In the name of God

Power Electronic Course

PWM Inverters

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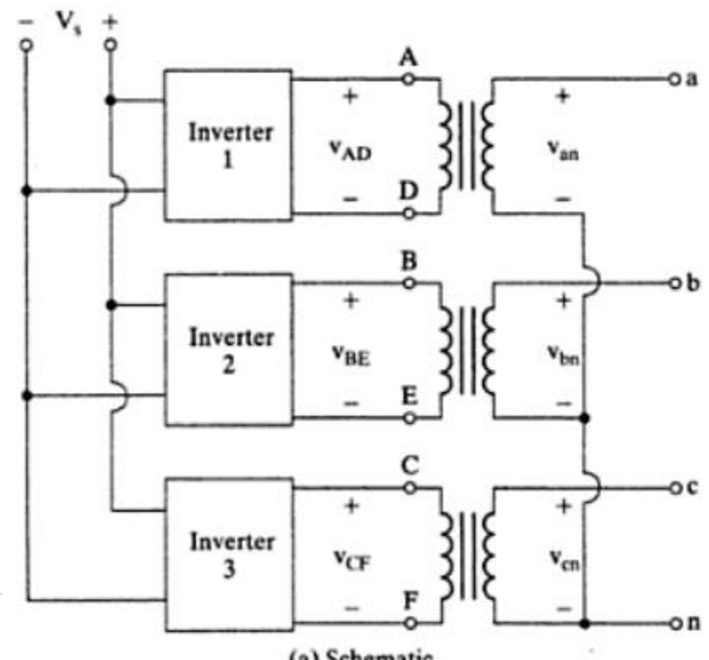
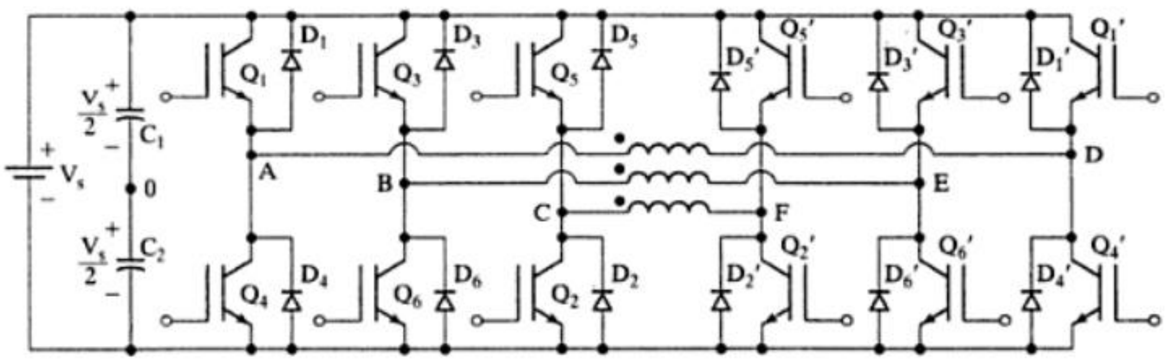
6.5 THREE-PHASE INVERTERS

Three-phase inverters are normally used for high-power applications. Three single-phase half (or full)-bridge inverters can be connected in parallel as shown in Figure 6.4a to form the configuration of a three-phase inverter. The gating signals of single-phase inverters should be advanced or delayed by 120° with respect to each other to obtain three-phase balanced (fundamental) voltages. The transformer primary windings must be isolated from each other, whereas the secondary windings may be connected in Y or delta. The transformer secondary is normally connected in delta to eliminate triplen harmonics ($n = 3, 6, 9, \ldots$) appearing on the output voltages and the circuit arrangement is shown in Figure 6.4b. This arrangement requires three single-phase transformers, 12 transistors, and 12 diodes. If the output voltages of single-phase inverters are not perfectly balanced in magnitudes and phases, the three-phase output voltages are unbalanced.

A three-phase output can be obtained from a configuration of six transistors and six diodes as shown in Figure 6.5a. Two types of control signals can be applied to the transistors: 180° conduction or 120° conduction. The 180° conduction has better utilization of the switches and is the preferred method.

