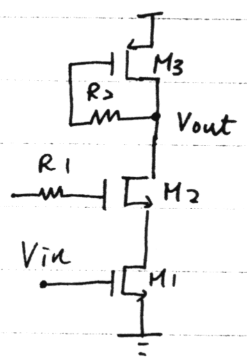
$A_v \cong -g_{m1} r_{o3} \to \infty$, if $\lambda = 0$

(ii) At high frequency, the equivalent circuit



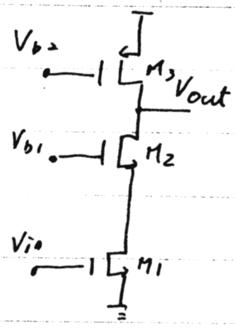
$A_V = -g_{m1}(r_{o1} // r_{o3}) \rightarrow \infty$ if $\lambda = 0$

(c) (i) At low frequency, the equivalent circuit



R_1, R_2 can be ignored

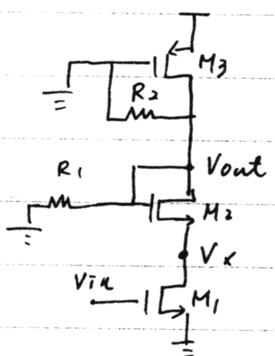
$\Rightarrow$



The impedance @ Vout $= \frac{1}{g_{m3}}$

$A_V \cong -g_{m1} \cdot {}^{1}\!/_{g_{m3}} = -\frac{g_{m1}}{g_{m3}}$

(ii) At high frequency,



$$\frac{V_X}{V_{in}} = -g_{m1} \cdot \underbrace{\frac{1}{g_{m2}}}$$

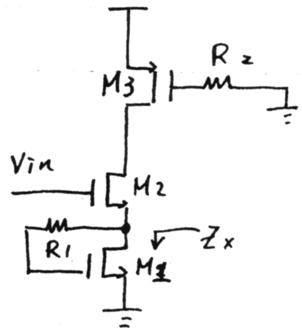
the impedance looking into V_X

The impedance @ Vout $= R_1 // R_2$

$$\therefore A_V = \left(-g_{m1} \cdot \frac{1}{g_{m2}}\right) \cdot g_{m2} \cdot (R_1 // R_2) = -g_{m1}(R_1 // R_2)$$

#

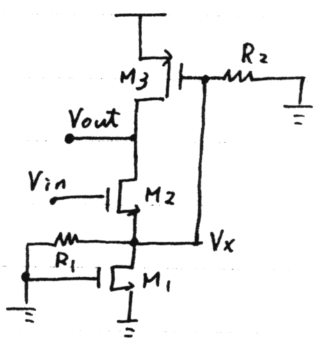
(d) (i) At low frequency, the equivalent circuit is



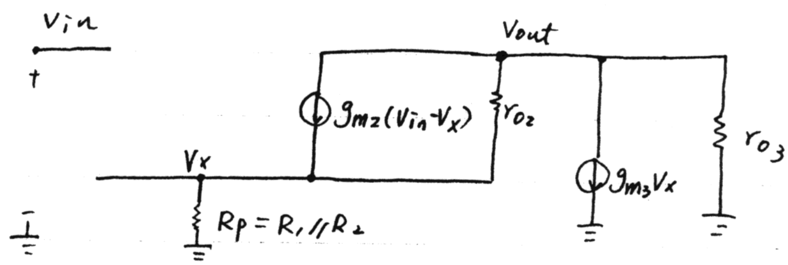
$$\frac{V_{out}}{V_{in}} = \frac{g_{m2}\left(r_{o3} // (1+g_{m2}r_{o2})Z_x\right]}{1+g_{m2}Z_x} \cong \frac{g_{m2}(r_{o3}//r_{o2})}{1+\frac{g_{m2}}{g_{m1}}} \to \infty \quad \text{if } \lambda = 0$$

$$Z_x = \frac{1}{g_m}$$

(ii) At high frequency



Small-signal model



$$\text{KCL @ } V_x, V_{out}: \quad \frac{V_x}{R_p} = g_{m2}(V_{in}-V_x) + \frac{V_{out}-V_x}{r_{o2}} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right)$$

$$\frac{V_x}{R_p} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right) \Rightarrow \frac{V_{out}}{V_x} = -r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)$$

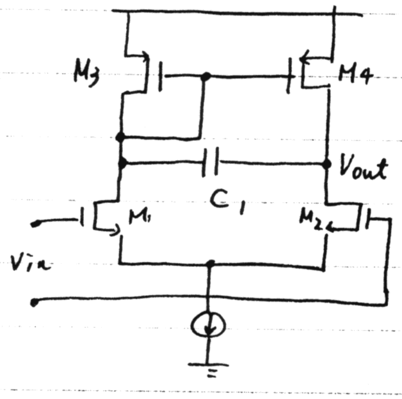
$$g_{m2}(V_{in}-V_x) + \frac{V_{out}-V_x}{r_{o2}} = \frac{V_x}{R_p} \Rightarrow g_{m2}V_{in} = \left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right)V_x + \frac{V_{out}}{r_{o2}}$$

$$\Rightarrow g_{m2}V_{in} = \left[-\left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right)\frac{1}{r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)} + \frac{1}{r_{o2}}\right]V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_{m2}r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)}{\left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right) - \frac{r_{o3}}{r_{o2}}\left(g_{m3}+\frac{1}{R_p}\right)} \to \infty \quad \text{if } \lambda = 0$$

#

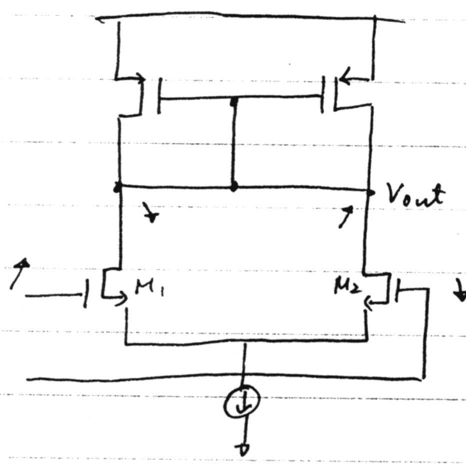
6.5 (a) (i) At low frequency



C_1 is like an open circuit @ very low frequency

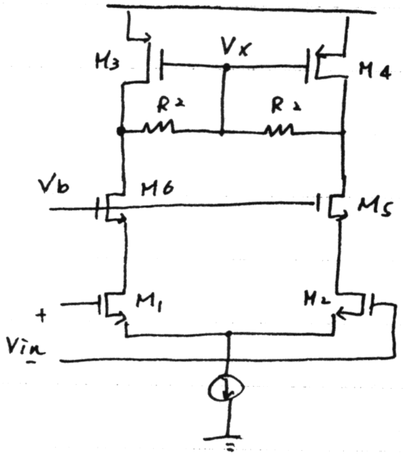
$$\Rightarrow \frac{V_{out}}{V_{in}} = -g_{m1}(r_{o2} // r_{o4}) \rightarrow \infty \text{ if } \lambda = 0$$

(ii) At very high frequency, C_1 is like a short circuit



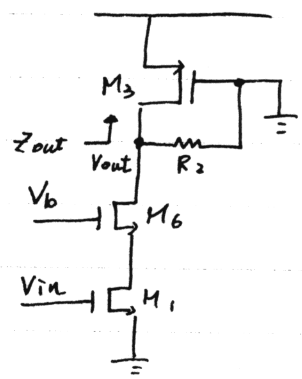
Gain = 0

(b) (i) At low frequency, the equivalent circuit is



half circuit →

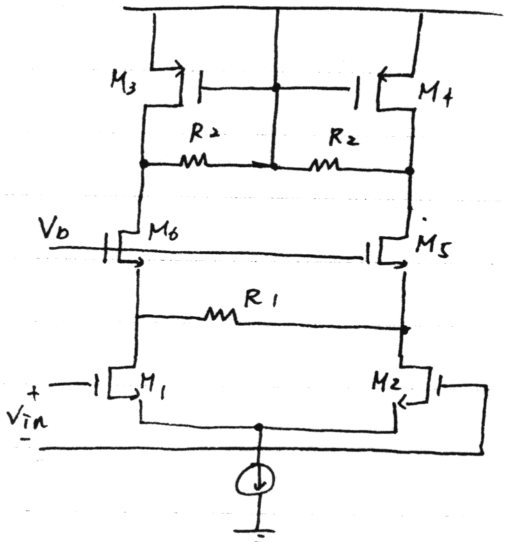
Vx is virtual ground



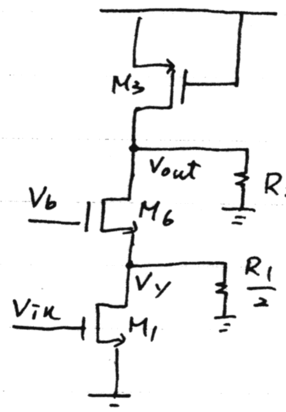
$$Z_{out} \cong r_{o3} // R_2 \cong R_2$$

$$A_v = -g_{m1} \cdot (R_2 // r_{o3}) \cong -g_{m1} R_2 \quad ※$$

(ii) At high frequency



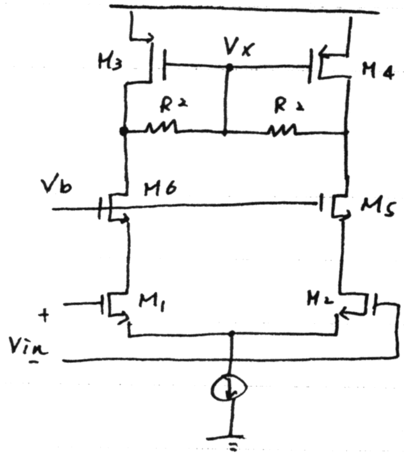
half circuit →



$$\frac{V_y}{V_{in}} = -g_{m1}\left(\frac{1}{g_{m6}} // \frac{R_1}{2}\right), \quad \frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$$

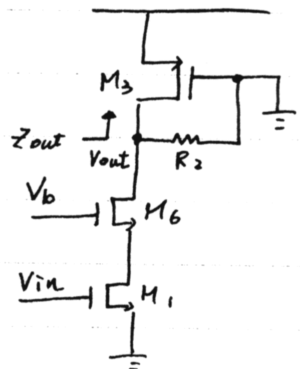
$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)} \quad ※$$

