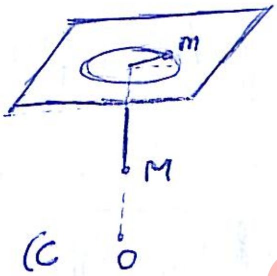
باسمه تعالی

امتحان اول - تابستان ۸۹ - دوره ی 23

$Y + L - r =$ ثابت $\rightarrow \boxed{\ddot{y} = \ddot{r}}$, $\boxed{y = r}$

1) a)

b)



$$\begin{cases} -T = (\ddot{r} - r\dot{\theta}^2)m \\ 0 = (2\dot{r}\dot{\theta} + r\ddot{\theta})m \\ T - Mg = M\ddot{y} \end{cases}$$

c)

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0 \rightarrow \frac{r^2\ddot{\theta} + (2r\dot{r})\dot{\theta}}{r} = 0 \rightarrow \frac{(r^2\dot{\theta})^{\cdot}}{r} = 0 \rightarrow \dot{\theta} = \frac{a^2 V_0}{r^2}$$

d)

$$E = mgL + \frac{1}{2}M\dot{y}^2 + Mgy + \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 \rightarrow E - mgL = \frac{(M+m)}{2}\dot{r}^2 + Mg(r) + \frac{m}{2}\cdot\frac{a^4V_0^2}{r^2} \rightarrow$$

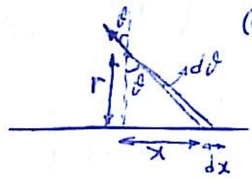
$$\frac{(E - mgL)2}{M+m} = \dot{r}^2 + \frac{2}{M+m}\left(\frac{2Mgr^3 + ma^4V_0^2}{2r^2}\right) \rightarrow U = \dot{r}^2 + \frac{2Mgr^3 + ma^4V_0^2}{(M+m)r^2} \rightarrow \boxed{f_{(r)} = \frac{2Mgr^3 + ma^4V_0^2}{(M+m)r^2}}$$

$$E = mgL + Mga + \frac{1}{2}mV_0^2 \rightarrow \boxed{U = \frac{2Mag + mV_0^2}{M+m}}$$

3) الف)

$$dE = \frac{2k\lambda dx}{x^2 + r^2}\cdot \cos\theta = \frac{2k\lambda\, dx}{r^2}\cdot\cos^3\theta \rightarrow \int dE = \frac{2k\lambda}{r}\int_0^{\pi/2}\cos\theta\, d\theta \rightarrow \boxed{E = \frac{2k\lambda}{r}}$$

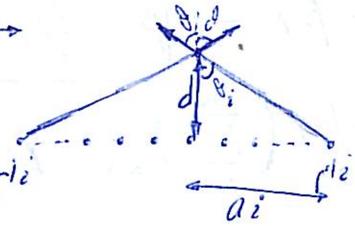
$$\tan\theta = \frac{x}{r} \rightarrow dx = \frac{r\,d\theta}{\cos^2\theta}$$



ب)

$$E_i = \frac{2k\lambda}{d}\cos^2\theta_i = \frac{2k\lambda}{d}\cdot\frac{d^2}{(d^2 + a^2i^2)} \rightarrow \sum E_i = 2k\lambda d\sum_{i=1}^{N}\frac{1}{d^2 + a^2i^2} \rightarrow$$

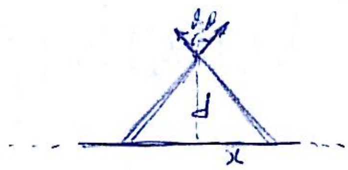
$$\sum E_i + \frac{2k\lambda}{d} = E \rightarrow \boxed{E = \frac{2k\lambda}{d}\left(1 + 2d^2\sum_{i=1}^{N}\frac{1}{d^2 + a^2i^2}\right)}$$



ج)

$$dE = \frac{2k\lambda}{d}\cos^2\theta = \frac{2kb}{d}\cos^2\theta\, dx \rightarrow dE = 2kb\, d\theta \rightarrow \boxed{E = \frac{b}{\varepsilon_0}}$$

$$\frac{x}{d} = \tan\theta \rightarrow dx = \frac{d}{\cos^2\theta}d\theta$$



امتحان دوم تابستان ۸۹ - دوره ی 23:

1)

$$r = k(\theta - \omega t) + r_1 \rightarrow \dot{r} = k(\dot{\theta} - \omega) \rightarrow \frac{r - r_1}{\theta - \omega t} = \frac{\dot{r}}{\dot{\theta} - \omega} \rightarrow \boxed{(r - r_1)(\dot{\theta} - \omega) = \dot{r}(\theta - \omega t)}$$

2) الف)

$$\varphi_{(\alpha)} = C \rightarrow \frac{q}{\varepsilon_0}\times\frac{2\pi R^2(1 - \cos\alpha)}{4\pi R^2} = \frac{(q + Q)}{\varepsilon_0}\cdot\frac{2\pi r^2(1 - \cos\beta)}{4\pi r^2} \rightarrow q(1 - \cos\alpha) = (q + Q)(1 - \cos\beta)$$

$$\cos\alpha \simeq 1 - \frac{\alpha^2}{2},\quad \cos\beta \simeq 1 - \beta^2/2 \rightarrow q\alpha^2 = (Q + q)\beta^2 \rightarrow \boxed{\beta = \sqrt{\frac{q}{Q + q}}\,\alpha}$$

ب)

$$\frac{kq}{(l - z)^2} = \frac{kQ}{R^2 + z^2}\cdot\frac{z}{\sqrt{R^2 + z^2}} \rightarrow qz^3\left(1 + \frac{R^2}{z^2}\right)^{3/2} = Qz(l^2 - 2lz + z^2) \rightarrow z^3 + \frac{3zR^2}{2} = z^3 + zl^2 - 2lz^2 \rightarrow$$

$$3R^2 = 2l^2 - 4lz \rightarrow \boxed{z = \frac{2l^2 - 3R^2}{4l}}$$

3) الف)

$$V_A = \dot{\varphi}\times\left(\frac{R}{\sin\alpha} - R\sin\alpha\right) \rightarrow V_A = R\frac{\cos^2\alpha}{\sin\alpha}\cdot\frac{2\pi}{T} \xrightarrow[\text{شرط غلتش}]{V_A = \dot{\psi}R} \boxed{\dot{\psi} = \frac{\cos^2\alpha}{\sin\alpha}\cdot\frac{2\pi}{T}}$$

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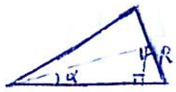
ب)

$$\vec{r}=\left(R\frac{\cos^2\alpha}{\sin\alpha}\cdot\cos\varphi\right)\hat{x}+\left(R\frac{\cos^2\alpha}{\sin\alpha}\sin\varphi\right)\hat{y}+(R\cos\alpha)\hat{z}\longrightarrow$$

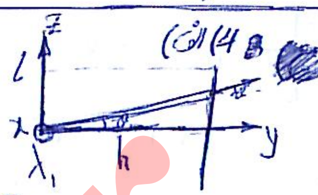


$$\boxed{\vec{r}=R\cos\alpha\left[\left(\cot\alpha\cos\left(\frac{2\pi t}{T}\right)\right)\hat{x}+\left(\cot\alpha\sin\left(\frac{2\pi t}{T}\right)\right)\hat{j}+\hat{z}\right]}\ \text{یا}\ \boxed{\vec{r}=\frac{R\cos^2\alpha}{\sin\alpha}\left(\cos\left(\frac{2\pi t}{T}\right)\hat{x}+\sin\left(\frac{2\pi t}{T}\right)\hat{y}\right)+(R\cos\alpha)\hat{z}}$$

(4 الف

$$d\varphi=E\cdot ds\cdot\cos\theta\longrightarrow d\varphi=\frac{2k\lambda}{\sqrt{h^2+z^2}}\cdot x\,dz\cdot\frac{h}{\sqrt{h^2+z^2}}\longrightarrow\varphi=2k\lambda hx\int_{-l}^{l}\frac{dz}{z^2+h^2}$$

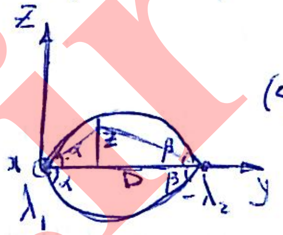
$$\longrightarrow\boxed{\varphi=\frac{\lambda}{\pi\varepsilon_0}\tan^{-1}\left(\frac{l}{h}\right)}$$



ب)

$$\varphi=C\longrightarrow d\varphi=(E_1\cos\theta_1+E_2\cos\theta_2)\cdot x\,dz\longrightarrow$$

$$d\varphi=\left(\frac{2k\lambda_1}{\sqrt{y^2+z^2}}\cdot\frac{y}{\sqrt{y^2+z^2}}+\frac{2k\lambda_2}{\sqrt{(D-y)^2+z^2}}\cdot\frac{(D-y)}{\sqrt{(D-y)^2+z^2}}\right)x\,dz\longrightarrow$$



$$\varphi=2kx\left(\lambda_1 y\int_{-z}^{z}\frac{dz}{y^2+z^2}+\lambda_2(D-y)\int_{-z}^{z}\frac{dz}{\sqrt{(D-y)^2+z^2}}\right)\longrightarrow\varphi=4kx\left(\lambda_1 y\tan^{-1}\left(\frac{z}{y}\right)+\lambda_2(D-y)\tan^{-1}\left(\frac{z}{D-y}\right)\right)$$

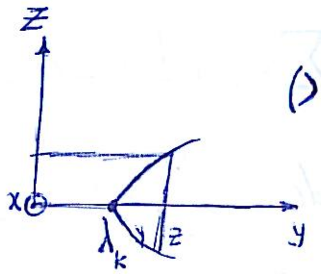
$$\varphi=\frac{\lambda_1 x}{\varepsilon_0}\times\frac{2\alpha Rx}{2\pi Rx}\longrightarrow\varphi=\lambda_1\alpha\cdot\frac{x}{\pi\varepsilon_0}\longrightarrow\boxed{\alpha\lambda_1=\lambda_1 y\tan^{-1}\left(\frac{z}{y}\right)+\lambda_2(D-y)\tan^{-1}\left(\frac{z}{D-y}\right)}$$

ج)

$$\alpha\lambda_1=\beta\lambda_2\longrightarrow\boxed{\beta=\left(\frac{\lambda_1}{\lambda_2}\right)\alpha}$$

د)

$$\varphi=kx\left(\sum_{k=1}^{N}\lambda_k(y-X_k)\tan^{-1}\left(\frac{z}{y-X_k}\right)\right)=\lambda_k\alpha_k\frac{x}{\pi\varepsilon_0}\longrightarrow$$



$$\boxed{\sum_{k=1}^{N}\lambda_k(y-X_k)\tan^{-1}\left(\frac{z}{y-X_k}\right)=\lambda_k\alpha_k}$$

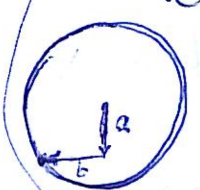
معادله‌ی خط میدانی که از بار k خارج شده یا به آن وارد شده.

