# Chapter 4: Electric Potential

- ✓ Electric Potential and Electrical Potential Energy
- ✓ Potential of a Point Charge
- ✓ Electric Potential for Multiple Charges
- ✓ Electric Potential for a Continuous Charge Distribution
- ✓ V Due to a Charged Conductor

## Session 10:

- ✓ **Electrical Potential Energy of Multiple Charges**
- ✓ **Electric Potential for a Continuous Charge Distribution**
- ✓ **V Due to a Charged Conductor**
- ✓ **Examples**

# Potential Energy of Multiple Charges

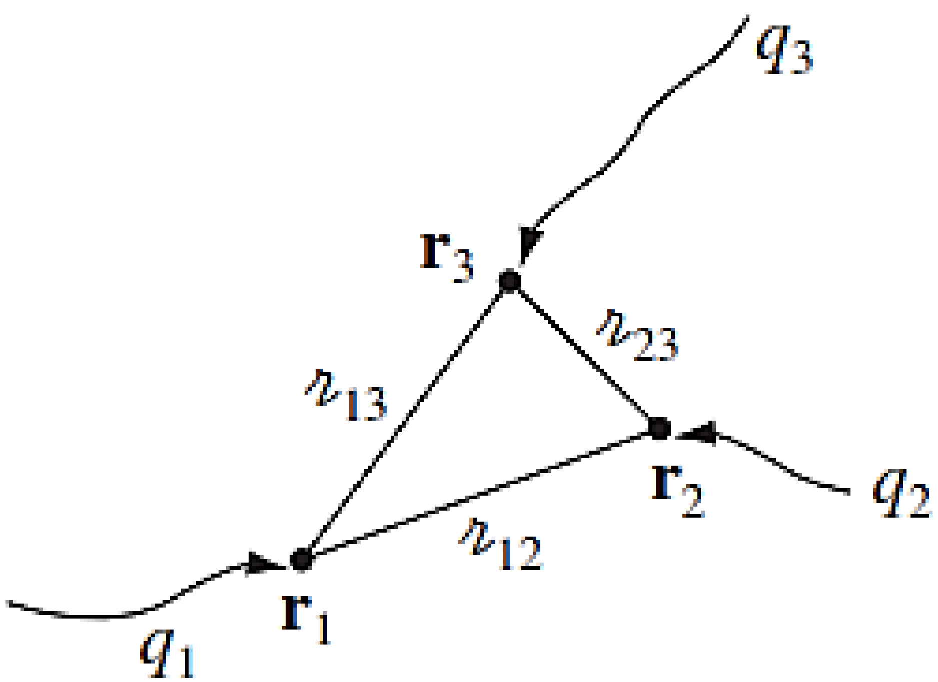
$$U_f - U_i = q_2(V_f - V_i)$$

$$V_i = 0 \ (at \ \infty) \ \ and \ \ V_f = \frac{q_1}{4\pi\varepsilon_0 r}$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$


Two-particle system

- **If the two charges are the same sign, *U* is positive and work must be done to bring the charges together.**
- **If the two charges have opposite signs, *U* is negative and work is done to keep the charges apart.**



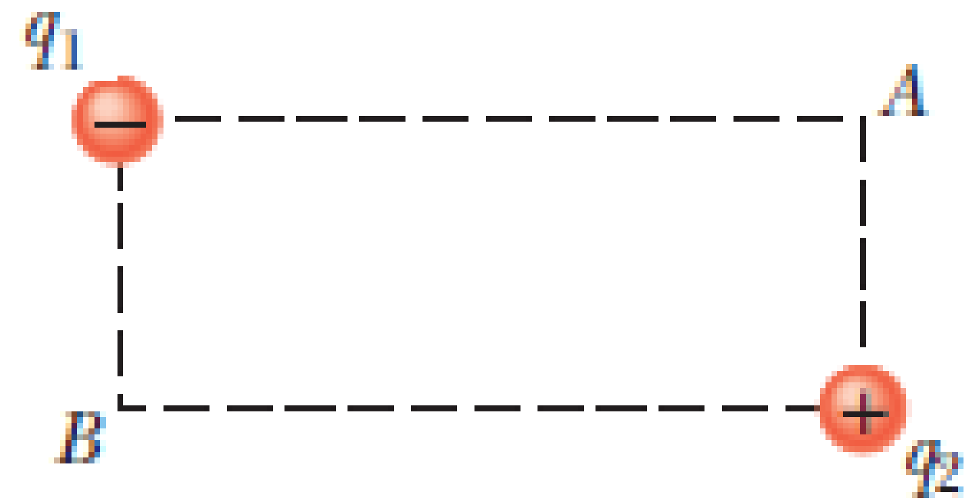

$$W_1 = 0$$

$$W_2 = q_2[V_1(\mathrm{r}_2) - \overbrace{V_1(\infty)}^{0}] = \frac{1}{4\pi\varepsilon_0} q_2 (\frac{q_1}{r_{12}})$$

$$W_3 = q_3 V_{1,2}(\mathrm{r}_3) = \frac{1}{4\pi\varepsilon_0} q_3 (\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}})$$

$$W = W_1 + W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}})$$

**Ex 3.** In the rectangle of Fig. 24- 55, the sides have lengths **5.0 cm** and **15 cm**, **$q_1$ = -5 mC**, and **$q_2$ = 2 mC**. With **V = 0 at infinity**, what is the electric potential at (a) corner A and (b) corner B? (c) How much work is required to move a charge **$q_3$ = 3 mC from B to A** along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if **$q_3$** is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?