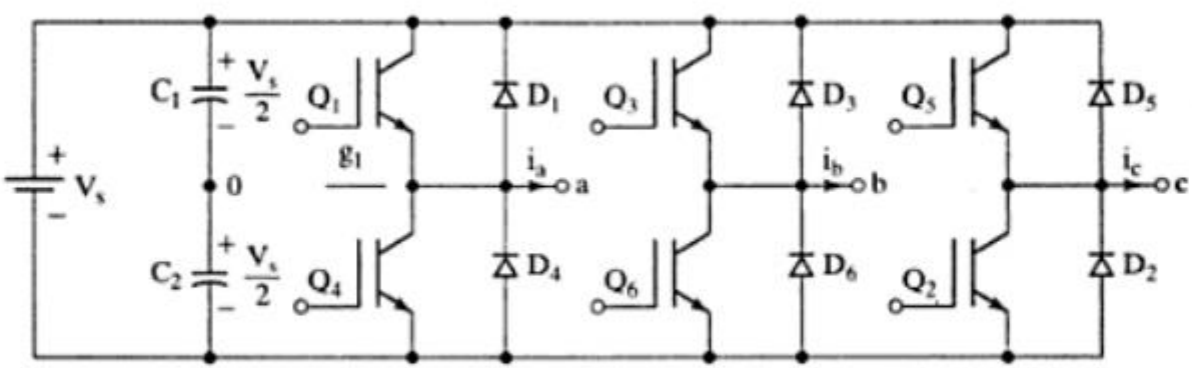


(a) Schematic

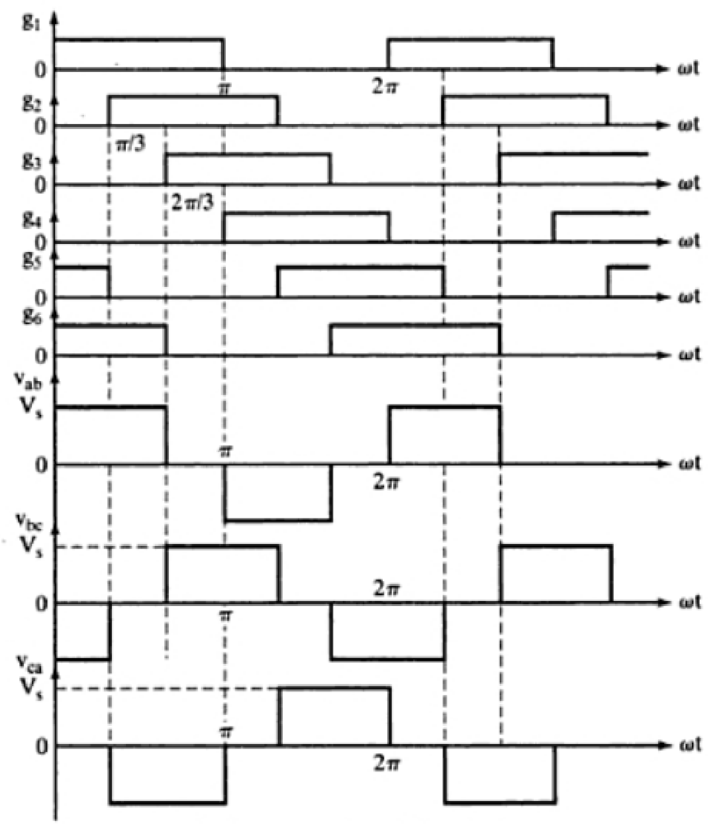


6.5.1 180-Degree Conduction

Each transistor conducts for 180°. Three transistors remain on at any instant of time. When transistor Q_1 is switched on, terminal *a* is connected to the positive terminal of the dc input voltage. When transistor Q_4 is switched on, terminal *a* is brought to the negative terminal of the dc source. There are six modes of operation in a cycle and the duration of each mode is 60°. The transistors are numbered in the sequence of gating the transistors (e.g., 123, 234, 345, 456, 561, and 612). The gating signals shown in Figure 6.5b are shifted from each other by 60° to obtain three-phase balanced (fundamental) voltages.