(3 ج

$$nT'=T\longrightarrow T=n\times\frac{2\pi}{\dot{\psi}}=n\times\frac{T\sin\alpha}{\cos^2\alpha}\longrightarrow\boxed{\cos\alpha=n\tan\alpha}\ ,\ \boxed{n\in\mathbb{N}}$$

د)

$$\vec{r}_B=\vec{r}_A+\vec{a}+\vec{b}=\left[\left(\frac{R\cos^2\alpha\cos\varphi}{\sin\alpha}\right)\hat{x}+\left(\frac{R\cos^2\alpha\sin\varphi}{\sin\alpha}\right)\hat{y}+(R\cos\alpha)\hat{z}\right]+$$



$$\left[(R\cos\psi\sin\alpha\cos\varphi)\hat{x}+(R\cos\psi\sin\alpha\sin\varphi)\hat{y}-(R\cos\psi\cos\alpha)\hat{z}\right]+$$

$$\left[(R\sin\varphi\sin\psi)\hat{x}-(R\sin\psi\cos\varphi)\hat{y}\right]\longrightarrow$$

درسته

$$\boxed{\vec{r}_B=R\left[\left(\frac{\cos^2\alpha\cos\varphi}{\sin\alpha}+\cos\psi\sin\alpha\cos\varphi+\sin\varphi\sin\psi\right)\hat{x}+\left(\frac{\cos^2\alpha\sin\varphi}{\sin\alpha}+\cos\psi\sin\alpha\sin\varphi-\sin\psi\cos\varphi\right)\hat{y}+\cos\alpha(1-\cos\psi)\hat{z}\right]}$$

$$\dot{\vec{r}}_B=R\left[\left(-\frac{\cos\alpha\sin\varphi\dot{\varphi}}{\sin\alpha}-\sin\alpha(\sin\psi\cos\varphi\dot{\psi}+\sin\varphi\cos\psi\dot{\varphi})+\sin\alpha\cos\psi\dot{\psi}\right)\hat{x}+\left(\frac{\cos^2\alpha\cos\varphi\dot{\varphi}}{\sin\alpha}+\sin\alpha(\cos\varphi\cos\psi\dot{\varphi}-\sin\varphi\sin\psi\dot{\psi})\right.\right.$$

$$\dot{\vec{r}}_B=R\left[\left(-\frac{\cos\alpha\sin\varphi\dot{\varphi}}{\sin\alpha}-\sin\alpha(\sin\psi\cos\varphi\dot{\psi}+\sin\varphi\cos\psi\dot{\varphi})+\sin\varphi\cos\psi\dot{\psi}+\sin\psi\cos\varphi\dot{\varphi}\right)\hat{x}+\left(\frac{\cos^2\alpha\cos\varphi\dot{\varphi}}{\sin\alpha}+\sin\alpha(\cos\varphi\cos\psi\dot{\varphi}-\sin\varphi\sin\psi\dot{\psi}\right.\right.$$

پاسخ نامه

ادامه امتحان دوم - تابستان ۸۹ - دوره ۲۳

$$\ldots + \sin\psi \sin\varphi\, \dot\varphi - \cos\varphi \cos\psi\, \dot\psi \Big]\hat y + \left(\cos\alpha \sin\psi\, \dot\psi\right)\hat z \Big] \longrightarrow$$

۳) د) $t = \frac{T}{2} : \varphi = \pi\ , \ \psi = \pi \frac{\cos^2\alpha}{\sin\alpha}$

$$\vec{\dot r}_{B(\frac{T}{2})} = R\dot\varphi\left[\left(-\sin\alpha\left(-\sin\psi \frac{\cos^2\alpha}{\sin\alpha}\right) - \sin\psi\right)\hat x + \left(-\frac{\cos^2\alpha}{\sin\alpha} - \sin\alpha\cos\psi + \cos\psi \cdot \frac{\cos^2\alpha}{\sin\alpha}\right)\hat y + \left(\sin\psi \frac{\cos^3\alpha}{\sin\alpha}\right)\hat z\right] \rightarrow$$

$$\boxed{\vec{\dot r}_{B(\frac{T}{2})} = \frac{2\pi R}{T}\left[\left(-\sin\psi \sin^2\alpha\right)\hat x + \left(\frac{\cos^2\alpha}{\sin\alpha}(\cos\psi - 1) - \sin\alpha\cos\psi\right)\hat y + \left(\frac{\sin\psi \cos^3\alpha}{\sin\alpha}\right)\hat z\right]} \ ; \ \boxed{\psi = \pi \frac{\cos^2\alpha}{\sin\alpha}}$$

غلط ۱

۵) الف) غلط

$$f = -m\alpha v^5 \xrightarrow{n=5} f = -m\alpha v^n \rightarrow f = -m\alpha v_0^n \rightarrow f \text{ ثابت} \rightarrow a = \text{ثابت} \rightarrow a = -(g + \alpha v_0^n) \rightarrow$$

$$v = at + v_0 \rightarrow t_A = \frac{-v_0}{a} \rightarrow \boxed{t_A = \frac{v_0}{g + \alpha v_0^n}} \rightarrow \boxed{t_A = \frac{v_0}{g + \alpha v_0^5}}$$

ب)

$$v^2 - v_0^2 = 2ay \rightarrow y = -\frac{v_0^2}{2a} \rightarrow \boxed{y = \frac{v_0^2}{2(g + \alpha v_0^n)}} \rightarrow \boxed{y = \frac{v_0^2}{2(g + \alpha v_0^5)}}$$

ج)

$$a = \text{ثابت} \rightarrow t_{\text{رفت}} = t_{\text{برگشت}} \rightarrow \boxed{t_B = \frac{v_0}{g + \alpha v_0^n}} \rightarrow \boxed{t_B = \frac{v_0}{g + \alpha v_0^5}}$$

د)

$$a = \text{ثابت} \rightarrow v_{\text{رفت}} = v_{\text{برگشت}} \rightarrow \boxed{v = v_0}$$

به شکل دارد؛ اینجا شکل دقیق ۱ نمره دارد، و شکل عمودی نیست.

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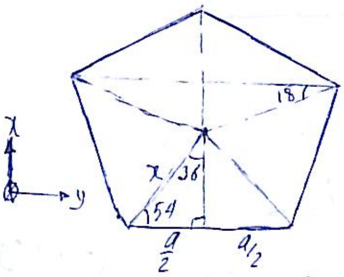
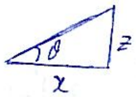


۲) ب)

$$\vec E = \left(\frac{5kq}{x^2 + z^2} \cdot \frac{z}{\sqrt{x^2 + z^2}}\right)\hat z + \frac{kq\cos\theta}{z^2 + x^2}\Big(\underbrace{2\cos(36) - 1 - 2\sin(18)}_{A}\Big)\hat x$$



$$A = 2\left(-2\sin^2 18 + 1\right) - 1 - 2\sin(18) = -4\sin^2(18) + 1 - 2\sin(18)$$

$$\rightarrow A = -\frac{6 - 2\sqrt5}{4} + 1 - \frac{\sqrt5 - 1}{2} \rightarrow A = 0 \rightarrow E_{(x)} = 0 \rightarrow \vec E = E_z \rightarrow$$

$$\vec E = \left(\frac{5kqz}{(x^2 + z^2)^{3/2}}\right)\hat z \quad , \ x = \frac{a_0}{2\sin(36)} \quad , \ \sin(36) = 2\sin(18)\cos(18) = \frac{(\sqrt5 - 1)\sqrt{16 - 6 + 2\sqrt5}}{8} = \frac{(\sqrt5 - 1)\sqrt{5 + \sqrt5}}{4\sqrt2}$$

$$\rightarrow \vec E_{(0,0,0)} = -5kq\left(\frac{z_1}{(x^2 + z_1^2)^{3/2}} + \frac{z_2}{(x^2 + z_2^2)^{3/2}} + \frac{1}{5(z_2 + z_1)^2}\right)\hat z$$

$$z_1^2 = a^2 - x^2 \rightarrow z_1 = \sqrt{a^2 - x^2} \rightarrow z_1 = a\sqrt{1 - \frac{1}{4\sin^2(36)}} = a\sqrt{1 - \frac{8}{(\sqrt5 - 1)(\sqrt5 - 1)\sqrt5(\sqrt5 + 1)}} = a\sqrt{1 - \frac{2}{5 - \sqrt5}} = a\frac{\sqrt{3 - \sqrt5}}{\sqrt{5 - \sqrt5}}$$

$$z_2 = z_1 + \frac{\sqrt3}{2}a \rightarrow z_2 = a\left(\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}} + \frac{\sqrt3}{2}\right)$$

$$\rightarrow \boxed{\vec F = -\frac{5kq^2}{a^2}\left(\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}} + \frac{\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}} + \frac{\sqrt3}{2}}{\left(\frac{7}{4} + \sqrt{\frac{3(3 - \sqrt5)}{5 - \sqrt5}}\right)^{3/2}} + \frac{1}{5\left(2\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}} + \frac{\sqrt3}{2}\right)^2}\right)\hat z}$$

الف)

$$\vec r_{(6)} = (x)\hat x + \left(\frac{\sqrt3}{2}a + a\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}}\right)\hat z \rightarrow \boxed{\vec r_{(6)} = a\left[\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}}\,\hat x + \left(\frac{\sqrt3}{2} + \sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}}\right)\hat z\right]}$$

$$\vec r_{(9)} = -\left(\sqrt{x^2 - \frac{a^2}{4}}\right)\hat x + \left(\frac{a}{2}\right)\hat y + \left(\frac{\sqrt3}{2}a + a\sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}}\right)\hat z \rightarrow \boxed{\vec r_{(9)} = a\left[-\left(\frac{1}{\sqrt{10(1 + \sqrt5)}}\right)\hat x + \left(\frac{1}{2}\right)\hat y + \left(\frac{\sqrt3}{2} + \sqrt{\frac{3 - \sqrt5}{5 - \sqrt5}}\right)\hat z\right]}$$

$$\cos(36) = 1 - 2\sin^2(18) = 1 - 2\cdot\frac{6 - 2\sqrt5}{16} \rightarrow \cos(36) = \frac{\sqrt5 - 1}{4} \rightarrow \cot(36) = \sqrt{\frac{2}{5(1 + \sqrt5)}}$$

