In The Name Of God

Power Electronics

Chopper Converters

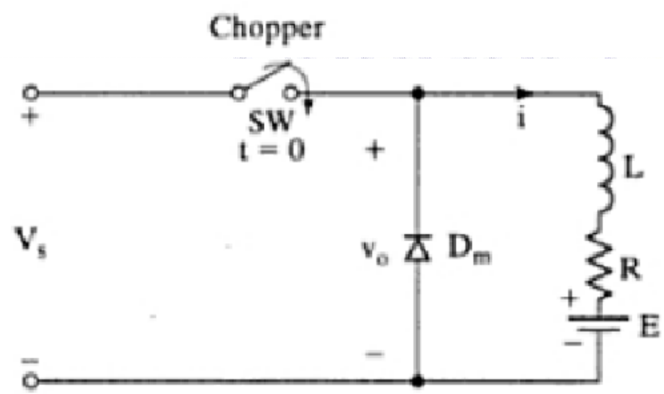
Behrooz Adineh

Fall 2015

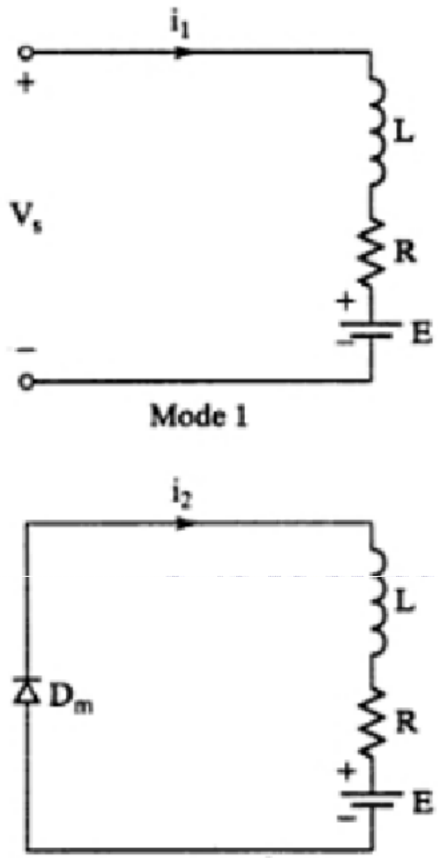
STEP-DOWN CONVERTER WITH *RL* LOAD

The operation of the converter can be divided into two modes. During mode 1, the converter is switched on and the current flows from the supply to the load. During mode 2, the converter is switched off and the load current continues to flow through freewheeling diode D_m.

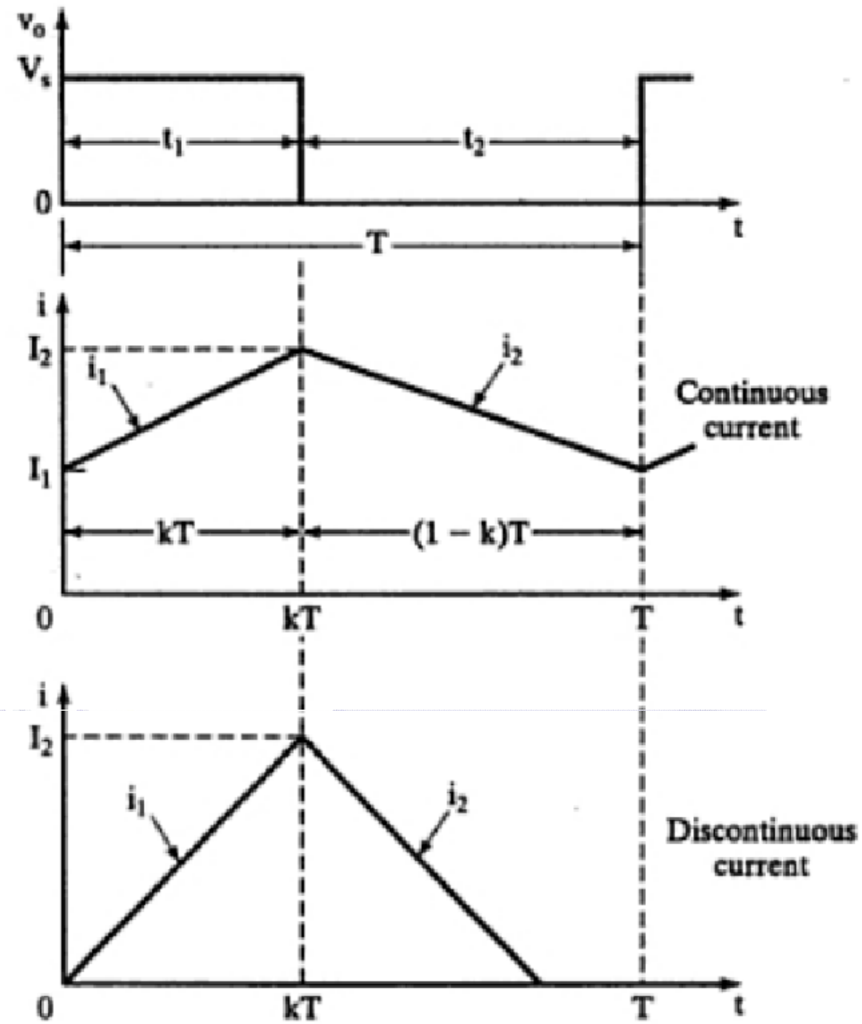
The load current and output voltage waveforms are shown in Figure with the assumption that the load current rises linearly. However, the current flowing through an *RL* load rises or falls exponentially with a time constant. The load time constant ($\tau = L/R$) is generally much higher than the switching period T. Thus, the linear approximation is valid for many circuit conditions and simplified expressions can be derived within reasonable accuracies.