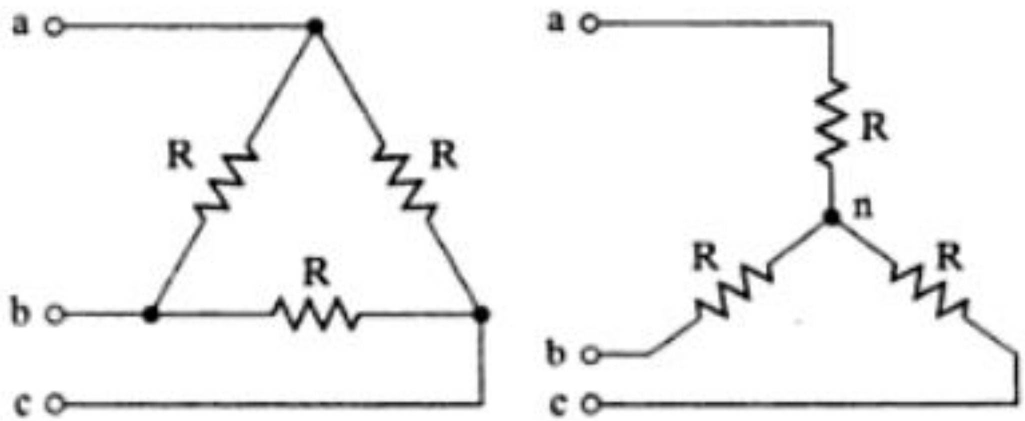


(a) Circuit



(b) Waveforms for 180° conduction

The load may be connected in Y or delta as shown in Figure 6.6. The switches of any leg of the inverter (S_1 and S_4, S_3 and S_6, or S_5 and S_2) cannot be switched on simultaneously; this would result in a short circuit across the dc link voltage supply. Similarly, to avoid undefined states and thus undefined ac output line voltages, the switches of any leg of the inverter cannot be switched off simultaneously; this can result in voltages that depend on the respective line current polarity.



(a) Delta connected (b) Y-connected

Table 6.2 shows eight valid switch states. Transistors $Q1$, $Q6$ in Figure 6.6a act as the switching devices $S1$, $S6$, respectively. If two switches: one upper and one lower conduct at the same time such that the output voltage is $\pm V_s$, the switch state is 1, whereas if these switches are off at the same time, the switch state is 0. States 1 to 6 produce nonzero output voltages. States 7 and 8 produce zero line voltages and the line currents freewheel through either the upper or the lower freewheeling diodes. To generate a given voltage waveform, the inverter moves from one state to another. Thus, the resulting ac output line voltages are built up of discrete values of voltages of V_s, 0, and $-V_s$. To generate the given waveform, the selection of the states is usually done by a modulating technique that should assure the use of only the valid states.

TABLE 6.2 Switch States for Three-Phase Voltage-Source Inverter (VSI)

State	State No.	Switch States	v_{ab}	v_{bc}	v_{ca}	Space Vector
S_1, S_2, and S_6 are on and S_4, S_5, and S_3 are off	1	100	V_S	0	$-V_S$	$\mathbf{V}_1 = 1 + j0.577 = 2/\sqrt{3}\ \angle 30°$
S_2, S_3, and S_1 are on and S_5, S_6, and S_4 are off	2	110	0	V_S	$-V_S$	$\mathbf{V}_2 = j1.155 = 2/\sqrt{3}\ \angle 90°$
S_3, S_4, and S_2 are on and S_6, S_1, and S_5 are off	3	010	$-V_S$	V_S	0	$\mathbf{V}_3 = -1 + j0.577 = 2/\sqrt{3}\ \angle 150°$
S_4, S_5, and S_3 are on and S_1, S_2, and S_6 are off	4	011	$-V_S$	0	V_S	$\mathbf{V}_4 = -1 - j0.577 = 2/\sqrt{3}\ \angle 210°$
S_5, S_6, and S_4 are on and S_2, S_3, and S_1 are off	5	001	0	$-V_S$	V_S	$\mathbf{V}_5 = -j1.155 = 2/\sqrt{3}\ \angle 270°$
S_6, S_1, and S_5 are on and S_3, S_4, and S_2 are off	6	101	V_S	$-V_S$	0	$\mathbf{V}_6 = 1 - j0.577 = 2/\sqrt{3}\ \angle 330°$
S_1, S_3, and S_5 are on and S_4, S_6, and S_2 are off	7	111	0	0	0	$\mathbf{V}_7 = 0$
S_4, S_6, and S_2 are on and S_1, S_3, and S_5 are off	8	000	0	0	0	$\mathbf{V}_0 = 0$

For a delta-connected load, the phase currents can be obtained directly from the line-to-line voltages. Once the phase currents are known, the line currents can be determined. For a Y-connected load, the line-to-neutral voltages must be determined to find the line (or phase) currents. There are three modes of operation in a half-cycle and the equivalent circuits are shown in Figure 6.7a for a Y-connected load.

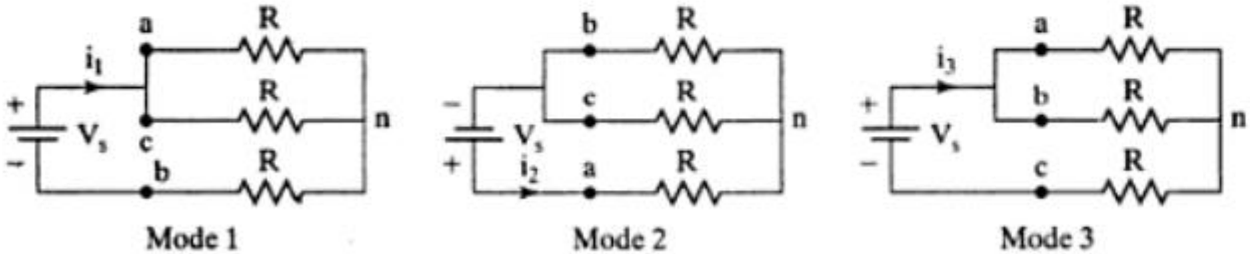
During mode 1 for $0 \le \omega t < \pi/3$, transistors Q_1, Q_5, and Q_6 conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_1 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = v_{cn} = \frac{i_1 R}{2} = \frac{V_s}{3}$$