(b) (i) At low frequency, the equivalent circuit is



half circuit →

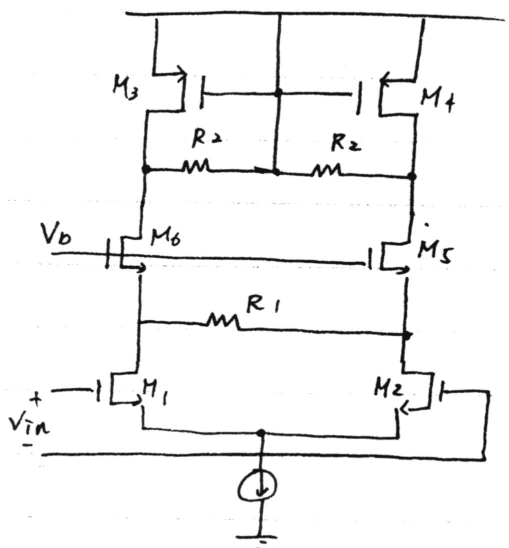
Vx is virtual ground



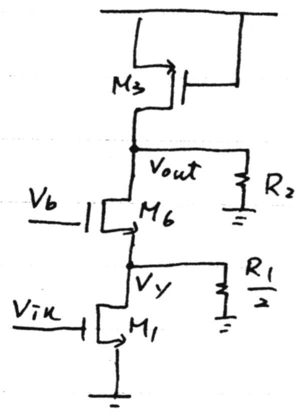
$$Z_{out} \cong r_{o3} /\!/ R_2 \cong R_2$$

$$A_v = -g_{m1} \cdot (R_2 /\!/ r_{o3}) \cong -g_{m1} R_2$$ *

(ii) At high frequency



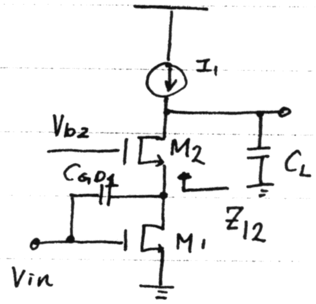
half circuit →



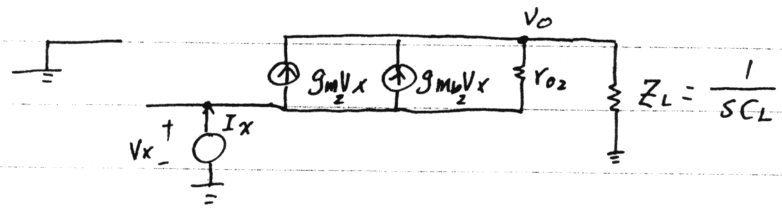
$$\frac{V_y}{V_{in}} = -g_{m1}\left(\frac{1}{g_{m6}} /\!/ \frac{R_1}{2}\right), \quad \frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$$ *

6.6



The impedance Z_{12} can be derived from the following small signal model



KCL @ V_o : $\frac{V_o}{Z_L} + \frac{V_o - V_x}{r_{o_2}} = (g_{m_2} + g_{mb_2})V_x \Rightarrow \left(\frac{1}{Z_L} + \frac{1}{r_{o_2}}\right)V_o = \left(g_{m_2} + g_{mb_2} + \frac{1}{r_{o_2}}\right)V_x$

$$\Rightarrow V_o = \left(\frac{g_{m_2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right)V_x$$

$$\Rightarrow I_x = \frac{V_o}{Z_L} = \left[\frac{g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right]V_x \Rightarrow \frac{V_x}{I_x} = Z_{12} = \frac{1 + \frac{Z_L}{r_{o2}}}{g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}}$$

$$\Rightarrow Z_{12} = \frac{r_{o2} + Z_L}{1 + (g_{m_2} + g_{mb2})r_{o2}}$$

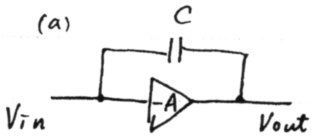
The miller multiplication for C_{GD1} = $1 + g_{m1}Z_{12}$

$$= 1 + \frac{g_{m1}(r_{o2} + Z_L)}{1 + (g_{m2} + g_{mb2})r_{o2}} \quad — ①$$

If C_L is relatively large $\Rightarrow \left|\frac{1}{sC_L}\right| << r_{o2}$

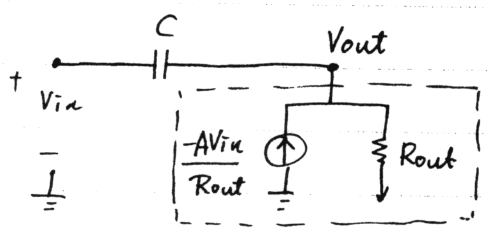
eq ① can be approximated as $\cong 1 + \frac{g_{m1}r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} \cong 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}$

6.7 (a)



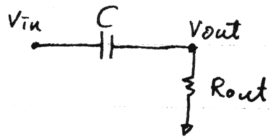
Assume the amplifier output resistance Rout

The small signal model is as follows



← The Norton equivalent circuit for amplifier

As we can see, the above circuit forms a high pass network



Thus, when there is a step ΔV at the input, output will follow Input, a step ΔV, first. Then, it will settle down to $-AV_{in}$ as the steady state

(b) KCL @ Vout :

$$-\frac{AV_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC\,(V_{out} - V_{in})$$

$$\Rightarrow \left(sC - \frac{A}{R_{out}}\right)V_{in} = \left(\frac{1}{R_{out}} + sC\right)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sCR_{out}}$$

✱

for the step response, $x(t) = u(t)$, $t \geq 0 \rightarrow X(s) = \frac{1}{s}$

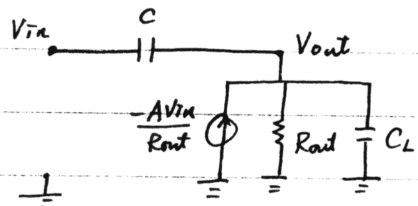
$$Y(s) = \frac{1}{s} \cdot \frac{V_{out}}{V_{in}}(s) = \frac{sCR_{out} - A}{s(1 + sCR_{out})} = \frac{-A}{s} + \frac{(A+1)\cdot R_{out}C}{1 + sCR_{out}}$$

$$\Rightarrow y(t) = -A\,u(t) + (A+1)\,e^{-\frac{t}{R_{out}C}}, \quad t \geq 0$$

For a ΔV input step, output $= -A\cdot\Delta V + (A+1)\cdot\Delta V\cdot e^{-\frac{t}{R_{out}C}}$

✱

6.8 (a) Small-signal circuit model



when input has ΔV jump, Vout will follow and the output jump $= \left(\frac{C}{C_L + C}\right)\Delta V$ ✱

(b) The transfer function $H(s)$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \quad : \quad KCL \text{ @ } V_{out}$$

$$\Rightarrow \quad -\frac{A V_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in}) + sC_L V_{out}$$

$$\Rightarrow \quad V_{in}\left(sC - \frac{A}{R_{out}}\right) = V_{out}\left(\frac{1}{R_{out}} + sC + sC_L\right)$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sR_{out}(C + C_L)}$$ ✱

step response

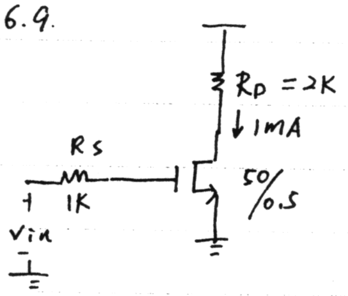
$$Y(s) = \frac{1}{s} \cdot \frac{sCR_{out} - A}{[1 + sR_{out}(C+C_L)]} = \frac{-A}{s} + \frac{[(A+1)C + AC_L]R_{out}}{1 + sR_{out}(C+C_L)} = \frac{-A}{s} + \frac{\frac{(A+1)C + AC_L}{C + C_L}}{s + \frac{1}{R_{out}(C+C_L)}}$$

$$y(t) = -A\,u(t) + \frac{(A+1)C + AC_L}{C + C_L} \cdot e^{-\frac{t}{R_{out}(C+C_L)}}\,u(t)$$

For a step ΔV @ the input

$$\text{output} = -A\,\Delta V + \left[\frac{(A+1)C + A\cdot C_L}{C + C_L}\right] \cdot \Delta V \cdot e^{-\frac{t}{R_{out}(C+C_L)}}$$ ✱

6.9.



$$\lambda = 0.1$$

$$C_{ox} = \frac{\epsilon_{SiO_2}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} = 3.835 \times 10^{-7}$$

$$\mu_n = 350$$

$$I_D = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2 \times 0.08} \right) (V_{gs} - V_T)^2 (1 + 0.1 \times V_{ds})$$

$$= 67.113 \times 10^{-6} \times \frac{50}{0.34} \times 1.1 \times (V_{in} - 0.7)^2$$

$$\Rightarrow V_{in} = 1.0035$$

$$g_m = \frac{2 I_D}{(V_{gs} - V_t)} = 6.59 \times 10^{-3}$$

$$C_{gs} = \frac{2}{3} C_{ox} W L + C_{ov} \cdot W$$

$$= \frac{2}{3} \times 3.835 \times 10^{-7} \times 50 \times (0.5 - 0.08 \times 2) \times 10^{-8} + 3.835 \times 10^{-7} \times 0.08 \times 10^{-4} \times 50 \times 10^{-4}$$

$$= 53.7 \times 10^{-15}$$

$$C_{gd} = C_{gdo} \cdot W = 0.4 \times 10^{-11} \times 50 \times 10^{-6} = 2 \times 10^{-16}$$

$$C_{dB} = \frac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{\left(1 + \frac{1}{0.9}\right)^{0.6}} + \frac{0.35 \times 10^{-11} \times 103 \times 10^{-6}}{\left(1 + \frac{1}{0.9}\right)^{0.2}} = 2.714 \times 10^{-14}$$

According to eq (20)

$$\text{zero} = \frac{g_m}{C_{gd}} = \frac{6.59 \times 10^{-3}}{2 \times 10^{-6}} = 3.3 \times 10^{13} \text{ rad/sec}$$

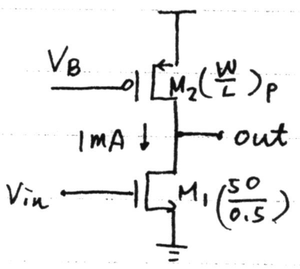
pole is the root of $R_S R_D (C_{gs} C_{gd} + C_{gs} C_{dB} + C_{gd} C_{dB}) s^2$
$+ [R_S (1 + g_m R_D) C_{gd} + R_S C_{gs} + R_D (C_{gd} + C_{dB})] s + 1$ ... from eq (6.20)

$$\Rightarrow 2.95 \times 10^{-21} s^2 + 1.112 \times 10^{-10} s + 1 = 0$$

$$\omega_{p1} = -14.82 \times 10^9 \text{ rad/sec}$$

$$\omega_{p2} \; -22.88 \times 10^9 \text{ rad/sec}$$

6.10



(a) The maximum output level = 2.6 V

$\rightarrow$ V_B can be as low as $2.6 - |V_{THP}| = 2.6 - 0.8 = 1.8V$

Let's choose output DC bias @ 1.5V, such that

$M_1, M2$ are both in saturation region

Thus, $I_{D1} = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2\times 0.08}\right)(V_{in} - 0.7)^2 (1 + 0.1 \times 1.5)$