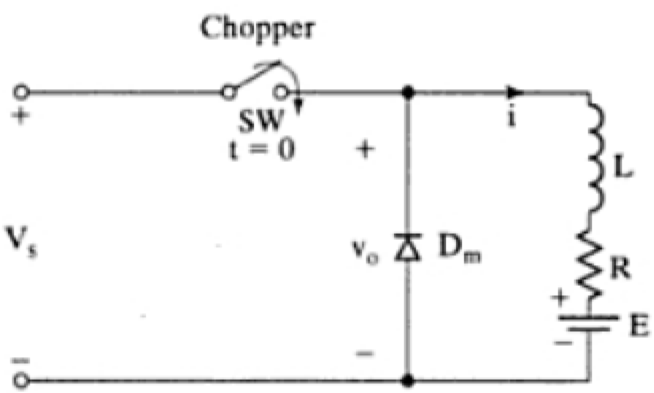
Rashid, Page 171



Mode 2
quivalent circuits



(b) Waveforms



Rashid, Page 172

The load current for mode 1 can be found from

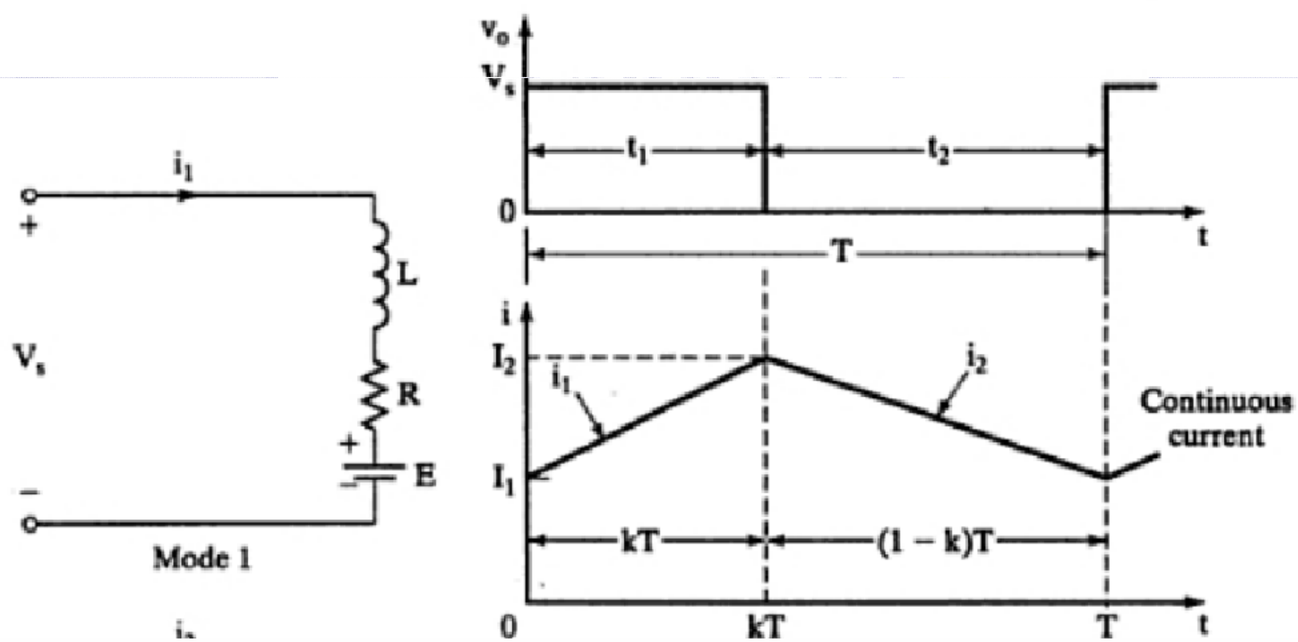
$$V_s = Ri_1 + L\frac{di_1}{dt} + E$$

which with initial current $i_1(t = 0) = I_1$ gives the load current as

$$i_1(t) = I_1 e^{-tR/L} + \frac{V_s - E}{R}(1 - e^{-tR/L}) \tag{5.11}$$

This mode is valid $0 \le t \le t_1$ $(= kT)$; and at the end of this mode, the load current becomes

$$i_1(t = t_1 = kT) = I_2 \tag{5.12}$$



Rashid, Page 172

The load current for mode 2 can be found from

$$0 = Ri_2 + L\frac{di_2}{dt} + E$$

With initial current $i_2(t = 0) = I_2$ and redefining the time origin (i.e., $t = 0$) at the beginning of mode 2, we have

$$i_2(t) = I_2 e^{-tR/L} - \frac{E}{R}(1 - e^{-tR/L}) \tag{5.13}$$

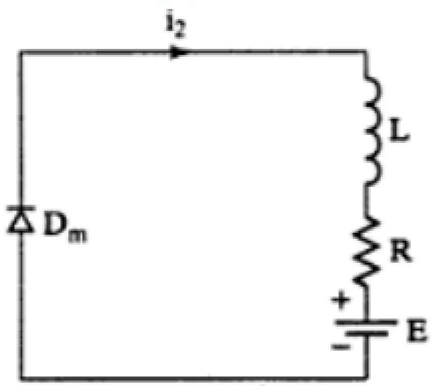