$$V_A = \frac{q_1}{4\pi\varepsilon_0 r_{1A}} + \frac{q_2}{4\pi\varepsilon_0 r_{2A}} = \frac{1}{4\pi(8.85\times10^{-12})}\left(\frac{-5\times10^{-3}}{15\times10^{-2}} + \frac{2\times10^{-3}}{5\times10^{-2}}\right) = 6\times10^{4}\ V$$

$$V_B = \frac{q_1}{4\pi\varepsilon_0 r_{1B}} + \frac{q_2}{4\pi\varepsilon_0 r_{2B}} = \frac{1}{4\pi(8.85\times10^{-12})}\left(\frac{-5\times10^{-3}}{5\times10^{-2}} + \frac{2\times10^{-3}}{15\times10^{-2}}\right) = -7.8\times10^{5}\ V$$

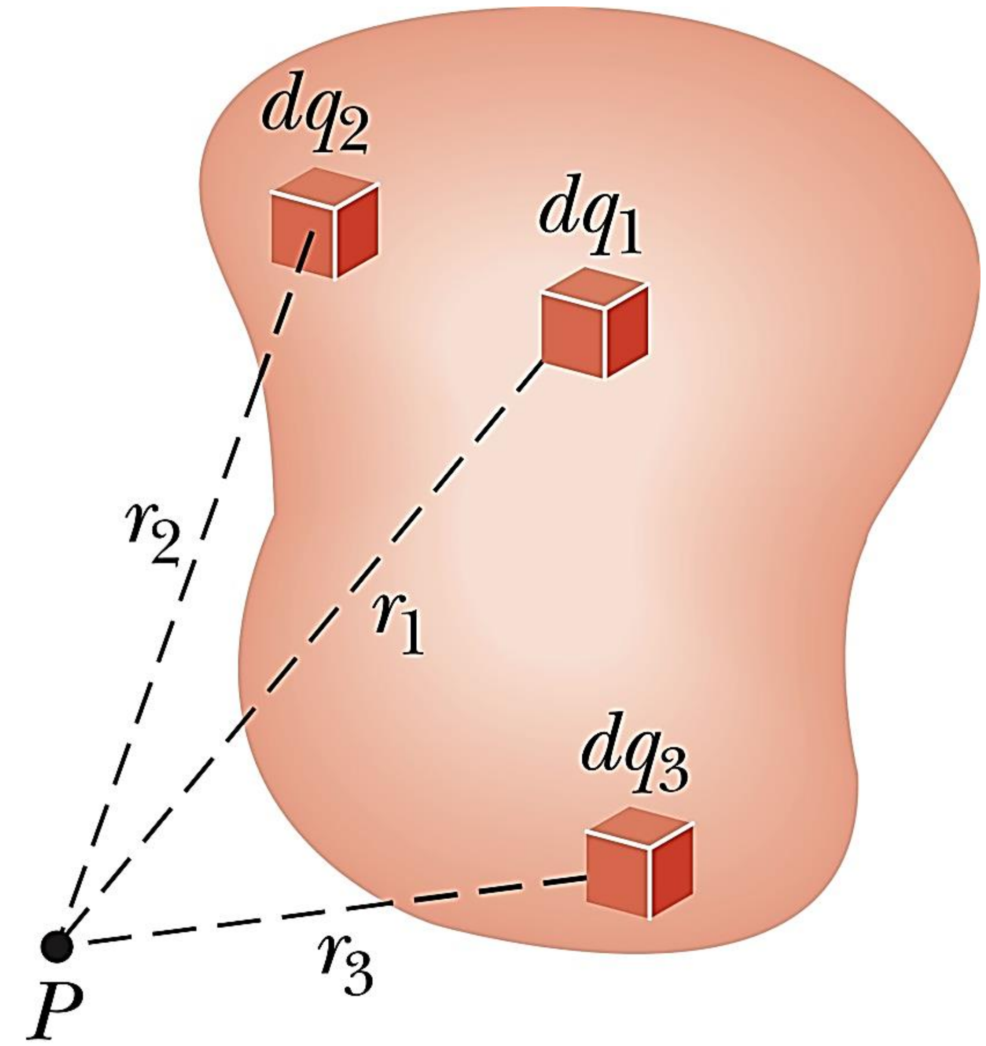
$$W = U_A - U_B = q_3(V_A - V_B) = 3\times10^{-3}(6\times10^{4} + 7.8\times10^{5}) = 2.5\ \ J$$

# Electric Potential for a Continuous Charge Distribution

$$dV = k_e \frac{dq}{r}$$

$$\boxed{V = k_e \int \frac{dq}{r}}$$

- For the length element: $dq = \lambda\, d\ell$
- For the surface element : $dq = \sigma\, dA$
- For the volume element : $dq = \rho\, dV$