$\Rightarrow V_{in} = 0.997 \cong 1V$

Also, $I_{D2} = 10^{-3} = \frac{1}{2} \times 100 \times 3.835 \times 10^{-7} \times \left(\frac{W_P}{0.5 - 2\times 0.09}\right)(3 - 1.8 - 0.8)^2 (1 + 0.2 \times 1.5)$

$W_P \cong 80.5 \mu m = 81 \mu m$

Therefore, we can choose $\left(\frac{W}{L}\right)_P = \left(\frac{81\mu m}{0.5\mu m}\right)$, with gate bias 1.8V

so that $M_1, M2$ are both in saturation region and $V_{out} \cong 1.5V$

$V_{out}, low = V_{in} - |V_{THN}| = 0.3V$

Thus, the maximum output peak-to-peak swing $= 2.6 - 0.3 = 1.3V$ ☆

(b) This problem is similar to problem 6.9 except

$R_D \rightarrow (r_{o1} // r_{o2})$

$C_{DB} = C_{DB1} + C_{DB2} + C_{gd2}$

$\therefore g_{m1} = \frac{2I_D}{V_{gs} - V_t} = 2\times10^{-3} / 0.297 = 6.73 \times 10^{-3}$

$r_{op} = 1 \Big/ \left(\frac{\lambda_p I_D}{1 + \lambda_p V_{DS}}\right) \cong 6.5K$

$r_{ON} = 1 \Big/ \left(\frac{\lambda_N I_D}{1 + \lambda_N V_{DS}}\right) = 11.5K$

$\therefore R_D = r_{op} // r_{ON} = 4.1K$

$R_S = 1K$

$C_{gs1} = \frac{2}{3} C_{ox} W_1 L_1 + C_{ox} W \cdot \Delta L = 58.8 \times 10^{-15} F$

$C_{gd1} = 2 \times 10^{-16}$

$$C_{dB_1} = \frac{0.56\times10^{-3}\times 75\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.6}} + \frac{0.35\times10^{-11}\times 103\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.2}} = 23.38\times10^{-15}F$$

$$C_{dB2} = \frac{0.94\times10^{-3}\times 121.5\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.5}} + \frac{0.32\times10^{-11}\times 165\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.3}} = 70.33\times10^{-15}F$$

$$C_{gd2} = 81\times0.3\times10^{-11}\times10^{-6} = 0.243\times10^{-15}F$$

$$\omega_z = -\frac{g_{m_1}}{C_{gd_1}} = -\frac{6.59\times10^{-3}}{2\times10^{-16}} = -3.3\times10^{13}\ \text{rad/sec}$$

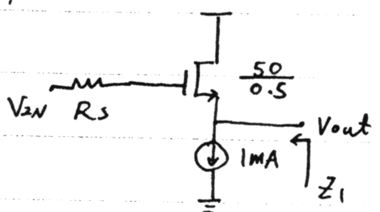
$\omega_{p_1}, \omega_{p_2}$ is the root of the equation

$$R_S R_D (C_{gs_1} C_{gd_1} + C_{gs_1} C_{dB} + C_{gd_1} C_{dB}) + [R_S(1+g_{m_1} R_D) C_{gd_1} + R_S C_{gs_1} + R_D (C_{gd_1} + C_{dB})] + 1$$

$\Rightarrow$ $\omega_{p_1} = -2.2\times10^{9}$ rad/sec

$\omega_{p_2} = -17.36\times10^{9}$ rad/sec

6.11



Assume $\gamma = 0$

$$g_m = 2I/(V_{gs}-V_t)$$

$$= \frac{2\times10^{-3}}{0.3} = 6.67\times10^{-3}$$

From eq (6.49) $\quad Z_1 = \dfrac{R_S C_{gs}\cdot S + 1}{g_m + C_{gs} S}$

Since $\dfrac{1}{g_m} < R_S$

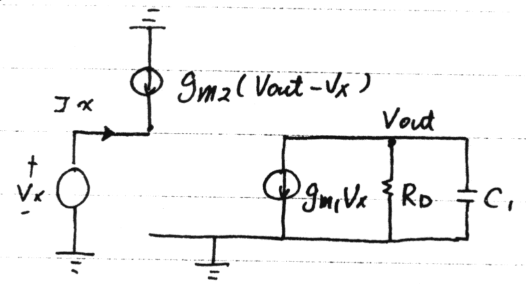
, thus Z_1 is inductive and the equivalent inductance is

$$= \frac{C_{gs}}{g_m}\left(R_s - \frac{1}{g_m}\right)$$

$C_{gs} = 58\times10^{-15}$ F

$\therefore L = 8.56\times10^{-8}$ H

6.12 (a)



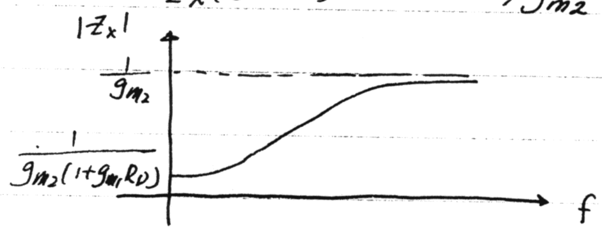
$$V_{out} = -g_{m1}V_X\left(R_D // \frac{1}{sC_1}\right)$$

$$I_X = -g_{m2}(V_{out}-V_x) = -g_{m2}V_{out} + g_{m2}V_x = \left[g_{m2}g_{m1}\left(R_D//\frac{1}{sC_1}\right) + g_{m2}\right]V_x$$

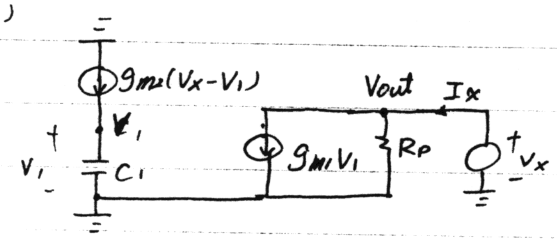
$$Z_X = \frac{V_X}{I_X} = \frac{1}{g_{m2}g_{m1}\frac{R_D/sC_1}{R_D+1/sC_1} + g_{m2}} = \frac{1}{g_{m2}\left[\left(\frac{g_{m1}R_D}{1+sR_DC_1}\right)+1\right]}$$

Thus, $Z_X(s\to 0) = 1/g_{m2}(1+g_{m1}R_D)$

$Z_X(s\to \infty) = 1/g_{m2}$



(b)



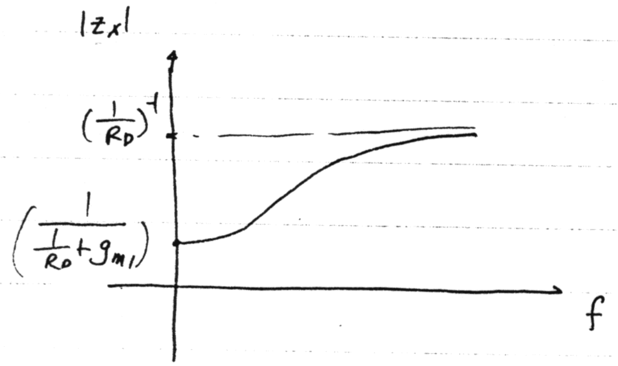
KCL @ V_1: $g_{m2}(V_X-V_1) = sC_1V_1$

$$\Rightarrow V_1 = \left(\frac{g_{m2}}{sC_1+g_{m2}}\right)V_X$$

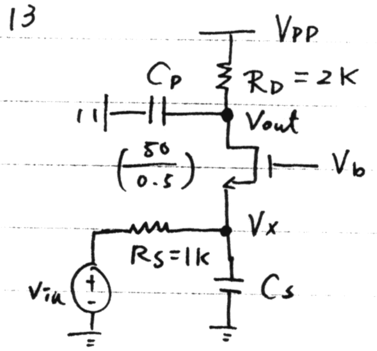
KCL @ V_{out}: $I_X = \frac{V_X}{R_D} + g_{m1}V_1$

$$= \frac{V_X}{R_D} + \frac{g_{m1}g_{m2}}{sC_1+g_{m2}}V_X$$

$$\Rightarrow Z_X = \frac{1}{\frac{1}{R_D} + \frac{g_{m1}g_{m2}}{sC_1+g_{m2}}}$$



6.13



from eq (6.53)

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_s} \frac{1}{\left(1 + \frac{C_s}{g_m + g_{mb} + R_s^{-1}} \cdot s\right)(1 + R_D C_D s)}$$

Assume V_b is chosen appropriately such that $V_x \sim 0$ (no body effect)

$g_m \sim 6.59 \times 10^{-3}$

$g_{mb} = \left[\gamma / 2\sqrt{2\phi_f} \right] g_m = 1.563 \times 10^{-3}$

$C_S = C_{SB} + C_{gs} = 42.4 \times 10^{-15} + 58.8 \times 10^{-15}$ } from problem 9

$C_D = C_{DB} = 27.14 \times 10^{-15}$ F

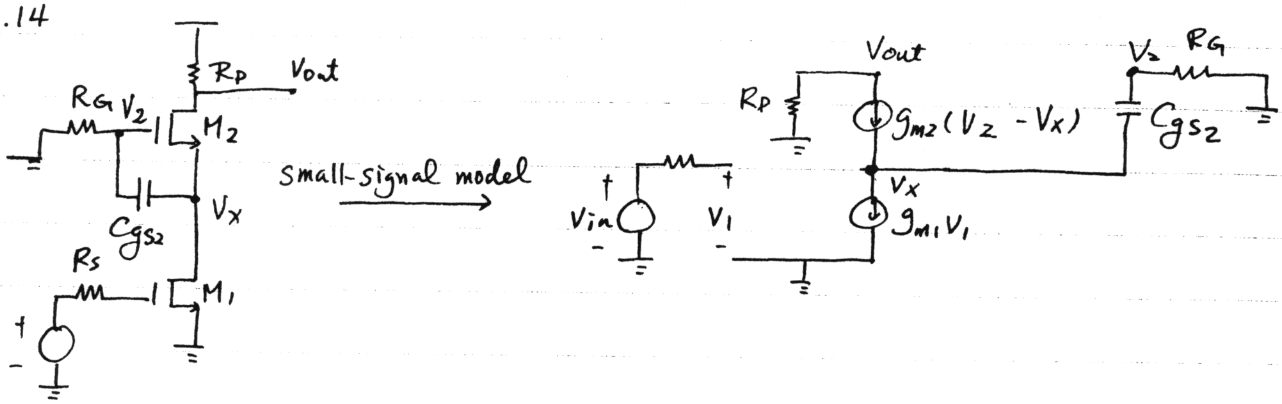
$$A_v\,(\text{low frequency}) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_s} = 1.44$$

$$\omega_{p_1} = -\frac{g_m + g_{mb} + R_s^{-1}}{C_s} = -\frac{6.59 \times 10^{-3} + 1.563 \times 10^{-3} + 10^{-3}}{42.4 \times 10^{-15} + 58.8 \times 10^{-15}} = -9.044 \times 10^{10}\ \text{rad/s}$$

$$\omega_{p_2} = -\frac{1}{R_D C_D} = -\frac{1}{2 \times 10^3 \times 2.714 \times 10^{-14}} = -1.84 \times 10^{10}\ \text{rad/s}$$