بسمه تعالی

$\vec{r}_{(9)}$ با توجه به : $\vec{r}_{(1)} = a\left[-\left(\frac{1}{\sqrt{10(1+\sqrt5)}}\right)\hat{x} + \left(\frac{1}{2}\right)\hat{y} + \left(\sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\right)\hat{z}\right]$, $\cos(18) = \frac{\sqrt{16-6+2\sqrt5}}{4} \to \cos(18) = \frac{\sqrt{10+2\sqrt5}}{4}$

$$\vec{r}_{(3)} = \left(x\sin(18)\right)\hat{x} - \left(x\cos(18)\right)\hat{y} + \left(\sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\right)\hat{z} \to \vec{r}_{(3)} = \left(\frac{a}{4\cos(18)}\right)\hat{x} - \left(\frac{a}{4\sin(18)}\right)\hat{y} + \left(\sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\right)\hat{z} \to$$

$$\boxed{\vec{r}_{(3)} = a\left[\left(\frac{1}{\sqrt{10+2\sqrt5}}\right)\hat{x} - \left(\frac{1}{\sqrt5-1}\right)\hat{y} + \left(\sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\right)\hat{z}\right]} , \quad \boxed{\vec{r}_{(11)} = a\left(2\sqrt{\frac{3-\sqrt5}{5-\sqrt5}} + \frac{\sqrt3}{2}\right)\hat{z}}$$

ب)

$$\vec{F}_{(1)} = -\frac{kq^2}{a^3}\cdot\frac{\vec{r}_{(1)}}{\left(\frac{1}{10(1+\sqrt5)} + \frac14 + \frac{3-\sqrt5}{5-\sqrt5}\right)^{3/2}} = -\frac{kq^2}{a^3}\cdot\frac{\vec{r}_1}{\left(\frac{2(\sqrt5-1)+5(1+\sqrt5)(\sqrt5-1)+4\sqrt5(1+\sqrt5)(3-\sqrt5)}{4\times5(\sqrt5+1)(\sqrt5-1)}\right)^{3/2}}$$

$$\vec{F}_{(1)} = -\frac{kq^2}{a^3}\cdot\frac{\vec{r}_{(1)}}{\left(\frac{2\sqrt5-2+20+12\sqrt5+60-20-20\sqrt5}{80}\right)^{3/2}} \to \boxed{\vec{F}_{(1)} = -\left(\frac{40}{29-3\sqrt5}\right)^{3/2}\frac{kq^2}{a^2}\left[-\left(\frac{1}{\sqrt{10(1+\sqrt5)}}\right)\hat{x} + \frac12\hat{y} + \sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\hat{z}\right]}$$

$$\vec{F}_{(6)} = -\frac{kq^2}{a^3}\cdot\frac{\vec{r}_{(6)}}{\left(\frac{3-\sqrt5}{5-\sqrt5} + \frac34 + \frac{3-\sqrt5}{5-\sqrt5} + \sqrt{\frac{3(3-\sqrt5)}{5-\sqrt5}}\right)^{3/2}} \to \boxed{\vec{F}_{(6)} = -\frac{kq^2}{a^2}\cdot\frac{\sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\hat{x} + \left(\frac{\sqrt3}{2} + \sqrt{\frac{3-\sqrt5}{5-\sqrt5}}\right)\hat{z}}{\left(2\cdot\frac{3-\sqrt5}{5-\sqrt5} + \frac34 + \sqrt{\frac{3(3-\sqrt5)}{5-\sqrt5}}\right)^{3/2}}}$$

$$\boxed{\vec{F}_{(11)} = -\frac{kq^2}{a^2}\cdot\frac{\hat{z}}{\left(2\sqrt{\frac{3-\sqrt5}{5-\sqrt5}} + \frac{\sqrt3}{2}\right)^2}}$$

5) الف)

$$f^{(1)} = -m\alpha v_{(0)}^5 \to f = -m\alpha(v_0 - gt)^5 \to -g - \alpha(v_0-gt)^5 = \ddot{y}_{(0)} + \ddot{y}_{(1)} \to \dot{y}_{(1)} = \frac{\alpha}{6g}\left[(v_0-gt)^6 - v_0^6\right] \to$$

$$\frac{\alpha}{6g}\left[(v_0-gt)^6 - v_0^6\right] + v_0 - gt = \dot{y}^{(1)} \to \frac{\alpha}{6g}\left((v_0 - gT'_{(1)})^6 - v_0^6\right) + v_0 - g(T'_{(0)} + T'_{(1)}) = 0 \to \boxed{T'^{(1)} = \frac{v_0}{g}\left(1 - \frac{\alpha v_0^5}{6g}\right)}$$

ب)

$$y_{(1)} = \frac{\alpha}{6g}\left[-\frac{1}{7g}\left[(v_0-gt)^7 - v_0^7\right] - v_0^6 t\right] \to h_{(1)} = \frac{\alpha}{6g}\left\{-\frac{1}{7g}\left(\left(\left(\frac{\alpha v_0^5}{6g}\right)^7 - 1\right)v_0^7 - \frac{v_0^7}{g}\left(1 - \frac{\alpha v_0^5}{6g}\right)\right)\right\} \to$$

$$h_{(1)} = -\frac{\alpha v_0^7}{6g^2}\left(\frac17\left(\frac{\alpha v_0^5}{6g}\right)^7 - \frac17 + 1 - \frac{\alpha v_0^5}{6g}\right) \to h^{(1)} = \frac{v_0^2}{2g} - \frac{\alpha v_0^7}{6g^2}\left(1 - \frac17\right) \to \boxed{h^{(1)} = \frac{v_0^2}{2g}\left(1 - \frac{2\alpha v_0^5}{7g}\right)}$$

ج)

$$y^{(1)} = 0 \to 0 = \frac{\alpha}{6g}\left(\frac{v_0^7}{7g} - \frac{2v_0^7}{g}\right) - \frac{gT^{(1)2}}{2} + v_0 T^{(1)} \to v_0 T_{(1)} - \frac{g}{2}\times\frac{2v_0}{g}\times 2T_{(1)} + \frac{\alpha v_0^7}{6g^2}\times\frac{-13}{7} = 0 \to T^{(1)} = \frac{2v_0}{g}\left(1 - \frac{13\alpha v_0^5}{42g}\right)$$

$$\to \Delta T^{(1)} = T^{(1)} - T'^{(1)} = \frac{v_0}{g} + \frac{\alpha v_0^6}{g^2}\left(\frac16 - \frac17\right) \to \boxed{\Delta T^{(1)} = \frac{v_0}{g}\left(1 + \frac{\alpha v_0^5}{42g}\right)}$$

د)

$$\dot{y}^{(1)} = v_0 - 2v_0 + \frac{\alpha v_0^6}{7g} + \frac{\alpha}{6g}\left(v_0^6 - v_0^6\right) \to \boxed{\dot{y}^{(1)} = -v_0\left(1 - \frac{\alpha v_0^5}{7g}\right)}$$

بسمه تعالی

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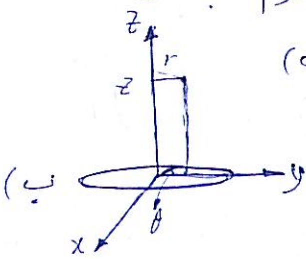
۱) الف)



$$dV = \frac{k\lambda R\, d\theta}{\sqrt{(R\cos\theta - r)^2 + R^2\sin^2\theta + z^2}} \longrightarrow V = k\lambda R \int_0^{2\pi} \frac{d\theta}{\sqrt{z^2+R^2+r^2-2rR\cos\theta}}$$

ب)

$$V = \frac{k\lambda R}{\sqrt{z^2+R^2}} \int_0^{2\pi} \frac{d\theta}{\sqrt{1+\frac{r^2-2rR\cos\theta}{R^2+z^2}}} = \frac{k\lambda R}{\sqrt{R^2+z^2}}\int_0^{2\pi}\left(1-\frac{r^2}{2(R^2+z^2)}+\frac{rR\cos\theta}{R^2+z^2}+\frac{3}{8}\cdot\frac{4r^2R^2\cos^2\theta}{(R^2+z^2)^2}\right)d\theta =$$

$$\frac{k\lambda R}{\sqrt{R^2+z^2}}\left(2\pi - \frac{\pi r^2}{(R^2+z^2)} + \frac{3\pi r^2R^2}{2(R^2+z^2)^2}\right) \longrightarrow V = \frac{k\lambda\pi R}{\sqrt{R^2+z^2}}\left(2+\frac{r^2(3R^2-2R^2-2z^2)}{2(R^2+z^2)^2}\right) \longrightarrow \boxed{V = \frac{k\lambda\pi R}{\sqrt{R^2+z^2}}\left(2+\frac{R^2-2z^2}{2(R^2+z^2)^2}\varepsilon^2\right)}$$

ج)

$$-\vec E = \vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac{1}{r}\frac{\partial V}{\partial\varphi}\hat\varphi + \frac{\partial V}{\partial z}\hat z \longrightarrow -\vec E = \frac{k\lambda\pi R(R^2-2z^2)}{(R^2+z^2)^2}\varepsilon\hat r - \frac{2k\lambda\pi R\cdot 2z}{2(R^2+z^2)^{3/2}}\hat z \longrightarrow$$

$$\boxed{\vec E = \frac{k\lambda\pi R}{(R^2+z^2)^{3/2}}\left((2z^2-R^2)\varepsilon\hat r + 2z(R^2+z^2)\hat z\right)}$$