$$v_{bn} = -i_1 R = \frac{-2V_s}{3}$$



(a) Equivalent circuits

During mode 2 for $\pi/3 \le \omega t < 2\pi/3$, transistors Q_1, Q_2, and Q_6 conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_2 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = i_2 R = \frac{2V_s}{3}$$

$$v_{bn} = v_{cn} = \frac{-i_2 R}{2} = \frac{-V_s}{3}$$

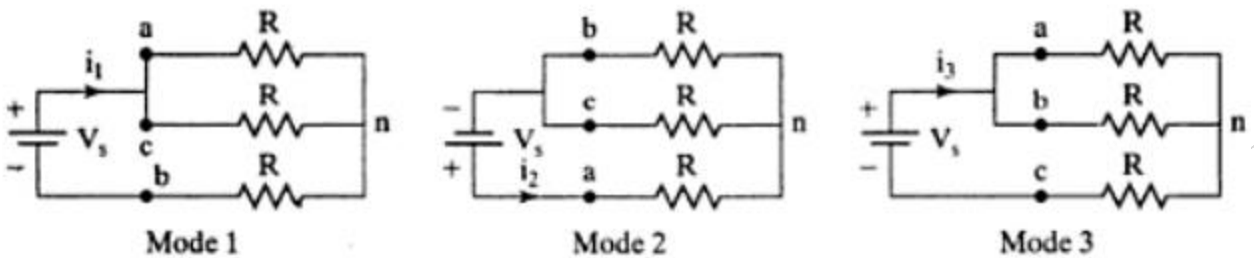
During mode 3 for $2\pi/3 \le \omega t < \pi$, transistors Q_1, Q_2, and Q_3 conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_3 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = v_{bn} = \frac{i_3 R}{2} = \frac{V_s}{3}$$

$$v_{cn} = i_3 R = \frac{-2V_s}{3}$$



(a) Equivalent circuits

The line-to-neutral voltages are shown in Figure 6.7b. The instantaneous line-to-line voltage v_{ab} in Figure 6.5b can be expressed in a Fourier series,

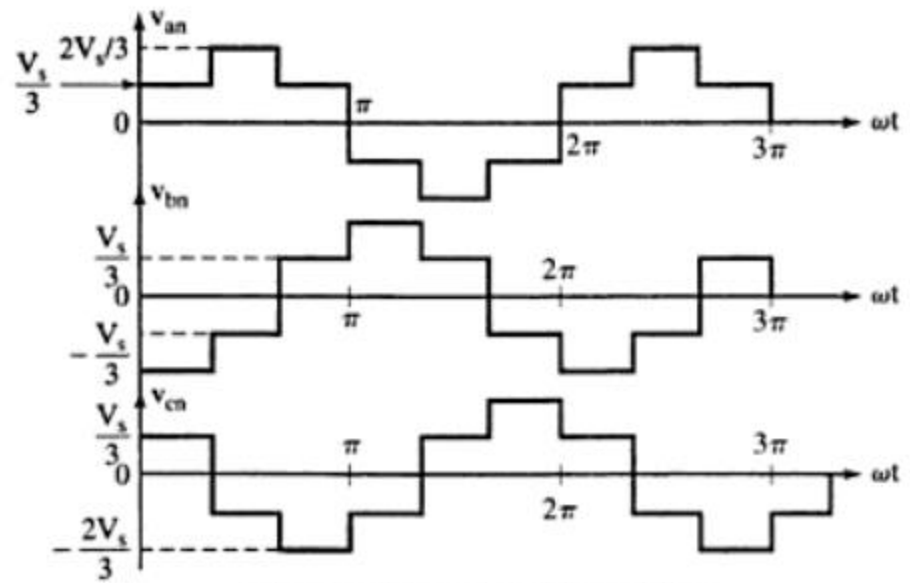
$$v_{ab} = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

Due to the quarter-wave symmetry along the x-axis, both a_0 and a_n are zero. Assuming symmetry along the y-axis at $\omega t = \pi/6$, we can write b_n as

$$b_n = \frac{1}{\pi}\left[\int_{-5\pi/6}^{-\pi/6} -V_s d(\omega t) + \int_{\pi/6}^{5\pi/6} V_s d(\omega t)\right] = \frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{3}\right)$$

which, recognizing that v_{ab} is phase shifted by $\pi/6$ and the even harmonics are zero, gives the instantaneous line-to-line voltage v_{ab} (for a Y-connected load) as

$$v_{ab} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \quad (6.16a)$$



(b) Phase voltages for 180° conduction

Both v_{bc} and v_{ca} can be found from Eq. (6.16a) by phase shifting v_{ab} by 120° and 240°, respectively,

$$v_{bc} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi} \sin \frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \tag{6.16b}$$

$$v_{ca} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi} \sin \frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \tag{6.16c}$$

We can notice from Eqs. (6.16a) to (6.16c) that the triplen harmonics ($n = 3, 9, 15, \ldots$) would be zero in the line-to-line voltages.