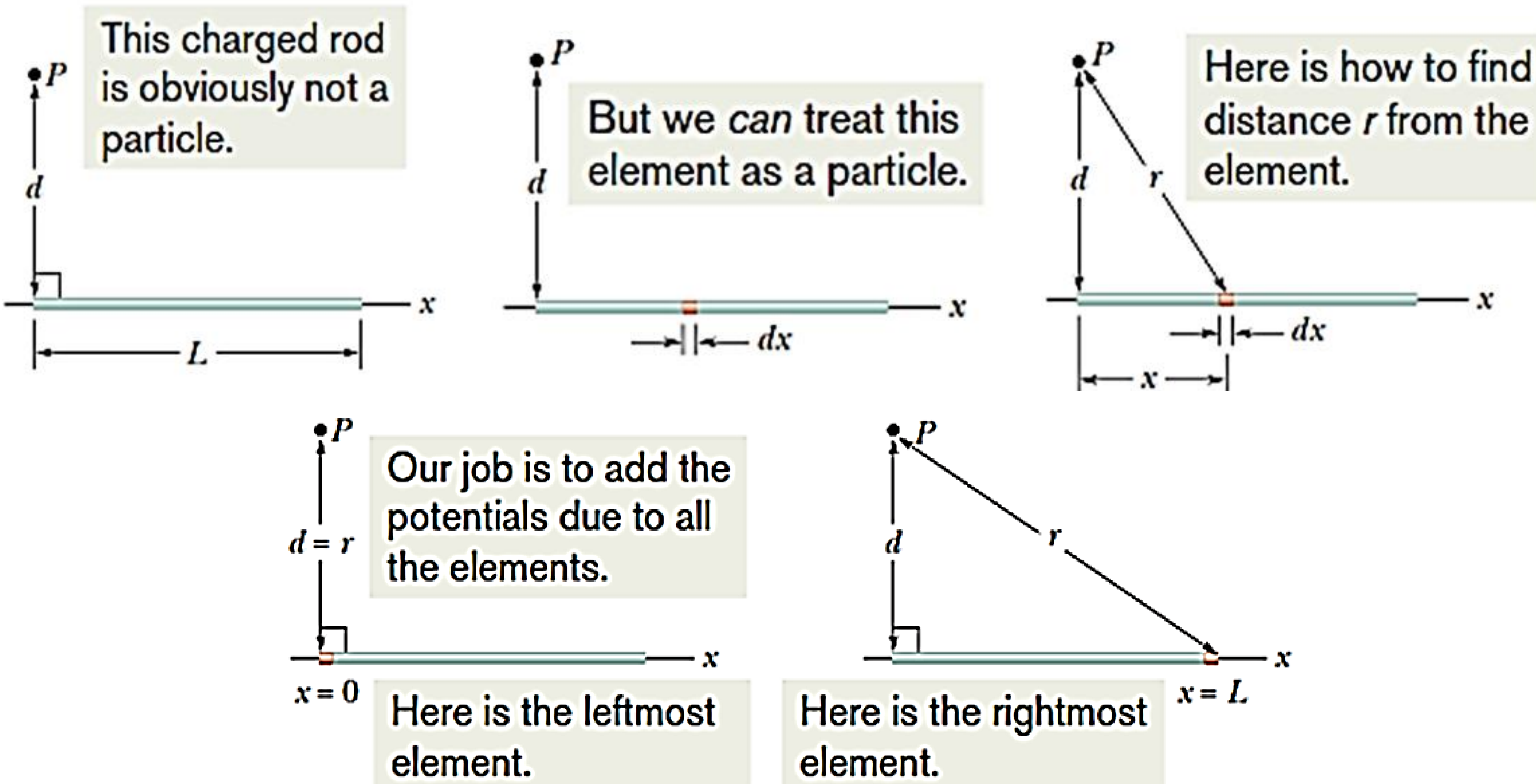

- If the **electric field is already known** from other considerations, the potential can be calculated using the original approach:

$$\boxed{\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$$

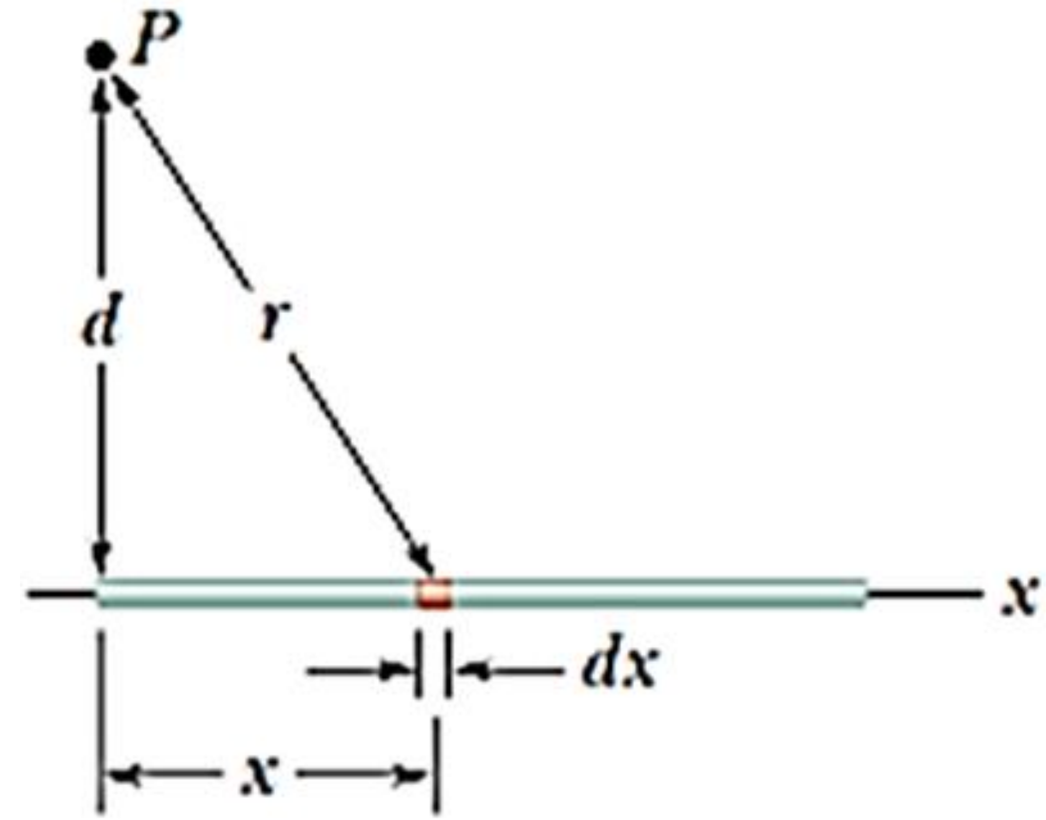
- Choose **V = 0** at some convenient point

# *V* for a Finite Line of Charge

**Ex 4.** A **non-conducting rod** of length **L = 10 cm** has a **line charge that uniformly** distributed along its length **λ = 2 μC/m**. What is the electric potential of the rod at point P, at distance **d = 1 cm** from the rod?



$$dV = k_e \frac{dq}{r}$$




$$dq = \lambda\, dx$$

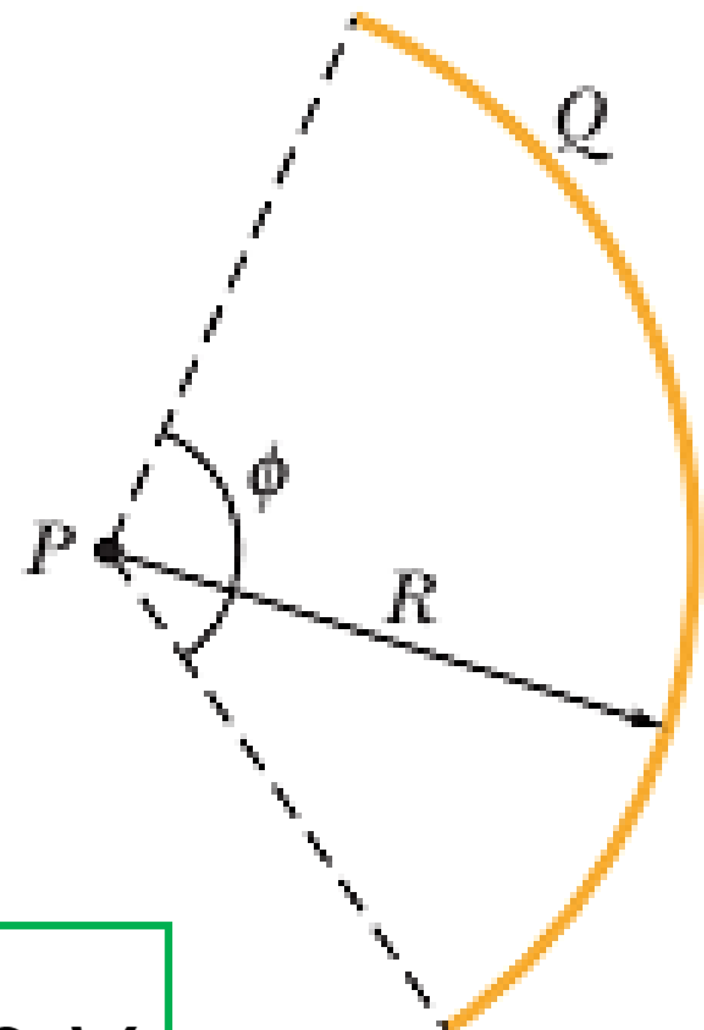
$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{d^2 + x^2}}$$

$$V = \int dV = k_e \lambda \int_0^L \frac{dx}{\sqrt{d^2 + x^2}} = k_e \lambda \left[ \ln\left(x + \sqrt{d^2 + x^2}\right) \right]_0^L$$

$$\boxed{V = k_e \lambda \ln\left(\frac{L + \sqrt{d^2 + L^2}}{d}\right) = 54 \times 10^3 \ \ V}$$