This mode is valid for $0 \leq t \leq t_2 [= (1 - k)T]$. At the end of this mode, the load current becomes

$$i_2(t = t_2) = I_3 \tag{5.14}$$

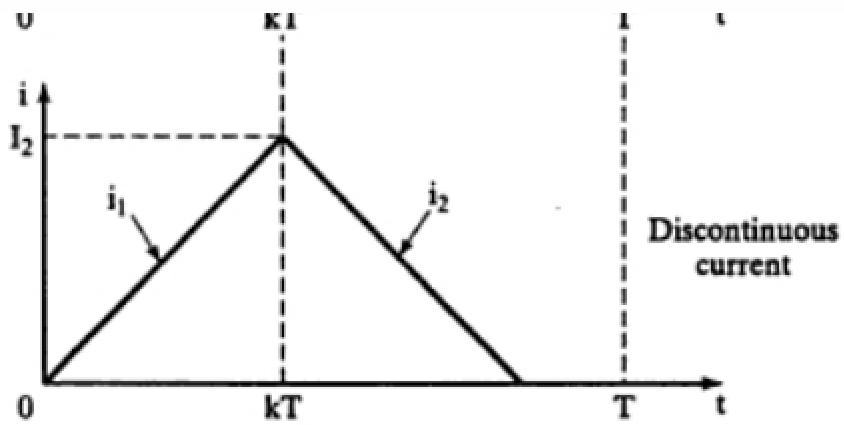
At the end of mode 2, the converter is turned on again in the next cycle after time, $T = 1/f = t_1 + t_2$.

Under steady-state conditions, $I_1 = I_3$. The peak-to-peak load ripple current can be determined from Eqs. (5.11) to (5.14). From Eqs. (5.11) and (5.12), I_2 is given by

$$I_2 = I_1 e^{-kTR/L} + \frac{V_s - E}{R}(1 - e^{-kTR/L}) \tag{5.15}$$



(a) Equivalent circuits



(b) Waveforms

Rashid, Page 172

From Eqs. (5.13) and (5.14), I_3 is given by

$$I_3 = I_1 = I_2 e^{-(1-k)TR/L} - \frac{E}{R}(1 - e^{-(1-k)TR/L}) \tag{5.16}$$

Solving for I_1 and I_2, we get

$$I_1 = \frac{V_S}{R}\left(\frac{e^{kz} - 1}{e^{z} - 1}\right) - \frac{E}{R} \tag{5.17}$$

where $z = \dfrac{TR}{L}$ is the ratio of the chopping or switching period to the load time constant.

