Chapter 7: kinetic Energy and Work

- ✓ kinetic Energy
- ✓ Work
- ✓ Work–Kinetic Energy Theorem
- ✓ Power

Session 14:

- ✓ Work–Kinetic Energy Theorem
- ✓ Power
- ✓ Examples

Work–Kinetic Energy Theorem

Work is an Energy Transfer

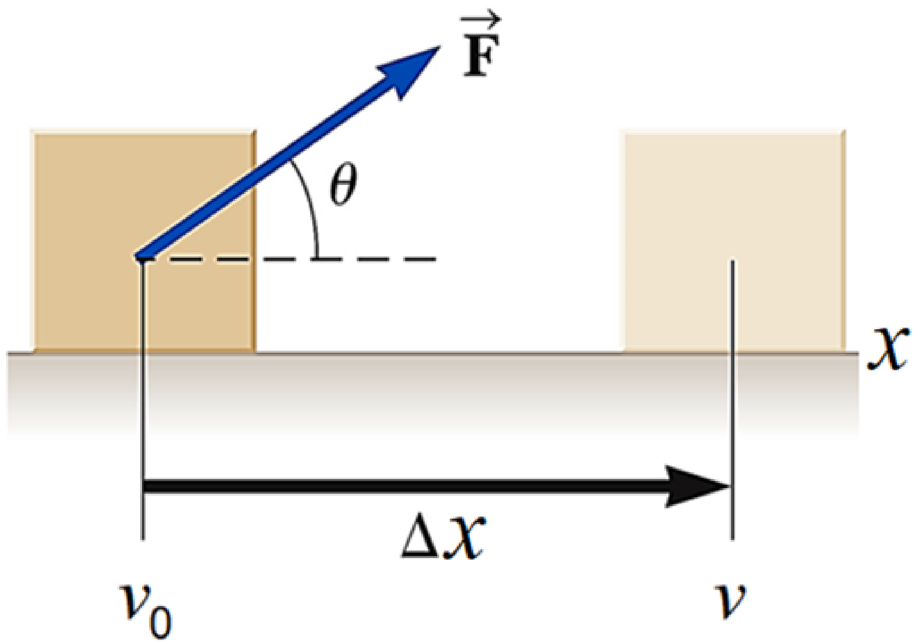
Positive work (W > 0): Energy transferred to the object

Negative work (W < 0): Energy transferred from the object

$$\mathbf{F}_{net,x} = ma_x = F\cos\theta$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2} = F\cos\theta\,\Delta x$$