# *V* for a Finite Line of Charge

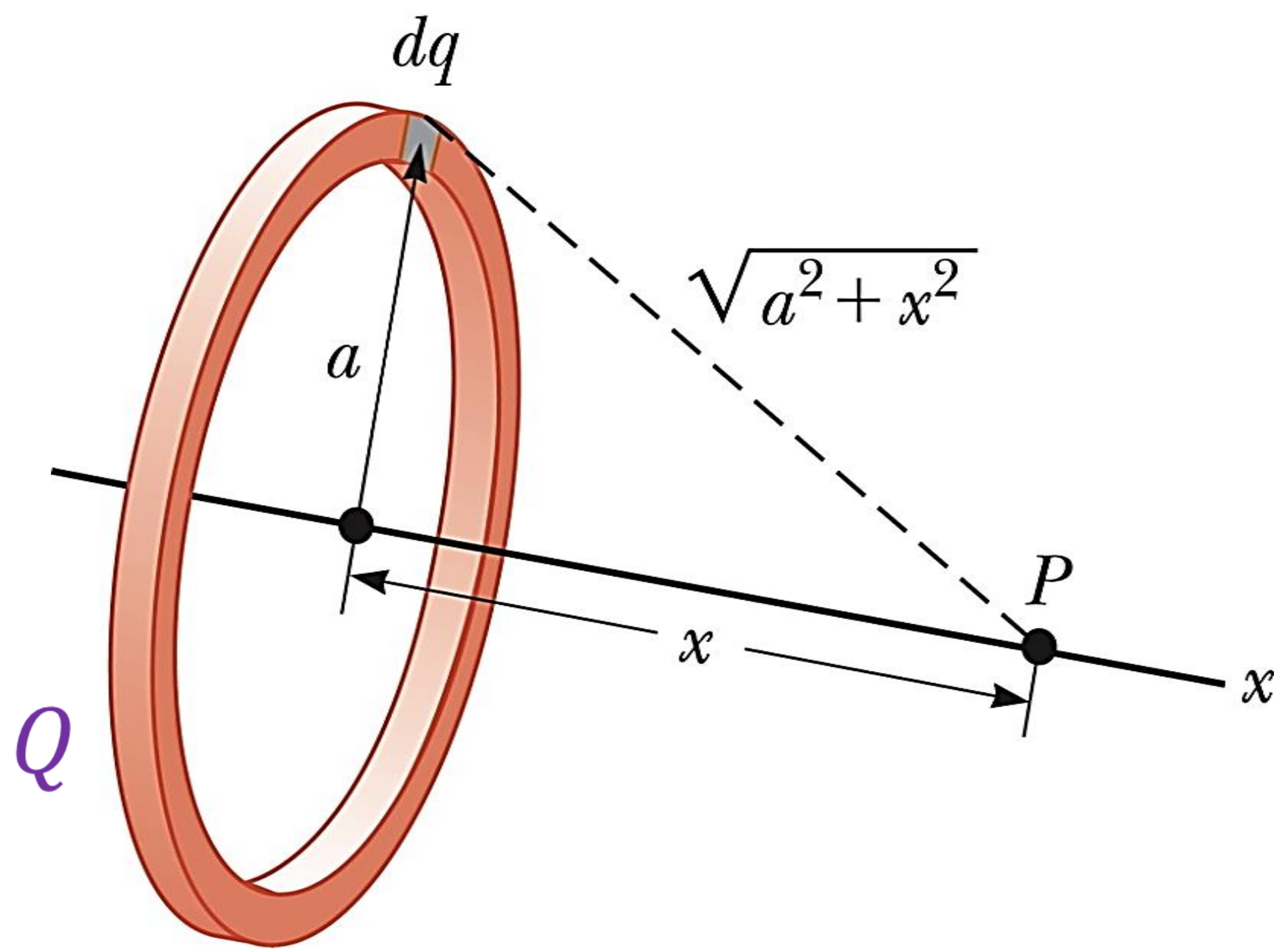
**Ex 5.** In Fig. 24-43, a **plastic rod having a uniformly distributed charge Q = -25.6 pC** has been bent into a circular arc of radius **R = 3.71 cm** and central angle **φ =120** With **V = 0 at infinity**, what is the electric potential at P, the **center of curvature** of the rod?




$$dV = k_e \frac{dq}{r} = k_e \frac{dq}{R}$$

$$\boxed{V = \int dV = k_e \int \frac{dq}{R} = \frac{k_e}{R} \int dq = \frac{k_e Q}{R} = -6.20\ V}$$

# *V* for a Uniformly Charged Ring




$$V = k_e \int \frac{dq}{\sqrt{a^2+x^2}} = \frac{k_e}{\sqrt{a^2+x^2}} \int dq = \frac{k_e Q}{\sqrt{a^2+x^2}}$$