د)

$$|\vec F| = 2k\pi\lambda R q\left(\frac{z+\frac{\delta}{2}}{\sqrt{R^2+(z+\frac{\delta}{2})^2}} - \frac{z-\frac{\delta}{2}}{\sqrt{R^2+(z-\frac{\delta}{2})^2}}\right) = \frac{2k\lambda\pi R q}{\sqrt{R^2+z^2}}\left(\left(z+\frac{\delta}{2}\right)\left(1-\frac{\delta z}{2(R^2+z^2)}\right) + \left(\frac{\delta}{2}-z\right)\left(1+\frac{\delta z}{2(R^2+z^2)}\right)\right) \longrightarrow \boxed{\vec E = \frac{\lambda R P(2z^2-R^2)}{2\varepsilon_0(R^2+z^2)^{5/2}}}$$ ✓

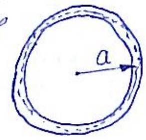
$$\vec F = \frac{\lambda R q}{2\varepsilon_0\sqrt{R^2+z^2}}\left(z+\frac{\delta}{2}-\frac{\delta z^2}{2(R^2+z^2)}+\frac{\delta}{2}-z-\frac{\delta z^2}{2(R^2+z^2)}\right)\hat z \longrightarrow \vec F = \frac{\lambda R\vec P}{2\varepsilon_0\sqrt{R^2+z^2}}\left(1-\frac{z^2}{R^2+z^2}\right) \longrightarrow \boxed{\vec F = \frac{\lambda R^3\vec P}{2\varepsilon_0(R^2+z^2)^{3/2}}}$$ X

$$\boxed{\vec F = \frac{\lambda R(2z^2-R^2)}{4\varepsilon_0(R^2+z^2)^{5/2}}\vec P}$$ ✓

(2)

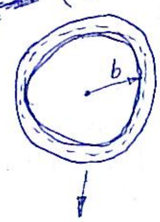
۲) الف) طبق قانون گاوس:

$$\int \vec E\cdot ds = \frac{q_{in\,a}}{\varepsilon_0} = 0 \longrightarrow \boxed{q_{a_{in}} = 0}$$



ج) تمامی خطوط میدان

د)



$$E_{(r)} = \begin{cases} 0 & r < a \\ \frac{kq_a}{r^2} & a < r < c \\ \frac{k(q_a+nq)}{r^2} & c < r < b \\ \frac{k(q_a+nq+q_b)}{r^2} & b < r \end{cases} \longrightarrow E_{(r)} = \begin{cases} 0 & r < a \\ -\frac{ka(c-b)}{c(a-b)r^2}nq & a < r < c \\ \frac{b(a-c)}{c(a-b)r^2}nq & c < r < b \\ 0 & b < r \end{cases}$$

$$\int E\cdot ds = \frac{q_a+q_c+q_{in\,b}}{\varepsilon_0} = 0 \quad (I)$$

$$r > b: \int E\cdot ds = \frac{\Sigma Q_i}{\varepsilon_0} = \frac{q_a+nq+q_{in\,b}+q_{out\,b}}{\varepsilon_0} = 0 \quad (II)$$

$$(I), (II) \longrightarrow \boxed{q_{out\,b} = 0}$$

$$\begin{cases} \frac{kq_a}{a}+\frac{nkq}{c}+\frac{kq_b}{b} = 0 \\ q_a+q_b = -nq \longrightarrow q_a = -nq-q_b \end{cases} \longrightarrow \frac{-nq-q_b}{a}+\frac{nq}{c}+\frac{q_b}{b} = 0 \longrightarrow q_b = \frac{\frac{1}{a}-\frac{1}{c}}{\frac{1}{b}-\frac{1}{a}}nq \longrightarrow \boxed{q_b = \frac{b(c-a)}{c(a-b)}nq} \longrightarrow q_a = -\left(\frac{b(c-a)}{c(a-b)}+1\right)nq$$

$$\longrightarrow \boxed{q_a = -\frac{a(c-b)}{c(a-b)}nq} \longrightarrow q_a+nq = nq\left(1-\frac{a(c-b)}{c(a-b)}\right) = \frac{b(a-c)}{c(a-b)}nq$$

ب)

$$V = \begin{cases} 0 & r < a \\ \frac{nkq}{c}+\frac{kq_b}{b}+\frac{kq_a}{r} & a < r < c \\ \frac{kq_b}{b}+\frac{k(q_a+nq)}{r} & c < r < b \\ 0 & b < r \end{cases} \longrightarrow V = \begin{cases} 0 & r < a \\ \frac{knq(c-b)}{c(a-b)} - \frac{ka(c-b)nq}{c(a-b)r} & a < r < c \\ \frac{k(c-a)}{c(a-b)}\left(1-\frac{b}{r}\right)nq & c < r < b \\ 0 & b < r \end{cases} \longrightarrow V = \begin{cases} 0 & r < a \\ \frac{k(c-b)nq}{c(a-b)}\left(1-\frac{a}{r}\right) & a < r < c \\ \frac{k(c-a)}{c(a-b)}\left(1-\frac{b}{r}\right)nq & c < r < b \\ 0 & b < r \end{cases}$$

ه)

$$\begin{cases} l^2+r_1^2-2lr_1\cos\theta=r_2^2 \\ L^2+r_1^2-2Lr_1\cos\theta=r^2 \end{cases} \rightarrow (l-L)+r_1^2\left(\frac{1}{l}-\frac{1}{L}\right)=\frac{r_2^2}{l}-\frac{r^2}{L} \rightarrow r=\sqrt{\frac{L^2(l)+(r_2^2-l^2-r_1^2)L+r_1^2 l}{l}}$$

$$dq_a=\frac{\lambda a}{b-a}\left(dL-b\frac{dL}{r}\right) \rightarrow q_a=\frac{\lambda a}{b-a}\left(l-b\sqrt{l}\int_0^l\left(\sqrt{l(L^2)+(r_2^2-l^2-r_1^2)L+r_1^2 l}\right)^{-1}dL\right) \rightarrow$$

$$dq_b=\frac{\lambda b}{a-b}\left(dL-a\frac{dL}{r}\right) \rightarrow q_b=\frac{\lambda b}{a-b}\left(l-a\sqrt{l}\int_0^l\left(\sqrt{l(L^2)+(r_2^2-l^2-r_1^2)L+r_1^2 l}\right)^{-1}dL\right)$$

~~$q_a=\frac{\lambda a}{b-a}\left(l-b\sqrt{l}\right)\Big|_0^l$, $q_b=\frac{\lambda b}{a-b}\left(l-a\sqrt{l}\right)\Big|_0^l$~~

$$\boxed{q_b=\frac{\lambda b}{b-a}\left(a\ln\left(\frac{2r_2 l+l^2+r_1^2+r_2^2}{r_1^2-l^2-r_2^2+2lr_1}\right)-l\right)} \Leftarrow \boxed{q_b=\frac{\lambda b}{b-a}\left(a\ln\left(\frac{(l+r_2)^2-r_1}{r_1^2-l^2-r_2^2+2lr_1}\right)-l\right)}$$

$$\boxed{q_a=\frac{\lambda a}{b-a}\left(l-b\ln\left(\frac{(l+r_2)^2+r_1^2}{r_1^2-l^2-r_2^2+2lr_1}\right)\right)}$$

3) a) تعریف

$$\frac{kg.m}{s^2}=\frac{[G]\times kg^2}{m^2} \rightarrow \boxed{[G]=\frac{m^3}{kg\times s^{-2}}}$$

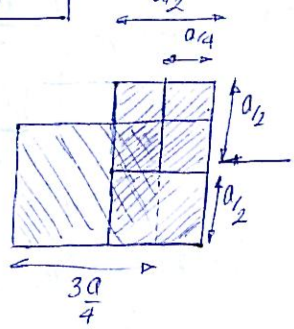
$$T=a^\alpha\times G^\beta\times M^\gamma\times f\left(\frac{b}{a}\right)=m^\alpha\times\frac{m^{3\beta}}{kg^\beta\times s^{-2\beta}}\times kg^\gamma = s \rightarrow \begin{cases}\alpha=-3\beta\\ \gamma=\beta\\ \beta=-\frac{1}{2}\end{cases} \rightarrow \begin{cases}\alpha=\frac{3}{2}\\ \gamma=\beta=-\frac{1}{2}\end{cases} \rightarrow \boxed{T=a\sqrt{\frac{a}{GM}}\,f\left(\frac{b}{a}\right)} \xrightarrow{a=b}$$

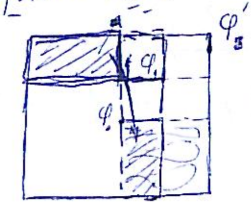
b)

$$\boxed{T=a\sqrt{\frac{a}{GM}}\,f_{(1)}}$$

c)

$$m\omega^2 a=\frac{GMm}{a^2} \rightarrow \omega=\frac{1}{a}\sqrt{\frac{GM}{a}} \rightarrow \boxed{T=a\sqrt{\frac{a}{GM}}(2\pi)} \rightarrow \boxed{f_{(1)}=2\pi}$$





4) a)

$$\begin{cases}\varphi_0=4\times k\times\frac{b}{\varepsilon_0}\times\frac{a}{2} \\ \varphi'=k\times\frac{b}{\varepsilon_0}\times a\end{cases} \rightarrow \boxed{\varphi'=\frac{\varphi_0}{2}}$$

b)

c)

$$\varphi_1=k\times\frac{b}{\varepsilon_0}\times\frac{3a}{4}+\frac{3\varphi_0}{2\times4}+\frac{x}{2}=\varphi_0\left(\frac{3}{8}+\frac{3}{8}\right)+\frac{x}{2} \rightarrow$$

$$\varphi_1-\frac{3+3}{8}\varphi_0=\frac{x}{2} \rightarrow \boxed{x=2\varphi_1-\frac{3}{2}\varphi_0} \quad (c,b)$$

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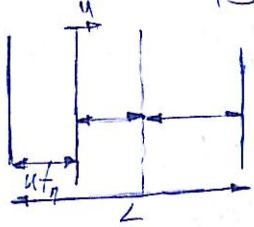
1) الف)

$$v_{n+1}=v_n+2u \rightarrow v_n=v_1+(n-1)2u \rightarrow \boxed{v_n=v_0+2nu}$$

ب)

$$t_{n+1}=t_n+\frac{L-ut_n}{u+v_n}+\frac{(L-ut_n)v_n}{(u+v_n)(v_n+2u)} \rightarrow t_{n+1}=t_n+\frac{L-ut_n}{u+v_0+2nu}+\frac{(L-ut_n)(v_0+2un)}{(v_0+u+2nu)(v_0+2u+2nu)} \rightarrow$$



$$t_{n+1}=t_n+\frac{(L-ut_n)\times2(v_n+u)}{(u+v_n)(v_n+2u)} \rightarrow \boxed{t_{n+1}=\frac{v_n t_n+2L}{v_n+2u}} \rightarrow \boxed{t_{n+1}=\frac{(v_0+2un)t_n+2L}{v_0+2(n+1)u}}$$