$$\boxed{W = k_f - k_i = \Delta k}$$

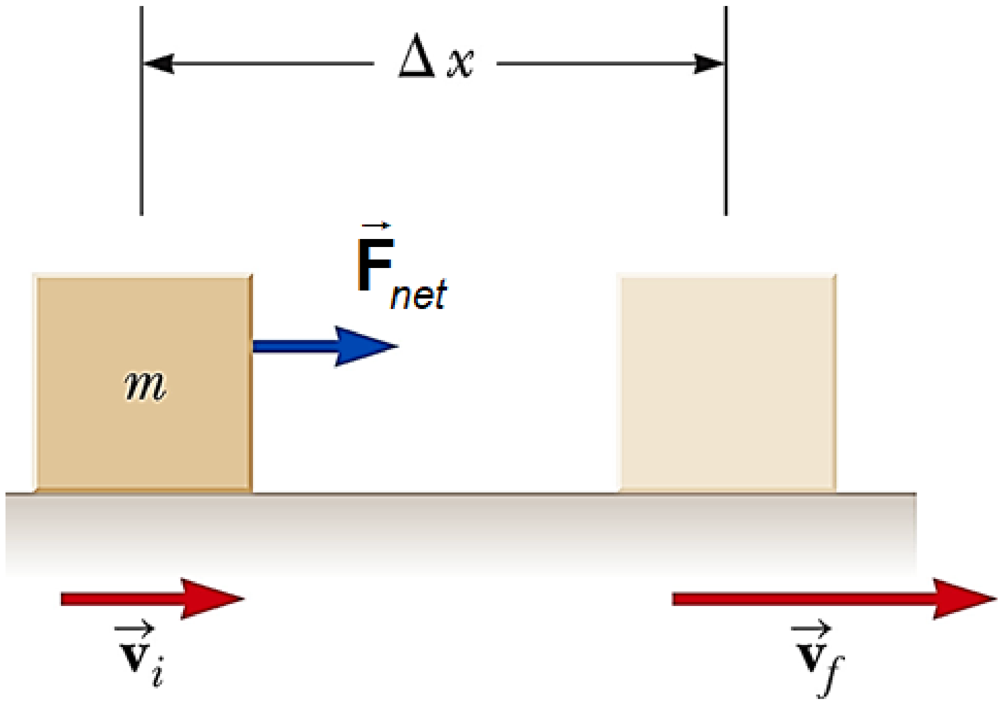


$$W_{ext} = \int_{x_i}^{x_f} \mathbf{F}_{net}\,dx = \int_{x_i}^{x_f} m\,a\,dx$$

$$W_{ext} = \int_{v_i}^{v_f} m\frac{dv}{dt}v\,dt = \int_{v_i}^{v_f} m\,v\,dv$$

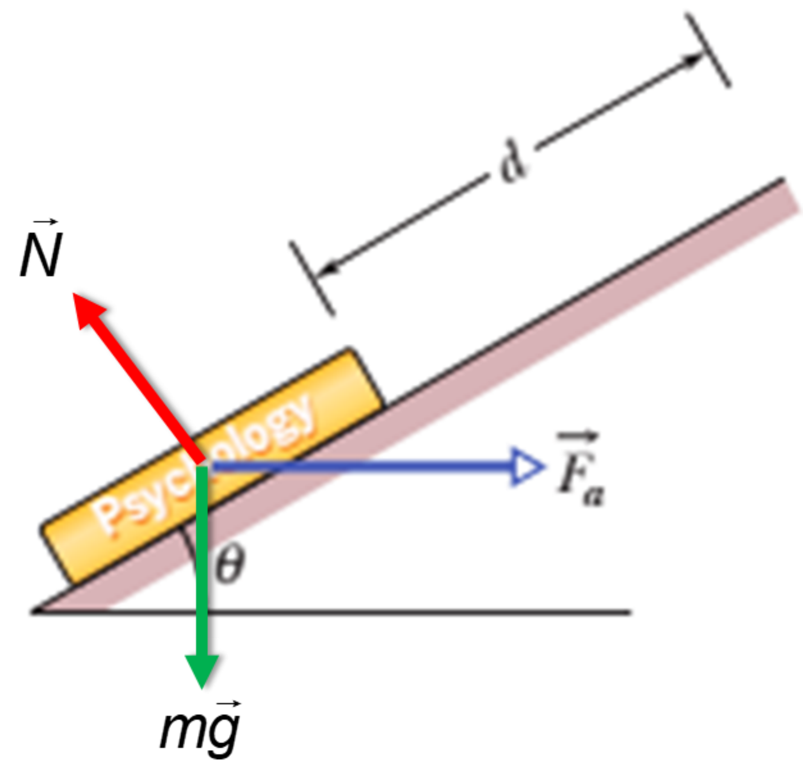
$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\boxed{W_{ext} = k_f - k_i = \Delta k}$$



Ex 4: (Problem 7.24 Halliday)

A horizontal force F_a of magnitude **20 N** is applied to a **3 kg** psychology book as the book slides a distance **d = 0.5 m** up a frictionless ramp at angle **θ = 30°** (a) During the displacement, what is the net work done on the book by F_a, the **gravitational force** on the book and the **normal force** on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?



$$\boxed{W = F\,\Delta r\cos\theta}$$

$$W_{F_a} = F_a\,d\cos 30 = 20\times 0.5\times 0.866 = 8.66 \text{ J}$$

$$W_{mg} = mg\,d\cos 120 = 3\times 9.8\times 0.5\times(-0.5) = -7.35 \text{ J}$$

$$W_N = N\,d\cos 90 = 0 \text{ J}$$

$$\boxed{W_{ext} = W_{F_a} + W_{mg} + W_N = 8.66 - 7.35 = 1.31 \text{ J}}$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \underbrace{\frac{1}{2}mv_i^2}_{0} = \frac{1}{2}mv_f^2$$

$$\boxed{v_f = \sqrt{\frac{2W_{ext}}{m}} = \sqrt{\frac{2\times 1.31}{3}} = 0.93\ m/s}$$

Ex 5 : An inclined plane of angle **θ = 20°** has a spring of force constant **k = 500 N/m** fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure. A block of mass **m = 2.50 kg** is placed on the plane at a distance **d = 0.300 m** from the spring. From this position, the block is projected downward toward the spring with speed **v = 0.750 m/s**. By what distance is the spring compressed when the block momentarily comes to rest?