$$E_x = \frac{k_e\, Q\, x}{(a^2+x^2)^{\frac{3}{2}}}$$

# $V$ for a Uniformly Charged Disk

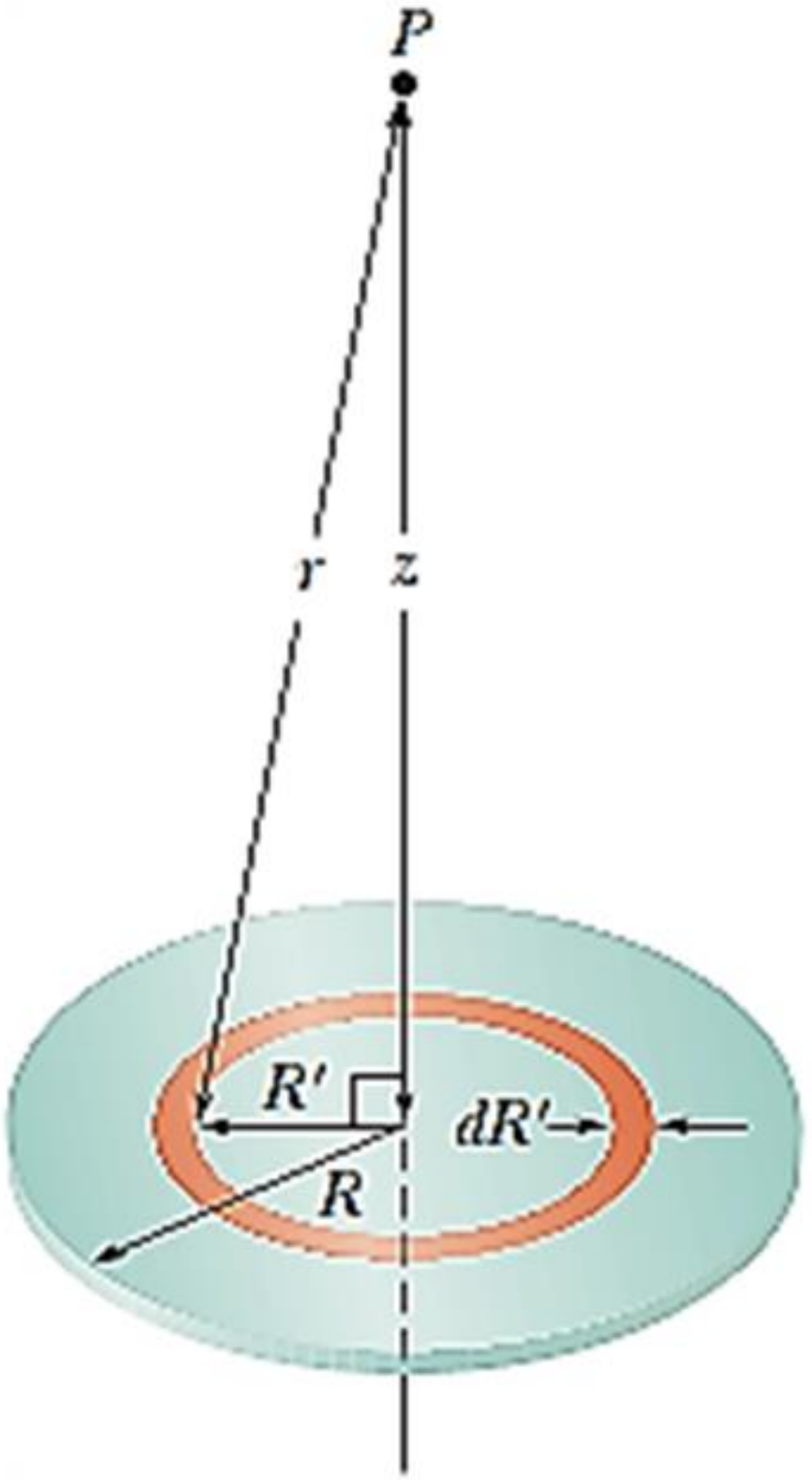
$$dV = k_e \frac{dq}{\sqrt{R'^2 + z^2}}$$

$$V = k_e \int_0^R \frac{dq}{\sqrt{R'^2 + z^2}} = k_e \int_0^R \frac{\sigma(2\pi R'\, dR')}{\sqrt{R'^2 + z^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{R'\, dR'}{\sqrt{R'^2 + z^2}} = \frac{\sigma}{2\varepsilon_0} \left[ \sqrt{R'^2 + z^2} \right]_0^R$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0} \left( \sqrt{R^2 + z^2} - z \right)}$$

$$\boxed{E_z = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{R^2 + z^2}} \right)}$$