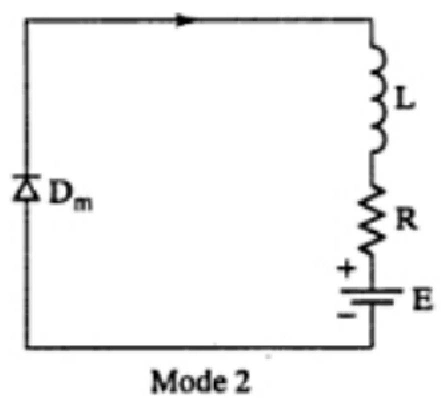
$$I_2 = \frac{V_s}{R}\left(\frac{e^{-kz} - 1}{e^{-z} - 1}\right) - \frac{E}{R} \tag{5.18}$$

The peak-to-peak ripple current is

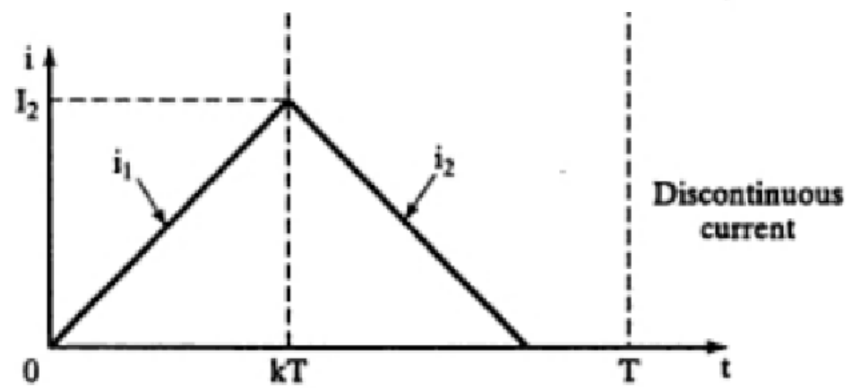
$$\Delta I = I_2 - I_1$$

which after simplifications becomes

$$\Delta I = \frac{V_s}{R}\frac{1 - e^{-kz} + e^{-z} - e^{-(1-k)z}}{1 - e^{-z}} \tag{5.19}$$



(a) Equivalent circuits



(b) Waveforms

Rashid, Page 172

The condition for maximum ripple,

$$\frac{d(\Delta I)}{dk} = 0 \tag{5.20}$$

gives $e^{-kz} - e^{-(1-k)z} = 0$ or $-k = -(1-k)$ or $k = 0.5$. The maximum peak-to-peak ripple current (at $k = 0.5$) is

$$\Delta I_{max} = \frac{V_s}{R} \tanh \frac{R}{4fL} \tag{5.21}$$

For $4fL >> R$, $\tanh \theta \approx \theta$ and the maximum ripple current can be approximated to

$$\Delta I_{max} = \frac{V_s}{4fL} \tag{5.22}$$

Note: Equations (5.11) to (5.22) are valid only for continuous current flow. For a large off-time, particularly at low-frequency and low-output voltage, the load current may be discontinuous. The load current would be continuous if $L/R >> T$ or $Lf >> R$. In case of discontinuous load current, $I_1 = 0$ and Eq. (5.11) becomes

$$i_1(t) = \frac{V_s - E}{R}(1 - e^{-tR/L})$$

and Eq. (5.13) is valid for $0 \le t \le t_2$ such that $i_2(t = t_2) = I_3 = I_1 = 0$, which gives

$$t_2 = \frac{L}{R} \ln\left(1 + \frac{RI_2}{E}\right)$$

Rashid, Page 173

Because $t = kT$, we get

$$i_1(t) = I_2 = \frac{V_s - E}{R}\left(1 - e^{-kz}\right)$$

which after substituting for I_2 becomes

$$t_2 = \frac{L}{R}\ln\left[1 + \left(\frac{V_s - E}{E}\right)\left(1 - e^{-z}\right)\right]$$