Compared with the pole locations in problem 9, the poles for Common-gate configuration are much larger because there is no Miller-effect for C_{gd} in this case

6.14



KCL @ V_2 : $\dfrac{V_2}{R_G} = sC_{gs2}(V_x - V_2)$

$$\Rightarrow \frac{V_2}{V_x} = \frac{sC_{gs2}}{\frac{1}{R_G} + sC_{gs2}} = \frac{sR_GC_{gs2}}{1+sR_GC_{gs2}} \quad \text{— ①}$$

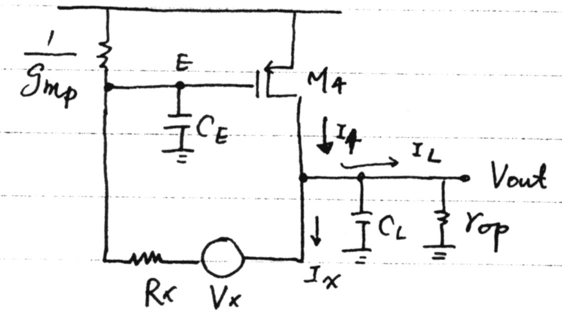
KCL @ V_{out} : $V_{out} = -g_{m2}(V_2 - V_x)R_D = \left[\dfrac{g_{m2}R_D}{1+sR_GC_{gs2}}\right]V_x$ — ②

KCL @ V_x : $g_{m1}V_1 = g_{m1}V_{in} = (g_{m2} + sC_{gs2})(V_2 - V_x)$

$$= \frac{-(g_{m2}+sC_{gs2})}{1+sR_GC_{gs2}}V_x \quad \text{— ③}$$

from ②, ③, $\dfrac{V_{out}}{V_{in}} = \dfrac{-g_{m1}g_{m2}R_D}{g_{m2}+sC_{gs2}}$ #

6.15



For zero frequency, $I_L = 0$ & $I_4 = I_x$

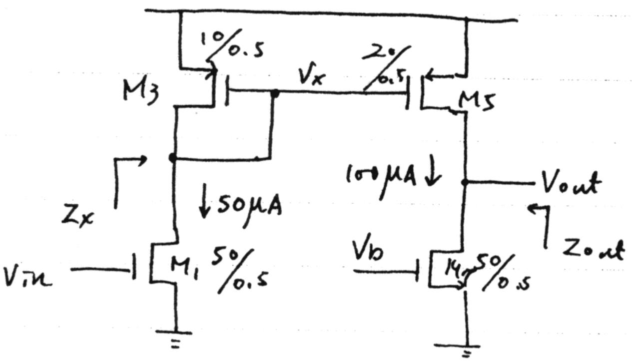
$I_4 = -g_{mp} V_E$

$I_x = V_E (g_{mp} + sC_E)$

$\therefore \quad I_4 = I_x \Rightarrow \quad -g_{mp} = g_{mp} + sC_E$

$S_z = \frac{-2g_{mp}}{C_E}$ ☆

6.18 Half circuit can be drawn as follows



Since $R_s = 0$, According to (6.20) & (6.76)

$$\frac{V_{out}}{V_{in}}(s) = -\frac{(C_{gd1}\cdot s - g_{m1})\cdot \frac{1}{g_{m3}}}{\frac{1}{g_{m3}}(C_{gd1}+C_X)s+1}\cdot g_{m5}\cdot(r_{o5}//r_{o7})\cdot\frac{1}{1+(r_{o5}//r_{o7})\cdot C_L\cdot s}$$

where $C_L = C_{dB7} + C_{gd7} + C_{dB5}$

$C_X = C_{gs3} + C_{dB1} + C_{gs5} + C_{dB3} + C_{gd5}(1+g_{m5}(r_{o5}//r_{o7}))$

$Z_{out} = (r_{o5}//r_{o7})$

$Z_X = \frac{1}{g_{m3}}$

First of all, let's calculate Vx operating point

$$I_{d3} = 50\times10^{-6} = \frac{1}{2}\times100\times3.835\times10^{-7}\times\frac{10}{0.5-0.09\times2}(3-V_X-0.8)^2(1+0.2(3-V_X))$$

$$V_X \doteqdot 1.94\,V$$

For Vin operating point

$$I_{d1} = 50\times10^{-6} = \frac{1}{2}\times350\times3.835\times10^{-7}\times\frac{50}{0.5-0.08\times2}(V_{gs1}-0.7)^2(1+0.1\cdot1.94)$$

$$V_{gs1} \doteqdot 0.765\,V$$

$$\Rightarrow g_{m1} = \frac{2I_{d1}}{(V_{gs1}-V_t)} \cong 1.54\times10^{-3}$$

$$g_{m3} = \frac{2I_{d1}}{(3-1.94-0.8)} \cong 3.73\times10^{-4} \Rightarrow g_{m5} = 2\cdot g_{m3} = 7.46\times10^{-4}$$

$g_{ds5} = 2\times 50\times 10^{-6}\cdot \lambda \Big/ (1+\lambda\cdot 1.06) = 10^{-4}\, 0.2 \Big/ 1.212 \cong 1.649\times 10^{-5}$

$g_{ds7} = 10^{-4}\times 0.1 \Big/ (1+0.196) \cong 8.36\times 10^{-6}$

$r_{o5} // r_{o7} = 40290$

$$C_L = C_{DB5} + C_{DB7} + C_{gd7} = \left[\frac{0.94\times 10^{-3}\times 30\times 10^{-12}}{(1+\frac{1.06}{0.9})^{0.5}} + \frac{0.32\times 10^{-11}\times 43\times 10^{-6}}{(1+\frac{1.06}{0.9})^{0.3}}\right]$$

$$+\left[\frac{0.56\times 10^{-3}\times 75\times 10^{-12}}{(1+\frac{1.94}{0.9})^{0.6}} + \frac{0.35\times 10^{-11}\times 103\times 10^{-6}}{(1+\frac{1.94}{0.9})^{0.2}}\right] + 50\times 0.4\times 10^{-17}$$

$$= 19.22\times 10^{-15} + 21.36\times 10^{-15} + 2\times 10^{-16} = 40.78\times 10^{-15}$$

$C_{gs3} = \frac{2}{3}\times 3.835\times 10^{-7}\times 10\times 0.32 + 3.835\times 10^{-7}\times 10\times 0.09 = 11.633\times 10^{-15}$

$C_{dB1} \cong C_{dB7} \cong 21.36\times 10^{-15}$

$C_{gs5} = 2\cdot C_{gs3} = 23.266\times 10^{-15}$

$C_{dB3} = \frac{1}{2}\cdot C_{DB5} = 9.61\times 10^{-15}$

$C_{gd5} = 6.3\times 10^{-11}\times 20\times 10^{-6} = 6\times 10^{-17}$

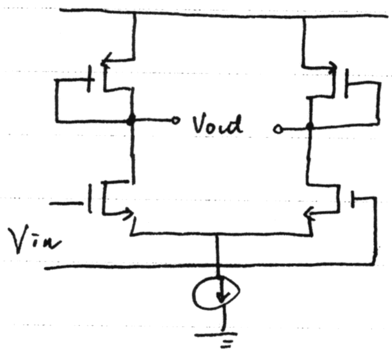
$C_X = \left[11.633 + 21.36 + 23.27 + 9.61 + 0.06\,(1+g_{m5}\cdot(r_{o5}//r_{o7}))\right]\times 10^{-15} = 67.732\times 10^{-15}$

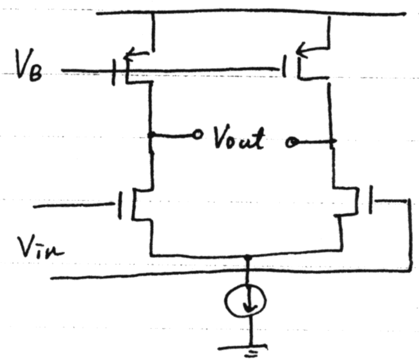
$$\therefore\ \omega_z = \frac{g_{m1}}{C_{gd1}} = 7.7\times 10^{12}\ \text{rad/sec}$$

$$\omega_{p1} = -\frac{1}{C_L\cdot(r_{o5}//r_{o7})} = -\frac{1}{40290\times 40.78\times 10^{-15}} = 6.08\times 10^{8}\ \text{rad/sec}$$

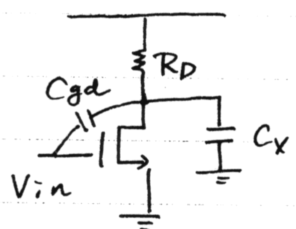
$$\omega_{p2} = -\frac{g_{m3}}{(C_{gd1}+C_X)} = \frac{-3.73\times 10^{-4}}{2\times 10^{-16} + 67.732\times 10^{-15}} = 5.5\times 10^{9}\ \text{rad/sec}$$

6.17 (a)





Both of these two differential pair can be simplified as common-source amplifier with different load resistance and capacitance



Since $R_S = 0$, equation (6.20) can be simplified as

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{gd}\,s - g_m)R_D}{s[R_D(C_{gd}+C_x)]+1}$$

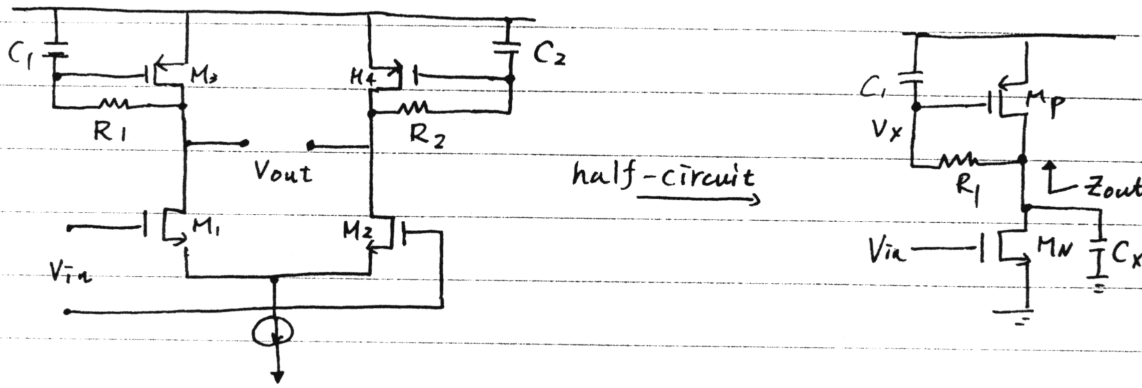
where $\begin{cases} R_D = \dfrac{1}{g_{mp}} & \text{for diode connected load} \\ C_x = C_{dBN} + C_{dBP} + C_{gsP} \end{cases}$

$\begin{cases} R_D \cong (r_{oN} \parallel r_{oP}) & \text{for current mirror load} \\ C_x = C_{dBN} + C_{dBP} + C_{gdP} \end{cases}$

Although there is right-half-plane zero, however this zero is much larger than the dominant pole.