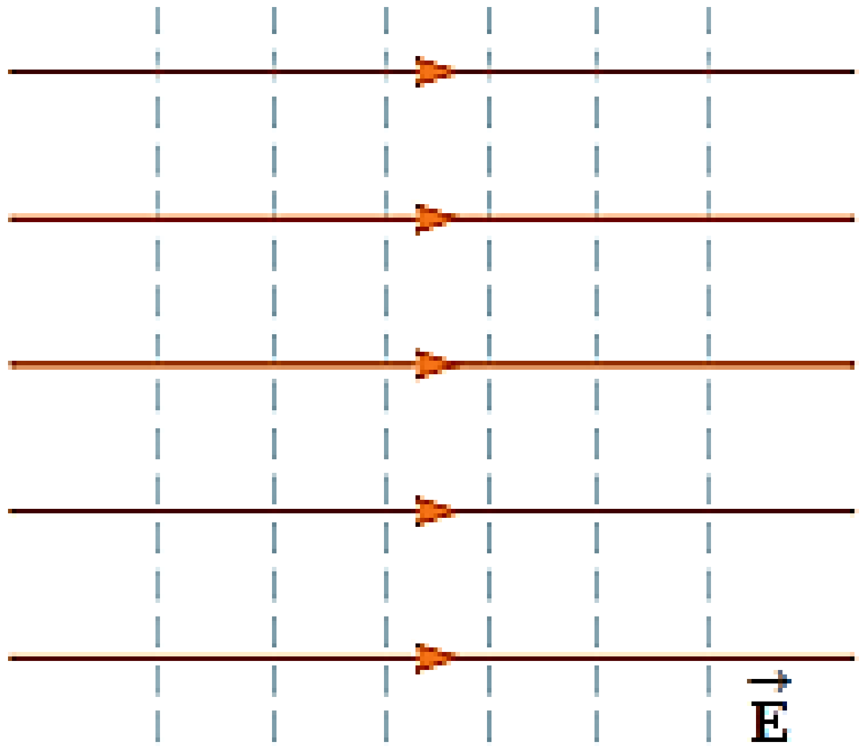

$$dq = \sigma\, dA = \sigma(2\pi R'\, dR')$$

# Finding E From *V*

$$V = -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \qquad dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

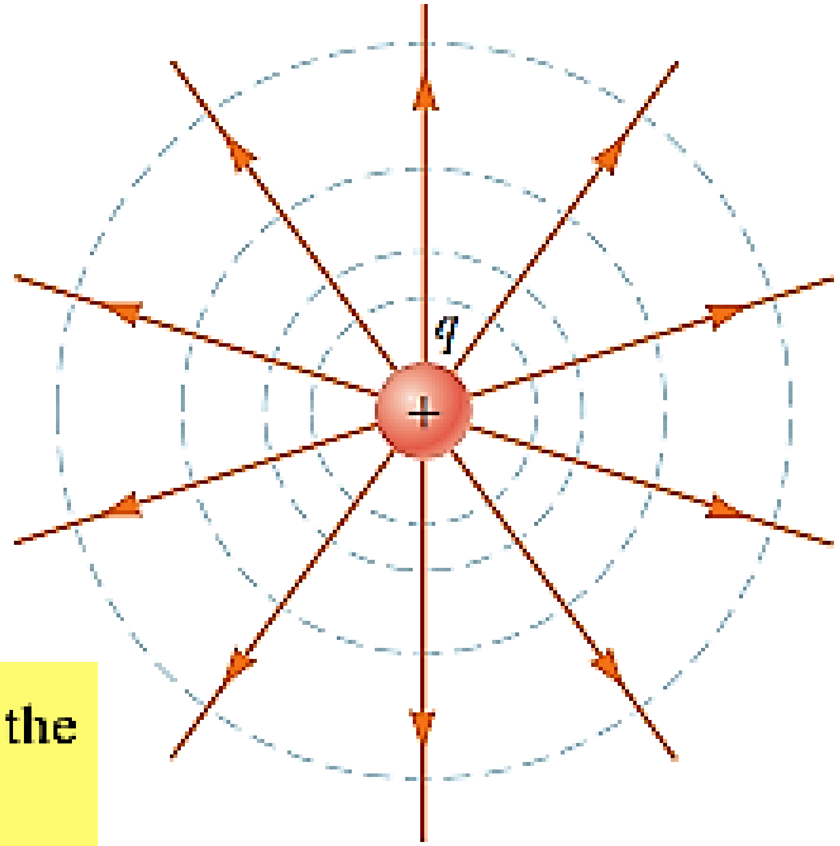
$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x\, dx. \qquad E_x = -\frac{dV}{dx}$$

$$\vec{\mathbf{E}} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$




$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_r\, dr \qquad E_r = -\frac{dV}{dr}$$