Condition for continuous current: For $I_1 \geq 0$, Eq. (5.17) gives

$$\left(\frac{e^{kz} - 1}{e^{z} - 1} - \frac{E}{V_s}\right) \geq 0$$

which gives the value of the load electromotive force (emf) ratio $x = E/V_s$ as

$$x = \frac{E}{V_s} \leq \frac{e^{kz} - 1}{e^{z} - 1} \qquad (5.23)$$

Rashid, Page 174

Example Finding the Currents of a Dc Converter with an *RL* Load

A converter is feeding an *RL* load as shown in Figure 5.3 with $V_s = 220$ V, $R = 5\ \Omega$, $L = 7.5$ mH, $f = 1$ kHz, $k = 0.5$, and $E = 0$ V. Calculate (a) the minimum instantaneous load current I_1, (b) the peak instantaneous load current I_2, (c) the maximum peak-to-peak load ripple current, (d) the average value of load current I_a, (e) the rms load current I_o, (f) the effective input resistance R_i seen by the source, (g) the rms chopper current I_R, and (h) the critical value of the load inductance for continuous load current. Use PSpice to plot the load current, the supply current, and the freewheeling diode current.

Solution

$V_s = 220$ V, $R = 5\ \Omega$, $L = 7.5$ mH, $E = 0$ V, $k = 0.5$, and $f = 1000$ Hz. From Eq. (5.15), $I_2 = 0.7165 I_1 + 12.473$ and from Eq. (5.16), $I_1 = 0.7165 I_2 + 0$.

a. Solving these two equations yields $I_1 = 18.37$ A.

b. $I_2 = 25.63$ A.

c. $\Delta I = I_2 - I_1 = 25.63 - 18.37 = 7.26$ A. From Eq. (5.21), $\Delta I_{max} = 7.26$ A and Eq. (5.22) gives the approximate value, $\Delta I_{max} = 7.33$ A.

d. The average load current is, approximately,

$$I_a = \frac{I_2 + I_1}{2} = \frac{25.63 + 18.37}{2} = 22\text{ A}$$

e. Assuming that the load current rises linearly from I_1 to I_2, the instantaneous load current can be expressed as

$$i_1 = I_1 + \frac{\Delta I t}{kT} \qquad \text{for } 0 < t < kT$$

The rms value of load current can be found from

$$I_o = \left(\frac{1}{kT}\int_0^{kT} i_1^2\,dt\right)^{1/2} = \left[I_1^2 + \frac{(I_2 - I_1)^2}{3} + I_1(I_2 - I_1)\right]^{1/2} \qquad (5.24)$$