$$\boxed{W_{ext} = \Delta k}$$

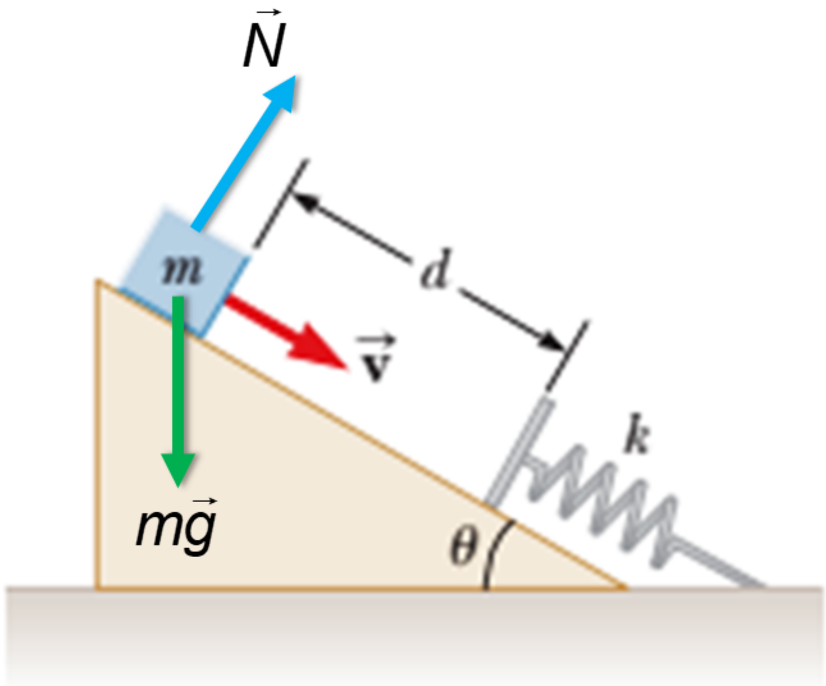
$$W_{ext} = W_{mg} + W_N + W_s$$

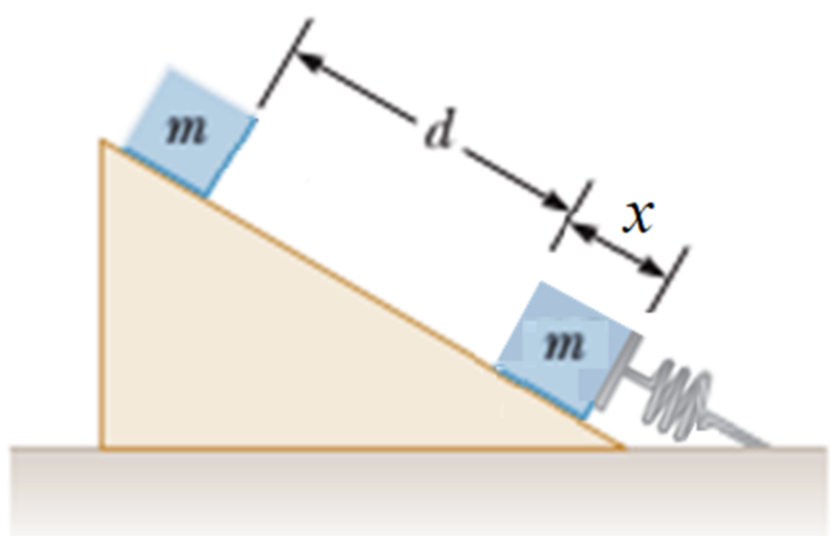
$$mg\sin\theta(d + x) - \frac{1}{2}k\,x^2 = 0 - \frac{1}{2}m v^2$$

$$\frac{1}{2}k\,x^2 - (mg\sin\theta)x - (\frac{1}{2}m v^2 + mg\sin\theta\, d) = 0$$

$$x = \frac{mg\sin\theta \pm \sqrt{(mg\sin\theta)^2 + 2k(\frac{1}{2}m v^2 + mg\sin\theta\, d)}}{k}$$

$$\boxed{x = 0.131\,m}$$





Ex 6: (Problem 7.31 Halliday)