ج)

$$t_{n+1}=\left(\left(\frac{2L}{v_0}+t_n\right)+2n\frac{u}{v_0}t_n\right)\left(1-\frac{2(n+1)u}{v_0}\right) \rightarrow t_{n+1}=\frac{2L}{v_0}\left(1-\frac{2(n+1)u}{v_0}\right)+\left(1-\frac{2u}{v_0}\right)t_n \rightarrow$$

$$t_n=A+Bn+Ct_{n-1} \rightarrow Ct_{n-1}=AC+B(n-1)C+C^2t_{n-2} \rightarrow C^2t_{n-2}=AC^2+BC^2(n-2)+C^3t_{n-3} \rightarrow \cdots \rightarrow C^j t_{n-j}=AC^j+BC^j(n-j)+C^{j+1}t_{n-j-1} \rightarrow$$

$$C^{n-2}t_2=AC^{n-2}+BC^{n-2}\times2+C^{n-1}t_1 \rightarrow t_n=A\left(1+C+C^2+\cdots+C^{n-2}\right)+B\left(1+C+C^2+\cdots+C^{n-2}\right)n-BC\left(1+2C+3C^2+(n-2)C^{n-3}\right)+C^{n-1}t_1$$

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۱) ادامه ج)

$t_n = (A+Bn)x - By + t_1 c^{n-1}$

$y - cy = (c+2c^2+3c^3+\dots+(n-2)c^{n-2}) - (c^2+2c^3+\dots+(n-2)c^{n-1}) = c+c^2+c^3+\dots+c^{n-2}+(n-2)c^{n-1} = c\frac{-c^{n-2}+1}{-c+1}+(n-2)c^{n-1}$

$\to y = \frac{c-c^{n-1}}{(1-c)^2} + \frac{(n-2)c^{n-1}}{1-c} \to t_n = (A+Bn)\frac{1-c^{n-2}}{1-c} - \frac{B}{1-c}\left((n-2)c^{n-1}+\frac{c-c^{n-1}}{1-c}\right)+t_1$

حل دقیق بدون در نظر گرفتن کوچک بودن $\frac{u}{v_0}$ بود. (در رابطه‌ی بازگشتی) حال که $\frac{u}{v_0}$ کوچک است نیاز به عبارت بالا نیست و آن جمع را تقریب می‌زنیم.

تقریب: $t_n = (A+Bn)(1+c+c^2+\dots+c^{n-2}) - B(c+2c^2+3c^3+\dots+(n-2)c^{n-2}) + c^{n-1}t_1$, $A=\frac{2L}{v_0}$, $B=-\frac{4uL}{v_0^2}$, $c=1-\frac{2u}{v_0} \to 1-c, B$

$\to t_n = (A+Bn)(1+(1-2\varepsilon)+(1-2\varepsilon)^2+\dots+(1-2\varepsilon)^{n-2}) - \frac{4L}{v_0}\varepsilon((1-2\varepsilon)+2(1-2\varepsilon)^2+\dots+(n-2)(1-2\varepsilon)^{n-2}) + c^{n-1}t_1$

$t_n = (A+Bn)(1+(1-2\varepsilon)+(1-4\varepsilon)+\dots+(1-2(n-2)\varepsilon)) - \frac{4L}{v_0}\varepsilon(1+2+3+\dots+n-2) + (1+2\varepsilon(n-1))t_1$

$t_n = (A+Bn)((n-1)-2\varepsilon(1+2+3+\dots+(n-2))) - \frac{4L}{v_0}\varepsilon\frac{(n-2)(n-1)}{2} + (1+2\varepsilon(n-1))t_1 \to$

$t_n = \frac{2L}{v_0}(1-\frac{2u}{v_0}n)(n-1) - \frac{4L}{v_0}\varepsilon\frac{(n-2)(n-1)}{2} - \frac{4L}{v_0}\varepsilon\frac{(n-2)(n-1)}{2} + (1+2\varepsilon(n-1))t_1 \to$

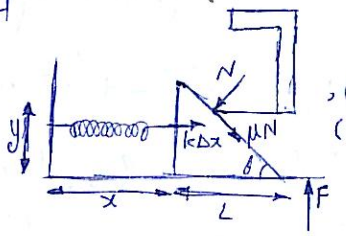
$t_n = \frac{2L}{v_0}(n-1) - \frac{4Lu}{v_0^2}n(n-1) - \frac{4Lu}{v_0^2}(n-2)(n-1) + \frac{2L}{v_0}(1-\frac{2u}{v_0})(1+\frac{2u}{v_0}(n-1)) \to$

$t_n = \frac{2L}{v_0}n - \frac{2L}{v_0} + \frac{2L}{v_0} + \frac{4Lu}{v_0^2}(n-2) - \frac{4Lu}{v_0^2}\times 2(n-1)^2 \to t_n = \frac{2L}{v_0}n + \frac{4Lu}{v_0^2}(n-2-2-2n^2+4n) \to t_n = \frac{2L}{v_0}\left(n+\frac{2u}{v_0}(5n-4-2n^2)\right)$

$t_0=0$, $t_1=\frac{2L}{v_0}(1-\frac{2u}{v_0})$, $t_2=\frac{4L}{v_0}(1-\frac{2u}{v_0})$, $t_3=\frac{2L}{v_0}(3-\frac{14u}{v_0})$, $t_4=\frac{8L}{v_0}(1-\frac{8u}{v_0})$, $t_5=\frac{2L}{v_0}(5-\frac{58u}{v_0})$

ت) $F_n = m\frac{2u}{\frac{2L}{v_0}(1+\frac{2u}{v_0}(5-2(1+2n)))} \to F_n=\frac{muv_0}{L}(1-\frac{2u}{v_0}(3-4n)) \to F_n=\frac{muv_0}{L}(1+\frac{2u}{v_0}(4n-3))$

۲) الف)، ب)



$N\cos\theta + \mu N\sin\theta = M\ddot y + F \to \frac{F+M\ddot y}{\mu\sin\theta+\cos\theta} = \frac{m\ddot y\cot\theta - k(L-y\cot\theta)}{\mu\cos\theta-\sin\theta}$

$k(D-x)+\mu N\cos\theta - N\sin\theta = m\ddot x$

$\tan\theta = \frac{\ddot y}{\ddot x} \to \ddot x = \ddot y\cot\theta \to x = D + y\cot\theta - L$

$\left(\frac{\mu\cos\theta-\sin\theta}{\mu\sin\theta+\cos\theta}\right)F + M\left(\frac{\mu-\tan\theta}{1+\mu\tan\theta}\right)\ddot y = m\ddot y\cot\theta - kL + ky\cot\theta \to \left(\left(\frac{\mu-\tan\theta}{1+\mu\tan\theta}\right)F+kL\right)L\tan\theta - \frac{kL^2}{2}\tan\theta = \left(m\cot\theta - \frac{M(\mu-\tan\theta)}{1+\mu\tan\theta}\right)\frac{v_0^2}{2}$

$\to v_0 = \sqrt{\frac{kL^2\tan\theta}{m\cot\theta - \frac{M(\mu-\tan\theta)}{1+\mu\tan\theta}}}$, $F=\frac{kL}{2}\left(\frac{1+\mu\tan\theta}{-\mu+\tan\theta}\right)$

$mg\sin\theta - T = mL\dot\theta^2 \to T = m(g\theta+L\dot\theta^2)$, $T = m(g\sin\theta+L\dot\theta^2)$

۳) الف) ب)

$\theta = \theta_0\cos(\omega t) \to \dot\theta = -\theta_0\omega\sin(\omega t) \to T = m(\omega^2\theta_0\cos(\omega t)+\theta_0^2\omega^2\sin^2(\omega t)) \to T = mg\theta_0(\cos(\omega t)+\theta_0\sin^2(\omega t))$

$\bar T = \frac{mg\theta_0}{\frac{2\pi}{\omega}}\left(\frac{1}{\omega}\sin(\omega t)\Big|_0^{2\pi} + \frac{\theta_0}{\omega}\frac{\theta}{2}\Big|_0^{2\pi}\right) \to \bar T = \frac{mg\theta_0}{2\pi}\times\pi\theta_0 \to \bar T = \frac{mg\theta_0^2}{2}$

پ) $E = -mgL\cos\theta_0 = -mgL\cos\theta + \frac{1}{2}mL^2\dot\theta^2 \to$

الف) $E = -mgL(1-\frac{\theta_0^2}{2})$ $mg\cos\theta - T = -mL\dot\theta^2 \to T = m(g(1-\frac{\theta^2}{2})+L\dot\theta^2)$, $T = m(g\cos\theta+L\dot\theta^2)$

ب) $\theta = \theta_0\cos(\omega t) \to T = m\left(g - \frac{g\theta_0^2}{2}\cos^2(\omega t) + L\theta_0^2\omega^2\sin^2(\omega t)\right) = mg\left(1+\frac{\theta_0^2}{2}(\sin^2(\omega t) - \cos(2\omega t))\right) =$

$mg\left(1+\frac{\theta_0^2}{2}(\frac{1}{2} - \frac{3}{2}\cos(2\omega t))\right) \to T = \frac{mg}{2}\left((2+\frac{\theta_0^2}{2}) - \frac{3}{2}\theta_0^2\cos(2\omega t)\right)$

پ) $\bar T = \frac{\omega}{2\pi}\times\frac{mg}{2}\left(\int_0^{\frac{2\pi}{\omega}}(2+\frac{\theta_0^2}{2})dt - \frac{3\theta_0^2}{2}\int_0^{\frac{2\pi}{\omega}}\cos(2\omega t)dt\right) \to \bar T = \frac{mg(4+\theta_0^2)}{4}$

ت) $W = \bar T\cdot dx = -\bar T\cdot dL = -mg(1+\frac{\theta_0^2}{4})dL \to dL + \frac{\theta_0^2}{4}dL = dL - \frac{\theta_0^2}{2}dL - \theta_0 L d\theta_0 \to$