The line-to-line rms voltage can be found from

$$V_L = \left[\frac{2}{2\pi}\int_0^{2\pi/3} V_s^2 \, d(\omega t)\right]^{1/2} = \sqrt{\frac{2}{3}}V_s = 0.8165V_s \tag{6.17}$$

From Eq. (6.16a), the rms nth component of the line voltage is

$$V_{Ln} = \frac{4V_s}{\sqrt{2}n\pi} \sin \frac{n\pi}{3} \tag{6.18}$$

which, for $n = 1$, gives the rms fundamental line voltage.

$$V_{L1} = \frac{4V_s \sin 60°}{\sqrt{2}\pi} = 0.7797V_s \tag{6.19}$$

The rms value of line-to-neutral voltages can be found from the line voltage,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{\sqrt{2}\,V_s}{3} = 0.4714V_s \tag{6.20}$$

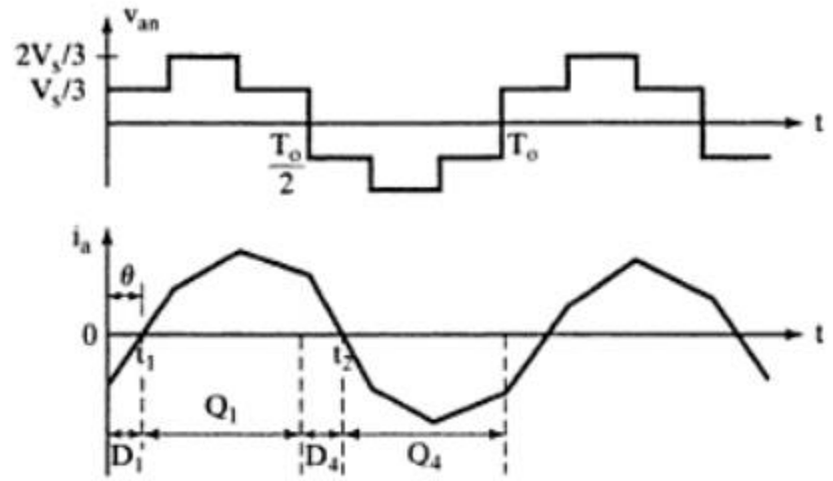
With resistive loads, the diodes across the transistors have no functions. If the load is inductive, the current in each arm of the inverter would be delayed to its voltage as shown in Figure 6.8. When transistor Q_4 in Figure 6.5a is off, the only path for the negative line current i_a is through D_1. Hence, the load terminal a is connected to the dc source through D_1 until the load current reverses its polarity at $t = t_1$. During the period for $0 \le t \le t_1$, transistor Q_1 cannot conduct. Similarly, transistor Q_4 only starts to conduct at $t = t_2$. The transistors must be continuously gated, because the conduction time of transistors and diodes depends on the load power factor.

respect to v_{ab}. Therefore, the instantaneous phase voltages (for a Y-connected load) are

$$v_{aN} = \sum_{n=1}^{\infty} \frac{4V_s}{\sqrt{3}n\pi} \sin\left(\frac{n\pi}{3}\right)\sin(n\omega t) \qquad \text{for } n = 1, 3, 5, \ldots \quad (6.21a)$$

$$v_{bN} = \sum_{n=1}^{\infty} \frac{4V_s}{\sqrt{3}n\pi} \sin\left(\frac{n\pi}{3}\right)\sin n\left(\omega t - \frac{2\pi}{3}\right) \qquad \text{for } n = 1, 3, 5, \ldots \quad (6.21b)$$

$$v_{bN} = \sum_{n=1}^{\infty} \frac{4V_s}{\sqrt{3}n\pi} \sin\left(\frac{n\pi}{3}\right)\sin n\left(\omega t - \frac{4\pi}{3}\right) \qquad \text{for } n = 1, 3, 5, \ldots \quad (6.21c)$$



Dividing the instantaneous phase voltage v_{aN} by the load impedance,

$$Z = R + jn\omega L$$

Using Eq. (6.21a), the line current i_a for an RL load is given by

$$i_a = \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{4V_s}{\sqrt{3}[n\pi\sqrt{R^2 + (n\omega L)^2}]} \sin\frac{n\pi}{3} \right] \sin(n\omega t - \theta_n) \qquad (6.22)$$

where $\theta_n = \tan^{-1}(n\omega L/R)$.