Therefore, the maximum phase shift it can achieve is $\sim 90°$ before the gain is down to unity.

(b)



(i) At low frequency, C_1 is open circuit,

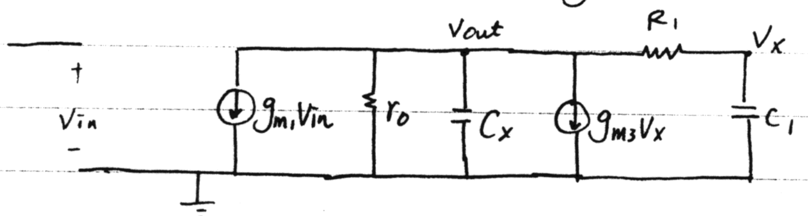
M_p is like a diode-connected device $\rightarrow Z_{out} \sim \frac{1}{g_{mp}}$

(ii) At high frequency, C_1 is short circuit.

M_p is like a current source device $\rightarrow Z_{out} \sim (r_{oN} // r_{op})$

Since $(r_{oN} // r_{op}) >> \frac{1}{g_{mp}}$, Z_{out} exhibits an inductive behavior

For transfer function, small-signal model



, $r_o = (r_{o1} // r_{o3})$

KCL @ V_X : $\frac{V_{out} - V_X}{R_1} = sC_1 V_X \Rightarrow V_{out} = (1 + sR_1C_1)V_X$

$$\Rightarrow V_X = \frac{V_{out}}{1 + sR_1C_1}$$

KCL @ V_{out} : $-g_{m1}V_{in} = V_{out}\left(\frac{1}{r_o} + sC_X\right) + \frac{1}{R_1}(V_{out} - V_X) + g_{m3}V_X$

$$= V_{out}\left(\frac{1}{r_o} + sC_X + \frac{1}{R_1}\right) + \left(g_{m3} - \frac{1}{R_1}\right) \cdot \frac{V_{out}}{1 + sR_1C_1}$$

$$-g_{m1}V_{in} = V_{out}\left(\frac{\frac{1}{r_o}+\frac{1}{R_1}+sC_x+\left(\frac{R_1}{r_o}+1\right)sC_1+s^2R_1C_1C_x+g_{m3}\frac{1}{R_1}}{1+sR_1C_1}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}(1+sR_1C_1)}{s^2R_1C_1C_x+s\left(C_1+\frac{R_1C_1}{r_o}+C_x\right)+\left(g_{m3}+\frac{1}{r_o}\right)}$$

From the above transfer function,

$$\omega_z = -\frac{1}{R_1C_1}$$

the sum of two poles $= -\frac{1}{R_1C_1C_x}\left(C_1+\frac{R_1C_1}{r_o}+C_x\right) = -\frac{1}{R_1C_1}\left(1+\frac{C_1}{C_x}\left(1+\frac{R_1}{r_o}\right)\right)$

usually $C_1 > C_x$, C_1 at least $= C_{gs3}$

Thus, the sum of two poles $> -\frac{2}{R_1C_1}$, which means that at least one of the poles are larger than zero $\Rightarrow$ It's quite impossible to produce 135° phase shift

Thus, this circuit still can't produce 135° phase shift.

However, it's more likely for it to generate 90° phase shift @ unity-gain frequency.

CHAPTER 7: NOISE

(7.1) $|A_V| = g_m R_D$

$$\overline{V_{n,out}^2} = \left(4KT\tfrac{2}{3}g_m + \tfrac{4KT}{R_D}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_V^2} = 4KT\left(\tfrac{2}{3}\tfrac{1}{g_m} + \tfrac{1}{g_m^2 R_D}\right)$$

$$\overline{V_{n,in}}_{TOT} = \sqrt{\overline{V_{n,in}^2}\cdot BW}$$

$$g_m = \sqrt{2I_D \mu_n C_{ox}(\tfrac{W}{L_{eff}})} = \sqrt{2(1mA)(134.28\tfrac{\mu A}{V^2})(\tfrac{50\mu m}{0.34\mu m})} \cong 6.28\ \tfrac{mA}{V}$$

$4KT = 1.656 \times 10^{-20}$ V-C, $R_D = 2K\Omega$, $BW = 100$ MHz

$$\therefore \overline{V_{n,in}}_{TOT} = \sqrt{(1.966\times10^{-18}\tfrac{V^2}{Hz})(100\ MHz)} \cong 14\ \mu V \text{ rms} \quad //$$

(7.2) using eqn. (7.57)

$$\overline{V_{n,in}^2} = 4KT\left(\tfrac{2}{3}\tfrac{1}{g_{m_1}} + \tfrac{2}{3}\tfrac{g_{m_2}}{g_{m_1}^2}\right)$$

$$\overline{V_{n,in}} = \sqrt{4KT\tfrac{2}{3}}\sqrt{\tfrac{1}{g_{m_1}} + \tfrac{g_{m_2}}{g_{m_1}^2}}$$

$$\frac{g_{m_2}}{g_{m_1}^2} = \left(\tfrac{1}{5}\right)^2\frac{1}{g_{m_1}} \Rightarrow g_{m_2} = \left(\tfrac{1}{5}\right)^2 g_{m_1}$$

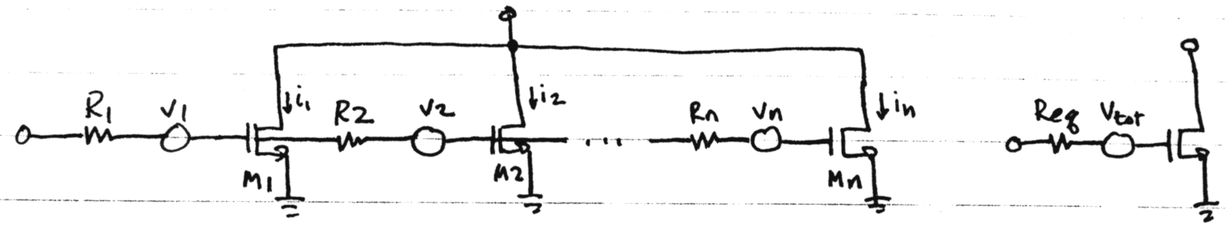
$$g_m = \frac{2I_D}{V_{gs}-V_T} \Rightarrow V_{gs}-V_T = \frac{2I_D}{g_m}$$

$$\therefore \text{Output Swing} = V_{DD} - (V_{gs_1}-V_{T_1}) - |(V_{gs_2}-V_{T_2})|$$
$$= V_{DD} - 2I_D\left(\tfrac{1}{g_{m_1}} + \tfrac{1}{g_{m_2}}\right)$$
$$= V_{DD} - 2I_D\left(\tfrac{1}{g_{m_1}}\right)(1+5^2)$$

$$g_{m_1} = \sqrt{2I_D\mu_n C_{ox}(\tfrac{W}{L_{eff}})} = \sqrt{2(1mA)(134.28\tfrac{\mu A}{V^2})(\tfrac{50\mu m}{0.34\mu m})} \approx 1.986\ \tfrac{mA}{V}$$

$$\therefore \text{Output Swing} = 3V - 2(1mA)\left(\tfrac{1}{1.986mA/V}\right)(26) \approx 0.38V \quad //$$

(7.3)



The drain noise current of M_1 resulting from the gate resistance is

$i_1 = g_{m1} v_1$ where v_1 is the noise voltage of R_1.

Similarly, $i_2 = g_{m2}(v_1 + v_2)$

Thus, for transistor M_j, $i_j = g_{mj}(v_1 + v_2 + \cdots + v_j)$

The total drain noise current is,

$$\begin{aligned} i_{tot} &= i_1 + i_2 + \cdots + i_n \\ &= g_{m1} v_1 + g_{m2}(v_1+v_2) + \cdots + g_{mn}(v_1+v_2+\cdots+v_n) \end{aligned}$$

If $g_{m1} = g_{m2} = \cdots = g_{mn} = \frac{g_m}{n}$ then

$$i_{tot} = \frac{g_m}{n}\left[n v_1 + (n-1)v_2 + \cdots + v_n\right]$$

Assuming $v_1, \ldots, v_n$ are uncorrelated,

$$\overline{i_{tot}^2} = \frac{g_m^2}{n^2}\left[n^2 \overline{v_1^2} + (n-1)^2 \overline{v_2^2} + \cdots + \overline{v_n^2}\right]$$

If $R_1 = R_2 = \cdots = R_n = \frac{R_g}{n}$ then $\overline{v_1^2} = \overline{v_2^2} = \cdots = \overline{v_n^2} = 4kTB\frac{R_g}{n}$

$$\begin{aligned} \overline{i_{tot}^2} &= \frac{g_m^2}{n^2}\,\frac{4kTBR_g}{n}\left[n^2 + (n-1)^2 + \cdots + 1\right] \\ &= g_m^2 (4kTB) R_g \frac{n(n+1)(2n+1)}{6n^3} \end{aligned}$$

As $n \to \infty$

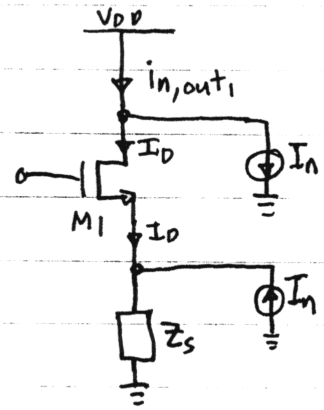
$$\overline{i_{tot}^2} = g_m^2 \left(4kTB\frac{R_g}{3}\right)$$

which can be referred to the input as

$$\overline{v_{tot}^2} = \frac{\overline{i_{tot}^2}}{g_m^2} = 4kTB\left(\frac{R_g}{3}\right)$$

$\Rightarrow$ lumped resistance $= \frac{R_g}{3}$ //

(7.4)



$I_{n,out_1} = I_D + I_n$, KCL@ drain

$I_D = \frac{-Z_S}{(\frac{1}{g_m} \| r_o) + Z_S} I_n$, current divider @ source

$\therefore I_{n,out_1} = \left(\frac{-Z_S}{(1/g_m \| r_o) + Z_S} + 1 \right) I_n$

$\therefore I_{n,out_1} = \frac{I_n}{Z_S(g_m + \frac{1}{r_o}) + 1}$ //

(7.5) $|A_v| = (g_{m1} + g_{m2})(r_{o1} \| r_{o2})$

$\overline{V_{n,out}^2} = 4kT\frac{2}{3}(g_{m1} + g_{m2})(r_{o1} \| r_{o2})^2$

$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1} + g_{m2}}\right)$

eqn(7.57) $\overline{V_{n,in}^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$

increasing g_{m2} increases $\overline{V_{n,in}^2}$ in eqn. (7.57) but reduces $\overline{V_{n,in}^2}$ for amplifier in figure 7.49.