$E = mgL(-1+\frac{\theta_0^2}{2}) \to dE = -mg(dL - \theta_0 L d\theta_0 - \frac{\theta_0^2}{2}dL)$ $-\frac{3}{4}\frac{dL}{L} = \frac{d\theta_0}{\theta_0} \to L^{3/4}\theta_0 =$ ثابت $\to \theta_0' = \frac{L^{3/4}\theta_0}{\dots}$

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ث)

$$W=-mg\left(-\Delta L+\frac{1}{4}\int_{L}^{L-\Delta L}\frac{\theta_0^2 L_0^{3/2}}{L^{3/2}}dL\right)\longrightarrow \boxed{W=mg\left(\Delta L+2L\theta_0^2\left(\sqrt{\frac{L_0}{L_0-\Delta L}}-1\right)\right)}\quad,\quad \boxed{\theta_0'=\left(\frac{L_0}{L_0-\Delta L}\right)^{3/4}\theta_0}$$

$$\int_{L}^{L-\Delta L}\frac{dL}{L^{3/2}}=\left(\frac{1}{\sqrt{L_0-\Delta L}}-\frac{1}{\sqrt{L_0}}\right)\times -2$$

۴) الف)

$$Q=2\pi\int_0^R r\sigma_{(r)}dr=\alpha\int_0^R\frac{rdr}{\sqrt{R^2-r^2}}=-\frac{\alpha}{2}(-R)\longrightarrow\boxed{\alpha=\frac{2Q}{R}}$$

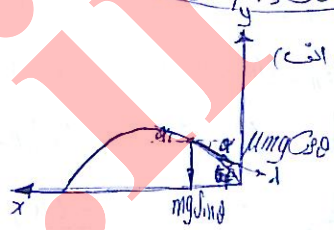
ب)

$$dV=\frac{k\times 2\pi r\times\sigma_{(r)}dr}{\sqrt{z^2+r^2}}\longrightarrow V=2k\pi\int_0^R\frac{2Q}{R}\times\frac{rdr}{\sqrt{(r^2+z^2)(R^2-r^2)}}\longrightarrow V=\frac{Q}{\varepsilon_0 R}\int_0^R\frac{d(\sqrt{r^2+z^2})}{\sqrt{R^2-r^2}}\longrightarrow V=\frac{Q}{\varepsilon_0 R}\int_0^R\frac{dx}{\sqrt{(R^2+z^2)-x^2}}=\frac{Q}{\varepsilon_0 R}\Big|_0^R\sin^{-1}\left(\sqrt{\frac{r^2+z^2}{R^2+z^2}}\right)$$

$$\longrightarrow V=\frac{Q}{\varepsilon_0 R}\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{z}{\sqrt{z^2+R^2}}\right)\right)\longrightarrow V=\frac{Q}{\varepsilon_0 R}\cos^{-1}\left(\frac{z}{\sqrt{z^2+R^2}}\right)\longrightarrow\boxed{V=\frac{Q}{\varepsilon_0 R}\cot^{-1}\left(\frac{z}{R}\right)}$$

امتحان پنجم 89

۱) الف)



$$\begin{cases}-\mu g\cos\theta\cos\alpha=\ddot{x}\\ -g(\sin\theta+\mu\cos\theta\sin\alpha)=\ddot{y}\\ \tan\alpha=\frac{dy}{dx}\end{cases}\longrightarrow y_{(1)}=-\frac{g\sin\theta}{2v_0^2\sin^2\lambda}x_{(1)}^2+\frac{x_{(1)}}{\tan\lambda}\longrightarrow\frac{dy_{(1)}}{dx_{(1)}}=-\frac{g\sin\theta}{v_0^2\sin^2\lambda}x_{(1)}+\cot\lambda$$

$$\longrightarrow\begin{cases}-\mu g\cos\theta\times\frac{v_0\sin\lambda}{\sqrt{(v_0\cos\lambda-g\sin\theta t)^2+v_0^2\sin^2\lambda}}dt=d\dot{x}_{(1)}\\ -g\left(\sin\theta+\mu\cos\theta\frac{v_0\cos\lambda-g\sin\theta t}{\sqrt{v_0^2\sin^2\lambda+(v_0\cos\lambda-g\sin\theta t)^2}}\right)dt=d\dot{y}_{(1)}\end{cases}$$

$$\int d\dot{x}_{(1)}=-v_0\sin\lambda\,\mu g\cos\theta\frac{dt}{\sqrt{(v_0\cos\lambda-g\sin\theta t)^2+v_0^2\sin^2\lambda}}\xrightarrow{v_0\cos\lambda-g\sin\theta t=u}\dot{x}_{(1)}=+v_0\mu\sin\lambda\cot\theta\int\frac{du}{\sqrt{u^2+v_0^2\sin^2\lambda}}+v_0\sin\lambda$$

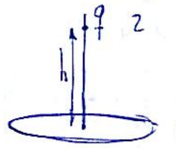
$$d\dot{y}_{(1)}=-g\left(\sin\theta dt+\mu\cos\theta\frac{(v_0\cos\lambda-g\sin\theta t)dt}{\sqrt{v_0^2\sin^2\lambda+(v_0\cos\lambda-g\sin\theta t)^2}}\right)\qquad \dot{y}_{(1)}=-g\left(t\sin\theta-\frac{\mu\cot\theta}{g}\int\frac{udu}{\sqrt{u^2+v_0^2\sin^2\lambda}}\right)+v_0\cos\lambda$$

$$\longrightarrow\begin{cases}\dot{x}_{(1)}=v_0\sin\lambda\left(1+\mu\cot\theta\ln\left(\frac{v_0\cos\lambda-g\sin\theta t+\sqrt{(v_0\cos\lambda-g\sin\theta t)^2+v_0^2\sin^2\lambda}}{v_0(1+\cos\lambda)}\right)\right)\ \checkmark\\ \dot{y}_{(1)}=v_0\cos\lambda+g\left(-t\sin\theta+\frac{\mu\cot\theta}{g}\left(\sqrt{(v_0\cos\lambda-g\sin\theta t)^2+v_0^2\sin^2\lambda}-v_0\right)\right)\ \checkmark\end{cases}$$

بالا رفتن

امتحان چهارم 89

۴) ب)



$$\begin{cases}V_1=P_{11}q_1+P_{12}q_2\\ V_2=P_{21}q_1+P_{22}q_2\end{cases}\ \Big|\ q_2=0\rightarrow\begin{cases}V_1=\left(\frac{\pi}{2\varepsilon_0 R}\right)Q\rightarrow\boxed{P_{11}=\frac{\pi}{2\varepsilon_0 R}}\\ V_2=\left(\frac{1}{\varepsilon_0 R}\cot^{-1}\left(\frac{h}{R}\right)\right)Q\rightarrow\boxed{P_{21}=\frac{\cot^{-1}\left(\frac{h}{R}\right)}{\varepsilon_0 R}}\end{cases}\ \Big|\ V_1=0\rightarrow q_1=-\frac{P_{12}}{P_{11}}q\rightarrow\boxed{q_1=-\left(\frac{2\cot^{-1}(h/R)}{\pi}\right)q}$$

امتحان پنجم 89

۱) ب)

$$v_0\cos\lambda-gt\sin\theta+\mu\cot\theta\left(v_0\sin\lambda-v_0\right)=0\longrightarrow\boxed{t=\frac{v_0}{g\sin\theta}\left(\cos\lambda+\mu\cot\theta(\sin\lambda-1)\right)}\ \checkmark$$

ب)

$$v_x=v_0\sin\lambda\left(1+\mu\cot\theta\ln\left(\frac{\sin\lambda}{1+\cos\lambda}\right)\right)\longrightarrow\boxed{v_{(x)}=v_0\sin\lambda\left(1+\mu\cot\theta\ln\left(\tan\frac{\lambda}{2}\right)\right)}\ \checkmark$$

$$y=\left(v_0\cos\lambda t-\frac{g\sin\theta t^2}{2}\right)-\mu\cot\theta\left(v_0 t+\frac{1}{g\sin\theta}\int\sqrt{v_0^2\sin^2\lambda+u^2}\,du\right)$$

$$\int\sqrt{u^2+v_0^2\sin^2\lambda}\,du=v_0\sin\lambda\int\frac{d\theta}{\cos^3\theta},\ \tan\theta=\frac{u}{v_0\sin\lambda}\rightarrow A=\frac{v_0^2\sin^2\lambda}{2}\left(\frac{\tan\theta}{\cos\theta}+\ln\left(\frac{1}{\cos\theta}+\tan\theta\right)\right)=\frac{v_0^2\sin^2\lambda}{2}\left(\frac{u}{v_0\sin\lambda}\sqrt{1+\frac{u^2}{v_0^2\sin^2\lambda}}+\ln\left(\sqrt{1+\frac{u^2}{v_0^2\sin^2\lambda}}+\frac{u}{v_0\sin\lambda}\right)\right)$$

$$=-\frac{v_0^2\sin^2\lambda}{2}\left(\cot\lambda\times\frac{1}{\sin\lambda}+\ln\left(\cot\lambda+\frac{1}{\sin\lambda}\right)\right)\longrightarrow y=\frac{v_0\cos\lambda}{2g\sin\theta}\times v_0\cos\lambda-\mu\cot\theta\left(\frac{v_0^2\cos\lambda}{g\sin\theta}-\frac{v_0^2\sin^2\lambda}{2g\sin\theta}\left(\frac{\cos^2\lambda}{\sin\lambda}+\ln\left(\cot\left(\frac{\lambda}{2}\right)\right)\right)\right)\longrightarrow$$

$$\boxed{y_{max}=\frac{v_0^2}{g\sin\theta}\left(\frac{\cos^2\lambda}{2}-\mu\cot\theta\left(\cos\lambda-\sin^2\lambda\left(\frac{\cos^2\lambda}{\sin\lambda}+\ln\cot\left(\frac{\lambda}{2}\right)\right)\right)\right)}\ \times$$

ت)

$$v_0\cos\lambda t_{(1)}-\frac{g\sin\theta}{2}\times 2t_{(1)}\times\frac{2v_0\cos\lambda}{g\sin\theta}=\mu\cot\theta\left(\frac{2v_0\cos\lambda}{g\sin\theta}+\frac{v_0^2\sin^2\lambda}{2g\sin\theta}A'\right)\longrightarrow -t_{(1)}v_0\cos\lambda=\frac{\mu\cos\theta v_0^2}{g\sin^2\theta}\left(2\cos\lambda-2\cos\lambda\ln\left(\tan\frac{\lambda}{2}\right)\right)$$