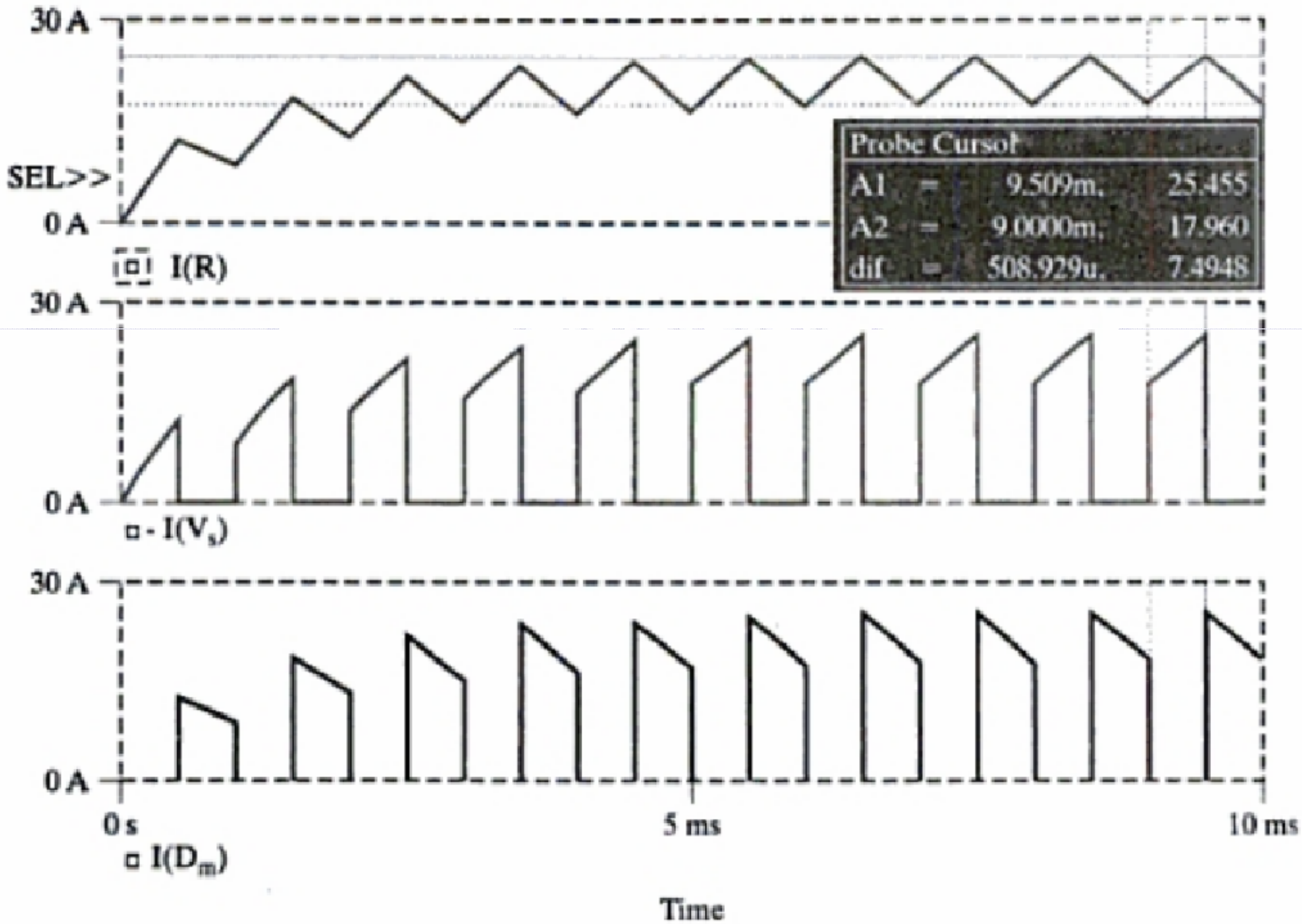
$$= 22.1\text{ A}$$

Rashid, Page 174

f. The average source current

$$I_s = kI_a = 0.5 \times 22 = 11 \text{ A}$$

and the effective input resistance $R_i = V_s/I_s = 220/11 = 20\ \Omega$.



Rashid, Page 175

DC Chopper

g. The rms converter current can be found from

$$I_R = \left(\frac{1}{T}\int_0^{kT} i_1^2\,dt\right)^{1/2} = \sqrt{k}\left[I_1^2 + \frac{(I_2 - I_1)^2}{3} + I_1(I_2 - I_1)\right]^{1/2} \tag{5.25}$$

$$= \sqrt{k}I_o = \sqrt{0.5} \times 22.1 = 15.63 \text{ A}$$

h. We can rewrite Eq. (5.23) as

$$V_S\left(\frac{e^{kz} - 1}{e^z - 1}\right) = E$$

which, after iteration, gives, $z = TR/L = 52.5$ and $L = 1 \text{ ms} \times 5/52.5 = 0.096$ mH. The SPICE simulation results [32] are shown in Figure 5.5, which shows the load current $I(E)$, the supply current $-I(V_s)$, and the diode current $I(D_m)$. We get $I_1 = 17.96$ A and $I_2 = 25.46$ A.

Rashid, Page 175

Example 5.3 Finding the Load Inductance to Limit the Load Ripple Current

The converter in Figure 5.3 has a load resistance $R = 0.25\ \Omega$, input voltage $V_s = 550$ V, and battery voltage $E = 0$ V. The average load current $I_a = 200$ A, and chopping frequency $f = 250$ Hz. Use the average output voltage to calculate the load inductance L, which would limit the maximum load ripple current to 10% of I_a.

Solution

$V_s = 550$ V, $R = 0.25\ \Omega$, $E = 0$ V, $f = 250$ Hz, $T = 1/f = 0.004$ s, and $\Delta i = 200 \times 0.1 = 20$ A. The average output voltage $V_a = kV_s = RI_a$. The voltage across the inductor is given by

$$L\frac{di}{dt} = V_s - RI_a = V_s - kV_s = V_s(1 - k)$$

If the load current is assumed to rise linearly, $dt = t_1 = kT$ and $di = \Delta i$:

$$\Delta i = \frac{V_s(1 - k)}{L}kT$$

For the worst-case ripple conditions,

$$\frac{d(\Delta i)}{dk} = 0$$

This gives $k = 0.5$ and

$$\Delta i\ L = 20 \times L = 550(1 - 0.5) \times 0.5 \times 0.004$$

and the required value of inductance is $L = 27.5$ mH.