Note: For a delta-connected load, the phase voltages (v_{aN}, v_{bN}, and v_{cN}) are the same as the line-to-line voltages (v_{ab}, v_{bc}, and v_{ca}) as shown in Figure 6.6b and as described by Eq. (6.16).

Dc supply current. Neglecting losses, the instantaneous power balance gives

$$v_s(t)i_s(t) = v_{ab}(t)i_a(t) + v_{bc}(t)i_b(t) + v_{ca}(t)i_c(t)$$

where $i_a(t)$, $i_b(t)$, and $i_c(t)$ are the phase currents in a delta-connected load. Assuming that the ac output voltages are sinusoidal and the dc supply voltage is constant $v_s(t) = V_s$, we get the dc supply current

$$i_s(t) = \frac{1}{V_s}\left\{ \begin{aligned} &\sqrt{2}V_{o1}\sin(\omega t) \times \sqrt{2}I_o\sin(\omega t - \theta_1) \\ &+ \sqrt{2}V_{o1}\sin(\omega t - 120°) \times \sqrt{2}I_o\sin(\omega t - 120° - \theta_1) \\ &+ \sqrt{2}V_{o1}\sin(\omega t - 240°) \times \sqrt{2}I_o\sin(\omega t - 240° - \theta_1) \end{aligned} \right\}$$

The dc supply current can be simplified to

$$I_s = 3\frac{V_{o1}}{V_s}I_o\cos(\theta_1) = \sqrt{3}\frac{V_{o1}}{V_s}I_L\cos(\theta_1) \qquad (6.23)$$

where $I_L = \sqrt{3}I_o$ is the rms load line current;
V_{o1} is the fundamental rms output line voltage;
I_o is the rms load phase current;
θ_1 is the load impedance angle at the fundamental frequency.

Thus, if the load voltages are harmonic free, the dc supply current becomes harmonic free. However, because the load line voltages contain harmonics, the dc supply current also contains harmonics.

Example 6.4 Finding the Output Voltage and Current of a Three-Phase Full-Bridge Inverter with an *RL* load

The three-phase inverter in Figure 6.5a has a Y-connected load of $R = 5\ \Omega$ and $L = 23$ mH. The inverter frequency is $f_0 = 60$ Hz and the dc input voltage is $V_s = 220$ V. (a) Express the instantaneous line-to-line voltage $v_{ab}(t)$ and line current $i_a(t)$ in a Fourier series. Determine (b) the rms line voltage V_L; (c) the rms phase voltage V_p; (d) the rms line voltage V_{L1} at the fundamental frequency; (e) the rms phase voltage at the fundamental frequency V_{p1}; (f) the THD; (g) the DF; (h) the HF and DF of the LOH; (i) the load power P_o; (j) the average transistor current $I_{Q(av)}$; and (k) the rms transistor current $I_{Q(rms)}$.

Solution

$V_s = 220$ V, $R = 5\ \Omega$, $L = 23$ mH, $f_0 = 60$ Hz, and $\omega = 2\pi \times 60 = 377$ rad/s.

a. Using Eq. (6.16a), the instantaneous line-to-line voltage $v_{ab}(t)$ can be written as

$$\begin{aligned} v_{ab}(t) = {} & 242.58 \sin(377t + 30°) - 48.52 \sin 5(377t + 30°) \\ & - 34.66 \sin 7(377t + 30°) + 22.05 \sin 11(377t + 30°) \\ & + 18.66 \sin 13(377t + 30°) - 14.27 \sin 17(377t + 30°) + \cdots \end{aligned}$$

$$Z_L = \sqrt{R^2 + (n\omega L)^2}\,\underline{/\tan^{-1}(n\omega L/R)} = \sqrt{5^2 + (8.67n)^2}\,\underline{/\tan^{-1}(8.67n/5)}$$

Using Eq. (6.22), the instantaneous line (or phase) current is given by

$$\begin{aligned} i_{a(t)} = {} & 14 \sin(377t - 60°) - 0.64 \sin(5 \times 377t - 83.4°) \\ & - 0.33 \sin(7 \times 377t - 85.3°) + 0.13 \sin(11 \times 377t - 87°) \\ & + 0.10 \sin(13 \times 377t - 87.5°) - 0.06 \sin(17 \times 377t - 88°) - \cdots \end{aligned}$$

b. From Eq. (6.17), $V_L = 0.8165 \times 220 = 179.63$ V.

c. From Eq. (6.20), $V_p = 0.4714 \times 220 = 103.7$ V.