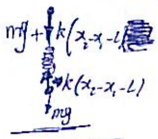
$$A'=-\cot\lambda\times\frac{1}{\sin\lambda}+\ln\left(\frac{1}{\sin\lambda}-\cot\lambda\right)-\frac{\cot\lambda}{\sin\lambda}-\ln\left(\frac{1}{\sin\lambda}+\cot\lambda\right)=-\frac{2\cos\lambda}{\sin^2\lambda}\ln\left(\frac{1-\cos\lambda}{1+\cos\lambda}\right)=-\frac{4\cos\lambda}{\sin^2\lambda}\ln\left(\tan\frac{\lambda}{2}\right)$$

$$\longrightarrow T=\frac{2v_0\cos\lambda}{g\sin\theta}+\frac{2\mu v_0\cos\theta}{g\sin^2\theta}\left(\ln\left(\tan\frac{\lambda}{2}\right)-1\right)\longrightarrow\boxed{T=\frac{2v_0}{g\sin\theta}\left(\cos\lambda+\frac{\mu\cos\theta}{\sin\theta}\left(\ln\left(\tan\frac{\lambda}{2}\right)-1\right)\right)}\ \times$$

باسمه تعالی

سال 89

(2



$$\begin{cases} -mg - k(x_2-x_1-l) = m\ddot{x}_2 \\ -mg + k(x_2-x_1-l) = m\ddot{x}_1 \end{cases}$$

(1) $\to \ddot x_1+\ddot x_2+2g=0 \to \dot x_1+\dot x_2 = -2gt \to x_1+x_2=-gt^2+l \to x_1=-x_2-gt^2+l$

(1) $\to \ddot x_2 + \frac{2k}{m}x_2 = -g - \frac{k}{m}(gt^2-2l) \to \ddot x_2+\frac{2k}{m}x_2 = -\frac{kg}{m}t^2+(\frac{2kl}{m}-g)$

$\ddot x+\omega^2 x = At^2+B \to x = A\sin\omega t + B\cos(\omega t) + \frac{At^2+B}{\omega^2} - \frac{2A}{\omega^4} \to x_2 = A\sin(\omega t)+B\cos(\omega t) - \frac{gt^2}{2}+l-\frac{mg}{2k}+\frac{mg}{2k}$

$t=0: x_2=l \to B+l=l \to \boxed{B=0}$, $t=0: \dot x_2=-v_0 \to -A\omega=v_0 \to \boxed{A=-\frac{v_0}{\omega}} \to \boxed{x_2 = -\frac{v_0}{\omega}\sin(\omega t) - \frac{gt^2}{2}+l} \to \boxed{x_1 = \frac{v_0}{\omega}\sin(\omega t) - \frac{gt^2}{2}}$

$x_2-x_1 = l - \frac{2v_0}{\omega}\sin(\omega t) \to l - \frac{2\sqrt{2gh}}{\sqrt{\frac{2k}{m}}} \geq l_0 \to \boxed{l-2\sqrt{\frac{mgh}{k}}\geq l_0}$ (a

$x_1=0 \to \boxed{\frac{gt_2^2}{2} = \sqrt{\frac{mgh}{k}}\sin(\sqrt{\frac{2k}{m}}t_2)}$ (b , $\boxed{x_2 = l - gt_2^2}$ (c

(3

$$\begin{cases} mg - T\frac{z}{l} = m\ddot z \\ -T\frac{\rho}{l} = m(\ddot\rho - \rho\dot\varphi^2) \\ 2\dot\rho\dot\varphi + \rho\ddot\varphi = 0 \to \rho^2\dot\varphi = \text{ثابت} \end{cases}$$ (a

$\rho = l\sin\theta \to \dot\rho = l\cos\theta\dot\theta \to \ddot\rho = l(\ddot\theta\cos\theta - \sin\theta\dot\theta^2)$

$z = l\cos\theta \to \dot z = -l\sin\theta\dot\theta \to \ddot z = -l(\ddot\theta\sin\theta+\cos\theta\dot\theta^2)$

$\to \frac{\ddot z - g}{\ddot\rho-\rho\dot\varphi^2}$ ~~$\cos\theta \to -l\ddot\theta\sin^2\theta - l\cos\theta\dot\theta^2 - g\cos\theta = l\ddot\theta\sin\theta\cos\theta - l\sin^2\theta\dot\theta^2 - l\sin\theta\dot\varphi^2$~~

$\frac{\ddot z-g}{\ddot\rho-\rho\dot\varphi^2} = \frac{\cos\theta}{\sin\theta} \to -l\ddot\theta\sin^2\theta - l\sin\theta\cos\theta\dot\theta^2 - g\sin\theta = l\ddot\theta\cos^2\theta - l\sin\theta\cos\theta\dot\theta^2 - l\sin\theta\cos\theta\dot\varphi^2 \to \boxed{\frac{g}{l}\sin\theta+\ddot\theta - \sin\theta\cos\theta\dot\varphi^2=0}$ (b

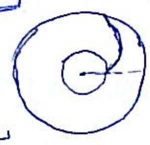
$\rho^2\dot\varphi = \text{ثابت} \to \sin^2\theta\dot\varphi = \frac{v_0}{l}\sin^2\theta_0 \to \dot\varphi = \frac{v_0}{l}\frac{\sin^2\theta_0}{\sin^2\theta} \to \boxed{\frac{g}{l}\sin\theta+\ddot\theta - \frac{v_0^2}{l^2}\sin^4\theta_0\frac{\cos\theta}{\sin^3\theta}=0}$ (c

$\frac{g}{l}(\sin\theta_0+\theta_{(1)}\cos\theta_0)+\ddot\theta_{(1)} - \frac{v_0^2\sin^4\theta_0}{l^2}\frac{\cos\theta_0-\theta_{(1)}\sin\theta_0}{(\sin\theta_0+\theta_{(1)}\cos\theta_0)^3}=0 \to \left(\frac{g}{l}\cos\theta_0 - \frac{v_0^2\sin\theta_0}{l^2}(-\sin\theta_0 - \frac{3\cos^2\theta_0}{\sin\theta_0})\right)\theta_{(1)}+\ddot\theta_{(1)}=0$ (e

ثابت $\theta \to \boxed{\cos\theta_0 = \frac{lg}{v_0^2}}$ (d

$\to \omega^2 = (\frac{g}{v_0})^2 + \frac{v_0^2}{l^2}(1-\frac{l^2g^2}{v_0^4}+3\frac{l^2g^2}{v_0^4}) \to \omega^2 = 3(\frac{g}{v_0})^2+(\frac{v_0}{l})^2 \to \boxed{\omega=\sqrt{3(\frac{g}{v_0})^2+(\frac{v_0}{l})^2}}$ ✗ $\boxed{\Omega=\sqrt{\frac{g}{l}(3\cos\theta_0+\frac{1}{\cos\theta_0})}}$

(4



الف)

$E\times 2\pi r L = \frac{Q}{\varepsilon_0} \to E = \frac{Q}{2\pi l\varepsilon_0}\frac{1}{r} \to V-0 = \frac{Q}{2\pi l\varepsilon_0}\ln(\frac{b}{a}) \to \frac{Q}{2\pi\varepsilon_0 L} = \frac{V}{\ln(\frac{b}{a})} \to \boxed{\vec E = (\frac{V}{\ln(\frac{b}{a})})\frac{1}{r}\hat r}$

ب)

$F_{\theta}=0 \to r^2\dot\theta = \text{ثابت} \to \boxed{r^2\dot\theta = au_0\sin\alpha}$, $\frac{1}{2}mv^2+U_e = E = \text{ثابت} \to \boxed{\frac{1}{2}m(\dot r^2+\frac{(au_0\sin\alpha)^2}{r^2}) + \frac{eV}{\ln(\frac{b}{a})}\ln(\frac{r}{a}) = \frac{1}{2}mu_0^2}$

$U_e = -\int_a^r \vec F_r\cdot d\vec r = \frac{eV}{\ln(\frac{b}{a})}\ln(\frac{r}{a}) \to U_e = U_0+\frac{eV}{\ln(\frac{b}{a})}\ln(\frac{r}{a})$

ج)

$v_\theta = \frac{au_0\sin\alpha}{b}$, ~~[illegible]~~ $v = \sqrt{u_0^2-\frac{2eV}{m}}$; $v\sin\beta = v_\theta \to \boxed{\sin\beta = \frac{au_0\sin\alpha}{b\sqrt{u_0^2-\frac{2eV}{m}}}}$

د)

$v_r^2+v_\theta^2=v^2 \to v_r^2 = u_0^2(1-\frac{a\sin\alpha}{b}) - \frac{2eV}{m} \to \boxed{u_{0_{min}} = \frac{2beV}{m(b-a\sin\alpha)}}$

ه)

$\sin\beta = (\frac{au_0}{b\sqrt{u_0^2-\frac{2eV}{m}}})\sin\alpha \to \cos\beta\frac{d\beta}{dN} = (\underbrace{\frac{au_0}{b\sqrt{u_0^2-\frac{2eV}{m}}}}_{A})\cos\alpha\frac{d\alpha}{dN} \to n' = \frac{n\cos\beta}{2\pi A\cos\alpha} \to \boxed{n' = \frac{n\sqrt{1-A^2\sin^2\alpha}}{2A\pi\cos\alpha}}$

باسمه تعالی

1)

$\ddot{S} = -\mu g\cos\theta + g\sin\theta\cos\varphi$

$\ddot{x} = g\sin\theta - \mu g\cos\theta\cos\varphi$

$\ddot{y} = -\mu g\cos\theta\sin\varphi$

$\rightarrow \begin{cases} \ddot{S} = g\sin\theta(\cos\varphi - \alpha) \\ \ddot{x} = g\sin\theta(1-\alpha\cos\varphi) \\ \ddot{y} = -g\alpha\sin\theta\sin\varphi \end{cases} \rightarrow \ddot{S} + \frac{g\sin\theta}{\alpha} = \frac{g\sin\theta - \ddot{x}}{\alpha} \rightarrow \alpha\ddot{S} + \ddot{x} = g\sin\theta(1-\alpha^2) \xrightarrow{\alpha=1} \boxed{\ddot{x}+\ddot{S}=0}$

$\rightarrow \boxed{\alpha\dot{S} + \dot{x} = (1-\alpha^2)g\sin\theta\, t + \alpha v_0} \xrightarrow{\dot{S} = \frac{\dot{x}}{\cos\varphi}}$