(7.6)(a) $|A_v| = \frac{g_m R_D}{1 + g_m R_S}$

$\overline{V_{n,out}^2} = 4kTR_D + 4kT\frac{2}{3}\frac{1}{g_m}\left(\frac{g_m R_D}{1 + g_m R_S}\right)^2 + 4kT\frac{1}{R_S}\left(\frac{R_S}{\frac{1}{g_m} + R_S}\right)^2 R_D^2$

$= 4kTR_D + 4kT\frac{2}{3}\frac{1}{g_m}\left(\frac{g_m R_D}{1 + g_m R_S}\right)^2 + 4kTR_S\left(\frac{g_m R_D}{1 + g_m R_S}\right)^2$

$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\frac{2}{3}\frac{1}{g_m} + 4kTR_S + 4kTR_D\left(\frac{1 + g_m R_S}{g_m R_D}\right)^2$ //

(7.6)(b) $|A_v| = g_m\left(\frac{1}{g_m} \| R_S\right)$

$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_m + 4kT\frac{1}{R_S}\right)\left(\frac{1}{g_m} \| R_S\right)^2$

$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\frac{2}{3}\frac{1}{g_m} + 4kT\frac{1}{g_m^2 R_S}$ //

(7.6)(c) $|A_V| = \frac{g_m}{1+(g_m+\frac{1}{R_F})R_S} \cdot R_{out}$

$R_{out} = R_S + (1+g_m R_S)R_F$

$$\overline{V_{n,out}^2} = \left(\frac{4kT\frac{2}{3}g_m}{(1+(g_m+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_F}}{(1+(g_m+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_m}//R_F)^2} \right) R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_m} + \frac{1}{g_m^2 R_F} + R_S\left(1+\frac{1}{g_m R_F}\right)^2\right) \quad //$$

(7.6)(d) $|A_V| = \frac{g_{m1}}{1+g_{m1}R_S} \cdot R_{out}$

$R_{out} = \frac{1}{g_{m2}}$

$$\overline{V_{n,out}^2} = 4kT\frac{2}{3}g_{m2}R_{out}^2 + 4kT\frac{2}{3}\frac{1}{g_{m1}}|A_V|^2 + 4kT\frac{1}{R_S}\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2 R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{2}{3}g_{m2}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2\right] \quad //$$

(7.6)(e) $|A_V| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_D}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) \quad //$$

M2 + R_F do not contribute noise because $r_{o1} = \infty$

(7.6)(f) $|A_V| = g_{m1}\left(\frac{g_{m2}R_S}{1+g_{m2}R_S}\right)R_D$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}\frac{1}{g_{m2}}\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2 + 4kT\frac{2}{3}\frac{1}{g_{m1}}\left(\frac{g_{m1}R_S}{\frac{1}{g_{m2}}+R_S}\right)^2 + 4kT\frac{1}{R_S}\left(\frac{R_S}{\frac{1}{g_{m2}}+R_S}\right)^2\right]\cdot R_D^2$$

note: $\frac{R_S}{\frac{1}{g_{m2}}+R_S} = \frac{g_{m2}R_S}{1+g_{m2}R_S}$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{1}{g_{m2}}\frac{1}{(g_{m1}R_S)^2} + \frac{1}{g_{m1}^2 R_S} + \frac{1}{g_{m1}^2 R_D}\left(\frac{1+g_{m2}R_S}{g_{m2}R_S}\right)^2\right] \quad //$$

(7.7)(a) $|A_v| = \dfrac{g_{m1} R_{out}}{1+(g_{m1}+\frac{1}{R_F})R_S}$

$R_{out} = R_S + (1+g_{m1}R_S)R_F$

$$\overline{V_{n,out}^2} = \left[\frac{4kT\frac{2}{3}g_{m1}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_F}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_{m1}}/\!/R_F)^2} + 4kT\frac{2}{3}g_{m2}\right]R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2R_F} + R_S\left(1+\frac{1}{g_{m1}R_F}\right)^2 + \frac{2}{3}g_{m2}\left(\frac{1+(g_{m1}+\frac{1}{R_F})R_S}{g_{m1}}\right)^2\right] //$$

(7.7)(b) $|A_v| = \left(g_{m2} + \dfrac{g_{m1}}{1+(g_{m1}+\frac{1}{R_F})R_S}\right)\cdot R_{out}$

$R_{out} = R_S + (1+g_{m1}R_S)R_F$

$$\overline{V_{n,out}^2} = \left[\frac{4kT\frac{2}{3}g_{m1}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_F}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_{m1}}/\!/R_F)^2} + 4kT\frac{2}{3}g_{m2}\right]R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{1}{g_{m1}+g_{m2}(1+(g_{m1}+\frac{1}{R_F})R_S)}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_F} + R_S\left(g_{m1}+\frac{1}{R_F}\right)^2 + \frac{2}{3}g_{m2}\left(1+(g_{m1}+\frac{1}{R_F})R_S\right)^2\right] //$$

(7.7)(c) $|A_v| = \left(\dfrac{g_{m1}}{1+g_{m1}R_S}\right)(1+g_{m2}R_S)(R_D)$

$$\overline{V_{n,out}^2} = 4kTR_D + 4kT\frac{2}{3}g_{m2}R_D^2 + 4kT\frac{2}{3}\frac{1}{g_{m1}}|A_v|^2 + 4kT\frac{1}{R_S}\left[\frac{R_S}{\frac{1}{g_{m1}}+R_S} - \frac{\frac{1}{g_{m1}}R_S}{\frac{1}{g_{m1}}+R_S}g_{m2}\right]^2 R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \left(\frac{1}{R_D}+\frac{2}{3}g_{m2}\right)\left(\frac{1+g_{m1}R_S}{g_{m1}(1+g_{m2}R_S)}\right)^2 + R_S(g_{m1}-g_{m2})^2\left(\frac{1}{g_{m1}(1+g_{m2}R_S)}\right)^2\right] //$$

(7.7)(d) $|A_v| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m3} + 4kT\frac{2}{3}g_{m1}\right]R_D^2$$

M2 does not contribute any noise because r_{o1} and $r_{o3} = \infty$.

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{g_{m3}}{g_{m1}^2} + \frac{1}{g_{m1}^2R_D}\right] //$$

(7.8)(a) $|A_v| = g_{m1} R_D$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + 4kTR_G g_{m1}^2\right]R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + R_G + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.8)(b) $|A_v| = (g_{m1} + \frac{1}{R_1})(R_1 // R_D)$

$$\overline{V_{n,out}^2} = \left[4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_1} + 4kT\frac{1}{R_D}\right](R_1//R_D)^2$$

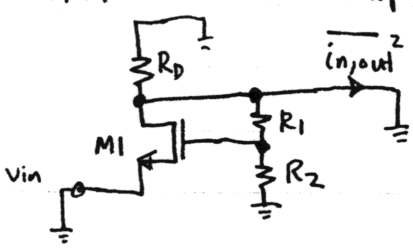
$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{1}{g_{m1} + \frac{1}{R_1}}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_1} + \frac{1}{R_D}\right] //$$

(7.8)(c) $A_v = (-g_{m1} + \frac{1}{R_F})(R_F // R_D)$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_F} + \frac{1}{R_D} + \frac{2}{3}g_{m1}\right)(R_F//R_D)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{1}{-g_{m1} + \frac{1}{R_F}}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_F} + \frac{1}{R_D}\right] //$$

(7.8)(d) Find short circuit output noise current $\overline{i_{n,out}^2}$

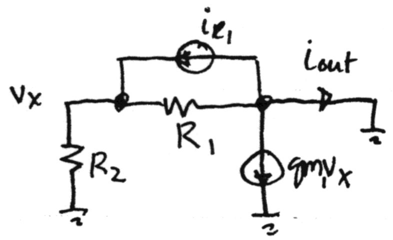


$$\overline{i_{n,out}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + \overline{i_{noise\,R_1}^2} + \overline{i_{noise\,R_2}^2}$$

$$\overline{i_{noise\,R_1}^2} = 4kT\frac{1}{R_1}|A_{I,R1}|^2$$

$$\overline{i_{noise\,R_2}^2} = 4kT\frac{1}{R_2}|A_{I,R2}|^2$$

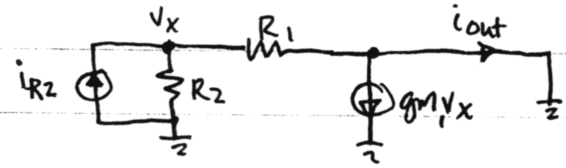
- small signal model used to find $A_{I,R1}$



$$A_{I,R_1} = \frac{i_{out}}{i_{R_1}} = -\left(1 + (R_1 // R_2)(g_{m1} - \frac{1}{R_1})\right)$$

(7.8) (d) cont.