d. From Eq. (6.19), $V_{L1} = 0.7797 \times 220 = 171.53$ V.

e. $V_{p1} = V_{L1}/\sqrt{3} = 99.03$ V.

f. From Eq. (6.19), $V_{L1} = 0.7797V_S$

$$\left(\sum_{n=5,7,11,\ldots}^{\infty} V_{Ln}^2\right)^{1/2} = (V_L^2 - V_{L1}^2)^{1/2} = 0.24236V_s$$

From Eq. (6.8), THD $= 0.24236V_s/(0.7797V_s) = 31.08\%$. The rms harmonic line voltage is

g. $$V_{Lh} = \left[\sum_{n=5,7,11,\ldots}^{\infty} \left(\frac{V_{Ln}}{n^2}\right)^2\right]^{1/2} = 0.00941V_s$$

From Eq. (6.9), DF $= 0.00941V_s/(0.7797V_s) = 1.211\%$.

h. The LOH is the fifth, $V_{L5} = V_{L1}/5$. From Eq. (6.7), $HF_5 = V_{L5}/V_{L1} = 1/5 = 20\%$, and from Eq. (6.10), $DF_5 = (V_{L5}/5^2)/V_{L1} = 1/125 = 0.8\%$.

i. For Y-connected loads, the line current is the same as the phase current and the rms line current,

$$I_L = \frac{(14^2 + 0.64^2 + 0.33^2 + 0.13^2 + 0.10^2 + 0.06^2)^{1/2}}{\sqrt{2}} = 9.91 \text{ A}$$

The load power $P_0 = 3I_L^2R = 3 \times 9.91^2 \times 5 = 1473$ W.

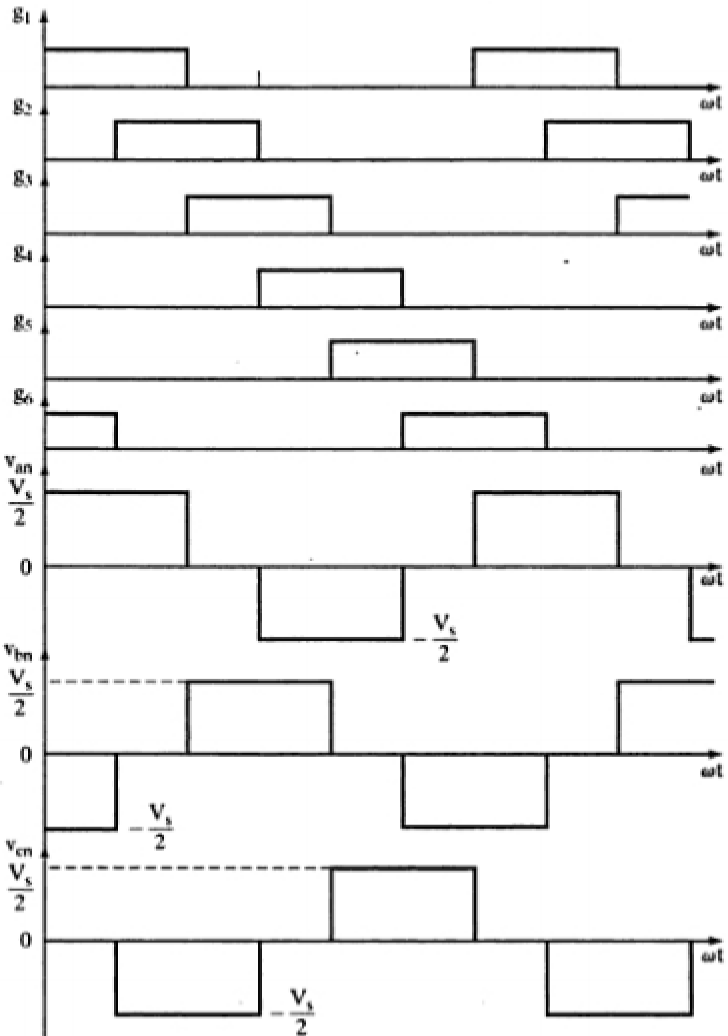
j. The average supply current $I_s = P_o/220 = 1473/220 = 6.7$ A and the average transistor current $I_{Q(av)} = 6.7/3 = 2.23$ A.