$\tan\varphi = \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \rightarrow$

$\left(\frac{\alpha}{\cos\varphi}+1\right)\dot{x} = (1-\alpha^2)g\sin\theta\, t + v_0\alpha \rightarrow \boxed{\dot{x} = \left(\frac{\cos\varphi}{\alpha+\cos\varphi}\right)\left((1-\alpha^2)g\sin\theta\, t + \alpha v_0\right)}$

$\ddot{y} = \ddot{x}\tan\varphi + \frac{\dot{x}\dot{\varphi}}{\cos^2\varphi} \rightarrow g\sin\theta\sin\varphi\left(\frac{1}{\cos\varphi} - \alpha\right) + \frac{\dot{x}\dot{\varphi}}{\cos^2\varphi} = -\alpha g\sin\theta\sin\varphi \rightarrow \boxed{\dot{x}\dot{\varphi} = -g\sin\theta\sin\varphi\cos\varphi} \rightarrow$

$\boxed{\dot{\varphi} = -\frac{g\sin\theta\sin\varphi(\alpha+\cos\varphi)}{\alpha v_0 + (1-\alpha^2)g\sin\theta\, t}} \xrightarrow{\alpha=1} \boxed{\dot{\varphi} = -\frac{g}{v_0}\sin\theta\sin\varphi(1+\cos\varphi)}$, $\boxed{\dot{x} = \frac{\cos\varphi}{1+\cos\varphi}v_0} \rightarrow \dot{y} = \frac{\sin\varphi}{1+\cos\varphi}v_0$

a) $\dot{\varphi}$ منفی است. پس φ در حال کاهش است و می‌دانیم که φ منفی نمی‌شود. پس φ به سمت صفر میل می‌کند $\leftarrow$ $\boxed{\varphi = 0}$, $\boxed{\dot{x} = \frac{v_0}{2}}$

$\rightarrow \dot{S} + \dot{x} = v_0 \rightarrow \boxed{\dot{S} = \frac{v_0}{2}} \rightarrow \boxed{\dot{y} = 0}$

b) می‌توانیم $\cos\varphi$ را هم تقریب بزنیم $\boxed{\dot{x} = \frac{(1-\alpha^2)g\sin\theta\cos\varphi}{\alpha+\cos\varphi}t}$

$\dot{S} = \dot{x} = 0 \rightarrow \boxed{T = \frac{\alpha v_0}{(\alpha^2-1)g\sin\theta}}$

c)

2) a)

$\begin{cases} r\,d\theta = ds \\ \tan\theta = \frac{dy}{dx} = f'_{(x)} \end{cases} \rightarrow \begin{cases} r\,d\theta = dx\sqrt{1+f'^2_{(x)}} \\ \frac{d\theta}{\cos^2\theta} = f''_{(x)}dx \rightarrow \frac{dx}{d\theta} = \frac{(1+f'^2)}{f''} \end{cases} \rightarrow r = \frac{(1+f'^2)^{3/2}}{f''} \rightarrow \boxed{R = -\frac{1}{2a}} \rightarrow \boxed{|R| = \frac{1}{2a}}$

$\frac{v_0^2}{R} \geq g \rightarrow \boxed{v_0 \geq \sqrt{\frac{g}{2a}}}$

b) $\boxed{\ddot{y} = -g}$, $\boxed{\ddot{x} = 0}$, $\frac{1}{2}m(\dot{x}^2+\dot{y}^2) + mgf_{(x)} = \frac{1}{2}mv_0^2 \rightarrow \dot{x}^2(1+f'^2_{(x)}) = v_0^2 - 2gf_{(x)} \rightarrow \boxed{\dot{x} = \sqrt{\frac{v_0^2-2gf_{(x)}}{1+f'^2_{(x)}}}}$, $\boxed{\dot{y} = f'_{(x)}\sqrt{\frac{v_0^2-2gf_{(x)}}{1+f'^2_{(x)}}}}$

c) روش اول :

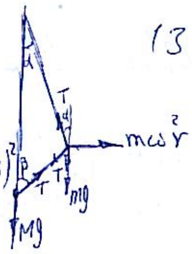
$\frac{1}{2}mv^2 + mgf_{(x)} = \frac{1}{2}mv_0^2 \rightarrow v^2 = v_0^2 - 2gf_{(x)}$,

$\frac{v^2}{R} = g\cos\alpha = -g \times \frac{1}{\sqrt{1+f'^2_{(x)}}} \rightarrow v_0^2 - 2gf_{(x)} = -\frac{g(1+f'^2_{(x)})}{f''_{(x)}} \rightarrow \boxed{f''_{(x)}\left(2f_{(x)} - \frac{v_0^2}{g}\right) = (1+f'^2_{(x)})}$

روش دوم :

$\ddot{x} = 0 \rightarrow \dot{x} = C\mathrm{o}l^t \rightarrow -2gf'_{(x)}(1+f'^2_{(x)}) - 2f'_{(x)}f''_{(x)}(v_0^2 - 2gf_{(x)}) = 0 \rightarrow f''_{(x)}\left(2f_{(x)} - \frac{v_0^2}{g}\right) = (1+f'^2_{(x)})$

3) آ, ب)



$T\cos\beta = Mg \rightarrow \boxed{T = \frac{Mg}{\cos\beta}}$

$m\omega^2 r = T(\sin\alpha + \sin\beta)$

$mg = T(\cos\alpha - \cos\beta)$

$\rightarrow \begin{cases} m = M\left(\frac{\cos\alpha}{\cos\beta} - 1\right) \rightarrow \boxed{\cos\beta = \frac{M}{M+m}\cos\alpha} \rightarrow \sin\beta = \frac{\sqrt{m^2 + M^2\sin^2\alpha + 2Mm}}{M+m} \\ m\omega^2 L\frac{\sin\alpha\sin\beta}{\sin\alpha+\sin\beta} = \frac{Mg}{\cos\beta}(\sin\alpha+\sin\beta) \rightarrow \left(\frac{m\omega^2 L}{Mg}\right)\sin\alpha\sin\beta\cos\beta = (\sin\alpha+\sin\beta)^2 \rightarrow \end{cases}$

$\begin{cases} (L_0 - L_1)\sin\beta = r \\ L_1\sin\alpha = r \end{cases} \rightarrow r = \frac{\sin\alpha\sin\beta}{\sin\alpha+\sin\beta}L$

$\sin\alpha\sin\beta(\gamma\cos\beta - 2) = \sin^2\alpha + \sin^2\beta \rightarrow \sin^2\beta\sin^2\alpha(\gamma\cos\beta-2)^2 = (\sin^2\alpha+\sin^2\beta)^2$

$\rightarrow (\sin^2\alpha+\sin^2\beta)^2 = \left(1-\cos^2\alpha + 1 - \frac{M^2}{(M+m)^2}\cos^2\alpha\right)^2 = \left(2 - \left(1+\frac{M^2}{(M+m)^2}\right)\cos^2\alpha\right)^2 \rightarrow$

$4 + \left(1+\frac{M^2}{(M+m)^2}\right)^2\cos^4\alpha - 4\left(1+\frac{M^2}{(M+m)^2}\right)\cos^2\alpha = \left(1-\frac{M^2}{(M+m)^2}\cos^2\alpha\right)(1-\cos^2\alpha)\left(\gamma\cos\beta(\gamma\cos\beta - 4) + 4\right) =$

$4 + \frac{4M^2}{(M+m)^2}\cos^2\alpha - 4\left(1+\frac{M^2}{(M+m)^2}\right)\cos^2\alpha + \gamma\frac{M}{M+m}\cos\alpha\left(\frac{\gamma M}{M+m}\cos\alpha - 4\right)(1-\cos^2\alpha)\left(1-\frac{M^2}{(M+m)^2}\cos^2\alpha\right)$

$\rightarrow \left(1-\frac{M^2}{(M+m)^2}\right)^2\cos^4\alpha = \frac{\gamma M}{M+m}\cos\alpha\left(\frac{\gamma M\cos\alpha}{M+m} - \frac{\gamma M^3}{(M+m)^3}\cos^3\alpha - \frac{\gamma M}{M+m}\cos^3\alpha + \frac{\gamma M^3}{(M+m)^3}\cos^5\alpha - 4 + 4\cos^2\alpha + \frac{4M^2}{(M+m)^2}\cos^2\alpha - \frac{4M^2\cos^4\alpha}{(M+m)^2}\right)$

$\left(\frac{\gamma M^3}{(M+m)^3}\right)\cos^5\alpha + \left(-\frac{4M^2}{(M+m)^2}\right)\cos^4\alpha + \left(-\frac{\gamma M}{M+m}\left(1+\frac{M^2}{(M+m)^2}\right) - \left(1-\frac{M^2}{(M+m)^2}\right)\frac{M+m}{\gamma M}\right)\cos^3\alpha + \left(4\left(1+\frac{M^2}{(M+m)^2}\right)\right)\cos^2\alpha + \frac{\gamma M}{M+m}\cos\alpha - 4 = 0$

پ) $L_1 = \frac{3}{4}L \rightarrow \frac{\sin\beta}{\sin\alpha+\sin\beta} = \frac{3}{4} \rightarrow \sin\beta = 3\sin\alpha \rightarrow 9(M+m)^2\sin^2\alpha = m^2 + M^2\sin^2\alpha + 2Mm \rightarrow \boxed{\sin\alpha = \sqrt{\frac{(2M+m)m}{9(M+m)^2 - M^2}}}$, $\boxed{\sin\beta = 3\sqrt{\frac{m(2M+m)}{9(M+m)^2 - M^2}}}$

ت) $\frac{m\omega^2 L}{Mg} \times 3\sin^2\alpha\cos\beta = 16\sin^2\alpha \rightarrow \omega^2 = \frac{16Mg}{3mL} \times \frac{1}{\cos\beta}$

$\rightarrow \boxed{\omega = \sqrt{\frac{4g}{3mL}\sqrt{2\left(9(M+m)^2 - M^2\right)}}}$

$\cos^2\beta = 1 - \frac{9m(2M+m)}{9(M+m)^2 - M^2} = \frac{9M^2 + 9m^2 + 18Mm - M^2 - 18Mm - 9m^2}{9(M+m)^2 - M^2} \rightarrow \cos\beta = 2M\sqrt{\frac{2}{9(M+m)^2 - M^2}}$

$\boxed{T = \frac{g}{2}\sqrt{\frac{9(M+m)^2 - M^2}{2}}}$