$$V = k_e q/r \qquad E_r = k_e q/r^2$$



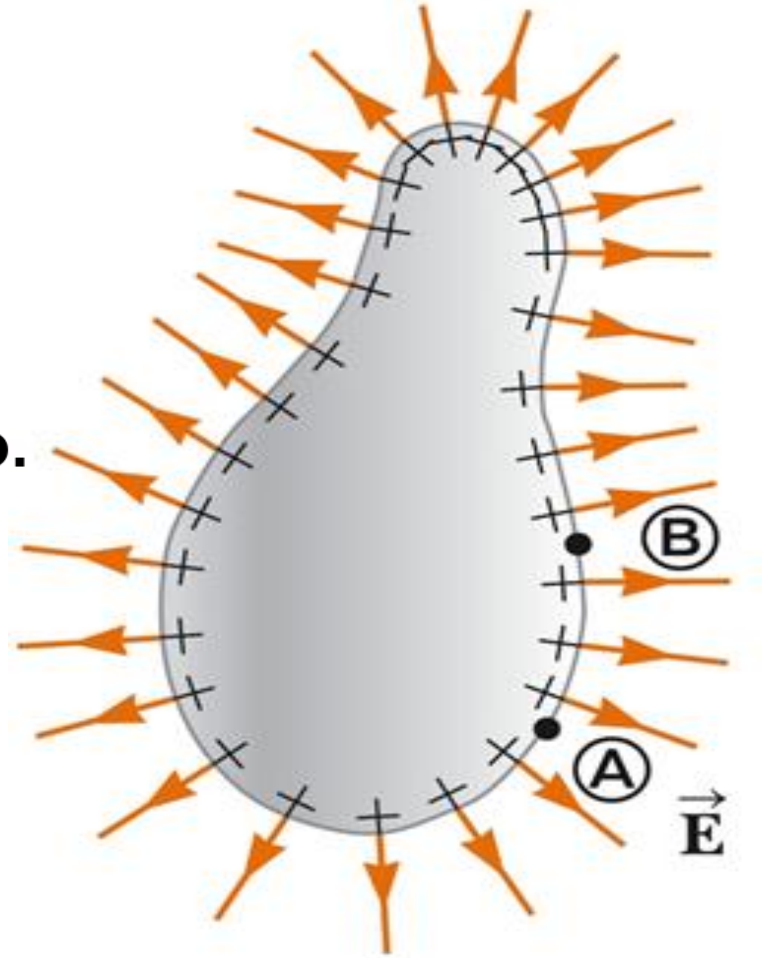

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

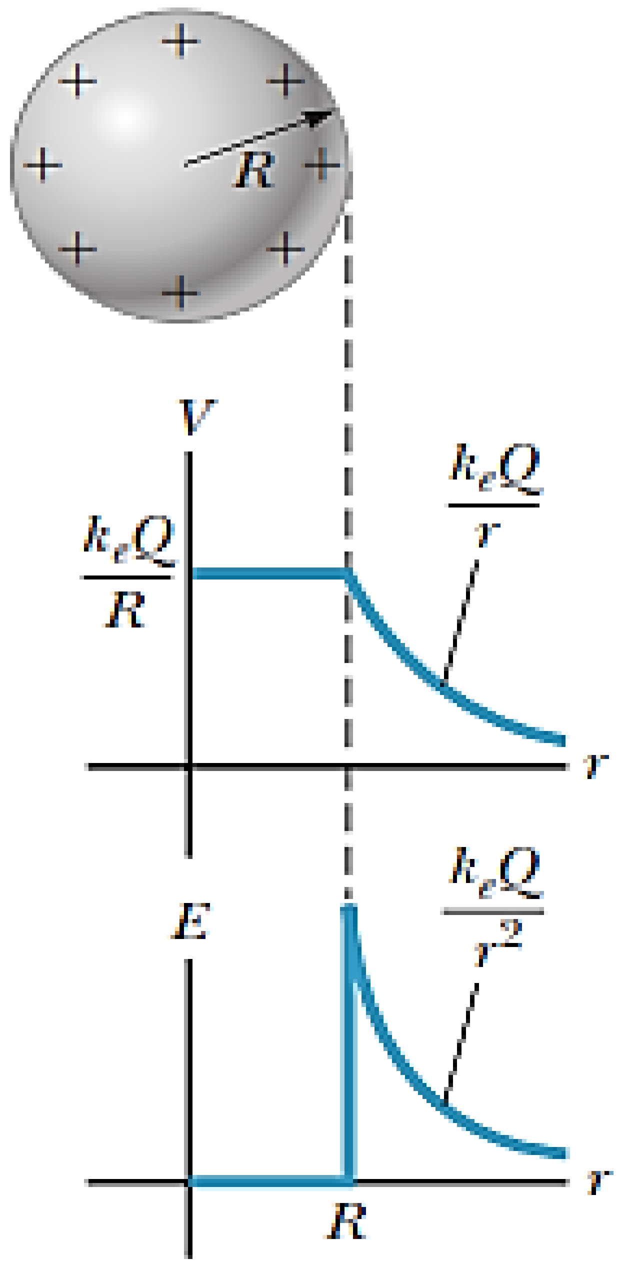
# *V* Due to a Charged Conductor

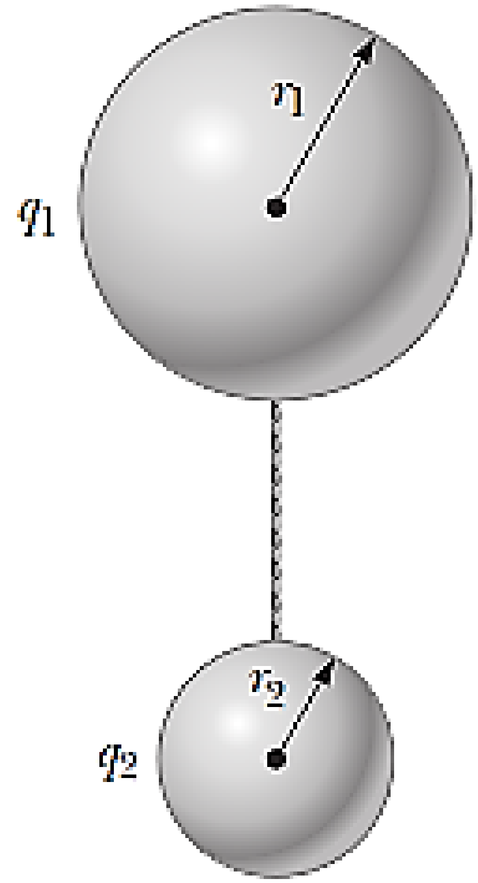
$\vec{\mathbf{E}}$ is always perpendicular to the displacement $d\vec{\mathbf{s}}$.

Therefore, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$

Therefore, the potential difference between *A* and *B* is also zero.




$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

$$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

# Cavity in a Conductor

- For all paths between $A$ and $B$,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

- A cavity surrounded by conducting walls is a **field-free region** as long as no charges are inside the cavity.