Note: For $\Delta I = 20$ A, Eq. (5.19) gives $z = 0.036$ and $L = 27.194$ mH.

Key Points of Section 5.3

- An inductive load can make the load current continuous. However, the critical value of inductance, which is required for continuous current, is influenced by the load emf ratio. The peak-to-peak load current ripple becomes maximum at $k = 0.5$.

Rashid, Page 175-176

PRINCIPLE OF STEP-UP OPERATION

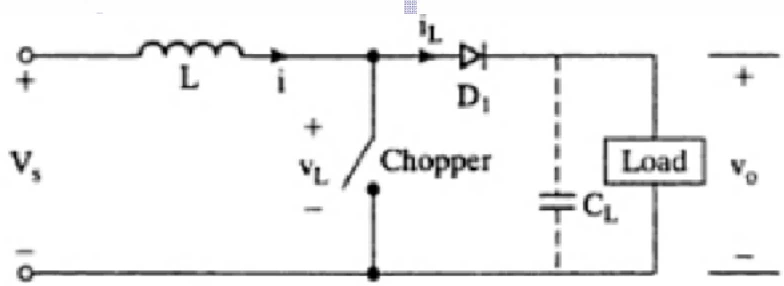
A converter can be used to step up a dc voltage and an arrangement for step-up operation is shown in Figure 5.6a. When switch SW is closed for time t_1, the inductor current rises and energy is stored in the inductor L. If the switch is opened for time t_2, the energy stored in the inductor is transferred to load through diode D_1 and the inductor current falls. Assuming a continuous current flow, the waveform for the inductor current is shown in Figure 5.6b.

When the converter is turned on, the voltage across the inductor is

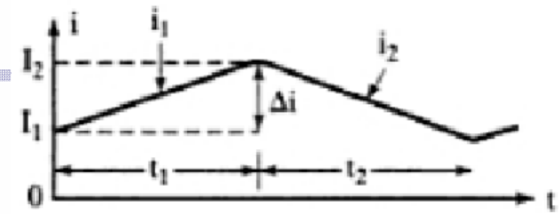
$$v_L = L\frac{di}{dt}$$

and this gives the peak-to-peak ripple current in the inductor as

$$\Delta I = \frac{V_s}{L}t_1 \qquad (5.26)$$



(a) Step-up arrangement



(b) Current waveform

Rashid, Page 176-177

The average output voltage is

$$v_o = V_s + L\frac{\Delta I}{t_2} = V_s\left(1 + \frac{t_1}{t_2}\right) = V_s\frac{1}{1-k} \quad (5.27)$$

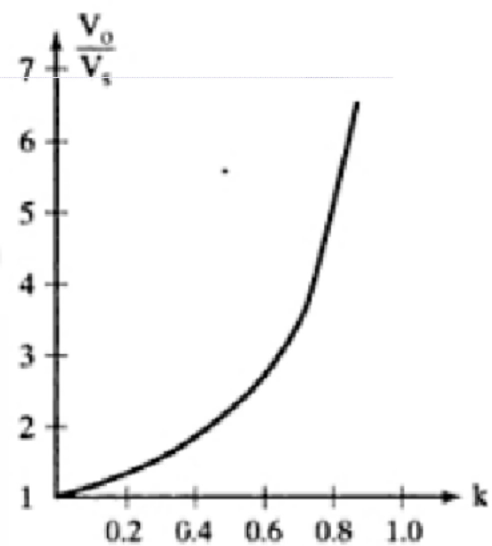
If a large capacitor C_L is connected across the load as shown by dashed lines in Figure 5.6a, the output voltage is continuous and v_o becomes the average value V_a. We can notice from Eq. (5.27) that the voltage across the load can be stepped up by varying the duty cycle k and the minimum output voltage is V_s when $k = 0$. However, the converter cannot be switched on continuously such that $k = 1$. For values of k tending to unity, the output voltage becomes very large and is very sensitive to changes in k, as shown in Figure 5.6c.