The only force acting on a **2 kg** body as it moves along a positive x axis has an x component **F_x = -6x N**, with x in meters. The velocity at ***x* = 3 m** is **8 m/s**. (a) What is the velocity of the body at ***x* = 4 m**? (b) At what positive value of x will the body have a velocity of **5 m/s**?

$$\boxed{W_{ext} = \Delta k}$$

$$W_{\text{ext}} = \int_{x_i}^{x_f} F(x)dx = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\int_3^4 -6x\,dx = \frac{1}{2}(2)(v_f^2 - 8^2) \Rightarrow -3x^2\Big|_3^4 = v_f^2 - 64 \Rightarrow -21 = v_f^2 - 64$$

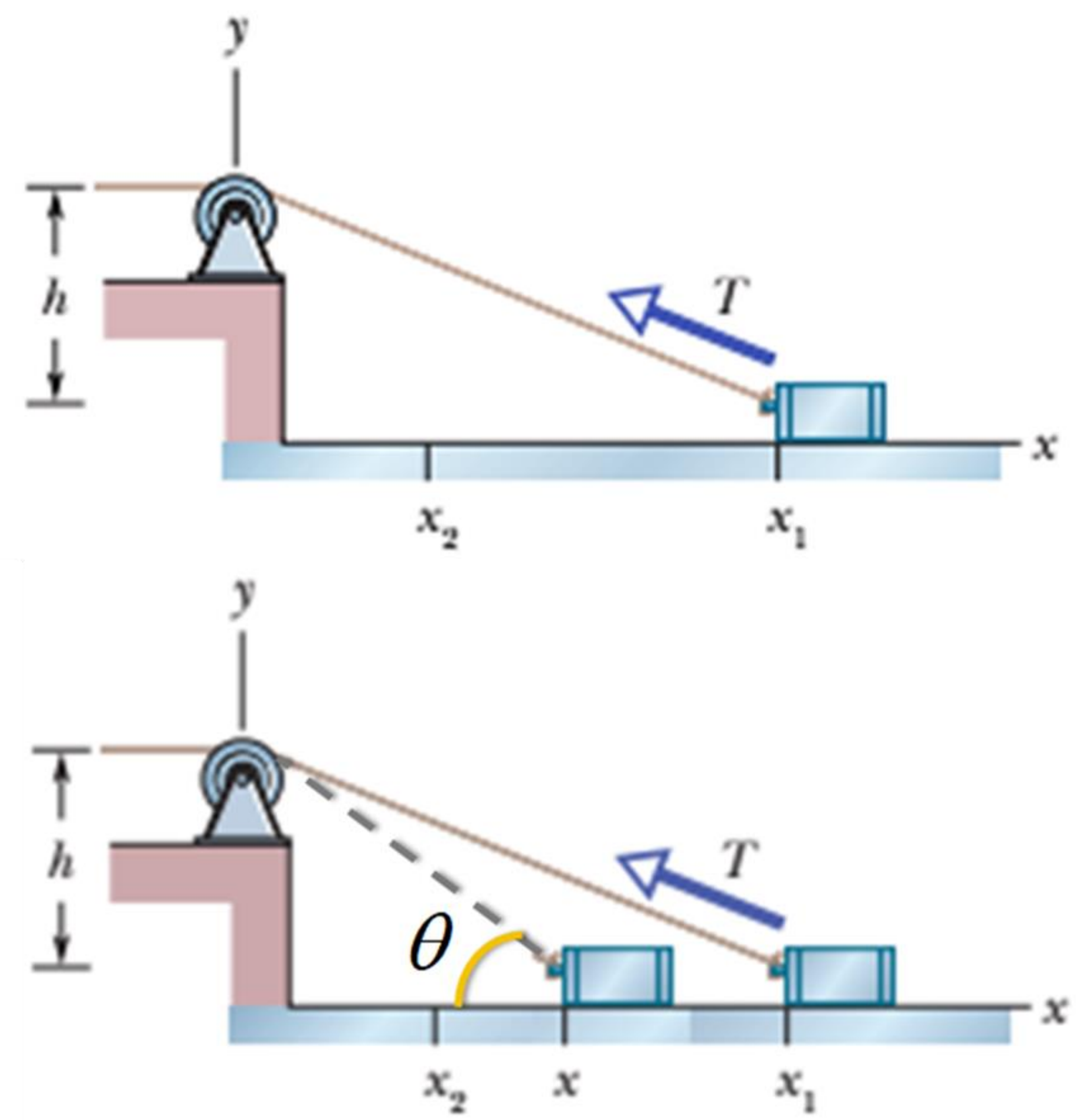
$$\boxed{v_f = \sqrt{43} = 6.6\ m/s}$$

$$\int_3^x -6x\,dx = \frac{1}{2}(2)(5^2 - 8^2) \Rightarrow -3x^2\Big|_3^x = 25 - 64 \Rightarrow 27 - 3x^2 = -39$$

$$\boxed{x = \sqrt{22} = 4.7\ \text{m}}$$

Ex 7: (Problem 7.42 Halliday)