- small signal model used to find $A_{I,R2}$



$$A_{I,R_2} = \frac{i_{out}}{i_{R2}} = \left[\frac{R_2}{R_1+R_2} - g_{m1}(R_1 // R_2)\right]$$

$$\therefore \overline{i_{n,out}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_1}\left(1+(R_1//R_2)(g_{m1}-\frac{1}{R_1})\right)^2 + 4kT\frac{1}{R_2}\left[\frac{R_2}{R_1+R_2} - g_{m1}(R_1//R_2)\right]^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{i_{n,out}^2}}{g_{m1}^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{1}{g_{m1}^2 R_1}\left(1+(R_1//R_2)(g_{m1}-\frac{1}{R_1})\right)^2 + \frac{1}{g_{m1}^2 R_2}\left(\frac{R_2}{R_1+R_2} - g_{m1}(R_1//R_2)\right)^2\right] //$$

(7.9)(a) $|A_v| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_D} + \frac{2}{3}g_{m1}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.9)(b) $|A_v| = g_{m1}(R_D // \frac{1}{g_{m2}})$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_D} + \frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)(R_D // \frac{1}{g_{m2}})^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right) //$$

(7.9)(c) $|A_v| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{2}{3}g_{m1} + \frac{1}{R_D}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.9)(d) $|A_v| = \frac{g_{m1}}{g_{m1}+g_{m2}}$

$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_{m1} + 4kT\frac{2}{3}g_{m2}\right)\left(\frac{1}{g_{m1}+g_{m2}}\right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right) //$$

(7.9)(e) $|A_v| = 1$, $\overline{V_{n,in}^2} = \overline{V_{n,out}^2} = 4kT\frac{2}{3}\frac{1}{g_{m1}}$ //

(7.10) • With the input shorted to ground,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4}\right](r_{o1} \| r_{o3})^2$$

$$|A_v| = (g_{m1} + g_{mb1})(r_{o1} \| r_{o3})$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4}\right]\frac{1}{(g_{m1} + g_{mb1})^2} \quad //$$

• With the input open,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f}\left[g_{m2}^2 k_n\left(\frac{1}{(WL)_0} + \frac{1}{(WL)_2}\right) + g_{m3}^2 k_p\left(\frac{1}{(WL)_3} + \frac{1}{(WL)_4}\right)\right] R_{out}^2$$

$$R_{out} \cong r_{o3} \| (g_{m1} r_{o1} r_{o2})$$

$$\overline{I_{n,in}^2} = \frac{1}{C_{ox} f}\left[g_{m2}^2 k_n\left(\frac{1}{(WL)_0} + \frac{1}{(WL)_2}\right) + g_{m3}^2 k_p\left(\frac{1}{(WL)_3} + \frac{1}{(WL)_4}\right)\right] \quad //$$

(7.11)

$$\overline{V_{n,out}^2} = \frac{K}{C_{ox}(WL)_1}\cdot\frac{1}{f}(g_{m1} R_{out})^2 + \frac{K}{C_{ox}(WL)_2}\cdot\frac{1}{f}(g_{m2} R_{out})^2$$

$$R_{out} = \left(\frac{1}{g_{m1}} \| \frac{1}{g_{mb1}} \| r_{o1} \| r_{o2}\right)$$

$$|A_v| = g_{m1} R_{out}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{K}{C_{ox} f}\left[\frac{1}{(WL)_1} + \frac{g_{m2}^2}{(WL)_2\, g_{m1}^2}\right] \quad //$$

(7.12)(a)

$$\overline{V_{n,out}^2} = 4KT\frac{2}{3}(g_{m1} + g_{m2} + g_{m3} + g_{m4} + g_{m5} + g_{m6}) R_{out}^2$$

$$|A_v| = g_{m1} R_{out}$$

$$g_{m1} = g_{m2}, \quad g_{m3,4} = 0.5\, g_{m5,6}$$

$$\overline{V_{n,in}^2} = 4KT\frac{2}{3}\left[\frac{2}{g_{m1}} + \frac{3 g_{m5}}{g_{m1}^2}\right] \quad //$$

(7.12)(b)

$$|A_v| = g_{m1}(r_{o2} \| r_{o4})$$

$$\overline{V_{n,out}^2} = 4KT\frac{2}{3}[g_{m1} + g_{m2} + g_{m3} + g_{m4}]$$

$$g_{m1} = g_{m2}, \quad g_{m3} = g_{m4}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left(\frac{2}{3}\right)\left[\frac{2}{g_{m1}} + \frac{2 g_{m3}}{g_{m1}^2}\right] \quad //$$

(7.13)(a) $|A_v| = \dfrac{g_{m1} R_D}{1 + g_{m1} R_S}$

$$\overline{V_{n,out}^2} = 4KT R_D + 4KT \frac{2}{3} \frac{1}{g_{m1}} |A_v|^2 + 4KT \frac{1}{R_S} \left(\frac{R_S}{R_S + \frac{1}{g_{m1}}} \right)^2 R_D^2$$

$$\overline{V_{n,in}^2} = 4KT \left[\frac{2}{3} \frac{1}{g_m} + R_S + \frac{1}{R_D} \left(\frac{1 + g_{m1} R_S}{g_{m1}} \right)^2 \right]$$

(7.13) (b) $IR_S = V_{GS} - V_T \Rightarrow R_S = \dfrac{V_{GS} - V_T}{I}$

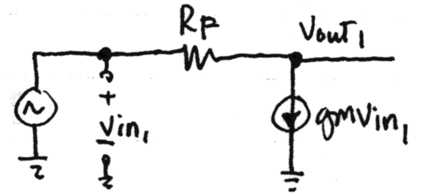
$$g_{m1} = \frac{2I}{V_{GS} - V_T} = \frac{2}{R_S}$$

$\therefore 4KT \left[\frac{2}{3} g_m \right] = 4KT \frac{R_S}{3}$ ← Thermal noise of M1

$4KT R_S$ ← Thermal noise of R_S

$\therefore$ R_S contributes 3x more noise power ($\overline{V_{n,in}^2}$) than M1 when $IR_S = V_{GS} - V_T$.

(7.14) Consider the following ckt with noise only due to resistor R_F:

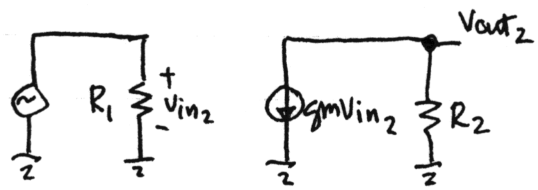


$$A_{v1} = \frac{V_{out1}}{V_{in1}} = -g_m R_F + 1$$

$$\overline{V_{n,out1}^2} = 4KT R_F$$

$$\overline{V_{n,in1}^2} = \frac{\overline{V_{n,out1}^2}}{|A_{v1}|^2} = 4KT R_F \left(\frac{1}{-g_m R_F + 1} \right)^2$$

using the Miller effect, we have the following ckt:



$$A_{v2} = \frac{V_{out2}}{V_{in2}} = -g_m R_2 = \frac{-g_m R_F A_{v1}}{A_{v1} - 1} = (-g_m R_F + 1) = A_{v1}$$

$$\overline{V_{n,out2}^2} = 4KT R_2$$

$$\overline{V_{n,in2}^2} = \frac{\overline{V_{n,out2}^2}}{|A_{v2}|^2} = 4KT \frac{1}{g_m^2 R_2} = 4KT \frac{1}{g_{m1}} \left(\frac{-1}{-g_{m1} R_F + 1} \right)$$

$$R_1 = \frac{R_F}{1 - A_{v1}} \qquad R_2 = \frac{R_F}{1 - \frac{1}{A_{v1}}}$$

notice: $\overline{V_{n,in1}^2} \neq \overline{V_{n,in2}^2} \Rightarrow$ cannot use Miller's Theorem //

(7.15) Using equation (7.26)

$$\overline{V_{n,out}^2} = 4kT\left(\tfrac{2}{3} g_m\right) r_o^2$$
$$= (1.656\times10^{-20}\,V\cdot C)\left(\tfrac{2}{3}\right)\left(4.44\,\tfrac{mA}{V}\right)(20\,k\Omega)^2$$
$$= 19.6\times10^{-15}\,\frac{V^2}{Hz}$$
$$\overline{V_{n,out_{TOT}}} = \sqrt{\left(19.6\times10^{-15}\tfrac{V^2}{Hz}\right)(50\,MHz)} \cong 990\,\mu V\ rms \quad //$$

(7.16) $|A_V|^2 = \dfrac{[g_m(R_D\|r_o)]^2}{1+(2\pi(R_D\|r_o)C_L f)^2}$

$$\overline{V_{n,out}^2} = 4kT\frac{(R_D\|r_o)^2}{R_D}\frac{1}{1+(2\pi(R_D\|r_o)C_L f)^2} + 4kT\frac{2}{3}g_m(R_D\|r_o)^2\frac{1}{1+(2\pi(R_D\|r_o)C_L f)^2}$$
$$+ \frac{k}{C_{ox}WL}g_m^2(R_D\|r_o)^2\frac{1}{1+(2\pi(R_D\|r_o)C_L f)^2}$$

$$\overline{V_{n,out_{TOT}}^2} = 4kT(R_D\|r_o)\left[\frac{(R_D\|r_o)}{R_D}+\frac{2}{3}g_m(R_D\|r_o)\right]\int_{f_L}^{f_H}\frac{df}{1+(2\pi(R_D\|r_o)C_L f)^2}$$
$$+ \frac{k g_m^2(R_D\|r_o)^2}{C_{ox}WL}\int_{f_L}^{f_H}\frac{df}{f(1+(2\pi(R_D\|r_o)C_L f)^2)}$$

$$\overline{V_{n,out_{TOT}}^2} = \frac{2kT}{\pi C_L}\left[\frac{(R_D\|r_o)}{R_D}+\frac{2}{3}g_m(R_D\|r_o)\right]\left[\tan^{-1}(2\pi(R_D\|r_o)C_L f_H) - \tan^{-1}(2\pi(R_D\|r_o)C_L f_L)\right]$$
$$+ \frac{k g_m^2(R_D\|r_o)^2}{C_{ox}WL}\int_{f_L}^{f_H}\frac{df}{f[1+(2\pi(R_D\|r_o)C_L f)^2]} \quad //$$

(7.17) Using equation number (7.57)

$$\overline{V_{n,in}^2} = 4kT\left(\tfrac{2}{3}\right)\left(\frac{1}{g_{m,1}} + \frac{g_{m2}}{g_{m,1}^2}\right)$$
$$g_{m1} = \sqrt{2(0.5mA)\left(134.29\tfrac{\mu A}{V^2}\right)\left(\tfrac{50}{.34}\right)} = 4.44\,\frac{mA}{V}$$
$$g_{m2} = \sqrt{2(0.5mA)\left(38.37\tfrac{\mu A}{V^2}\right)\left(\tfrac{50}{.34}\right)} = 2.36\,\frac{mA}{V}$$
$$\therefore \overline{V_{n,in}^2} = (1.656\times10^{-20}\,V\cdot C)\left(\tfrac{2}{3}\right)(225.23\,\Omega + 119.71\,\Omega)$$
$$\cong 3.81\times10^{-18}\,\frac{V^2}{Hz}$$
$$\overline{V_{n,in}} \cong 1.95\,\frac{nV}{\sqrt{Hz}} \quad //$$

(7.18) (a) $|A_v| = g_{m1} R_{out}$

$$R_{out} = r_{o1} \,//\, (R_S + (1+g_{m2}R_S) r_{o2})$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_S}\left(\frac{R_S}{R_S+\frac{1}{g_{m2}}}\right)^2 + 4kT\frac{2}{3}\frac{1}{g_{m2}}\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2 + 4kT\frac{2}{3}g_{m1}\right] R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2\frac{1}{g_{m1}^2} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\left(\frac{1}{1+g_{m2}R_S}\right)^2\right]$$

(b) R_S large

(7.19) Neglecting body effect and using eqns 7.60 and 7.61

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3}\frac{1}{g_m^2} + \frac{1}{g_m^2 R_D}\right)$$