This principle can be applied to transfer energy from one voltage source to another as shown in Figure 5.7a. The equivalent circuits for the modes of operation are shown in Figure 5.7b and the current waveforms in Figure 5.7c. The inductor current for mode 1 is given by

$$V_s = L\frac{di_1}{dt}$$

and is expressed as

$$i_1(t) = \frac{V_s}{L}t + I_1 \quad (5.28)$$



(c) Output voltage

Rashid, Page 177

This principle can be applied to transfer energy from one voltage source to another as shown in Figure 5.7a. The equivalent circuits for the modes of operation are shown in Figure 5.7b and the current waveforms in Figure 5.7c. The inductor current for mode 1 is given by

$$V_s = L\frac{di_1}{dt}$$

and is expressed as

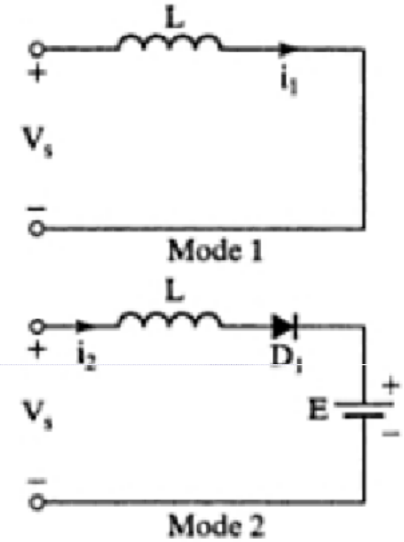
$$i_1(t) = \frac{V_s}{L}t + I_1 \qquad (5.28)$$

where I_1 is the initial current for mode 1. During mode 1, the current must rise and the necessary condition,

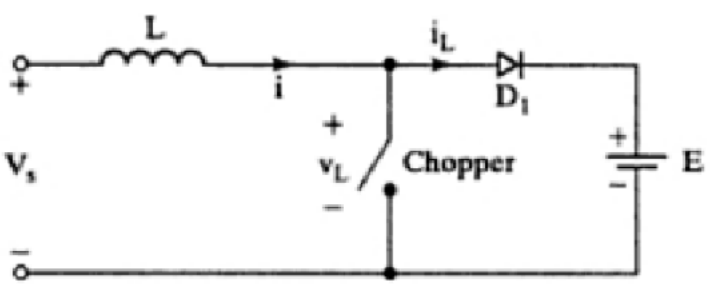
$$\frac{di_1}{dt} > 0 \quad \text{or} \quad V_s > 0$$

The current for mode 2 is given by

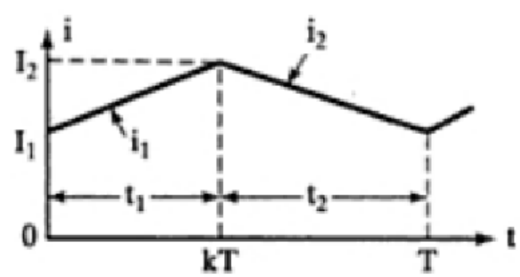
$$V_s = L\frac{di_2}{dt} + E$$



(b) Equivalent circuits



(a) Circuit diagram



(c) Current waveforms

Rashid, Page 178

and is solved as

$$i_2(t) = \frac{V_s - E}{L} t + I_2 \tag{5.29}$$

where I_2 is initial current for mode 2. For a stable system, the current must fall and the condition is

$$\frac{di_2}{dt} < 0 \quad \text{or} \quad V_s < E$$

If this condition is not satisfied, the inductor current continues to rise and an unstable situation occurs. Therefore, the conditions for controllable power transfer are

$$0 < V_s < E \tag{5.30}$$

Equation (5.30) indicates that the source voltage V_s must be less than the voltage E to permit transfer of power from a fixed (or variable) source to a fixed dc voltage. In electric braking of dc motors, where the motors operate as dc generators, terminal voltage falls as the machine speed decreases. The converter permits transfer of power to a fixed dc source or a rheostat.

When the converter is turned on, the energy is transferred from the source V_s to inductor L. If the converter is then turned off, a magnitude of the energy stored in the inductor is forced to battery E.

Note: Without the chopping action, v_s must be greater than E for transferring power from V_s to E.

Key Points of Section 5.4

- A step-up dc converter can produce an output voltage that is higher than the input. The input current can be transferred to a voltage source higher than the input voltage.