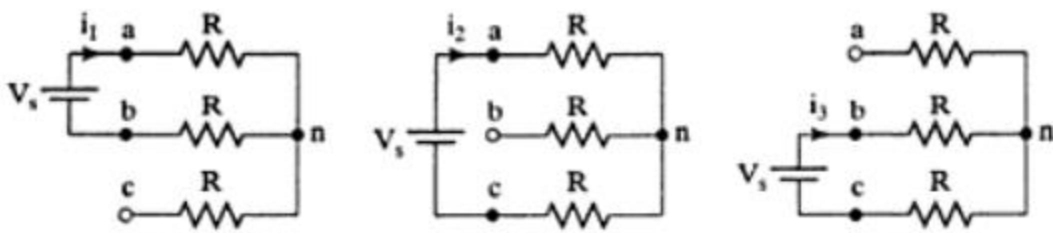
k. Because the line current is shared by three transistors, the rms value of a transistor current is $I_{Q(rms)} = I_L/\sqrt{3} = 9.91/\sqrt{3} = 5.72$ A.

6.5.2 120-Degree Conduction

In this type of control, each transistor conducts for 120°. Only two transistors remain on at any instant of time. The gating signals are shown in Figure 6.9. The conduction sequence of transistors is 61, 12, 23, 34, 45, 56, 61. There are three modes of operation in one half-cycle and the equivalent circuits for a Y-connected load are shown in Figure 6.10. During mode 1 for $\leq \omega t \leq \pi/3$, transistors 1 and 6 conduct.

$$v_{an} = \frac{V_s}{2} \qquad v_{bn} = -\frac{V_s}{2} \qquad v_{cn} = 0$$





(a) Mode 1 (b) Mode 2 (c) Mode 3

During mode 2 for $\pi/3 \leq \omega t \leq 2\pi/3$, transistors 1 and 2 conduct.

$$v_{an} = \frac{V_s}{2} \qquad v_{bn} = 0 \qquad v_{cn} = -\frac{V_s}{2}$$

During mode 3 for $2\pi/3 \leq \omega t \leq 3\pi/3$, transistors 2 and 3 conduct.

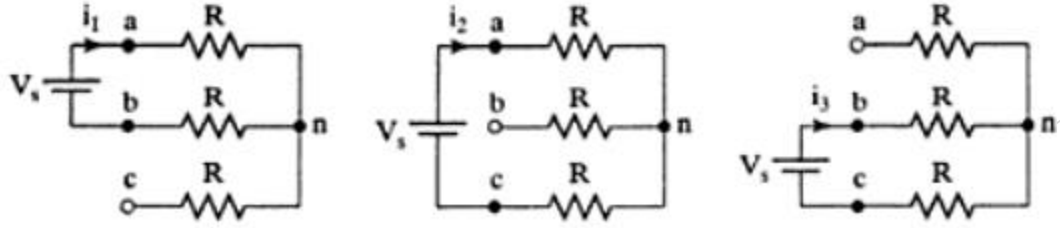
$$v_{an} = 0 \qquad v_{bn} = \frac{V_s}{2} \qquad v_{cn} = -\frac{V_s}{2}$$

The line-to-neutral voltages that are shown in Figure 6.9 can be expressed in Fourier series as

$$v_{an} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \tag{6.24a}$$

$$v_{bn} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \tag{6.24b}$$

$$v_{cn} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \tag{6.24c}$$



(a) Mode 1 (b) Mode 2 (c) Mode 3

The line a-to-b voltage is $v_{ab} = \sqrt{3}\, v_{an}$ with a phase advance of 30°. Therefore, the instantaneous line-to-line voltages (for a Y-connected load) are

$$v_{ab} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{3}\right) \sin n\left(\omega t + \frac{\pi}{3}\right) \quad \text{for } n = 1, 3, 5, \ldots \qquad (6.25a)$$

$$v_{bc} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{3}\right) \sin n\left(\omega t - \frac{\pi}{3}\right) \quad \text{for } n = 1, 3, 5, \ldots \qquad (6.25b)$$

$$v_{ca} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{3}\right) \sin n\,(\omega t - \pi) \quad \text{for } n = 1, 3, 5, \ldots \qquad (6.25c)$$

There is a delay of $\pi/6$ between the turning off Q_1 and turning on Q_4. Thus, there should be no short circuit of the dc supply through one upper and one lower transistors. At any time, two load terminals are connected to the dc supply and the third one remains open. The potential of this open terminal depends on the load characteristics and would be unpredictable. Because one transistor conducts for 120°, the transistors are less utilized as compared with those of 180° conduction for the same load condition. Thus, the 180° conduction is preferred and it is generally used in three-phase inverters.

Key Points of Section 6.5

- The three-phase bridge inverter requires six switching devices and six diodes. The rms fundamental component V_{L1} of the output line voltage is $0.7798V_s$ and that for phase voltage is $V_{p1} = V_{L1}/\sqrt{3} = 0.45V_s$ for 180° conduction. For 120° conduction, $V_{P1} = 0.3898V_s$ and $V_{L1} = \sqrt{3}\,V_{P1} = 0.6753V_s$. The 180° conduction is the preferred control method.
- The design of an inverter requires the determination of the average, rms, and peak currents of the switching devices and diodes.