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$

$$g_m = \sqrt{2(1mA)(134.29\frac{\mu A}{V^2})(\frac{50}{.34})} = 6.28\frac{mA}{V}$$

$$\therefore \overline{V_{n,in}^2} = (1.656\times10^{-20} V\cdot C)(\frac{2}{3}\, 159.24\Omega + 25.36\Omega) \cong 2.18\times10^{-18}\frac{V^2}{Hz} \;//$$

$$\overline{I_{n,in}^2} = \frac{(1.656\times10^{-20} V\cdot C)}{1k\Omega} = 16.56\times10^{-24}\frac{A^2}{Hz} \;//$$

(7.20) (a) $\overline{I_{n,in}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m2}$

$$\overline{I_{n,in}} = \sqrt{4kT(\frac{1}{R_D} + \frac{2}{3}g_{m2})}$$

$$\therefore \frac{2}{3}g_{m2} = (\frac{1}{5})^2(\frac{1}{R_D})$$

$$\Rightarrow g_{m2} = (\frac{1}{25})(\frac{1}{1000})(\frac{3}{2}) = 60\frac{\mu A}{V}$$

$$\left(\frac{W}{L}\right)_2 = \frac{g_{m2}^2}{2I_D\mu_n C_{ox}} = \frac{(60\frac{\mu A}{V})^2}{2(.05mA)(134.29\frac{\mu A}{V^2})} \cong 0.268 \;//$$

(b) $g_{m2} = \frac{2I_D}{(V_{gs}-V_T)_2} \Rightarrow (V_{gs}-V_T)_2 = \frac{2I_D}{g_m} = \frac{2(.05mA)}{60\frac{\mu A}{V}} \cong 1.67\ V$

$$(V_{gs}-V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})_1}} = \sqrt{\frac{2(.05mA)}{134.29\frac{\mu A}{V^2}(\frac{50}{.34})}} \cong 71.2mV$$

neglecting body effect

$$V_b = (V_{gs}-V_t)_2 + V_{GS1} = 1.67 + 0.0712 + 0.7 = 2.4412V \;//$$

$$\text{Output Swing} = V_{CC} - (V_{gs}-V_t)_1 - (V_{gs}-V_t)_2 = 3 - 1.67 - 0.0712 = 1.2588V \;//$$

Note: output swing is not symmetric.

(7.21) Neglecting body effect and using the result of eqn. 7.60

$$\overline{V_{n,in}} = \sqrt{4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)} = 3\,\frac{nV}{\sqrt{Hz}}$$

$$\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} = 543.4\,\Omega$$

note:

$$\frac{1}{g_{m1}} = \frac{(V_{GS}-V_T)}{2I_D} \triangleq \frac{\Delta V}{2I_D} \qquad \text{also define } R_N = 543.4\,\Omega$$

$$\Rightarrow \Delta V^2 + \frac{4}{3} I_D R_D \Delta V - 4 I_D^2 R_D R_N = 0 \quad ; \quad I_D = 0.5\text{mA}$$

One possible answer assumming $(\frac{W}{L})_1 = (\frac{W}{L})_2$ and a 3V supply

$$\Delta V_1 = \Delta V_2 = 562\text{mV} \quad ; \quad R_D = 1875\,\Omega$$

$$\Rightarrow \text{Output swing} = 2 \cdot I_D R_D = 1.875\text{ V} \quad //$$

$$g_{m1} = g_{m2} = \frac{2I_D}{\Delta V} = 1.78\ \text{mA/V}$$

$$\left(\frac{W}{L_{eff}}\right) = \frac{g_m^2}{2I_D \mu_n C_{ox}} = \frac{(1.78\,\text{mA/V})^2}{2(0.5\text{mA})(134.29\frac{\mu A}{V^2})} \cong 23.6 \quad //$$

$$V_b = \Delta V_2 + V_{GS2} \cong 0.562\text{ V} + 0.562\text{V} + 0.7\text{V} = 1.824\text{ V} \quad //$$

(7.22) Neglecting body effect and using eqns 7.64 + 7.65

$$\overline{V_{n,in}^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right)$$

$$\overline{I_{n,in}^2} = 4kT\frac{2}{3}(g_{m2} + g_{m3})$$

$$g_{m1} = g_{m2} = \sqrt{2(0.5\text{mA})(134.29\tfrac{\mu A}{V^2})(\tfrac{50}{.34})} \cong 4.44\ \frac{mA}{V}$$

$$g_{m3} = \sqrt{2(0.5\text{mA})(38.37\tfrac{\mu A}{V^2})(\tfrac{50}{.34})} \cong 2.38\ \frac{mA}{V}$$

$$\therefore \quad \overline{V_{n,in}^2} = (1.656\times10^{-20}\,\text{V}\cdot\text{C})\left(\frac{2}{3}\right)(225.23\Omega + 120.73\Omega) \cong 3.82\times10^{-18}\ \frac{V^2}{Hz} \quad //$$

$$\overline{I_{n,in}^2} = (1.656\times10^{-20}\,\text{V}\cdot\text{C})\left(\frac{2}{3}\right)\left(4.44\,\frac{mA}{V} + 2.38\,\frac{mA}{V}\right) \cong 75.3\times10^{-24}\ \frac{A^2}{Hz} \quad //$$

(7.23) Neglect body effect and use eqn. 7.65

$$\overline{I_{n,in}^2} = 4KT\,\tfrac{2}{3}\,(g_{m2} + g_{m3})$$

$$g_{m1} = \sqrt{2(0.5mA)(134.29\,\tfrac{\mu A}{V^2})(\tfrac{50}{.134})} = 4.44\,\tfrac{mA}{V}$$

$$(V_{gs} - V_t)_1 = \frac{2I_D}{g_{m1}} = \frac{2(0.5mA)}{4.44\,mA/V} \cong 225.23\ mV$$

define $\Delta V_x = |(V_{gs} - V_t)|_x$

$\therefore$ Output Swing $= V_{CC} - (\Delta V_1 + \Delta V_2 + \Delta V_3) = 2V$; $V_{CC} = 3V$

$$\Rightarrow \Delta V_2 + \Delta V_3 = 774.77\ mV$$

note: $g_m = \dfrac{2I_D}{\Delta V}$

$$\therefore \overline{I_{n,in}^2} = 4KT\left(\tfrac{2}{3}\right)2I_D\left(\frac{\Delta V_2 + \Delta V_3}{\Delta V_2 \Delta V_3}\right)$$

to minimize $\overline{I_{n,in}^2}$ let $\Delta V_2 = \Delta V_3$ $\Rightarrow g_{m2} = g_{m3} = \dfrac{2(0.5mA)}{0.387V} = 2.58\,\dfrac{mA}{V}$

$$\left(\frac{W}{L_{eff}}\right)_2 = \frac{(g_{m2})^2}{2I_D\mu_n C_{ox}} = \frac{(2.58\,\frac{mA}{V})^2}{2(0.5mA)(134.29\,\frac{\mu A}{V^2})} \cong 49.7 \;//$$

$$\left(\frac{W}{L_{eff}}\right)_3 = \frac{(g_{m3})^2}{2I_D\mu_p C_{ox}} = \frac{(2.58\,\frac{mA}{V})^2}{2(0.5mA)(38.37\,\frac{\mu A}{V^2})} \cong 174 \;//$$

(7.24) (a) Neglecting body effect and r_{o1}, r_{o2}

$$R_{out} = \frac{1}{g_{m1}} = \frac{1}{\sqrt{2 I_D \mu_n C_{ox} \left(\frac{W}{L}\right)_1}} = 100\Omega$$

$$\left(\frac{W}{L_{eff}}\right)_1 = \frac{1}{2 I_D \mu_n C_{ox} R_{out}^2} = \frac{1}{2(1mA)(134.29\frac{\mu A}{V^2})(100\Omega)^2} \cong 3723$$

(b) Using the result from eqn 7.73

$$\overline{V_{n,in}} = \sqrt{4KT\left(\frac{2}{3}\right)\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)}$$

$$\frac{g_{m2}}{g_{m1}^2} = \left(\frac{1}{5}\right)^2 \frac{1}{g_{m1}} \Rightarrow g_{m2} = \left(\frac{1}{5}\right)^2 g_{m1} = \frac{1}{2500\Omega}$$

$$(V_{gs}-V_T)_2 = \frac{2I_D}{g_{m2}} = 0.5V \qquad \left(\frac{W}{L_{eff}}\right)_2 = \frac{(g_{m2})^2}{2 I_D \mu_n C_{ox}} = \frac{(400\mu A/V)^2}{2(1mA)(134.29\frac{\mu A}{V^2})} \cong 5.96$$

$$(V_{gs}-V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.1mA)}{(134.29\mu A)(3723)}} \cong 20mV$$

neglecting body effect

$$V_{GS1} \approx 0.7 + 0.02 = 0.72$$

$\therefore$ Output swing $\approx V_{CC} - V_{GS1} - (V_{GS}-V_T)_2 = 3 - 0.72 - 0.5 = 1.78V$

(7.25) $|A_v|^2 = g_{m1}^2 \left(\frac{g_{m2}}{\omega C_X}\right)^2 \left(\frac{1}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2}\right) R_D^2$ $\qquad \omega = 2\pi f$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}\frac{1}{g_{m2}}\frac{g_{m2}^2}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2} + 4kT\frac{2}{3}g_{m1}\left(\frac{g_{m2}}{\omega C_X}\right)^2\left(\frac{1}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2}\right)\right]R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{1}{g_{m2}}\left(\frac{\omega C_X}{g_{m1}}\right)^2 + \frac{1}{R_D}\left(\frac{g_{m2}^2 + (\omega C_X)^2}{g_{m1}^2 g_{m2}^2}\right)\right] \quad //$$

(7.26) (a) $M_1 = M_2$, M_3 does not contribute differential noise

$$|A_v| = \frac{g_{m1}R_1}{1+g_{m1}R_S}$$

$$\overline{V_{n,out_a}^2} = \left[2(4kT)\frac{1}{R_D} + 2\left(4kT\frac{2}{3}\right)\frac{1}{g_{m1}}\left(\frac{g_{m1}}{1+g_{m1}R_S}\right)^2 + 2\left(4kT\frac{1}{R_S}\right)\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2\right]R_D^2$$

$$\overline{V_{n,in_a}^2} = \frac{\overline{V_{n,out_a}^2}}{|A_v|^2} = 2(4kT)\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{1}{R_D}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2\right] \quad //$$

(b) $M1 = M2$, $M_3 = M_4$

$$\overline{V_{n,out_b}^2} = \overline{V_{n,out_a}^2} + 2(4kT)\left(\frac{2}{3}\right)g_{m3}\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2 R_D^2$$

$$\overline{V_{n,in_b}^2} = \frac{\overline{V_{n,out_b}^2}}{|A_v|^2} = 2(4kT)\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{1}{R_D}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2 + \frac{2}{3}g_{m3}R_S^2\right] \quad //$$

$$\therefore \overline{V_{n,in_b}^2} = \overline{V_{n,in_a}^2} + 2(4kT)\left(\frac{2}{3}g_{m3}R_S^2\right)$$

M3 + M4 contribute differential noise in (b)