Figure 7-41 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height **h = 1.20 m**, so the cart slides from $\mathbf{x_1 = 3\ m}$ to $\mathbf{x_2 = 1\ m}$. During the move, the tension in the cord is a constant **25 N**. What is the change in the kinetic energy of the cart during the move?



$$\boxed{W_{ext} = \Delta k} \qquad W_{ext} = \int_{x_1}^{x_2} F_x\, dx = \Delta k$$

$$F_x = -T\cos\theta = -T\frac{x}{\sqrt{h^2 + x^2}}$$

$$W_{ext} = \int_{x_1}^{x_2} -T\frac{x}{\sqrt{h^2 + x^2}}\, dx$$

$$W_{ext} = -T\int_{x_1}^{x_2} \frac{x\, dx}{\sqrt{h^2 + x^2}} = -T\sqrt{h^2 + x^2}\Big|_{x_1}^{x_2}$$

$$W_{ext} = T\left(\sqrt{h^2 + x_1^{\,2}} - \sqrt{h^2 + x_2^{\,2}}\right) \Rightarrow \boxed{W_{ext} = \Delta k = 41.7\ \text{J}}$$

Power

❑ **Power is the time rate of energy transfer.**

Average Power:

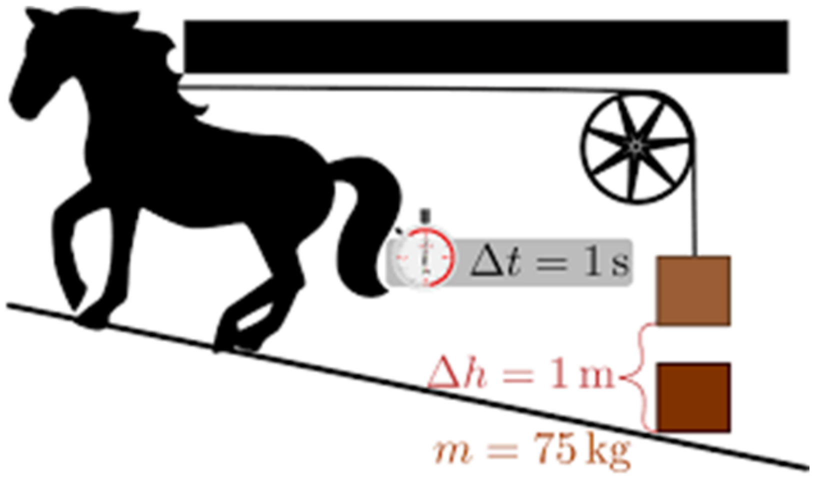
$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous Power:

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

1 Watt = 1 Joule / second = 1 kg · m^2 / s^3

Horsepower; 1 hp = 746 W



1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J

Ex 8 : An elevator car has a mass of **1600 kg** and is carrying passengers having a combined mass of **200 kg**. A constant friction force of **4000 N** retards its motion. **(a)** How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of **3 m/s**? **(b)** What power must the motor deliver at the instant the speed of the elevator is $\boldsymbol{v}$ if the motor is designed to provide the elevator car with an upward acceleration of **1 m/s²**?

$$\mathbf{F}_{net,y} = ma_y \Rightarrow T - Mg - f = Ma_y$$

$$if\ v = \textbf{constant};\ a_y = 0 \Rightarrow T = Mg + f$$

$$\boxed{P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \mathbf{T}v = (Mg + f)v = 6.49 \times 10^4\ W}$$

$$T = Mg + f + Ma_y = M(a_y + g) + f$$

$$\boxed{P = \mathbf{T}v = \left[M(a_y + g) + f\right]v = (2.34 \times 10^4)\ v}$$

$$\boxed{P = (2.34 \times 10^4)\ (3) = 7.02 \times 10^4\ W}$$